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Unemployment and the Dynamic Effects of Factor Income Taxation

Shouyong Shi   Quan Wen

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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by

Shouyong Shi
Queen's University

and

Quan Wen
University of Windsor

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Shouyong Shi
Department of Economics
Queen's University
Kingston, Ontario
Canada, K7L 3N6

Quan Wen
Department of Economics
University of Windsor
Windsor, Ontario
Canada, N9B 3P4

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Abstract

This paper introduces search unemployment into an intertemporal maximization model with capital accumulation. It characterizes the decentralized search equilibrium, examines the dynamic effects of factor income taxation and calculates the welfare cost of the taxation. Four tax policies are considered: labor income taxation, capital income taxation, the subsidy to job search and the subsidy to hiring. It is found that the dynamic effects and welfare costs of these policies are quite different from the standard model without unemployment. The differences illustrate the importance of the labor market frictions.

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1. Introduction

Dynamic equilibrium models have been constructed in the past 15 years to evaluate the welfare cost of factor income taxation. Because of inadequate analytical tools to integrate unemployment into a dynamic model, these models have typically omitted the important fact of unemployment.[1] The omission seriously undermines the predictive power of these models and delivers biased policy recommendations on tax reforms. On the other hand, most models of unemployment do not examine capital accumulation in the utility-maximizing framework, with the exception of Merz (1993) discussed below. In this paper, we hope to construct a tractable dynamic model that incorporates both unemployment and capital accumulation in a utility-maximizing framework. Giving reasonable parameter values, we then examine the dynamic effects and the welfare costs of four taxation policies: labor income taxation, capital income taxation, the subsidy to job search and the subsidy to hiring.

Our analysis has two building blocks, and the references to each can be found in the representative works cited below. The first is a framework to evaluate the welfare cost. For this, we follow Chamley (1981), Judd (1985, 1987), and Auerbach and Kotlikoff (1987) to focus on the change in intertemporal utility caused by tax changes. In particular, a measure of the marginal deadweight loss proposed by Judd (1987) is used to measure the welfare cost of taxation. The second is a theory of unemployment. To choose among competing theories of unemployment for the present investigation, we require that the theory allow capital accumulation and be easy to have a long horizon. Based on these criteria, we choose to introduce unemployment through an extended version of the search model described by

[1] See for example Judd (1985, 1987) and Auerbach and Kotlikoff (1987).

Mortensen (1992) and Pissarides (1990).

The extension of the search model follows Merz (1993). Extending the search unemployment model to incorporate capital accumulation and intertemporal utility maximization, Merz characterizes the planner's optimal allocation and calibrates the model. Our objectives are different, we directly characterize the decentralized equilibrium and examine distortionary taxes. A direct characterization of equilibrium is necessary for the issues interested in this paper. In an economy where workers and firms are matched by a matching technology, there are externalities in the labor market. Only under very strong conditions can the externalities be internalized so that the planner's allocation in Merz (1993) can be decentralized as outcomes of equilibrium. There is no a priori reason to believe that those conditions are satisfied in reality. When distortionary taxes are present in addition to the labor market externalities, as in the economy described in this paper, there is little hope for the equilibrium allocation to be efficient.

The characterization of equilibrium contributes to the search theory of unemployment in two respects. First, it provides an integrated framework for welfare analysis. In a typical search model, it is difficult to evaluate welfare because it is rather arbitrary to impose a set of welfare weights on the employed, the unemployed and the firms. Here we use a representative household utility function to integrate naturally the different types of agents. Second, by embedding agents' search decisions into the household's intertemporal utility maximization, we link the reservation wage to the household's marginal rate of substitution between leisure and consumption.

Our framework generates the following results. First, job vacancies and unemployment are among the first variables to respond immediately to tax changes. These variables overshoot along their dynamic paths. Labor employment and the capital stock respond only slowly and typically follow nonmonotonic adjustment paths. For example, a labor income tax induces first a rising capital stock and then a falling one. In contrast, the standard model without unemployment predicts a monotonic adjustment in the capital stock. The responses in employment and the capital stock also take a longer time in our model. Thus tax changes have more laggard and prolonged effects on output in an economy with unemployment than without.

Second, the introduction of search allows agents to substitute searching for working. This further allows employment to move together with leisure. Thus tax changes can create the stylized positive comovement between consumption and labor employment. This stylized positive comovement cannot be induced by taxes only in the standard model with time separable preferences (Barro and King 1984).

Third, the welfare cost of capital income taxation is smaller than in previous models. With a set of parameter values used by Judd (1987), we find that the marginal deadweight loss of capital income taxation is below 50 cents in real income for a dollar increase in the tax revenue. In contrast, Judd (1987) found that the loss exceeds 50 cents and easily exceeds a dollar. He also found that "when *any* proposed estimates are used for taste and technology parameters, welfare would be improved substantially at the margin by moving away from capital income taxation toward higher labor income taxation." The figures in Judd (1987) indicate that capital income taxation is 3 to 4 times as costly as labor income taxation. We find much smaller welfare gains from such a switch. For some reasonable parameter values, labor income taxation can even be more costly than capital income taxation.

Fourth, a job search subsidy is very inefficient and a hiring subsidy is very efficient. The search subsidy reduces employment and increases unemployment. Reducing the search subsidy increases governmental revenue *and* welfare. The marginal welfare gain from reducing the search subsidy easily exceeds 50 cents. Cutting the search subsidy to finance a cut in either the labor income tax or the capital income tax can easily raise welfare by 50 cents in real income. Such financing method also increases labor employment. In contrast, a hiring subsidy reduces unemployment and increases employment. It typically raises revenue and welfare. The marginal welfare gain of a hiring subsidy often exceeds 1.5 dollars and can run as high as 714 dollars.

Finally, the welfare costs of taxation depend on labor market conditions. We examine two of such conditions. One is the bargaining power of labor in wage determination; the other is the efficiency of search in creating jobs. A high bargaining power of labor increases the welfare costs of factor income taxes and subsidies to search. A high search efficiency reduces these welfare costs.

This paper is organized as follows. Section 2 builds the dynamic model. Section 3 examines how taxes and subsidies affect the steady state. Section 4 specifies the simulation method and parameterizes the model. Section 5 analyzes the dynamic effects of taxation. Section 6 reports the welfare cost of factor taxation. Section 7 concludes this paper.

2. Decentralized Economy

2.1. Households

Consider an economy with many identical households. The number of households is normalized to one. Each household consists of many agents. An agent is infinitely-lived, endowed with a flow of one unit of time and chooses to work for wages, search for jobs or enjoy leisure.[2] Search generates matches between job vacancies and agents. For a specific agent who is searching for jobs, the timing of a match is random. Random matching results in uncertain streams of income and leisure for any specific agent.

Randomness of such type causes analytical complexity. Directly tackling this problem would require complicated dynamic programming with uncertainty and the model would be too complicated to be easily parameterized as in Judd (1987). A similar difficulty exists in the literature of indivisible labor. There, various authors have shown that employment lotteries can be used to smooth agent's consumption across states of employment (see Rogerson 1988, Hansen 1985, and Rogerson and Wright 1988). This technique has its limitation, particularly in a model with capital accumulation and elastic labor supply. Optimal capital accumulation will depend on the state of employment, and dynamic programming with uncertainty is still required. To circumvent this analytical problem, we assume that each household consists of a continuum of agents with measure one and all members care only about the household's utility. Thus individual risks in consumption and leisure are completely smoothed within each household. Similar assumptions are used in the closely related paper by Merz (1993) and in other well-known macroeconomic models.[3]

The utility function of a household is

$$U = \int_0^\infty u(C, 1 - n - s)e^{-\rho t} dt \tag{2.1}$$

where $u(\cdot,\cdot)$ has the standard properties, with an additional simplifying assumption $u_{12} = 0$.

[2]For tractability, we choose to adopt an infinite-horizon model instead of an overlapping generations model. For examinations on welfare costs of taxation in overlapping generations models, see Auerbach and Kotlikoff (1987).

[3]For example, in a monetary model Lucas (1990b) assumes that a household consists of several members, each conducting a different actitivity in a different market. At the end of each period, receipts are pooled in the household.

C is the household consumption, n the proportion of hours in work and s the proportion of hours in search. Since the size of a household is one, n and s can be equivalently interpreted as the proportion of household members in work and in search respectively. With this interpretation, the notation s conforms with the notion of unemployment; and $n+s$ with the labor force participation. The rate of unemployment is $s/(n+s)$.

Different from a standard representative agent model but in common with search models of unemployment, the variable n is predetermined at each given time. It can change only gradually as workers quit or searching agents find jobs. The law of motion of n is

$$\dot{n} = ms - \theta n. \tag{2.2}$$

θ is the (constant) rate of natural separation from jobs; m is the rate at which searching agents find job matches. As discussed later, m depends on the aggregate vacancy and aggregate number of search agents. However, individual agents take m as given.

It is worthwhile emphasizing the following features of (2.2). Since $\theta > 0$, some agents are unemployed even when $\dot{n} = 0$. Moreover, there are inflows and outflows into the state of unemployment all the time, even in the steady state. In this sense it is reasonable to think that the identities of agents who are unemployment in each household are changing all the time. This observation helps clarify on some issues later.

A representative household's maximization problem is

$$(PH) \qquad \max_{(C,n,s,K)} U \qquad s.t.$$

$$\begin{aligned}
&(2.2)\ \textit{holds};\\
&\dot{K} = (1-\tau_K)(rK+\Pi) + (1-\tau_W)wn + \tau_u ws - C + L;\\
&K(0) = K_0,\ n(0) = n_0\ \textit{given}.
\end{aligned} \tag{2.3}$$

Here r is the rental rate of capital, w the wage rate, K the household's capital stock, and Π the dividend defined later. τ_K and τ_W are the tax rates on capital and labor incomes; τ_u is the subsidy to search or the replacement ratio. Finally, L is the lump-sum transfer from the government to the household.

For all the following examinations, we will focus on the case where $\tau(t)$ is piece-wise constant overtime so that $\dot{\tau} = 0$. That is, we will examine only permanent changes in the taxes.[4] Let the current-value shadow price of (2.2) be Ω. The shadow price of the constraint (2.3) can be computed as u_1. Standard optimization techniques generate the following equations:

$$\dot{C} = \frac{u_1}{u_{11}}[\rho - r(1-\tau_K)]; \tag{2.4}$$

$$\Omega = (u_2 - u_1 \tau_u w)/m; \tag{2.5}$$

$$\dot{\Omega} = (\theta + \rho)\Omega + u_2 - u_1(1-\tau_W)w. \tag{2.6}$$

2.2. Firms

There are many identical firms in the economy. Each firm has job vacancies. The unit cost of maintaining a vacancy is a constant b (> 0). The rate at which vacancies turn into job matches is μ. As m, μ depends on the numbers of aggregate job vacancies and searching agents. But an individual firm takes μ as given. Let v be the number of vacancies. A firm's

[4] For temporary or anticipated future changes in taxes, see Judd (1987). The assumption of a constant $\tau_u(t)$ may seem unrealistic at the first glance because the replacement ratio typically falls with the unemployment duration. However, as we clarified above, the identities of the unemployed agents are changing overtime. Thus it is not entirely unreasonable to interpret τ_u as the replacement ratio on a newly unemployed agent. In this sense, a constant $\tau_u(t)$ can be consistent with the negative dependence of replacement ratio on unemployment duration.

labor employment evolves according to

$$\dot{n} = \mu v - \theta n. \tag{2.7}$$

An individual firm takes as given the wage rate w offered by other firms. It also takes as given the wage rates offered by itself to its existing workers (see later discussion for a justification). Let the production function $F(K, n)$ be linearly homogenous and exhibit the standard properties: increasing and concave in each argument. The representative firm maximizes its value:

$$(PF) \max_{(v,K,n)} H = \int_0^\infty e^{-\int_0^t [1-\tau_K(s)]r(s)ds} \Pi(t) dt$$

$$\begin{aligned} &s.t. \\ &\Pi = F(K, n) - (r + \delta)K - wn - b(1 - \tau_v)v; \\ &(2.7) \textit{ holds and } K(0) = K_0,\ n(0) = n_0 \textit{ given.} \end{aligned} \tag{2.8}$$

δ is the rate of depreciation of capital and τ_v is the rate of subsidy to hiring. Optimization implies that

$$F_1 = r + \delta; \tag{2.9}$$

$$\frac{\dot{\mu}}{\mu} = \frac{\mu}{b(1 - \tau_v)}(F_2 - w) - [\theta + r(1 - \tau_K)]. \tag{2.10}$$

Dividends can be positive. It can be so because firms must maintain job vacancies in order to employ labor. In this sense, a positive dividend resembles that in the Tobin's q model which has costs to adjust the capital stock. However, the two models differ in their long-run implications. In the adjustment cost model, the adjustment cost disappears when the economy approaches the steady state. In the current model, the cost is positive even in the steady state because job vacancies are positive in the steady state.

2.3. Matching and Wage Determination

Job vacancies and searching agents create job matches. The number of matches is assumed to be a function of v and s:

$$M(v,s) = Av^{\alpha}s^{1-\alpha}, \qquad \alpha \in (0,1). \tag{2.11}$$

The matching technology exhibits constant-returns to scale. Besides its apparent similarity to the usual production technology, constant-returns to scale matching technology has also be supported empirically (see Pissarides 1986, and Blanchard and Diamond 1989). The Cobb-Douglas form is adopted for analytical simplicity. We call $(1-\alpha)$ the *search efficiency* in creating job matches. With constant-returns to scale, the number of matches per vacancy or searching agents depends only on the vacancy-unemployment ratio. Let $x = v/s$ be such a ratio. A small x represents a tight labor market. Then

$$m = m(x) = Ax^{\alpha}, \qquad \mu = \mu(x) = m(x)/x. \tag{2.12}$$

Once a searching agent is matched with a vacancy, the agent and the firm decide current *and* future wage rates for this agent[5]. These wage rates are assumed to be determined by a Nash bargaining which maximizes a product of weighted surpluses of the household and firm. To be precise, let T be the time when the match is created. Denote by $\{w^*(t)\}_{t\geq T}$ the path of wage rates to be determined for the new worker conditional on the continuation of employment of the worker. Then having an additional member working at the wage schedule increases the household's utility at $t \geq T$ by $[(1-\tau_W(t))w^*(t)u_1(t) - u_2(t)]dn$. Hiring an

[5]That a firm has power to determine a new worker's wage is not inconsistent with our earlier assumption that a firm takes as given the wage rates offered to existing workers. The same approach has been taken by Pissarides (1990, pp. 11–12).

additional worker dn with the wage schedule increases the firm's current-valued surplus at time $t \geq T$ by $[F_2(t) - w^*(t)]dn$. With normalization, the Nash bargaining solution solves

$$\max_{w^*(t)}[F_2(t) - w^*(t)]^{1-\lambda}[w^*(t) - \frac{u_2(t)}{(1-\tau_W(t))u_1(t)}]^\lambda, \quad for\ t \geq T.$$

The parameter $\lambda \in (0,1)$ can be interpreted as the bargaining power of labor.

It is well known that the Nash bargaining formulation proposed above generates the same outcome as some noncooperative sequential bargaining games in the steady state (Wolinsky 1987). In a non-steady state environment, the equivalence no longer holds. But it is still possible to find some other Nash bargaining formulation which delivers the same outcome as the sequential bargaining game for any time t and the same steady state outcome as the Nash formulation proposed above (see Coles and Wright 1994). Thus the proposed form is a useful approximation around the steady state of the underlying noncooperative bargaining framework. The simple formulation has an advantage. Since the wage rate $w^*(t)$ solves only the period-t problem, the wage path $\{w^*(t)\}_{t \geq T}$ is time-consistent. That is, workers and firms do not have incentive to reopen negotiation in the future. This justifies our earlier assumption that firms take as given the wages rates offered to existing workers.

The quantity $u_2/[u_1(1-\tau_W)]$ can be interpreted as the agent's reservation wage. Thus different from the usual search model of unemployment such as Pissaridis (1990), the current model closely ties the reservation wage to the marginal rate of substitution between consumption and leisure. The actual wage rate is a weighted sum of the this reservation wage and the marginal product of labor, with the bargaining power of labor being the weight. Solving the Nash bargaining problem gives this wage formula:

$$w^*(t) = \lambda F_2(t) + (1-\lambda)\frac{u_2(t)}{(1-\tau_W(t))u_1(t)}. \tag{2.13}$$

Since all firms are identical, they must offer the same wage rate in any symmetric equilibrium. Also, since the wage path is time consistent, two workers who are hired by the same firm at different times must be paid the same wage at any given time. So $w^* = w$.

Around the steady state (2.10) and (2.13)$\Rightarrow$ $F_2 > w > u_2/[(1-\tau_W)u_1]$. The wage rate equals neither the marginal product of labor nor the after-tax marginal rate of substitution between leisure and consumption. In contrast, all three equal in the standard representative agent model. Also around the steady state (2.6)$\Rightarrow$ $\Omega > 0$ and (2.5)$\Rightarrow$ $u_1\tau_u w < u_2 < u_1(1-\tau_W)w$. The condition $\tau_u < 1-\tau_W$ is required for an equilibrium to exist and hence imposed throughout this paper. Under this condition, a searching agent strictly prefers working to searching if he is offered at least the reservation wage.

2.4. The Government

The government faces the following budget constraint:

$$L \leq \tau_K(rK + \Pi) + \tau_W wn - \tau_u ws - b\tau_v v - g. \tag{2.14}$$

Any changes in revenue caused by taxes and subsidies are rebated to households through the lump-sum transfer L. Government bonds are abstracted from the model for simplicity. As stated earlier, all changes in taxes are assumed to be permanent.[6]

3. Search Equilibrium and Steady State

A *search equilibrium* is a collection $\{C(t), s(t), x(t), n(t), K(t), r(t), w(t), \Pi(t)\}_{t=0}^{\infty}$ such that (i) given $\{r(t), w(t), \Pi(t)\}$, $\{C(t), s(t), n(t), K(t)\}$ solve the problem (PH);

[6]When tax changes are compensated by lump-sum transfers as in the current model, the existence of government bonds adds little to the discussion of the welfare cost of taxation. Also, for an analysis of anticipated tax changes, see Judd (1987).

(ii) given $\{w(t), r(t)\}$, $\{v(t), n(t), K(t)\}$ solve the problem (PF) with $v(t) = s(t)x(t)$;

(iii) $w(t)$ solves the Nash bargaining problem;

(iv) $r(t)$ and $\Pi(t)$ satisfy (2.9) and (2.8) respectively;

(v) the government budget constraint (2.14) is satisfied.

The equilibrium conditions are (2.2)–(2.6), (2.8)–(2.10) and (2.12)–(2.14). After suitable substitutions, they give rise to a dynamic system of $Y \equiv (C, s, x, n, K)^T$. In particular,

$$\dot{x} = \frac{1}{1-\alpha}\left\{[\theta + (F_1 - \delta)(1-\tau_K)]x - \frac{(1-\lambda)m}{b(1-\tau_v)}\left(F_2 - \frac{u_2}{(1-\tau_W)u_1}\right)\right\}; \tag{3.1}$$

$$\begin{aligned}\dot{s} = -\dot{n} - \tfrac{1}{u_{22}}[(\theta + \rho + \tfrac{\dot{m}}{m})(u_2 - u_1\tau_u w) + \tau_u w u_{11}\dot{C} \\ + m(u_2 - u_1(1-\tau_w)w) + u_1\tau_u\dot{w}];\end{aligned} \tag{3.2}$$

$$\dot{K} = F - \delta K - bsx - C - g. \tag{3.3}$$

(3.1) is derived from (2.10); (3.2) from (2.5) and (2.6); (3.3) from (2.3), (2.8) and (2.14). Other equations of the dynamic system are given by (2.2) and (2.4). To emphasize the dependence of the dynamic system on $\tau = (\tau_K, \tau_W, \tau_u, \tau_v)$, denote this dynamic system by

$$\dot{Y} = h(Y, \tau). \tag{3.4}$$

Before studying the dynamic equilibrium, we remark on its efficiency. Besides distortionary taxes, there are externalities in the labor market. The number of job matches per search, m, depends on search effort s, and the number of job matches per job vacancy, μ, depends on job vacancies v. However, individual agents and firms do not take the dependence into account when they make their decisions. In general the search equilibrium will not deliver efficient allocations even when there are no distortionary taxes. These externalities can only be internalized with special matching functions and wage bargaining rules.

Hosios (1990) specifies the conditions for the internalization in an environment with no capital accumulation and no distortionary taxes. With capital accumulation and no taxes, the conditions are similar (see Merz 1993). That is,

$$vM_1(v,s)/M = 1-\lambda, \quad sM_2(v,s)/M = \lambda.$$

With the Cobb-Douglas matching function, these two conditions are equivalent to a single condition $\lambda = 1-\alpha$. To examine the importance of labor market conditions, we do not force this condition to hold, but will explore its implications in Section 6.

We now determine the steady state of the equilibrium dynamic system. The steady state $Y^* = (C^*, s^*, x^*, n^*, K^*)^T$ is given by the solution to $h(Y^*, \tau) = 0$:

$$F_1 = \delta + \frac{\rho}{1-\tau_K}; \tag{3.5}$$

$$\lambda x^* + (\theta+\rho)\left(1 - \frac{(1-\lambda)\tau_u}{1-\tau_W}\right)\frac{x^*}{m(x^*)} = \frac{1-\lambda}{b(1-\tau_v)}\left(1 - \frac{\tau_u}{1-\tau_W}\right)F_2; \tag{3.6}$$

$$s^* = \frac{\theta n^*}{m(x^*)}; \tag{3.7}$$

$$C^* = \left(\frac{F-\delta K^*}{n^*} - \frac{b\theta x^*}{m(x^*)}\right)n^* - g; \tag{3.8}$$

$$\frac{u_2}{u_1} = \frac{(\theta+\rho)\tau_u + (1-\tau_W)m(x^*)}{(\theta+\rho)\left[1 - \frac{(1-\lambda)\tau_u}{1-\tau_W}\right] + \lambda m(x^*)} \cdot \lambda F_2. \tag{3.9}$$

In particular, (3.6) and (3.9) are derived from $\dot{x} = 0$ and $\dot{s} = 0$. Note that equation (3.7) gives the Beveridge curve, a negative relationship between v^* and s^*.

The equation system (3.5)–(3.9) can be solved sequentially to determine a unique steady state. First, note that F_1 and F_2 are functions of only the capital-labor ratio, so (3.5) uniquely solves for K^*/n^*. Second, after substituting the value for K^*/n^* (3.6) becomes an equation of only x^* and its left-hand-side is an increasing function of x^*. The equation solves for a unique x^* under the condition $\tau_u < 1 - \tau_W$. Third, after substituting the values of K^*/n^* and x^* into (3.7) and (3.8), s^* and C^* become linear functions of n^*. Substituting these functions, (3.9) becomes an equation of only n^*. In particular, u_2/u_1 is an increasing function of n^*. Thus (3.9) uniquely solves for n^*. Finally, s^*, C^* and K^* can be recovered from (3.7), (3.8) and the ratio K^*/n^*.

The policies $\tau's$ all have unambiguous effects on the long-run tightness of the labor market. An increase in the hiring subsidy τ_v increases x^*. An increase in τ_W, τ_K or τ_u reduces x^*. These effects are intuitive. A hiring subsidy increases x^* because it provides an incentive for firms to maintain more vacancies. A search subsidy reduces x^* because it brings more agents to search. A labor income tax reduces x^* because it raises the reservation wage and the equilibrium wage. A higher wage rate increases the cost of hiring and reduces job vacancies. A capital income tax reduces x^* because it reduces the capital-labor ratio, reduces the marginal product of labor and hence reduces the benefit for firms to hire workers.

In contrast to the unambiguous effects of τ on x^*, the effects of τ on n^* are all analytically ambiguous. This ambiguity makes it difficult to assess analytically the effects of tax changes on the steady state. The same problem exists for the dynamic analysis. Although it may be possible to characterize analytically the locally stable dynamic path that the system will follow after tax changes, the analytical results are not illuminating. For this reason, we resort to numerical exercises. The next section sets up the framework to do so.

4. Simulation Method

Following Judd (1987), we model marginal changes in taxes by letting $\tau(t) = \tau_0 + \Delta \cdot \tau_1(t)$, where τ_0 is the initial tax and Δ is an arbitrarily small number. Since we only examine permanent tax changes, $\tau_1(t) = \tau_1$ for all t. All tax changes take place at time 0 unexpectedly. Denote by $Y_\Delta(t)$ the change of $Y(t)$ with respect to the tax change and Y^*_Δ the corresponding change of the steady state. Since the tax changes are infinitesimal and permanent, the dynamic path $\{Y_\Delta(t)\}_{t=0}^{\infty}$ can be calculated in the following way.

First, differentiate the dynamic system (3.4) with respect to Δ and approximate the resulted system by a liner system evaluated at $\Delta = 0$. This procedure creates

$$\dot{Y}_\Delta(t) = J(Y_\Delta(t) - Y^*_\Delta). \tag{4.1}$$

J is a 5×5 matrix defined by $J = h_Y(Y^*, \tau_0)$. Second, solve the linear differential system (4.1) under the initial conditions $K_\Delta(0) = n_\Delta(0) = 0$. Since there are two predetermined variables, stability requires that matrix J have two stable eigenvalues and three unstable eigenvalues.[7] Let the stable eigenvalues be ω_1 and ω_2. Let $Z_i = (Z_{i1}, Z_{i2}, Z_{i3}, Z_{i4}, Z_{i5})^T$ be the eigenvector of J corresponding to ω_i. Then the stable dynamic path of (4.1) is

$$Y_\Delta(t) - Y^*_\Delta = (Z_1, Z_2)\begin{pmatrix} a_1 e^{\omega_1 t} \\ a_2 e^{\omega_2 t} \end{pmatrix} \tag{4.2}$$

where a_1 and a_2 are uniquely determined by the two initial conditions $K_\Delta(0) = n_\Delta(0) = 0$.

Now, the change in the intertemporal utility is

$$U_\Delta = u_1 \int_0^\infty C_\Delta(t) e^{-\rho t} dt - u_2 \int_0^\infty [n_\Delta(t) + s_\Delta(t)] e^{-\rho t} dt.$$

Utilizing (4.2), U_Δ can be computed analytically. This change in utility is equivalent to a change U_Δ/u_1 in the present value of real income. We measure this change in real income against the governmental revenue raised by the tax change. Since additional tax revenue is

[7]Under certain regularity conditions specified in Scheinkman (1976) and Epstein (1987), the correct number of stable eigenvalues is also sufficient for local stability. Those regularity conditions are met in the following numerical exercises.

rebated through lump-sum transfers, the present value of the additional tax revenue is

$$R \equiv \int_0^\infty L_\Delta(t) e^{-\rho t} dt.$$

The function $L_\Delta(t)$ can be derived from the government budget constraint and (4.2). The marginal deadweight loss (MDL) proposed by Judd (1987) takes the following form:[8]

$$MDL = -\frac{U_\Delta}{u_1 R}. \tag{4.3}$$

For example, the marginal deadweight loss of capital taxation is computed by setting $\tau_{K1} \neq 0$ and $\tau_{W1} = \tau_{u1} = \tau_{v1} = 0$. Denote the marginal deadweight loss of capital taxation (labor taxation, search subsidy, hiring subsidy) by MDL_K (MDL_W, MDL_u, MDL_v). In a normal case, MDL is a positive number.

To calculate MDL numerically, we parameterize the model. To facilitate comparison, most of the parameter values used below are taken from Judd (1987). On the production side, we let the production function be $F(K,n) = K^\gamma n^{1-\gamma}$ with a capital share $\gamma = 0.25$. The rate of capital depreciation δ is such that capital consumption allowance is 12 percent of the net output. On the preference side, we let the rate of time preference be $\rho = 0.01$. If one interprets the unit of time as a quarter, this value of ρ gives roughly a 4 percent steady state annual interest rate. The instantaneous utility function is

$$u(C, 1-n-s) = \frac{C^\sigma}{\sigma} - \beta\frac{(n+s)^\eta}{\eta}.$$

We let σ vary from -0.1 to -5.0. This range gives a relative risk aversion from 1.1 to 6, which is consistent with the macro-econometric literature (for example, see Hansen and

[8]Judd defines MDL by $\rho U_\Delta/(u_1 R)$. The results reported in Judd (1987) are actually calculated with the definition $MDL = U_\Delta/(u_1 R)$. We correct this typograghical error in our definition. Also, we add a negative sign to Judd's definition in order to report welfare losses as positive numbers.

Singleton 1983). η is chosen such that the elasticity of labor supply $\varepsilon = 1/(\eta - 1)$ covers the range from 0.1 to 0.6, a range consistent with Killingsworth (1983). To determine β through (3.9), we let the steady state employment n^* be 0.28 to match the empirical observation by Christiano (1988). On the policy side, we choose $\tau_{K0} = \tau_{W0} = 0.3$ to be consistent with King and Fullerton (1984). We later allow $\tau_{K0} = 0.5$ and $\tau_{W0} = 0.4$. The rate of search subsidy τ_{u0} is chosen to vary between 0.45 and 0.55 to cover a reasonable subset of the replacement ratio observed in different states of the U.S. (see footnote 4 for a clarification). τ_v is chosen to be zero. Government spending is assumed to be 20 percent of the output.

There are five parameters related to search and unemployment, $(\theta, A, \alpha, \lambda, b)$. We normalize $A = 1$ and let $\theta = 0.05$. The value of θ resembles the quarterly rate of transition from employment to unemployment used by Mortensen and Pissarides (1992).[9] To determine the parameter b through (3.6), let the steady state unemployment rate $s^*/(s^* + n^*)$ be 6 percent. To check the response of MDL to the labor market conditions, we let α and λ vary from 0.2, 0.4, 0.6 to 0.8. This covers the value $\alpha = 0.6$ estimated by Blanchard and Diamond (1989).

5. Dynamic Effects of Factor Income Taxation

5.1. Increases in τ_u and τ_W

Figures 1 *and* 2 *here.*

To a working agent, an increase in the search subsidy is similar to an increase in the labor income tax. Both increase the opportunity cost of working. The similarity is represented by the similar responses of variables in Figures 1 and 2. Both policies decrease labor

[9]Burdett et al (1984) indicate a different estimate of $\theta = 0.15$. We have done simulation also with this estimate but found results similar to the ones reported here.

employment along the entire dynamic path. Wage rates increase to compensate for the increased opportunity cost of working. High wage rates discourage hiring and hence reduce job vacancies. The labor market gets tighter.[10]

The main difference between the two policies lies in how they affect the agents who are not working. An increase in the search subsidy encourages more agents to search and hence increases labor force participation along the entire dynamic path. A labor income tax does the opposite. It reduces the after-tax wage rate even though it increases the gross wage rate. Because of a lower net income from future employment, some agents drop out of search at the beginning of transition. Even though the number of searching agents gradually rises along the dynamic path, employment falls and the labor force participation is lower.

The second difference between the two policies is in the magnitude of the reduction in employment. Although both reduce employment, a labor income tax reduces more. To increase employment, a policy maker may find it attractive at the margin to cut the search subsidy to finance a cut in the labor income tax. Figures 1 and 2 suggest that such a policy increases employment along the entire dynamic path. The policy immediately increases unemployment when it is carried out, but reduces unemployment after a short time. In Section 6, we will see that this policy also increases welfare.

An important feature of the dynamic response in the current model is that job vacancy and unemployment are among the first variables to respond to changes in taxes. Output responds slowly because it takes time to accumulate capital and to create employment. This pattern seems realistic. In contrast, a standard representative model such as Judd (1987) assumes that labor employment and output respond to tax changes immediately.

[10]In Figures 1 and 2, $x_{\Delta}(t)$ is negative but its absolute value is too big to fit into the diagram.

The response pattern in the current model typically has overshooting job vacancy and unemployment (see Figure 2). To explain this behavior, consider an increase in the labor income tax. As analyzed above, an increase in the labor income tax increases the wage rate. Firms want to reduce labor employment. Since the employment level cannot be reduced suddenly, firms must reduce job vacancy to a level lower than the new long-run level. As the employment level gradually decreases along the transition path, job vacancy rises. The overshooting behavior of unemployment can be explained similarly by the household's decision to reduce the labor supply.

Overshooting job vacancies and unemployment are accompanied by nonmonotonic adjustments in the capital stock and labor employment. In both Figures 1 and 2, the capital stock first rises because of the increase in savings created by the fall in job vacancy and in consumption (see (3.3)). It then falls because of the falling marginal product of capital and the decreased savings created by rising job vacancies. On the opposite, labor employment first falls because fewer agents are newly employed as a result of lower vacancies. It then rises as job vacancies rise. The nonmonotonic adjustment of the capital stock is in sharp contrast with the standard model without unemployment. In the standard model, the capital stock falls monotonically after a labor income tax. Nonmonotonic capital stock and labor employment also imply that output responds to changes in taxes much longer in the economy with search unemployment than without.

5.2. Increases in τ_K and τ_v

Figures 3 *and* 4 *here.*

Similar to the labor income tax, an increase in the capital income tax immediately reduces job vacancies and search effort, creating overshooting in these variables. Different from the labor income tax, the capital income tax reduces search effort proportionally more than vacancies and hence decreases the tightness of the labor market immediately.

A capital income tax also differs from a labor income tax in its immediate effect on consumption. Recall that a labor income tax immediately reduces consumption and increases investment. On the opposite, a capital income tax immediately increases consumption and reduces investment. Investment falls because the net return to investment is reduced by the capital income tax. Since job vacancies also fall, consumption must rise to absorb the extra resource created by lower investments and lower vacancies.

A novel feature of our model is that the capital income tax can raise the long-run employment level. This occurs despite its negative effect on the long-run wage rate. In a standard model, however, a capital income tax reduces the long-run employment level. An explanation for our result is that consumption falls by less in our model than in the standard model. When consumption falls by only a moderate amount but the capital stock falls by a large amount, labor employment must rise to maintain the resource constraint (3.3) in the steady state. There are two possible reasons for why consumption falls less in our model. First, because the wage rate does not equal the marginal product of labor, the fall in the marginal product of labor caused by the reduction in the capital stock is only partially transmitted into the wage rate. Second, since search effort and job vacancies are complementary in the matching technology, the rising vacancy during the transition induces a rising search effort. Even thought the wage rate falls eventually, the receipts from the search subsidy fall by less and hence the household income falls by less.

Finally, an increase in the hiring subsidy τ_v has the opposite effects to the search subsidy.

In particular, it increases consumption, job vacancies and labor employment. It also reduces unemployment (see Figure 4).[11] These opposite effects are hardly surprising, because a subsidy to hiring creates incentives (or disincentives) to the two sides of the labor market that are just opposite to a search subsidy.

5.3. Comovement between consumption and employment

In our model, taxes can generate positive comovement between consumption and labor employment (see Figures 1 and 2 for example). In contrast, taxes generate negative comovement between the two variables in the standard model when preferences are (weakly) separable over time (Barro and King 1984). This negative comovement has led to the perception that disturbances in taxes cannot generate the stylized positive comovement between consumption and labor employment during business cycles. A formal argument for the negative comovement is as follows. When the capital stock is the only state variable, the saddle path can be written as

$$C_\Delta - C_\Delta^* = A_1 (K_\Delta - K_\Delta^*), \qquad n_\Delta - n_\Delta^* = -A_2 (K_\Delta - K_\Delta^*).$$

Normality of consumption and leisure requires $A_1 > 0$ and $A_2 > 0$. Therefore, $\dot{C}_\Delta = -(A_1/A_2)\dot{n}_\Delta$.

Two elements of our model invalidate the above argument. The first is the substitution between search effort and employment. The substitution creates the possibility that labor employment moves in the same direction as leisure. So even when leisure and consumption always move in the same direction, it is possible that labor employment also moves in the

[11] The positive magnitude of $c_\Delta(t)$ is too small and the positive magnitude of $x_\Delta(t)$ too big to be visible in Figure 4.

same direction as consumption. The second element is the presence of labor employment as a state variable. The saddle path (4.2) gives

$$\dot{C}_{\Delta} = (Z_{11}, Z_{21}) \begin{pmatrix} Z_{14} & Z_{24} \\ Z_{15} & Z_{25} \end{pmatrix}^{-1} \begin{pmatrix} \dot{K}_{\Delta} \\ \dot{n}_{\Delta} \end{pmatrix}.$$

Thus consumption does not necessarily move in the opposite direction to labor employment.

6. Welfare Costs of Factor Income Taxation

6.1. Comparison between different τ's

The welfare results are presented in Tables 1 and 2. Table 1 reports marginal deadweight losses for different values of ε, σ and τ_{u0} when $\tau_{K0} = \tau_{W0} = 0.3$. The numbers confirm the intuition that marginal deadweight losses from capital income taxation and labor income taxation increase with labor supply elasticity ε and consumption demand elasticity $1/(1-\sigma)$.

Tables 1 and 2 here.

Three conclusions can be drawn from Table 1. First, the marginal deadweight loss of capital income taxation is much lower than in previous models. In Table 1, no figure of MDL_K exceeds 50 cents. In contrast, Judd (1987) found that MDL_K could easily exceed 50 cents and very often exceed one dollar. Capital income taxation is less costly here because firms job vacancies as a cushion to reduce the impact of a capital income tax. Facing an increase in capital income taxation, firms reduce the number of vacancies first and then gradually adjust the capital stock and labor employment. Such a cushion is not available in a standard model. Instead labor employment reacts immediately to absorb the impact of a capital income tax.

Although such a cushion also exists for the labor income tax, the labor income tax is more costly in the current model than in the standard model. This is because such a tax reduces labor employment more in the current model by inducing agents to substitute searching for working. Consequently, the labor income tax has a much narrower welfare gain over the capital income tax than in the standard model. For the case $\tau_u = 0.55$ in Table 1, the labor income tax can be as costly as or even more costly than the capital income tax. With different estimates $(\tau_{K0}, \tau_{W0}) = (0.5, 0.4)$ and reasonable values for search efficiency $1 - \alpha$ (≤ 0.6), Table 2 indicates a reversal of the welfare ranking between the capital income taxation and labor income taxation. There is a large marginal welfare gain from switching taxes away from labor income to capital income. The gain can be as big as 4 dollars. In Judd (1987), such welfare gains never occur and MDL_K is three to four times as large as MDL_W.

The second conclusion from Table 1 is that MDL_u is negative. The negativity is quite robust with respect to changes in ε, σ and τ_{u0}. The absolute value of MDL_u is above 50 cents and increases with τ_{u0}. The corresponding values of R_Δ and U_Δ are negative. Thus an increase in the search subsidy reduces the governmental revenue *and* intertemporal utility. In other words, a reduction in the search subsidy *increases* intertemporal utility and the governmental revenue. There is a large marginal welfare gain from cutting the search subsidy to finance a cut in the labor income tax. This welfare gain, measured by $MDL_W - MDL_u$, can easily exceed 50 cents for $\tau_u = 0.45$ and exceed a dollar for $\tau_u = 0.55$.

It is natural that the search subsidy reduces the governmental revenue. What is surprising at the first glance is that it also reduces intertemporal utility. Nevertheless, this utility-reducing effect can be explained. As discussed in subsection 5.1, an increase in the search subsidy decreases consumption and increases the labor force participation. Since both consumption and leisure are lower, utility falls. The critical element for this negative welfare effect is the increase in the labor force participation. For this to occur, the search subsidy must stimulate search by more than it discourages working. In this respect the negative welfare consequence of the search subsidy depends on the labor market frictions summarized by the parameters λ and α. We will see this dependence in the next subsection.

The third result from Table 1 is that the hiring subsidy has the opposite welfare effect to the search subsidy. The hiring subsidy increases welfare in all cases. It also increases the governmental revenue in all but the few cases marked with $*$ in Tables 1 and 2. For the hiring subsidy to increase the governmental revenue, it must have increased capital income and labor income sufficiently so that the additional revenue from the taxes on these incomes outweighs the subsidy. It is noteworthy that the magnitude of MDL_v is large. It runs as high as 714.512 when $\tau_{u0} = 0.45$. Although the magnitude falls when $\tau_{u0} = 0.55$, it still exceeds 1.8 dollars. Therefore cutting the search subsidy to subsidize hiring has a marginal welfare gain of at least 2.5 dollars and could have a gain of 715 dollars! Compared with this magnitude, the marginal gains from switching between other taxes are pale. The sizeable marginal gain from the hiring subsidy is analogous to the gain from the investment tax credit found in Judd (1987). Since firms must "invest" in vacancies in order to increase employment, the hiring subsidy is a subsidy on the investment in employment.

6.2. Importance of labor market frictions

Table 2 enables us to examine how marginal deadweight losses depend on the labor market conditions summarized by (α, λ). Looking across Table 2 vertically, we can find that capital income taxation, labor income taxation and subsidy to search all become more inefficient when λ increases. Basing on the interpretation of λ as the bargaining power of labor in wage determination, we can supply the following explanation. Facing a high bargaining power of labor, firms find it optimal to maintain few vacancies and hire few workers. Because capital and labor are complementary under the assumed technology, equilibrium capital stock is low as well. That is, labor income and capital income are both low when λ is high. An increase in either τ_K or τ_W raises smaller revenue and generates higher marginal welfare loss. In this case, encouraging agents to work by reducing the replacement ratio generates large gains.

Looking across Table 2 horizontally, we can find that capital income taxation, labor income taxation and search subsidy become more inefficient when α increases. Since α is the efficiency of job vacancies in the matching technology, the result can be explained as follows. An increase in τ_K, τ_W and τ_u all results in a fall in job vacancies (See Section 5). A given magnitude of the fall in job vacancies reduces job matches more significantly when α is larger. In the case of an increase in τ_W or τ_u, this implies that labor employment and hence capital will decrease more significantly when α is larger, generating a larger welfare loss. In the case of an increase in τ_K, labor employment could eventually rise but rise by less when α is larger. The capital stock falls more significantly, also generating a larger welfare loss.

An interesting alternative explanation exists for the dependence of MDL_u on λ and α. Recall that the condition $\lambda = 1 - \alpha$ is required for the search equilibrium to internalize the labor market externalities when there are no distortionary taxes (see Section 3). When $\tau_u \neq 0$ and $\tau_v = 0$ as in the current numerical exercise, the labor market is distorted even when $\lambda = 1 - \alpha$. There is too much search subsidy and too little hiring subsidy. If we fix λ and reduce α from the level $1 - \lambda$, the resulting increase in search efficiency $1 - \alpha$ reduces the labor market friction and reduces the welfare cost of τ_u. On the other hand, increasing α from the level $1 - \lambda$ exacerbates the labor market friction and increases the welfare cost of τ_u. This argument supports the positive dependence of MDL_u on α found in Table 2. Similarly the welfare cost of τ_u increases with λ. This argument also explains why there are cases where a search subsidy increases welfare, although there are only a few of such cases. A low λ and a low α are necessary for those cases. One example is $\lambda = 0.2$ and $\alpha = 0.2$.

The dependence of MDL_v on (α, λ) is nonmonotonic. Part of the reason for the nonmonotonic dependence is that the additional revenue raised by a hiring subsidy can switch signs as the parameters α and λ change.

Table 2 also reports the sensitivity of the results with respect to changes in the base values of taxes (τ_{K0}, τ_{W0}). When τ_{K0} changes from 0.3 to 0.5 and τ_{W0} from 0.3 to 0.4, the welfare costs of capital income taxation and labor income taxation increase significantly. This sensitivity is common in the dynamic models (see Judd 1987). Different from Judd, the change in (τ_{K0}, τ_{W0}) increases the welfare cost of labor income taxation by a much larger margin than the welfare cost of capital income taxation. The labor income tax becomes more costly than the capital income tax when α is 0.6 or larger. Despite this sensitivity, the results on MDL_u and MDL_v are robust. MDL_u continues to be negative and exceeds 50 cents in absolute values in most cases. MDL_v continues to be negative and exceeds a dollar in absolute values.

7. Conclusion

Building on Pissarides (1990), Mortensen (1992) and particularly Merz (1993), we have integrated unemployment into an equilibrium dynamic model with capital accumulation.

The model generates dynamic effects and welfare costs of taxation that are quite different from the standard model. The differences illustrate that unemployment and labor market conditions are important for the examination of factor income taxation.

We hope that the model constructed here provides a benchmark for future research in a broad area. In general it is useful for issues which involve unemployment and capital accumulation. One can extend the model to compute the gain or loss of abolishing a certain tax as in Lucas (1990a). One can also use the model to examine the normative questions of optimal taxation. Standard models without unemployment such as Chamley (1986) have shown that optimal capital income tax converges to zero when the economy converges to the steady state, but optimal labor income tax does not converge to zero. The key argument is that capital supply is perfectly inelastic in the short-run but perfectly elastic in the long-run. In the present model, labor employment has similar features as capital. This may imply tax smoothing between capital and labor income even in the steady state.

The question of optimal search subsidy is also interesting. The dynamic framework of this paper provides a novel avenue along which this old question can be examined (see Topel and Welch 1980 for a survey of the old arguments). The results in this paper indicate that for given tax rates on capital and labor income, the search subsidy is inefficient. It is interesting to determine the optimal subsidy when factor taxes are chosen optimally. Similarly, optimal hiring subsidy may also be different in the current framework of intertemporal maximization.

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Table 1. MDL of Permanent Changes in τ

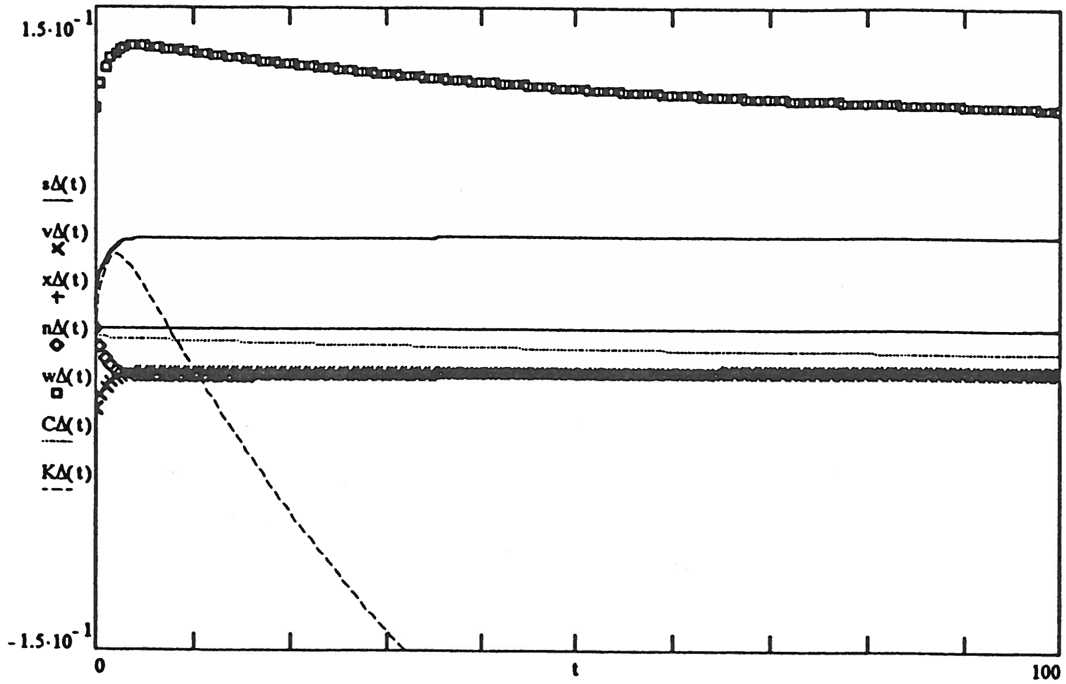
		$\tau_{u0} = 0.45$				$\tau_{u0} = 0.55$			
ε	σ	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0.1	-0.1	0.108	0.338	-0.585	-8.269	0.232	0.350	-0.742	-1.815
	-0.5	0.105	0.307	-0.582	-9.313	0.226	0.318	-0.740	-1.835
	-2.0	0.096	0.239	-0.572	-15.047	0.211	0.248	-0.732	-1.899
	-5.0	0.085	0.181	-0.560	-103.191	0.192	0.188	-0.723	-1.997
0.4	-0.1	0.197	0.410	-0.551	-7.810	0.328	0.422	-0.723	-1.861
	-0.5	0.178	0.361	-0.547	-10.364	0.302	0.372	-0.719	-1.910
	-2.0	0.137	0.271	-0.538	-68.058	0.245	0.281	-0.710	-2.048
	-5.0	0.102	0.211	-0.529	*14.207	0.200	0.220	-0.701	-2.210
0.6	-0.1	0.240	0.444	-0.535	-7.621	0.374	0.457	-0.714	-1.882
	-0.5	0.209	0.384	-0.532	-10.810	0.334	0.396	-0.710	-1.942
	-2.0	0.149	0.284	-0.526	*714.512	0.256	0.294	-0.702	-2.101
	-5.0	0.106	0.222	-0.522	*11.138	0.202	0.231	-0.695	-2.271

$\gamma = 0.25$, $\theta = 0.05$, $\alpha = 0.6$, $\lambda = 0.4$, $\tau_{K0} = 0.3, \tau_{W0} = 0.3$, $\tau_{v0} = 0$. Column (1) ((2), (3), (4)) reports MDL_W (MDL_K, MDL_u, MDL_v respectively). $*$: $R < 0$.

Table 2. Dependence of MDL on (α, λ) and (τ_{K0}, τ_{W0})

	(τ_{K0}, τ_{W0})	(0.3,0.3)				(0.5,0.4)			
λ	α	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
0.2	(1)	-0.072	0.036	0.164	0.320	0.047	0.473	1.134	4.090
	(2)	0.216	0.236	0.255	0.275	0.641	0.673	0.707	0.743
	(3)	2.377	0.348	-0.386	-0.766	0.476	-0.470	-0.764	-0.908
	(4)	-0.736	-0.187	*1.641	-22.334	-0.448	-2.105	-1.561	-1.452
0.4	(1)	0.054	0.142	0.245	0.368	0.260	0.714	1.640	4.583
	(2)	0.249	0.265	0.281	0.298	0.672	0.702	0.734	0.768
	(3)	0.271	-0.404	-0.710	-0.884	-0.392	-0.752	-0.883	-0.951
	(4)	-0.341	-3.959	-2.048	-1.792	-1.358	-1.210	-1.185	-1.175
0.6	(1)	0.099	0.180	0.274	0.384	0.337	0.801	1.749	4.762
	(2)	0.262	0.276	0.291	0.307	0.683	0.712	0.744	0.776
	(3)	-0.217	-0.617	-0.809	-0.923	0.618	-0.838	-0.922	-0.966
	(4)	-2.545	-1.536	-1.440	-1.404	-1.133	-1.113	-1.108	-1.106
0.8	(1)	0.122	0.199	0.288	0.392	0.376	0.846	1.805	4.855
	(2)	0.268	0.282	0.296	0.312	0.688	0.718	0.749	0.781
	(3)	-0.434	-0.718	-0.858	-0.942	0.722	-0.879	-0.940	-0.973
	(4)	-1.359	-1.295	-1.278	-1.271	-1.080	-1.077	-1.075	-1.075

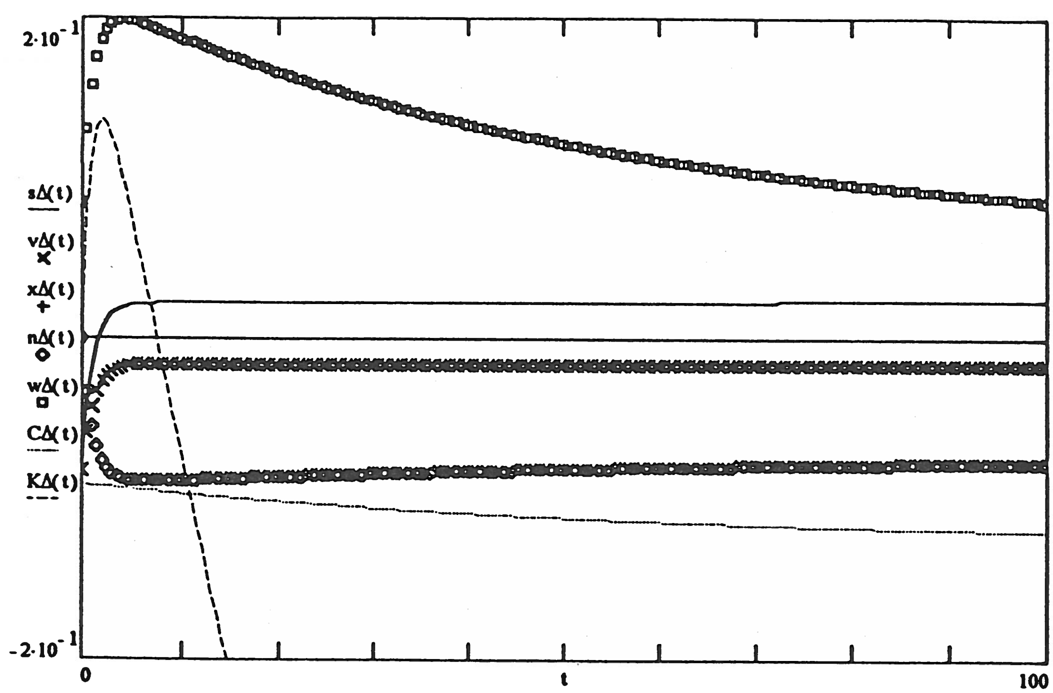
$\sigma = -2$, $\varepsilon = 0.4$, $\gamma = 0.25, \theta = 0.05$, $\tau_{u0} = 0.55$, $\tau_{v0} = 0$. Row (1) ((2), (3), (4)) reports MDL_W (MDL_K, MDL_u, MDL_v respectively). $*$: $R < 0$.



ε := 0.4 σ := - 2 λ := 0.4 α := 0.6 τk := 0.3 τw := 0.3

τu := 0.45 τu1 := 1 τk1 := 0 τw1 := 0 τv1 := 0

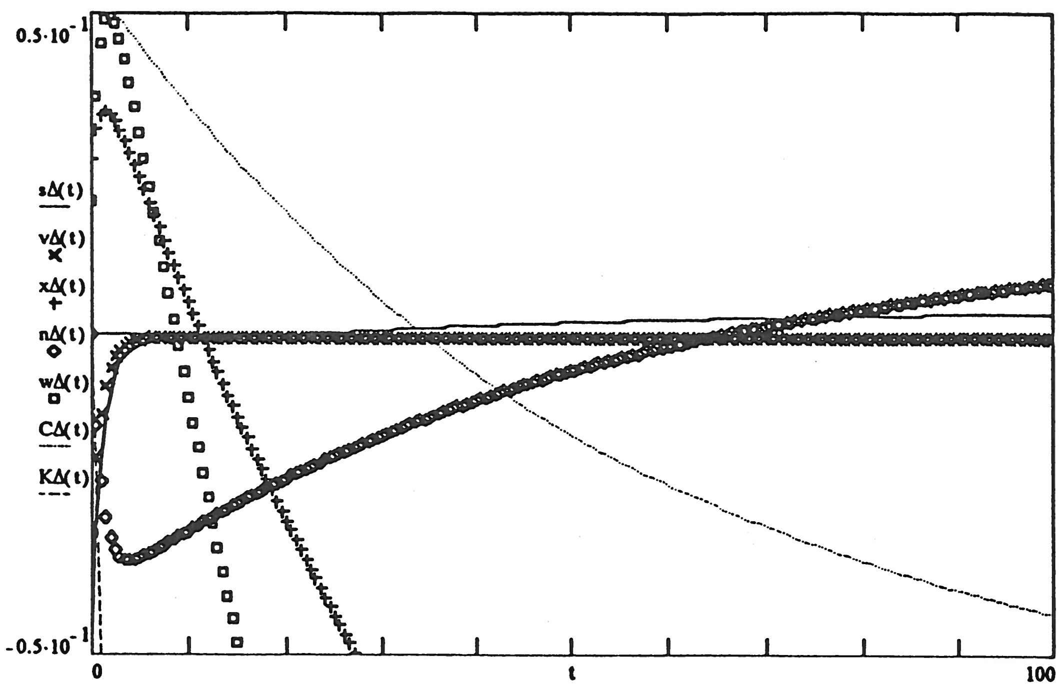
Figure 1



ε := 0.4 σ := - 2 λ := 0.4 α := 0.6 τk := 0.3 τw := 0.3

τu := 0.45 τw1 := 1 τk1 := 0 τu1 := 0 τv1 := 0

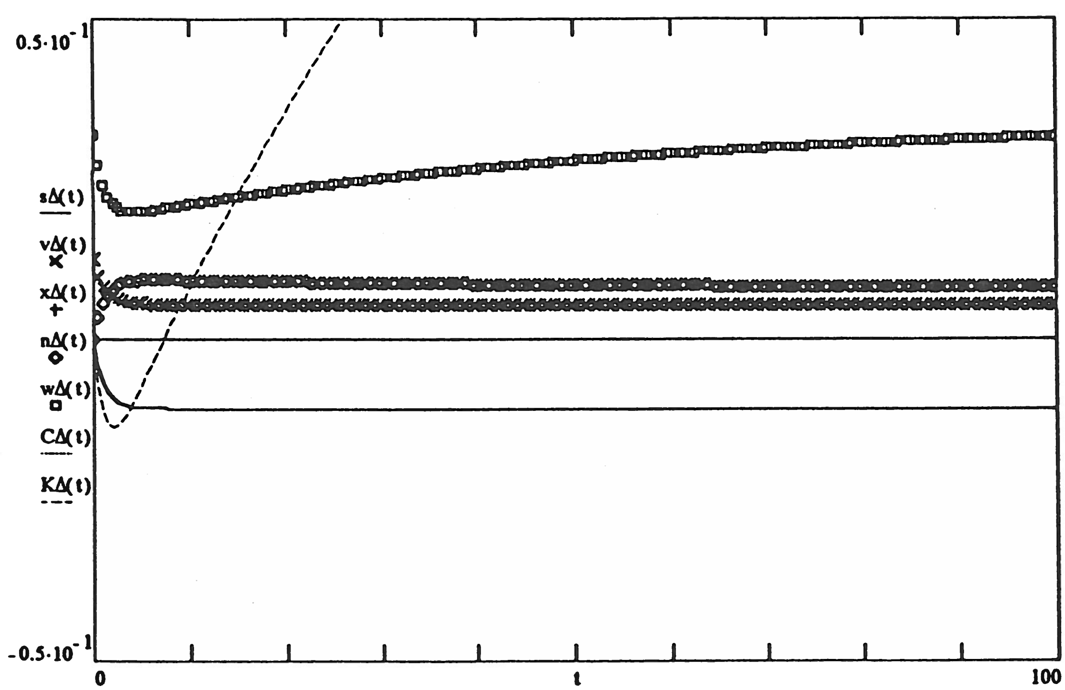
Figure 2



$\varepsilon := 0.4$ $\sigma := -2$ $\lambda := 0.4$ $\alpha := 0.6$ $\tau k := 0.3$ $\tau w := 0.3$

$\tau u := 0.45$ $\tau k1 := 1$ $\tau w1 := 0$ $\tau u1 := 0$ $\tau v1 := 0$

Figure 3



$\varepsilon := 0.4$ $\sigma := -2$ $\lambda := 0.4$ $\alpha := 0.6$ $\tau k := 0.3$ $\tau w := 0.3$

$\tau u := 0.45$ $\tau v1 := 1$ $\tau k1 := 0$ $\tau u1 := 0$ $\tau w1 := 0$

Figure 4