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STRUCTURAL SHIFT WITH AN INTER-STRUCTURAL TRANSITION FUNCTION*

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Economic analysis of time-series data typically employs regression techniques on the assumption that an underlying relationship is stable over time. However, in many cases the underlying structure may not be constant, and methods of detecting and facilitating structural change in the relationship must be incorporated into the analysis. The usual practice of economists in such cases is to employ F-tests, as outlined by Chow (1960), to test the equality of regression coefficients (all coefficients or a subset) in two or more regressions. There are two major problems with such a procedure: the identification of the exact point of structural shift and the assumption that the entire shift is accomplished in one interval of time. The problem of estimating the point in time at which a switch in structures (regimes) takes place has been investigated by Quandt (1958, 1960, and 1972), Hudson (1966), Hinkley (1969), and Farley and Hinich (1970). Recently, Bacon and Watts (1971) have employed Bayesian techniques to analyze the problem where the switching point is unknown and where the switch in regimes occurs either abruptly or smoothly during a transition period defined over both regimes.

The purpose of this paper is to suggest a simple model for the regression problem of a known switching date in structures which consumes several intervals of time to complete the shift. This model, then, permits a smooth transition from an old to a new structure, and is easily tested by employing standard t and F tests. Before presenting the model, a few examples of the type of economic problems for which this model is applicable. First, the establishment of an Incomes Policy may take a number of time intervals to affect a change in the underlying structure (e.g., inflation expectations). Or, the price response of exporters and importers to a revaluation of a fixed foreign exchange rate may not occur simultaneously, but rather may be partially absorbed in profit margins over a transition period. Finally, corporation adjustments to a new industrial policy may be extended over a number of years. As illustrated below, the model is successfully applied to a structural shift problem of this latter variety arising from the signing of the Canada-U.S. Automotive Agreement.

In each of the above illustrations, the termination point of the "hypothesized" old structure is known, and the shift to the "hypothesized" new structure is unlikely to occur in one interval of time. The task of the researcher is to test the null hypothesis that there has been no structural shift against the hypothesis that there are two distinctly different structures separated by a transition period. A misspecification which fails to allow for a transition phase may bias the results in favour of the null hypothesis of no structural shift.[1]

One might, of course, simply discard the "troublesome" inter-structural observations. The costs of such action are twofold. First, valuable information is lost and/or degrees of freedom are sacrificed. Second, the use of simulation techniques, a frequent tool in economics, is placed in jeopardy when structural relationships are undefined over certain intervals of time. The establishment of a parameter transition function over the inter-structural period minimizes these costs, as well as permitting an analysis of the nature

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1. To illustrate this point, consider the simple case where the intercept shifts to a higher value (the slope coefficients are unaffected) over an inter-structural transition phase of "r" intervals of time. Assuming that the observations for the explanatory variables have a secular time trend and no pronounced "overshooting" of the structural shift occurs in the transition period, then the inclusion of these r observations in the new structure (or in the old structure, or dividing them between the two structures)will reduce the "observed" difference between the intercepts in the two structural periods.

and characteristics of the inter-structural transition phase (assuming the null hypothesis has been rejected).

Assume that a relationship spanning two distinct structures can be defined in the following manner:

$$(1) \qquad y_t = a + b_t X_t + u_t$$

$$\text{or} \quad (2) \qquad y_t = \begin{cases} a + b_0 X_t + u_t & (t = 1,2, \ldots , m) \\ a + b_i X_t + u_t & (t = m+i \ ; \ i = 1,2, \ldots , r) \\ a + b_r X_t + u_t & (t = m+r+1, \ldots , T) \end{cases}$$

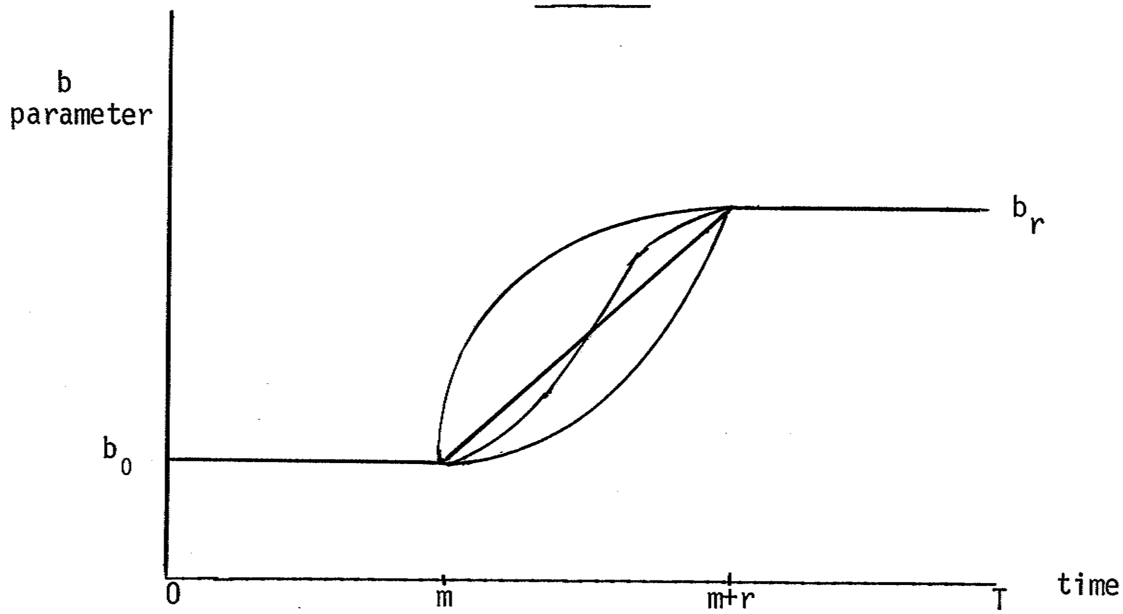
where u_t are independently and normally distributed errors with mean zero and constant variance, i.e. $\{u_1, \ldots u_T\} \sim N(0, \sigma^2 I)$, and where there are T independent pairs of observations $(y_1, X_1, \ldots y_T, X_T)$. For simplicity it is assumed that the hypothesized structural shift, commencing at time m+1, occurs only for the b coefficient, taking on values of b_0 in the first structure and b_r in the second structure. Both m, the termination point of the old structure, and r, the transition interval in units of time, are assumed to be known.

The behaviour of the b parameter over the transition phase (b_i) is assumed to be approximated by an n^{th} order time polynomial, i.e. the b parameter is assumed to move from b_0 to b_r by following some time polynomial over the r period transition phase.

$$(3) \qquad b_i = b_0 + c_1 i + c_2 i^2 + \ldots + c_n i^n \qquad (i = 1,2, \ldots , r)$$

Several possible shapes for this parameter transition function are given in Chart I.

CHART I



The specification of the b parameter in (2) can be simplified by noting that when $i = 0$, then b_i is equal to b_0; and when $i = r$, then b_i is equal to a constant, say b_r. Define a vector Z of T elements which consist of m zeros followed by a time trend to r and then a series of (T-m-r) values of the parameter r.

$$(4) \qquad Z = \{0, 0, \ldots . 0, 1, 2, \ldots , r, r, \ldots , r\}$$

Thus the b_t in (1) can be represented simply as

$$(5) \qquad b_t = b_0 + c_1 Z_t + c_2 Z_t^{\,2} + \ldots . . + c_n Z_t^{\,n} \qquad (t = 1, 2, \ldots , T)$$

Substituting (5) into (1) produces the following result:

$$(6) \qquad y_t = a + (b_0 + c_1 Z_t + c_2 Z_t^{\,2} + \ldots + c_n Z_t^{\,n})\, X_t + u_t$$

which can be re-arranged as

$$(7) \qquad y_t = a + b_0 X_t + c_1 (Z_t X_t) + c_2 (Z_t^2 X_t) + \ldots + c_n (Z_t^n X_t) + u_t$$

The hypothesis that there has been a structural shift occurring over an r period transition phase can be verified by the calculation of an F-value to test whether the set of c parameters is significantly different from zero.

To employ this test, three parameters are assumed to be known; m, r and n. As discussed above, there are many economic problems where the beginning of a potential shift in structure is well-known. The selection of a value for r is akin to the specification of the length of lag in a distributed lag problem in econometrics, and usually one will have some information concerning the length of the adjustment (transition) phase. The specification of the degree of polynomial can either be done on economic or *a priori* grounds, or by employing t-tests to test the significance of various c parameters. However, higher order polynomials are likely to increase the likelihood of multicollinearity between the transition phase variables.

One notes that the gains from utilizing this technique for the shift in one parameter, as opposed to slope dummies for each of r time intervals, depend on the value of r and n. For example, if $r = 1$, then this model simply reduces to the standard Chow test. If $r - n = 0$, then the model is equivalent to simply employing slope dummies. On the other hand, if $r > n$ then this technique minimizes the loss in degrees of freedom on the assumption that the transition phase adjustment for the particular parameter can be approximated by an n-th order polynomial. One also notes that if $r < n$, then the parameters of the polynomial are not linearly estimatable. Finally, this technique can be applied to as many coefficients in a regression as desired (although collinearity and degrees of freedom problems are a greater likelihood), and can be utilized for multiple structural shifts, with differing transition functions, over extend sample periods.

To illustrate this technique, the Canadian import function for U.S. produced automobiles is examined in the context of the Canada-United States Automotive Agreement. This Agreement, signed on January 16, 1965, permitted "controlled" free trade[2] in automotive products; and, in essence, sought to rationalize the production of automotive products in North America. Prior to the Agreement, a 17½% tariff resulted in a wide range of automobiles being manufactured in Canada by subsidiaries of U.S. automotive companies. The Agreement encouraged the Canadian production (and exportation) of a few makes and models, and the importation of a great variety of U.S. produced vehicles duty-free from the larger parent industry. The structure of inter-country automotive trade was clearly altered by the signing of the Agreement, although industry adjustments extended well beyond the initial year of the Agreement.

The model employed to explain the aggregate movement of U.S. produced automobiles into Canada focusses on the conventional aggregate trade generating factors: domestic income-expenditure (activity), relative prices, and domestic supply constraints. These three factors are represented by Canadian consumer expenditure on all automobiles (E), the Canadian wholesale price index of domestic automobile production divided by a foreign exchange rate corrected U.S. wholesale price index of automobiles (P), and thousands of man days lost in strike activity in the Canadian automobile industry (SDL). The dependent

2. The Agreement differs from free trade in automotive products in that tariff exemptions were only given to producers which met certain conditions. For further details, see Beigie (1970).

variable, import flows of automobiles from the U.S., and the expenditure explanatory variable are measured in millions of 1961 Canadian dollars.

As shown in Table 1, this model provides a reasonably good explanation for U.S. produced automobile imports for the seventeen years prior to the signing of the Agreement. All explanatory variables have the correct sign and are significant at the .01 level (t-values are given below the estimated coefficients). However, the model's performance seriously deteriorates when the sample period is extended through the Agreement years (column 2). Both consumer expenditure and price relative variables are insignificant and first-order autocorrelation is a serious problem, presumably a manifestation of missing structural factors (e.g. the effects of the new Agreement).

To incorporate the change in structure occasioned by the signing of the Automotive Agreement into the regression model, a parameter transition function is hypothesized for the expenditure variable (E) over the 1965-68 period.[3] The existence of only four observations in the transition phase necessitates the restriction of parameter transition functions to one explanatory variable[4] and to low order degrees for the time polynomial. Columns 3 to 5 in Table 1 present estimates for linear, quadratic and cubic parameter transition functions for the expenditure explanatory variable. The linear transition function clearly adds significant explanatory power

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3. The transition function parameters m and r are therefore seventeen and four in this particular example.

4. Given the substantial change in Canadian production and import mix under the Agreement, a new expenditure elasticity may clearly emerge in the Agreement structure. Price sensitivity and domestic strike activity are less likely to be influenced by the signing of the Agreement.

TABLE 1

Canadian Imports of U.S. Produced Automobiles
(1948-70, unless otherwise indicated)

	(1)*	(2)	(3)	(4)	(5)	(6)
Constant	-479	693	-343	-414	-506	-681
E	.0295 (3.56)	.2601 (3.40)	.0107 (.37)	.0117 (.37)	.0384 (1.86)	.0494 (.58)
P	499 (4.60)	-861 (.73)	379 (1.06)	360 (.87)	522 (1.98)	684 (.66)
SDL	.0456 (6.01)	.0776 (1.20)	.0128 (.66)	.0122 (.58)	.0329 (2.38)	.0592 (1.15)
Z*E			.0812 (14.23)	.0787 (2.94)	-.0891 (2.44)	
Z^2*E				.0006 (.10)	.1227 (5.13)	
Z^3*E					-.0205 (5.18)	
S*E						.2232 (3.49)
S.E.E.	11.1	146.3	42.9	44.2	27.8	116.0
F	14.5	9.4	127.7	100.5	220.8	14.4
$\bar{R}^2$	.775	.611	.966	.965	.986	.755
D.W.	2.31	.57	1.52	1.50	1.72	1.21

* 1948-1964

to the underlying model (column 3 compared to column 2). While the addition of the quadratic argument to the transition function provides no significant increase in explanatory power, the cubic function (column 5) does provide significant additional explanatory power over the linear version[5]. Besides the F-test calculations, one also notes the improvements in the t-statistics, the Durbin-Watson statistic, and the estimates of the initial structural parameters for the cubic transition function model.

Table 2 presents the expenditure parameters for the new structural period and for the transition phase (1965-1967). The dramatic shift in the expenditure parameter (.03 to .33) produces a post-Agreement expenditure elasticity[6] of .99 (compared to .54 in the pre-Agreement structure). This is presumably a reflection of the change in Canadian domestic production mix[7] resulting from the rationalization of the industry. Under the cubic transition function hypothesis, most of the adjustment occurs in the two middle years (1966-67).

As a final check on these results, the comparable Chow type model is estimated employing a constant shift in the expenditure parameter commencing in the year 1965. While this conventional slope dummy variable (S*E) is

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5. The calculated F-value for the null hypothesis that the parameters associated with Z^2*E and Z^3*E are insignificantly different from zero is 13.3.

6. Elasticities are computed utilizing mean values for M and E over the two structural periods (i.e., 1948-64 and 1968-70).

7. Canadian production in the post-Agreement era has tended to specialize in small, inexpensive vehicles (e.g. Dart, Maverick, Vega, etc.), requiring the importation of a wide variety of higher-priced automobiles from the parent U.S. industry.

significant (see column 6, Table 1), this equation is clearly inferior to comparable transition function models. For example, the standard error of estimate is more than four times larger than that obtained for the cubic transition function model.

In conclusion, a simple regression model has been suggested to cope with the problem of a non-abrupt structural shift which occurs over a number of intervals of time. In spite of a limited observation range arising from the use of annual data, the transition function approach provides reasonable estimates for the gradual structural shift in automotive import flows resulting from the signing of the Canada-U.S. Automotive Agreement.

TABLE 2

Consumer Expenditure Parameter
Over the Sample Period

Regression	Old Structure	Transition Phase			New Structure
	(1948-64)	1965	1966	1967	(1968-70)
(1) No shift (17 obs.)	.0295	--	--	--	--
(2) No shift (23 obs.)	.2601	.2601	.2601	.2601	.2601
(3) Linear Transition function	.0107	.0918	.1730	.2542	.3353
(4) Quadratic Transition function	.0117	.0910	.1715	.2531	.3358
(5) Cubic Transition function	.0384	.0514	.1871	.3234	.3346
(6) Slope dummy shift	.0494	.2726	.2726	.2726	.2726

REFERENCES

BACON, David W., and Donald G. Watts, "Estimating the Transition Between Two Intersecting Straight Lines", Biometrika (1971), Vol. 58, pp. 525-534.

BEIGIE, C. E., The Canada-U.S. Automotive Agreement: An Evaluation, Canadian-American Committee (Montreal), 1970.

CHOW, G. C., "Tests of Equality Between Sets of Coefficients in Two Linear Regressions", Econometrica, July 1960, Vol. 28, pp. 591-605.

FARLEY, J. U., and M. J. Hinich, "A Test for a Shifting Slope Coefficient in a Linear Model", Journal of the American Statistical Association, September 1970, Vol. 65, No. 331, pp. 1320-1329.

FISHER, F. M., "Tests of Equality Between Sets of Coefficients in Two Linear Regressions: An Expository Note", Econometrica, March 1970, Vol. 38, pp. 361-366.

HINKLEY, D. V., "Inference About the Intersection in Two-Phase Regression", Biometrika, 1969, Vol. 56, No. 3, pp. 495-504.

HUDSON, D. J., "Fitting Segmented Curves Whose Join-Points Have to be Estimated", Journal of the American Statistical Association, December 1966, Vol. 61, pp. 1097-1129.

QUANDT, R.E., "The Estimation of the Parameters of a Linear Regression System Obeying Two Separate Regimes", Journal of the American Statistical Association, 1958, Vol. 53, pp. 873-880.

__________, "Tests of the Hypothesis That a Linear Regression System Obeys Two Separate Regimes", Journal of the Royal Statistical Association, 1960, Vol. 55, pp. 324-330.

__________, "A New Approach to Estimating Switching Regressions", Journal of the American Statistical Association, June 1972, Vol. 67, No. 338, pp. 306-310.