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A Divisible Search Model of Fiat Money

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Abstract

This paper extends the Kiyotaki-Wright search model of fiat money to allow for divisible money and goods. The extension allows me to examine the standard issues in monetary economics, such as the neutrality and super-neutrality of money, by severing the artificial link in the Kiyotaki-Wright model between the money supply and the number of money holders. It is shown that money is neutral, but not super-neutral. Money growth generates a trading opportunity effect: it changes the fraction of different agents in the economy and hence changes the probability with which agents have a successful match. In addition, money growth has a negative effect on the real money balance that is familiar in Walrasian monetary models. The balance of the two effects can imply a positive optimal money growth rate.

Keywords: search, fiat money, neutrality, super-neutrality, trading opportunity, coincidence of wants.

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1. Introduction

In a series of influential papers, Kiyotaki and Wright (1989, 1991, 1993) have analyzed a monetary model that uses random matching to represent the trading process. The model, called the search model of money, offers a novel alternative to the conventional Walrasian monetary models.[1] By decentralizing the trading process, the model abandons the Walrasian fiction and captures such realistic features of markets as the time-consuming trading process. In particular, exchanges must be *quid pro quo* in the sense that a coincidence of wants is required for each bilateral trade. The model naturally generates a transaction demand for money that has been articulated ever since Jevons. That is, when agents must spend time to look for a coincidence of wants, the use of money speeds up transactions by alleviating the difficulty of a coincidence of wants.

Given these desirable features, one hopes that the search model can be used to analyze such standard issues in monetary economics as the neutrality and super-neutrality of money.[2] However, there are indivisibility restrictions in the search model that limit its ability to analyze these issues. In the Kiyotaki-Wright model, both goods and money are indivisible so that the only form of exchange is a one-to-one swap of the indivisible inventories. Since the nominal and the real prices are unity, the model has no predictive power on prices. Subsequently Shi (1993a,b) and Trejos and Wright (1995) have relaxed the assumption of indivisible goods and introduced sequential bargaining to determine prices. However, money is still kept indivisible. That is, each money holder is restricted to holding one unit of money and exchanging the entire unit of money in trade. This restriction forces the number of agents who hold money to equal the stock of money. Since output responds to changes in the number of agents who hold money, this restriction artificially generates the non-neutrality of money. It also delivers unnatural predictions about the super

[1] Some influential Walrasian monetary models include the overlapping generations model (Wallace 1980); the cash-in-advance model (Lucas 1980) and the model of spatially separated markets (Townsend 1980).

[2] Further motivations for studying a non-Walrasian monetary model are provided by Diamond (1984) and Clower and Howitt (1992). Besides the search model, another non-Walrasian monetary model is the trading-post model with market games by Shubik (1990) (see Hayashi and Matsui (1991) and Alonso (1991) for applications). Shubik's model is suitable for explaining why certain markets do not exist, but not for analyzing the time-consuming trading process. Like Walrasian models, Shubik's model assumes instantaneous trading.

non-neutrality of money. Positive money growth, no matter how small, implies that the fraction of money holders grows exponentially so that, within a finite time, all agents in the economy will hold only money. But then no one produces, so money is useless.[3]

The first goal of this paper is to extend the Kiyotaki-Wright model to incorporate divisible money and goods; the second is to examine the neutrality and super-neutrality of money. Relaxing the restriction of indivisible money presents a technical difficulty. The matching technology generates randomness in the timing of a successful match and induces a non-degenerate distribution of money holdings across agents. Keeping track of this distribution is difficult, as Diamond and Yellen (1990) have illustrated in a search model with indivisible goods and cash-in-advance constraints, and may be intractable in the current environment. For tractability, the present paper assumes that each household consists of a continuum of members that pool their trade receipts each period. This assumption eliminates aggregate uncertainty for households and allows us to focus on symmetric equilibria where the distribution of money holdings is degenerate across households.[4] However, our results below should be interpreted more generally as ones arising from an economy with a large finite number of agents, where the matches are correlated in such a way that eliminates aggregate uncertainty in each household's matches (see, for example, Gale (1986) for a similar interpretation). We choose not to analyze this finite economy because such correlated matching technology is cumbersome to construct.

With this simplifying assumption, a basic model is constructed. The basic model severs the artificial link between the money supply and the fraction of money holders in the Kiyotaki-Wright model by exogenously fixing the latter fraction. The model reproduces two results that are familiar in Walrasian monetary models. First, money is neutral: a once-and-for-all change in the money supply has no effect on real variables. Second, money is not super-neutral: money growth reduces

[3]With this undesirable feature, previous search models such as Li (1992) have examined inflation in the following way. The government injects money to producers and confiscates money from money holders. To maintain a constant money stock, money injection is made equal to money confiscation. The rate of confiscation is interpreted as the rate of inflation, even though the total stock of money and the nominal price level are fixed.

[4]See Bergin and Bernhardt (1992) for a more formal definition of no aggregate uncentainty. Lucas (1990) also uses a similar modelling strategy to induce a degenerate wealth distribution across households.

the real money balance and hence reduces real output. We call this the *real balance effect.* As a result, the optimum quantity of money obeys the Friedman rule: the money growth rate that maximizes the steady-state utility requires the money supply to fall at the discount rate.[5]

We then extend the basic model to endogenize the fraction of money holders in the economy by allowing each household to choose its own fraction of money holders. Although money is still neutral and super non-neutral, the extension uncovers a new element of super non-neutrality. Money growth has a *trading opportunity effect*: it changes agents' probability of having a successful match. In particular, when the money growth rate is moderate and a coincidence of wants is unlikely, money growth increases the fraction of money holders in the economy. If, in addition, money holders have a small bargaining weight on the terms of trade in their matches, money growth increases the overall trading opportunity, output and welfare. In this case, the Friedman rule does not hold and the optimal money growth rate can be positive.

The trading opportunity effect arises because of an externality in the trading process. The trading opportunity depends on all households' choices of the fraction of money holders. One household's choice affects other households' probability of a successful trade, but the household ignores this externality because it views itself as negligible in the economy. The equilibrium fraction of money holders differs from the optimal level. When money holders have a small bargaining weight, each household has too few money holders and the equilibrium fraction of money holders is too low. An increase in the money growth rate from a low level induces households to trade money away by increasing the fraction of money holders. In so doing money growth increases the average trading opportunity. The trading opportunity effect is a non-Walrasian feature. If every possible trade were carried out by the Walrasian auctioneer, the chance of trade would be independent of the composition of different agents in the market.

A positive welfare effect of inflation also arises in previous search models, such as Benabou (1988, 1992), Diamond (1993) and Li (1992), but for different reasons. Benabou and Diamond generate the positive welfare effect through non-degenerate price dispersion which does not exist

[5]See Woodford (1990) for a survey on the literature of optimum quantity of money.

in the current model. Li (1992) generates the positive welfare effect by endogenizing search effort in the Kiyotaki-Wright model. Because money is indivisible in Li's model, inflation forces a change in the fraction of different agents in the market (see footnote 3). It is such a forced change, not the endogenous search intensity, which is necessary for the trading opportunity effect. Incorporating search intensity in the basic model, we show that money growth *reduces* search intensity.

The remainder of this paper is organized as follows. Section 2 specifies the basic model. Section 3 examines symmetric monetary equilibria. Section 4 extends the basic model to examine the trading opportunity effect of money growth. Section 5 compares the present model with previous ones and examines some modelling assumptions. Section 6 concludes the paper. Appendix D endogenizes search intensity. Other appendices provide necessary proofs.

2. Basic Model

2.1. Tastes and technology

Time is discrete. There are a continuum of goods, identified by points along a circle of circumference 2. There is no capital and goods are perishable across periods. There is a storable object called money which is intrinsically useless. There are a continuum of households with measure one. For tractability reasons described in the introduction, let us assume that each household consists of a continuum of members with measure one and all members share the same consumption. An exogenously fixed fraction N of members are money holders; others are producers. We endogenize the fraction in section 4.[6] Money holders cannot produce. For a household i, let A_i be the set of money holders and A_i^c the set of producers.

Household i can derive utility from goods within the arc length z from the good i. Call these goods household i's *consumption goods* and denote them by the set $D_i = \left\{j : \widehat{j\,i} \leq z\right\}$, where

[6]The fraction N can loosely be interpreted as the fraction of time that a household spends shopping with money. A formal implementation of such a dynamic interpretation is, however, problematic. First, it is cumbersome to construct a matching technology that generates no aggregate uncertainty in matching probabilities throughout the trading day. Potential measurability problems à la Judd (1985) and Feldman and Gilles (1985) have to be finessed. Second, strategies are more difficult to detail, because of the sequential nature inherited in the time interpretation. This alternative interpretation is quantitatively indistinguishable from the one adopted here, at least at the macroeconomic level. Both correspond to the fraction of agents in the economy who shop with money.

$z \in [0,1]$ is a constant. To simplify discussion, assume that all goods in D_i are equally preferred by household i. The utility of consuming q units of a good $j \in D_i$ is $u(q) = aq$ where $a > 0$ is a constant.[7] The good household i produces is determined by random shocks. In each period t, a random shock selects a good i_t^* uniformly and independently from the circle for household i to produce. Call i_t^* household i's *production good* in period t. Production takes no time but incurs a utility cost $\phi(q)$ for producing q units of goods, with $\phi(0) = 0$, $\phi' > 0$, and $\phi'' > 0$. To avoid some analytical difficulties, I follow Diamond (1984) and assume that agents never consume their production goods. This makes exchange the only way to acquire consumption goods.[8]

The random shocks that determine the production good for each household are realized at the beginning of each period. Then the household divides money balances evenly among money holders. After the division of money balances, each member of a household is randomly matched to one agent from other households. Depending on the match type, matched agents decide whether to trade. Two types of trade are possible: barter and monetary trade. The terms of trade satisfy the bargaining incentives described in subsection 2.3. After exchange, members of each household bring their receipts back to the household. Then the household allocates goods evenly to members for consumption. After consumption, the household receives a lump-sum monetary transfer τ. The lump-sum transfer keeps the money supply per household, denoted $\hat{M}$, growing at a constant (gross) rate γ. That is, $\tau_t = (\gamma - 1)\hat{M}_t, \gamma > 0$. After the transfer, time proceeds to the next period and the sequence of events repeats.

Money and goods are divisible. In particular, a money holder can trade any fraction of his money holdings. The divisibility of money improves upon previous search models such as Shi (1993a,b) and Trejos and Wright (1995). Allowing divisible goods improves upon the model of Diamond and Yellen (1990), which has divisible money but requires agents to exchange a fixed

[7]Non-uniform tastes over D can be modelled along the line of Kiyotaki and Wright (1991). The linear utility function is adopted to ease the exposition of the results.

[8]If agents are allowed to consume their own products, they can be self-sufficient with probability z. The probability for an agent to meet a desirable trader depends on how close his production good is to his consumption goods. This makes the analytical characterization intractable within the current framework. However, there are other variations of the model which can be used to address this self-sufficiency problem. Shi (1993c) provides such a variation and shows that self-sufficiency does not preclude valuable fiat money.

quantity of goods in each exchange.

We assume that households face no aggregate uncertainty in their matches so that the distribution of different types of matches for each household is almost surely non-random, although each member in the household is uncertain about the kind of agent he will meet. Then the parameter z captures a coincidence of wants. Consider two randomly-selected producers from households i and $-i$. A producer from household i can produce consumption goods for household $-i$ iff $i^* \in D_{-i}$; a producer from household $-i$ can provide consumption goods for household i iff $(-i)^* \in D_i$. Each event occurs with probability z so that the two households have a double coincidence of wants with probability z^2.

2.2. Household's decision problem

Consider the decision problem in household i. Denote an arbitrary household other than i by $-i$. Let j be a typical member of household i and $-j$ be the member of household $-i$ with whom agent j is matched. There are three types of matches that result in trade, as depicted in Figure 1. The arrows indicate the flow of consumption goods. If the arrow is unidirectional, a monetary match takes place; if the arrow goes in both directions, a barter match takes place.

Figure 1.

Agents in household i who successfully trade can be classified further into sets I_b, I_p and I_m. The set I_b consists of matched producers who successfully barter; I_p consists of matched producers who successfully trade goods for money and I_m consists of matched money holders who successfully trade money for goods:

$$\begin{aligned} I_b &= \{j \in A_i^c : -j \in A_{-i}^c; i^* \in D_{-i}, (-i)^* \in D_i\}, \\ I_p &= \{j \in A_i^c : -j \in A_{-i}; i^* \in D_{-i}\}, \\ I_m &= \{j \in A_i : -j \in A_{-i}^c; (-i)^* \in D_i\}. \end{aligned}$$

Let the measure of agents in a set I be $\mathcal{M}(I)$. Under the assumption of no aggregate uncertainty, we have:

$$\mathcal{M}(I_b) = z^2(1-N)^2; \ \mathcal{M}(I_p) = zN(1-N) = \mathcal{M}(I_m).$$

For example, the measure of I_b is $z^2(1-N)^2$ because there are $1-N$ producers in each household and each producer can successfully barter with probability $z^2(1-N)$.

In each period t, household i chooses its consumption C_{it} and future money balance M_{it+1}, taking as given the terms of trade prevailing in the economy which are denoted $(\hat{q}^m, \hat{q}^b, \hat{L})$. In a monetary match, the money holder exchanges $\hat{L}$ units of money for $\hat{q}^m$ units of goods; in a barter match, $\hat{q}^b$ units of goods are exchanged by each party. We assume, and emphasize by the hat, that these terms of trade are taken as given by households, because each household and its ex ante (before match) influence on the terms of trade are negligible. Of course, each agent has bargaining power on the match-specific terms of trade because he is facing only one agent. We examine this ex post bargaining power in subsection 2.3. Although the terms of trade $(\hat{q}^m, \hat{q}^b, \hat{L})$ must, *in equilibrium*, coincide with the match-specific terms of trade, the household's decision problem is formulated below for any given non-negative triple $(\hat{q}^m, \hat{q}^b, \hat{L})$:[9]

$$(PH1) \quad \max_{(C_{it},\, M_{it+1})} \sum_{t=0}^{\infty} \beta^t [u(C_{it}) - \Phi_{it}] \quad s.t.$$

$$C_{it} \le Y_{it} \equiv \int_{j \in I_{mt}} \hat{q}_t^m(j)dj + \int_{j \in I_{bt}} \hat{q}_t^b(j)dj; \tag{2.1}$$

$$\Phi_{it} = \int_{j \in I_{pt}} \phi(\hat{q}_t^m(j))dj + \int_{j \in I_{bt}} \phi(\hat{q}_t^b(j))dj; \tag{2.2}$$

$$M_{it+1} \le M_{it} + \tau_t + \int_{j \in I_{pt}} \hat{L}_t(j)dj - \int_{j \in I_{mt}} \hat{L}_t(j)dj; \tag{2.3}$$

$$\hat{L}_t(j) \le M_{it}/N, \quad \forall j \in I_{mt}. \tag{2.4}$$

Condition (2.1) gives the expected trade receipts of consumption goods from monetary exchanges (the first integral) and from barter (the second integral). Condition (2.2) specifies the cost of production in monetary exchanges (the first integral) and in barter (the second integral). Condition (2.3) specifies the law of motion of the household's money balance, the first integral being the amount of money received by producers in monetary exchanges and the second integral being the

[9] I am grateful to a referee for insisting on the clarification of these assumptions on the terms of trade. These assumptions resemble those in search models of unemployment such as Pissarides (1990), where each firm takes the wage rate as given when it decides on the levels of labor and capital employment but has bargaining power on the wage rate with its matched workers.

amount of money paid by money holders. Condition (2.4) is a cash-in-advance constraint, stating that each money holder who has a successful trade is constrained by his money holdings. Because matching takes place simultaneously, a money holder who finds it desirable to make a transaction with more than his money holdings cannot make use of other members' money balances. That is, some money holders may find the constraint binding even though other money holders in the household do not use their money holdings as a result of unsuitable matches. This feature differs from other simultaneous trading models such as Engineer and Bernhardt (1991), Alonso (1991) and Hayashi and Matsui (1991), where the cash-in-advance constraint never binds if there are left-over money balances in the household.

With the assumption of no aggregate uncertainty, the formulation of a household as a continuum of members simplifies the maximization problem to one of certainty. The variables C_{it}, Φ_{it} and M_{it+1} represent the household's actual consumption, cost of production and the future money stock despite the fact that each household member is randomly matched. This simplification also avoids solving for the inventory distribution of money holdings. In the symmetric equilibrium described later, all households hold the same quantity of money.

Since the terms of trade are exogenous to the household, so are Y_{it} and Φ_{it}. The household's optimal consumption decision is simply $C_{it} = Y_{it}$. To characterize the optimal condition for M_i, let ω_{it} and $\lambda_{it}(j)$ be the current-value Lagrangian multipliers in $(PH1)$ of the constraints (2.3) and (2.4) respectively. Note that (2.4) holds for every $j \in I_{mt}$. We have:

$$\omega_{it} = \beta[\omega_{it+1} + \frac{1}{N}\int_{j \in I_{mt}} \lambda_{it+1}(j)dj]; \tag{2.5}$$

$$\lambda_{it}(j)[M_{it}/N - L_{it}(j)] = 0 \quad \forall j \in I_{mt}. \tag{2.6}$$

(2.5) is the optimal condition for M; (2.6) the Kuhn-Tucker condition on (2.4).

2.3. Bargaining solutions

Let us suppress the time index in this subsection. When an agent is matched with another agent, he has an incentive to bargain over how much to trade. To ensure that the terms of trade

$(\hat{q}^m, \hat{q}^b, \hat{L})$ are consistent with this ex post incentive, they must coincide with the terms of trade that would result from bilateral bargaining. Denote the latter by (q^m, q^b, L). When a member is engaged in bargaining, he takes as given the economy-wide and household-wide variables. In particular, he takes the Lagrangian multipliers (ω, λ) as given because these multipliers depend only on the household variables (C, M) and the economy-wide variables $(\hat{q}^m, \hat{q}^b, \hat{L})$.

We assume that the bargaining outcomes maximize the Nash product $S_i^{\Theta} S_{-i}^{1-\Theta}$ where S is the surplus from trade and $(\Theta, 1-\Theta)$ are the weights for the two agents.[10] Because barter involves two symmetric agents, it is reasonable to assume $\Theta = 1/2$ in a barter match. For a monetary trade, however, there is no a priori reason for selecting a particular value for Θ. In this case, let $\Theta = \theta \in (0,1)$ and term θ the *bargaining weight* of the money holder. We assume that θ is a constant for now and explore the implications of endogenizing θ in section 5.

Let us first examine a barter match between two producers from households i and $-i$. The two bargain over the quantities of goods to be produced, (q_i^b, q_{-i}^b). In the sense made precise in Appendix A, the trade increases household i's utility by $[aq_{-i}^b - \phi(q_i^b)]$ and increases the partner household's utility by $[aq_i^b - \phi(q_{-i}^b)]$, where a is the marginal utility of consumption. The pair (q_i^b, q_{-i}^b) is determined by

$$(Pb) \quad \max_{(q_i^b, q_{-i}^b)} \left[aq_{-i}^b - \phi(q_i^b)\right]^{1/2} \left[aq_i^b - \phi(q_{-i}^b)\right]^{1/2}.$$

The solution to (Pb) is

$$q_i^b = q_{-i}^b = q^N \equiv \phi'^{-1}(a) \quad \forall i. \tag{2.7}$$

Barter is ex post efficient in the sense that the marginal cost of production equals the marginal utility of consumption.

Now consider a monetary trade between a money holder from household i and a producer from household $-i$. The money holder pays L_i units of money for q_{-i}^m units of consumption goods.

[10] In a stationary environment, the Nash solution coincides with the solution to some non-cooperative sequential bargaining game. (For a general discussion see Osborne and Rubinstein (1990); for an application in search models of money, see Shi (1993a,b) and Trejos and Wright (1995)). Although the coincidence does not hold strictly in a non-stationary environment, the Nash solution can be viewed as an approximation around the steady state of the underlying more complicated non-cooperative characterizations (see Coles and Wright (1994)).

The trade increases household $i's$ utility by $[aq^m_{-i} - (\omega_i + \lambda_i)L_i]$ and the partner household's utility by $[\omega_{-i}L_i - \phi(q^m_{-i})]$. The pair (L_i, q^m_{-i}) solves

$$(Pm) \quad \max_{(L_i, q^m_{-i})} [aq^m_{-i} - (\omega_i + \lambda_i)L_i]^\theta [\omega_{-i}L_i - \phi(q^m_{-i})]^{1-\theta},$$

where $(\omega_i, \omega_{-i}, \lambda_i)$ are taken as given. The solutions for (q^m, L) are given by:

$$\phi'(q^m_{-i}) = \frac{a\omega_{-i}}{\omega_i + \lambda_i}, \tag{2.8}$$

$$L_i = [\theta\phi(q^m_{-i}) + (1-\theta)q^m_{-i}\phi'(q^m_{-i})]/\omega_{-i}. \tag{2.9}$$

Condition (2.9) states that the producer's surplus $(\omega_{-i}L_i - \phi)$ is a fraction $1-\theta$ of the total surplus $(aq^m_{-i} - \phi)$. Condition (2.8) deserves some attention. If $\omega_i = \omega_{-i}$ as in a symmetric equilibrium and if $\lambda_i > 0$, the condition implies $\phi'(q^m) < a$ so that $q^m < q^N$. That is, fewer goods are exchanged in a monetary match than in a barter match. This is because a barter match involves two symmetric agents but a monetary match involves two asymmetric agents. In a monetary match, the producer's marginal valuation of money, ω_{-i}, is lower than the money holder's marginal cost of money, $\omega_i + \lambda_i$, whenever $\omega_i = \omega_{-i}$ and $\lambda_i > 0$.[11] By exchanging money for goods, the money holder not only gives up the purchasing power commanded by money in the future, but also suffers a tighter cash-in-advance constraint. In contrast, by accepting money the producer gains only the purchasing power of money. He cannot use the receipts to ease the cash-in-advance constraint that is currently binding in the monetary matches experienced by the money holders of his household. Because the producer values money by less than the money holder, he is only willing to sacrifice up to the margin, $\phi'(q^m)$, below the money holder's marginal valuation of goods, a.

The result $q^m < q^N$ is also a feature of search models with indivisible money such as Shi (1993a,b) and Trejos and Wright (1995). However, the underlying reason is different there and lies in the difference between the two agents' timing of consumption. That is, by exchanging money for goods the money holder can consume immediately; by exchanging goods for money

[11]Despite such difference, there is a room for trade becasue, at $(L_i, q^m_{-i}) = (0,0)$, the money holder values the producer's good more than his own money and the producer values money more than his own good.

the producer must await the next suitable match to exchange money for good and then consume. This difference in the timing of consumption is absent in the present model because every agent consumes in each period.

It is worth emphasizing that agents' threat points in the bargaining problem (Pm) are endogenous. They depend on $(\omega_i, \omega_{-i}, \lambda_i)$, which are endogenous in equilibrium as conditions (2.5) and (2.6) illustrate. Therefore, although the bargaining weights are assumed constant, the bargaining outcomes respond endogenously to such market pressures as the relative scarcity of goods and money in the market.

Finally, the bargaining problem between a producer from household i and a money holder from household $-i$ is symmetric to (Pm), with the subscripts i and $-i$ being interchanged. Note that the member index j is suppressed in (2.7)-(2.9). If $\lambda(j) = \lambda(j')\ \forall j, j' \in I_m$ as in a symmetric equilibrium described below, (2.7)-(2.9) hold for all j.

3. Symmetric Equilibrium

We will focus on symmetric monetary equilibrium, defined below:[12]

Definition 3.1. *A symmetric monetary equilibrium is a collection* $\{C_{it}, M_{it}, \omega_{it}, \lambda_{it},\ L_t, q_t^b, q_t^m, \hat{L}_t, \hat{q}_t^b, \hat{q}_t^m\}_{i,t}$ *with* $q^m > 0$ *such that*

(i) $C_{it} = C_t, M_{it} = M_t, \omega_{it} = \omega_t, \lambda_{it} = \lambda_t\ \forall i$ *and* $\lambda(j) = \lambda(j')\ \forall j, j' \in I_{mt}$;

(ii) (C_{it}, M_{it}) *solve* $(PH1)$ *for given* $(\hat{L}_t, \hat{q}_t^b, \hat{q}_t^m)$;

(iii) $(\hat{L}_t, \hat{q}_t^b, \hat{q}_t^m) = (L_{it}, q_{it}^b, q_{it}^m)$ *for all* i, *where* (q_i^b, q_{-i}^b) *solve* (Pb) *and* (q_{-i}^m, L_i) *solve* (Pm) *for given* $(C_t, M_t, \omega_t, \lambda_t)$;

(iv) $M_t = \hat{M}_t$.

Condition (i) is a symmetry condition. Together with (2.8)-(2.9), it implies that (q_i^m, q_i^b, L_i) are the same for all i. Thus I will suppress the household subscript i. Condition (ii) is self-explanatory. Condition (iii) requires that the terms of trade $(\hat{L}, \hat{q}^b, \hat{q}^m)$ be consistent with the

[12]Besides the monetary equilibrium, a non-monetary equilibrium corresponds to $q^m = \hat{q}^m = 0$ and always exists if all agents believe that money has no value.

after-match bargaining incentive. Condition (iv) requires that all supplied money be demanded. Because of conditions (iii) and (iv), I will suppress the hat on the variables. Then we have:

$$\begin{aligned} C_t &= z(1-N)[Nq_t^m + z(1-N)q^N], \\ \Phi_t &= z(1-N)[N\phi(q_t^m) + z(1-N)\phi(q^N)], \\ M_{t+1} &= M_t + \tau_t = \gamma M_t. \end{aligned} \tag{3.1}$$

Let us further restrict attention to the equilibrium where constraint (2.4) binds (i.e. $\lambda_t > 0$) and call such equilibrium a *binding monetary equilibrium.*[13] Since (2.8) implies

$$\lambda_t = \left(\frac{a}{\phi'(q_t^m)} - 1\right)\omega_t, \tag{3.2}$$

a binding equilibrium exists only if $q_t^m < q^N$. To characterize the binding equilibrium, define $g : R_+ \to R_+$ by

$$g(q) = (1-\theta)q\phi'(q) + \theta\phi(q).$$

Clearly g is an increasing function and hence invertible.

Proposition 3.2. *A symmetric binding equilibrium is characterized by (3.1), (3.2), $L_t = M_t/N$, $\omega_t = N[\theta\phi(q_t^m) + (1-\theta)q_t^m\phi'(q_t^m)]/M_t$ and the following equation:*

$$q_t^m = G(q_{t+1}^m), \quad G(q) \equiv g^{-1}\left(g(q)\frac{\beta}{\gamma}\left[1 + z(1-N)\left(\frac{a}{\phi'(q)} - 1\right)\right]\right). \tag{3.3}$$

Proof. Set $\lambda_t > 0$. Then (2.6)$\Rightarrow L_t = M_t/N$ and (2.9)$\Rightarrow \omega_t$. (3.2) and (2.5) imply

$$\frac{\omega_t}{\omega_{t+1}} = \beta\left[1 + z(1-N)\left(\frac{a}{\phi'(q_{t+1})} - 1\right)\right].$$

Substituting for ω and noticing $M_{t+1}/M_t = \gamma$, one obtains (3.3). Since g is invertible, G is well-defined. ■

[13] It can be verified that there exist a continuum of monetary equilibria in which $\lambda_t = 0\ \forall t$. In those equilibria, $q_t^m = q_t^b = q^N\ \forall t$, $\omega_t = \beta\omega_{t+1}$ and the nominal price of goods $p_t \equiv L_t/q_t^m$ falls over time at the rate $1-\beta$. These equilibria differ from each other only in the initial price (or equivalently in ω_0). Each of these equilibria is a self-fulfilling bubble on money in the following sense. Agents keep an increasing proportion of their money as a store of value, expecting the value of money ω to increase. When an increasing proportion of money is withdrawn from exchange, the remaining money commands a higher purchasing power that supports the expectation. We choose to focus on the binding equilibrium because its price behavior is more realistic.

The key to the existence of a binding monetary equilibrium is a solution to equation (3.3). Inspection of (3.3) confirms the following corollary on the existence of the steady state:[14]

Corollary 3.3. *The binding monetary steady state, denoted by q^*, exists if and only if $\gamma > \beta$, where q^* is given by*

$$\phi'(q^*) = a\left(1 + \frac{\gamma/\beta - 1}{z(1-N)}\right)^{-1}. \tag{3.4}$$

The condition $\gamma > \beta$ is required for $\phi'(q^*) < a$ and hence for $\lambda > 0$. Notice that q can be interpreted as the real money balance transacted in the monetary trade. Then (3.4) implies a negative effect of money growth on the real balance: An increase in the money growth rate γ reduces q^*. Producers offer fewer units of goods for money because money growth reduces the value of money. The negative real balance effect implies super non-neutrality. By reducing the real money balance, money growth reduces consumption and output in the steady state. It can be shown that setting $\gamma = \beta$ maximizes steady-state utility. That is, the Friedman rule holds.

The equilibrium dynamics crucially depend on the slope of the function G at the binding monetary steady state. Compute this slope as follows:

$$G'(q^*) = 1 - \left[1 - \frac{\beta}{\gamma}(1 - z(1-N))\right] \frac{\phi''(q^*)}{\phi'(q^*)} \frac{g(q^*)}{g'(q^*)}. \tag{3.5}$$

Clearly $G'(q^*) < 1$. If also $G'(q^*) > 0$, as in the example where $\phi = q^\sigma$ $(\sigma > 1)$, then for any $q \in (0, q^N)$ the sequence $\{G^s(q)\}_{s=0}^{\infty}$ converges to the binding monetary steady state. If $G'(q^*) < 0$, limit cycles can occur (see Grandmont (1985)).

4. An Extension with Endogenous N

4.1. Description and equilibrium

Endogenizing N allows the probability of a successful trade to depend on household's choices. If money growth can affect such choices, then it can affect trading opportunities. To begin, let each household choose its fraction of money holders and let n_i denote the fraction of money holders in

[14]Since $g(0) = 0$, $q = 0$ is also a steady state of (3.3), which corresponds to the non-monetary equilibrium.

household i. The fraction of money holders in other households is denoted by N. In a symmetric equilibrium, of course, $n_i = N$. Each money holder in household i holds a money balance M_{it}/n_{it} in period t. We have:

$$\mathcal{M}(I_{bt}) = z^2(1-N_t)(1-n_{it});\ \mathcal{M}(I_{pt}) = zN_t(1-n_{it});\ \mathcal{M}(I_{mt}) = z(1-N_t)n_{it}.$$

Keeping in mind the distinction between n_i and N, we can formulate the household's maximization problem similarly to $(PH1)$, with n_i as an additional choice variable. Conditions similar to (2.5) and (2.6) can be derived. With symmetry, the choice of n is reduced to the following problem:

$$\begin{aligned}(Pn) \quad & \max_{(n_t)} \sum_{t=0}^{\infty} \beta^t \{aC(n_t, N_t) - \Phi(n_t, N_t) + \lambda_t z(1-N_t)n_t(M_t/n_t - L_t) \\ & +\omega_t [M_t + \tau_t - M_{t+1} + zN_t(1-n_t)L_t - z(1-N_t)n_t L_t]\}\end{aligned}$$

where

$$\begin{aligned} C(n_t, N_t) &= z(1-N_t)[n_t q_t^m + z(1-n_t)q^N]; \\ \Phi(n_t, N_t) &= z(1-n_t)[N_t\phi(q_t^m) + z(1-N_t)\phi(q^N)]. \end{aligned}$$

The symbols $C(n, N)$ and $\Phi(n, N)$ emphasize that these variables depend on (n, N) in equilibrium. The variables (N, q^m, q^N, L) are taken as given by the household in (Pn) where the hat on the terms of trade is suppressed. The derivative of the maximand with respect to n_t is $z(\omega_t L_t - \phi(q_t^m))[(1-N_t)\Gamma_t - 1]$, where

$$\Gamma_t = \frac{a(q_t^m - zq^N) - \phi(q_t^m) + z\phi(q^N) - \lambda_t L_t}{\omega_t L_t - \phi(q_t^m)}. \tag{4.1}$$

Since $\omega_t L_t - \phi(q_t^m) > 0$ (see (2.9)), n_t is given by the following best response correspondence:

$$n_t \begin{cases} = 0, & if\ \ N_t > 1 - \Gamma_t^{-1}\ \ or\ \ \Gamma_t < 1 \\ = 1, & if\ \ N_t < 1 - \Gamma_t^{-1} \\ \in [0,1], & if\ \ N_t = 1 - \Gamma_t^{-1}\ \ and\ \ \Gamma_t \geq 1. \end{cases} \tag{4.2}$$

This correspondence intuitively states that a household is willing to shop with money only if money holders are likely to trade, i.e., if $z(1-N_t)$ is large.

The bargaining solutions are still given by (2.7)-(2.9). We can redefine a symmetric equilibrium as in section 3 by adding n_{it} into the household's choice set and imposing an additional restriction $n_{it} = N_t \in (0,1]\ \forall i,t$. As before, I examine only the binding monetary equilibrium which requires $\lambda_t > 0$ and hence $q_t^m < q^N\ \forall t$.

Proposition 4.1. *A symmetric binding monetary equilibrium with endogenous N is characterized by (2.7), (3.1), $L_t = M_t/N_t$, $\omega_t = N_t g(q_t^m)/M_t$, $n_t = N_t = 1-\Gamma_t^{-1}$, and the following:*

$$N_t g(q_t^m) = N_{t+1} g(q_{t+1}^m) \cdot \frac{\beta}{\gamma}\left\{1 + z(1-N_{t+1})\left(\frac{a}{\phi'(q_{t+1}^m)} - 1\right)\right\}. \tag{4.3}$$

Such an equilibrium exists only if

$$q_t^m - \phi(q_t^m)/\phi'(q_t^m) > \frac{z}{\theta}\left[q^N - \phi(q^N)/a\right] \quad \forall t. \tag{4.4}$$

Proof. Let $n_t = N_t\ \forall t$. By (4.2), $n_t = N_t = 0$ if $\Gamma_t \le 1$ and $n_t = N_t = 1-\Gamma_t^{-1}$ if $\Gamma_t > 1$. Thus a monetary equilibrium requires $\Gamma_t > 1$. Using the conditions (2.7)-(2.9) to substitute for ω and λ in (4.1), we can transform Γ_t into a function of q_t^m:

$$\Gamma(q_t^m) = 1 + \frac{\theta}{1-\theta}\cdot\frac{a}{\phi'(q_t^m)} - \frac{z}{1-\theta}\cdot\frac{aq^N - \phi(q^N)}{q_t^m\phi'(q_t^m) - \phi(q_t^m)}. \tag{4.5}$$

Thus $\Gamma_t > 1 \Leftrightarrow$(4.4). Finally, the condition (4.3) comes from (2.5), (3.2) and $N_t = 1-\Gamma_t^{-1}$. ■

Remark 1. *Because $q^m < q^N$ and $q - \phi(q)/\phi'(q)$ is an increasing function, the condition $z < \theta$ is necessary for (4.4). Define q_0 by the following equation:*

$$q_0 - \phi(q_0)/\phi'(q_0) = \frac{z}{\theta}\left[q^N - \phi(q^N)/a\right]. \tag{4.6}$$

Then $z < \theta \Leftrightarrow q_0 < q^N$, and (4.4)$\Leftrightarrow q^m > q_0$.

As before our focus is on the steady state. To determine the steady state value of q^m, write N_t as $N(q_t^m) = 1 - [\Gamma(q_t^m)]^{-1}$ where $\Gamma(q_t^m)$ is defined by (4.5). From (4.3), the steady state value of q^m, denoted as q^*, is given by:

$$F(q^*) \equiv z[1 - N(q^*)]\cdot\left(\frac{a}{\phi'(q^*)} - 1\right) = \frac{\gamma}{\beta} - 1. \tag{4.7}$$

$F(\cdot)$ is a decreasing function as shown in the proof of the following proposition (see Appendix B).

Proposition 4.2. *There exists a binding monetary steady state if and only if* $z < \theta$ *and*

$$\beta < \gamma < \beta\left[1 + z\left(\frac{a}{\phi'(q_0)} - 1\right)\right] \equiv \bar{\gamma}. \tag{4.8}$$

Proposition 4.2 states that the money growth rate γ must be moderate for a binding monetary steady state to exist. Too high a money growth rate severely decreases the purchasing power of money and drives money out of the economy; too low a money growth rate makes the cash-in-advance constraint non-binding.

The condition $z < \theta$ states that a coincidence of wants must be unlikely for a monetary steady state to exist. Intuitively, money will not be valuable if agents have a good chance to barter. To further explain the necessity of the condition, consider the special environment where shopping with money is most desirable for household i. This is when producers are the easiest to find ($N = 0$) and when a monetary trade exchanges as many units of good as barter ($q^m = q^N$). Clearly for a monetary equilibrium to exist, household i must be willing to allocate some members to trade with money in this special environment. In this environment, allocating a member to trade with money increases the chance of trade by $z(1-z)$ and hence increases the household's utility by a margin $z(1-z)aq^N$. The allocation also saves some production cost, $z^2\phi(q^N)$. The total utility gain is $z[(1-z)aq^N + z\phi(q^N)]$. The total cost of such allocation is the value of the money balance, ωL, multiplied by the chance of a monetary trade, z (note that $\lambda = 0$ in this special case). According to (2.9), $\omega L = (1-\theta)aq^N + \theta\phi(q^N)$ in this special environment. Therefore it pays for household i to allocate some members to trade with money only if $z < \theta$.

Because q^m depends on z, the equilibrium effect of z on the existence of equilibrium is more complicated than in the above example. In particular, a smaller z reduces the probability $z(1-N)$ of having a successful monetary trade and hence reduces q^m. This terms-of-trade effect reduces the benefit of trading with money over trading with goods, since the terms of trade in barter do not vary with z. When a coincidence of wants is unlikely, it is possible that making it easier can help a monetary steady state to exist. That is, $\bar{\gamma}$ may increase with the coincidence of wants parameter, z, when z is very small. For example, when $\phi(q) = q^\sigma$ ($\sigma > 1$), $\bar{\gamma}$ increases in z when

$\sigma < 2$ and $z < \theta(2-\sigma)^{1/(\sigma-1)}$. (Notice that q_0 depends positively on z.)

As in the basic model, money growth generates a negative real balance effect. This is clear from (4.7) because $F(\cdot)$ is a decreasing function. In contrast to the basic model, money growth also changes agents' trading opportunities by changing the fraction N. We now explore this trading opportunity effect.

4.2. Trading opportunity effect

Money growth can increase the fraction of money holders in the market. To facilitate arguments in the remainder of section 4, we will employ the functional form $\phi(q) = q^\sigma$ $(\sigma > 1)$. Denote $x = q^*/q^N$ and rewrite F and N as functions of x instead of q:

$$F(x) = z(x^{1-\sigma} - 1)(1 - N(x)); \; N(x) = 1 - [1 + \frac{\theta}{1-\theta}x^{1-\sigma} - \frac{z}{1-\theta}x^{-\sigma}]^{-1}. \tag{4.9}$$

The analysis in the last subsection implies $F'(x) < 0$ and $\partial x/\partial\gamma < 0$. The conditions for the existence of a binding monetary steady state are

$$z < \theta \quad and \quad \beta < \gamma < \overline{\gamma} = \beta(1 + \theta^{\sigma-1}z^{2-\sigma} - z).$$

The condition on γ can be equivalently written in terms of x as $x \in (z/\theta, 1)$. Since $\partial x/\partial\gamma < 0$, the fraction of money holders increases with money growth iff $N'(x) < 0$. Differentiating (4.9), we have $N'(x) < 0 \Leftrightarrow 1 > x > z\sigma/[\theta(\sigma-1)]$. Since $F'(x) < 0$ and $\gamma/\beta - 1 = F(x)$, we have:

$$N'(x) > 0 \Leftrightarrow z < \theta(\sigma-1)/\sigma \quad and \quad \gamma < \beta[1 + F(\frac{z\sigma}{\theta(\sigma-1)})].$$

The interpretation of the above result is as follows. Because a higher money growth rate reduces the purchasing power of money, it increases the incentive for the household to trade money away quickly by choosing a large fraction of its members as money holders. However, increasing money holders is also costly because it reduces the number of producers and so reduces the quantity of consumption goods that the household acquires through barter. Only when the benefit of increasing money holders dominates the cost does the household increase its money holders to respond to a higher money growth rate. This requires that the coincidence of wants be unlikely and the money growth rate be low. Unlikely coincidence of wants makes barter difficult and hence reduces the amount of consumption goods acquired from barter that must be sacrificed if the household increases its money holders. A low money growth rate induces a high purchasing power of money and hence increases the amount of consumption goods that can be acquired by the increased money holders.

Through N a change in the monetary growth rate affects agents' trading opportunities. However, the effects are non-uniform across different types of agents. To examine the trading opportunity effect, let us measure the *average trading opportunity* by the total number of successful trade that a household has each period in equilibrium. This measure is

$$\pi = zN(1-N) + z^2(1-N)^2 + z(1-N)N = z(1-N)[2N + z(1-N)].$$

It is possible that money growth increases this average trading opportunity, thus raising output and welfare. To confirm, compute the steady state aggregate output Y and utility V as follows:

$$Y = C = z(1-N)[z(1-N) + Nx]q^N;$$

$$V = z(1-N)\{(\sigma-1)z(1-N) + N(\sigma x - x^\sigma)\}\phi(q^N)/\beta.$$

The proof of the following proposition in given in Appendix C.

Proposition 4.3. *When $\sigma > 2, \theta < \frac{1}{2}(1-\frac{1}{\sigma-1})$ and*

$$z < \theta \cdot \frac{(\sigma-1)(1-2\theta)-1}{\sigma(1-2\theta)-1}, \tag{4.10}$$

steady state output and utility increase with the money growth rate γ when γ is close to β and decrease with γ when γ is close to the upper bound $\bar{\gamma}$. There is $\gamma_0 \in (\beta, \bar{\gamma})$ that maximizes the steady state utility. The above conditions are sufficient for $dN/d\gamma > 0$ and $d\pi/d\gamma > 0$ when γ is close to β. On the other hand, if $\theta > 1/2$ and z is close to θ then Y and V monotonically increase with γ for all $\gamma \in [\beta, \bar{\gamma}]$. In this case the optimal money growth rate is $\bar{\gamma}$, which implies $N = 0$.

Proposition 4.3 states that the optimal money growth rate can deviate from the Friedman rule. The optimal net money growth rate can be positive, as shown later with some parameter values.

Before explaining the conditions in the above proposition, let us explain why an increase in the money growth rate can raise welfare. The positive welfare effect arises because the equilibrium fraction of money holders is inefficiently low and an increase in the money growth rate can increase this fraction. In turn, the equilibrium fraction of money holders is inefficiently low because there is an externality in the trading process. That is, each household ignores the effect that its choice of n has on the trading probability another household has.

To be more specific about the externality, let us reconsider the household's choice of n characterized by problem (Pn) in subsection 4.1. The externality is represented by the effect of N that is taken as given by the household. There are two ways in which the externality affects the household's utility. First, it affects the level of consumption and the production cost. By ignoring this effect, the household exaggerates the effect of n on the steady state utility by

$$\begin{aligned}\Delta_1 &\equiv -\frac{\partial[u(C(n,N)) - \Phi(n,N)]}{\partial N} \mid_{n=N} \\ &= z\left[aNq^m + (1-N)\phi(q^m) + z(1-N)(aq^N - \phi(q^N))\right].\end{aligned}$$

Second, N affects the household's choice of money balance through the term associated with ω_t in (Pn).[15] By ignoring this effect, the household overlooks the importance of n by

$$\Delta_2 \equiv \omega\frac{\partial}{\partial N}[zN(1-n)L - z(1-N)nL] \mid_{n=N} = z\omega L = z[\theta\phi(q^m) + (1-\theta)q^m\phi'(q^m)].$$

Overall, the externality affects the household's steady state utility by $(\Delta_2 - \Delta_1)/(1-\beta)$. The equilibrium value of N is deficient if and only if $\Delta_2 > \Delta_1$.

Now we can interpret the conditions in Proposition 4.3. First, θ must be low to induce a positive optimal money growth rate. When money holders have a low bargaining weight, households tend to allocate few members to trade with money. In this case, the equilibrium fraction of money holders is deficient and an increase in the fraction of money holders improves output and welfare. In fact, when $\gamma = \beta$, the equilibrium fraction of money holders is $N = (\theta - z)/(1-z)$, which is low if θ is low, and $\Delta_2 > \Delta_1$ if and only if $\theta < 1/2$.

[15] Although N also enters problem (Pn) through the term associated with λ, this effect is zero at the margin because $\lambda_t(M_t/n_t - L_t) = 0$.

Second, a coincidence of wants must be unlikely in order to induce a positive optimal money growth rate. That is, z must be small. Only when a coincidence of wants is unlikely does an increase in the fraction of money holders increase the average trading opportunity. As discussed in the last subsection, a small z helps the existence of a monetary equilibrium only under the condition $\sigma > 2$ so the latter is required in Proposition 4.3.

On the other hand, if θ is large and a coincidence of wants is likely, the negative externality in the choice of n can be so prominent that the optimal money growth rate is the upper bound $\bar{\gamma}$ which induces households to choose no one to shop with money.

4.3. A numerical example

There are reasonable parameter values that deliver a positive optimal money growth rate $\gamma_0 - 1$. We choose $a = 1$, $\beta = 0.99$, $\sigma = 8$, $z = 0.1$ and $\theta = 0.4$. The value of a is a normalization. The selected value of β implies a discount rate, $1 - \beta = 0.01$, that roughly equals the average of quarterly real interest rates in the U.S.. This equality between the discount rate and the real interest rate is an equilibrium requirement of a growth model that possesses a steady state.

To justify the selected value of σ in the cost function ϕ, note that the cost appears as disutility. Thus we can interpret ϕ as the disutility of the time spent in production. If the production function takes the Cobb-Douglas form as it is usually assumed in numerical exercises, then $q = K^{\delta}l^{1-\delta}$ where l is the labor input and K is the capital input. Since capital is implicitly assumed to be fixed in our model, we set $K = 1$ without loss of generality. In this case $\phi = l^{\sigma(1-\delta)}$ so that the labor supply elasticity is $[\sigma(1-\delta) - 1]^{-1}$. The estimate for this elasticity varies across sections of the working force (see Killingsworth (1983)). We choose a value 0.2 in the estimated range. A realistic value for δ is 0.25. These two values imply $\sigma = 8$.

A justification for the selected value of z would require disaggregated data on transactions. Although we do not have such data, we feel that the value 0.1 is reasonable.[16] For the value of θ, we identify it through equilibrium conditions. To do this, let the money growth rate be

[16]For example, the number of candidates interviewed by a typical economics department in a regular recruiting season may be around ten percent of the number of applicantions received by the department.

$\gamma = 1.0122$, which corresponds to the average of quarterly inflation rates in the U.S.. Let the fraction of producers be $1 - N = 0.6$, which is a realistic value for the U.S.[17] With these values and other identified parameters, the steady state equation $F(x) = \gamma/\beta - 1$ solves for x. Then the equation $1 - N(x) = 0.6$ implies $\theta = 0.4$.

For the selected parameter values, Figure 2 plots $\pi(x(\gamma)), V(x(\gamma))$ and $Y(x(\gamma))$ with the normalization $\pi(x(\beta)) = V(x(\beta)) = Y(x(\beta))$. All three variables increase with γ when γ is close to β and then decrease with γ when γ is large. The optimal money growth rate is $\gamma_0 - 1 = 1.6\%$. Since the parameter values are selected with reference to the quarterly data, this optimal rate should also be interpreted as the quarterly rate. It is reasonable to believe that the optimal rate will be lower if other competing media of exchange, such as credit, are introduced into the model.

Figure 2.

5. Discussion

In this section we compare our model with four classes of previous models. Then we relax the assumption of a constant bargaining weight θ used so far.

The first comparison is with the standard monetary models. As surveyed by Woodford (1990), it is possible to add frictions to the Walrasian model to invalidate the Friedman rule.[18] Quite different from these variations of the Walrasian model, the present model generates a positive optimal money growth rate because of the non-Walrasian, time-consuming trading process modelled by random matching. This positive role of money growth is absent in the standard model because all desirable trades are carried out there instantaneously by the Walrasian auctioneer.

The second comparison is with search models that have divisible goods but indivisible money, such as Shi (1993a,b) and Trejos and Wright (1995). Besides its ability to analyze money growth, the present model differs from these models mainly in the neutrality of money. In these models,

[17] For example, the fraction of employees in agriculture, mining, construction, manufacturing and services together was about 60% of the total U.S. employment in 1993 (see U.S. Bureau of Sensus (1994, p412)). I thank a referee for suggesting this method to identify the parameters.

[18] Williamson (1994) also shows that a sufficiently strong liquidity effect can imply an optimal money growth rate that differs from the Friedman rule.

money is not neutral: a once-and-for-all increase in the money supply increases output when a coincidence of wants is unlikely. In contrast, a once-and-for-all increase in the money supply is neutral in the present model, although money growth is not. The non-neutrality in previous models arises from the assumption of indivisible money. When money is indivisible and each money holder is restricted to holding and exchanging one unit of money, an increase in the total money stock forces an increase in the fraction of money holders. Since an exogenous increase in N affects output as our model shows:

$$\frac{\partial Y}{\partial N} \sim (1-2N)x - 2z(1-N),$$

output increases with N if z is small and $N < 1/2$. However, when money is divisible as in the present model, a once-and-for-all increase in the money stock has no effect on N or on any other real variables.

The third comparison is with Li (1992). Adding search intensity into the model of Kiyotaki and Wright (1991, 1993), Li argues that a moderate inflation rate can increase steady-state utility by inducing more intensive search. Since goods and money are indivisible in Li's model, the positive optimal inflation rate does not translate into the usual notion of increasing nominal prices (see footnote 3). It does not translate into the money growth rate either, because a zero money growth rate is assumed in his paper. In these respects, our result is more general. More importantly, the indivisibility restriction in Li's model implies that inflation forces a change in the fractions of producers and money holders in the market. The present analysis suggests that this forced change in the fraction of agents, not the endogenous search intensity, is the source of a positive optimal inflation rate. To support the argument, Appendix D endogenizes search intensity in the basic model with exogenous N. Contrary to Li's result, money growth reduces search intensities.

The fourth comparison is with search models that also illustrate a potential positive welfare effect of inflation, such as Benabou (1988, 1992) and Diamond (1993). The models in Benabou (1988) and Diamond (1993) are similar, focusing on the relationship between inflation and price dispersion. Benabou (1992) extends the two by using heterogeneous buyers and more general preferences to show that inflation can also have a negative welfare effect of increasing buyers' search cost. In these models, firms have some monopoly power on inventories and inflation can reduce the monopoly power by increasing the price dispersion and inducing more intensive search.

The present model differs from those models mainly along two dimensions. First, the present model models money as a valuable object arising from search frictions. In contrast, those models treat money as a vehicle for the determination of nominal prices, with no explicit modelling of how money can be valued. In this sense the present model is a search model of fiat money while those models are ones of prices. Second, the reason for the positive welfare effect of inflation differs in the two classes of models. In the present model the positive welfare effect of inflation arises from the externality generated by households' decision on whether to shop with money or to barter. In those models the positive welfare effect of inflation arises from the price dispersion and the monopoly power that firms have on their inventories. Such price dispersion or monopoly power does not exist in the present model.

We now turn to the assumption of a constant bargaining weight θ in monetary matches. Although in previous sections the bargaining outcomes in the monetary match respond endogenously to market conditions as agents' threat points in the bargaining do, one may argue that the exogenous bargaining weight limits the scope of such endogenous response. A suitably endogenized bargaining weight might induce the equilibrium outcome to approach competitive outcomes and eliminate the positive welfare effect of money growth. To check whether this argument is valid, we endogenize θ below.[19]

There can be many sensible ways to specify θ as functions of the endogenous variables. To be specific we choose the fraction of money holders N to be the variable upon which the bargaining weight depends. In particular, we assume:

$$\theta(N) = \theta_0 - \theta_1 N$$

[19]I thank a referee and an editor for pressing on this idea. For a comparison between the search market equilibrium and the competitive equilibrium, see Osborne and Rubinstein (pp 123-136).

where $\theta_0 \in (0,1)$ and $\theta_0 - \theta_1 \in [0,1]$. These restrictions on (θ_0, θ_1) ensure that $\theta(N) \in [0,1]$ for all $N \in [0,1]$. If, in addition, $\theta_1 > 0$, the bargaining weight of the money holder decreases when the fraction of money holders in the economy increases. This is the case where the bargaining outcomes are likely to approach the Walrasian ones. Despite such intuitive logic for restricting θ_1 to be positive, we examine both the case $\theta_1 > 0$ and the case $\theta_1 < 0$.

Since agents and households take N as given, they also take $\theta(N)$ as given. Thus the characterization of equilibrium in section 4 continues to hold, with θ being replaced by $\theta(N)$. Substituting the form of $\theta(N)$ into the equation $N = 1 - \Gamma^{-1}$ where Γ is defined in (4.5), we have:

$$h(1-N) \equiv \theta_1(\frac{a}{\phi'} - 1)(1-N)^2 + B(1-N) - 1 = 0$$

where

$$B \equiv 1 - \theta_0 + 2\theta_1 + (\theta_0 - \theta_1)\frac{a}{\phi'} - z \cdot \frac{aq^N - \phi(q^N)}{q^m\phi' - \phi}.$$

It can be verified that there is a unique solution $N \in (0,1]$ to the equation $h(1-N) = 0$ if $h(1) > 0$. This condition is equivalent to (4.4), with θ in the condition being replaced by θ_0. If $\theta_1 > 0$, the condition is also necessary for the existence of the solution. We impose this condition in the following discussion and note that it requires $z < \theta_0$. The solution for N is:

$$N(q^m) = 1 - \frac{\sqrt{B^2 + 4\theta_1(1-\theta_0+\theta_1)(a/\phi'-1)} - B}{2\theta_1(a/\phi'-1)}.$$

With this new function $N(q^m)$, the steady state value of q^m, denoted as q^*, is still given by (4.7). We use numerical exercises to illustrate the property of the equilibrium. As in subsection 4.2, let $\phi(q) = q^\sigma$ ($\sigma > 1$). With this functional form, the existence condition for the solution of N becomes $x > z/\theta_0$ where $x = q^*/q^N$ as before. Choose $a = 1$, $z = 0.1$, $\sigma = 8$, $\beta = 0.99$, $\theta_0 = 0.4$ as in subsection 4.3 and let θ_1 have different values in the range $(-0.35, 0.35)$. For all these values of θ_1, the following two properties in section 4 continue to hold. First, the function $F(q)$ defined in (4.7) is a decreasing function in the admissible range $q \in (q^N z/\theta_0, q^N)$ so that q^* decreases with the money growth rate γ. Second, the function $N(q)$ is non-monotonic. Starting from $q = q^N z/\theta_0$, the function first increases with q and then decreases. Since q^* decreases with

γ, the fraction N increases with γ when γ is close to β and decreases with γ when γ is close to the admissible upper bound.

With the parameter values, the optimal (gross) money growth rate γ_0 is greater than the discount factor β if $\theta_1 \geq -0.23$ and equals β otherwise. The net rate $\gamma_0 - 1$ can be positive or negative, depending on θ_1. We report the dependence in Table 1 below.

Table 1 Dependence of $(\gamma_0 - 1)$ on θ_1

θ_1	-0.23	-0.20	-0.15	-0.05	-0.02	0.02	0.05	0.15	0.25	0.35
γ_0-1 (%)	$(\beta-1)\times 100$	-0.76	-0.25	0.92	1.32	1.88	2.34	4.03	6.00	8.17

The optimal money growth rate increases with θ_1. In particular, when $\theta_1 > 0$, the optimal money growth rate is larger than in the baseline case with $\theta_1 = 0$. That is, the positive externality in the choice of n becomes stronger as the bargaining weight of money holders decreases with their population in the market. This result is opposite to the informal argument that motivated the exercise. To understand the result, recall that the optimal money growth rate can be positive because the equilibrium fraction of money holders is deficient, which occurs if a household's equilibrium valuation of money is too low. If $\theta_1 > 0$, the bargaining power of money holders is reduced for any $N > 0$, inducing households to reduce further their money holders. That is, the deficient number of money holders is exacerbated by the negative dependence of money holders' bargaining power on the fraction of money holders. It is then not surprising that this case supports a higher optimal money growth rate. If instead, $\theta_1 < 0$, the resulting stronger bargaining power of money holders encourages households to allocate more members to shop with money. This corrects some of the deficiency in the equilibrium choice of n and hence reduces the room for money growth to improve welfare. When θ_1 is negative and large in the absolute value, the correction can be overdone. In this case, the Friedman rule applies.

6. Conclusion

This paper extends the Kiyotaki-Wright search model of fiat money to incorporate divisible money and goods. By discarding the indivisibility restriction, we have focused on the time-consuming trading process that arises from the lack of double coincidence of wants in barter. Because agents are randomly matched and because trade between two agents must be *quid pro quo*, matches can fail to generate trade. We have shown that money growth can increase output and welfare by increasing agents' trading opportunities. These results demonstrate that the search monetary model can be a useful analytical alternative to Walrasian monetary models.

Whether the search model is also a useful quantitative alternative is beyond the scope of this paper and remains to be explored. Although we used some numerical exercises to verify the analytical results, some extensions must be made in order to calibrate the model as Cooley and Hansen (1989) have done for a standard cash-in-advance model. For example, credit must be introduced to compete against money and capital accumulation must be introduced to allow inflation to affect output through the capital stock. These extensions seem feasible. Credit has been introduced by the author (Shi 1993b) into a search model with divisible goods and indivisible money. A simple version of capital accumulation was introduced into an earlier version of the present model (Shi (1995)), and is the focus of current research. With divisibility, the present model can incorporate capital accumulation more easily than previous search models of money. Since the trading opportunity effect is present, we expect inflation to have a different effect on the capital stock from that in Cooley and Hansen (1989).

For analytical applications, the present model can be useful for issues of dual currencies and exchange rates. These issues have been examined in search models by Shi (1993a) and Zhou (1993) under various restrictions on divisibility. In particular, it has been restricted that no agent can hold two different currencies at the same time. It would be interesting to see how the equilibrium exchange rate behaves without those technical restrictions.

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Appendix

A. Individual's Contribution

In this appendix we define and calculate the contribution of an individual member's action to the household's aggregate variables. Strictly speaking, when the household consists of a continuum of members, an individual is negligible in the household. However, our definition below can loosely be interpreted as the limit of each member's contribution in a finite economy as the number of members in the household goes to infinity. With a change of the member index to the time index, our definition is analogous to the Volterra derivative that has been used in the literature to compute the marginal utility of consumption at a particular time in a continuous-time framework (see, for example, Epstein and Hynes (1983) for an application).

Let I be a subset of member indices with measure $\mathcal{M}(I) > 0$. Let the actions of members be $\{R(j)\}_{j\in I}$ and the aggregate variable be:

$$\Omega = H(\int_{j\in I} h(R(j))dj)$$

where h and H are continuously differentiable. For given $\{R(j)\}_{j\in I}$, consider a change in action by a group of members around member $k \in I$. As the size of the group gets arbitrarily small, we take the limit of the group's average contribution to the aggregate variable as member k's contribution. Formally, let $S(k) \subset I$ be a neighborhood of k with a measure $\mathcal{M}(S(k)) = B$ where B is a sufficiently small number. For given $\{R(j)\}_{j\in I}$ and a constant $d > 0$, construct a new action $\{Rd(j)\}_{j\in I}$ as follows:

$$Rd(j) = \begin{cases} R(j) + d, & if\ j \in S(k) \\ R(j), & otherwise. \end{cases}$$

That is, all agents in the neighborhood $S(k)$ increase their actions by d. The path $\{Rd(j)\}_{j\in I}$ is depicted in Figure $A1$ for a continuous graph $R(j)$, although continuity is unnecessary for our definition and calculation. We define the contribution to Ω by individual k's change in action from $R(k)$ to $Rd(k)$ as:

$$\Omega^d(k) \equiv \lim_{B\to 0} \frac{1}{B}\left\{H(\int_{j\in I} h(Rd(j))dj) - H(\int_{j\in I} h(R(j))dj)\right\}.$$

Figure A1.

To calculate $\Omega^d(k)$, first notice that

$$\Delta \equiv \int_{j\in I} h(Rd(j))dj - \int_{j\in I} h(R(j))dj$$

$$
\begin{aligned}
&= \int_{j\in S(k)} [h(R(j)+d) - h(R(j))]dj \\
&= B\cdot[h(R(\xi)+d) - h(R(\xi))] \quad for\ some\ \xi \in S(k).
\end{aligned}
$$

$\Delta \to 0$ and $\xi \to 0$ as $B \to 0$. Then

$$
\begin{aligned}
\Omega^d(k) &= \lim_{B\to 0}\frac{1}{B}\left\{H(\int_{j\in I} h(R(j))dj + \Delta) - H(\int_{j\in I} h(R(j))dj)\right\} \\
&= \lim_{B\to 0}[h(R(\xi)+d) - h(R(\xi))]\cdot \lim_{\Delta\to 0}\frac{1}{\Delta}\left\{H(\int_{j\in I} h(R(j))dj + \Delta) - H(\int_{j\in I} h(R(j))dj)\right\} \\
&= [h(R(k)+d) - h(R(k))]\cdot H'(\int_{j\in I} h(R(j))dj).
\end{aligned}
$$

For example, with the choices of $H = u, h(R) = R, R(k) = 0$ and $d = q(k)$, the quantity $\Omega^d(k)$ gives the contribution of individual k to the household utility when he acquires $q(k)$ units of goods. Similarly, with $h = \phi$ and H being the identity function, the quantity $\Omega^d(k)$ gives the increase in the cost Φ made by individual k's production of $q(k)$ units of goods.

B. Proof of Proposition 4.2

Proof. Compute

$$
\begin{aligned}
\frac{dF(q^*)}{dq^*} &\sim -a\left[q^* - \frac{\phi(q^*)}{\phi'(q^*)}\right]^2 + z\left[q^* - \frac{a\phi(q^*)}{(\phi'(q^*))^2}\right]\left[aq^N - \phi(q^N)\right] \\
&< -a\left[q^* - \frac{\phi(q^*)}{\phi'(q^*)}\right]^2 + z\left[q^* - \frac{\phi(q^*)}{\phi'(q^*)}\right]\left[aq^N - \phi(q^N)\right] \\
&\sim z\left[q^N - \phi(q^N)/a\right] - \left[q^* - \frac{\phi(q^*)}{\phi'(q^*)}\right] < 0.
\end{aligned}
$$

The first inequality follows from $a > \phi'(q^*)$; the second inequality from (4.4). Since $F'(q^*) < 0$, there is a unique solution to (4.7). For this solution to be qualified as a steady state of the binding monetary equilibrium, it must satisfy (4.4) and the condition $\lambda > 0$. That is, $q_0 < q^* < q^N$ or equivalently $F(q_0) > \frac{\gamma}{\beta} - 1 > F(q^N)$. These conditions are equivalent to $z < \theta$ and (4.8). The condition $z < \theta$ also guarantees a nonempty interval (q_0, q^N). ■

C. Proof of Proposition 4.3

Proof. Note that $dx/d\gamma < 0$. Compute:

$$
\begin{aligned}
\frac{dY}{d\gamma} &\sim -[2z(1-N) - (1-2N)x]\frac{dN}{d\gamma} + N(1-N)\frac{dx}{d\gamma} \\
&\sim [x - 2z - \frac{x^{1-\sigma}}{1-\theta}(\theta x - z)][\theta(\sigma-1)x - z\sigma] - x(\theta x - z)[1 + \frac{x^{-\sigma}}{1-\theta}(\theta x - z)];
\end{aligned}
$$

$$
\begin{aligned}
\frac{dV}{d\gamma} &\sim -[2(\sigma-1)z(1-N) - (1-2N)(\sigma x - x^\sigma)]\frac{dN}{d\gamma} + N(1-N)\sigma(1-x^{\sigma-1})\frac{dx}{d\gamma} \\
&\sim \left[2\cdot\frac{(\sigma-1)z - \sigma x + x^\sigma}{1+\frac{\theta}{1-\theta}x^{1-\sigma} - \frac{z}{1-\theta}x^{-\sigma}} + \sigma x - x^\sigma\right]\frac{z\sigma - \theta(\sigma-1)x}{1-x^{\sigma-1}} - \sigma x(\theta x - z).
\end{aligned}
$$

Under the conditions in the proposition, we have:

$$\frac{dY}{d\gamma}\mid_{x=1}\sim \theta[(\sigma-1)(1-2\theta)-1]-z[\sigma(1-2\theta)-1]>0,$$

$$\frac{dV}{d\gamma}\mid_{x=1}\sim (\sigma-1)(1-2\theta)(\theta(\sigma-1)-z\sigma)>0.$$

Since $x\mid_{\gamma=\beta}=1$, $dY/d\gamma\mid_{\gamma=\beta}>0$ and $dV/d\gamma\mid_{\gamma=\beta}>0$. Thus Y and V are increasing functions of γ for γ sufficiently close to β. Similarly, one can show under the conditions of the proposition that Y and V are decreasing functions of γ when γ is sufficiently close to the upper bound $\bar{\gamma}$. Thus there exists γ_0 close to β that maximizes V.

Furthermore, (4.10) implies $z\sigma-\theta(\sigma-1)<0$. For γ sufficiently close to β, $z\sigma-\theta(\sigma-1)x<0$ and hence $dN/d\gamma>0$. In this case,

$$\frac{d\pi}{d\gamma}\mid_{x=1}\sim (2-z-\frac{1}{1-N})\mid_{x=1}=\frac{1-\theta(2-z)}{1-\theta}>\frac{z}{2(1-\theta)}>0.$$

The first inequality follows from $\theta<1/2$. Thus $d\pi/d\gamma>0$ for γ sufficiently close to β.

On the other hand, if $\theta>1/2$ and $\theta>z>\theta\cdot\frac{1+(\sigma-1)(2\theta-1)}{1+\sigma(2\theta-1)}$, it can be verified that $dY/d\gamma>0$ for all $\gamma\in[\beta,\bar{\gamma}]$. Similarly, $dV/d\gamma>0$ for all $\gamma\in[\beta,\bar{\gamma}]$ when $\theta>1/2$ and z is sufficiently close to θ. In this case the money growth rate that maximizes V is $\bar{\gamma}$, which implies $N=0$. ■

D. Endogenous Search Intensity

This appendix extends the basic model in section 2 to allow households to choose search intensity. We show that an increase in the money growth reduces the search intensity. Let household i choose a search intensity s_{mi} for each of its money holders and s_{pi} for each producer. For symmetric equilibrium we suppress the subscript i. Let S_m be the average search intensity of money holders and S_p the average search intensity of producers. Define an average search intensity by

$$S=NS_m+(1-N)S_p.$$

The variables (S_m,S_p,S) are taken as given by individual households.

Let $h(s,S)$ be the probability with which an agent with search intensity s is matched with another agent. Assume that h is strictly increasing and concave in each of its arguments and that h is linearly homogeneous. Linear homogeneity implies that $h_1(s,S)$ and $h(s,S)/s$ are functions of the ratio s/S only. The two are constants when $s=S$. Denote these constants by

$$h_1=h_1(s,S)\mid_{s=S};\quad h_a=h(s,S)/s\mid_{s=S}.$$

We show $s_p = s_m \equiv s$. Note that each money holder meets a producer with probability $(1-N)h(s_m, S)$ so that the total number of meetings between money holders and producers is $N(1-N)h(s_m, S)$. Since each producer meets a money holder with probability $Nh(s_p, S)$, the total number of meetings between money holders and producers is also $(1-N)Nh(s_p, S)$. Therefore $h(s_m, S) = h(s_p, S)$ and hence $s_m = s_p$. By the definition of S, we have $S = s$ for any symmetric equilibrium.

Increasing search intensity is costly. Let the cost of search in terms of utility be $\Omega(s)$ with the properties $\Omega'(s) > 0$, $\Omega''(s) > 0$ for $s > 0$ and $\Omega(0) = \Omega'(0) = 0$. The household's decision problem is similar to that in section 2. The additional choice variable of the household is s, which enters the maximization problem through the measures of the sets I_b, I_p and I_m:

$$\mathcal{M}(I_b) = h(s,S)z^2(1-N)^2,\ \mathcal{M}(I_p) = h(s,S)zN(1-N) = \mathcal{M}(I_m).$$

The first order condition for s is

$$\Omega'(s_t) = h_1 z(1-N)\left[N(aq_t^m - \phi(q_t^m)) + z(1-N)(aq^N - \phi(q^N))\right].$$

The bargaining solutions are the same as in section 2.3. The dynamic equation for q^m is

$$g(q_t^m) = g(q_{t+1}^m)\frac{\beta}{\gamma}\left\{1 + sh_a z(1-N)\left(\frac{a}{\phi'(q_{t+1}^m)} - 1\right)\right\}.$$

In the steady state the last two equations determine the steady state pair (s^*, q^*):

$$\Omega'(s^*) = h_1 z(1-N)\left[N(aq^* - \phi(q^*)) + z(1-N)(aq^N - \phi(q^N))\right];$$

$$s^* = \frac{\gamma/\beta - 1}{z(1-N)h_a}\cdot\frac{\phi'(q^*)}{a - \phi'(q^*)}.$$

In general, there are an odd number of positive solution pairs to these equations. We focus on the smallest (or the largest) positive solution.

It can be shown that an increase in the money growth rate γ decreases the real balance q^*, as in the basic model. Also, an increase in γ reduces the search intensity s^* (a diagram can confirm this). This result is opposite to that in Li (1992). Nevertheless, an exogenous increase in N can increase s^* and q^* if z is sufficiently small.

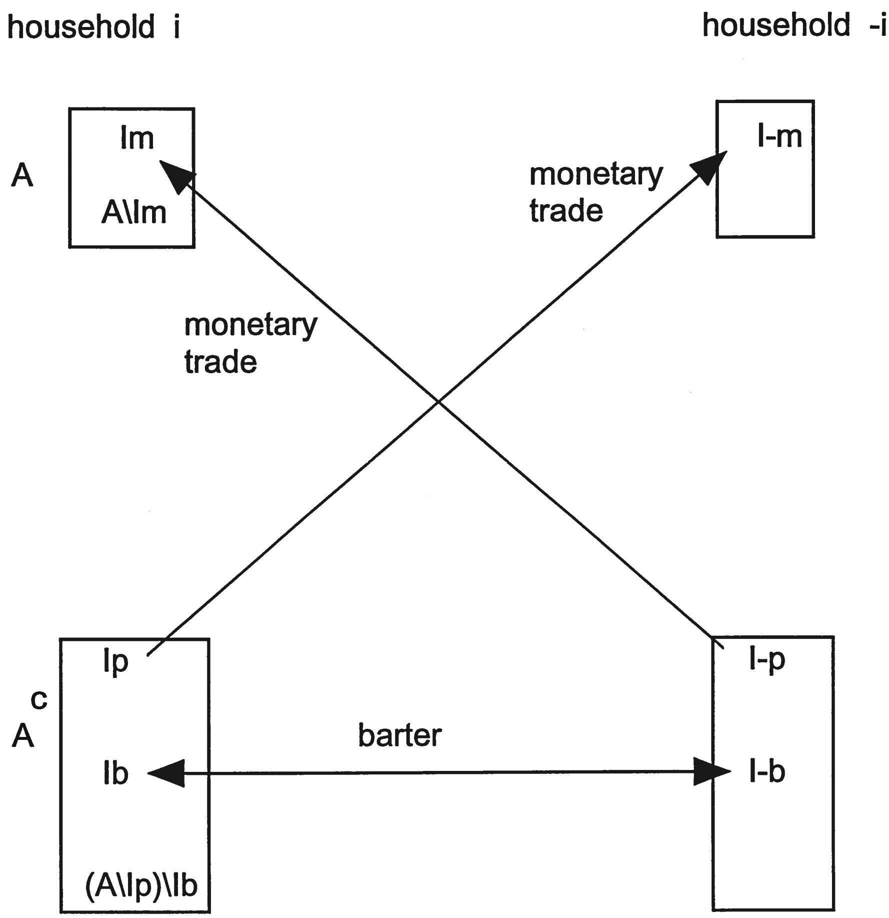

Figure 1 Exchanges

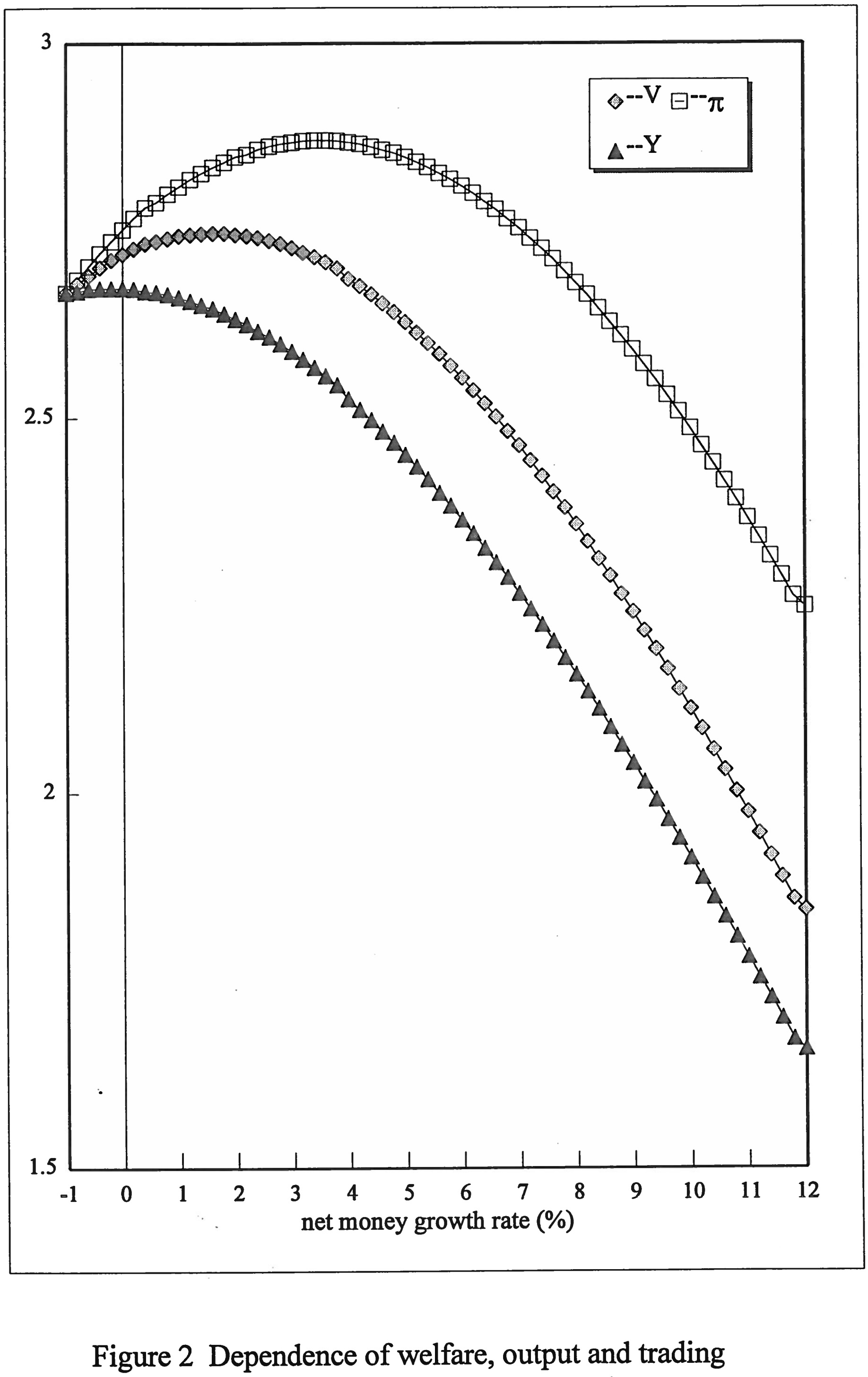

Figure 2 Dependence of welfare, output and trading opportunities on the money growth rate

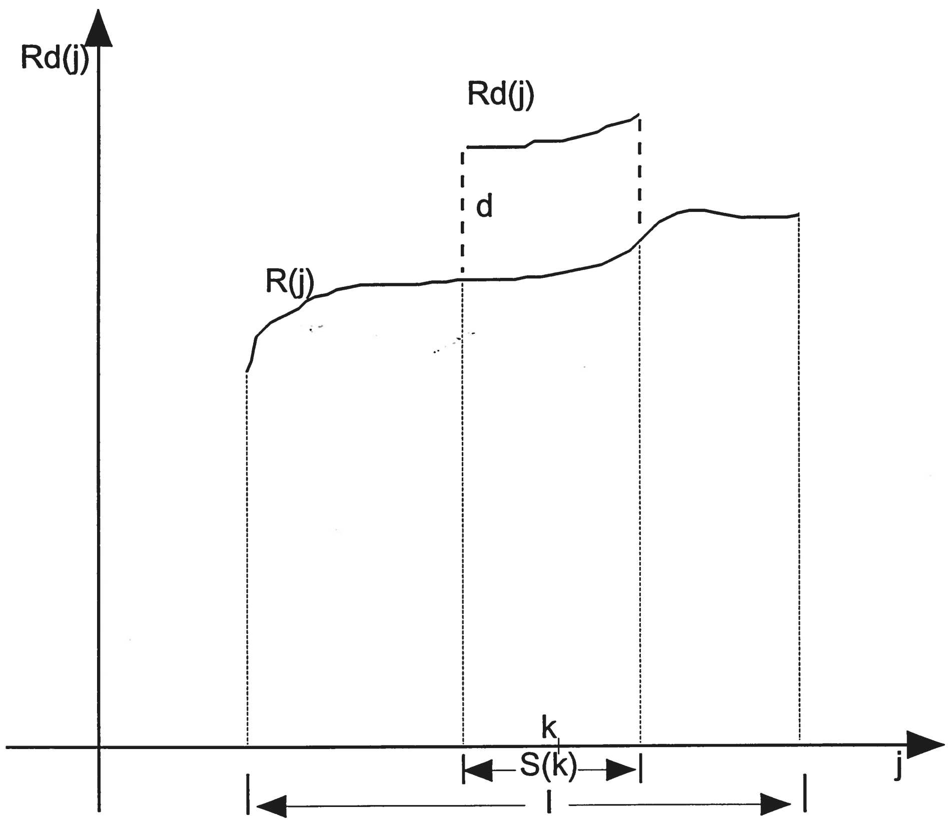

Figure A1