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A Dynamic Model of Endogenous Trade Policy

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Abstract

A puzzle in the analysis of trade policy is why free trade outcomes, which maximize world income, are not more often observed. One reason is that economic agents with special interests affect both the form and level of international protective policies. This paper investigates the dynamic interaction of special interest groups and highlights endogenous links between current policies and future policies. We explore these intertemporal links in a general equilibrium environment in which lobbying activity and tariff policies are the outcome of a dynamic game among interest groups located in both countries. In the Markov perfect equilibrium, future policies are affected by current policies as the current trade regime affects the ability and the willingness of these groups to lobby for future policies which favor their interests. An unusual feature of the economy is that the transition probabilities over trading regimes are *endogenous*. The average length of a trade war can be computed as a function of these probabilities, and we show that it is increasing in the agents' degree of risk aversion. In addition, due to deadweight losses associated with lobbying, expected aggregate welfare in a lobbying equilibrium is generally below autarkic welfare.

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I. Introduction

A puzzle in the analysis of trade policy is why free trade outcomes, which maximize world income, are not more often observed. Despite a trend towards freer trade over successive decades of GATT rounds, tariff based and non-tariff based forms of protection are still abundantly evident. One reason for the continued presence of these barriers is that economic agents with special interests affect both the form and level of international protective policies. In particular, economic prescriptions typically assume frictionless redistributive instruments, while political activity involves manipulation of very imperfect, costly, redistributive mechanisms.

Voting behavior and resulting policies have been studied by Mayer (1984), Baldwin (1982), Brock and Magee (1980) and others. The median voter approach adopted by Mayer assumes that voters are perfectly informed, together with single peaked preferences. Equilibrium tariffs will reflect the preferred tariff of the median voter. An alternative methodology which we adopt is based on an imperfectly informed electorate. Different trade policies create rents for different groups, and an incentive for rent seeking in the form of *lobbying* exists. The mechanism by which lobbying expenditures translate into votes is usually not modeled explicitly, but operates through the dissemination of information (one could appeal to models of advertising for a modeling framework). The mechanism can operate at the level of individual voters, or in representative democracies on the representatives in legislative bodies. See, for example, Young and Magee (1986), Findlay and Wellisz (1982), Brock and Magee (1978), and especially Magee, Brock, and Young (1989) for examples of this approach..

This paper investigates the dynamic interaction of special interest groups and highlights endogenous links between current policies and future policies. We explore these intertemporal links in a general equilibrium environment in which lobbying activity and tariff policies are the outcome of a dynamic game among interest groups located in both countries. Future policies are affected by current policies as the current trade regime affects both the ability and the willingness of these groups to lobby for future policies which favor their interests.

We study an infinite horizon 2x2x2 Heckscher-Ohlin-Samuelson trade model. Interest groups are associated with factor ownership and divide their factor endowment between productive activity and lobbying. There are two potential types of governments in each country with each type favoring a particular factor. Lobbying by factors within a country affects the probability of election of the type of government in power in that country in the following period. The four interest groups (two in each country) play a dynamic Nash game in Markov strategies and choose levels of lobbying activity to maximize their expected discounted lifetime utility. The benefit of lobbying results from increasing the probability that the policy-making government which favors the lobby's interests will be elected in the next period. The cost of lobbying reflects foregone current consumption. The equilibrium concept is that of Markov perfection with the current pair of governments as the state variable.

Generally, factors' lobbying expenditures in any period will depend on the prevailing policies in that period. The degree of relative risk aversion of factor groups will determine whether factors lobby more or less when current policies favor their interests than when current policies do not favor their interests. In particular, factor groups will lobby more in favorable states than in unfavorable states if their preferences exhibit a high degree of relative risk aversion. The dependence of lobbying activity on current policies implies that the probabilities of future policies are affected by current policies. This link can be summarized by a transition probability matrix. These probabilities are *endogenous* and depend on the economy's underlying preferences, technologies, and endowments.

The relationship between the degree of risk aversion and equilibrium lobbying activity suggests a link between preferences and the average length of a trade war. Trading systems comprised of agents with high relative risk aversion will exhibit longer trade wars, on average, than economies comprised of agents with lower relative risk aversion. Furthermore, the pattern of endowments across groups will affect whether trade wars last longer, on average, than do periods of free trade.

The presence of dynamically interacting special interest groups also has important normative implications. In order to aggregate the utility of different groups, we restrict our normative comparisons to the case of linearly homogeneous utility functions, such that the distribution of wealth between groups has no effect on the aggregate measure.

Under this assumption, expected aggregate utility in a country in an environment with lobbying is below that in a free trade equilibrium without lobbying. This is a standard Prisoner's Dilemma result and is present in static models of tariffs such as those presented in Magee, Brock, and Young (1989). In addition, aggregate welfare is generally lower in the presence of lobbying than if the economies were to operate in autarky in every period with no lobbying. This results from the deadweight losses associated with lobbying and occurs even though free trade is a possible outcome in the economy with lobbying.[1] If the factor which favors free trade has a high enough endowment, then the probability of free trade in the lobbying equilibrium may outweigh the deadweight losses from lobbying. In this case, expected aggregate welfare will be higher in the presence of lobbying than if the economies were to operate in autarky indefinitely.

The remainder of the paper is organized as follows. Section II describes the economy and characterizes the link between current trade policies and lobbying activity. Section III presents numerical results for example economies and suggests relationships among economy parameters and equilibrium lobbying, the average length of trade wars, and aggregate welfare. Section IV concludes and offers suggestions for future work.

II. The Model

II.1: The Economy

We examine a two factor, two good, two country Heckscher-Ohlin-Samuelson model of trade. Technologies exhibit constant returns to scale and are identical across countries. Factors, goods, and countries are labelled so that good j uses factor j relatively intensively, and country j is relatively well endowed with factor j, for $j = 1, 2$. The Heckscher-Ohlin-Samuelson Theorem holds in this economy and implies that country j will be a net exporter of good j and a net importer of good $i \neq j$.

Consumers are differentiated with respect to factor ownership and can be separated into two interest groups in each country. Agents in group 1 own only factor 1 and those in group 2 own only factor 2. The measure of each group in each country is normalized to one so that group aggregates equal individual levels within a group. This allows us to

[1] This is similar to the black and gray holes discussed by Magee, Brock, and Young (1989).

focus on representative group members. The endowment each period of factor j to group j in country k is denoted $\bar{l}_j^k$. Consumers have identical preferences both within and across countries. These preferences are represented by the following period utility function

$$U(c_{j1}^k, c_{j2}^k),$$

where c_{ji}^k denotes consumption of good i by group j in country k . $U(\cdot,\cdot)$ is a Von Neumann-Morgenstern utility function, and is assumed to be homothetic, concave, and increasing in both its arguments.

There are two potential government types in each country. Government j favors group j and seeks to maximize the utility of that group through its choice of tariff policy. Which government is in power is the result of an election, whose outcome is stochastic. The probability of a particular government being elected in a country depends on lobbying expenditures by groups in that country. The optimal level of lobbying expenditures chosen by groups is discussed in detail below.

We focus on equilibria which are characterized by full employment and fully diversified production in both countries. In addition, we assume the absence of a Metzler paradox so that increases in the tariff rate on a country's imports will increase the domestic price of the imported good. Tariff revenues are distributed lump-sum to factors in proportion to post-tariff factor income shares. This ensures that the revenue distribution will not have any redistributive effects on factors' incomes. Under these assumptions, the Stolper-Samuelson Theorem implies that an increase in a tariff imposed by country k will decrease the real income of group k (the group which owns the factor used relatively intensively in production of that country's export good) and will increase the real income of group $j \neq k$ (the group which owns the factor used relatively intensively in production of that country's import-competing good).

II.2: Government Tariff Game

In each period, the government currently in power in each country plays a static tariff game against the government in the other country. Given the income effects of tariffs discussed above, the solution to the tariff game is straightforward. The optimal tariff policy for government k in country k is to set a zero tariff since that government completely favors the export group k. On the other hand, the optimal tariff policy for government $j \neq k$ in country k is to set a prohibitive tariff since that government completely favors the import group j. These policies are optimal independent of the tariff policy of the other country.

Let τ^k denote the tariff rate in country k and let $\bar{\tau}^k$ denote the (prohibitive) tariff rate in country k which eliminates imports. Letting G_j^k signify that government j is in power in country k, then Table 1 depicts the unique Nash equilibria of the static tariff game between governments in each country.

Table 1: Government Game

	G_1^2	G_2^2
G_1^1	$\tau^1 = 0 \quad \tau^2 = \bar{\tau}^2$ *(Autarky)*	$\tau^1 = 0 \quad \tau^2 = 0$ *(Free Trade)*
G_2^1	$\tau^1 = \bar{\tau}^1 \quad \tau^2 = \bar{\tau}^2$ *(Autarky)*	$\tau^1 = \bar{\tau}^1 \quad \tau^2 = 0$ *(Autarky)*

Hence, free trade will be the outcome only when both governments favor their respective export groups. Prohibitive tariffs and autarky will result under any other distribution of governments across the two countries.

The *state* in any period is described by the type of government in power in each country. Let $s \in \{s_{11}, s_{12}, s_{21}, s_{22}\} \equiv \mathcal{S}$ denote this state, where the first subscript denotes the government type in country 1 and the second subscript denotes the government type in country 2. Let $\mathcal{S}^a \equiv \{s_{11}, s_{21}, s_{22}\}$ denote the states in which the trading environment is autarky. We also let $z \in \{z^a, z^f\}$ indicate the trading environment, i.e. whether the countries are operating in autarky or in free trade. Hence, $z = z^a$ if $s \in \mathcal{S}^a$ and $z = z^f$ if $s = s_{12}$.

II.3: Equilibrium Lobbying

Each type of representative factor owner divides their specialized endowment between productive and lobbying activity. Let the vector of lobbying activity in country k be given by $\lambda^k \equiv (\lambda_1^k, \lambda_2^k)$ where λ_j^k denotes the amount of the endowment of factor j in country k which is devoted to lobbying. The world-wide vector of lobbying is written $\lambda \equiv (\lambda^1, \lambda^2)$. For a given level of lobbying activity, the equilibrium vector of goods prices in country k

in trading environment z is written:

$$p^k(z,\lambda) \equiv (p_1^k(z,\lambda), p_2^k(z,\lambda))$$

and the vector of equilibrium factor prices is written:

$$w^k(z,\lambda) \equiv (w_1^k(z,\lambda), w_2^k(z,\lambda)).$$

Note that in the fully diversified free trade equilibrium ($z = z^f$), goods prices and factor prices will be equalized across countries.

The timing of the game is as follows. The four factor groups move first, simultaneously choosing how much of their endowments to allocate to lobbying activities to maximize their expected discounted lifetime utility. The pair of governments which are subsequently elected implement the Nash equilibrium tariff policies shown in Table 1. Thus, factor groups behave as Stackelberg leaders with respect to potential governments. Given their lobbying activity and the current trading regime, income from productive activity is allocated over consumption of the two goods to maximize period utility. For a given trading environment, z, we write the final stage problem for a representative member of group j in country k as follows:

$$\max_{c_{j1}^k, c_{j2}^k} U(c_{j1}^k, c_{j2}^k)$$

$$\textit{subject to} \quad p_1^k(z,\lambda)c_{j1}^k + p_2^k(z,\lambda)c_{j2}^k = w_j^k(z,\lambda)(\bar{l}_j^k - \lambda_j^k).$$

The consumer takes goods and factor prices as given when solving this problem.

The solution to the above period problem gives Marshallian demand for good i by a member of group j in country k. We can write these demands as functions of the trading environment and the vector of lobbying:

$$d_{ji}^k(z,\lambda) \equiv c_{ji}^k(p^k(z,\lambda), I_j^k(z,\lambda)), \tag{1}$$

where $I_j^k(z,\lambda) = w_j^k(z,\lambda)(\bar{l}_j^k - \lambda_j^k)$. Substituting these demands into the utility function allows us to write an indirect period utility function as a function of the trading environment and the vector of lobbying:

$$W_j^k(p^k(z,\lambda), I_j^k(z,\lambda)). \tag{2}$$

We now examine the choice of lobbying expenditures. Let $\pi_j^k(\lambda^k)$ be the probability that the government of type j will be elected in country k in the next period given the current level of lobbying. $\pi_j^k(\cdot)$ satisfies the following properties:

$$0 \leq \pi_j^k(\cdot) \leq 1 \qquad \sum_{j=1}^{2} \pi_j^k(\cdot) = 1 \tag{3.1}$$

$$\frac{\partial \pi_j^k(\cdot)}{\partial \lambda_j^k} > 0 \qquad \frac{\partial \pi_j^k(\cdot)}{\partial \lambda_i^k} < 0 \ \ for\ i \neq j \tag{3.2}$$

$$\frac{\partial^2 \pi_j^k(\cdot)}{(\partial \lambda_j^k)^2} < 0 \qquad \frac{\partial^2 \pi_j^k(\cdot)}{(\partial \lambda_i^k)^2} > 0 \ \ for\ i \neq j. \tag{3.3}$$

This implies that increases in lobbying activity by a group, ceteris paribus, increases, at a decreasing rate, the probability that the government which favors its interests will be elected and decreases, at a decreasing rate, the probability that the other government will be elected. These probabilities imply that the probability of a given state is well defined and is given by

$$\Pi(s_{ij}, \lambda) = \pi_i^1(\lambda^1)\pi_j^2(\lambda^2). \tag{4}$$

Each member of each group in each country chooses lobbying activity to maximize their expected discounted lifetime utility. We write this problem as a dynamic programming problem as follows:

$$V_j^k(s) = \max_{\lambda_j^k} \left\{ W_j^k(p^k(z,\lambda), I_j^k(z,\lambda)) + \beta \sum_{s' \in \mathcal{S}} \Pi(s',\lambda) V_j^k(s') \right\} \tag{5}$$

where $z = z^a$ when $s \in \mathcal{S}^a$ and $z = z^f$ when $s = s_{12}$. In addition, $0 < \beta < 1$ is the common discount factor and $V_j^k(\cdot)$ is the value function of a member of group j in country k. Each agent takes the lobbying activity of all other groups as given when solving their problem. As noted above, the agent does not take into account how their lobbying expenditures affect equilibrium prices.

The solution to the above problem is a best response function as a function of the state and the vector of other agents' lobbying activities, λ_{jo}^k:

$$\hat{\lambda}_j^k(s, \lambda_{jo}^k).$$

We focus on Markov strategies and Markov perfect equilibria. The Markov perfect equilibrium vector of lobbying activity as a function of the state, $\lambda^*(s)$, satisfies:

$$\lambda_j^{k*}(s) = \hat{\lambda}_j^k(s, \lambda_{jo}^{k*}(s)) \;\; j = 1,2; \; k = 1,2. \tag{6}$$

Since the state space is discrete and current lobbying activity affects only the *probabilities* of future states, the Markov perfect equilibrium can be written as the solution to a system of 16 equations. This can be seen as follows. Since autarky prevails for $s \in \mathcal{S}^a$, we can write Markov perfect equilibrium lobbying choices as functions of the trading environment:

$$\tilde{\lambda}(z^a) = \lambda^*(s) \;\; for \; s \in \mathcal{S}^a$$

$$\tilde{\lambda}(z^f) = \lambda^*(s) \;\; for \; s = s_{12},$$

where the payoff relevant state variable is now the *trading environment* (autarky or free trade), and not the pair of governments which produced it. In a similar manner, we can define a value function over the two possible trading environments:

$$\tilde{V}_j^k(z^a) \equiv V_j^k(s) \qquad for \; s \in \mathcal{S}^a$$

$$\tilde{V}_j^k(z^f) \equiv V_j^k(s) \qquad for \; s = s_{12}.$$

From equation (5), these functions must satisfy:

$$\begin{aligned} \tilde{V}_j^k(z) =& W_j^k(p^k(z,\lambda), I_j^k(z,\lambda)) \\ &+ \beta\Big\{\Big[\sum_{s'\in\mathcal{S}^a} \Pi(s', \tilde{\lambda}(z))\Big] \tilde{V}_j^k(z^a) + \Pi(s_{12}, \tilde{\lambda}(z))\tilde{V}_j^k(z^f)\Big\}. \end{aligned} \tag{7}$$

This represents 8 equations: one for each of the 2 trading environments and one for each of the 4 groups. In addition, the functions must satisfy the first order conditions:

$$\frac{\partial W_j^k}{\partial I_j^k}\frac{\partial I_j^k}{\partial \lambda_j^k} + \beta\Big\{\Big[\sum_{s'\in\mathcal{S}^a} \frac{\partial \Pi}{\partial \lambda_j^k}(s', \tilde{\lambda}(z))\Big] \tilde{V}_j^k(z^a) + \frac{\partial \Pi}{\partial \lambda_j^k}(s_{12}, \tilde{\lambda}(z))\tilde{V}_j^k(z^f)\Big\} = 0 \tag{8}$$

This also represents 8 equations. Hence equations (7) and (8) are 16 equations in the 16 unknowns: $\{\tilde{\lambda}_j^k(z), \tilde{V}_j^k(z)\}$ for $j = 1,2; k = 1,2; z = z^a, z^f$.

Using the properties of the probability functions given by equation (3), and rearranging allows the first order conditions to be written as:

$$\beta \frac{\partial \pi_k^k}{\partial \lambda_j^k} \pi_i^i [\tilde{V}_j^k(z^f) - \tilde{V}_j^k(z^a)] = \left[\frac{-\partial W_j^k(p^k(z,\lambda), I_j^k(z,\lambda))}{\partial I_j^k} \right] \left[\frac{\partial I_j^k}{\partial \lambda_j^k} \right] \qquad (i \neq k). \quad (9)$$

The left-hand side of this equation is the marginal benefit of lobbying to factor j in country k and the right-hand side is the marginal cost of lobbying. Consider each term.

The marginal benefit term can be written as a function of vector of lobbying:

$$MB_j^k(\lambda) \equiv \beta \frac{\partial \pi_k^k}{\partial \lambda_j^k} \pi_i^i [\tilde{V}_j^k(z^f) - \tilde{V}_j^k(z^a)] \qquad (i \neq k). \qquad (10)$$

This term reflects the benefits from increasing the probability that the government which favors the group's interests will be elected in the next period. Note that since the current state does not *directly* affect these probabilities, then the marginal benefit of lobbying is independent of the state. Since $\pi_k^k(\cdot)$ is increasing in λ_k^k and decreasing in $\lambda_j^k, j \neq k$, and since $\tilde{V}_k^k(z^f) - \tilde{V}_k^k(z^a) > 0$ and $\tilde{V}_j^k(z^f) - \tilde{V}_j^k(z^a) < 0$ for $j \neq k$, then the marginal benefit term is positive for all groups. The properties of the second derivatives of the probability functions given by equation (3.3), imply that the marginal benefit is a decreasing function of lobbying activity.

The marginal cost of lobbying reflects the foregone current period consumption associated with lobbying. Homotheticity and concavity of the utility function implies that the indirect utility function can be written as:

$$W_j^k(p^k(z,\lambda), I_j^k(z,\lambda)) = g(\omega(p^k)\, I_j^k), \qquad (11)$$

where $\omega(p^k)$ is a non-increasing, quasiconvex function and $g(\cdot)$ is an increasing, concave function.

Recall that individuals treat prices parametrically. By evaluating the right-hand side of equation (9) and using (11), we obtain the marginal cost of lobbying for factor j in country k as a function of the trading environment and the vector of lobbying:

$$MC_j^k(z,\lambda) = \frac{1}{\bar{l}_j^k - \lambda_j^k} g'(\omega(p^k)\, I_j^k) \omega(p^k)\, I_j^k \qquad (12)$$

or

$$MC_j^k(z,\lambda) = \frac{1}{\bar{l}_j^k - \lambda_j^k} g'(\tilde{v}(z))\, \tilde{v}(z), \tag{13}$$

where $\tilde{v}(z) = \omega(p^k(z, \tilde{\lambda}(z)))\, I_j^k(z, \tilde{\lambda}(z))$.

The coefficient of relative risk aversion for a factor is given by:

$$\sigma \equiv -\frac{g''(\tilde{v}(z))\tilde{v}(z)}{g'(\tilde{v}(z))}. \tag{14}$$

The following proposition illustrates that differences in lobbying activity by a group across trading environments can be characterized according to the value of σ.

Proposition :

A group will expend more resources on lobbying in the state which is favorable (unfavorable) to them if and only if $\sigma > (<) 1$ for consumption levels in the range between autarkic consumption and free trade consumption. A group's expenditures on lobbying will be independent of the state if and only if $\sigma = 1$ for all consumption levels.

Proof :

See Appendix A.

Corollary :

If the utility function is homogeneous of degree $\theta > 0 (< 0)$, then agents will lobby less(more) in the state which is favorable to their interests than in the unfavorable state.

Proof :

See Appendix A.

The corollary implies that if preferences are Cobb-Douglas or CES, then groups will lobby less in the state which is favorable to their interests.

The intuition behind these results is as follows. When current trading policies favor a factor's interests, that factor earns a higher real return. This has two opposing effects on the marginal cost of lobbying. Since lobbying uses resources of the factor's endowment, the price of lobbying relative to consumption is high when that factor's real return is high. This implies a higher marginal cost of lobbying in favorable states. This effect is reflected in the term $\tilde{v}(z)$ in equation (13) which is high in favorable states. On the other hand, since the factor's income is high in a favorable state, their marginal utility of income will be low (or unaffected in the case of linearly homogeneous preferences), and marginal foregone consumption will be less valuable than in an unfavorable state. This implies a lower marginal cost of lobbying in favorable states. The latter effect is reflected in the term $g'(\tilde{v}(z))$ in equation (13), where $g(\cdot)$ is concave. The degree of the factor's relative risk aversion will determine which effect dominates with the strength of the latter effect increasing in the degree of relative risk aversion. Figures 1.1 and 1.2 illustrate the effects of relative risk aversion on the marginal cost of lobbying, and, hence, on equilibrium lobbying.

Another interpretation of this result is that agents view lobbying as a form of insurance. The more risk averse agent will be willing to purchase more insurance when the government which favors their interests is in power, in order to smooth their consumption stream across states. Less risk averse agents are less concerned with consumption smoothing and increase their consumption when the government in power favors their interests, and hence decrease their lobbying expenditures.

These results have implications for the properties of transitional probabilities across the trading environments. We now turn to those properties and the related issue of the average lengths of trade wars and period of free trade.

II.4: Endogenous Probabilities

Equilibrium lobbying expenditures determine equilibrium probabilities over the four states contingent on the current trading environment:

$$\Pi^*(s' \mid z) = \Pi(s', \tilde{\lambda}(z)). \tag{15}$$

These imply a Markov probability matrix given by

$$\tilde{\Pi} = \begin{bmatrix} \tilde{\Pi}_{aa} & \tilde{\Pi}_{af} \\ \tilde{\Pi}_{fa} & \tilde{\Pi}_{ff} \end{bmatrix} \tag{16}$$

where $\tilde{\Pi}_{ij}$ denotes the probability of moving from trading environment i to trading environment j and are given by:

$$\tilde{\Pi}_{aa} = \sum_{s' \in S^a} \Pi^*(s' \mid z^a) \qquad \tilde{\Pi}_{af} = \Pi^*(s_{12} \mid z^a)$$

$$\tilde{\Pi}_{fa} = \sum_{s' \in S^a} \Pi^*(s' \mid z^f) \qquad \tilde{\Pi}_{ff} = \Pi^*(s_{12} \mid z^f)$$

These transition probabilities allow us to compute the expected number of periods the current trading environment is expected to last:

$$N^a = 1 + \sum_{t=1}^{\infty} t(\tilde{\Pi}_{aa})^t \tag{17}$$

$$N^f = 1 + \sum_{t=1}^{\infty} t(\tilde{\Pi}_{ff})^t. \tag{18}$$

Both N^a and N^f can be shown to converge to finite, bounded values. N^a is the expected length of a trade war, and N^f is the expected length of a regime of free trade. Since the structure of preferences affects factors' relative lobbying expenditures across different current trading environments, preferences will affect the average number of periods of either trading environment. The proposition suggests that if factor groups have high relative risk aversion and therefore lobby more in favorable states, then current trade policies can be expected to last longer than if the factor groups were less risk averse. In other words trade policies will be *persistent* whereas less risk averse factor groups will induce more *cyclical* trade policies. These effects are evident in the numerical examples presented in Section III.

We also seek to characterize long-run probabilities. Under fairly general conditions (see Stokey, Lucas, and Prescott (1989)), the Markov matrix given by (16) will converge to an invariant distribution, $\bar{\Pi}$, which gives the unconditional probabilities of each trading environment in any period. This invariant distribution is given by any row of the limit matrix:

$$\lim_{n\to\infty} \tilde{\Pi}^n$$

and satisfies

$$\bar{\Pi}(z^a) + \bar{\Pi}(z^f) = 1.$$

These probabilities allow us to determine ex ante expected discounted lifetime utility of group j in country k in an economy with lobbying:

$$\mathbf{UL}_j^k = [\bar{\Pi}(z^a) \;\; \bar{\Pi}(z^f)] \; [I + \sum_{t=1}^{\infty} \beta^t \tilde{\Pi}^t] \begin{bmatrix} W_j^k(p^k(z^a, \tilde{\lambda}(z^a)), I_j^k(z^a, \tilde{\lambda}(z^a))) \\ W_j^k(p^k(z^f, \tilde{\lambda}(z^f)), I_j^k(z^f, \tilde{\lambda}(z^f))) \end{bmatrix} \tag{19}$$

where I is the 2x2 identity matrix. We seek to compare this with discounted utility in the autarkic and free trade equilibria without lobbying:

$$\mathbf{UA}_j^k = \sum_{t=0}^{\infty} \beta^t W_j^k(p^k(z^a,0), I_j^k(z^a,0)) \tag{20}$$

and

$$\mathbf{UF}_j^k = \sum_{t=0}^{\infty} \beta^t W_j^k(p^k(z^f,0), I_j^k(z^f,0)) \tag{21}$$

Since changes in trading regime affect the distribution of income between factor groups, for comparisons of aggregate welfare it is convenient to restrict ourselves to utility functions such that aggregate welfare is independent of the distribution of income. Thus, we consider only the case of linearly homogeneous utility. We can then define aggregate utility in a country (for each trading regime) as the sum of utilities across the groups within a country:

$$\mathbf{UL}^k = \sum_{j=1,2} \mathbf{UL}_j^k \tag{22.1}$$

$$\mathbf{UA}^k = \sum_{j=1,2} \mathbf{UA}_j^k \tag{22.2}$$

$$\mathbf{UF}^k = \sum_{j=1,2} \mathbf{UF}_j^k \tag{22.3}$$

Given that there exist gains from trade (and given our restriction on preferences), we have the following relationships $\forall\ k$:

$$\mathbf{UA}^k < \mathbf{UF}^k \qquad\qquad \mathbf{UL}^k < \mathbf{UF}^k.$$

This reflects the standard prisoner's dilemma result of trade wars and has been noted by others in static environments. We cannot, however, determine whether or not expected utility in the lobbying equilibrium is higher than utility in the autarkic equilibrium because there are deadweight losses associated with lobbying. If the equilibrium is such that a high enough probability is placed on the free trade outcome in the lobbying equilibrium, then the lobbying equilibrium may result in higher aggregate welfare than autarky. However, it is possible that the deadweight loss associated with lobbying outweighs the benefits of a positive probability of free trade, and aggregate welfare is actually lower in the lobbying equilibrium than in autarky. It is shown below that both cases are possible.

We seek to determine the relationships between economy parameters (preferences, technologies, and endowments) and lobbying activity, average length of trade wars and regimes of free trade, and aggregate welfare. Given the non-linearities of the equilibrium system of equations, it is difficult to derive analytical results for these relationships. In the next section, we apply numerical solution methods to example economies and suggest relationships based on those experiments.

III. Applications

Suppose technologies are Leontief:

$$y_j = min\left\{\frac{v_{1j}}{a_{1j}}, \frac{v_{2j}}{a_{2j}}\right\}, \tag{23}$$

where v_{ij} is input of factor i in production of good j. Let preferences be given by:

$$U(c_1, c_2) = \frac{1}{\gamma}\left[c_1^{\alpha} c_2^{1-\alpha}\right]^{\gamma}, \tag{24}$$

where $0 < \alpha < 1$ and $\gamma \in [-1, 1]$. With these preferences, the coefficient of relative risk aversion is given by $\sigma = 1 - \gamma$. Autarkic and free trade equilibrium goods prices, factor prices, outputs, and indirect utility functions for these preferences and technologies are given in Appendix B.

Let the probability functions be given by:

$$\pi_i^k(\lambda^k) = \frac{\lambda_i^k}{\lambda_1^k + \lambda_2^k}. \tag{25}$$

This is a standard specification of the probability function and has been used elsewhere to model dichotomous choices (see for example, Magee, Brock, and Young (1989) and Young and Magee (1986)). With these specifications and specific parameter values, equations (7) and (8) were solved using a numerical non-linear equation solver. We consider a benchmark economy and vary economy parameters to explore the effects of preferences, technologies, and endowments on equilibrium variables in this economy. Table 2 gives benchmark parameter values. These parameters are consistent with interior solutions and reflect symmetry across countries.

TABLE 2: Benchmark Parameters

Preferences	*Technologies*		*Endowments*	
$\beta = .98$	$a_{11} = 4$	$a_{12} = 1$	$\bar{l}_1^1 = 18$	$\bar{l}_2^1 = 12$
$\alpha = .5$	$a_{21} = 1$	$a_{22} = 4$	$\bar{l}_1^2 = 12$	$\bar{l}_2^2 = 18$

Ideally, we would like to perform comparative dynamics analytically on the endogenous variables, but the equation system is not tractable to analytical manipulation. By varying the economy parameters in our numerical simulations, however, we are able to obtain indications of how the system responds in equilibrium. Tables 3.1 and 3.2 illustrate the relationships among economy parameters and lobbying activity, average duration of a trading environment, and unconditional probabilities as suggested by the numerical experiments.[2] The tables demonstrate that the interactions in our general equilibrium system are complex. To give one example: one might expect that a stronger preference for the import good, which makes the import factor better off, and the export factor worse off, would lead to increased "compensatory" lobbying by the export group in order to protect its real income.[3] But in fact lobbying by the export group and the import group fall, reflecting a sort of strategic complementarity between the export and import group's lobbying activity. Nevertheless, we have reported the signs of the changes in endogenous variables, where these appear to be monotonic, for the sake of completeness.

Figure 2 depicts lobbying expenditure by each group as a function of γ for three levels of endowment of the export factor. The diagram illustrates the result suggested by the Proposition. For high degrees of risk aversion (γ low), each group expends *more* when the current trading regime favors their interests than when it does not. In contrast, for low degrees of risk aversion, each group expends *less* when the current trading regime favors their interests than when it does not.

Figure 3 illustrates how these lobbying expenditures affect the expected duration of a particular trading regime. As suggested in the previous section, economies with agents with a high degree of risk aversion will exhibit more persistent policies than economies with less risk averse agents. The figure also shows that the average length of a trade war is

[2] The entries in Table 3.1 are applicable to both trading regimes.

[3] The idea of a compensation effect is discussed in Magee, Brock, and Young (1989), Chapter 11.

decreasing in the endowment of the factor which favors free trade while the average length of a regime of free trade is increasing in this factor.

Figure 4 provides a welfare comparison when preferences are linearly homogeneous in income ($\gamma = 1$). As noted above, in this case the sum of individual utilities is an appropriate measure of aggregate welfare. The figure depicts aggregate welfare in each country as a function of the endowment of the export factor in country 1 for three cases: (i) autarky in every period and no lobbying (equation (22.2)), (ii) free trade in every period and no lobbying (equation (22.3)), and (iii) the economy with lobbying (equation (22.1)). The interesting result here is that the lobbying equilibrium will dominate autarky only when the probability of the free trade outcome is large enough. In all other cases, lobbying is an unproductive activity resulting in deadweight losses which not only makes each country worse off than in free trade but worse off than in autarky. Although aggregate welfare comparisons are difficult with other values of γ, a similar result should hold in those cases.

IV. Conclusions and Extensions

This paper has highlighted endogenous feedback effects between current trade policies and future policies. Current trading regimes affect agents' abilities and willingness to lobby for future regimes which favor their interests. This gives rise to an endogenous probability distribution over future trade policies which is conditional on current policies. The relative risk aversion of factor groups determines whether factors lobby more or less when current policies favor their interests than when current policies do not favor their interests. More risk averse factor groups imply equilibria with a longer duration of trading regimes, on average, than economies with less risk averse factor groups. In addition, due to deadweight losses associated with lobbying, expected aggregate welfare in an environment is generally below autarkic welfare.

Extensions of this work include examining economies with production technologies other than Leontief (such as Cobb-Douglas). As the Proposition is independent of technologies, the curvature of the utility function will still be important for determining the average duration of a trading regime, but gains from lobbying may differ from the Leontief model presented here. We have also limited our analysis to import tariffs. A natural extension would include examining dynamic lobbying behavior and trade policy in the presence of other policies such as export subsidies and quotas.

TABLE 3.1: Comparative Statics

Parameter	Lobbying by Domestic Export Factor	Lobbying by Domestic Import Factor	Lobbying by Foreign Export Factor	Lobbying by Foreign Import Factor
Preferences				
Discount Factor	+	+	+	+
Preference for Export Good	N	+	−	−
Preference for Import Good	−	−	N	+
Technologies				
Export Factor Requirement in Export Good	−	+	−	−
Import Factor Requirement in Export Good	+	−	+	+
Export Factor Requirement in Import Good	+	+	+	−
Import Factor Requirement in Import Good	−	−	−	+
Endowments				
Endowment of Export Factor	+	N	+	+
Endowment of Import Factor	−	N	−	−

Notes:

- The entries in the first column are from the domestic country's viewpoint.
- A $+$ indicates a positive relationship and a $-$ indicates a negative relationship between the parameter and the equilibrium variable. An **N** indicates a non-monotonic relationship.

TABLE 3.2: **Comparative Statics (cont.)**

Parameter	Expected Duration of Autarky	Expected Duration of Free Trade	Unconditional Probability of Autarky	Unconditional Probability of Free Trade
Preferences				
Discount Factor	NA	NA	NA	NA
Preference for Export Good	N	N	N	N
Preference for Import Good	N	N	N	N
Technologies				
Export Factor Requirement in Export Good	+	−	+	−
Import Factor Requirement in Export Good	−	+	−	+
Export Factor Requirement in Import Good	−	+	−	+
Import Factor Requirement in Import Good	+	−	+	−
Endowments				
Endowment of Export Factor	−	+	−	+
Endowment of Import Factor	+	−	+	−

Notes:

- The entries in the first column are from the domestic country's viewpoint.
- A + indicates a positive relationship and a − indicates a negative relationship between the parameter and the equilibrium variable. An N indicates a non-monotonic relationship and an **NA** indicates that the parameter has a negligible effect on the variable.

APPENDIX A

Proof of Proposition:

Define the following function:

$$f(v) = g'(v)v.$$

Since $\tilde{v}(z)$ is higher in trading environments which favor the group, then, for a given level of lobbying, the marginal cost of lobbying is higher(lower) in the state which favors the group if $f(v)$ is an increasing(decreasing) function for $v \in [\tilde{v}(z^a), \tilde{v}(z^f)]$. Differentiating $f(\cdot)$ gives

$$f'(v) = g''v + g',$$

and

$$Sign(f'(v)) = Sign(1 - \sigma).$$

Hence, whether the marginal cost of lobbying is higher, lower, or the same in the favorable state as in the unfavorable state depends on whether σ is less than, greater than, or equal to 1 in the relevant range. Since the marginal benefit of lobbying is independent of the trading environment and agents equate marginal benefit to marginal cost, agents will lobby less in the high cost environment. *Q.E.D.*

Proof of Corollary:

If $U(\cdot,\cdot)$ is homogeneous then $g(\cdot)$ must take the following form:

$$g(v) = Av^\theta,$$

where $A > 0$ if $\theta > 0$ and $A < 0$ if $\theta < 0$. In this case, we have

$$\sigma = 1 - \theta.$$

The Proposition then implies the result. *Q.E.D.*

APPENDIX B

With Leontief technologies given by equation (20) and preferences by (21), autarkic and free trade equilibrium price ratios in country k are given by:

$$\bar{p}^k(z^a,\lambda) \equiv \frac{p_1^k(z^a,\lambda)}{p_2^k(z^a,\lambda)} = \left[\frac{\alpha}{1-\alpha}\right]\left[\frac{y_2^{ka}}{y_1^{ka}}\right] \tag{B1}$$

and

$$\bar{p}^k(z^f,\lambda) \equiv \frac{p_1^k(z^f,\lambda)}{p_2^k(z^f,\lambda)} = \left[\frac{\alpha}{1-\alpha}\right]\left[\frac{y_2^{1f}+y_2^{2f}}{y_1^{1f}+y_1^{2f}}\right] \tag{B1'}$$

where for $m \in a, f$

$$y_1^{km} = \frac{1}{A}\left[a_{22}l_1^{km} - a_{12}l_2^{km}\right] \qquad y_2^{km} = \frac{1}{A}\left[a_{11}l_2^{km} - a_{21}l_1^{km}\right], \tag{B2}$$

$$A = a_{11}a_{22} - a_{12}a_{21}.$$

Recall that l_j^{km} is the portion of the endowment of group j in country k which is allocated to production and will vary across trading environments. In addition, the ratio of factor prices to the price of good 2 in trading environment z are given by:

$$\bar{w}_1^k(z,\lambda) \equiv \frac{w_1^k(z,\lambda)}{p_2^k(z,\lambda)} = \frac{1}{A}\left[a_{22}\bar{p}^k(z,\lambda) - a_{21}\right] \tag{B3}$$

$$\bar{w}_2^k(z,\lambda) \equiv \frac{w_2^k(z,\lambda)}{p_2^k(z,\lambda)} = \frac{1}{A}\left[a_{11} - a_{12}\bar{p}^k(z,\lambda)\right] \tag{B3'}$$

Using these, we write period utility as a function of z and λ:

$$W_j^k(p^k(z,\lambda), I_j^k(z,\lambda)) = \frac{1}{\gamma}\left[\alpha^\alpha(1-\alpha)^{1-\alpha}\bar{p}^k(z,\lambda)^\alpha\bar{w}_j^k(z,\lambda)(\bar{l}_j^k - \lambda_j^k)\right]^\gamma \tag{B4}$$

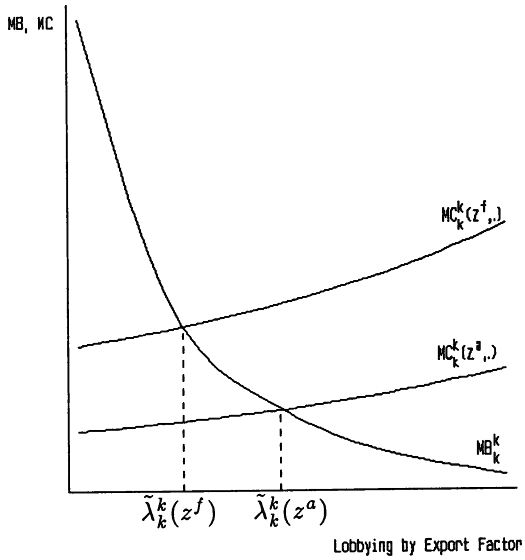

Figure 1.1: Low Risk Aversion

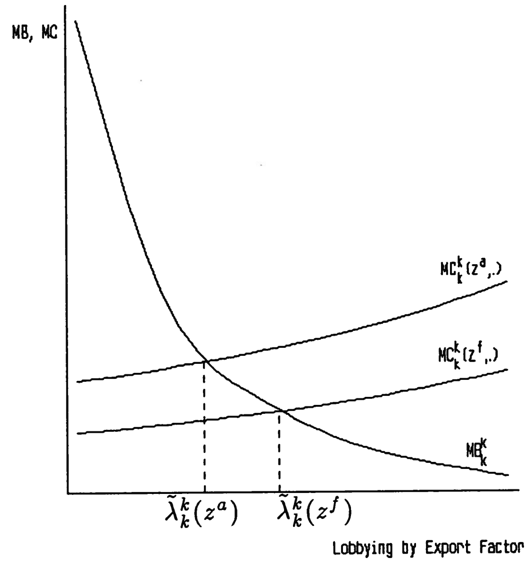

Figure 1.2: High Risk Aversion

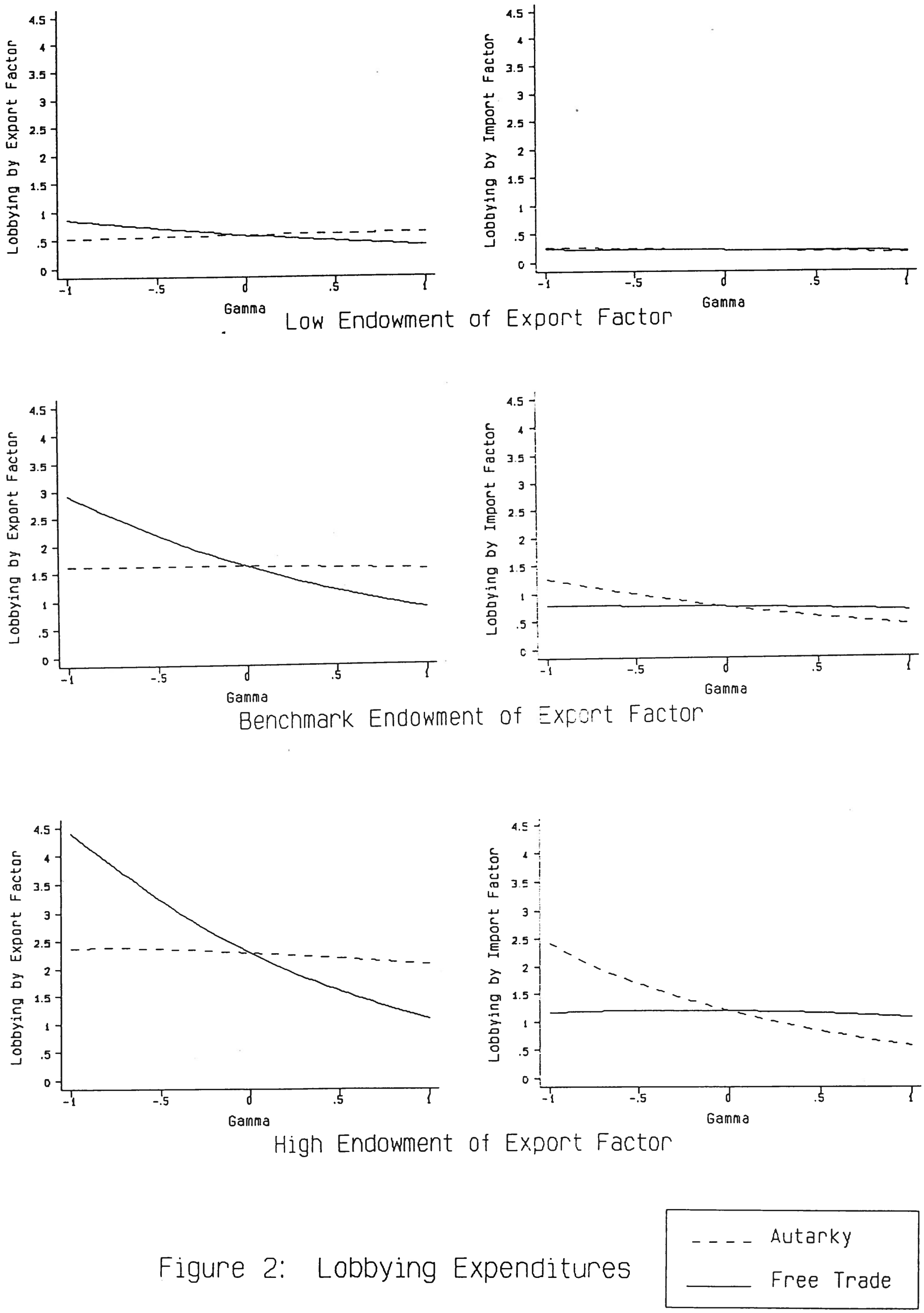

Figure 2: Lobbying Expenditures

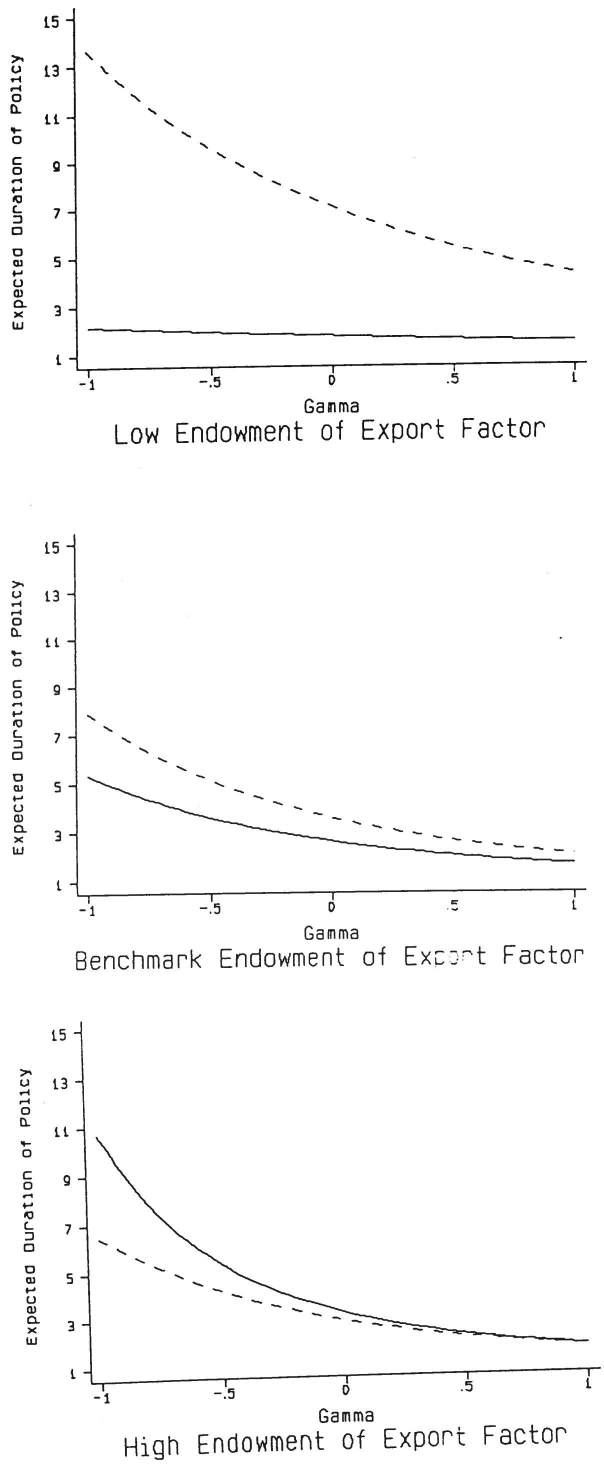

Figure 3: Expected Duration of Trade Policy

- - - Autarky
——— Free Trade

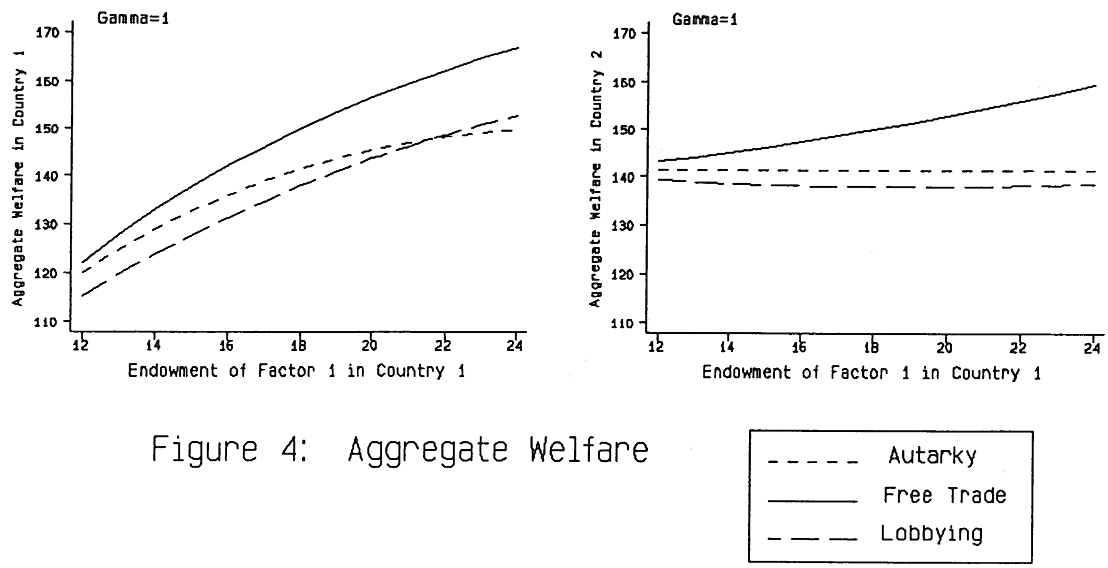

Figure 4: Aggregate Welfare

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