Queen's Economics Department Working Paper No. 916

Money and Prices: A Model of Search and Bargaining

Shouyong Shi

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

1-1995

Discussion Paper #916

Money and Prices: A Model of Search and Bargaining

by

Shouyong Shi
Queen's University

January 1995

Money and Prices: A Model of Search and Bargaining*

by

Shouyong Shi

Department of Economics
Queen's University
Kingston, Ontario
Canada, K7L 3N6

* I thank Randy Wright, Quan Wen, Neil Wallace, Alberto Trejos, Ellen McGratten, Nobu Kiyotaki, Merwan Engineer, Paul Anglin and the participants of workshops at various institutions for helpful comments. Financial support from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged. All errors are mine.

ABSTRACT

This paper makes commodities divisible and incorporates bargaining into the search-theoretic model of money to determine the purchasing power of money (or price). It is shown that two monetary equilibria always coexist where fiat money is universally accepted. The two equilibria differ in price, output, welfare and the velocity of money. Sunspot monetary equilibria exist in which money is universally accepted in all states of the economy. Multiplicity has novel implications on the effectiveness of currency substitution and exchange market intervention. Journal of Economic Literature Classification Numbers: C78, E40.

1. INTRODUCTION

An old and yet challenging question in monetary economics is what determines the purchasing power of money or the nominal price. As early as J.S. Mill [20, p.267], economists have argued that the price level may be indeterminate and that indeterminacy is an independent cause of economic fluctuations. Indeterminacy is supported by the three common monetary models--the overlapping generations model (Grandmont [9]), the cash-in-advance model (Woodford [29]) and the money-in-the-utility-function model (Matsuyama [18]). However, these models are often criticized for their severe restrictions imposed for indeterminacy and the existence of valuable fiat money. More importantly, these models assume competitive markets and do not examine how the purchasing power of money is determined by the strategic behavior of traders in a bilateral exchange.[1]

A new monetary model uses search to examine the bilateral exchange, but with limited success in explaining the nominal price. Two examples are Diamond [6] and Kiyotaki and Wright [15, 16] (see references therein). Assuming indivisible commodities, both models impose that the traders in a bilateral exchange swap their inventories one-for-one regardless of the type of trade. When prices are briefly discussed, they are calculated by the rule of splitting the trade surplus between the traders. Although this rule can be interpreted as a Nash bargaining outcome, it is unclear what economies satisfy the axioms underlying the Nash bargaining solution. Since the specification of the economy is important for understanding the role for fiat money, it is necessary to explicitly describe the strategic process which

[1] The assumption of competitive markets is also imposed by Townsend's [25] model of spatial separation of markets and other models such as [1, 8, 11] that use incomplete markets to deliver valued fiat money.

determines prices. Superimposing the Nash bargaining solution oversimplifies the strategic behavior and precludes the examination of how the role of money changes with the underlying environment.[2]

Each of the two search models cited above has further specific limitations. To generate multiple monetary equilibria, Diamond [6] imposes the cash-in-advance constraint on all transactions and increasing-returns-to-scale on the search technology. Kiyotaki and Wright [15, 16] do not rely on these devices but find that there is a unique pure monetary equilibrium, namely that in which money is accepted with probability one. As a consequence, sunspot equilibria in Wright [30] involve at least one state of the economy in which money is not universally accepted by commodity producers.

This paper adopts the framework of Kiyotaki and Wright [15, 16] but abandons three of their restrictions. First, we allow for production of any desirable quantity of commodities. Second, the terms of trade in a bilateral exchange are determined by a bargaining process with alternating proposals (Rubinstein [22]). Third, there is no transaction cost other than time discounting. The first two devices allow us to examine prices; the absence of fixed transaction costs greatly simplifies the analysis.

The main results of the paper are as follows. First, there are two pure monetary equilibria which differ in the purchasing power of money. Multiplicity emerges here even though there always exists a unique solution to the bilateral bargaining problem. The fundamental reason for multiple monetary equilibria is that the gains from a monetary

---

[2] After completing the first draft of this paper, the author became aware of a closely related paper independently written by Trejos and Wright [26]. A brief discussion on their model and its connection to this paper is provided near the end of Section 3.

exchange depend on the values of producing commodities and holding money which in turn depend on the gains from the exchange. Different beliefs on these values can be self-fulfilling. Some interesting features of the multiple monetary equilibria are that they always coexist and that they do not rely on increasing-returns-to-scale. Thus multiplicity cannot be excluded by restricting the parameter values of the model, as can be done in the three common monetary models cited earlier.

Second, the purchasing power of money has very different features in the two equilibria, generating different velocity of money, output and welfare. In particular, a decrease in the rate of time preference or an increase in the frequency of trade has opposite effects on the purchasing power of money in the two equilibria. The purchasing power of money increases with the money supply in the two equilibria when the money supply is sufficiently small but decreases in one of the equilibria when the supply is high. Also, the lack of double coincidence of wants reduces the purchasing power of money in one of the equilibria. This seemingly counter-intuitive result can be explained from the nature of the bargaining outcomes.

Third, multiple monetary equilibria give rise to monetary sunspot equilibria. Similar to Wright [30], sunspot equilibria emerge without complicated preferences or technology. Different from Wright [30], however, money is universally accepted in <u>every</u> state of the sunspot equilibria. We also show that there are no sunspot barter equilibria of the sort and hence the use of money makes the economy more vulnerable to extrinsic shocks.

Finally, the model, extended to incorporate two currencies, has strong welfare implications on exchange policies such as currency substitution and exchange market intervention. We show that two fiat monies with different purchasing powers can both be universally accepted. Whether currency substitution improves welfare depends on the strength of the purchasing powers of the national currency and the competing currency. If both currencies have high purchasing powers, the nominal exchange rate is one. In this case currency substitution and exchange market intervention have no welfare effect. If the national currency has a low purchasing power, introducing a currency with higher purchasing power for competition increases welfare, but such substitution yields welfare lower than pegging the national currency onto the strong foreign currency.

The four sets of results are shown, respectively, in Sections 3, 4, 5 and 6. Section 2 sets up the model and examines the barter equilibrium. Section 7 concludes the paper. The appendix supplies some of the proofs.

2. THE BARTER ECONOMY

2.1 Preferences and Production

Consider an economy with a continuum of differentiated commodities identified by a circle with circumference two, and a continuum of infinitely-lived agents with unit mass identified by points on the same circle.[3] Agents differ in tastes. Agent i has her ideal commodity indexed by i, and her preferences represented by $\Sigma_{t(i)}U_{t(i)}\exp(-rt(i))$, where t(i) denotes the time points at which agent i consumes and r is the constant rate of time preference. All agents have the same rate of time preference. The instantaneous utility

[3] The description is similar to that in Kiyotaki and Wright [15, 16]. The readers who are unfamiliar with this setting are suggested to check these references.

function U is described as follows. If I(j,i) is the length of the arc between commodities j and i along the circle and q is the quantity consumed, the utility derived from consuming commodity j is U(q,I(j,i)). To maintain tractability, we impose

Assumption 2.1 The function U takes the form:

$$U(q,I(j,i)) = \begin{cases} u(q), & \text{if } I(j,i) \leq z \ (<1) \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where u is twice differentiable with $u'>0$, $u''<0$, $u'(0)=\infty$ and $u'(\infty)=0$.

The specification, that all commodities within the distance z generate the same utility, simplifies the discussion of the role of money as a medium of exchange.[4] The parameter z indicates the severity of the lack of double coincidence of wants. A smaller z implies that two randomly selected agents are less likely to have mutually acceptable commodities. The restrictions on the function u are imposed for various existence results in the paper and are satisfied, for example, by the function $u(q)=q^{1-\sigma}$ $(0<\sigma<1)$.

The production technology is characterized as follows. Production opportunities arrive instantly to those who are not trading or holding any production opportunities.[5] An opportunity is characterized by a commodity type drawn randomly and uniformly from the circle. The unit cost of production is assumed to be a constant c in terms of utility, common to all commodities. Production opportunities can be held indefinitely as long as no production

---

[4] If, instead, $U_2(q,I)<0$, there will be a distribution of surpluses in a certain type of trade and hence a distribution of prices. With (1), the distribution degenerates to a point.

[5] Allowing production opportunities to arrive at a finite rate considerably complicates the calculation of the measures of agents in production and trade.

has occurred. Also, producers can produce as much as they want each time using the same production opportunity. However, once the commodity is produced and traded, the corresponding opportunity vanishes. To simplify results, we assume that an agent does not consume what she produces and that all commodities are perishable and hence are produced for immediate consumption.[6] Therefore, a producer will produce only after she has reached an agreement on the terms of exchange with her trading partner.

An important feature of this production technology, distinguished from Diamond [6] and Kiyotaki and Wright [15, 16], is that output is determined endogenously rather than being fixed. Allowing for arbitrary output permits the examination of prices. Accordingly, a more careful description of how trade occurs is necessary.

2.2 Matching and Bargaining

Agents meet through a random matching process. To describe the matching technology, let e and p be, respectively, the state in which agents are bargaining and waiting for trading partner. An agent can only be in one of the two states.[7] Let V_p be the present value of being in state p. For the agents in state p, they are randomly matched in pairs over time according to a Poisson process with a constant arrival rate β. Since the arrival rate is

---

[6] The first assumption is common in search models of money but unnecessary for the existence of valuable fiat money (see [4]). The second assumption prevents some commodities from being used as commodity money. The same objective is achieved in [15,16] by imposing transaction cost. Later we assume that money can be stored. This may give an impression that money has the store-of-value function which commodities do not have. However, this asymmetry is not severe since production opportunities can also be stored.

[7] Throughout the paper, we ignore the degenerate case where all production opportunities are discarded. Thus the measure of agents in state p is always positive.

constant, the matching technology does not exhibit increasing-returns-to-scale which are crucial for Diamond's [6] results. Also, an agent is matched to exactly one other agent at any given time.

Once two agents, say i and j, are matched, they know immediately what the partner can produce and what commodities the partner desires. However, the trading history of each agent is private. Let agents i and j be holding the opportunities for producing i' and j' respectively. Without loss of generality, we assume that producers only trade commodities, not production opportunities.[8] Since all proceeds from trade are consumed immediately, both agents are willing to trade only if $I(j',i) \leq z$ and $I(i',j) \leq z$. If both agents are willing to trade, they bargain over the quantities each produces.

Bargaining takes place in the form of alternating proposals, as depicted in Figure 1. Each proposal consists of a pair of quantities, (q_i, q_j), where q_k $(k=i,j)$ is the quantity proposed for agent k to produce. Let agent i make the first proposal.[9] Given a proposal, agent j responds immediately by accepting, quitting or rejecting the proposal (choices Y, Q and N respectively in Figure 1), where rejection implies staying in the game. If the proposal is accepted, the two agents immediately produce the agreed quantities and consume the partner's output. There is no fixed transaction cost as in Kiyotaki and Wright [15, 16]. The payoffs to

[8] When the two commodities are mutually acceptable to the two producers, trading commodities generates higher utility to both producers than trading either commodity or production opportunity for another production opportunity (since $x^* > V_p$ in subsection 2.3). When at least one of commodities is not acceptable by the partner, the two traders may swap their production opportunities. Doing so merely re-labels the owners of the production opportunities but does not change the equilibrium, since a producer cannot consume what she produces no matter what production opportunity she uses to produce.

[9] In the limit case where the interval between two alternating proposals approaches zero, equilibrium will be independent of who makes the first proposal.

agents i and j are $u(q_j)-cq_i+V_p$ and $u(q_i)-cq_j+V_p$ respectively. If agent j quits, each agent retains V_p, the present value of having no partner. If agent j rejects the proposal, agent i can quit immediately and both agents get V_p. If agent i decides to stay (choice C), agent j makes the next proposal after a fixed interval Δ. The game continues in such a manner until a proposal is accepted or quitting occurs. Once a game is terminated, the two agents depart and can no longer recognize each other. Most of this paper deals with the bargaining outcomes when $\Delta \to 0$.

Figure 1 here.

The bargaining framework is the familiar one in Rubinstein [22] but with a nonlinear bargaining frontier. A similar model is studied by Binmore [3]. The limit bargaining outcome is studied by Binmore [2] and Gale [10]. Besides simplicity, the Rubinstein framework seems to be particularly natural for modeling the markets where agents meet in pairs. Unlike these cited models, however, the present framework has an endogenous bargaining frontier. That is, the outside payments are endogenously determined in equilibrium, although they are exogenously given to the agents in bargaining. This endogeneity is important for multiple equilibria (see Section 3).

The bargaining framework depicted in Figure 1 is adopted for specificity. It is by no means the only possible one for the purpose of the current paper. For example, we could allow agents to search for new partners in the interval Δ as in Rubinstein and Wolinsky [23] and Wolinsky [27]. As publicized by Osborne and Rubinstein [21], changing agents' options in bargaining can affect equilibrium outcomes. While the same sensitivity may appear here, allowing agents to search during bargaining does not eliminate multiple equilibria (see the discussion near the end of Section 3).

We examine the subgame perfect equilibrium of the bargaining game between two agents, say agents i and j. Let the two agents have double coincidence of wants, i.e., agent i's commodity i' and agent j's commodity j' satisfy $I(j',i)\leq z$ and $I(i',j)\leq z$. Begin by examining a typical maximization problem which agent i faces with in the bargaining:

$$\phi(x) \equiv \max_{(q_i,q_j)} \{u(q_j)-cq_i+V_p:\ u(q_i)-cq_j+V_p \geq x\}. \tag{2}$$

$\phi(x)$ is the maximum value obtainable by agent i given the partner's value x. It can be verified that $\phi'=-c/u'(q_i)<0$ and $\phi''<0$. Also, $\phi(\phi(x))=x$ so $\phi^{-1}=\phi$.

To examine the limit case $\Delta\to 0$, we first examine the case $\Delta>0$. Define

$$f(x) \equiv \phi(x) - e^{-r\Delta}\phi(e^{-r\Delta}x). \tag{3}$$

It can be verified that $f'(x)<0$ and $f(\hat{x})=0$ for a unique $\hat{x}$ (>0). $\phi(e^{-r\Delta}\hat{x})=\hat{x}$. For $x=e^{-r\Delta}\hat{x}$, let $(\hat{q},\hat{Q})$ be the corresponding maximizers (q_i,q_j) in (2). Then we have:

Lemma 2.1 If Δ is sufficiently close to zero, then there is a unique subgame perfect equilibrium (SPE for short) whose strategy profile is as follows: An agent always proposes the quantity of production $\hat{q}$ for herself and $\hat{Q}$ for the partner, accepts any proposal which yields utility no less than $e^{-r\Delta}\hat{x}$, rejects any other proposals and stays in the game. The proposing agent never quits after the respondent's rejection. The proposal $(\hat{q},\hat{Q})$ is accepted immediately which gives payoffs $(\hat{x},e^{-r\Delta}\hat{x})$.

Proof First, we show that the potential surplus of trade is positive. It is so if

$$u(q_j)-cq_i > 0, \quad and \quad u(q_i)-cq_j > 0.$$

Since the two inequalities are symmetric with respect to the line $q_i=q_j$, they are satisfied by some pairs of (q_i,q_j) if and only if $A\neq\varnothing$ where

$$A = \{(q_i,q_j): u(q_i)-cq_j>0 \ and \ q_j>q_i\}.$$

However, $A\neq\varnothing$ if and only if $\max_q[u(q)-cq]=u(q^*)-cq^*>0$, which is satisfied.

A positive potential surplus of trade implies $\phi(V_p)>V_p$. In this case, $\phi(V_p)>e^{r\Delta}V_p$ for sufficiently small Δ. Therefore for sufficiently small Δ,

$$\phi(e^{-r\Delta}\hat{x}) - \hat{x} = 0 < \phi(e^{-r\Delta}\cdot e^{r\Delta}V_p) - e^{r\Delta}V_p.$$

Since $\phi(e^{-r\Delta}x)-x$ is decreasing in x, $e^{-r\Delta}\hat{x}>V_p$. The latter result implies that both agents will obtain positive surpluses from the proposal $(\hat{x},e^{-r\Delta}\hat{x})$.

The proof for the remainder of Lemma 2.1 closely follows that presented by Osborne and Rubinstein [21, Proposition 3.5, pp.56-58] for a simpler bargaining problem and hence is omitted. In particular, if x_i (x_j) is the equilibrium value which agent i (j) can obtain in the subgame where she makes the first proposal, then

$$x_i=\phi(e^{-r\Delta}x_j) \ \text{ and } \ x_j=\phi(e^{-r\Delta}x_i).$$

Thus $x_i=x_j=\hat{x}$. Since both agents obtain positive surpluses, the number of cases in [21] reduces to the one described in Lemma 2.1. ■

In Lemma 2.1, agents obtain the Rubinstein [22] payoffs. When $\Delta=0$, however, $f(x)=0$ for all x. To pass the Rubinstein solution to the limit $\Delta\to0$, define F(x) by

$$F(x) \equiv \lim_{\Delta\to0} f(x)/(1-e^{-r\Delta}) = \phi(x)+x\phi'(x) \tag{4}$$

and take the solution to $F(x)=0$ as the limit of the solution to $f(x)=0$.[10] It can be shown that $F'<0$ and there is a unique value, denoted x^*, such that $F(x^*)=0$. Also, $x^*=u(q^*)-cq^*+V_p$ where q^* is the solution to

$$u'(q^*) = c. \tag{5}$$

Note that the properties of u in Assumption 2.1 imply $q^*>0$ and $u(q^*)-cq^*>0$.

Taking limit of the results in Lemma 2.1 gives the following proposition:

Proposition 2.1 When $\Delta\to 0$, there is a unique subgame perfect equilibrium. The proposed quantities converge to (q^*,q^*) which give a present value x^* to both agents. An agent accepts any proposal which gives her no less than x^*, rejects any other proposals but stays in the bargaining. The first proposal (q^*,q^*) is accepted immediately.

The quantities exchanged in barter are completely determined by the cost of production, irrespective of the market value V_p. In contrast, the quantities in a monetary exchange depend more sensitively on the market values, as shown in Section 3.

2.3 Barter Equilibrium

We now determine the equilibrium value of V_p. As is made implicit in subsection 2.2, we focus on stationary equilibrium where this value is constant. Also, we consider only symmetric equilibrium where agents have the same beliefs. Since agents accept a randomly selected commodity with probability z, then in equilibrium a producer should rationally

---

[10] In the simplest case where two agents with the same rate of time preference bargain over one unit of surplus, $f(x)=(1-e^{-2r\Delta})x-(1-e^{-r\Delta})$ and $F(x)=2x-1$. The Rubinstein solution $x=(1+e^{-r\Delta})^{-1}$ is the solution to $f(x)=0$. The limit of this solution when $\Delta\to 0$, 1/2, is the solution to $F(x)=0$.

believe that a randomly encountered producer is willing to accept her commodity with probability z. Because a successful trade occurs when both a producer likes her partner's commodity and the partner likes her commodity, and because each of these events occurs independently with probability z, the probability of a successful trade between two randomly selected producers is z^2. Since a trading partner arrives at t with probability $\beta e^{-\beta t}$ and a successful trade gives the trader a value $x^*=u(q^*)-cq^*+V_p$, then

$$V_p = \int_0^\infty \beta e^{-(r+\beta)t}\left(z^2(u(q^*)-cq^*+V_p)+(1-z^2)V_p\right)dt.$$

Integration leads to

$$rV_p = \beta z^2[u(q^*)-cq^*]. \tag{6}$$

For stationarity of equilibrium, we require the measures of agents in all states to be constant. Since bargaining is resolved immediately, all agents are in state p.

Definition 2.1 A barter equilibrium is a vector (q^*,V_p) which satisfies (5) and (6).

Directly solving q^* and V_p from (5) and (6) proves the following proposition:

Proposition 2.2 There is a unique barter equilibrium.

There are no multiple barter equilibria. In contrast, there are multiple monetary equilibria, as shown in Section 3 below.

3. MONETARY ECONOMY

Now let a measure M (<1) of agents hold fiat money, each holding one unit. It is maintained that a money holder trades either none or her entire unit of money. Such an assumption is restrictive but necessary for tractability (see Section 7 for a discussion). An agent at a given time can either hold money or production opportunity but not both. Let m be the state in which an agent holds money and V_m be the present value of being in state m. Since bargaining is resolved immediately as shown by Proposition 2.1 and also by Proposition 3.1 below, the measure of producers who are searching for trading partners is 1-M.

The bargaining problem between two producers is identical to the one in Section 2 and hence Proposition 2.1 still holds. Bargaining between a money holder and a producer follows the same process as depicted in Figure 1, differing in that each proposal now consists of only the quantity q which the producer produces. If a proposal q is accepted, the payoffs are $u(q)+V_p$ to the money holder and V_m-cq to the producer. Quitting gives V_m and V_p to the two agents respectively.

To economize on notation, use the same notation ϕ to define

$$\phi(x) \equiv \max_q \{u(q)+V_p : V_m - cq \geq x\}. \tag{7}$$

$\phi(x)$ is the money holder's maximum utility given the producer's value x. It can be verified that $\phi'<0$ and $\phi''<0$. Similarly, define the maximum utility to the producer by

$$\Phi(x) \equiv \max_q \{V_m - cq : u(q)+V_p \geq x\}. \tag{8}$$

It can be directly verified that $\phi(\Phi(x))=x$ and hence $\Phi^{-1}=\phi$. With the new meaning of ϕ, define F(•) by (4). Then F'<0 and there is a unique solution, denoted x_p^*, such that $F(x_p^*)=0$. A similar proof to Proposition 2.1 shows the following (see the appendix):

Proposition 3.1 Let $\Delta \to 0$. A money holder and a producer trade only if

$$\phi(V_p) = u\left(\frac{V_m - V_p}{c}\right) + V_p \geq V_m. \tag{9}$$

Given V_p, (9) is satisfied by some values of V_m. Let (9) hold with strict inequality, there is a unique subgame perfect equilibrium in the following cases:[11]

(a) $V_p < x_p^* < \phi^{-1}(V_m)$: The proposal is $q_H = (V_m - x_p^*)/c$. The payoff is $\phi(x_p^*)$ to the money holder and x_p^* to the producer. The first proposal is accepted immediately.

(b) $x_p^* < V_p$: Replace x_p^* by V_p and q_H by $q_L = (V_m - V_p)/c$ in case (a).

(c) $x_p^* > \phi^{-1}(V_m)$: Replace x_p^* by $\phi^{-1}(V_m)$ and q_H by $u^{-1}(V_m - V_p)$ in case (a).

The equilibrium strategy profile is specified in the proof in the appendix.

Remark In case (b), the proposal is accepted immediately although the proposal gives the producer zero surplus of trade. Mixed strategies are <u>not</u> robust to small perturbations in the proposed values. In particular, there is a positive surplus of trade in the bargaining if (9) holds with strict inequality. A proposal which gives the producer $(V_p + \varepsilon)$, where ε is positive and sufficiently small, is feasible and will be accepted immediately.

Given the values V_p and V_m, there is a unique subgame perfect equilibrium to the bargaining problem. However, the equilibrium is different in different cases. Which case emerges depends critically on the values V_p and V_m. Case (a) delivers the limit of the

---

[11] We ignore the borderline cases. For example, when $x_p^* = \phi^{-1}(V_m)$, the bargaining problem has a unique value as the one in case (c) but the strategies may not be unique. In particular, if a proposal which gives the money holder V_m, the money holder may accept, or reject and stay in the game.

Rubinstein payoffs. In case (b), the Rubinstein payoffs are infeasible since they give the producer less than the outside payment V_p. In this case, the subgame perfect equilibrium strategies give the producer exactly V_p and the money holder the entire surplus of trade. Case (c) is opposite to case (b). The dependence of the subgame perfect equilibrium on outside payments is illustrated by Osborne and Rubinstein [21, pp.56-58] with a linear bargaining frontier. The dependence did not arise in barter because we have focused on barter with symmetric producers. With symmetry, there are positive surpluses to both traders (see the proof of Lemma 3.1) and hence the Rubinstein payoffs are feasible.

To determine V_p and V_m, we specify agents' beliefs in equilibrium. Similar to Section 2, a rational producer should believe that a randomly selected producer or money holder accepts her commodity with probability z. Let π be the probability in the mixed strategies with which a producer accepts money. Similar to (6), we have:

$$\begin{aligned} rV_p &= \beta(1-M)z^2[u(q^*)-cq^*] + \beta Mz \max_\pi \pi(V_m-cq-V_p) \\ rV_m &= \max\{rV_p, \beta(1-M)z\pi[u(q)+V_p-V_m]\} \end{aligned} \tag{10}$$

where $\beta(1-M)$ is the rate at which a money holder meets producers. The second equation of (10) has incorporated the assumption that money is free disposal ($V_m \geq V_p$).

Focusing on the equilibrium with $q>0$, we adopt

Definition 3.1 A monetary equilibrium is a vector $(V_p,V_m;q^*,q;\pi)$ with $\pi>0$, $q>0$, $V_p>0$ which satisfies (5) and (10) and conforms with Proposition 3.1.

Lemma 3.1 If $q>0$, then $\pi=1$ and

$$u(q)+V_p>V_m,\quad V_m-V_p\geq cq. \tag{11}$$

Moreover, case (c) of Proposition 3.1 will never occur in a monetary equilibrium.

Proof The first inequality implies that (9) holds with strict inequality and hence π=1 by the remark following Proposition 3.1. To show $u(q)+V_p>V_m$, suppose that it does not hold. Then the second equation of (10) $\Rightarrow V_m=V_p \Rightarrow u(q)\leq 0 \Rightarrow q\leq 0$ (since $u(0)=0$ and $u'>0$ by Assumption 2.1). This contradicts the condition $q>0$. Similarly, for the second inequality in the lemma, suppose $V_m-cq<V_p$, then $\pi=0$ in (10), which leads to $V_m=V_p$ and the contradiction $q\leq 0$. Now, case (c) of Proposition 3.1 cannot occur in equilibrium because it requires $u(q)=V_m-V_p$ which violates the first inequality of the lemma. ■

By Lemma 3.1, all monetary equilibria are pure (π=1). In contrast, in models with indivisible commodities such as Kiyotaki and Wright [16], money can be partially accepted ($0<\pi<1$). Using Lemma 3.1 in (10) yields

$$\begin{aligned} V_m &= \frac{k_3}{1-k_1k_3}\left[u-k_1cq+k_2(u^*-cq^*)\right] \\ V_P &= \frac{1}{1-k_1k_3}\left[k_1(k_3u-cq)+k_2(u^*-cq^*)\right], \end{aligned} \tag{11}$$

where

$$k_1 = \frac{\beta zM}{r+\beta zM}, \quad k_2 = \frac{\beta z^2(1-M)}{r+\beta zM}, \quad k_3 = \frac{\beta z(1-M)}{r+\beta z(1-M)}. \tag{12}$$

To present the central proposition, define q_H and q_L by the following equations:

$$u'(q_H) = \frac{c}{k_3}\left\{1+\frac{(k_3^{-1}-k_1)cq_H}{u(q_H)-k_3^{-1}cq_H+k_2(u^*-cq^*)}\right\} \tag{13}$$

$$L(q_L) \equiv u(q_L)-k_3^{-1}cq_L-z(u^*-cq^*) = 0, \quad L'(q_L)>0. \tag{14}$$

Emphasize the dependence of k and q_L on r by writing them as functions of r. Define r^* by

$$u'(q_L(r*)) = \frac{c}{k_3(r*)}\left\{1 + \frac{(1-k_1(r*))cq_L(r*)}{k_2(r*)(u^*-cq^*)}\right\}. \tag{15}$$

The proof of Proposition 3.2 below shows that q_L, q_H and r^* are well-defined.

Proposition 3.2 A monetary equilibrium exists if and only if $r \le r^*$. When $r<r^*$, two monetary equilibria coexist with different purchasing powers of money q_H and q_L ($<q_H<q^*$). When $r \to r^*$, $q_L \to q_H$. Moreover, the critical value r^* has the following properties:

$$dr^*/d\beta>0, \quad dr^*/dM<0 \quad \text{and} \quad dr^*/dz \text{ ambiguous.} \tag{16}$$

<u>Sketch of proof</u>: The proof consists of five parts. The first part is to rewrite the condition $V_m - cq_H > V_p$ as $L(q_H)>0$ and the condition $V_p > x_p^*$ as the following:

$$u'(q_L) > \frac{c}{k_3}\left\{1 + \frac{(1-k_1)cq_L}{k_2(u^*-cq^*)}\right\}. \tag{17}$$

If we ignore for the moment the borderline case $V_p=x_p^*$ in Proposition 3.1, then the equilibrium with q_H exists iff (13) holds and $L(q_H)>0$; the equilibrium with q_L exists iff $L(q_L)=0$ and (17) holds. (17) implies $L'(q_L)>0$. The second part of the proof finds a necessary condition for existence: $r<r_0$ for some r_0. Under this condition, Part 3 shows that there is a unique solution to (13). Parts 4 and 5 can be pictured by Figure 2, where RHS1(q) and RHS2(q) denote the right-hand sides of (13) and (17) respectively.

Figure 2 here.

Part 4 shows that an equilibrium exists iff the curve u'(q) intersects RHS2(q) before it intersects RHS1(q). Part 5 shows that this indeed happens if $r<r^*$ ($<r_0$).

The borderline case $V_p=x_p^*$ corresponds to the limit $r \rightarrow r^*$ where $q_L \rightarrow q_H$. ■

A monetary equilibrium exists only if the rate of time preference is low. This result is intuitive. When a producer trades for commodities, she can consume immediately. But if she trades for money, she can consume only after she exchanges her money back into some desirable commodities. There is an extra time cost in the monetary exchange, which increases with the rate of time preference. If the rate of time preference is sufficiently large, barter gives the producer higher utility and hence money will not be accepted.

The critical value for the rate of time preference, r^*, depends on β and M in intuitive ways: When agents meet more frequently (higher β) or when fewer agents hold money (M smaller), it is more likely to have monetary equilibrium. Surprisingly, however, the sign of dr^*/dz is ambiguous and in particular can be positive when z is sufficiently small. That is, when the lack of double coincidence of wants is severe, a further increase in the severity can eliminate monetary equilibrium. This result arises since a lower z makes a money holder more difficult to meet producers with acceptable commodities and hence reduces the value of holding money (see (10)).

Proposition 3.2 states that the two monetary equilibria always coexist (except the special case $r=r^*$). The multiplicity cannot be excluded by restricting parameters values, as in the common monetary models, or by restricting the value of holding money to be strictly positive, as in Kiyotaki and Wright [15, 16]. Also, the multiplicity does <u>not</u> rely on assumptions such as increasing-returns-to-scale in Diamond [6]. Nevertheless, multiplicity relies on the opportunity to barter. If barter is excluded, then $q_L=0$.

Since the equilibrium with q_L corresponds to the constrained case (b) in Proposition 3.1, it raises the question whether multiplicity disappears under different bargaining frameworks which eliminate the constrained case. A particularly relevant alternative framework is to allow agents to search for new partners between bargaining rounds, as specified in [23] and [27]. With this alternative framework and suitable conditions, Trejos and Wright [26] have shown that the sequential bargaining solution in a monetary trade is equivalent to the following Nash bargaining solution:[12]

$$q = argmax\left(u(q)+V_p-V_M\right)\left(V_m-cq-V_p\right).$$

Thus when (9) holds with strict inequality, both the producer and the money holder obtain positive surpluses. That is, there is no constrained bargaining outcome like case (b) or (c) in Proposition 3.1. However, multiple monetary equilibria still appear.

The fundamental reason for multiplicity in this paper and in [26] is that the payoffs from bargaining depend on the values V_p and V_m which in turn depend on agents' beliefs on the payoffs from bargaining. If a producer believes that monetary exchange does not significantly increase her utility, as in case (b) of Proposition 2.1, she will exchange fewer commodities for money. And if all producers believe so then the low purchasing power of money is self-fulfilled. This is why the resulting purchasing power of money is low in case

[12] Trejos and Wright [26] have also examined the monetary equilibrium within the same bargaining framework as in this paper and investigated its dynamic features. Binmore [2] has shown the equivalence between the sequential bargaining solution and the Nash bargaining solution in isolated bargaining problems.

(b) even though the money holder obtains the entire surplus of trade in that case. The dependence of equilibrium on agents' beliefs makes the economy vulnerable to extrinsic shocks, as shown in Section 5.

The equilibria in this paper are closely related to those in the search models of money with fixed price. In this paper, there are two pure monetary equilibria where money is accepted with probability one. In addition, there is a nonmonetary equilibrium with q=0 where no one accepts money, which has been precluded by our focus on monetary equilibrium. Taking together, there are self-fulfilling beliefs on three levels of the purchasing power of money, $(0,q_L,q_H)$. In models with fixed prices, prices cannot vary by assumption, but the probability with which money is accepted can vary. Accordingly there are self-fulfilling beliefs on three levels of probability, (0,z,1) (see [16]).

4. PROPERTIES OF MONETARY EQUILIBRIUM

The purchasing power of money in the two equilibria has very different dependence on parameters, as shown by the following:[13]

$$\partial q_H/\partial r<0,\quad \partial q_H/\partial \beta>0,\quad \partial q_H/\partial M \textit{ ambiguous},\quad \partial q_H/\partial z>0. \tag{18}$$

$$\partial q_L/\partial r>0,\quad \partial q_L/\partial \beta<0,\quad \partial q_L/\partial M>0,\quad \partial q_L/\partial z \textit{ ambiguous}. \tag{19}$$

To understand the features of q_H in (18), take the sign of $\partial q_H/\partial r$ for example. As is explained in Section 3, a high rate of time preference makes monetary exchange more costly and hence

[13] All proofs of the properties and propositions in this section are straightforward manipulations and hence are omitted.

tends to reduce V_m. A decrease in V_m reduces the bargaining power of the money holder by decreasing her own outside payments, decreasing the value of a proposal to the producer, V_m-cq, and making the producer less eager to accept the proposal. However, an increase in r also increases the cost of barter, reduces V_p and affects the bargaining power of the two traders in the opposite direction to a decrease in V_m. Overall, an increase in r reduces the relative bargaining power of the money holder.

The purchasing power of money q_L has the opposite dependence on the rate of time preference. The perverse feature stems from the nature of the bargaining outcome in case (b) of Proposition 3.1, which gives the producer the outside value V_p and the money holder the entire surplus of trade. Since V_p decreases with r, the surplus of trade and hence the purchasing power of money increases with r.

Two other features in (18) and (19) are noteworthy. The first is that the purchasing power can increase with the money supply in both equilibria. In particular, it can be verified from (13) that $\partial q_H/\partial M > 0$ if r and M are close to zero and if

$$z < \left[(u(q^*)cq^*)^{1/2} - cq^*\right]/(u(q^*)-cq^*).$$

The positive effect reflects the fact that money facilitates exchange. However, the effect becomes negative when too much money is chasing too few goods. The second feature in (18) is the seemingly counter-intuitive result $\partial q_H/\partial z > 0$. That is, the lack of double coincidence of wants <u>reduces</u> the purchasing power of money in the equilibrium with q_H. Nevertheless, since a decrease in z tends to reduce both V_m and V_p, the result can be explained along the same line used to explain the sign of $\partial q_H/\partial r$.

The remainder of this section examines three implications of the differences between

q_H and q_L. The first is on the income velocity of money. Note that the flow of output in a monetary equilibrium is[14]

$$Y_m = \beta z(1-M)[(1-M)zq^* + Mq]. \tag{20}$$

Let us take 1/q as the price level. The income velocity of money is

$$v \equiv Y_m/(qM) = \beta z(1-M)\left[1 + \frac{(1-M)zq^*}{Mq}\right]. \tag{21}$$

The properties in (18) and (19) imply that the velocity of money is an increasing function of r in the equilibrium with q_H but a decreasing function of r in the equilibrium with q_L. If the rate of time preference equals the long-run real interest rate, as is often the case in an infinite horizon growth model, then the velocity of money increases with the real interest rate in the equilibrium with q_H but decreases in the equilibrium with q_L.

The second implication of the differences between q_H and q_L is on how money affects output. Note that the flow of output in a barter economy is $Y_B=\beta z^2 q^*$.

Proposition 4.1 $Y_m<Y_B$ if either $z>1/2$ or $M>M_1\equiv(1-2z)/(1-z)$. When $z<1/2$ and r is sufficiently close to zero, there exists an $M_H<M_1$ such that $Y_m>Y_B$ for $M<M_H$ in the equilibrium with q_H. If, in addition,

$$u(2zq^*) - 2zcq^* - z(u^* - cq^*) < 0 \tag{22}$$

then there exists an $M_L<M_H$ such that $Y_m>Y_B$ for $M<M_L$ in the equilibrium with q_L.

Therefore, using money can increase aggregate output in both equilibria when the rate

---

[14] Since all commodities are consumed immediately, consumption equals output. (20) can be more easily interpreted as consumption of a representative trader.

of time preference and the money supply are sufficiently small. However, when the lack of double coincidence is sufficiently severe, (22) is violated and money can increase aggregate output only in the equilibrium with q_H.

Finally, the two equilibria differ in efficiency. To examine efficiency, use superscripts B and m to index the value V_p in the barter and the monetary equilibria. Note that $V_m > V_p^m$ (Lemma 3.1). If $V_p^m > V_p^B$, the monetary equilibrium Pareto dominates the barter equilibrium because using money makes all agents better off. If $V_p^B > V_m$, all agents are worse off in the monetary economy than in the barter equilibrium. One monetary equilibrium Pareto dominates another monetary equilibrium if the first equilibrium has higher values V_p and V_m .

Proposition 4.2 The monetary equilibrium with q_H Pareto dominates the monetary equilibrium with q_L. The monetary equilibrium with q_L cannot dominate the barter equilibrium. When r is sufficiently close to zero, the barter equilibrium dominates the monetary equilibrium with q_L, but is dominated by the monetary equilibrium with q_H if $z<1/2$ and $M<M_1$, where M_1 is defined in Proposition 4.1.

The conditions $z<1/2$ and $M<M_1$ are required for the monetary equilibrium with q_H to dominate the barter equilibrium. This is because money crowds out production opportunities in the economy. The conditions ensure that the exchange-facilitating role of money dominates the opportunity-crowding effect. Under suitable conditions, the optimal quantity of money can be positive in the equilibrium with q_H. To see this, let the social welfare function be a weighted average of the values of the agents in different states: $W=(1-M)V_p+MV_m$. The limit $\lim_{r\to 0}(rW)$ is maximized at $M=M_1=(1-2z)/(1-z)$. Thus when $z<1/2$ and the rate of time preference is small, the optimal quantity of money is positive and an increasing function of the lack of double coincidence of wants.

5. SUNSPOT EQUILIBRIA

We now show that a monetary economy is more vulnerable to extrinsic shocks than a barter economy. This is done by showing that sunspot equilibria exist in a monetary economy but not in a barter economy. As remarked in the introduction, Wright [30] have established the existence of sunspot monetary equilibria which involve one state of economy where money is not universally accepted. For contrast, we focus on those sunspot equilibria where money is always accepted.

Let the extrinsic shocks have two realizations, denoted by ω (=1,2) and called the states of the aggregate economy. (They should be distinguished from the states p and m.) The state evolves as follows. When the state is ω, it switches into Ω ($\neq\omega$) following a Poisson process at the rate λ_Ω. The shocks have no effect on the fundamentals such as the preferences, technology and the trading process, but change agents' beliefs on the values V_p and V_m. In state ω, the beliefs on these values are V_p^ω and V_m^ω.

First consider a barter economy. Let (q_i^ω, q_j^ω) be the quantities proposed in a bilateral exchange between agents i and j in state ω. Define ϕ^ω by adding a superscript ω to the variables (including V_p) in (2). Let $\hat{x}_i^\omega$ and $\hat{x}_j^\omega$ be the solutions to

$$\hat{x}_i^\omega = \phi^\omega\left(e^{-(r+\lambda_\Omega)\Delta}\hat{x}_j^\omega + e^{-r\Delta}(1-e^{-\lambda_\Omega\Delta})\hat{x}_j^\Omega\right), \quad i\neq j;\ \omega\neq\Omega,\ \ \omega,\Omega\in 1,2. \tag{23}$$

$\hat{x}_i^\omega$ ($\hat{x}_j^\omega$) is the Rubinstein payoff to agent i (j) in state ω when she makes the first proposal. The argument in ϕ^ω is the expected present value to agent j in state ω if she rejects agents i's proposal. By rejecting the proposal, agent j can obtain $\hat{x}_j^\omega$ after an interval Δ if the state of the economy is ω and $\hat{x}_j^\Omega$ if the state is Ω. The present value is calculated with the discount factor $\exp(-r\Delta)$ and the probability $\exp(-\lambda_\Omega\Delta)$ with which the state is ω after Δ.[15]

Focus on $\hat{x}_i^\omega$. Similar to (4), one can show from (23) that $x_i^{*\omega} \equiv \lim_{\Delta\to 0}\hat{x}_i^\omega$ solves

$$\begin{aligned} F^\omega(x^\omega, x^\Omega) &= 0, \quad \Omega \neq \omega, \ \omega, \Omega = 1,2 \\ F^\omega(x^\omega, x^\Omega) &= \frac{r+\lambda_\Omega}{r}\left[\phi^\omega(x^\omega)+x^\omega\phi^{\omega\prime}(x^\omega)\right]-\frac{\lambda_\Omega}{r}\left[\phi^\Omega(x^\Omega)+x^\Omega\phi^{\omega\prime}(x^\omega)\right] \end{aligned} \tag{24}$$

where the subscript i of x is suppressed. Similar to Proposition 2.1, $x_i^{*\omega}$ is the subgame perfect equilibrium payoff to agent i in state ω if $x_i^{*\omega} > V_p^\omega$ for $\omega=1,2$. If $x_i^{*\omega} \leq V_p^\omega$, there is no trade in state ω. The following proposition rules out sunspot barter equilibria:

Proposition 5.1 The only barter equilibrium with positive trade in all states of the economy is such that $x_i^{*\omega}=x_j^{*\omega}=x^*=u(q^*)-cq^*+V_p$.

<u>Proof</u> First, we show that $x_i^{*\omega}=x_j^{*\omega}=x^{*\omega}$ for $\omega=1,2$. Since $\phi^\omega(x_j^{*\omega})=x_i^{*\omega}$ and $\phi^\omega(x_i^{*\omega})=x_j^{*\omega}$ by (23), agent i obtains $x_i^{*\omega}$ and agent j obtains $x_j^{*\omega}$ no matter who makes the first proposal. Now follow the derivation for (6) to obtain the expected value of being a producer in state ω. For agent i,

[15] Strictly speaking, the probability with which the state is ω after Δ includes not only the probability with which the event Ω never occurs in Δ, expressed as $\exp(-\lambda_\Omega\Delta)$, but also that with which the event Ω occurs during Δ but is eventually replaced by ω. The second probability is in the order of Δ^2 and hence is negligible.

$$rV_p^{\omega} = \beta z^2(x_i^{*\omega}-V_p^{\omega})+\lambda_{\Omega}\left(V_p^{\Omega}-V_p^{\omega}\right), \quad \Omega\neq\omega. \tag{25}$$

For agent j, replace $x_i^{*\omega}$ by $x_j^{*\omega}$. Since the value V_p^{ω} is the same for both agents, we must have $x_i^{*\omega}=x_j^{*\omega}$. Now we have $x^{*\omega}=\phi^{\omega}(x^{*\omega})$ which, with the definition of $\phi^{\omega}(x)$, implies $x^{*\omega}=u^*-cq^*+V_p^{\omega}$. That is, $x^{*\omega}-V_p^{\omega}$ is independent of ω. Then (25) implies that V_p^{ω} and hence $x^{*\omega}$ is independent of ω. ■

For the monetary economy, suppose agents believe that the purchasing power of money is so low in state 2 that the producer obtains V_p^2, as in case (b) of Proposition 3.1. We show that this belief can be supported by equilibrium. Define ϕ^{ω} and Φ^{ω} by adding superscript ω to (7) and (8).Then

$$\begin{aligned} x_m^1 &= \phi^1\left(e^{-(r+\lambda_2)\Delta}x_p^1+e^{-r\Delta}(1-e^{-\Delta\lambda_2})V_p^2\right) \\ x_m^2 &= \phi^2\left(e^{-(r+\lambda_1)\Delta}V_p^2+e^{-r\Delta}(1-e^{-\Delta\lambda_1})x_p^1\right) \\ x_p^1 &= \Phi^1\left(e^{-(r+\lambda_2)\Delta}x_m^1+e^{-r\Delta}(1-e^{-\Delta\lambda_2})x_m^2\right) \end{aligned} \tag{26}$$

where x_m^{ω} is the equilibrium payoff to a money holder in state ω when she makes the first proposal; x_p^1 is the equilibrium payoff to a producer in state 1 when she makes the first proposal. For $(x_m^1,x_p^1,x_m^2,V_p^2)$ indeed to be equilibrium payoffs, we require

$$x_p^1>V_p^1, \quad V_p^2>\Phi^2\left(e^{-(r+\lambda_1)\Delta}x_m^2+e^{-r\Delta}(1-e^{-\Delta\lambda_1})x_m^1\right). \tag{27}$$

In particular, the second inequality requires that in state 2 the Rubinstein payoff be infeasible and hence that the producer obtain V_p^2. We have:

Proposition 5.2 Let $\Delta\to 0$. When λ_1 and λ_2 are sufficiently small, there are sunspot monetary equilibria where the purchasing power of money is q^{ω} (>0) in state ω. $(q^1,q^2)\approx(q_H,q_L)$. The quantity of trade between two commodity producers is still q^*.

Proof Let $\Delta\to 0$ in (26). We have

$$\begin{aligned} x_m^1 &= \phi^1(x_p^1), \quad x_m^2 = \phi^2(V_p^2) \\ 0 = F(x_p^1,V_p^2) &\equiv \frac{r+\lambda_2}{r}\left[\phi^1(x_p^1)+x_p^1\phi^{1'}(x_p^1)\right]-\frac{\lambda_2}{r}\left[\phi^2(V_p^2)+V_p^2\,\phi^{1'}(x_p^1)\right]. \end{aligned} \tag{28}$$

Since F(•,•) is decreasing in the first argument, then (27) is equivalent to

$$\begin{aligned} &F(V_p^1,V_p^2) > 0 \\ &(r+\lambda_1)\left[\phi^2(V_p^2)+V_p^2\,\phi^{2'}(V_p^2)\right]-\lambda_1\left[\phi^1(x_p^1)+x_p^1\phi^{2'}(V_p^2)\right] < 0. \end{aligned} \tag{29}$$

Similar to (10), we have

$$\begin{aligned} rV_p^1 &= \beta(1-M)z^2(u^*-cq^*)+\beta zM(x_p^1-V_p^1)+\lambda_2(V_p^2-V_p^1) \\ rV_p^2 &= \beta(1-M)z^2(u^*-cq^*)+\lambda_1(V_p^1-V_p^2) \\ rV_m^{\omega} &= \beta(1-M)z\left[u(q^{\omega})+V_p^{\omega}-V_m^{\omega}\right]+\lambda_{\Omega}\left(V_m^{\Omega}-V_m^{\omega}\right), \quad \omega\neq\Omega, \quad \omega,\Omega=1,2. \end{aligned} \tag{30}$$

Let $\lambda_1\to\lambda_2\to 0$. Since $x_p^1=V_m^1-cq^1$ and $V_p^2=V_m^2-cq^2$, one can verified that the quantities $(q^1,q^2)=(q_H,q_L)$ give payoffs which satisfy (28), (29) and (30). By continuity, there exist $(q^1,q^2)\approx(q_H,q_L)$ that induce values $(x_p^1, V_p^{\omega}, V_m^{\omega})$ to satisfy the same conditions when $(\lambda_1,\lambda_2)\approx(0,0)$. ■

When the purchasing power of money fluctuates in the sunspot equilibria, so do the aggregate output and welfare. It can be shown that the sunspot equilibria are Pareto dominated by the deterministic equilibrium with q_H, a result similar to that in Cass and Shell [5]. Therefore policies which stabilize the purchasing power of money at q_H are Pareto improving in the event of sunspot equilibria.

6. EXCHANGE RATES AND EXCHANGE POLICY

We now extend the model to incorporate two currencies. Different from previous search models with two currencies such as [19, 16, 31], we study the differences in the purchasing powers of the currencies rather than their acceptability. The extension enables us to examine the exchange rate and exchange policies. We intend to answer the following questions: (a) Is the nominal exchange rate determinate? (b) Do currency substitution and exchange intervention improve welfare? (c) Does a floating exchange regime improve welfare relative to a fixed regime? The answers to these questions are contrasted with those obtained in the common monetary models. We delay all the proofs for this section to the appendix.

Consider an economy with two fiat monies, red and green. A money holder can hold only one type of money at any given time. Let M be the total supply of the two monies and μ_J be the proportion of money J (=R,G). Let q^J be the purchasing power of money J. The (implicit) nominal exchange rate between the two monies is $E \equiv q^R/q^G$.[16] With a superscript J, define ϕ^J and Φ^J similarly to (7) and (8). Let x^J satisfy

$$F^J(x^J) \equiv \phi^J(x^J) + x^J \phi^{J\prime}(x^J) = 0, \quad J = R, G. \tag{31}$$

Similar to Proposition 3.1, if $x^J > V_p$, then x^J is the equilibrium payoff to the producer in the bargaining with a holder of money J (when $\Delta \to 0$). Otherwise V_p is the payoff. Since money J has a higher purchasing power when $x^J > V_p$ than when $x^J \leq V_p$, we say that it is <u>strong</u> if $x^J > V_p$. Since the function $F^J(\bullet)$ is decreasing, money J is strong iff

[16] The exchange rate is implicit because there is no direct exchange between the two monies in the model constructed here. Direct exchange can be generated by introducing asymmetric tastes into the model, as Zhou [31] has done in a model of fixed price.

$$F^J(V_p) > 0, \quad J=R,G. \tag{32}$$

This section focuses on two pure monetary equilibria, one in which both currencies are strong and another in which the green currency is strong but the red currency is weak.[17] Comparing the two equilibria, we draw implications on exchange policies.

Since trading with a money holder gives a producer either x^J or V_p, then

$$cq^J = V_J - \max(x^J, V_p), \quad J=R,G, \tag{33}$$

where V_J is the present value of holding money J. In equilibrium, these values satisfy

$$\begin{aligned} rV_p &= \beta(1-M)z^2(u^* - cq^*) + \beta z\mu_R M(V_R - cq^R - V_p) + \beta z\mu_G M(V_G - cq^G - V_p) \\ rV_J &= \beta(1-M)z(u(q^J) + V_p - V_J), \qquad J=R,G \end{aligned} \tag{34}$$

where we have used the results $u(q^J)+V_p-V_J>0$ and $V_J-V_p \geq cq^J$, which can be shown by the same arguments for Lemma 3.1.

Proposition 6.1 The only equilibrium in which both monies are strong is such that $q^R=q^G=q_H$. Such an equilibrium exists if $r<r^*$, where r^* is defined in (15).

When both monies are strong, the nominal exchange rate is fixed at one and is independent of the money supply. The economy delivers the same outcome as the monetary equilibrium with one strong money. In this case, currency substitution and exchange market intervention have no effect on the exchange rate or on welfare. The result that the exchange rate must be fixed if both monies are strong is in accordance with the result obtained by

[17] There are two other monetary equilibria, one in which both currencies are weak and another in which the red currency is strong but the green currency is weak.

Kareken and Wallace [14] in an overlapping generations model,[18] but the specific equilibrium value for the exchange rate contrasts with the continuum of values obtained by those authors.

Now we search for an equilibrium with a strong money and a weak money. Define $\tilde{q}^G$ and $\tilde{q}^R$ by:

$$u'(\tilde{q}^G) = \frac{c}{k_3}\left\{1 + \frac{[1-k_1(k_3\mu_G+\mu_R)]c\tilde{q}^G}{(1-k_1\mu_R)(k_3u^G-c\tilde{q}^G)+k_2k_3(u^*-cq^*)}\right\} \tag{35}$$

$$k_3u(\tilde{q}^R)-c\tilde{q}^R-\frac{1-k_3}{1-k_1(k_3\mu_G+\mu_R)}\left\{k_1\mu_G(k_3u^G-c\tilde{q}^G)+k_2(u^*-cq^*)\right\} = 0. \tag{36}$$

Proposition 6.2 If $r<r^*$, there is an equilibrium where the green money is strong and the red money is weak. The purchasing power of money J is $\tilde{q}^J$ ($\tilde{q}^G>\tilde{q}^R$). Furthermore,

$$\frac{d\tilde{q}^R}{d\mu_R} < 0, \quad \frac{d\tilde{q}^G}{d\mu_R} < 0 \quad \textit{and when} \quad r\to 0, \quad \frac{dE}{d\mu_R} < 0. \tag{37}$$

A weak currency and a strong currency can both be universally accepted. The nominal exchange rate is less than one. The coexistence of two currencies with different purchasing power is a novel feature of the model. In contrast, two currencies are universally accepted in Matsuyama, et al. [19] only when they exchange for the same quantity (one unit) of commodities. Another interesting feature of the equilibrium is that an increase in the proportion of the weak currency not only reduces its own purchasing power, but also reduces

---

[18] Extending [14] by allowing the exchange rate to depend on infinite history, Manuelli and Peck [17] show that the exchange rate follows a martingale process.

the purchasing power of the strong currency. To explain this, note from (34) that the difference V_J-V_p (J=R, G) is positively related to the "average" purchasing power of money, $\mu_R q^R+\mu_G q^G$. Thus for given quantities q^R and q^G, an increase in the proportion of the weak currency reduces V_J-V_p and hence reduces the relative bargaining power of the money holders (see Section 4 for a discussion).

Now an exchange market intervention, which reduces the share of the weak money but maintains the total money supply constant, has complicated effects on the exchange rate. However, when the rate of time preference is sufficiently close to zero, the intervention increases the exchange rate. The welfare effect, on the other hand, is much simpler. Manipulations of (34) and (36) show that a reduction in the share of the weak money increases V_p, V_R and V_G and hence has a positive welfare effect. By reducing μ_R to 0, policies can induce the economy toward the equilibrium with q_H. For the same welfare result, a country with a weak currency can peg its currency one-to-one on the strong foreign currency.

Propositions 6.1 and 6.2 imply that whether currency substitution improves welfare depends on whether the national currency is already strong and whether the currency introduced for competition is strong. If both currencies are strong, currency substitution has no welfare effect, as argued before. If the national currency is strong, introducing a weak foreign currency reduces welfare. If the national currency is weak, however, introducing a strong currency for competition increases welfare. Since the equilibrium with a strong money and a weak money is Pareto dominated by the equilibrium with a single strong money, such substitution is Pareto dominated by the policy of pegging the national currency onto the strong foreign currency.

The welfare effect of exchange policies contrasts with those obtained by Helpman [12], who shows in a cash-in-advance model that both fixed and floating exchange regimes are efficient and deliver the same allocations as in the barter equilibrium. Also in a cash-in-advance model, Woodford [28] argues that currency substitution can generate indeterminate exchange rate and presumably reduce welfare. Our result allows a positive welfare effect of currency substitution.

7. CONCLUSIONS

In a model of search and bargaining, this paper has examined how the purchasing power of money or the nominal price is determined by the strategic behavior of traders in bilateral exchanges. Using the Rubinstein [22] framework to model the strategic behavior, the paper shows that two pure monetary equilibria always coexist which differ in the purchasing power of money and in output, despite uniqueness of the subgame perfect equilibrium for the bargaining problem. The multiplicity has interesting implications on welfare, the velocity of money, the vulnerability of the economy to extrinsic shocks and the effectiveness of exchange policies.

The paper can be extended in two major ways. The first extension is to abandon the assumption that a money holder spends all her money in each trade. The restrictive nature of this assumption is that we cannot disentangle an increase in the money supply from an increase in the measure of money holders. For the purpose of examining the effects of money, it is more appropriate to allow money holders to spend only part of their money. The difficulty involved in this extension is that agents' strategies depend on their money holdings and hence on their histories of trading. In general, there will be a distribution of money holdings and hence of prices. One possible way to find this distribution is to apply the technique in Diamond and Yellen [7]. However, since commodities are indivisible in [7], the technique has to be modified to deal with divisible commodities.

The second extension is to introduce other assets and allow portfolio choices. This extension will enhance our understanding of the robustness of valuable fiat money found in search-theoretic models. The research along this line has already begun. Credit has been introduced into the model with fixed price by Hendry [13] and into the model with endogenous price by Shi [24]. In both models, money can still be valuable despite the competition of credit and multiple monetary equilibria continue to appear.

APPENDIX

1. Proof of Proposition 3.1

To see that (9) is necessary for the money holder and the producer to trade, note that any acceptable proposal q must satisfy $u(q)+V_p \geq V_m$ and $V_m - cq \geq V_p$. Otherwise at least one of the agents will quit. These two inequalities give rise to (9). Given V_p, (9) is satisfied, with strict inequality, by $V_m = V_p + cq^*$.

To specify the strategy profile for each case of the proposition. Let $\Delta>0$ and Δ be sufficiently close to zero. Define

$$x_m = \phi\left(e^{-r\Delta}x_p\right) \quad and \quad x_p = \Phi\left(e^{-r\Delta}x_m\right). \tag{A.1}$$

Then $f(x_p)=0$ where f takes the form of (3) but with ϕ defined in (7). Since $\phi'<0$ and $\phi''<0$, it can be shown that there is a unique solution to $f(x_p)=0$. Although $\phi(V_p)>V_p$, as in the proof of Lemma 2.1, the inequality no longer implies $e^{-r\Delta}x_p > V_p$. As a result, the SPE depends on the values of V_p and V_m. The strategy profile for Proposition 3.1 is obtained by taking the limit $\Delta \rightarrow 0$ of the following Lemma.

Lemma A1.1 If (9) holds with strict inequality and if Δ is sufficiently close to zero, then there is a unique SPE of the bargaining between the producer and the money holder in the following cases.

(a) $e^{-r\Delta}V_p < x_p < \phi^{-1}(V_m)$: The money holder always proposes $q=(V_m - e^{-r\Delta}x_p)/c$, accepts any proposal which gives her no less than $e^{-r\Delta}x_m$, rejects any other proposals but stays in the game. The producer always proposes q such that $u(q)+V_p = e^{-r\Delta}x_m$, accepts any proposal which gives her no less than $e^{-r\Delta}x_p$, rejects any other proposals but stays in the game. Also, the proposing agent never quits after a rejection of the proposal. The first proposed values, $(x_m, e^{-r\Delta}x_p)$ by the money holder or $(e^{-r\Delta}x_m, x_p)$ by the producer, are accepted immediately.

(b) $x_p<e^{-r\Delta}V_p$: Replace $e^{-r\Delta}x_p$ by V_p and x_m by $\phi(V_p)$ in (a). In this case, the producer rejects any proposal which gives her less than V_p and quits.

(c) $x_p>\phi^{-1}(V_m)$: Replace $e^{-r\Delta}x_m$ by V_m and x_p by $\phi^{-1}(V_m)$ in (a). In this case, the money holder rejects any proposals which give her less than V_m and quits.

Proof Again follow closely that presented by Osborne and Rubinstein [21, pp.56-58]. ■

2. Proof of Proposition 3.2

Ignore for a moment the borderline case $V_p=x_p^*$ in Proposition 3.1. We divide the proof into five parts.

Part 1 The monetary equilibrium with q_H exists iff (13) holds and $L(q_H)>0$ where the function L is defined in (14). The equilibrium with q_L exists iff (14)and (17) hold.

Proof Note that $x_p^*=V_m-cq_H$ if $x_p^*>V_p$. Condition (13) is derived from $F(x_p^*)=0$. Since $V_m-cq_H>V_p$, one can verify with (11) that $L(q_H)>0$. For the equilibrium with q_L, $V_m-cq_L=V_p$ and hence $L(q_L)=0$. The condition required for such an equilibrium to exist is $V_p>x_p^*$ or equivalently $F(V_p)<0$. This latter condition and the equation $L(q_L)=0$ imply (17) after the substitution of (11).

Part 2 Use the symbols $k_3(r)$ and $L(q,r)$ to emphasize the dependence of k_3 and the function L on r. Define $q^0(\bullet): R_+\rightarrow R_+$ by

$$u'(q^0(r)) = c/k_3(r). \tag{A.2}$$

Define r_0 as the solution to $L(q^0(r),r)=0$. A monetary equilibrium exists only if $r<r_0$.

Proof It is clear that $q^{0\prime}(r)<0$ and $q^0(r)<q^*$ $\forall r>0$. Since $L(q^0(r),r)$ is a decreasing function of r and

$$\lim_{r\to 0} L(q^0(r),r) = (1-z)(u^*-cq^*) > 0, \quad \lim_{r\to\infty} L(q^0(r),r) < 0,$$

then r_0 is well-defined and $L(q^0(r),r)>0 \Leftrightarrow r<r_0$.

Now (13) and (17) indicate that $u'(q)>c/k_3$ for $q=q_H$ and q_L, and hence $q_H<q^0(r)$ and $q_L<q^0(r)$. Since both monetary equilibria require $L(q)\geq 0$ and since $L(q)$ attains maximum at $q^0(r)$, it is necessary that $L(q^0(r),r)>0$, or equivalently $r<r_0$.

Part 3 Let $r<r_0$. There is a unique solution to (13).

Proof Call the right-hand-side of (13) $RHS1(q_H)$. We show that $RHS1'(q)>0$. Note first that if $r<r_0$ then

$$u(q^0)-k_3^{-1}cq^0+k_2(u^*-cq^*) > 0. \tag{A.3}$$

Next, it is easy to verify that $RHS1'(q)>0 \Leftrightarrow$

$$s(q) \equiv \left[u(q)+k_2(u^*-cq^*)\right]/q - u' > 0. \tag{A.4}$$

Suppose $s(q)\leq 0$. Since $s'(q)=-u'' - s(q)/q$, then for all q such that $s(q)=0$ we have $s'(q)>0$. Therefore if there is any solution to $s(q)=0$, that solution must be unique. Suppose there is such a solution and denote it by q^+. Then $s(q)\leq 0$ if and only if $q\leq q^+$. Since $s'(q)>0$ if $s(q)\leq 0$, then $s'(q)>0$ for $q\leq q^+$. This implies $s(q^+)>s(0)$. Since $\lim_{q\to 0} u'q/u<1$ by Assumption 2.1, then $\lim_{q\to 0} s(q)>0$ and hence $s(q^+)>0$. A contradiction. Therefore there is no such q (>0) that $s(q)=0$. In this case, $s(q)<0$ $\forall q>0$ and hence $s(q^0)<0$, a contradiction to (A.3). Therefore $s(q)>0$ and $RHS1'(q)>0$.

Now it is easy to see that there is a unique solution to (13) if there is any solution at all. The following facts show that there is at least one solution to (13):

$$u'(0) = \infty > c/k_3 = RHS1(0)$$
$$u'(q^0) = c/k_3 < RHS1(q^0)$$

Part 4 Denote the right-hand-side of (17) by RHS2(q). Define q_{cr} by

$$u'(q_{cr}) = RHS2(q_{cr}). \tag{A.5}$$

A monetary equilibrium exists if and only if $q_{cr}>q_L$. If $q_{cr}>q_L$, both the equilibrium with q_H and that with q_L exist. Moreover, $q_L<q_{cr}<q_H<q^0$.

__Proof__ First, one can verify that RHS2(q)>RHS1(q) if and only if L(q)>0. Thus we can depict a situation of existence in Figure 2. The condition $q_{cr}>q_L$ is sufficient for the two equilibria to exist: It implies that $u'(q_L)>u'(q_{cr})=RHS2(q_{cr})>RHS2(q_L)$ and that $L(q_H)>L(q_{cr})>L(q_L)>0$. For necessity, suppose $q_{cr}\leq q_L$. Since RHS1(q)>RHS2(q) if and only if $q<q_L$ and since u' is decreasing, then $q_H<q_{cr}$. In this case, $L(q_H)<L(q_L)=0$ and hence the equilibrium with q_H does not exist. At the same time, $u'(q_L)<u'(q_{cr})=RHS2(q_{cr})<RHS2(q_L)$ and hence the equilibrium with q_L does not exist either.

Part 5 $q_{cr}>q_L \Leftrightarrow r<r^*$ where r^* is defined in (15). r^* has the properties in (16).

__Proof__ The condition $r<r_0$ is required for q_L to be well-defined. Let $r<r_0$. Emphasize the dependence of q_{cr} and q_L on r. It can be shown from the definitions of q_L and q_{cr} that $dq_{cr}/dr<0$ and $dq_L/dr>0$. It can also be shown that

$$\lim_{r\to r_0} q_L = q^0(r_0) > \lim_{r\to r_0} q_{cr}, \quad and \quad \lim_{r\to 0} q_L < q^* = \lim_{r\to 0} q_{cr}.$$

Thus there is a unique solution $r^* < r_0$ to (15), and $q_{cr} > q_L \Leftrightarrow r < r^*$. The properties in (16) can be directly verified from (15) and (14). This completes the proof of Part 5.

The borderline case $V_p = x_p^*$ corresponds to the limit case $r \to r^*$. In this limit case, $q_L = q_H = q_{cr}$. ■

3. Proofs of Propositions 6.1 and 6.2

First, derive the following formulas from (34):

$$V_p = \frac{1}{1-k_1k_3}\{\mu_R k_1(k_3u^R - cq^R) + \mu_G k_1(k_3u^G - cq^G) + k_2(u^* - cq^*)\} \tag{A.6}$$

$$V_R = \frac{k_3}{1-k_1k_3}\{(1-\mu_G k_1k_3)u^R - \mu_R k_1cq^R + \mu_G k_1(k_3u^G - cq^G) + k_2(u^* - cq^*)\} \tag{A.7}$$

$$V_G = \frac{k_3}{1-k_1k_3}\{(1-\mu_R k_1k_3)u^G - \mu_G k_1cq^G + \mu_R k_1(k_3u^R - cq^R) + k_2(u^* - cq^*)\}. \tag{A.8}$$

Under these formulas, (31) and (32) (for J=R) are, respectively, equivalent to

$$u'(q^R) = \frac{c}{k_3}\left\{1 + \frac{(k_3^{-1} - k_1)cq^R}{(k_3^{-1} - \mu_G k_1)(k_3u^R - cq^R) + \mu_G k_1(k_3u^G - cq^G) + k_2(u^* - cq^*)}\right\} \tag{A.9}$$

$$[1 - k_1(k_3\mu_G + \mu_R)]\,(k_3u^R - cq^R) - k_1(1-k_3)\mu_G(k_3u^G - cq^G) - k_2(1-k_3)(u^* - cq^*) > 0. \tag{A.10}$$

Similar conditions can be obtained for J=G. To economize on space, we label these conditions (A.11) and (A.12) but do not write them down.

Next, we show that given q^G, there is a unique solution to (A.9) for q^R. The proof follows Part 3 of the proof of Proposition 3.2 and hence is omitted. Denote the solution by

$Q^R(q^G)$. The following properties can be verified:

$$\partial Q^R(q^G)/\partial q^G > 0, \quad Q^R > 0, \quad Q^R(\infty) = q^0, \tag{A.13}$$

where q^0 is defined in (A.2). Similarly, for given q^R, there is a unique solution to (A.11). The solution, denoted by $Q^G(q^R)$, has similar properties to those in (A.13). Furthermore, one can verify that for all possible solutions (q^R,q^G) to (A.9) and (A.11),

$$Q^{R\prime}(q^G) < 1/Q^{G\prime}(q^R). \tag{A.14}$$

Thus the curves $q^R=Q^R(q^G)$ and $q^G=Q^G(q^R)$ have a unique intersection. Since the pair (q_H,q_H) satisfies both (A.9) and (A.11), it must be the only pair which satisfies the two equations. That is, $q^R=q^G=q_H$.

Now both (A.10) and (A.12) are reduced to $L(q_H)>0$, which is satisfied under $r<r^*$. Thus (q_H,q_H) is the only equilibrium. This completes the proof of Proposition 6.1.

For Proposition 6.2, the condition (36) is derived from (A.6)-(A.8) and the condition $V_R-cq^R=V_p$, which is an implication of a weak red money. For the red money to be indeed weak in equilibrium, we also need $F(V_p)<0$. This inequality, under (36) and (A.6)-(A.8), is equivalent to

$$u'(q^R) > \frac{c}{k_3}\left\{1 + \frac{(1-k_3)cq^R}{k_3 u^R - cq^R}\right\}. \tag{A.15}$$

Also, with (36), the condition for the green money to be strong is $L(q^G)>0$ where the function L is defined in (14). At the same time, (A.11) is reduced to (35). Thus for the green money to be strong and the red money to be weak, it suffices to find a pair (q^R,q^G) which satisfies (36), (A.15), (35) and $L(q^G)>0$.

Similar to Parts 3 and 5 of the proof of Proposition 3.2, we can show that there is a unique solution to (35) and the solution satisfies $L(q^G)>0$. Denote the solution by $\tilde{q}^G$. There is also a unique solution to (36) for given q^G. Denote this solution at $q^G=\tilde{q}^G$ by $\tilde{q}^R$. Then

$$q_L < \tilde{q}^R < \tilde{q}^G \quad (<q^0).$$

Since the right-hand-side of (A.15) is an increasing function of q^R, for the existence of equilibrium, it suffices to show that (A.15) holds when q^R is replaced by $\tilde{q}^G$. However, when q^R is replaced by $\tilde{q}^G$, condition (A.15) reduces to $L(\tilde{q}^G)>0$ under (35), which we proved to hold. Thus the equilibrium specified in Proposition 6.2 exists.

Finally, the properties in (37) can be verified from (35) and (36). This completes the proof of Proposition 6.2. ■

REFERENCES

1. I. Alonso, Patterns of exchange, fiat money, and the welfare costs of inflation, unpublished manuscript, University of Minnesota, 1991.

2. K. Binmore, Nash bargaining theory II, in "The Economics of Bargaining," ed. K. Binmore and P. Dasgupta, Basil Blackwell, Oxford, UK, 1987.

3. K. Binmore, Perfect equilibria in bargaining models, in "The Economics of Bargaining," ed. K. Binmore and P. Dasgupta, Basil Blackwell, Oxford, UK, 1987.

4. K. Burdett, M. Coles, N. Kiyotaki and R. Wright, Buyers and sellers, unpublished manuscript, University of Pennsylvania, 1993.

5. D. Cass and K. Shell, Do sunspots matter? J. Polit. Econ. 91 (1983), 193-227.

6. P. Diamond, Money in a search equilibrium, Econometrica 52 (1984), 1-20.

7. P. Diamond and J. Yellen, Inventories and money holdings in a search economy, Econometrica 58 (1990), 929-950.

8. M. Engineer and D. Bernhardt, Money, barter, and the optimality of legal restrictions, J. Polit. Econ. 99 (1991), 743-773.

9. J. M. Grandmont, On endogenous competitive business cycles, Econometrica 53 (1985), 995-1045.

10. D. Gale, Limit theorem for markets with sequential bargaining, J. Econ. Theory 43 (1987), 20-54.

11. F. Hayashi and A. Matsui, Fiat money, barter, and optimal monetary policy with capital, unpublished manuscript, University of Pennsylvania, 1991.

12. E. Helpman, An exploration in the theory of exchange-rate regimes, J. Polit. Econ. 89 (1981), 865-890.

13. S. Hendry, Credit in a search model with money as a medium of exchange, unpublished manuscript, University of Western Ontario, 1992.

14. J. Kareken and N. Wallace, On the indeterminacy of equilibrium exchange rates, Quart. J. Econ. 96 (1981), 207-222.

15. N. Kiyotaki and R. Wright, A contribution to a pure theory of money, J. Econ. Theory 53 (1991), 215-235.

16. N. Kiyotaki and R. Wright, A search-theoretic approach to monetary economics, Amer. Econ. Rev. 83 (1993), 63-77.

17. R. Manuelli and J. Peck, Exchange rate volatility in an equilibrium asset pricing model, Int. Econ. Rev. 31 (1990), 559-574.

18. K. Matsuyama, Sunspot equilibria (rational bubbles) in a model of money-in-the-utility -function, J. Monet. Econ. 25 (1990), 137-144.

19. K. Matsuyama, N. Kiyotaki and A. Matsui, Toward a theory of international currency, Rev. Econ. Stud. 62 (1993), 283-307.

20. J. S. Mill, Essays on economics and society (1830), in "Collected Works of John Stuart Mill," books IV-V, ed. J.M. Robson, University of Toronto press, Toronto, 1967.

21. M. J. Osborne and A. Rubinstein, "Bargaining and Markets," Academic Press, San Diego, 1990.

22. A. Rubinstein, Perfect equilibrium in a bargaining model, Econometrica 50 (1982), 97-109.

23. A. Rubinstein and A. Wolinsky, Equilibrium in a market with sequential bargaining, Econometrica 53 (1985), 1133-1150.

24. S. Shi, Credit and money in a search model with divisible commodities, unpublished manuscript, Queen's University, 1993.

25. R. M. Townsend, Models of money with spatially separated agents, in "Models of Monetary Economies," ed. J.H. Kareken and N. Wallace, Federal Reserve Bank of Minneapolis, Minneapolis, 1980.

26. A. Trejos and R. Wright, Search, bargaining, money and prices, CARESS working paper 93-13, University of Pennsylvania, 1993.

27. A. Wolinsky, Matching, search, and bargaining, J. Econ. Theory 42 (1987), 311-333.

28. M. Woodford, Does competition between currencies lead to price level and exchange rate stability? NBER working paper 3441, 1990.

29. M. Woodford, Monetary policy and price level determinacy in a cash-in-advance economy, unpublished manuscript, University of Chicago, 1988.

30. R. Wright, A note on sunspot equilibria in search models of fiat money, J. Econ. Theory (forthcoming, 1993).

31. R. Zhou, Currency exchange in a random search model, unpublished manuscript, Federal Reserve Bank of Minneapolis, Minneapolis, 1993.

Not for Publication: Supplementary Appendix for

"Money and Prices: A Model of Search and Bargaining" by Shouyong Shi

S1 The completion of the proof of Lemma 2.1

We show that the strategy profile is first an SPE and then the only SPE. Let G_i (G_j) be a typical subgame with agent i (j) making the first proposal. Consider G_i. Given agent j's strategies, if agent i proposes (q_i, q_j) which gives agent j less than $e^{-r\Delta}\hat{x}$, then the proposal will be rejected and agent j proposes $(\hat{Q}, \hat{q})$ after Δ. The counter proposal gives agent i a present value $e^{-2r\Delta}\hat{x}$. If agent i quits in G_i, she obtains V_p. Both options yield less than $\hat{x}$ which is delivered by the proposal $(\hat{q}, \hat{Q})$. Moreover, given that agent j accepts no less than $e^{-r\Delta}\hat{x}$, agent i can obtain at most $\phi(e^{-r\Delta}\hat{x}) = \hat{x}$. Hence $(\hat{q}, \hat{Q})$ is agent i's best proposal. Given this proposal, agent j can reject and make a counter proposal after Δ, which gives her at most $e^{-r\Delta}\hat{x}$ in present value, or she can quit, which gives her a value V_p. Since $e^{-r\Delta}\hat{x} > V_p$, neither option is better than accepting the proposal. Thus the strategies are a Nash equilibrium in G_i. Similarly, the strategies are a Nash equilibrium in G_j and hence an SPE of the bargaining.

For uniqueness, we show that all possible SPE's give the same payoffs as those specified in the proposition. Then the argument in Osborne and Rubinstein [21, p.47] shows that the first proposal is accepted immediately in all possible SPE's and hence the SPE is unique. To show uniqueness of the payoffs in the SPE's, denote by X_i (X_j) the supremum of the payoffs to agent i (j) generated by all SPE's of G_i (G_j). Let x_i (x_j) be the corresponding infimum. We establish some inequalities. Consider G_i. If agent j quits, she gets V_p. If she rejects the proposal and makes a counter proposal after Δ, she gets a present value no less than $e^{-r\Delta}x_j$. Thus the proposal by agent i should give agent j no less than $\max(V_p, e^{-r\Delta}x_j)$. That is,

$$\text{(S1.1)} \quad X_i \le \max\{V_p, \phi(\max(V_p, e^{-r\Delta}x_j))\}$$

Similarly,

$$x_i \ge \max\{V_p, \phi(\max(V_p, e^{-r\Delta}X_j))\} \quad \text{(S1.2)}$$

$$X_j \le \max\{V_p, \phi(\max(V_p, e^{-r\Delta}x_i))\} \quad \text{(S1.3)}$$

$$x_j \ge \max\{V_p, \phi(\max(V_p, e^{-r\Delta}X_i))\} \quad \text{(S1.4)}$$

Since the strategy profile specified in case (a) is an SPE and has payoff $\hat{x}$, then

$$X_i \ge \hat{x} \ge x_i; \quad X_j \ge \hat{x} \ge x_j. \quad \text{(S1.5)}$$

Examine (S1.1). Since ϕ is a decreasing function, then

$$\phi(\max(V_p, e^{-r\Delta}x_j)) \le \phi(e^{-r\Delta}x_j).$$

Since $x_j \le \hat{x}$, then $\phi(e^{-r\Delta}x_j) \ge \phi(e^{-r\Delta}\hat{x}) = \hat{x} > V_p$ and hence (S1.1) $\Rightarrow$

$$X_i \le \phi(e^{-r\Delta}x_j). \quad \text{(S1.6)}$$

Similarly, since $e^{-r\Delta}X_i \ge e^{-r\Delta}\hat{x} > V_p$, then (S1.4) $\Rightarrow$

$$x_j \ge \phi(e^{-r\Delta}X_i). \quad \text{(S1.7)}$$

Note that $\phi^{-1} = \phi$ and ϕ is decreasing. Combine (S1.6) and (S1.7) to obtain $f(X_i) \ge 0$ where the function f is defined in (3). Since f is a decreasing function and $f(\hat{x}) = 0$, then $X_i \le \hat{x}$. Then (S1.5) implies $X_i = \hat{x}$. In this case, (S1.7) and (S1.5) imply $x_j = \hat{x}$.

Similarly, one can show $X_j = x_i = \hat{x}$. ■

S2 The sign of $\partial q_H/\partial M$

Since the RHS of (13), denoted RHS1, is increasing in q, it is clear that

$$\mathrm{sgn}(\partial q_H/\partial M) = -\,\mathrm{sgn}(\partial RHS1/\partial M).$$

Denote $B=(1-M)^{-1}$, $D=r/(\beta z)$ and

$$A_1(B) = u + \frac{z}{DB+B-1}(u^*-cq^*) - \frac{B-1}{DB+B-1}cq$$

$$A_2(B) = u + \frac{z}{DB+B-1}(u^*-cq^*) - (DB+1)cq$$

Then $A_1 > A_2 > 0$. A_1 and A_2 are decreasing in B. For some coefficient $\theta>0$, we have

$$\theta \cdot \frac{\partial RHS1}{\partial M} = A_2 + (B+D^{-1})\left(\frac{A_2}{A_1}\frac{\partial A_1}{\partial B} - \frac{\partial A_2}{\partial B}\right)$$

$$> A_2 + (B+D^{-1})\frac{\partial(A_1-A_2)}{\partial B} = A_2 + (DB+1)[1-(DB+B-1)^{-2}]cq$$

If $DB+B-1 \geq 1$, or equivalently if $B \geq 2/(D+1)$, then $\partial q_H/\partial M < 0$. Thus for $\partial q_H/\partial M > 0$, both M and r have to be small. Note that $\lim_{r\to 0} q = q^*$ and

$$\lim_{r,M\to 0} \theta \cdot \frac{\partial RHS1}{\partial M} = \frac{u^*-cq^*}{D^2}\left[z^2(u^*-cq^*) + (2z-1)cq^*\right]$$

then, as stated in Section 4, $\partial q_H/\partial M > 0$ if both r and M are sufficiently small and if

$$z < \left(\sqrt{u(q^*)cq^*} - cq^*\right) / \left(u(q^*) - cq^*\right)$$

S3 Proofs of Propositions 4.1 and 4.2

For Proposition 4.1, $Y_m > Y_B \Leftrightarrow$

$$q > ([1+(1-M)^{-1}]zq^* \qquad \text{(S3.1)}$$

Since $q<q^*$, (S3.1) is satisfied only if $[1+(1-M)^{-1}]z<1$, which is violated if either $z>1/2$ or $M>M_1$. Let $z<1/2$ and call the right-hand side of (S3.1) RHS(M). For $q=q_H$, note

$$\lim_{M\to 1} \mathrm{RHS}(M)=\infty > \lim_{M\to 1} q_H \quad \text{and} \quad \mathrm{RHS}'(M)>0.$$

If $q_H|_{M=0} > 2zq^*$, then there is a value, M_H, with which (S3.1) holds with equality and (S3.1) is satisfied if $M<M_H$. However, when $r\to 0$, $q_H\to q^*$ and hence $q_H>2zq^*$ for $z<1/2$. Similarly, M_L exists for $q=q_L$ where (22) is required for $q_L>2zq^*$ when $r\to 0$. $M_L<M_H$ since q_L is bounded by q_H. This completes the proof of Proposition 4.1.

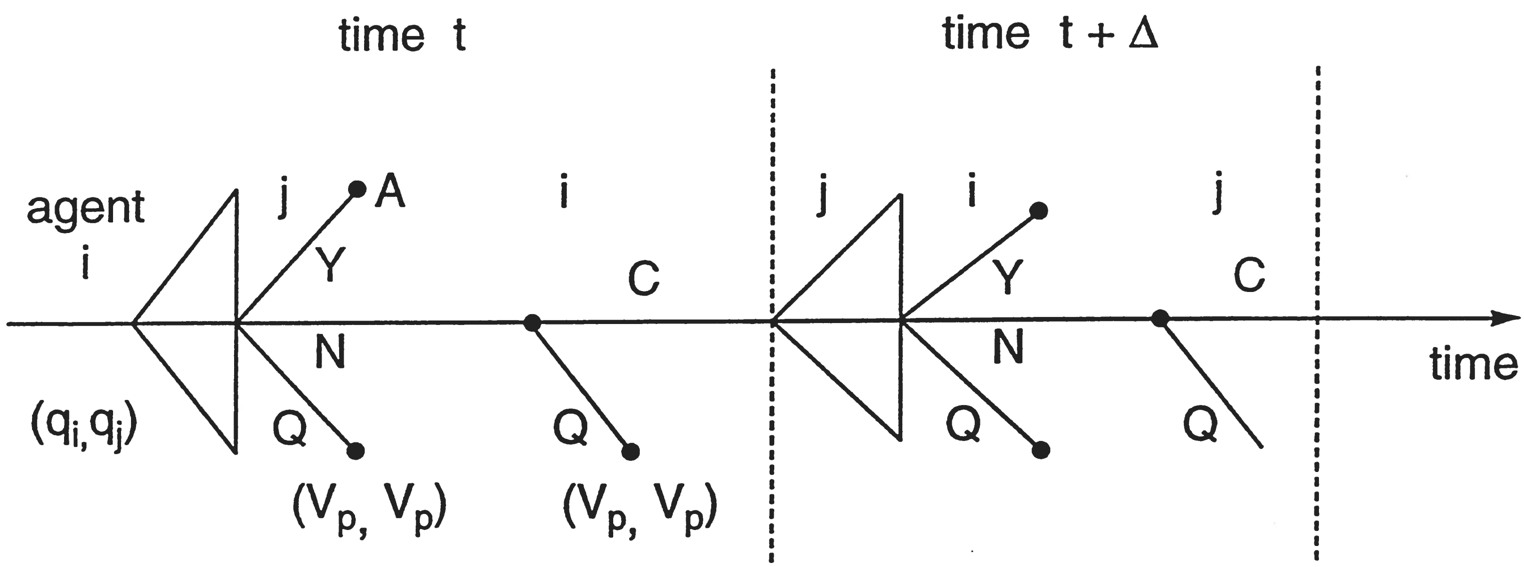
For Proposition 4.2, it is clear from (11) that the monetary equilibrium with q_H Pareto dominates the equilibrium with q_L. Also it is straightforward to verify that the monetary equilibrium with q_L cannot dominate the barter equilibrium. The barter equilibrium dominates the monetary equilibrium with q_L if and only if

$$u(q_L)-k_1 cq_L-\left[(k_3^{-1}-k_2)\beta z^2/r - k_2\right](u^*-cq^*) < 0,$$

which, with (14), becomes $cq_L<(\beta z^2 M/r)(u^*-cq^*)$. Since $\lim_{r\to 0} q_L$ is bounded, the inequality is clearly satisfied when $r\to 0$. Finally, the monetary equilibrium with q_H dominates the barter equilibrium if and only if

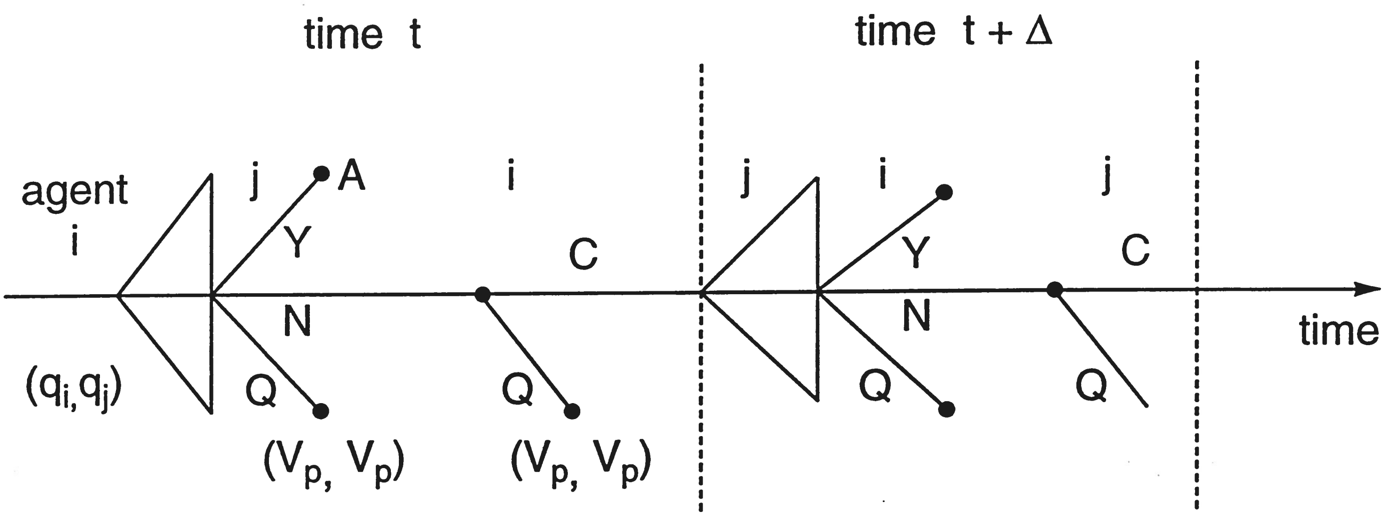
$$k_3 u(q_H)-cq_H+k_1^{-1}\left[k_2-\frac{\beta z^2}{r}(1-k_1 k_3)\right](u^*-cq^*) > 0.$$

Since $\lim_{r\to 0} q_H=q^*$, the above inequality approaches $[1+(1-M)^{-1}]z<1$ when $r\to 0$. This latter inequality is satisfied if $z<1/2$ and $M<M_1$. ■



$$A: \left(u(q_j) - cq_i + V_p,\; u(q_i) - cq_j + V_p\right)$$

Figure 1



$$A: \left(u(q_j) - cq_i + V_p,\ u(q_i) - cq_j + V_p\right)$$

Figure 1

Journal: Journal of Economic Theory
Author: SHOUYONG SHI

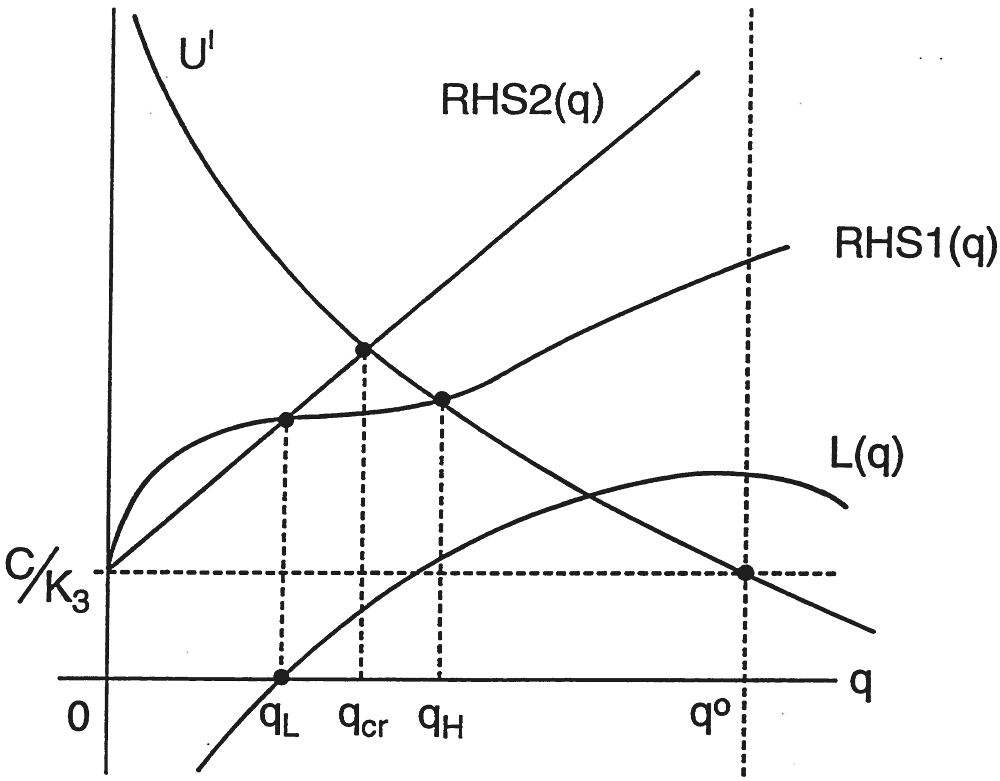


Figure 2

Journal: Journal of Economic Theory
Author: SHOUYONG SHI