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National Wealth and NNP and Natural Resources and National Wealth and NNP

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and

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National Wealth and NNP

Abstract

Corresponding to current national wealth as discounted future optimal consumption (Weitzman [1976]) we observe current national wealth to be the value of net accumulated stocks of capital. It follows that NNP can be interpreted as interest on national wealth. A constant value of wealth implies no net investment and NNP equals the value of consumption, the Hicksian notion of "income".

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National Wealth and NNP

Introduction

Solow [1986] established that corresponding to constant consumption programs with essential exhaustible resource stocks there was a capital value that was remaining constant. He thus obtained a Hick's type of result, namely current consumption left capital intact. Solow was using Weitzman [1976] as a point of departure. It turns out that there is a quite general Solow-type of result, namely NNP(t) is "interest" on the value of accumulated capital, V(t), where V(t) can include diminutions in natural resource stocks and the interest rate is the social discount rate. The Weitzman [1976] result is that NNP(t) is "interest" on W(t) where W(t) is discounted future optimal consumption, the interest rate being the discount rate. There is then an attractive duality here: forward views of capital (discounted future optimal consumption programs at date t) coincide with backward views of capital (current value of accumulated stocks at date t). First we observe the Weitzman result as the solved Bellman equation in dynamic programming. This leads directly to the result that NNP(t) is interest on the value of accumulated capital. We then re-work the derivations with an essential exhaustible resource and then with endogenously produced "knowledge".

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NNP(t) and Capital

An economy has an initial stock of machine capital K(0) and labor services N(0). The latter remains unchanged.[1] Current output $\dot{Q}(t) = F(K(t),N(0))$ is divided between current aggregate consumption[2] C(t) and net investment $\dot{K}(t)$. That is

$$\dot{K}(t) = F(K(t),N(0)) - C(t)$$

A planner (or the invisible hand) maximizes $\int_0^\infty e^{-\rho t} U(C(t))dt$ where U(.) is a concave utility function with $U'(0) = \infty$ and $U(0) = 0$. The Bellman equation for this problem is

$$\rho V(t) = \max_{\{C(t)\}} [U(C) + V_K(t) [F(K(t), N(0)) - C(t)]]$$

Carry out the maximization and observe $U_C(t) = V_K(t)$. Then solve for {C*(t)}, {K*(t)} and {V*(t)}. We then have

$$\rho V^*(t) = U(C^*(t)) + U_C(C^*(t)) \dot{K}^*(t) \qquad (1)$$

The RHS of (1) is Weitzman's NNP(t) in utils. The LHS is $\rho \int_t^\infty e^{-\rho(s-t)} U(C^*(s))ds$. Thus NNP(t) is "interest" on V*(t) where the interest rate is ρ.

Now $[U(C^*(t)) + U_C(C^*(t))\dot{K}^*(t)]/\rho = \int_t^\infty U(C^*(s))e^{-\rho(s-t)}ds$. Hence $U_C(C^*(t))\dot{K}^*(t) = \rho \int_t^\infty e^{-\rho(s-t)} [U(C^*(s)) - U(C^*(t))]ds$ or the current value of investment is proportional to the discounted gain in the value of consumption, period by period, over the current value of

[1] If N(t) moved exogenously over time, then we would have a non-autonomous optimization problem. See Lozada [1992] for an analysis of the Weitzman result in non-autonomous environments including those with a non-constant discount rate. See also Asheim [1992] who works in discrete time.

[2] We could interpret K(t) as a vector of distinct capital stocks. With many consumption goods C(t) must be valued as U(C(t)). Weitzman worked with C(t) a scaler and $U(C(t)) \equiv C(t)$. Then the RHS of (1) is $C^*(t) + \dot{K}(t)$. This NNP(t) looks more familiar and can be interpreted as either util-valued or dollar valued.

consumption $U(C^*(t))$. Investment in NNP(t) is a proxy for the discounted value of extra consumption to be realized, in excess of the value of current consumption. This sheds light on the meaning of NNP(t) as a welfare measure in competitive economies. It is obvious what $U(C(t))$ is representing but $\dot{K}(t)$ has a less obvious interpretation. We see that the value of $\dot{K}(t)$ is precisely a proxy for future gains in consumption.

We can differentiate $V^*(t)$ to obtain $\dot{V}^*(t) = \rho V^*(t) - U(C^*(t))$. We substitute this expression for $\rho V^*(t)$ to obtain

$$\dot{V}^*(t) = U_C(C^*(t))\dot{K}^*(t)$$

and

$$V^*(t) - V^*(0) = \int_0^t U_C(C^*(s))\dot{K}^*(s)ds$$

This indicates that $V^*(t) - V^*(0)$ is the value of cumulative net investment in K. Hence NNP(t) is interest on the current value of cumulative investment including the initial stocks $V^*(0)$ or the current value of the net capital stock K(t).[3] If current net investment $\dot{K}(t) = 0$, then $V^*(t)$ remains unchanged or capital value is preserved intact. In this case, current NNP(t) is Hicksian income, namely that level of potential current consumption that leaves capital intact.

From (2) and the definition of $V^*(t)$ following (1), we have

$$V^*(0) = \int_t^\infty e^{-\rho(v-t)}U(C^*(v))dv - \int_0^t U_C(C^*(v))\dot{K}^*(v)dv \tag{3}$$

or for each t, $0 < t < \infty$, the difference in the discounted utility of optimal consumption and the current value of net capital <u>accumulated</u> to t is constant, the constant being the value of initial capital stock at time zero. This is a variant of the proposition in Dorfman, Samuelson, and

[3] Solow [1986] obtained this result in an economy with exhaustible resources and a constant consumption profile.

Solow [1958; p.322]: "At every point of time the value of the capital stock at current efficiency prices, discounted back to the initial time, is a constant, equal to the initial value." This is valid for "any efficient capital program and its corresponding profile of prices and own-rates".

An Exhaustible Resource Stock

Suppose oil R(t) is an essential input. Production becomes $Q(t) = F(K(t),N(0),R(t)) = C(t) + \dot{K}(t) - g(R(t))$ where g(R(t)) is the cost of extracting R(t) and processing it for use in the production of Q(t). R(t) is drawn from current stock S(t). Then $R(t) = -\dot{S}(t)$. There is an initial stock S(0) and we assume the {R(t)} path is asymptotic to zero. We can preserve the autonomous structure of the problem if S(t) never becomes zero. The invisible hand maximizes

$$\int_0^\infty U(C(t)e^{-\rho t}dt$$

subject to initial conditions and $\dot{K}=F(.) - C(t) - g(R(t))$ and $\dot{S} = -R(t)$. The Bellman equation for this problem is

$$\rho V(t) = \max_{\{C(t),R(t)\}} [U(C(t)) + V_K(t)[F(K(t),N(0),R(t)) - C(t) - g(R(t))] - V_S(t)R(t)]$$

The maximization yields $U_C(t) = V_K(t)$ and $F_R(t) - g_R(t) = V_S(t)/V_K(t)$. Along the optimal path we will have

$$\rho V(t) = U(C^*(t)) + U_C(C^*(t))\dot{K}^*(t) - U_C(C^*(t))[F_R(t)-g_R(t)]R^*(t)$$

The RHS is NNP(t) in utils. $F_R(t) - g_R(t)$ is Hotelling rent per ton extracted. Using up oil S(t) shows up as a debit in NNP(t), an economic depreciation of stock S(t) as it is drawn down by $\dot{S}(t)$ (equal to -R(t)). V*(t) is defined as $\int_t^\infty e^{-\rho(s-t)}U(C^*(s))ds$ as in the problem above. We have then a Weitzman type result, namely current NNP(t) is "interest" on W(t). We can also obtain $U_C(C^*(t))[\dot{K}^*(t) + [F_R(t) - g_R(t)]\dot{S}^*(t)] = \rho \int_t^\infty U(C^*(s)) - U(C^*(t))] e^{-\rho(s-t)}ds$. The value of net investment is proportional to the discounted value of future extra consumption over the current value of consumption. (We omit *'s on V(t) to avoid clutter.)

As above $\dot{V}(t) = \rho V - U(C^*(t))$ and thus $\dot{V}(t) = U_C(C^*(t))[\dot{K}^*(t) + (F_R^*(t) - g_R^*(t))\dot{S}^*(t)]$ which implies

$$V(t) - V(0) = \int_0^t U_C(C^*(s))[\dot{K}^*(s) + (F_R^*(s) - g_R^*(s))\dot{S}^*(s)]ds.$$

Thus we obtain the result that current NNP(t) in utils is interest on the value of cumulative net investment (including disinvestment $\dot{S}(s)$) plus the value of the initial capital stocks. The investment of exhaustible resource rents, i.e. $\dot{K}(t) = (F_R(t) - g_R(t))R(t)$, implies $\dot{V}(t) = 0$. Also NNP(t) will equal U(C(t)) in this case. This is the case Solow [1986] focused attention on.[4]

Technical Change and the Knowledge Stock

Weitzman made the important point that perfect foresight and endogenous technical change would result in technical change showing up in NNP as a knowledge stock increase, the stock change valued in marginal terms. We can illustrate this point by drawing on the model in Hartwick [1992, 1992a]. Technical change $\dot{A} = \alpha(\bar{L}-L_p)A$ is labor augmenting.[5] A is the stock of knowledge, $\bar{L}-L_p$ the workers in knowledge production and α a positive parameter. $\bar{L}$ is the total labor in the economy. Goods production $Q(t) = F(K(t),AL_p)$ where L_p are production

[4] Land is a durable natural stock and as such shares certain essential characteristics with durable "exhaustible" resources which can be extracted and re-cycled indefinitely. Hung [1992] has analyzed the national accounting implications of durable exhaustible resources.

[5] One could develop a model in which the knowledge stock was a private good as in $Q = F(K,L_p,A)$ rather than a public input. (See Hartwick [1993].) The above approach presumes that knowledge once produced "leaks" freely to all workers. There is no individual cost of acquiring the new knowledge, once it is produced.

workers. $\dot{K} = F(K, AL_p) - C$. Given initial conditions for K and A as K(0) and A(0) respectively, the invisible hand maximizes $\int_0^\infty U(C)e^{-\rho t}dt$. The Bellman equation for this problem is

$$\rho V(t) = \max_{\{C, L_p\}} \{U(C) + V_K[F(K, AL_p) - C] + V_A \alpha(\bar{L} - L_p)A\}$$

Maximization yields $V_K = U_C$ and $F_{AL_p} = \alpha V_A / V_R$. We then obtain, for the optimum,

$$\rho V(t) = U(C) + U_C(t)\dot{K} + U_C(t)F_{AL_p} \bullet (L - L_p)A$$

We have omitted *'s indicating optimal values to avoid clutter. The RHS is util-valued NNP and includes an expression for $V_A\dot{A}$, the value of the increase in the knowledge stock, equal to the wage bill of the workers in knowledge production, namely $U_C F_{AL_p}(L - L_p)A$. Since $\dot{V}(t) = -U(C(t)) + \rho V(t)$, we get $\dot{V}(t) = U_C(t)\left\{\dot{K}(t) + F_{AL_p} \bullet (L - L_p)A\right\}$ and the value of accumulated capital is

$$V(t) - V(0) = \int_0^t U_C(s)\left\{\dot{K}(s) + F_{AL_p}(s)[\bar{L} - L_p(s)]A(s)\right\}ds$$

which leaves us with NNP(t) as interest $\rho V(t)$ on the value of net accumulated capital, including knowledge capital, plus the value of initial stocks. If one were to do growth accounting with this model one would have

$$g_Q = [KF_K/Q]g_K + [(AL_p F_{AL_p})/Q][g_{L_p} + g_A]$$

where $g_x \equiv \dot{x}/x$. In this case $g_A = \alpha(L - L_p)$ or the growth rate of knowledge is proportional to the number of workers in knowledge production.

Concluding Remarks

We formalized Hicks' idea, so obvious in partial equilibrium, in general equilibrium, namely NNP can be viewed as interest on the current value of accumulated capital. We noted that exhaustible natural resource capital and knowledge capital appear "symmetrically" with machine capital. Sustainable consumption programs become formally linked to the preservation of the value of capital programs.

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Natural Resources and National Wealth and NNP

Abstract

We introduce natural resources into competitive dynamic general equilibrium models and examine the concepts of NNP and national wealth which are seen to sustain the efficient paths. This procedure leads to "formulas" for amending traditional statements of NNP in national accounting to allow for natural resource stock depletion and degradation. We consider depletion of exhaustible resource stocks and economic depreciation, pollution stock increase (and Pigovian taxes), renewable resource stock decline, and land use change (deforestation). This later inquiry leads to the appearance of capital gains in NNP. In the end we consider an oil exporter's NNP and optimal investment strategy for exhaustible resource rents. Throughout, we link NNP to the formal notion of national wealth or the value of accumulated capital.

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Natural Resources and National Wealth and NNP

Introduction

Out of the debates between Pigou and Hayek in the 1930's emerged the notion of Hicksian income - that level of current consumption which could be indulged in without reducing capital. This conforms with modern notions of net national product (NNP) as gross national product net of an amount of investment which replaces capital used up over the accounting period. Simply put, Hicksian income is the current net product of capital. It is an implicit statement that one should consume today no more than the interest on capital or wealth. This leaves the issue of an appropriate split between current actual consumption and savings out of net product unaddressed, this being "the optimal savings question" most satisfactorily treated by Ramsey [1928]. The Pigou-Hayek-Hicks legacy is one of being precise about distinguishing among income, consumption, and capital or wealth. We must distinguish flow concepts such as income and consumption from stock concepts such as capital or wealth.

But "national income" should indicate some notion of current national well-being and of course the latter is defined in terms of consumption realized, not potential consumption. If "national income" is a measure of national well-being, what should one do with net investment I in NNP = C+I. One's natural inclination is to define I as some measure of future consumption. Then "national income" or NNP becomes a welfare measure, capturing a composite of current consumption and a measure of future consumption. This was formalized in a classic paper of Weitzman [1976].

Weitzman Defines NNP(t) and I(t)

Weitzman [1976] asked us to consider a competitive economy pursuing optimal savings. That is, {C(t)} satisfies maximization of $\int_0^\infty C(t)e^{-\rho t}dt$ subject to $C(t) = F(K(t),N) - \dot{K}$ where C(t) is aggregate consumption,[1] ρ is the social discount rate, K(t) is the machine capital stock and N is the constant labor force. K(0) and N are given exogenously. If C(t) is optimal, then it satisfies the Bellman equation.[2]

$$\rho V(t) = \max_{C(t)}\{C(t) + V_K \bullet (F(K,N) - C(t)\} \tag{1}$$

where $V(t) = \int_t^\infty e^{-\rho(s-t)}C^*(s)ds$. The * indicates an optimal value. It follows that V_K above equals 1. Thus the above Bellman equation becomes

$$\rho\int_t^\infty e^{-\rho(s-t)}C^*(s)ds = C^*(t) + \dot{K}^*(t). \tag{2}$$

The RHS of (2) with the solution values[3] is NNP(t)[4] or C(t) + I(t). Thus current NNP(t) is "interest" on V(t) where ρ is the "interest rate". V(t) must be current wealth or social capital in

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[1] We are of course treating the consumer as a single entity. Many complications arising from aggregating many heterogeneous consumers are being abstracted from.

[2] This statement of the Bellman equation is valid for autonomous problems, those with the horizon infinite and time appearing only in $e^{-\rho t}$. See Kamien and Schwartz [1991; pp. 262-63] for detail. Thus we should state that exhaustion of S(t) occurs asymptotically and in so doing we preserve the autonomousness of the problem. Exhaustion in finite time would complicate matters somewhat. Lozada [1992] considers non-autonomous versions of Weitzman's analysis as well as issues involving non-autonomousness and exhaustible resource depletion. See also Asheim [1992]. In particular exogenously changing discount rates raise interesting complications.

[3] The point is that along the optimal path (the solution values inserted) the RHS of the Bellman equation becomes the NNP in utils. The Bellman equation is traditionally viewed as an equation to solve in order to obtain the optimal solution.

[4] In Hartwick [1990] and Maler [1991], the current value Hamiltonian for the solved problem was defined as NNP(t). See also Weitzman [1976] and Cass and Shell [1976]. The solved Hamiltonian and Bellman equation will be the same for regular problems with interior solutions.

the Irving Fisher sense of capital being discounted future returns. Hence Weitzman's argument shows that NNP(t) is the return to capital, another instance of Hicksian income. The new element is of course the appearance of "capital", V(t).

Consider dividing (2) by ρ. Now $C(t)/\rho = C(t)\int_t^{\infty} e^{-\rho(s-t)}ds.$ Thus

$$\dot{K}(t) = \rho\int_t^{\infty} e^{-\rho(s-t)}[C^*(s) - C(t)]ds. \tag{3}$$

This expression in (3) answers the question: what is the welfare interpretation of net investment I(t) (= $\dot{K}$(t) above) in NNP(t) = C+I. Current investment is ρ multiplied by the present value of the difference between each future consumption and current consumption. Investment is the discounted future gain in consumption, multiplied by "interest rate", ρ.

V(t), social capital, is the new concept of capital or wealth. Let us reflect on it for a moment. Its derivative is $\dot{V}(t) = \rho V(t) - C(t)$. Since $\rho V(t) = C(t) + I(t)$, the Weitzman result, we obtain $\dot{V}(t) = I(t)$. Thus current net investment in machines, I(t), is the current augmentation in social capital (discounted future consumption), V(t). But it follows that

$V(t) = \int_0^t I(s)ds + \gamma = K(t) + \gamma$, where γ is a constant of integration (initial K). Current wealth can thus be expressed as cumulative past net investment (Solow [1986]). A different statement of the Weitzman result is then: $NNP(t) = \rho[K(t) + \gamma]$. This is of course quintessentially Hicksian.

Weitzman was careful to point out that his K(t) could be a vector of different types of capital, including depleting natural resource stocks but he did not pursue this point. Dasgupta and Heal [1979, pp. 244-45] followed up in a slightly different framework. Solow [1986] pursued the Weitzman suggestion in the context of constant consumption programs. We leave these constant consumption programs at this time and return to our earlier Weitzman framework and now we introduce exhaustible resource use. At each date there is a stock of S(t) say tons of oil remaining to use. At each date R(t) tons are used in say production, at extraction and refining cost g(R(t)), measured in terms of composite output Q(t) = F[K(t),N,R(t)]. Then

$$\dot{K} = F(K,N,R) - C - g(R) \tag{4}$$

and

$$\dot{S} = -R. \tag{5}$$

These are given initial values for K and S. N is constant. The invisible hand maximizes $\int_0^\infty e^{-\rho t}C(t)dt$. This optimization satisfies the Bellman equation

$$\rho V(t) = \max_{\{C,R\}} \{C(t) + V_K(t)[F(K,N,R) - C - g(R)] - V_s(t)R\}. \tag{6}$$

Carrying out the maximization yields $V_K(t) = 1$ and $V_s = V_K.[F_R - g_R]$. Thus we obtain

$$\rho V(t) = C(t) + \dot{K}(t) - [F_R - g_R]R(t) \tag{7}$$

The RHS is our new statement of NNP(t). The definition of V(t) is the same as earlier, namely $\int_t^\infty C^*(s)e^{-\rho(s-t)}ds$, where * indicates optimal paths. The central result in (7) is the deduction in NNP(t) for the using up of S(t) by amount R(t). This depletion of S(t) is valued at $[F_R - g_R]$ or price minus marginal cost of a unit of R or S. This marginal valuation is multiplied by the amount used, namely $R = -\dot{S}(t)$. So true NNP(t) is net of natural resource use in the particular sense of $[F_R - g_R]R(t)$. It is a disinvestment in S(t) whereas $\dot{K}$ is a net investment in K. We have now two capital goods in the Weitzman "model", one produced (namely K) and one natural (namely S(t)) and each enters with a net price (1 for K and $[F_R - g_R]$ for S) and a change in stock, namely $\dot{K}$ and $\dot{S}$. The treatment of each stock is the same except here one is declining and the other is growing. That is, $(F_R - g_R)\dot{S}$ is a negative entry in (7).

The LHS of (7) is again an "interest rate" ρ multiplied by wealth, V(t). Since $\dot{V}(t) = -C(t) + \rho V(t)$, we can substitute from (7) to get $\dot{V}(t) = \dot{K}(t) + [F_R - g_R]\dot{S}$. Then upon integrating, we get $V(t) = \int_0^t \dot{K}(s)ds + \int_0^t [F_R(s) - g_R(s)]\dot{S}(s)ds + \eta$ which is $K(t) + \int_0^t [F_R(s) + g_R(s)]\dot{S}(s)ds + \eta$, the dollar value of current wealth, expressed in terms of stocks of K and S(0) depleted. Note that $\dot{S}$ will be negative and so V(t) will be net investment in K, accumulated to t minus the amount of S(0) used up to date t, each unit valued at its current net price.[5] η is the value of initial stocks. Thus the current wealth V(t) is the value of net <u>accumulation</u>[6] up to date t. Remaining stock S(t) does not appear. NNP(t) = ρV(t) is then "interest" on the value of past stocks accumulated to t. Since V(t) is also discounted future optimal consumption, we see plainly that potential future discounted consumption is reduced when exhaustible stocks such as oil are depleted, period by period. In other words, the current using up of exhaustible stocks contributes positively to current consumption via increased current output and negatively to current wealth represented either by discounted future consumption or by past "accumulation" of physical capital. Though between time 0 and t, the initial stock of the exhaustible resources is run down, the current remaining stock S(t) appears as part of current wealth because it is captured by S(0) minus past decumulations of the original stock. Thus current wealth is measured by the value of remaining S(t) at t and K accumulated to the current

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[5] Observe that national wealth here includes <u>remaining</u> exhaustible resource stocks. That is, current stock S(t) is captured by S(0) in η minus past "withdrawls" from S(0) as time passed.

[6] Solow [1986] has a similar exploration of the definition of current social capital in the context of economies with constant consumption programs. My investigation of NNP as "interest" on accumulated (or depleted) capital was inspired by this Solow article. I have greatly benefitted from correspondence with Robert Solow.

level K(t). The prices reflect the constraint $\int_t^\infty R(s)ds \leq S(t)$ via perfect foresight on the part of agents. We are left with that Hicksian result, namely current NNP(t) is "interest" on national wealth V(t) where ρ is the "interest rate" and V(t) can be defined in a forward-looking sense (discounted future optimal consumption) or in a backward-looking sense (past "accumulations" of capital stocks valued at their current prices, in addition to initial stocks).[7]

In the special case of net investment $\dot{K}(t)$ being financed out of exhaustible rents $(F_R - g_R).R(t)$, the NNP(t) = C(t) and V(t) will remain constant. This implies that C(t) remains constant since $C(t) = \rho V(t)$ in this case. Thus investing resource rents only in the expansion of K(t) implies (a) constant C(t) and (b) constant wealth V(t). Since N(t) is constant (autonomousness), then per capita consumption remains constant (as in Hartwick [1977]). If we added the assumption of the production function being Cobb-Douglas with a larger exponent on R(t) than K(t), then C(t)/N(t) would remain constant indefinitely (Solow [1974]). Constant consumption programs are linked directly to constant wealth programs (Solow [1986]). We have seen this clearly above.

The Practice of National Accounting

Data arrive at the central bureau of statistics from firms and the accounts are then built up from these entries. Our principal message is that for mining (extracting) firms economic depreciation of the stocks depleted currently should show up in NNP. We illustrate in Figure 1.

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[7] Recall footnote 5.

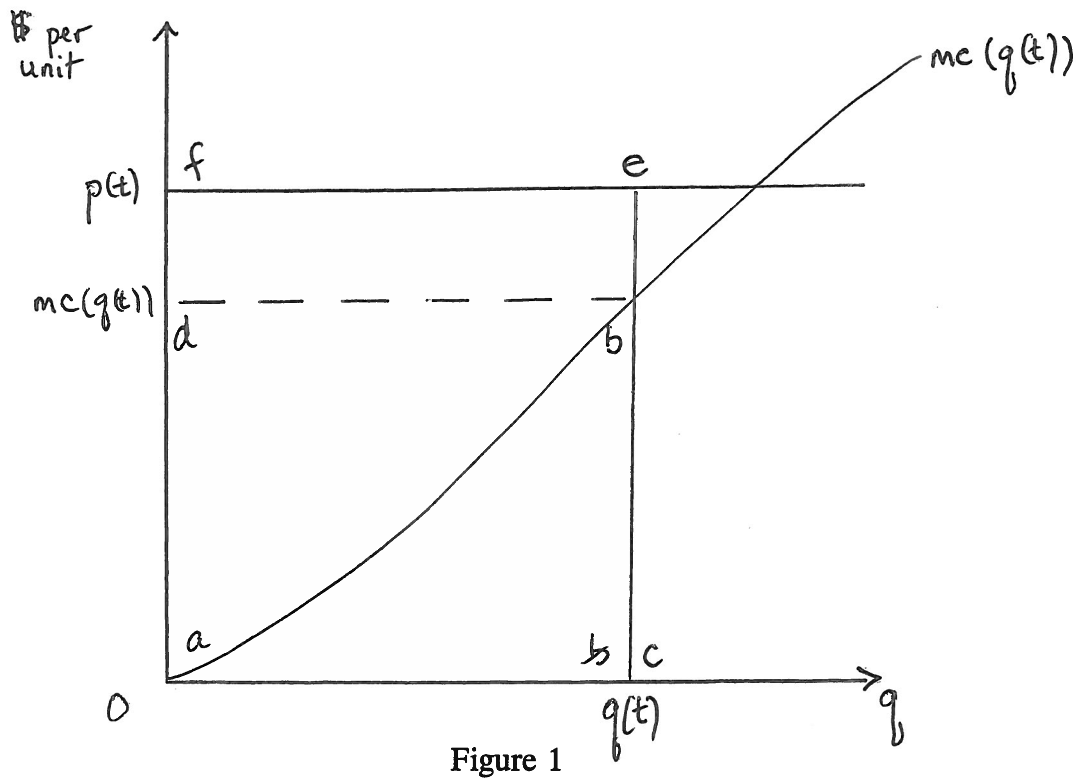

Figure 1

The firm currently extracts q(t) which sells for p(t) per unit. Area abc is total variable extraction costs (capital and labor). abd is net profit (site rent) and dbef is Hotelling exhaustible resource rent, equal to economic depreciation.

In Figure 1, we have the extracting firm currently producing q(t) over the accounting period and receiving price p(t) per unit. Total variable extraction costs are area abc under the marginal extraction cost curve. Area abd is net profit (net of the "depletion allowance") or site rent. Area dbef is Hotelling exhaustible resource rent, equal to current economic depreciation of the stock of resource owned by the mine. This area is of course [p(t) - mc (q(t))]q(t), our rent-economic depreciation entity.

The accountant enters the net value of product on the product side of the national accounts, that is pq - [p-mc]q, and the net value of inputs on the inputs side of the accounts. See Table 1.

Table 1

Value of Inputs	Value of Product
site rent: abd	p(t)q(t) - [p(t) - mc(q(t))]q(t)
inputs in extraction: abc	

If one were dealing with a firm such as a farm, then q(t) in Figure 1 would move up below g. There would be zero Hotelling rent or economic depreciation. The value of inputs would be site rent (net profit) plus the costs of variable inputs (capital and labor) used in farm production.

Technical Change and Exhaustible Resource Discoveries

Weitzman [1976] noted that if technical change were endogenous (produced from observed inputs) then there would be a stock of knowledge and increments to the stock over time. Now R&D would be a produced good and there would be another capital good, represented by the knowledge stock. This stock could be treated like other stocks and NNP(t) would be defined with a new good (new R&D) and a new stock (useful knowledge). This seems correct except that knowledge has an inherent public goods property, namely i's use does not constrain j's use. The knowledge stock is best viewed as a public input and this raises well-known difficulties for the functioning of a decentralized price system. (See for example Hartwick [1992].) Roughly speaking, the public goodsness of knowledge introduces inherent scale economies to an economy and these scale economies engender familiar problems in the operation of a decentralized price system.

Weitzman's general argument is that endogenous technical change and perfect foresight leave his notion of NNP(t) essentially unchanged but unanticipated technical changes will raise insoluble problems. Simply put, asset prices today reflect future stock scarcities and if those scarcities change unexpectedly, current asset prices have reflected the wrong scarcities. Current NNP(t) is then a wrong reflection of current welfare because, roughly speaking, current prices reflect future events inaccurately. No one could argue with this fundamental observation. Discoveries of new stocks of exhaustible resources works in a parallel fashion. If the discoveries are correctly anticipated, if only in an expected value sense, then current asset prices reflect true scarcities. Pindyck [1978] treats discoveries in this way. (See also Hartwick [1991].) Exploration costs in his framework are really locating or finding costs and are not ex ante expenditures linked to ex post discoveries with no probability density function, this latter being unanticipated discoveries. The finding cost approach to discovery of new stock parallels the endogenous technical change argument above. Unanticipated technical change corresponds to unanticipated discovery of new exhaustible resource stock.

One can view a discovery as a site value rising in value from u^0 dollars per hectare to u^1 dollars per hectare. The capital gain is then $[u^1 - u^0]$ multiplied by the area in question. In an equilibrium model of search, this dollar value gain will on average equal the marginal cost of discovery of the valuable site. This suggests treating discovery of new mineral in the national accounts as one treats land improvement, which we discuss below. Note that we are implicitly separating the extraction activity (and extracting firms) from the discovery activity (and exploration firms).

Pollution as an Environmental Stock

Though we generally think of stocks as useful, pollution as a stock is not. Yet it can be treated as other stocks above (eg. K(t) and S(t)) in defining true NNP(t). The new difficulty which pollution raises is the appropriate pricing of inputs and outputs when the pollution stock has no owner willing to ration its production in response to compensation. A Pigovian tax scheme is required. Consider the case of mining causing pollution as in

$$\dot{Z} = f(R) - b(Z)$$

where Z is the stock of pollution, f(R) is new pollution caused by mining R tons of oil and b(Z) is natural abatement of pollution. Thus if mining ceased (R=0), Z(t) would decline, eventually to zero, say. In a steady state, $\dot{Z}=0$ and natural abatement matches new pollution from mining. Suppose then that the pollution stock Z(t) inhibits goods production in Q = F(K,N,R,Z) where $\partial F/\partial Z < 0$. Otherwise our earlier problem is unchanged. Now the Bellman equation is

$$\rho V(t) = \max_{\{C,R\}} \{C + V_R \bullet [F(K,N,R,S) - g(R) - C] - V_S R + V_Z \bullet [f(R) - b(Z)]\}$$

The maximization yields $V_K = 1$, $V_S = F_R - g_R + V_Z f_R$. Then

$$\rho V(t) = C + \dot{K} - [F_R - g_R + V_Z f_R].R + V_Z \dot{Z}$$

V_Z will be negative and $V_Z f_R$ is the Pigovian tax on quantity mined. Mineral rent is now $[F_R - g_R + V_Z f_R].R$, a lower value than would occur with no pollution. Each unit extracted yields rent $F_R - g_R$ and a marginal increase in pollution f_R which must be valued and taxed appropriately to sustain a first best solution. This effect makes NNP(t) larger than it would be with no pollution! However the net increase in Z(t) is an additional cost and gets valued at $V_Z \dot{Z}$ in NNP(t). For $\dot{Z}>0$, this will be a negative entry in NNP(t). V_Z is the marginal value of extra Z in V(t), a negative marginal value since extra pollution is a marginal bad. Pollution shows up in NNP(t) as a Pigovian tax term $V_Z f_R R$ and as a negatively valued stock increase $V_Z \dot{Z}$. If mining were pursued sustainably with respect to pollution, then $\dot{Z}$ would be zero, and only the Pigovian tax on stock Z would appear in the pollution accounting.

We solve for social wealth

$$V(t) = K(t) + \int_0^t [F_R(s) - g_R(s) + V_Z(s) f_R(s)] \dot{S}(s) ds + \int_0^t V_Z(s) \dot{Z}(s) ds + \delta.$$

The last two integrals will each be negative because $\dot{S}(s) < 0$ and $V_Z(s) < 0$. We assume $\dot{Z}(s) > 0$. The accumulation of pollution lowers social wealth as does the decumulation of exhaustible resource stocks. Since V(t) also equals $\int_t^\infty e^{-\rho(s-t)} C^*(s) ds$, we are observing a lower future consumption program from increased pollution. The cost of pollution in our model is in terms of a higher cost for producing output Q(t). The consequences of a lower path of future Q(t) is a lower path of future C(t).

NNP in Utils or Dollars

If consumption C(t) is a vector of more than one distinct good, then we must weigh the different goods by some procedure. The standard way is to invoke utility of the bundle as in U(C(t)). Then our asset prices $V_K(t)$ etc. are defined in utils and we end up with an NNP(t) measure in util units. For the original Weitzman model, we would have

$$\rho V(t) = U(C(t)) + U_C(t) \dot{K}$$

in place of the $\rho V(t) = C + \dot{K}$ earlier. A dollar valued NNP(t) would be $U(C)/U_C + \dot{K}$, where U_C is the price of a unit of consumption goods. Recall that Weitzman set U(C) to C and this implies $U_C = 1$. With two commodities consumed, say wheat C and fish Y, one will end up with dollar valued NNP(t) = $U(c(t), Y(t))/U_C(t) + \ldots$. A natural inclination is to linearize

$U(C(t),Y(t))$ as $CU_C + YC_Y$ at the optimal bundle. Then $NNP(t) = C + [U_Y/U_C]Y + \ldots$ where the dollar price of wheat is 1 and of fish is U_Y/U_C. One also redefines the util-valued asset prices V_K, V_S etc. as dollar-valued asset prices V_K/U_C, V_S/U_C, etc. This is the standard procedure in welfare economics of defining the same bundle in terms of utils or numeraire units. This is entirely uncontroversial. However when we compare changes in NNP we have two bundles to value before comparisons can be made. Ideally we would have $\hat{NNP}_2 - \hat{NNP}_1 = k.\{NNP_2 - NNP_1\}$ where $\wedge$'s indicate util valued NNP and k is constant, independent of the position in the space of bundles on which NNP is defined. Matters are never so simple. There are a host of well-understood index number problems associated with moving from comparisons in util units to comparisons in dollar units. The sought after constant k is a wil-o-the-wisp. The problem can be put another way: in comparing two dollar valued NNPs, should one use prices from the second situation in the comparison or prices in the first situation? A different k will emerge for the different price vectors. We have nothing new to contribute to these important index number issues. We are of course attracted to work in terms of dollar-valued NNP(t).

Consider then the renewable resource, fish. If J is the current stock, then $\dot{J} = h(J) - Y$ where h(J) is the natural increment in stock at a date (gross births in excess of deaths of identically sized fish) and Y is harvest at t. Then $\dot{J}$ is the net change in stock J (the bio mass). The invisible hand maximizes $\int_0^\infty U(C,Y)e^{-\rho t}dt$ where C(t) is now a composite consumer good, distinct from fish. The output comprising C can be invested in durable stock K. There is no exhaustible resource in our economy. Nor is there pollution. There are initial values for K, and J. The Bellman equation is

$$\rho V(t) = \max_{\{C,Y\}} \{(C,Y) + V_K[F(K,N) - C] + V_J[h(J) - Y]\}$$

Maximization yields $V_K + U_C$ and $U_Y = V_J$. Then we have

$$\rho V(t) = U(C,Y) + U_C(t)\dot{K}(t) + U_Y(t)\dot{J}(t).$$

The RHS is the util-valued NNP. It is "interest" on util-valued wealth V(t), where $V(t) = \int_t^\infty U(C^*(s),Y^*(s))e^{-\rho(s-t)}ds$. As earlier $\dot{V}(t) = \rho V(t) - U(C,Y)$. Thus $\dot{V}(t) = U_C\dot{K}(t) + U_J\dot{J}(t)$ and $V(t) = \int_0^t U_C(s)\dot{K}(s)ds + \int_0^t U_J(s)\dot{J}(s)ds$. Thus $\hat{NNP}(t)$ is again "interest" on accumulated capital. Now $\hat{NNP}(t)$ and capital are valued in utils, not dollars. For $\dot{K}(t)$ positive, we might expect $\dot{J}(t)$ negative since J is bounded above by the carrying capacity of the aquatic environment to sustain a fish stock. Thus $\hat{NNP}(t)$ $(= U(C,Y) + \dot{K}U_C + \dot{J}U_Y)$ would include a negative entry for declining fish stocks. The dollar valued NNP would be $C + (U_Y/U_C)Y + \dot{K} + (U_Y/U_C)\dot{J}$ where U_Y/U_C is the price of a unit of fish in terms of the composite good C. If we introduced a cost of catching and processing Y, say W(Y), then $(U_Y/U_C)\dot{J}$ would become $(U_Y/U_C - W_Y)\dot{J}$ where $U_Y/U_C - W_Y$ is price minus marginal cost of fish. Since $CU_C + YU_Y$ is an <u>approximation</u> to U(C,Y), we lose the firm link between $\hat{NNP}(t)$ and $\rho V(t)$ when we switch to dollar valued NNP(t). Dollar valued NNP(t), is then only approximately "interest" on dollar valued wealth, V(t).

<u>Capital Gains and NNP (Deforestation)</u>[8]

So far we have observed no price changes in our expressions for NNP. Eisner [1988; p. 1627] proposes that revaluations or capital gains on land be included in NNP. Land is a natural capital good of constant size but changing uses in a growing economy. A change in use

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[8] This is based in part on Hartwick [1992]. Hung [1992] deals with national accounting and durable exhaustible resources.

is associated with a new activity out-bidding a current activity for the use of the land. Such out-bidding is of course a land price change as one activity makes way for a higher valued activity. Much deforestation activity can be viewed as say sugar cane cultivation displacing tree growing on land. Consider an economy with three activities. Composite commodity Q is produced and it is used in accumulating durable capital K and it can be consumed as C(t). Then there is cane production requiring land L and labor N_C; $Q^C = l(L,N_C)$. Forested land $\bar{L}$-L yields sustainable produce $m(\bar{L}-L)$. In a growing economy land R(t) will be being drawn into cane production. Thus $\dot{L}=R$. We assume that it <u>costs</u> v(R) to clear R hectares. Consumption includes the composite commodity C(t), cane, Q^C, and forest products $m(\bar{L}-L)$. The Bellman equation is

$$\rho V(t) = \max_{\{C,N^C,R\}} U(C,Q^C,m(\bar{L}-L)) + V_K[F(K,N-N^C) - C - \psi(R)] + V_L R$$

Maximization implies $U_{Q^C} l_{N^C} = V_K F_{N-N^C}$, $V_K \psi_R = V_L$, and $U_C = V_K$. This leads to

$$\rho V(t) = U(t) + U_C \dot{K} + U_C \psi_R R$$

and $V(t) = \int_0^t U_C(s)\dot{K}(s)ds + \int_0^t U_C(s)\psi_R(s)\dot{L}(s)ds + \xi$ where $N\hat{N}P(t)$ in utils is U(t) + $U_C(t)\dot{K} + U_C(t)\psi_R\dot{L}$. ψ_R is the difference in the price of land in forestry and in cane cultivation. Cane growing out-bids forestry for R hectares. ψ_R is the marginal cost of clearing R hectares. Deforestation leads to a capital gain on the land which experiences a change in use and this capital gain shows up in NNP. On the product side of the accounts, one activity (forestry product cultivation) is replaced by cane growing on a parcel of land, R. These capital gains seem to capture Eisner's idea. They are firmly linked to a change in land use. We are really dealing with changes in land use at the margin. (A technology shock which changed land use in the large would require a different analysis.) The dollar valued NNP(t) would be $U(t)/U_C(t) + \dot{K} + \psi_R\dot{L}$.

The entry of capital gains on land into the national accounts is a departure from traditional practice. The firm (eg. farm) must submit the hectares of changed land use multiplied by the marginal cost of changing the land use over the current accounting period. One way to view this is that the farmer directs two firms, a production of crops firm and a production of new land firm. These two entities can be treated completely separately for accounting purposes. We need focus only on the latter here. The "new land" is the product of the firm. This is entered in the value of product side of the national accounts. See Table 2. The price per unit of output is the change in land value. On the input side we enter MC(q)q where q is the hectarage "produced". See Figure 2.

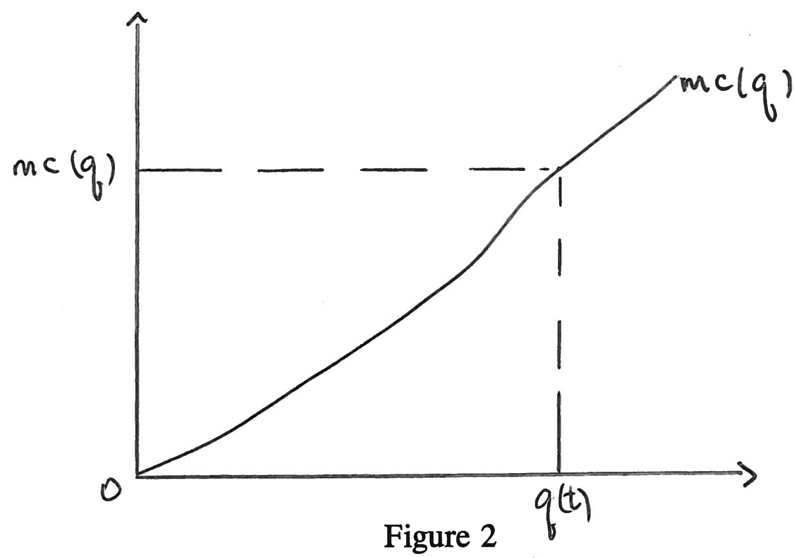

Figure 2

q(t) is land area up-graded over the accounting period.
MC(q(t)) is the marginal cost of "producing" the land.

In Figure 2, the area under the marginal cost is the variable cost of "producing" the "new land". The remaining area is a rent, part of the cost of "producing". These areas comprise the value of inputs in Table 2.

Table 2

Value of Inputs	Value of Product
mc(q(t))q(t)	$[u^1 - u^0]q(t)$

In Table 2, $u^1 - u^0$ is the change in the price (capital value) of land as improvements are made. We note that the entries in Table 2 are not naturally collected. The value of inputs comes to the total cost of up-grading ("producing") land. As a practical matter, one could use the total expenditures on land improvement as a proxy for value of <u>output</u> of this activity.

Recall that mineral discoveries could be viewed as site "improvement" (a jump up in the value of site). Then the marginal cost of exploration is the marginal cost of "site improvement". Our procedures for dealing with capital gains on land can then be appealed to, to take account of mineral discoveries.

Biodiversity has been associated with forests in their natural state. One can then model declining biodiversity as a decline in the species stock, this latter being linked to the size of current natural forests (Hartwick [1993]). The policy prescription in this case is to assign a social price or premium to forested land to account for the indirect services (provision of biodiversity) of the forested land. An interesting variant of this approach is to treat the loss in biodiversity (linked to forests) as a trigger. At an uncertain date the trigger is tripped and there is a structural collapse in say agricultural productivity.[9] In this case a Pigovian tax on land removed from forest slows the squeezing of the trigger or in the sense of expected value, delays

[9] Cropper [1976], Heal [1984], and Hung [1992] consider increasing pollution as a trigger leading to a once-over structural collapse.

the structural collapse. This model leads to a risk-adjusted NNP(t) at any date (Hung [1992]) and interesting relations between the level of risk and the revenue from the current Pigovian taxes.

An Oil-Exporting Nation's NNP and Sustainable Consumption

The Oil Republic (OR) exports R(t) of oil currently in return for consumption goods C(t) from abroad. World price of oil is p(t), defined in terms of the numeraire price, for C(t), and p(t) is assumed to remain constant.[10] This rise in oil prices implies a terms of trade improvement for the OR. The stock of oil, is homogeneous and finite with S(t) remaining at any date. Thus $R(t) = -\dot{S}(t)$. h(R(t)) is the cost of extracting R(t). h(0)=0 and h′ and h′′ are positive. The OR invests (disinvests) savings abroad at a constant interest rate r so that

$$H(t) = \int_0^t e^{r(t-s)}[p(s)R(s) - h(R(s)) - C(s)]ds. \tag{8}$$

This implies

$$\dot{H}(t) = rH(t) + p(t)R(t) - h(r(t)) - C(t) \tag{9}$$

We assume H(0) = 0.

The zero lifetime (of the OR) borrowing-lending constraint is

$$\int_0^\infty [rH(s) + p(s)R(s) - h(R(s)) - C(s)]ds = 0. \tag{10}$$

Clearly one feasible strategy is for the OR to build up H(t) abroad and when the oil stock is exhausted, to live on the interest, rH (=C). (10) defines the limit of the OR's borrowing and lifetime consumption.

The OR's planner's problem is to maximize $\int_0^\infty e^{-\rho t}U(C(t))dt$ given the equation of

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[10] The case of p(t) increasing is a non-autonomous problem and has been dealt with by Lozada for a closed economy. Asheim [1993] deals with capital gains in a two country, trade framework.

motion for H in (9) and for S in $\dot{S}$=-R. Initial stock S(0) is given at S_0 and H(0)=0. The current value Hamiltonian for this problem is

$$\mathcal{H}(t) = U(C(t)) + \lambda(t)[rH(t) + p(t)R(t) - h(R(t)) - C(t)] - \psi(t)R(t)$$

where $\lambda(t)$ is the co-state variable (util asset price) for H(t) and $\psi(t)$ is the co-state variable (util asset price) for S(t). Necessary conditions for an optimum are

$$\frac{\partial H}{\partial C} = 0 \Rightarrow U_C = \lambda \tag{11}$$

$$\frac{\partial H}{\partial R} = 0 \Rightarrow \lambda(t)[p(t) - h'] = \psi(t) \tag{12}$$

$$-\frac{\partial H}{\partial H} = \dot{\lambda} - \rho\lambda \Rightarrow -\lambda r = \dot{\lambda} - \rho\lambda \tag{13}$$

$$-\frac{\partial H}{\partial S} = \dot{\psi} - \rho\psi \Rightarrow \dot{\psi} - \rho\psi = 0 \tag{14}$$

Case[11] I (ρ=r)

In this case, we have $\dot{\lambda}$=0 from (13) and $\dot{C}$=0 from (11). From (12) we obtain the r% rule on p(t) - h′, Hotelling rent or marginal profit per ton. There are many levels of constant C attainable. The level of C depends on lifetime borrowing possibilities. Given (10) there will be a unique sustainable level C*. C>C* and C constant implies perpetual borrowing in the limit. C<C* implies perpetual lending in the limit. In these latter cases, the LHS of (10) is not

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[11] In the Cass-Koopmans model of optimal accumulation of producible capital, the steady state is achieved with the constant social rate of discount equal to the interest rate. But the latter is endogenous. In the Dasgupta and Heal [1974] and Solow [1974] models the endogenous interest rate tends to zero as time passes. Our assumption of the interest rate constant and positive is not only "buying" a constant interest rate but one that is not near zero in the long run. Svensson [1986] expressed concern about the robustness of results on investing resource rents when discount rates were not constant. See Asheim [1992] for an exploration of cases with non-constant discount rates. See also Lozada [1992].

solved equal to zero. The lifetime borrowing/lending constraint is violated.

With perfect foresight and the interest rate constant, the OR can achieve the C* optimal solution by selling S(t) at any date and living on the interest r[H(t) + p(t)S(t)] where

$$p(t)S(t) = \int_t^T p(s)R(s) - h(R(s))e^{-r(s-t)}ds \tag{15}$$

and $S(t) = \int_t^T R(s)ds$. This includes selling at t=0. Then C* = [p(0)S(0)]r.

Observe that (12), (13), and (14) imply that $\frac{\bullet}{p(t)-h'(R(t))} = [p(t)-h'(R(t))]r$ which is the Hotelling [1931] r% rule or the dynamic efficiency condition for extraction from S(t).

Define the dollar value Hamiltonian as $\mathcal{H}(t)/U_C(t)$. Then NNP(t) = $\mathcal{H}(t)/U_C(t) = U(C)/U_C + \dot{H} - [p(t) - h'(R(t))]R(t)$. Since $1/U_C$ is the price of a unit of consumption, we will express $U(C)/U_C$ as C. The Hicksian concept of income is potential consumption $\hat{C}$ out of current product which leaves "capital" intact. Thus if NNP(t) = $\hat{C}$, we have

$$\hat{C} - C = \dot{H} - [p(t) - h'(R(t))]R(t) \tag{16}$$

From (16) we have C(t) = $\hat{C}$ when $\dot{H}$ equals total exhaustible resource rent. This latter is also the dollar value decline in S(t) from extracting R(t) (economic depreciation of S(t)). Thus $\hat{C}=C$ is associated with capital value intact. Savings abroad, namely H(t), grow by an amount equal to the decline in dollar value of S(t). In the sustainable case, C(t)=C* and the OR's lifetime budget constraint is satisfied. This case echoes the Solow [1974] - Hartwick [1977] case. C remains constant and exhaustible resource rents are invested in other "capital". However this case is partial equilibrium since r and p(t) are exogenously given. But the discount rate ρ is not zero.

There is a simple picture of the situation at one instant. See Figure 3.

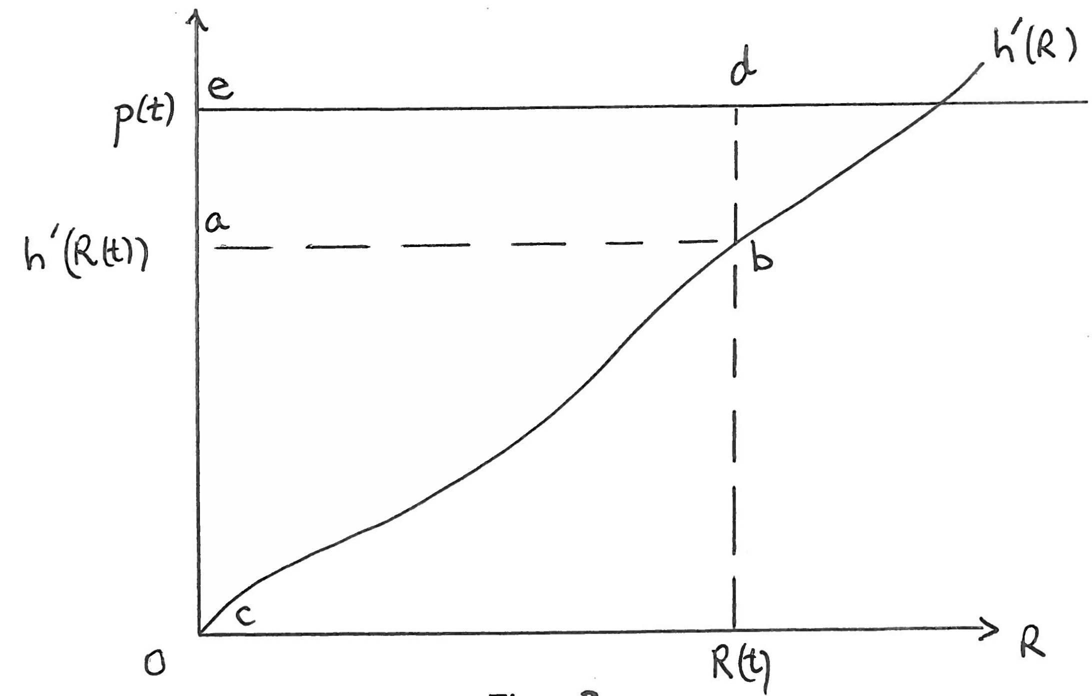

Figure 3

At date t, C(t) equals area abc plus interest on H(t) abroad. Area abde equals [p(t) - h'(R)]R(t) which is invested abroad. Area cbj is extraction cost associated with extraction R(t). Up to exhaustion of the stock, H grows by area abde, resource rent (economic depreciation of S(t)), and consumption remains constant at level rH(t) + R(t)h'(R(t)) - h(R(t)).

Case II ($\rho > r$)

From (13) we obtain $\dot{\lambda}/\lambda = (\rho\text{-}r)$ and from (11) $U_{CC}\dot{C}/U_C = (\rho\text{-}r)$. (12), (13) and (14) still imply the r% rule on resource rent, p-h'. Since $U_{CC}<0$, in this case $\dot{C}<0$ and the gap ρ-r implies decline in C(t). Investment in H(t) is not proceeding rapidly enough to compensate for the loss in value of S(t) as extraction proceeds. Given satisfaction of the budget constraint in (10), this high discount case implies C(0) > C*. Early consumption is high (and declining) relative to the sustainable case. The long term behavior of consumption depends on the form of

U(C). Three cases are asymptotic decline in C to zero, decline in C to zero in finite time, or asymptotic decline in C to a positive bound.

Case III $(r>\rho)$

Now $U_{CC}\dot{C}/U_C = (\rho\text{-}r) < 0$ and $\dot{C}>0$. In this case C(0) is below C* from Case I and C(t) grows. This is a case of relatively large savings abroad. The low initial value of C(0) frees income to be invested abroad. Clearly $\dot{C}>0$ is not sustainable given the budget constraint in (10) unless $\lim_{t\to\infty} C(t)$ is finite. The form of U(C) is crucial. It is clearly feasible for $\lim_{t\to\infty} C(t)$ to approach C* in Case I from below. But for C(t) increasing without bound is incompatible with the lifetime budget constraint in (10).

Concluding Remarks

We have shown that NNP is an inherent part of a dynamic general equilibrium system. Also NNP is "interest" on wealth or capital, also in an essential sense. The NNPs which emerge in growth models with different types of natural resources indicate the appropriate formulas for adjusting "traditional" NNP to incorporate changes in stocks, and prices in some cases, of national resources. The general adjustment is marginal profit (price minus marginal cost) multiplied by current physical stock size change. With land use change, we observed land value changes (capital gains) in NNP. We also noted that declining biodiversity could be modelled as a species of deforestation (land use change) model. In closing, we considered an oil exporting country's NNP and Hicksian income. In this case natural resource revenues could be accumulated as financial capital abroad. We considered investment-abroad policies to sustain a constant level of consumption into the indefinite future.

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