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A SECOND BEST THEORY OF A FISCAL FEDERAL SYSTEM

Motohiro Sato
Queen’s University

Department of Economics
Queen’s University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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Queen's University / Hitotsubashi University

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Abstract

This paper investigates properties of the second best allocation in a fiscal federal system in which both federal tax and intergovernmental grants are involved and the taxation is distortionary. We extend the analysis of Boadway and Keen (1996) and Dahlby and Wilson (1994) by introducing both imperfect mobility and heterogeneous regions. In contrast to the outcomes in the existing works, we find; (i) the second best does not require the equalization of the conventional MCPFs between regions; (ii) in order to replicate the second best, matching grants based on either the local tax rates or tax revenues should be introduced to internalize the tax externality; and (iii) federal tax policy is redundant once the intergovernmental grants are optimized. The irrelevancy of the federal tax implies that optimal fiscal gap is indeterminate. Therefore, it will be argued that the standard framework of fiscal federal model in the literature does not provide a rationale for either decentralization or centralization of the tax system once the federal government is allowed to use sufficient instruments of inter-regional transfers.

Keywords: tax externality, equalization of MCPFs, matching grants, optimal fiscal gap

JEL classification: H7

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1. Introduction

There is a large literature on efficiency aspects in a fiscal federal system initiated by Flatters, Henderson and Mieszkowski (1974). In a relatively simple framework, some key results have been obtained. Boadway and Flatters (1982) show that the existence of the fiscal externality arising from individual free mobility leads to an inefficient allocation of population among regions; they also gives an explicit formula of the equalization grants that internalize this fiscal externality. An analogous argument is also made by Stiglitz (1977), Atkinson and Stiglitz (1980) and Hartwick (1980). On the other hand, Myers (1990) argues that the fiscal federal system can ensure migration efficiency through voluntary inter-regional transfers when local authorities are allowed to use such an instrument. Myers has extended his argument to the urban setting model (Myers and Papageorgiou (1993)) and the case of heterogeneous individuals (Burbidge and Myers (1994a)).

Although these works have significant implication for policy making, they are still in the world of the first best: head and rent taxes are assumed to be available so we do not have to care about the excessive cost associated with local or national tax policies. Surprisingly, there are few works on the properties of a fiscal federal system in the presence of distortionary tax system. A classical exception is Gordon (1983). He analyzes the characteristics of both perfectly coordinated and non-cooperative fiscal federal systems, and identifies several sources of inefficiency associated with the non-cooperative circumstance. Although he uses a relatively general framework, his analysis does not give an explicit formula of federal policy to implement the perfectly coordinated outcome in a decentralized setting. Nor does his model include individual locational decision making in a consistent way. Wildasin (1983) also consider a general equilibrium model in the presence of distortive taxation with mobile and immobile households. Although he examines the welfare effect of marginal change in the grants policy by the central government, his analysis is limited to comparative statics; the optimization of intergovernmental grants is not attempted.

When examining the excessive cost associated with distortional local and national taxes, we should not overlook the significance of tax assignment problem. In some federal nations including Canada and the United States, major tax bases such as personal and corporate income tax are shared by federal and provincial/states governments. In this circumstance, a form of tax externality can arise, which will distort decision making at each level of government. In a simple partial equilibrium framework, Dahlby (1994) shows that if a local government ignores the effect of its tax policy on the central government's tax revenue, the MCPF (marginal cost of public fund) is underestimated and there may be an excessive supply of local public goods.[1] An analogous conclusion is derived by Johnson (1988). He studies income redistribution policy in a federal system and points out that when both local and central governments share this task, the former will underestimate the cost of income redistribution. The former argument is extended to a general equilibrium model by Dahlby and Wilson (1994). They establish that when only efficiency issue is concerned, the second best allocation requires the social MCPFs to be equalized not only among tax bases in each local jurisdiction, but also among regions. The explicit form of equalization grants to achieve this objective is also presented. In their model, however, immobility of households and capital is assumed as well as perfect cooperation among governments. When the MCPF is underestimated by local authorities, they may raise local tax rates beyond the second best levels without binding agreement. To correct the tax externality, a policy instrument is needed which is analogous to Pigovian tax. Dahlby (1996) recommends the introduction of matching grants to internalize the tax externality: the matching grants should be imposed on the local tax rate, or tax revenue. As long as the externality exerts a negative impact on the tax revenues of other governments, the rate of the grants reflecting the marginal external cost should be negative. His idea gives a rise to the following question: how should the lump-sum and matching grants be combined? The case of mobile households is considered by Boadway and Keen (1996). They rise a quite important question: what is an optimal level of fiscal gap between local and central governments? They examine an economy consisting of homogeneous regions in which a labor income tax is co-occupied by local and central governments. They conclude that the existence of the tax externality makes the optimal fiscal gap negative in a fairly standard circumstance. However, their model is restrictive in the sense that they pay attention only on the symmetric equilibrium, and only a lump-sum transfer is available for the central government.

This paper examines the second best policy at the federal level in the presence of heterogeneous regions and imperfect individual mobility. As cited above, so far, most authors have proceeded under the assumption of either immobility of households or/and homogeneous regions. This is due to the apparent complexity of analysis when relaxing these assumptions: readers familiar with the fiscal externality literature will recognize how complicated it is to treat asymmetric equilibrium with individual mobility even without distortionary taxes. We extend the analysis of Boadway and Keen (1996) and Dahlby and Wilson (1994). As a notion of imperfect mobility, we employ the home-attachment model of Mansoorian and Myers (1993): by changing the degree of home-attachment, we can treat immobility and perfect mobility as polar cases. The basic framework of our model is fairly standard. The economy consists of two regions and there are two levels of governments (local and federal governments). Individuals are homogeneous in all respects but the degree

[1] The MCPF is defined by the ratio of the change in household's welfare (measured by the marginal utility of income) to the change in the tax revenue due to an additional increase in the tax payment. The well-known formula is MCPF $= 1/(1-\varepsilon)$ where ε is an uncompensated elasticity of a taxed good with respect to tax rate. If distribution is an issue, as we will see below, this formula needs to be modified. In the present paper, we use the term "MCPF" to designate the formula $1/(1-\varepsilon)$ or its generalization to include welfare weights.

of home-attachment. As in Boadway and Keen (1996), we suppose that a labor tax is shared by the two governments in each region. In terms of game theory, we describe the central government as a first mover and the local governments as followers. The local government provides a single local public good financed by local tax revenue and the transfer from the central government. The central government designs the inter-regional transfer and federal tax schemes in addition to providing a national public good. The transfer scheme can involve both lump-sum and matching grants. The issue of the tax externalities is closely related to how the local governments act in a non-cooperative setting. We follow Boadway and Keen (1996) and Dahlby (1994, 1996) in assuming partially myopic behavior at the local level in the following sense: the local authority does not incorporate budget constraints of other governments in its optimization, implying that the effects of migration induced by the regional policies are not fully captured at the local level. Note that any migration induced by one local government's policy can alter the size of tax bases of other governments including the federal one, and it can lead to changes in public expenditures and in the welfare of the residents in both regions, which may lead to further migration.[2] Of course, in equilibrium all budgets must be in balance.

We begin by characterizing the second best outcome in the economy. We shall see that in the presence of perfect or imperfect mobility of households, the heterogeneity of the regions casts a new light on its characteristics. Despite the recommendation of Dahlby and Wilson(1994), the second best does not generally require the equalization of the MCPF in familiar fashion between regions. In a decentralized system (non-cooperative setting), we will examine the federal policy needed to replicate the second best allocation. A matching grants scheme as a function of the local tax rate or tax revenue should be employed to internalize the tax externalities arising due to the misperception of the social MCPFs by the local authorities. A lump-sum transfer is also needed to equate the shadow prices of raising public funds among the governments to ensure the second best national public good and population allocation. Furthermore, it will be shown that the optimal fiscal gap is indeterminate, a consequence of the fact that federal tax policy is redundant as an instrument for the purpose of achieving the second best. This result is sharply in contrast with Boadway and Keen (1996) in which only the lump-sum transfer along with the federal taxation is considered.

This paper is organized as follows. Section 2 provides a basic framework of our model. The characterization of second best allocation is attempted in section 3. Federal policy to implement the second best outcome is discussed in section 4. Section 5 concludes.

[2] There is another formulation of local authority's behavior. Boadway (1982), Myers (1990), Wellisch (1994) suppose that each local jurisdiction takes into account regional resource constraints in other jurisdictions. In the economy in the presence of distortionary taxation, this assumption can be restated that each local government incorporates the revenue constraints of other governments. The author also considered this alternative case. The results of this case are available upon request from the author.

2. The basic structure of the model

We consider a federation consisting of two regions denoted by k =A, B. The national population is normalized to unity. Following Mansoorian and Myers (1993), we introduce imperfect mobility by supposing heterogeneous preferences with respect to home attachment. The type of households is denoted by n and is assumed to be distributed uniformly on [0, 1] in the economy. We write the utility function of type n-household by:

$$U(x_A, h_A) + b(g_A) + B(G) + a(1 - n)$$

if the household resides in region A,

$$U(x_B, h_B) + b(g_B) + B(G) + an$$

if he resides in region B, where x_k is a private good, h_k is labor supplied, g_k denotes a local public good and G is a national public good. Preferences are strictly concave, strictly increasing in x_k, g_k and G, and strictly decreasing in h_k. The benefit of the local public good does not spill over across regions, while those of the national public good accrues to all households irrespective of where they reside. The assumption of separability in the utility function implies that g_k and G do not affect the leisure-consumption decision making of individuals. Moreover, individual residential choice will turn out to be independent of the national public good. "a" designates the degree of home attachment. The difference in the degree of home-attachment influences only inter-regional migration, but not individual decision making in each region. Without loss of generality, we suppose that initially the households with $n < 1/2$ (resp. $> 1/2$) reside in region A (resp. B). For $a = 0$, there will be prefect mobility as is familiar in the literature. Complete immobility can be described as another extreme case ($a = \infty$).

The decision making of the individuals can be divided into two stages: choice of labor supply in each region and locational choice. The latter is done by comparing the maximized utilities for given local and federal policies, taking as given the size of population in each region. The local government chooses the local tax rate and the amount of a local public good subject to regional budget balance, and taking into account individual migration. The budget balance of other governments is, however, ignored: from a regional viewpoint, the public policies of other governments including the tax rate and the public expenditures are taken as given. Following most of the literature, we assume that the central government is a first mover and thus incorporates the effects of its decision making on the Nash equilibrium. In other words, we can consider the central government as a Stackelberg leader in this game. Both federal tax and intergovernmental grants, including a matching component, are available for the federal government. They will be used to manipulate the regional governments' decisions and individual migration in order to enhance efficiency. A key to understanding one of our main arguments, indeterminacy of optimal fiscal gap, lies in the difference between the number of federal instruments and the number of economic variables which the federal authority attempts to manipulate. It will be seen that the former exceeds the latter, which implies that one instrument is presumably redundant.

Household's optimization

Each household's utility maximization in region k (=A, B) is expressed by:

$$\max_{x_k, h_k} \quad U(x_k, h_k) + b(g_k) + B(G) \quad \text{subject to} \quad x_k = (w_k - \tau_k)h_k$$

where w_k is the wage rate and τ_k is the per unit tax on labor. The latter includes both the federal tax, T and the local tax, t_k, so $\tau_k = t_k + T$. Following Boadway and Keen (1996), we assume that rent income does not accrue to the households. In the present paper, the federal tax rate is assumed to be uniform across regions.[3] The assumption of the per unit tax system is just for simplicity. The essence does not change even if we replace the per unit tax system by the ad valorem tax (wage income tax). Solving the above optimization yields labor supply function, $h(w_k - \tau_k)$. Throughout this paper, we assume $h'(w_k - \tau_k) > 0$ for all $w_k - \tau_k$. The assumption can be justified when the income effect is not so significant relative to the substitution effect.

The production side of the economy is simple. Output, which can be used for private consumption, local and national public good provision on one to one basis, is produced by both labor and a fixed factor (land). The technology is represented by an increasing and strictly concave production function, $f_k(n_k h_k)$, where n_k is the population in the region. Our analysis includes the case that both regions are heterogeneous with respect to technology or fixed factor supply. The wage is equated to the marginal productivity of labor so in equilibrium, we have:

$$w_k = f_k'(n_k h(w_k - \tau_k)). \tag{1}$$

Solving (1) for w_k yields a regional market clearing wage as a function of n_k and τ_k. We denote it by $w_k(\tau_k, n_k)$.

Rent is defined as a residual, or

$$r_k(\tau_k, n_k) = f(n_k h(w(\tau_k, n_k) - \tau_k)) - w(\tau_k, n_k) n_k h(w(\tau_k, n_k) - \tau_k). \tag{2}$$

We assume that all the rent accrues to the public sector and denote the proportion of the rent accruing to the federal government and the local government by θ and $1 - \theta$, respectively. This assumption implies either public ownership of the fixed factor or 100 percent tax on the rent income. In the latter case, θ is the federal tax rate on the rent income. Throughout this paper, θ is assumed to be fixed.

[3] This uniformity seems to reflect a realistic restriction on the national tax policy: generally the central government is not allowed to treat individuals differently based on their residence. Of course, by allowing the local tax payment to be deducted from the federal tax base, it is possible to differentiate individual treatment in the federal tax system to some extent although such a deduction generally cannot be equivalent to regionally differentiated federal tax.

We write maximized utility (excluding the part of home-attachment) as $v_k(\tau_k, n_k)$ $+b(g_k) + B(G)$. The indirect function has the following properties:

$$\frac{dv_k}{d\tau_k} = -\frac{\lambda_k h_k}{D_k} < 0 \tag{3}$$

$$\frac{dv_k}{dn_k} = \frac{\lambda_k f_k'' h_k^2}{D_k} < 0 \tag{4}$$

where λ_k is the marginal utility of income, $h_k = h(w_k - \tau_k)$ and $D_k = 1 - n_k f_k'' h_k' > 0$.

Next we turn to locational choice. Households differ in their attachment to a region so migration equilibrium for given policy instruments can be characterized by the marginal household who is just indifferent between the two regions. The type of the marginal household is equivalently the population of region A due to the assumption of the uniform distribution of the types:

$$v_A(\tau_A, n_A) + b(g_A) + B(G) + a(1 - n_A) = v_B(\tau_B, 1 - n_A) + b(g_B) + B(G) + an_A. \tag{5}$$

Households with $n < n_A$ locate in region A and those with $n > n_A$ reside in region B.[4]

Optimization by local governments

Local governments provide the local public good, g_k and finance it by the labor tax and the transfer they receive in lump-sum and matching forms from the federal government. The local revenue constraint for region k is thus:

$$g_k = R_k(t_k, T, n_k, m_k, S_k) \equiv t_k n_k h(w(\tau_k, n_k) - \tau_k) + (1 - \theta) r_k(\tau_k, n_k) + m_k t_k + S_k \tag{6}$$

where m_k is the matching grant on the local tax rate, S_k is the lump-sum transfer. We assume that both m_k and S_k can be of either sign; $m_k < 0$ implies that the federal government taxes the local tax rate, while $S_k < 0$ designates the lump-sum tax on the local government.[5]

Following Burbidge and Myers (1994b) and Wellich (1994), we employ the residents' utility excluding home-attachment, $v(\tau_k, n_k) + b(g_k)$ $+B(G)$ (k=A, B) as the regional objective. This may correspond to the median voter objective (Mansoorian and Myers (1995),

[4] For simplicity, we assume that migration equilibrium is always interior and unique although it is well-known that there can be multiple equilibria (Atkinson and Stiglitz (1980)). Including such a multiplicity or possibility of a corner solution would substantially complicate our analysis.

[5] The matching grants correspond to the revenue grants examined by Dahlby (1996). Precisely, the latter is the grant related to the local tax revenue, R_k, rather than the local tax rate, t_k. Both tax rate and revenue matching grants are equivalent, however: we prefer the present formulation because it makes our analysis more tractable.

Wellisch (1994)).[6] Each voter locating in one region will prefer a regional policy which maximizes his own utility. Since all voters are identical except for locational preferences, and the latter is a parameter, the maximization of each voter's utility is equivalent to maximizing $v(\tau_k, n_k) + b(g_k) + B(G)$. Therefore, there will be unanimous agreement for the choice of t_k and g_k.

Local decision making incorporates the migration function, which can be obtained by solving (5) with respect to n_A and substituting (6). For future reference, we present its derivatives with respect to t_k, S_k and T:

$$\frac{\partial n_k}{\partial t_k} = -\frac{1}{D}\left(\frac{\partial v_k}{\partial \tau_k} + b_k' \frac{\partial R_k}{\partial t_k}\right) \tag{7}$$

$$\frac{\partial n_k}{\partial S_k} = -\frac{1}{D} b_k' \tag{8}$$

$$\frac{\partial n_A}{\partial T} = -\frac{1}{D}\left\{\left(\frac{\partial v_A}{\partial \tau_A} + b_A' \frac{\partial R_A}{\partial T}\right) - \left(\frac{\partial v_B}{\partial \tau_B} + b_B' \frac{\partial R_B}{\partial T}\right)\right\} \tag{9}$$

where $n_B = 1 - n_A$ and

$$D = \sum_{k=A,B} \left(\frac{\partial v_k}{\partial n_k} + b_k' \frac{\partial R_k}{\partial n_k}\right) - 2a. \tag{10}$$

Stability requires $D < 0$.[7] When this condition is ensured, we can verify that $\partial n_k / \partial S_k > 0$ while $\partial n_A / \partial T$ is ambiguous. The sign of the first equation is also ambiguous: while the direct impact of higher tax rate decreases the resident's welfare, the expansion of g_k following an increase in the tax revenue improves it.

Although most of the existing works in the literature include the migration function explicitly in the local optimization (Boadway (1982), Myers (1990)), we find it more convenient to use n_A as a regional control variable by including (5) in the regional optimization as a constraint. With respect to interaction between local authorities, we assume competition of Nash-Cournot type. As cited above, we assume that the local governments take as given both public expenditure and tax rate of other governments

For the local government k (= A, B) , t_k and g_k should be chosen to maximize per capita utility, $v(\tau_k, n_k) + b(g_k) + B(G)$ subject to its own local budget constraints (6) and the migration constraint (5). The control variables are t_k, g_k, n_A. Note, however, that n_A is artificial. Formally, the local optimization is expressed by:

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[6] An alternative objective may be the total utility of residents. However, as cited by Mansoorian and Myers (1995), this formulation implies that each local authority has a preference for the population size, which may lead to inefficiency in the population allocation.

[7] In a fiscal federal model, it is well-known that the stability issue is closely related to the sign of aggregate tax rate, τ_k. We examine this problem in section 4.

$$\max_{t_k, g_k, n_k} \quad L_k = v(\tau_k, n_k) + b(g_k) + B(G) + \mu_k\{v(\tau_k, n_k) + b(g_k) + a(1-2n_k)$$

$$-v(\tau_j, n_j) - b(g_j)\} + \gamma_k^k\{R_k(t_k, T, n_k, m_k, S_k) - g_k\} \tag{11}$$

where $k, j =$A, B, $k \neq j$ and $n_B = 1 - n_A$. The multiplier associated with (6), γ_k^k, represents a regional shadow price of raising marginal tax revenue, which will not be coincident with the social value. For given federal policies, $\Theta \equiv \{T, S_k, m_k(k = A, B)\}$ and G, the Nash equilibrium requires the decisions of the local governments to be consistent each other in the following sense: (i) t_j and g_j taken as given by another local authority k $(\neq j)$ should be chosen by j, (ii) the value of the overlapped control variable, n_A should be the same between the regions. Throughout this paper, uniqueness of Nash equilibrium for each federal policy is supposed. Due to the separability of the utility function, however, G does not affect the structure of the Nash equilibrium. We can define the values in the equilibrium as functions of Θ: say, $t_k(\Theta)$ (k=A, B). As for the local public good, we can write $g_k(\Theta) = R_k(t_k(\Theta), T, n_k(\Theta), m_k, S_k)$. The federal policy exerts direct and indirect impacts on the local public expenditure: the latter is done through the change in the local tax rate and the induced migration. We can also write the welfare level in region k in the resulting Nash equilibrium by $V_k(\Theta, G, t_j(\Theta), g_j(\Theta))$ $(k, j = A, B, j \neq k)$. Recall that the policy parameters of the other local government are taken as given and the changes in these parameters led by changes in Θ can affect the region k's welfare. This is why we include $t_j(\Theta)$ and $g_j(\Theta)$. The properties of this function will be fully examined in section 4.

Central government

Following Burbidge and Myers (1994b) and Wellisch (1994), the objective of the central government is assumed to be given by the weighted average of regional welfare [8]:

$$\delta\{v(\tau_A, n_A) + b(g_A) + B(G)\} + (1-\delta)\{v(\tau_B, n_B) + b(g_B) + B(G)\}, \ \delta \in [0, \ 1] \tag{12}$$

As suggested by Mansoorian and Myers (1993) and Wellisch (1994), in the presence of home-attachment ($a > 0$), we can trace the second best frontier of regional utilities by changing the welfare weight, δ. Below, we discuss the general characterization of the second best allocation. However, it should be kept in mind that the second best policy that the federal government attempts to implement relies on the value of δ. The task of the

[8] The social welfare function may be defined by:

$$\int_0^{n_A} \omega_n\{v(\tau_A, n_A) + b(g_A) + B(G) + a(1-n)\}dn + \int_{n_A}^1 \omega_n\{v(\tau_B, n_B) + b(g_B) + B(G) + an\}dn$$

where $\int_0^1 \omega_n dn = 1$ (Mansoorian and Myers (1995)). Using the migration constraint, (5), however, we can reduce this to (12).

federal government is to design federal policy so as to maximize the social welfare subject to the budget constraint:

$$G = R_F(t_A, t_B, n_A, T, S_A, S_B, m_A, m_B)$$

$$\equiv T \sum_{k=A,B} n_k h(w(\tau_k, n_k) - \tau_k) + \theta \sum_{k=A,B} r_k(\tau_k, n_k) - \sum_{k=A,B} m_k t_k - \sum_{k=A,B} S_k \tag{13}$$

along with the dependency of Nash equilibrium on Θ and G. Or formally:

$$\max_{\Theta, G} \quad \delta V_A(\Theta, G, t_B(\Theta), g_B(\Theta)) + (1-\delta) V_B(\Theta, G, t_A(\Theta), g_A(\Theta))$$

$$+\gamma_F\{R_F(t_A(\theta), t_B(\Theta), n_A(\Theta), \Theta) - G\}. \tag{14}$$

As mentioned above, the federal instruments are used to manipulate the incentives of the local authorities and internalize externalities such as tax externalities arising in the federal system.

3. The second best allocation

In this section, we characterize the second best allocation in a fiscal federal system. A similar exercise has been undertaken by Dahlby and Wilson (1994) and Boadway and Keen (1996) in the context of their simplified models. In the former, the well-known Ramsey tax rule can be applied, given the immobility of households: from a efficiency view point, the conventional MCPF should be equalized not only among tax bases in each region, but also between regions and between local and federal governments. A similar conclusion is obtained by Boadway and Keen (1996) with perfect mobility under the restriction of a symmetric equilibrium. We will see, however, that when there is (perfect or imperfect) inter-regional movement and regions are not homogeneous, the equalization of the "standard" formula of MCPFs across regions does not necessarily hold.[9]

At the outset, we should make a careful distinction between the "conventional" MCPF and the "economic" one. Let ε be the uncompensated elasticity of the taxed good (labor in this model) with respect to tax rate. The conventional MCPF is given by $1/(1-\varepsilon)$. This form represents the value of the multiplier associated with the government's revenue constraint in an economy without mobility (Atkinson and Stiglitz (1980)). This multiplier should be regarded as economic or true MCPF. Our argument is that, although we can still expect the equalization of the values of the multipliers associated with local budget constraints in the second best, it does not imply the equalization of conventional MCPFs across regions. Henceforth we refer to the multipliers as shadow prices to avoid possible confusion. But it should be born in mind that the multipliers (shadow prices) should be the marginal cost of public fund from an economics viewpoint. We use the term "MCPF" to express $1/(1-\varepsilon)$ because this terminology is widely used. It should also be noted that

[9] For the derivation of conventional form of MCPF and its general properties, see Usher (1984) and Wildasin (1984).

throughout this section, "MCPF" is defined in the social sense as reflecting all relevant costs associated with an increase in the labor tax rate.

The necessary conditions for the second best resource and population allocations are derived by maximizing the social welfare function (12) subject to the migration constraint (5) and the unified revenue constraint:

$$g_A + g_B + G = \sum_{k=A,B} \{\tau_k n_k h(w(\tau_k, n_k) - \tau_k) + r_k(\tau_k, n_k)\}. \tag{15}$$

The control variables here are τ_k, g_k (k=A, B), n_A and G. Formally, the second best optimization problem is expressed by :

$$\max_{\tau_A, \tau_B, g_A, g_B, n_A, G} \quad \delta\{v(\tau_A, n_A) + b(g_A) + B(G)\} + (1-\delta)\{v(\tau_B, n_B) + b(g_B) + B(G)\}$$

$$+\mu\{v(\tau_A, n_A) + b(g_A) + a(1-2n_A) - v(\tau_B, n_B) - b(g_B)\}$$

$$+\gamma\left\{\sum_{k=A,B} \{\tau_k n_k h(w(\tau_k, n_k) - \tau_k) + r_k(\tau_k, n_k)\} - \sum_{k=A,B} g_k - G\right\}.$$

We can establish the following first order conditions:

$$-(\delta+\mu)\lambda_A h_A + \gamma n_A(h_A - \tau_A h'_A) = 0 \quad (\tau_A) \tag{16}$$

$$-(1-\delta-\mu)\lambda_B h_B + \gamma n_B(h_B - \tau_B h'_B) = 0 \quad (\tau_B) \tag{17}$$

$$(\delta+\mu)b'_A - \gamma = 0 \quad (g_A) \tag{18}$$

$$(1-\delta-\mu)b'_B - \gamma = 0 \quad (g_B) \tag{19}$$

$$(\delta+\mu)B' + (1-\delta-\mu)B' - \gamma = 0 \quad (G) \tag{20}$$

$$\frac{1}{D_A}\{f''_A h_A^2(\lambda_A(\delta+\mu) - \gamma n_A) + \gamma\tau_A h_A\} - \frac{1}{D_B}\{f''_B h_B^2(\lambda_B(1-\delta-\mu) - \gamma n_B) + \gamma\tau_B h_B\}$$
$$= 2a\mu \quad (n_A). \tag{21}$$

The variables shown in parentheses are the instruments being optimized. Combining (16) and (17) with (18) and (19), respectively, yields the necessary conditions for the second best provision of the local public goods:

$$n_k \frac{b'(g_k)}{\lambda_k} = \left(1 - \tau_k \frac{h'_k}{h_k}\right)^{-1}, \ k = A, B. \tag{22}$$

This is a well known modified Samuelson condition where $(1-\tau_k h'_k/h_k)^{-1}(k = A, B)$ is the "conventional" MCPF. Equation (22) requires that the provision of the local public good should be made so as to equate the marginal gain with the MCPF, which includes the marginal excess burden in addition to the resource cost. Solving (16) for $\delta + \mu$ and

(17) for $1-\delta-\mu$, respectively and inserting them into (20) gives the modified Samuelson condition for the national public good:

$$\sum_{k=A,B} n_k \frac{B'}{\lambda_k}\left(1-\tau_k \frac{h'_k}{h_k}\right) = 1. \tag{23}$$

In the above, the marginal gain from G in each region is weighted by reciprocal of the MCPF in each region. Only if the MCPFs are equalized across regions, will (23) reduce to a standard form of the Samuelson condition for the second best economy as in (22). Substituting (16) and (17) into (21) and making some manipulations establish the necessary condition for the second best population allocation:

$$\tau_A h_A - \tau_B h_B = 2a\left\{\frac{(1-\delta)n_A}{\lambda_A}\left(1-\frac{\tau_A h'_A}{h_A}\right) - \frac{\delta n_B}{\lambda_B}\left(1-\frac{\tau_B h'_B}{h_B}\right)\right\}. \tag{24}$$

(24) is analogous to the necessary condition in the economy with non-distortionary taxes, as derived by Wellish (1994). Using the individual budgets constraint, we can write $\tau_k h_k = w_k h_k - x_k$, which is the net social product (Boadway and Flatters (1982)). Thus the left hand side of (24) represents the difference in the net social product between the two regions. When mobility is perfect so $a=0$, (24) reduces to:

$$\tau_A h_A = \tau_B h_B \tag{24'}$$

The residence-based tax payment is the same in both regions, which characterize the efficiency of population allocation in the first best economy: note that (24′) also provide the formula for inter-regional transfers (equalization payments) (See Boadway and Flatters (1982)).[10] The next proposition summarizes the above discussion.

Proposition 1: *The second best in the economy is characterized by Eq. (22) - (24).*

An alternative way of deriving the second best is to assume full policy coordination among the governments. As examined by Dahlby and Wilson (1994), when policy coordination is possible, the optimization is characterized by the maximization of the social welfare function subject to the set of revenue constraints.[11] The lump-sum grants should

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[10] The sign of the second best tax rate cannot be seen only from (24) or (24′). For $\tau_k > 0$ to result, the net social product must be positive, which implies that the economy is under-populated. Therefore, the second best is compatible with a positive tax rate only in the economy as such. Whether or not the economy is under-populated is closely related to the stability issue and we will turn to it in section 4.

[11] According to this alternative formulation, the Lagrange function can be written as:

$$L = \delta\{v(\tau_A, n_A) + b(g_A) + B(G)\} + (1-\delta)\{v(\tau_B, n_B) + b(g_B) + B(G)\} + \mu\{v(\tau_A, n_A) + b(g_A)$$

$$+a(1-2n_A) - v(\tau_B, n_B) - b(g_B)\} + \sum_{k=A,B} \gamma_k\{R_k(t_k, T, n_k, m_k, S_k) - g_k\}$$

be included in the instruments being optimized. Since all externalities are incorporated, tax matching grants are not required. Let γ_k be the multiplier associated with local budget constraint (6) and let γ_F be the one for federal constraint (13). From this alternative approach, we get an additional condition:

$$\gamma_A = \gamma_B = \gamma_F. \tag{25}$$

Corollary 1 to Proposition 1: *In the second best, the shadow prices of taxation are equalized among governments.*

Equation (25) is imbedded in the integrated form of revenue constraint (15): note that the equalization of the shadow prices implies the unification of the budget constraints.

Let us turn to the issue of relevancy of conventional MCPFs. In the case of immobility, we can verify that the argument of Dahlby and Wilson (1994) favoring the equalization of MCPFs holds. Immobility implies $\mu = 0$. Solving (16) and (17) for γ:

$$\gamma = \frac{\delta\lambda_A}{n_A}\left(1 - \frac{\tau_A h'_A}{h_A}\right)^{-1} = \frac{(1-\delta)\lambda_B}{n_B}\left(1 - \frac{\tau_B h'_B}{h_B}\right)^{-1} \tag{26}$$

where $\delta\lambda_A/n_A$ and $(1-\delta)\lambda_B/n_B$ represent distributional concerns[12]. Thus the last two terms may be called the MCPF with ditstibutional weights, which is a generalized form of the conventional MCPF. If only efficiency is considered, these weights should be equal and then (26) reduces to the familiar formula:

$$\left(1 - \frac{\tau_A h'_A}{h_A}\right)^{-1} = \left(1 - \frac{\tau_B h'_B}{h_B}\right)^{-1}. \tag{26'}$$

Thus we can establish:

Corollary 2 to Proposition 1 (Dahlby and Wilson (1994)): *If labor is immobile, the second best requires the MCPFs with distributional weights to be equated across regions. Moreover, if distributional issue is not a concern, the condition reduces to the equalization of the conventional MCPFs.*

In accordance with this corollary, the interregional transfers needed to realize the second best should be made from the region which would otherwise enjoy lower MCPF to the one with higher MCPF initially. It is worth noting that (26) or (26′) is sharply contrast

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$$+\gamma_F\{R_F(t_A, t_B, n_A, T, S_A, S_B, m_A, m_B) - G\}$$

Optimizing with respect to S_A and S_B establishes (25), and the last three terms of the Lagrangian can reduce to (15) with the multiplier, $\gamma = \gamma_F$.

[12] In Dahlby and Wilson (1994), the identical size of regions (measured by population) is implicitly assumed. However, in the more general case allowing for differences in the size of regions, a scale adjustment is needed since the social welfare weight in our definition does not reflect the size of population in each region.

to (24′): the latter implies that the interregional transfer should equate per capita tax payments across regions. These two can be consistent with each other in the case of symmetric regions as supposed in Boadway and Keen (1996). Otherwise, it is (24′) and therefore, the equalization of per capita tax payments that should be the criterion of the interregional transfers in the presence of perfect mobility. In more general circumstances involving imperfect mobility and heterogeneous regions, using (16) and (17), the difference in the MCPFs with distributional weights can be represented by:

$$\frac{\delta\lambda_A}{n_A}\left(1-\frac{\tau_A h'_A}{h_A}\right)^{-1}-\frac{(1-\delta)\lambda_B}{n_B}\left(1-\frac{\tau_B h'_B}{h_B}\right)^{-1}=\gamma\left(\frac{\delta}{\delta+\mu}-\frac{1-\delta}{1-\delta-\mu}\right). \tag{27}$$

The right hand side cannot vanish unless $\mu = 0$, which is not a general property in the presence of mobility. Equation (27) indicates that the equalization of the MCPFs with distributional weights is not an appropriate criterion of equalization payments when inter-regional mobility is present. Rather, the equalization formula should follow (24), which implies on the second best frontier:

$$-2a\frac{n_B}{\lambda_B}\left(1-\tau_B\frac{h'_B}{h_B}\right)\leq \tau_A h_A-\tau_B h_B\leq 2a\frac{n_A}{\lambda_A}\left(1-\tau_A\frac{h'_A}{h_A}\right). \tag{28}$$

Therefore, the inter-regional resource transfer should be made so that the difference in the per-capita tax payment across regions is bounded by reciprocal of the conventional MCPFs weighted by $2an_k/\lambda_k$ $(k = A, B)$.[13]

Our conclusion does not, however, eliminate all use of the conventional form of the MCPF with or without distributional weights. It is still valid for evaluating the marginal cost of expanding the local public expenditure: we still have the familiar form of the modified Samuelson condition for local public goods. This is not unusual in the literature. The presence of inter-regional mobility does not change the criterion for efficient intra-regional resource allocation: in the first best world, the Samuelson condition holds for a local public good even when there is free mobility (Boadway (1982), Boadway and Flatters (1982)). However, mobility imposes the additional condition for efficient population allocation among regions and the inter-regional resource allocation must be designed to ensure this: the equalization of the MCPFs is not compatible with this purpose.

4. Second best policy in a fiscal federal system

The second best allocation achieves the maximum social welfare given that only distortional taxation is available. The question is: can this be achieved in a decentralized framework? In the present model, it turns out that the central government can replicate the second best. To establish this argument, we begin with examining the characteristics of the Nash equilibrium for a given federal policy. Then we present the second best federal policy which involves both tax policy and an intergovernmental transfer program.

[13] Of course, the precise value of the difference should be dependent on δ.

For a given Θ and G, solving the local government problem (11) for region k with respect to the regional policy instruments yields:

$$\lambda_k h_k(1+\mu_k) = n_k h_k \gamma_k^k \left(1 - \tau_k \frac{h_k'}{h_k}\right) + \gamma_k^k\{n_k h_k' T - n_k^2 h_k h_k' f_k" \theta + D_k m_k\} \quad (t_k) \qquad (29)$$

$$b_k'(1+\mu_k) - \gamma_k^k = 0 \quad (g_k) \qquad (30)$$

$$\frac{h_k}{D_k}\left\{\gamma_k^k t_k + f_k'' h_k\{(1+\mu_k)\lambda_k - (1-\theta)\gamma_k^k n_k\}\right\} - 2a\mu_k = -\frac{h_j}{D_j} f_j'' h_j \mu_k \lambda_j \quad (n_k) \qquad (31)$$

where $j \neq k$. The variables being optimized are shown in parentheses. Recall that n_A is an artificial instrument; therefore (31) explains how population should be allocated from a regional view point. Inserting (29) into (30) establishes the first order condition for an optimal provision of g_k from a regional viewpoint:

$$n_k \frac{b_k'}{\lambda}\left\{\left(1 - \tau_k \frac{h_k'}{h_k}\right) + \frac{n_k h_k'(T - n_k h_k f_k''\theta) + D_k m_k}{n_k h_k}\right\} = 1, \; k = A, B. \qquad (32)$$

The bracket term in the LHS corresponds to the reciprocal of the regional MCPF. The difference between the regional and social MCPFs in conventional form is represented by the second term in the bracket which can be rewritten as:

$$n_k h_k'(T - n_k h_k f_k''\theta) + D_k m_k = D_k\left(-\frac{\partial R_F}{\partial t_k}\right), \quad k = A, B. \qquad (33)$$

In terms of Dahlby (1994 , 1996), $\partial R_F/\partial t_k$ represents a tax externality that the local government imposes on the federal budget. When $m_k = 0$, $\partial R_F/\partial t_k < 0$ from (33) and thus as argued by Dahlby (1994) and Boadway and Keen (1996), the MCPF is underestimated by the local government.

Note that (32) can be restated as:

$$\frac{\partial v_k}{\partial \tau_k} + b_k' \frac{\partial R_k}{\partial t_k} = 0. \qquad (32')$$

From (7), this implies $\partial n_k/\partial t_k = 0$: in the regional optimum, a marginal increase in t_k has no impact on the migration. This property simplifies our analysis in this section substantially. For a given federal policy $\{\Theta, G\}$, the Nash equilibrium can be obtained by solving the system of the equation involving (29) – (31) with the migration constraint (5) and the revenue constraints of the local governments (6) for $t_k, g_k, n_A, \mu_k, \gamma_k^k$. This system consists of 9 equations for the same number variables so we expect the system can be solved. As cited in section 2, the values in the equilibrium can be expressed as functions of Θ.

Before turning to the optimal federal policy, we analyze the welfare effects of the lump-sum transfers, S_k. Higher S_k influences $g_k = R_k(t_k(\Theta), T, n_k(\Theta), m_k, S_k)$ in two different ways. First, it increases the regional revenue R_k on one to one basis. Second, it leads to the changes in t_k and n_k:

$$\frac{dg_k}{dS_k} = 1 + \frac{\partial R_k}{\partial t_k}\frac{dt_k}{dS_k} + \frac{\partial R_k}{\partial n_k}\frac{dn_k}{dS_k}, \quad k = A, B. \tag{34}$$

Note that $dn_B/dS_B = -dn_A/dS_B$. By (32′), dn_k/dS_k involves only a direct effect on migration: $dn_k/dS_k = \partial n_k/\partial S_k$. Local expenditure is also influenced by the lump-sum transfer to another region:

$$\frac{dg_k}{dS_j} = \frac{\partial R_k}{\partial t_k}\frac{dt_k}{dS_j} + \frac{\partial R_k}{\partial n_k}\frac{dn_k}{dS_j} \tag{35}$$

where $dn_k/dS_j = \partial n_k/\partial S_j$ and $j \neq k$. We can describe the effects of other federal instruments on g_k in a similar fashion. Now, using the above results and the envelope theorem, we can show how the regional welfare is altered by S_k:

$$\frac{dV_k}{dS_k} = \frac{\partial L_k}{\partial S_k} + \frac{\partial L_k}{\partial t_j}\frac{dt_j}{dS_k} + \frac{\partial L_k}{\partial g_j}\frac{dg_j}{dS_k} = \gamma_k^k - \mu_k b_j' \frac{\partial R_j}{\partial n_j}\frac{dn_j}{dS_k} \tag{36}$$

$$\frac{dV_k}{dS_j} = \frac{\partial L_k}{\partial t_j}\frac{dt_j}{dS_j} + \frac{\partial L_k}{\partial g_j}\frac{dg_j}{dS_j} = -\mu_k b_j' \left\{1 + \frac{\partial R_j}{\partial n_j}\frac{dn_j}{dS_j}\right\}. \tag{37}$$

In the final equalities of (36) and (37), we make use of (32′). They imply that an increase in S_k (k=A, B) is followed by expansion of local expenditures of the recipient and a reallocation of the population, which in turn leads to a change in welfare in both regions.

We now turn to the federal policy needed to replicate the second best outcome. Denote the values in the second best by asterisks, $*$. To replicate the second best, $\Theta \equiv \{T, S_A, S_B, m_A, m_B\}$ and G are required to satisfy the following relations:

$$\tau_k^* = t_k(\Theta) + T, \quad g_k^* = g_k(\Theta) = R_k(t_k(\Theta), T, n_k(\Theta), m_k, S_k) \quad (k = A, B)$$

and

$$G^* = R_F(t_A(\Theta), t_B(\Theta), n_A(\Theta), \Theta).$$

Note that migration equilibrium is ensured for each federal policy and once $\tau_k = \tau_k^*$, $g_k = g_k^*$ and $G = G^*$ hold, n_A is also its second best value. Equivalently (and more clearly), Θ, G should give a system of equations consisting of (13), (22), (23) and (24) with the Nash equilibrium values of the local policy instruments, say, $t_k = t_k(\Theta)$. The system of equations for Θ should be solvable and, as a matter of fact, one federal instrument seems to be redundant: there are six federal instruments variables for five equations. In the rest of this section, we show that this conjecture is correct.

It is obvious that the matching grants should be used to internalize the tax externality (Dahlby (1996)). In other words, m_k should be set so as to cause $\partial R_F/\partial t_k$ to vanish in (33). Therefore, we can establish:

$$m_k = \frac{n_k h_k}{D_k}\left(n_k h_k' f_k'' \theta - \frac{h_k'}{h_k} T\right), \quad k = A, B. \tag{38}$$

With (38), (32) becomes coincident with the second best condition for g_k, (22). The value of m_k depends on T as well as θ. At least, however, it should be negative. This is because the tax externality has a negative impact on the federal budget constraint.

The corresponding formula is obtained when we replace the above matching grants by the revenue matching grants (Dahlby (1996)). Let q_k be the grant on the local tax revenue $R_k (k = A, B)$. For m_k and q_k to be equivalent, the following relation should hold:

$$m_k = q_k \frac{\partial R_k}{\partial t_k} \equiv q_k \frac{n_k h_k}{D_k} \left\{ 1 - \frac{h'_k}{h_k} t_k - n_k h'_k f''_k \theta \right\}, \quad k = A, B \tag{39}$$

The right hand side represents the amount of the change in the matching grant payment following the change in the local tax rate. Equating (39) with (38) yields the second best formula for q_k:

$$q_k \left(1 - \frac{h'_k}{h_k} t_k - n_k h'_k f''_k \theta \right) = n_k h'_k f''_k \theta - \frac{h'_k}{h_k} T \tag{40}$$

Let us now turn to the optimal formula for lump-sum grants. By solving the federal optimization using the fact that $\partial R_F / \partial t_k = 0$ under (38), the necessary condition for optimal level of S_k (k=A, B) can be obtained from:

$$\gamma_F = \delta \gamma_A^A - (1 - \delta) \mu_B b'_A + H \frac{dn_A}{dS_A} = -\delta \mu_A b'_B + (1 - \delta) \gamma_B^B + H \frac{dn_A}{dS_B} \tag{41}$$

where

$$H = \delta \mu_A b'_B \frac{\partial R_B}{\partial n_B} - (1 - \delta) \mu_B b'_A \frac{\partial R_A}{\partial n_A} + \gamma_F \frac{\partial R_F}{\partial n_A}.$$

An additional increase in g_k is worth γ_k^k from a region k's stand-point, while region j ($\neq k$) put the value of $-\mu_j g'_k$ on it. Thus, the first two terms in (41) represent the social evaluation of the marginal increment in g_k due to one dollar transfer to region k. The transfer followed by the increase in g_k also induces interregional migration, which alters the size of the tax base of all governments and, therefore, public expenditures. Such an induced migration exerts a first-order effect on social welfare as summarized by H. To summarize, the middle and the right-hand sides of (41) represent the social (aggregate) net gain from exogenous revenue increase in region k (=A, B). We can say that they are the shadow prices of the local tax revenues. γ_F is the shadow price of the federal tax revenue and therefore, we have the analogous expression to (25).

By Eq.(41) and (38), we can see that both the provision of the national public good and the population allocation are in the second best. From (30), $\gamma_A^A = (1 + \mu_A) b'_A$. Substituting it into (41) gives:

$$\delta(1 + \mu_A) - (1 - \delta) \mu_B = \frac{\gamma_F}{b'_A} - \frac{H}{b'_A} \frac{dn_A}{dS_A} = \frac{n_A \gamma_F}{\lambda_A} \left(1 - \tau_A \frac{h'_A}{h_A} \right) - \frac{H}{b'_A} \frac{dn_A}{dS_A} \tag{42}$$

In the last equality, we use the fact that g_A satisfies (22). Similarly,

$$-\delta \mu_A + (1 - \delta)(1 + \mu_B) = \frac{\gamma_F}{b'_B} - \frac{H}{b'_B} \frac{dn_A}{dS_B} = \frac{n_B \gamma_F}{\lambda_B} \left(1 - \tau_B \frac{h'_B}{h_B} \right) - \frac{H}{b'_B} \frac{dn_A}{dS_B} \tag{43}$$

By adding up (42) and (43), we find:

$$\sum_{k=A,B} \frac{n_k \gamma_F}{\lambda_k} \left(1 - \tau_k \frac{h'_k}{h_k}\right) = 1 + H \left(\frac{1}{b'_A} \frac{dn_A}{dS_A} + \frac{1}{b'_B} \frac{dn_A}{dS_B}\right). \tag{44}$$

It is straightforward to see that $B'(G) = \gamma_F$. Moreover, from $dn_k/dS_k = \partial n_k/\partial S_k$ (k=A, B) and (8), the bracket in the RHS vanishes. Thus the modified Samuelson condition for G, (23), can be obtained.

To show that (24), the second best condition for the population allocation, is satisfied, we multiplying (31) for region A, B by δ and $1 - \delta$, respectively and add them up. Then by substituting (42) and (43) and rearranging, we obtain:

$$\tau_A h_A - \tau_B h_B - \frac{2a}{\gamma_F} \{\delta \mu_A - (1-\delta)\mu_B\}$$

$$= \frac{H}{\gamma_F} \left\{1 + \sum_{k=A,B} \frac{h_k}{D_k} \left\{t_k + n_k h_k f''_k \left(\theta - \tau_k \frac{h'_k}{h_k}\right)\right\} \frac{\partial n_k}{\partial S_k}\right\}. \tag{45}$$

It can be shown that when (22) holds, D, the denominator of $\partial n_k/\partial S_k$, can be written as:

$$D = \sum_{k=A,B} \frac{h_k}{D_k} b'_k \left\{t_k + n_k h_k f''_k \left(\theta - \tau_k \frac{h'_k}{h_k}\right)\right\} - 2a. \tag{46}$$

Using the above, the RHS of (45) reduces to

$$The\ RHS = \frac{H}{\gamma_F} \left\{1 + \sum_{k=A,B} \frac{h_k}{D_k} \left\{t_k + n_k h_k f_k" \left(\theta - \tau_k \frac{h'_k}{h_k}\right) \left(-\frac{b'_k}{D}\right)\right\}\right\} = -\frac{2a}{D} \frac{H}{\gamma_F}.$$

From (42) and (43), we can establish:

$$\delta \mu_A - (1-\delta)\mu_B = (1-\delta)\{\delta(1+\mu_A) - (1-\delta)\mu_B\} - \delta\{-\delta\mu_A + (1-\delta)(1+\mu_B)\}$$

$$= (1-\delta)\gamma_F \frac{n_A}{\lambda_A} \left(1 - \tau_A \frac{h'_A}{h_A}\right) - \delta \gamma_F \frac{n_B}{\lambda_B} \left(1 - \tau_B \frac{h'_B}{h_B}\right) + \left\{-(1-\delta) \frac{H}{b'_A} \frac{dn_A}{dS_A} + \delta \frac{H}{b'_B} \frac{dn_A}{dS_B}\right\}. \tag{47}$$

The final bracket term can reduce to H/D. Therefore, inserting (47) back to (45) establishes (24).

To conclude, the second best can be attainable by matching grants scheme, (38), and lump-sum grants satisfying (41). The following proposition summarizes the above results.

> **Proposition 2:** *The federal government can replicate the second best by choosing the federal policy so as to fulfill (38) and (41).*

We can define the second best transfer scheme involving both lump-sum and matching components as:

$$\bar{S}_k = S_k + m_k t_k \quad \text{or} \quad \bar{S}_k = S_k + q_k R_k, k = A, B$$

where m_k and S_k $(k = A, B)$ are the optimized values. q_k is the revenue matching grants and is related to the second best value of m_k by (38).

In the present context, the matching and the lump-sum grants do different jobs. The former internalizes the tax externalities, which arise from the tax base sharing, while the latter unifies the revenue constraints of all governments by equalizing their shadow prices. This unification has a few economic implications. First, the lump-sum transfer serves to realize the one point on the second-best frontier depending on the value of δ. Second, S_k should be used to minimize the excess burden associated with the leisure-consumption decision by spreading it across the governments' budgets. Suppose that there is no mobility. Then (41) becomes coincident with (26): as argued in the previous section, we obtain the equalization of the conventional MCPFs with distributional weights. Although this relation cannot be extended to the case of mobility, (41) still implies the minimization of the excess burden under the restriction of individual mobility. Finally, the inter-regional transfer is required to resolve the inefficiency associated with migration. The residence-based tax (labor tax) will distort the households' locational decision making and therefore the population allocation. This kind of distortion should be carefully distinguished from the one resulting from the leisure-consumption choice. In the presence context, the individuals can avoid higher tax payments not only by decreasing the labor supply, but also by moving to the other region, which provides a lower tax rate. S_k works to remove the latter incentive.

It may be worth noting the difference between the formula of the grants derived in the present paper and the one of equalization in Canada. The latter also includes a matching component, but it is based on local tax bases, rather than local tax revenues or tax rates. Such a difference will have a substantial implication for the tax externality issue. Smart (1996) argues that under the Canadian equalization formula, the regional MCPF is reduced further since the shrink of the regional tax base due to an increase in local tax rate is effectively compensated by equalization payments. An extreme is the case that the regionally perceived MCPF become unity: this will be true for equalization receiving provinces whose tax rate is the same as the national average. It is straightforward from the above discussion that matching grants must be imposed on local tax revenue or tax rates in order to resolve the tax externalities. Insofar as the equalization payment is dependent on the tax base, it makes the situation worse.

What is about the federal tax? The above proposition holds for any value of T once the matching and lump-sum grants are optimized. This leads to the conclusion that the federal tax is irrelevant for achieving the second best. In fact, using the envelope theorem and (41), we can establish the following:

$$\frac{dSW}{dT} = H\left\{\frac{dn_A}{dT} - \sum_{k=A,B}\frac{1}{D_k}\{n_k h_k' T - n_k h_k + (1-\theta)n_k^2 h_k h_k' f_k''\}\frac{dn_A}{dS_k}\right\}. \qquad (48)$$

Substituting (8) and (9) above, we can show $dSW/dT = 0$, that is, the increase in T has no impact on the social welfare.[14] This result is closely related to the issue of optimal fiscal gap examined by Boadway and Keen (1996). In the present context, the fiscal gap can be defined by the difference between the federal tax revenue minus the public expenditure net of transfers:

$$Z = T \sum_{k=A,B} n_k^* h(w(\tau_k^*, n_k^*) - \tau_k^*) + \theta \sum_{k=A,B} r_k(\tau_k^*, n_k^*) - G^* \tag{49}$$

where the values of the targeted second best are denoted by asterisks, *. Z is equivalent to the total amount of the transfer from the central to the local governments and thus $Z > 0$ (< 0) designates that the fiscal gap is positive (negative). Since the federal tax is irrelevant, the following is immediate:[15]

> **Proposition 3:** *Federal tax policy is redundant in the optimum and therefore the optimal fiscal gap is indeterminate.*

The degree of decentralization of the tax system may be measured by the fiscal gap: higher Z implies a relatively centralized tax system and vice versa. The above proposition argues that when federal governments are equipped with sufficient instruments of intergovernmental grants, the existence of inefficiency in the federal system does not justify either centralization or decentralization of tax policies. Put differently, it may be the lack of instruments or other restrictions abstracted from the present model that determines the optimal fiscal gap. The conclusion of Boadway and Keen (1996) in favor of negative fiscal gap comes from the fact that matching grants are not available their model.[16]

Proposition 3 does not necessarily deny any significance to the optimal fiscal gap issue. In the present model, we assume that the federal government can conduct the regionally differentiated grants policy. In some circumstance, the federal government may

[14] In the second best equilibrium, dn_A/dT can be written as:

$$\frac{dn_A}{dT} = -\frac{1}{D}\left\{ \frac{b_A}{D_A}\{Tn_A h'_A - n_A h_A + (1-\theta)n_A^2 h_A h'_A f''_A\} \right.$$
$$\left. -\frac{b_B}{D_B}\{Tn_B h'_B - n_B h_B + (1-\theta)n_B^2 h_B h'_B f''_B\} \right\}$$

[15] Since the second best value of τ_k is given, the indeterminacy of T implies that so is t_k, the local tax rate. For a given τ_k^* , an increases in T is followed by the reduction in t_k by exactly the same amount.

[16] The second best formula of the federal tax rate derived by Boadway and Keen (1996) is still valid in the heterogeneous region case when the regionally differentiated tax policy is allowed at the federal level (Lin (1995)). Let T_k be the federal tax rate applied to region k (=A, B) and let $T_k = n_k h_k f''_k \theta$. From (38), this corresponds to the case of $m_k = 0$. With (41), the second best can be realized and the fiscal gap is unique. It may be said that the matching grants and the regionally differentiated tax policy are substitute instruments.

be restricted to uniform grants. That is, S_k or/and m_k (k=A, B) may be limited to be invariant across regions, which may be the case when there is asymmetric information about local characteristics such as preference for the public good between the central and local governments: as is familiar from the optimal income taxation literature, the imperfectness of information makes it infeasible to conduct differentiated policy for different agents (Stiglitz (1982)). This kind of restriction decreases the number of federal instruments available, which can prevent the federal government from replicating the second best or/and make the coordination between the inter-regional transfer and federal tax policies essential. If so, uniqueness of the optimal fiscal gap may result.[17]

It should be also mentioned that the irrelevancy of federal tax policy partly relies on the following: if necessary, the federal government can make use of negative lump-sum grants for raising the revenue. The negative value of S_k does not cause an additional distortion in the process of transferring the resource from the local to the central governments. In other words, the federal government has an option to tax on the local governments instead of the households. If S_k or $\bar{S}_k$ is limited to be non-negative, T must be sufficiently high to finance the grants payment, which will impose a lower-bound for the optimal fiscal gap.

Before closing this section, we should make a comment on the stability issue of the second best equilibrium in the presence of migration. As discussed by Stiglitz (1977) and Boadway and Flatters (1982), the stability of efficient equilibrium is not necessarily ensured in fiscal federal models. The well-known requirement for the stability in the economy without distortionary taxation is that the economy as a whole be over-populated in that the equilibrium size of each region gives a negative value for the net social product. This condition may seem not to be compatible with a positive labor tax rate in the second best, which is the most interesting case: for the second best tax rates to be positive, the net social product must be positive, which implies under-population of the economy. However, in the present model with two levels of governments and imperfect mobility, we can find that the positive aggregate tax rates ($\tau_k > 0$) can result in the stable second best equilibrium.

The dynamic system of migration may be described by the following equation:

$$\frac{dn_A}{dt} = \alpha\{v_A(\tau_A, n_A) + b(g_A) + a(1 - n_A) - v_B(\tau_B, 1 - n_A) - b(g_B) - an_A\} \tag{50}$$

where $\alpha > 0$. The RHS can be linearized around the second best values denoted by *:

$$RHS\ of\ Eq.(50) = \alpha\left\{\left(\frac{\partial v_A}{\partial n_A} + b'_A\frac{\partial R_A}{\partial n_A}\right) + \left(\frac{\partial v_B}{\partial n_B} + b'_B\frac{\partial R_B}{\partial n_B}\right) - 2a\right\}(n_A - n_A^*)$$

$$= \alpha\left\{\Sigma_{k=A,B}\frac{h_k}{D_k}b'_k\left\{(\tau_k^* - T) + n_k h_k f''_k\left(\theta - \tau_k^*\frac{h'_k}{h_k}\right)\right\} - 2a\right\}(n_A - n_A^*).$$

[17] Gilbert and Picard (1996) points out that when information is perfect, under a linear (matching) subsidy, any degree of decentralization can do the same job as the unitary nation. Although they establish a model in a different context, their argument seems to be true here.

Stability requires that the initial population can be restored after the perturbation if τ_k^* and S_k^* $(k = A, B)$ are kept: as mentioned in section 2, this implies that the largest bracket term ($=D$) should be negative.[18] It is immediate that with sufficient degree of home-attachment, a, the bracket term can be negative as a whole: as cited by Wellisch (1994), the home-attachment can improve the stability issue. Or if θ is sufficiently high or/and the elasticity of labor supply is low, the second term in the smallest bracket is likely to be negative and so is the sign of D.

Interestingly, federal tax policy can ensure the stability of the second best equilibrium: when T is set high enough, $D < 0$ will result. This implies that if necessarily the federal tax policy should be conducted to stabilize the targeted second best equilibrium. Although the optimal fiscal gap is not unique yet, it should be bounded below since T should satisfy:

$$\sum_{k=A,B} \frac{h_k}{D_k} b'_k \left\{ \tau_k^* + n_k h_k f''_k \left(\theta - \tau_k^* \frac{h'_k}{h_k} \right) \right\} - 2a < T \sum_{k=A,B} \frac{h_k}{D_k} b'_k. \tag{51}$$

The intuition behind the above argument is as follows. As shown in (4), an increase in n_k reduces $v_k(\tau_k, n_k)$ because of decrease in the wage rate, while it expands the local tax base, R_k. The latter raises the local expenditure and therefore improves welfare of the residents. For a given second best rate of τ_k^*, higher T implies lower t_k. This in turn leads to lower value of $\partial R_k / \partial n_k$: that is, an additional resident does not enlarge the local tax base so much. Thus for sufficiently high T, the former effect outweighs the latter at least in one region ($\partial v_k / \partial n_k + b'_k \partial R_k / \partial n_k < 0$), which will let $D < 0$. To summarize:[19]

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[18] Alternatively, suppose g_k is fixed, while t_k is adjusted to balance the budget. In this case, (50) should be rewritten as:

$$\frac{dn_A}{dt} = \alpha \left\{ \left(\frac{\partial v_A}{\partial n_A} + \frac{\partial v_A}{\partial t_A} \frac{dt_A}{dn_A} \right) + \left(\frac{\partial v_B}{\partial n_B} + \frac{\partial v_B}{\partial t_B} \frac{dt_B}{dn_B} \right) - 2a \right\} (n_A - n_A^*)$$

where

$$\frac{dt_k}{dn_k} = -\frac{\partial R_k}{\partial n_k} \left(\frac{\partial R_k}{\partial t_k} \right)^{-1} \quad (k = A, B).$$

Under (32′), we have

$$\frac{\partial v_k}{\partial t_k} \frac{dt_k}{dn_k} = b'_k \frac{\partial R_k}{\partial t_k} \frac{\partial R_k}{\partial n_k} \left(\frac{\partial R_k}{\partial t_k} \right)^{-1} = b'_k \frac{\partial R_k}{\partial n_k}.$$

The alternative expression can reduce to (50). Therefore, whether t_k or g_k is adjusted in the perturbation of n_A does not matter for the discussion of stability.

[19] This corollary is relevant even when the taxation is not distortionary ($h'_k = 0$). It is well known that in such a circumstance, federal taxation is equivalent to a negative lump-sum grant to the local authority: the federal government can achieve exactly the same allocation by either taxing individuals or local governments. The corollary implies, however, that the two instruments have different implication with respect to stability in the economy.

Corollary to Proposition 3: *For the second best equilibrium to be stable, T should be set so as to satisfy (51) and this gives a lower bound to the optimal fiscal gap.*

We can say the second best equilibrium can be stable even in the case of $\tau_k^* > 0$, that is, the economy is under-populated. The above corollary leads us to the conclusion that the centralization of the tax system to some degree may be justified for the purpose of ensuring stability, rather than achieving the second best.

5. Conclusion

In the present paper, we have attempted to generalize the existing works of Dalhby and Wilson (1994) and Boadway and Keen (1996) by introducing imperfect mobility and heterogeneous regions. In contrast to the familiar argument, the second best allocation does not require the equalization of the conventional form of the MCPFs across regions in the presence of imperfect or perfect mobility: rather, the inter-regional transfer should take into account the efficient allocation of the population. Regarding the implementation of the second best allocation in a fiscal federal setting, we have examined the case where local governments ignore the budget constraints of other governments. We can characterize the second best matching grants scheme, which internalizes the tax externalities, while lump-sum grants should be provided so as to equalize the shadow prices of governments' revenues. Under the behavioral assumption of the local governments considered in the present paper, the lump-sum grants must be combined with the matching grants irrespective of the degree of home-attachment and the value of δ. The optimal fiscal gap is indeterminate: in other terms, we can establish an intergovernmental transfer scheme for any level of the federal tax rate to achieve a given second best allocation. The indeterminacy implies that the optimal fiscal gap, if it exists, comes from restrictions extraneous to our model. We have also shown that when stability is concerned, there is a minimum level of federal tax needed to stabilize the second best equilibrium, which may give a rationale for centralization of tax system to some extent even in the presence of matching grants.

The possibility of replicating the second best outcome by the federal policy and the indeterminacy of the optimal fiscal gap may be striking. But some caution must be exercised. As cited above, in some circumstance, the regionally differentiated grants program may be implausible. Imperfect information about some local characteristics can lead to such a case. In addition, the establishment of an intergovernmental transfer scheme may not be exclusive to the federal government. So far, we have assumed that the grants can be of either sign: if necessary, the federal government can tax on the local governments. In a federation consisting of relatively small number of provinces (like in Canada), however, it can be the case that the provincial governments have a significant influence on the decision making at the federal level. The relation between the federal and provincial governments may be modeled more appropriately in the framework of bargaining.

Before closing this paper, we should mention a few restrictions in our framework of analysis. First, the introduction of imperfect mobility is done in a rather restrictive form. We have assumed the psychological cost of mobility (home attachment) introduced by Mansoorian and Myers (1993). However, there should be another way of formalizing the imperfectness of mobility. One alternative will be to assume resource cost related to the mobility as discussed by Boadway and Wildasin (1990). It might be possible that the different formulations of the imperfect mobility bring different characteristics in the second best. Second, we are abstracted from capital mobility, which is another essential feature of a fiscal federal system. So far, there are only a few attempts to incorporate both labor and capital mobility (Burbidge and Myers (1994b), Wellisch and Wildasin (1996)). Introducing two sorts of mobility may give new insight on our analysis. Finally, in our model, the households are homogeneous except locational preference. So essentially, only efficiency issue is concerned. Recently, Boadway *et al* (1996), Burbidge and Myers (1994a) and Wildasin (1994) examine income redistribution policy in a fiscal federal system. By extending our model to include heterogeneous agents, we may find different formula for the second best federal policy. These extensions and the consideration for imperfect information issue and the bargaining process between governments remain for future research.

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