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MACROECONOMIC FLUCTUATIONS AND THE LORENZ CURVE*

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In this paper a new method is forwarded of estimating the effect of short-run macroeconomic fluctuations on concentration in the size distribution of personal income. In particular, the impacts of changes in unemployment and participation rates and in the level of wage and salary income upon the shape of a Lorenz curve are analyzed for each of three age groups in the United States. It is found that increases in the participation rate and decreases in the unemployment rate are consistent with an upward shift of the Lorenz curve, while increases in the overall level of wage and salary income have differing effects on the shape of the Lorenz curve depending on the age and relative position in the income distribution of a group.

I. Introduction

Recent years have seen a growing discussion of the distributional impact of cyclical fluctuations in macroeconomic activity. Johnson [4], Tobin [13], and Hollister and Palmer [3] have argued the importance of low unemployment policies largely on distributional grounds. And Schultz [10], Thurow [12], and Metcalf [8] have attempted to measure exactly what the impacts are of macroeconomic fluctuations upon inequality in the size distribution of income. This paper is written in the spirit of the latter contributions, but offers an approach that differs substantially from the earlier ones. Whilst Schultz focused on aggregate inequality measures such as the Gini coefficient, this paper examines inequality behaviour at a disaggregative level within distributions. Metcalf based his study on fitting lognormal density functions to empirical income distributions; but the present approach is distribution-free in that it does not constrain the data to satisfy any particular distribution. Furthermore, none of the earlier papers examined the functional mechanism linking income inequality to short-run cyclical fluctuations, so that one of the objectives of this paper is to identify the various channels through which macro fluctuations affect distributional inequality by means of a simple a priori model.

The approach used in this study involves first analyzing inequality changes in a set of quantile income levels spanning a distribution, expressing standard inequality measures in terms of the income quantiles, and then deriving the behaviour of these measures implicit by the behaviour of the underlying set of income quantiles. The inequality measure used in this paper is the standard Lorenz curve. Fluctuations in economic aggregates thus affect the shape of the Lorenz curve only indirectly via their impacts on a set of income quantiles. Such an "indirect quantile approach" makes more efficient use of the distributional data available than previous studies, while offering substantially more flexibility.

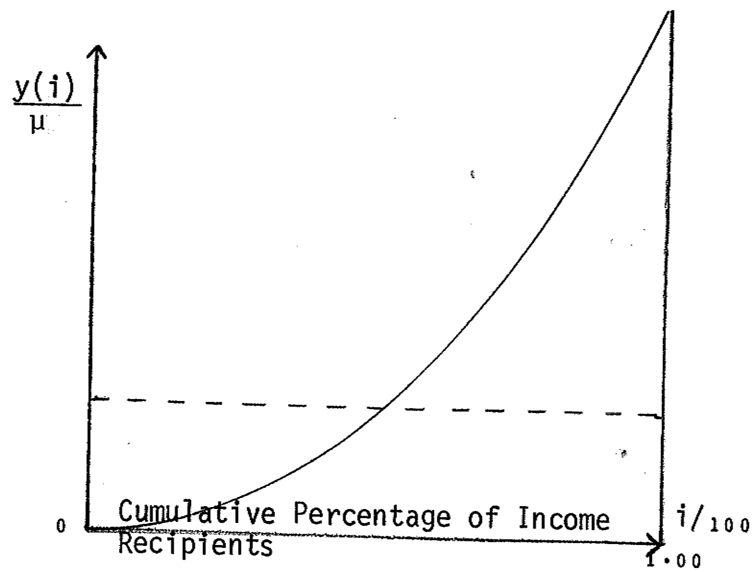
The outline of the paper is as follows. The next section presents the basic model of income changes within a distribution. Section III discusses the estimation procedure and results based on annual Bureau of the Census data for 1947-1970. Section IV compares these results with some of Metcalf's findings. Section V examines the implications of the results for the Lorenz curve of a distribution. And Section VI summarizes the principal findings of the paper.

II. A Model of Distributional Changes

One possible approach to studying the behaviour of a Lorenz curve is to examine the behaviour of its derivative, the relative mean income curve illustrated in Figure I. The ordinate of this curve is simply $y(i)/\mu$ where $y(i)$ (for $0 \leq i \leq 100$) denotes the i'th quantile income level and μ is the mean income for the distribution [5]. If one is able to explain the behaviour of mean income and a set of income quantiles, one can then explain

FIGURE I

Relative Mean Income Curve



their ratio; i.e., one can explain the fluctuations of a set of points on the relative mean income curve, and thus of the curve itself. One can then integrate up and analyze how the associated Lorenz curve and implied income shares fluctuate. The essential idea is that a study of fluctuations in the shape of the Lorenz curve can be reduced to an analysis of the behaviour of mean income and a set of income quantiles of a distribution, and of the channels through which macro fluctuations affect these individual mean and quantile levels. Consequently, we now turn to the development of a simple empirical model of the channels through which macro activity affects the incomes of individual recipients.

In the analysis that follows the basic income receiving unit will be the individual rather than the family. This avoids complications associated with changing composition of family units[1], and with cyclical fluctuations in the number of family units.[2] As an empirical convenience, the quantiles selected for study will be the nine income deciles corresponding to i = 10, 20, ..., 90; and to allow for a more detailed analysis, individuals will also be disaggregated by age. Thus we have a population of individual income recipients who are ranked by age (represented by the variable a) and by their relative position in the income distribution (indicated by the decile index i), and we can let y (a,i) represent the average gross money income of individuals of age a at the i 'th decile position in the income distribution. Now y(a,i) can be decomposed according to a stochastic identify into components derived from different sources:

$$y(a,i) = YE(a,i) + YU(a,i) + YPB(a,i) + YPF(a,i) + YTR(a,i) + YTP(a,i) + YK(a,i) + v(a,i) \qquad (1)$$

where YE(a,i) is the average income received from employment; YU(a,i) is average unemployment benefits received; YPF(a,i) and YPB(a,i) represent farm proprietary income and business and professional proprietary income; YTR(a,i) and YTP(a,i) are relief transfers and pension transfers; YK(a,i) is average capital income in the form of rents, interest, and dividends; and v(a,i) is a random term assumed to represent remaining minor sources of income. The first two labour income components can be further factored into

$$YE(a,i) = PR(a,i) \,.\, ER(a,i) \,.\, W(a,i)$$

and

$$YU(a,i) = PR(a,i) \,.\, UR(a,i) \,.\, UB(a,i), \qquad (2)$$

1. See [10], p. 78.
2. See [11].

where PR(a,i) is the group's average labour force participation rate; ER(a,i) = 1 - UR(a,i) is its employment rate; W(a,i) is wage and salary income per employed person in the group; and UB(a,i) is the average unemployment benefits per unemployed person. Content is now given to these relationships by (a) relating the age - and decile - specific variables in (1) and (2) to economic aggregates, and (b) selecting which variables in (1) and (2) to retain or drop for particular age and income groups.

Since the right-hand side variables in (1) and (2) are generally unobserved, they will be assumed to be nondecreasing linear functions of observed aggregates. That is,

$$\begin{aligned}
PR(a,i) &= \delta_{PR}(a,i) + \alpha_{PR}(a,i)\ PR(a) & \alpha_{PR}(a,i) &\geq 0 \\
ER(a,i) &= \delta_{ER}(a,i) + \alpha_{ER}(a,i)\ ER(a) & \alpha_{ER}(a,i) &\geq 0 \\
W(a,i) &= \delta_{W}(a,i) + \alpha_{W}(a,i)\ W & \alpha_{W}(a,i) &\geq 0 \\
UB(a,i) &= \delta_{UB}(a,i) + \alpha_{UB}(a,i)\ W & \alpha_{UB}(a,i) &\geq 0 \\
YPB(a,i) &= \delta_{PB}(a,i) + \alpha_{PB}(a,i)\ W & \alpha_{PB}(a,i) &\geq 0 \qquad (3) \\
YPF(a,i) &= \delta_{PF}(a,i) + \alpha_{PF}(a,i)\ YPF & \alpha_{PF}(a,i) &\geq 0 \\
YTR(a,i) &= \delta_{TR}(a,i) + \alpha_{TR}(a,i)\ YTR & \alpha_{TR}(a,i) &\geq 0 \\
YTP(a,i) &= \delta_{TP}(a,i) + \alpha_{TP}(a,i)\ YTP & \alpha_{TP}(a,i) &\geq 0 \\
YK(a,i) &= \delta_{K}(a,i) + \alpha_{K}(a,i)\ YK & \alpha_{K}(a,i) &\geq 0.
\end{aligned}$$

PR(a) and ER(a) are average participation and employment rates for members of age group a, while the rest of the right-hand side variables in (3) are averages over all age groups. It will be noted that unemployment benefits have been assumed a function of wage income, W, because of the institutional way that individuals' unemployment benefits are closely tied to their recent wage earnings. Business and professional proprietary income has also been assumed dependent on W because the supply curve of labour for the self-employed is assumed elastic to wages in alternative employment opportunities.

When the relationships in (2) and (3) are substituted into (1), the resulting equations can be written compactly for all n observations as

$$y(a,i) = x(a)\ \beta\ (a,i) + v(a,i) \tag{4a}$$

where y(a,i) is now a column vector of n observations on the i'th decile for the a'th age group; x(a) is an n x 12 matrix of observations on terms involving the right-hand side variables in (3); β(a,i) is a conformable column vector of coefficients derived from the alpha and delta coefficients in (3); and v(a,i) is now an n-dimensional column vector of random terms. In analogous fashion, it will be assumed that we can also write a mean income equation

$$\mu(a) = x(a)\beta(a,\mu) + v(a,\mu) \tag{4b}$$

for each age group. In general, the elements of x(a) include observations on composite terms such as PR(a).W or PR(a).ER(a) as well as on simple terms such as YPF and YTR. Consequently, (4) is linear in the betas, but nonlinear in the aggregate variables of the analysis. The betas are also, in general, nonlinear functions of the coefficients in (3). The non-negativity constraints on the alphas imply that the coefficients corresponding to the simple terms YPF, YTR, YTP, and YK are also non-negative. Expressing the mean and decile incomes in the form of (4), one can then characterize changes in the overall income distribution for each age group by the set of one mean and nine decile equations

$$\begin{bmatrix} y(a,10) \\ \vdots \\ y(a,90) \\ \mu(a) \end{bmatrix} = \begin{bmatrix} x(a) & & \\ & \ddots & \\ & & x(a) \end{bmatrix} \begin{bmatrix} \beta(a,10) \\ \vdots \\ \beta(a,90) \\ \beta(a,\mu) \end{bmatrix} + \begin{bmatrix} v(a,10) \\ \vdots \\ v(a,90) \\ v(a,\mu) \end{bmatrix}$$

or more compactly,

$$y(a) = X(a)\beta(a) + v(a) \quad (5)$$

In summary, then, equation (5) with its appropriately signed coefficients sets out in formalistic fashion the channels through which quantile changes occur in the income distribution.

We should not be very optimistic about estimating (5) with a paucity of observations, however, since it contains twelve coefficients for each quantile equation. Consequently a second set of constraints specifying some of the beta coefficients to be zero has been imposed. In particular, three specifications of the labour terms in (4) have been used. In Case I (applicable to low decile groups), it is assumed that both participation and employment rates have an impact on decile incomes, and that these impacts occur via proportional response functions; i.e., $\delta_{PR} = \delta_{ER} = \delta_{UB} = 0$, so that the labour income terms in (4) simplify to

$$\beta_3(a,i)\ PR(a).ER(a) + \beta_5(a,i)\ PR(a).W + \beta_7(a,i)\ PR(a).ER(a).W. \quad (6)$$

In Case II (applicable largely to secondary workers), everyone in the labour force in the decile group is assumed to be employed, but not everyone is a full-time participant in the labour force; i.e., $\delta_{PR} = \alpha_{ER} = 0$, and $\delta_{ER} = 1$. And in Case III, all the members of the decile group are assumed to be in the labour force and fully employed, so that $\delta_{PR} = \delta_{ER} = 1$ and $\alpha_{PR} = \alpha_{ER} = 0$.

The nonlabour terms also need to be further constrained. Farm proprietary income and relief transfers will occur in equations only for low decile income levels. Pension benefits are received only by those of retirement age. Capital income accrues largely to the top quantile groups and the retired. Thus only a fraction of the terms in equations (4) will appear in the equation for any particular decile or mean, and the matrix of independent variables in (5) may be rewritten as

$$X(a) = \begin{bmatrix} x(a,10) & & & \\ & \ddots & & \\ & & x(a,90) & \\ & & & \ddots \; x(a,\mu) \end{bmatrix}$$

Finally there exists a third set of constraints upon the derivatives of the independent variables and thus upon the coefficients of the equations in (5). For each age group, there exists an income density function f(a;y) with an associated mean income of

$$\mu(a) = \int_{i=0}^{100} y(a;i)\ f(a;y)\ dy\ (a;i). \qquad (7)$$

Thus if W, say, were to shift up by a dollar, the resulting distribution of the shift among income groups in the f(a;y) density would have to satisfy the adding-up constraint

$$\int_i \left(\frac{\partial y(a;i) f(a;y)}{\partial W}\right) dy\ (a;i) = \frac{\partial \mu(a)}{\partial W}\ . \qquad (8)$$

Analogous constraints can also be formulated for each of the other independent variables. However, since the distribution is being approximated by only nine deciles in this paper, the left-hand side of (8) must be replaced by some weighted average of these deciles such as

$$(.10) \sum_{j=1}^{9} \left(\frac{\partial y(a;10j)}{\partial W}\right),$$

with an adjustment on the right-hand side for the weights summing to only

.90.[1] The approximation is assumed also to hold only at the means so that the independent variables appearing in the constraint itself are evaluated at their means over the period of analysis.

In summary, then, the "seemingly unrelated" equations in (5) together with the three sets of constraints introduced above constitute the model of decile income behaviour that has been estimated.

III. Estimation of the Model

In the empirical work to follow, the model is estimated for individual males of three age groups, 20-24 years (a=1), 35-44 years (a=2), and 55-64 years (a=3), although the model could easily be estimated for any other age and sex categories. Estimates for the income deciles were obtained by linear interpolation and estimates of the means were taken directly from the U.S. Bureau of the Census Series P-60 on Consumer Income for each year from 1947 to 1970 and from the Bureau's Technical Paper No. 17 on "Trends in the Income of Families and Persons in the United States: 1947-1964." Since the estimates of the dependent variables in the above model

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1. Implicit in this weighting scheme is the assumption that the mean income for a distribution with the top five percent and bottom five percent truncated is the same as the mean income for the whole distribution. But for a skewed distribution this is an invalid assumption. Since the mean of the truncated income distribution would be expected to be slightly less than the mean of the overall distribution, the weight on the right-hand side of (8) has been reduced from .90 to .85 so that the constraint actually imposed is

$$(.10) \sum_{j=1}^{9} \left(\frac{\partial y(a;10j)}{\partial W} \right) = .85 \frac{\partial \mu(a)}{\partial W} \quad . \qquad (9)$$

are assumed to be related to the true means and decile levels by an additive error term, the equation in (5) can be rewritten as

$$\hat{y}\ (a) = X(a)\beta\ (a) + u(a), \qquad (10)$$

where the intercept terms have been adjusted so that u(a) has zero mean.

Data on the independent variables in (10) have been drawn from the U.S. annual national accounts in the Survey of Current Business (and deflated by adult population figures drawn from the Census Bureau's Series P-25 on Population Estimates) and from the Handbook of Labour Statistics. Thus YTR, for example, is mean relief transfers per adult in the United States. W, on the other hand, is the mean wage and salary income per year over only the employed adult male population.

Since the adding-up constraints outlined in the last section are only approximations, they have been imposed upon the coefficients of only those variables that appear in almost all the ten equations for a given age group. Consequently the only constraints that have been imposed are those corresponding to the wage income and participation rate variables for the first age group, and to W alone for the remaining two groups.

Several econometric problems now arise in estimating the model specified in (9) and (10). First of all, inspection of (6) reveals that, in the Case I situation, two of the labour income terms (with coefficients β_5 and β_7) are extremely collinear. The approach that has been taken to this multicollinearity problem is simply to specify the $\beta_5(a,i)$ coefficients a priori, and then fit the equations subject to these restrictions. This procedure is facilitated by the fact that β_5 is simply $\alpha_{PR}(a,i).\alpha_{UB}(a,i)$, where α_{UB} = UB(a,i)/W and α_{PR} = PR(a,i)/PR(a). Thus, given observations on PR(a) and W, it is sufficient to specify values for only UB(a,i), the unemployment benefits members of a particular age and decile group receive on average, and PR(a,i), their average participation rate. The resulting _a priori_ beta coefficients are shown in Table I below with no "t-ratios."

Secondly, since the study uses time series data, it would be expected that the residuals in (10) would be serially interdependent as well as contemporaneously correlated. Parks [9] suggested a procedure for handling first-order serial correlation in the framework of Zellner's seemingly unrelated equations without cross-equation coefficient constraints. The estimators used in this study generalize Parks' procedure so as to incorporate the adding-up constraints. The resulting estimation procedure can be summarized briefly as follows: (1) run the constrained Zellner seemingly unrelated regressions to obtain estimates of the residuals and thence of the autocorrelation coefficients; (2) transform the variables in each equation according to a first-order Markov autoregressive scheme; and (3) calculate the constrained Zellner estimates of the beta coefficients of the transformed equations. Under conventional assumptions, it can be shown[1] that the resulting "constrained Parks" estimators are consistent and asymptotically efficient. This procedure has been used to estimate simultaneously the ten equations in each of the three age groups subject to the adding-up constraints discussed above.

The coefficient estimates together with their "t-ratios" are set out in Table I with each block of results corresponding to a different age

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1. See [1], pp. 125-126.

TABLE I

Constrained Parks Estimates of the Regression Coefficients

Ages 20-24 (a=1)

	$\hat{y}(1,10)$	$\hat{y}(1,20)$	$\hat{y}(1,30)$	$\hat{y}(1,40)$	$\hat{y}(1,50)$	$\hat{y}(1,60)$	$\hat{y}(1,70)$	$\hat{y}(1,80)$	$\hat{y}(1,90)$	$\hat{\mu}(1)$
Constant	-1112. (2.637)	-1345. (3.024)	-638.0 (1.161)	477.1 (0.825)	967.5 (2.072)	1396. (3.463)	1657. (1.875)	3086. (4.819)	2700. (2.892)	852.8 (2.562)
PR(1).ER(1)	1774. (3.387)	2284. (4.208)	1692. (2.564)	386.7 (0.559)	-121.5 (0.217)	-602.0 (1.241)				-29.83 (0.075)
W.PR(1).ER(1)	-.0952 (6.128)	-.1102 (7.463)	-.1195 (6.699)	-.0153 (1.016)	.0613 (4.274)	.1582 (11.52)				.1151 (11.77)
PR(1)							-857.7 (0.894)	-2439. (3.496)	-1942. (1.913)	
PR(1).W	.095	.153	.239	.253	.267	.278	.5362 (31.23)	.6552 (52.99)	.8093 (59.30)	.267
YTR	7.887 (2.658)	9.418 (3.250)	8.032 (2.920)							
YPF	.9360 (1.276)	1.185 (1.750)								

Ages 35-44 (a=2)

	$\hat{y}(2,10)$	$\hat{y}(2,20)$	$\hat{y}(2,30)$	$\hat{y}(2,40)$	$\hat{y}(2,50)$	$\hat{y}(2,60)$	$\hat{y}(2,70)$	$\hat{y}(2,80)$	$\hat{y}(2,90)$	$\hat{\mu}(2)$
Constant	-6122. (3.963)	-2447. (2.156)	806.1 (1.315)	224.7 (3.098)	236.0 (2.876)	94.60 (1.405)	-247.5 (1.655)	-328.9 (1.558)	-1138. (2.874)	66.88 (0.655)
PR(2).ER(2)	5961. (3.510)	2441. (1.982)	-694.2 (1.070)							
W.PR(2).ER(2)	.2039 (10.54)	.3431 (32.04)	.4067 (48.56)							
PR(2).W	.174	.230	.279							
W				.7531 (70.38)	.8517 (72.05)	.9834 (100.8)	1.171 (55.09)	1.368 (44.71)	1.892 (32.73)	.9819 (74.98)
YTR	11.74 (3.394)									
YPF	2.140 (2.203)	1.044 (1.461)								

TABLE I (Continued)

Constrained Parks Estimates of the Regression Coefficients

Ages 55-64 (a=30)

	$\hat{y}(3,10)$	$\hat{y}(3,20)$	$\hat{y}(3,30)$	$\hat{y}(3,40)$	$\hat{y}(3,50)$	$\hat{y}(3,60)$	$\hat{y}(3,70)$	$\hat{y}(3,80)$	$\hat{y}(3,90)$	$\hat{\mu}(3)$
Constant	-2283. (2.741)	-1521. (1.032)	-1701. (1.309)	872.9 (1.014)	3153. (3.061)	258.0 (2.702)	213.5 (1.701)	-73.06 (0.250)	65.18 (0.133)	330.7 (3.811)
PR(3).ER(3)	2028. (1.992)	1299. (0.717)	1734. (1.103)	-1193. (1.180)						
W.PR(3).ER(3)	.0816 (4.718)	.1908 (6.185)	.2851 (11.93)	.4183 (22.97)						
PR(3)					-3577. (3.141)					
PR(3).W	.137	.185	.260	.301	.8109 (47.11)					
W						.7815 (58.21)	.9173 (51.27)	1.164 (27.86)	1.520 (23.17)	.7842 (59.36)
YTR	9.358 (2.831)	14.06 (2.434)	11.66 (2.660)							
YPF	2.538 (3.504)	1.353 (1.122)	0.371 (0.403)							

group. The dependent variables for each set of equations are listed across the top of each block, and the terms appearing on the right-hand side of each equation are listed down the left-hand column. Thus by glancing down a column, one can read off the coefficient estimates for any equation within a given block. All the coefficients with a priori expected signs turn out to have their correct signs. F-tests of the adding-up constraints resulted in a rejection at the ninety-five percent level of confidence in only one case (for the second age group). But since the constraints in (8) are essentially identities, they have been retained in the regression analysis. The estimated autocorrelation coefficients of the regressions vary between -.089 and .670 with twelve of the thirty coefficients having t-ratios of 2.0 or more.

More interesting, however, are the partial elasticities (evaluated at the mean) of the mean and decile income levels implicit in these estimated equations. These are presented in Table II which is in the same format as Table I. Thus by scanning horizontally across the page, one can read off the profile of wage income elasticities, say, as one moves from lower to higher deciles. According to the model developed in Section II, all of these elasticities should be non-negative; and as can be seen in Table II, all are.

To assist in interpreting these figures, several highlights should perhaps be commented upon. As one would expect, participation and employment rate elasticities have their greatest impact at the bottom ends of the distribution. Most of the males of these ages, however, are fully employed and are receiving incomes almost entirely in the form of wages and salaries.

TABLE II

Estimated Mean and Decile Income Elasticities

Ages 20-24(a=1)

	$\hat{y}(1,10)$	$\hat{y}(1,20)$	$\hat{y}(1,30)$	$\hat{y}(1,40)$	$\hat{y}(1,50)$	$\hat{y}(1,60)$	$\hat{y}(1,70)$	$\hat{y}(1,80)$	$\hat{y}(1,90)$	$\hat{\mu}(1)$
PR(1)	2.504	2.017	1.340	.774	.631	.563	.563	.306	.485	.701
ER(1)	1.648	1.246	.513	.117	.079	.091	0	0	0	.194
W	.063	.258	.445	.620	.670	.737	.766	.797	.804	.710
YTR	.187	.125	.073	0	0	0	0	0	0	0
YPF	.171	.121	0	0	0	0	0	0	0	0

Ages 35-44(a=2)

	$\hat{y}(2,10)$	$\hat{y}(2,20)$	$\hat{y}(2,30)$	$\hat{y}(2,40)$	$\hat{y}(2,50)$	$\hat{y}(2,60)$	$\hat{y}(2,70)$	$\hat{y}(2,80)$	$\hat{y}(2,90)$	$\hat{\mu}(2)$
PR(2)	3.690	1.705	.804	0	0	0	0	0	0	0
ER(2)	3.203	1.294	.405	0	0	0	0	0	0	0
W	1.033	1.005	.963	.951	.953	.981	1.030	1.034	1.103	.981
YTR	.078	0	0	0	0	0	0	0	0	0
YPF	.110	.034	0	0	0	0	0	0	0	0

Ages 55-64 (a=3)

	$\hat{y}(3,10)$	$\hat{y}(3,20)$	$\hat{y}(3,30)$	$\hat{y}(3,40)$	$\hat{y}(3,50)$	$\hat{y}(3,60)$	$\hat{y}(3,70)$	$\hat{y}(3,80)$	$\hat{y}(3,90)$	$\hat{\mu}(3)$
PR(3)	2.973	1.632	1.543	.755	.268	0	0	0	0	0
ER(3)	2.217	1.107	1.048	.313	0	0	0	0	0	0
W	1.190	1.045	1.028	1.034	.989	.945	.960	1.006	.989	.933
YTR	.140	.108	.060	0	0	0	0	0	0	0
YPF	.292	.080	.015	0	0	0	0	0	0	0

Thus it should not be surprising that their wage income elasticities are approximately unity.

More interesting, however, are the patterns of the wage income elasticities across deciles for the three distributions. For the first group, males twenty to twenty-four, the elasticities rise from .063 to .804. Several possible reasons can be suggested for this increase. From (1) to (2) and the substitution constraints in (3), one can see that

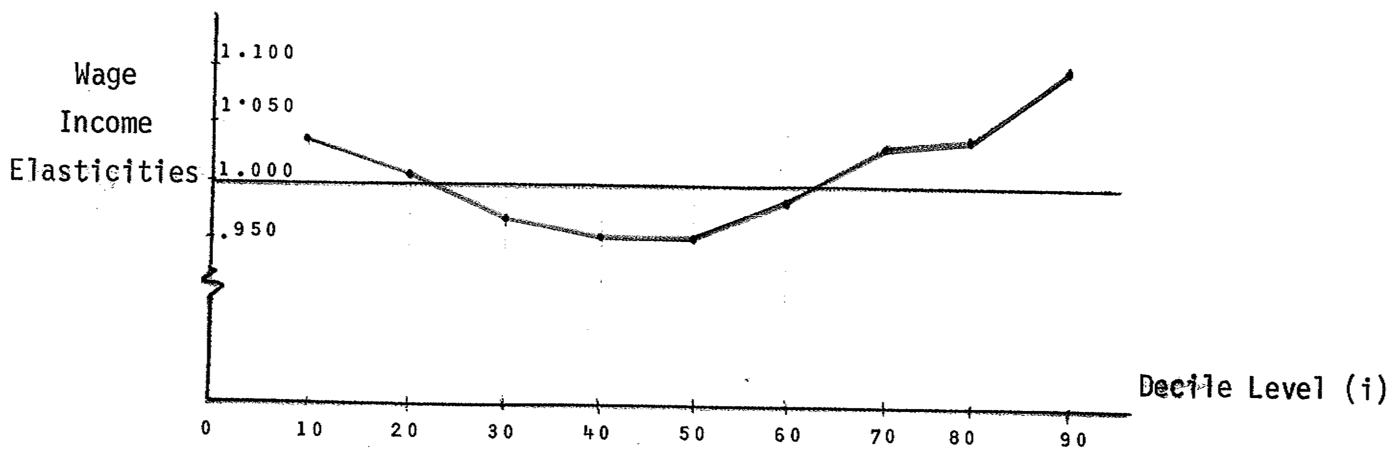
$$\frac{\partial y}{\partial W}(1,i) = \alpha_W(1,i)\ PR(1,i)\ ER(1,i) + \alpha_{UB}(1,i)\ PR(1,i)\ UR(1,i),$$

so that two contributing factors for low wage income elasticities at the bottom end of the distribution may be low participation rates for members of these decile groups and low unemployment benefits (i.e. a small $\alpha_{UB}(1,i)$) young workers receive. Furthermore, a small response of decile wage income to changes in overall wage levels (i.e. a small $\alpha_W(1,i)$) may be due to the dampening effect of relatively high unemployment among low-income secondary-aged workers. As one moves up the range of deciles, one would expect the depressing effect of these factors upon the wage income elasticities to be attenuated so that the elasticities should tend toward unity.

For the second age group, and to some extent for the third as well, the wage income elasticities appear to be slightly U-shaped across deciles (see Figure II). One possible hypothesis to account for the dip in the elasticities over the middle decile groups may be the rigidifying effects that union agreements have upon the wage incomes their members receive. Lewis [6] among others has found evidence to suggest that unions tend to make their members' money wages somewhat rigid against short-run movements

FIGURE II

Profile of Wage-Income Elasticities for Group Aged 35-44



in the incomes of non-unionized workers. And a recent study by Massad [7] indicates that the effects of unionism are concentrated in the middle deciles of the income distribution for adult males. Consequently, a slight dip in the elasticities around the middle of the second and third distributions would appear quite reasonable.

In summary, then, the estimation results appear on the whole to be quite consistent with a priori expectations and established empirical findings, and at the same time reveal some interesting new distributional detail.

IV. Comparison With Metcalf's Results

In a recent paper in the American Economic Review [8], Metcalf also obtained some estimates of mean and decile elasticities that may be compared with those in Table II. His approach, however, differed in a number of respects from the one in this paper. In particular, he estimates three equations for each of six distributions of the real incomes of families classified by labour force participation and marital status, whereas the present study concerns the money income of individual males classified by age. Thus to ease comparison, only Metcalf's results for families with a male head and wife not in the labour force will be examined. In addition, the age-specific participation and employment rate elasticities in Table II have been transformed into elasticities with respect to average participation and employment rates for all adult males.[1] A second difference between the two studies is that two of the three dependent variables in Metcalf's regressions are decile ratios $z_{10} = \hat{y}(10)/y(50)$ and $z_{90} = \hat{y}(90)/\hat{y}(50)$, rather than decile levels used in this study. Third, the estimation procedures used in the two studies differ substantially in that Metcalf follows a two-stage least squares principal components approach that does not adjust for serially correlated errors or a priori constraints on

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1. The transformation was obtained by using a constrained Parks procedure to regress simultaneously the logarithms of each of the age-specific participation and employment rates on the logarithms of their respective overall average rates subject to the explicit adding-up constraint (thus yielding estimates of the elasticity factors of PR(a) and ER(a) with respect to PR and ER respectively), and then multiplying the age-specific elasticities in Table II by these elasticity factors:

$$\text{e.g.} \quad \eta_{y(a,i),\,PR} = \eta_{y(a,i),PR(a)} \cdot \eta_{PR(a),PR}.$$

For details, see [1], pp. 149-153.

the regression coefficients. And fourth, Metcalf's results were obtained over the period 1949-1965, while the present study covers the lengthier period 1947-1970. With these differences in mind, one may compare in Table III the elasticities of z_{10}, $\hat{\mu}$, and z_{90} implicit in Metcalf's results and in the results of Table II.

TABLE III

Comparison of Metcalf's Elasticities with Those of this Study*

	$\eta_{z_{10},PR}$	$\eta_{\hat{\mu},PR}$	$\eta_{z_{90},PR}$	$\eta_{z_{10},ER}$	$\eta_{\hat{\mu},ER}$	$\eta_{z_{90},ER}$
Metcalf's Results	0	0	0	-.969	.544	-.518
Results of this Study	1.797	.673	-.140	2.964	.366	-.149
	2.651	0	0	2.363	0	0
	2.096	0	-.207	1.737	0	0

	$\eta_{z_{10},W}$	$\eta_{\hat{\mu},W}$	$\eta_{z_{90},W}$	$\eta_{z_{10},P}$	$\eta_{\hat{\mu},P}$	$\eta_{z_{90},P}$
Metcalf's Results	1.354	.933	-1.000	1.828	0	-.967
Results of this Study	-.607	.710	.134	0	0	0
	.080	.981	.150	0	0	0
	.201	.933	0	0	0	0

*$z_{10} = \hat{y}(10)/\hat{y}(50)$, $z_{90} = \hat{y}(90)/\hat{y}(50)$.

As can be seen from the table, while the income level elasticities (i.e. of $\hat{\mu}$) are of roughly similar magnitudes, the income ratio elasticities (i.e. of z_{10} and z_{90}) frequently differ substantially. To evaluate their reasonableness, it is useful to derive the decile level elasticities implicit in the ratio elasticities. Since the mean and median family income levels for the distribution Metcalf is working with do not differ greatly in size, it should not affect the results materially to assume that, as a rough approximation, the median income elasticities are equal to their respective mean income elasticities (presented in Table III). From this assumption, one can estimate Metcalf's elasticities for $\hat{y}(10)$ and $\hat{y}(90)$ implicit in Table III; and they are presented along with the corresponding elasticities of this study in Table IV. Examination of these figures reveals that both of Metcalf's wage elasticities are very different from the figure of approximately unity that one would expect. Furthermore, three of Metcalf's

TABLE IV

Comparison of Decile Income Elasticities

	$\eta_{y(10),PR}$	$\eta_{y(90),PR}$	$\eta_{y(10),ER}$	$\eta_{y(90),ER}$
Metcalf's Results	0	0	-.425	.026
Results of this Study	2.402	.465	3.114	0
	2.651	0	2.363	0
	2.303	0	1.737	0

	$\eta_{y(10),W}$	$\eta_{y(90),W}$	$\eta_{y(10),P}$	$\eta_{y(90),P}$
Metcalf's Results	2.287	-.067	1.828	-.967
Results of this Study	.063	.806	0	0
	1.033	1.103	0	0
	1.190	.989	0	0

elasticities have wrong signs according to the model specified earlier in this paper. A rise in the employment rate should raise income levels, and substantially so in the case of the bottom deciles. The wage income elasticity for the ninth decile should be positive and close to unity. And a rise in the price level, ceteris paribus, should reduce real income proportionally so that both of Metcalf's price elasticities should be approximately -1. The conclusions that follow from these rather curious results are (1) it would seem that the procedure of estimating separate equations for income levels rather than for income ratios yields much more satisfactory results; and (2) Metcalf's principal conclusions that "increases in real wages and employment rates tend... to lower the relative, but not the absolute, position of high income families" and that "increases in the price level have a parallel effect"[1] are suspect.

V. Short-Run Fluctuations in the Lorenz Curve

According to the "indirect quantile approach" outlined at the beginning of this paper, income deciles are the basic objects of regression analysis, but it is the standard inequality measures which are ultimately the subjects of concern. In particular, this paper focuses on how the shape of the Lorenz curve fluctuates in response to short-run changes in wage income, participation, and employment rates. To carry out such an analysis, one need only express the ordinates corresponding to $i = 10, 20, \ldots, 90$ on the Lorenz curve in terms of the underlying income levels $y(10), y(20), \ldots, y(90)$ and the mean, and then evaluate the derivatives of the ordinates in terms of the derivatives implied in Table I. Specifically,

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1. Metcalf [8], p. 667.

it is shown in the Appendix to this paper that if B(i) represents the proportion of total income received by the lower i percent of the distribution, the Lorenz curve ordinate

$$B(a,i) \doteq .10 \sum_{j=1}^{i/10} \left(\frac{y(a,10(j-1)) + y(a,10j)}{2\mu(a)} \right)$$

is simply the area under the mean income curve (see Figure I) over the interval [0,i]. Consequently, the wage income derivatives, say, of the Lorenz curve ordinates are

$$\frac{\partial B(a,i)}{\partial W} \doteq .10 \sum_{j=1}^{i/10} \left[\left(\frac{1}{2\mu(a)} \right) \left(\frac{\partial y(a,10(j-1))}{\partial W} + \frac{\partial y(a,10j)}{\partial W} \right) - \left(\frac{y(a,10(j-1)) + y(a,10j)}{2\mu(a)^2} \right) \left(\frac{\partial \mu(a)}{\partial W} \right) \right].$$

Calculating estimates of the decile and mean derivatives from the results in Table I and evaluating the decile and mean income levels at their means over the period 1947-1970, one obtains the figures in Table V. To illustrate the interpretation of these figures, consider the employment rate derivative of the Lorenz curve ordinate B(2,30) which can be seen to be $20.31(10)^{-2}$ or .2031. That is, a one percentage point fall in the unemployment rate for this age group would be expected to result in a rise in the income share of the lower thirty percent in the distribution of .2031 percentage points (for example, from an average income share of 12.40 percent to 12.60 percent). Similarly, the estimated increase in the income share of the lower fifty percent of income recipients in the oldest age group of a $100 increase in average wage and salary income is $3.795(10)^{-2}$ or .038 of a percentage point (thus raising the group's income share, say, from an average of 22.17 percent to 22.21 percent).

TABLE V

Estimated Derivatives of the Lorenz Curve*

Ages 20-24(a=1)

	B(1,10)	B(1,20)	B(1,30)	B(1,40)	B(1,50)	B(1,60)	B(1,70)	B(1,80)	B(1,90)
PR(1)	2.141	7.073	11.84	14.11	14.04	12.81	10.91	6.405	.602
ER(1)	1.662	5.472	8.568	9.211	8.335	7.146	5.135	2.103	-1.541
W	-1.130	-3.672	-6.288	-8.037	-8.885	-8.938	-8.064	-6.322	-3.715

Ages 35-44 (a=2)

	B(2,10)	B(2,20)	B(2,30)	B(2,40)	B(2,50)	B(2,60)	B(2,70)	B(2,80)	B(2,90)
PR(2)	6.639	18.05	25.63	28.44	28.44	28.44	28.44	28.44	28.44
ER(2)	5.800	15.24	20.31	21.74	21.74	21.74	21.74	21.74	21.74
W	.150	.409	.416	.119	-.284	-.495	-.031	1.016	3.323

Ages 55-64(a=3)

	B(3,10)	B(3,20)	B(3,30)	B(2,40)	B(2,50)	B(2,60)	B(3,70)	B(3,80)	B(3,90)
PR(3)	3.204	9.831	18.08	25.96	30.32	31.62	31.62	31.62	31.62
ER(3)	2.149	6.390	11.43	15.52	16.66	16.66	16.66	16.66	16.66
W	.398	1.133	1.850	2.815	3.795	4.292	4.643	5.733	7.427

*The partial derivatives of the Lorenz curve ordinates are obtained by multiplying each of the above figures by $(10)^{-2}$. The figures for the participation and employment rate derivatives show the effect of a one percentage point rise in each of these variables upon the ordinates (expressed in percentage terms), while the figures for the wage income derivatives show the impact of a \$100. increase in W on the Lorenz curve ordinates.

More generally, several highlights of the figures in Table V may be commented upon. First of all, participation rate increases shift the Lorenz curves up towards the "absolute equality" diagonal thus resulting in a decrease in the Gini concentration ratio, while increases in the unemployment rates have the opposite effect.

Secondly, increases in the overall wage level appear to have differing impacts depending on the age and position in the income distribution of the group being studied. For young workers, the Lorenz curve shifts out implying a more unequal distribution; for older workers it shifts in implying the opposite; and for middle-aged income recipients, the central portion of the curve (corresponding to the trough of the elasticity profile in Figure II) tends to shift down while the rest of the curve moves up toward the equality diagonal. Thus during periods of economic expansion, the falling unemployment rate and rapidly rising wage level have conflicting impacts on the shapes of the Lorenz curves for younger workers and for middle-income middle-aged workers, and have reinforcing impacts on the curves for older workers and for the rest of the middle-aged recipients. During recessionary periods with increasing unemployment rates and gradually rising wage levels, just the opposite occurs: for the first two groups, the impacts of these factors reinforce each other in lowering the Lorenz curve, while for the remaining two groups, these factors have conflicting impacts.

Third, since the Lorenz curve for the middle-aged group does not fluctuate in or out uniformly in response to an increase in W, the Gini coefficient (or indeed any single measure of overall income inequality) is not a particularly appropriate tool for measuring inequality change in such a case since it indicates only the net effect of a set of conflicting impacts. This underscores the usefulness of the approach in this paper of analyzing changes in income concentration in a disaggregated fashion since this allows one to see (a) whether or not analysis of a Gini coefficient is appropriate in a certain situation, and (b) where within a distribution the changes in income concentration occur. That is, a disaggregated approach allows one to analyze the whole pattern of inequality change within a distribution.

VI. Summary

The problem this paper has addressed is the estimation of how the pattern of inequality in the size distribution of income fluctuates with changes in aggregate economic activity, and particularly with changes in unemployment rates, participation rates, and wage and salary income.

It has been argued that a useful approach to analyzing changes in the pattern of income concentration as illustrated by the fluctuating shape of a Lorenz curve is to consider the behaviour of a set of income quantiles. This indirect procedure of first analyzing variations in the quantiles rather than directly estimating an equation to explain some aggregate index of concentration has the advantages of being very general and flexible, of aiding in the specification of regression equations, and of yielding much more detailed results than previously obtained.

A simple empirical model of the channels through which changes in aggregate economic variables affect a set of income deciles was developed with attention paid to age and income differences, and was estimated by a constrained Parks regression procedure. The estimation results turn out to be very reasonable and quite consistent with a priori expectations. One of the principal findings was that the impact pattern of changes in the level of wage and salary income differed among age groups. In particular, younger workers did not appear to benefit as much from higher overall wage and salary levels as older workers, and the wage income elasticity profile across deciles for the middle aged income recipients appeared to dip noticably over the middle deciles.

The results of this decile analysis were then compared with some analogous regression results recently obtained by Metcalf by a somewhat different approach. It was found that in his equations where the dependent variables were decile ratios rather than decile levels a number of implied elasticity coefficients were of unreasonable sign and magnitude.

The estimation results of this paper were then used to derive the implied cyclical behaviour of the Lorenz curve for each age group. In particular, it was found that, while changes in participation and unemployment rates essentially move the Lorenz curves uniformly in or out in an a priori expected fashion, changes in the level of wage and salary income have differing effects depending on the age and quantile level of a group. Consequently an analysis of disaggregated change in income concentration appears to be more informative and useful then some summary measure of overall concentration such as the Gini coefficient.

APPENDIX

Since the derivative of a Lorenz curve is a relative mean income curve illustrated in Figure I of this paper, to find an estimate of the Lorenz curve ordinate B(i), one need merely integrate the area under an estimated relative mean income curve. If $y(i)/\mu$ and $y(i-10)/\mu$ are the ordinates corresponding to two adjoining decile levels, then the area under the relative mean income curve over this decile interval may be approximated by the area of the trapezoid given by joining the ordinates by a straight line:

$$(.10)\frac{(y(i-10)/\mu) + (y(i)/\mu)}{2}.$$

Therefore the total area under the relative mean income curve over the interval [0,i] can be approximated simply by the sum of a set of adjoining trapezoidal areas:

$$\sum_{j=1}^{i/10} (.10) \left(\frac{y(10(j-1)\ ) + y(10j)}{2\mu}\right).$$

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