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Financing R & D with Knowledge Stock Rentals

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We set out an endogenous growth model along the lines of Romer [1990] and investigate the implications of financing new knowledge production (R&D) with rental income accruing to the knowledge stock used in goods production. The knowledge stock is a non-public input in goods production. The balanced growth rate under optimal growth can be greater or less than that under the invest knowledge stock rentals regime. The balanced growth rate in the invest knowledge stock rentals regime depends on the parameters of the production function and not on the parameters of preference.

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Introduction

Romer [1990] focused attention on how the funding of knowledge production affected the growth path of an economy. His market-type model involved a leading firm in a monopolistic competition paying for new knowledge via a mark-up of output price over marginal cost. This institutional funding scheme for R&D resulted in the growth path for his market economy differing from that path under optimal growth.[1] The growth rate in the market model was less than that for the optimal model. Implicit in the optimal model is tax revenue from goods production supporting the knowledge production sector (KPS) or R&D sector. Here we set up an endogenous growth model with its distinct KPS.[2] The knowledge stock is a non-public input to goods production.[3] We consider the institutional mechanism of funding the KPS with rentals from the knowledge stock in the goods production sector (GPS). In general the growth path differs from that under optimal growth. The optimal steady state growth rate can exceed or be less than that emerging under the regime of funding R&D with knowledge stock rentals.

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[1] Grossman and Helpman [1991] have R&D paid for by a winning firm in an R&D race. Their model shares a similar structure to Romer's.

[2] Our KPS is formally the same as that in Romer [1990] and Hartwick [1992]. It differs slightly from that in Lucas [1988].

[3] In Hartwick [1992] the knowledge stock was a labor-augmenting public input to both goods production and knowledge production.

We intentionally made the knowledge stock a non-public input[4] here and thus we are dealing with two seemingly standard sorts of capital, machines K and knowledge A. However to visualize matters concretely, we might view A as a stock of plant designs (generalized blue prints). As the economy grows or K increases, we can envisage a new design for each new plant (increased K). One must not view this in terms of one standard design per standard plant since $\dot{A}$ and $\dot{K}$ are substitutable. With a particular amount of new K one can employ much or little design (amount of new A). There will be a cost minimizing combination of K and A at each date. In fact in balanced growth in our model A grows faster than K implying a design-deepening in the economy over time. This seems to be a stylized fact of modern industrial economies.[5]

The Model

There are a fixed number of workers N to be divided between new knowledge production, $\dot{A} = b(1-a)NA$ and goods production $Q = K^{\alpha}(aN)^{\beta}A^{1-\alpha-\beta}$ where a is the fraction of workers in goods production, b is a positive constant, A is the current stock of knowledge used for production, and K is the stock of machine capital. α, β and $1-\alpha-\beta$ are positive constants. At each date goods output Q is divided between current aggregate consumption C and machine

[4] Actually the knowledge stock enters jointly into goods production (as a non-public input) and into knowledge production so it has a particular sort of spillover similar to that in Romer [1990]. We are interested in the non-publicness of the knowledge stock in goods production here.

[5] About one tenth of the budget for building the new library at Queen's University is for design and consulting services.

investment $\dot{K}$. That is $\dot{K} = YK^{\alpha}a^{\beta}A^{1-\alpha-\beta} - C$ where $Y \equiv N^{\beta}$, a constant. If balanced growth exists, we will have

$$\frac{\dot{K}}{K} = Ya^{\beta}Z - \frac{C}{K} = g \tag{1}$$

where $Z = A^{1-\alpha-\beta}/K^{1-\alpha}$ and g is a growth rate. For g to remain constant, we can search for a path with C growing at rate g, a constant, and Z constant. In this case

$$\frac{\dot{Z}}{Z} = (1-\alpha-\beta)\frac{\dot{A}}{A} - (1-\alpha)\frac{\dot{K}}{K} \tag{2}$$

will equal zero and the growth rate of A will be $(1-\alpha)g/(1-\alpha-\beta)$. Since $\dot{A} = b(1-a)NA$, we will have the balanced growth 'a' defined in $b(1-a)N = (1-\alpha)g/(1-\alpha-\beta)$. Since these hypothesized relationships seem 'sustainable', we press forward.

To solve for the savings rate, we formulate our problem as one of optimal savings. That is $\int_0^{\infty} e^{-\rho t}U(C)dt$ is maximized subject to $A(0) = A_0$, $K(0) = K_0$, $\dot{K} = YK^{\alpha}a^{\beta}Z^{1-\alpha-\beta} - C$ and $\dot{A} = b(1-a)NA$. We shall specialize U(C) to $[C^{1-\sigma}/(1-\sigma)] - 1$. The current value Hamiltonian is

$$H(t) = [C^{1-\sigma}/(1-\sigma)] - 1 + \lambda(t)[YK^{\alpha}a^{\beta}A^{1-\alpha-\beta} - C] + \psi(t)[b(1-a)NA]$$

$$\frac{\partial H}{\partial C} = 0 \Rightarrow C^{-\sigma} = \lambda \tag{3}$$

and $$\frac{\dot{\lambda}}{\lambda} = -\sigma\frac{\dot{C}}{C} \tag{4}$$

$$\frac{\partial H}{\partial a} = 0 \Rightarrow \lambda\beta Q = \psi bAaN \tag{5}$$

$$-\frac{\partial H}{\partial K} = \dot{\lambda} - \rho\lambda \Rightarrow \frac{\dot{\lambda}}{\lambda} = \rho - \frac{\alpha Q}{K} \tag{6}$$

$$-\frac{\partial H}{\partial A} = \dot{\psi} - \rho\psi \Rightarrow \frac{\dot{\psi}}{\psi} = \rho - b(1-a)N - \frac{\lambda}{\psi}(1-\alpha-\beta)\frac{Q}{A} \tag{7}$$

Transversality conditions are $\lim_{T\to\infty} \lambda(T)K(T) \to 0$ and $\lim_{T\to\infty} \psi(T)A(T) \to 0$. Using (5), (6) and (7) we obtain

$$\frac{\dot{\lambda}}{\lambda} - \frac{\dot{\psi}}{\psi} = \frac{\dot{A}}{A} - \frac{\alpha Q}{K} + \frac{baN}{\beta}(1-\alpha-\beta) \tag{8}$$

Differentiating (5) yields

$$\frac{\dot{\lambda}}{\lambda} - \frac{\dot{\psi}}{\psi} = \frac{\dot{A}}{A} - \frac{\dot{a}}{a} - \frac{\dot{Q}}{Q} \tag{9}$$

Combining (8) and (9), and substituting for $\dot{Q}$ yields

$$\frac{\dot{a}}{a} = \left\{ baN\left[\frac{1-\alpha-\beta}{\beta}\right] + (1-\alpha-\beta)b(1-a)N - \alpha c \right\} / (1-\beta) \tag{10}$$

where $c = C/K$. Combining (4) and (6) yields

$$\dot{c} = c - \frac{\rho}{\sigma} - \left[\frac{\sigma-\alpha}{\sigma}\right] Ya^{\beta}Z \tag{11}$$

Combining (1) and (2) and substituting for $\dot{A}/A$ yields

$$\frac{\dot{Z}}{Z} = (1-\alpha-\beta)(1-a)Nb - (1-\alpha)Ya^{\beta}Z + (1-\alpha)c \tag{12}$$

Equations (10), (11) and (12) define the dynamics of a, c, and Z. Balanced growth is defined by a^*, c^* and Z^* in (10), (11) and (12) which solve (10), (11) and (12) for $\dot{a} = \dot{c} = \dot{Z} = 0$.

<u>Balanced Growth</u>

Consider now (10), (11) and (12) for the case $\dot{a} = \dot{c} = \dot{Z} = 0$. Substitute for c from (11) into (10) and observe that

$$a = \frac{\beta}{bN(1-\alpha-\beta)}\left[\frac{\rho(1-\alpha)+bN(1-\alpha-\beta)(\sigma-1)}{(1-\alpha-\beta)+\sigma\beta}\right] \tag{13}$$

Clearly $\sigma > 1$ is sufficient for $a > 0$. Since $g = \frac{(1-\alpha-\beta)}{(1-\alpha)} \frac{\dot{A}}{A}$ we obtain, given (13)

$$g = \frac{bN(1-\alpha-\beta)-\beta\rho}{1-\alpha-\beta+\sigma\beta}. \tag{14}$$

We are interested in the case of $g > 0$. For a and g positive we require

$$\rho > \left[\frac{1-\sigma}{1-\alpha}\right] (1-\alpha-\beta)bN$$

and

$$bN(1\text{-}\alpha\text{-}\beta)/\beta > \rho.$$

A necessary condition is then $(1\text{-}\alpha)/\beta > (1\text{-}\sigma)$. (1-a) must also be positive, since it is the fraction of total workers working in the KPS. Since g is defined as $b(1\text{-}a)N(1\text{-}\alpha\text{-}\beta)/(1\text{-}\alpha)$, g positive implies (1-a) positive. Note that bN large in (14) implies g large but then 'a' in (13) will be negative when say $\sigma = \alpha$. This same window of parameters or boundedness in g by $a > 0$ was observed in Hartwick [1992] also.

(14) indicates that dg/db and dg/dN are positive. That is, a larger economy or a more productive KPS implies more rapid growth in the economy. Also $dg/d\rho$ and $dg/d\sigma$ are negative. That is, a higher discount rate and a higher risk aversion parameter in utility each imply slower growth. These results are also observed in other endogenous growth models, including Hartwick [1992].

Given 'a' in (13) we can solve for the steady state c in (10). It is clearly positive for $0 < a < 1$. Then Z follows in (12). Z is also obviously positive for $0 < a < 1$ since $0 < \alpha < 1$ by definition.

Xie [1991, 1992] suggests investigating the case of $\sigma = \alpha$ because dynamics become tractable in this case.

Case $\alpha = \sigma$:

From (13) we obtain

$$a = \frac{\beta(1-\alpha)}{bN(1-\alpha-\beta)} \left[\frac{\rho - bN(1-\alpha-\beta)}{(1-\alpha-\beta+\alpha\beta)}\right].$$

Clearly ρ - bN(1-α-β) must be positive. Recall also that (1-a) must be positive. From (14) we obtain

$$g = \frac{bN(1-\alpha-\beta)-\beta\rho}{1-\alpha-\beta+\beta\alpha}.$$

In this case of $\sigma = \alpha$, we obtain from (11) $c = \rho/\sigma$ and Z from (12). c and Z will be positive for $0 < a < 1$.

Dynamics

Since (10) - (12) is a system of three simultaneous differential equations in a, c, and Z, the dynamics can be quite complicated. Matters are much simplified if we restrict attention to the case of $\alpha = \sigma$. In this case C and K move in unison and c remains constant over time at value ρ/σ (see (11)). From (10) we obtain a differential equation in 'a' alone, namely

$$\frac{\dot{a}}{a} = \left\{\frac{(1-\beta)baN(1-\alpha-\beta)}{\beta} + (1-\alpha-\beta)bN - \rho\right\}/(1-\beta).$$

We have

$$\dot{a} = Aa - Ba^2$$

where $A = [(1-\alpha-\beta)bN - \rho]/(1-\beta)$ and $B = -bN(1-\alpha-\beta)/\beta$. This equation can be written

$$\dot{a} = Ba(a^* - a)$$

where $a^* = A/B$. Recall that for $a > 0$, A must be negative. Hence a^* is positive. Clearly for an initial $a_0 > a^*$, $\dot{a} > 0$. For $a_0 < a^*$, $\dot{a} < 0$ since $B < 0$. Hence the dynamics require a_0 set equal to a^*. Also given K_0, C_0 must be set to satisfy $C_0/K_0 = \rho/\sigma$.

Given $c = \rho/\sigma$ and $a_0 = a^*$, we can solve for the time path of Z from (12). We have then

$$\dot{Z} = \gamma Z - \eta Z^2$$

where $\gamma = (1-\alpha-\beta)(1-a)Nb + (1-\alpha)\rho/\sigma$ and $\eta = (1-\alpha)Ya^{\beta}$. This can be expressed as

$$\dot{Z} = \eta Z(Z^* - Z)$$

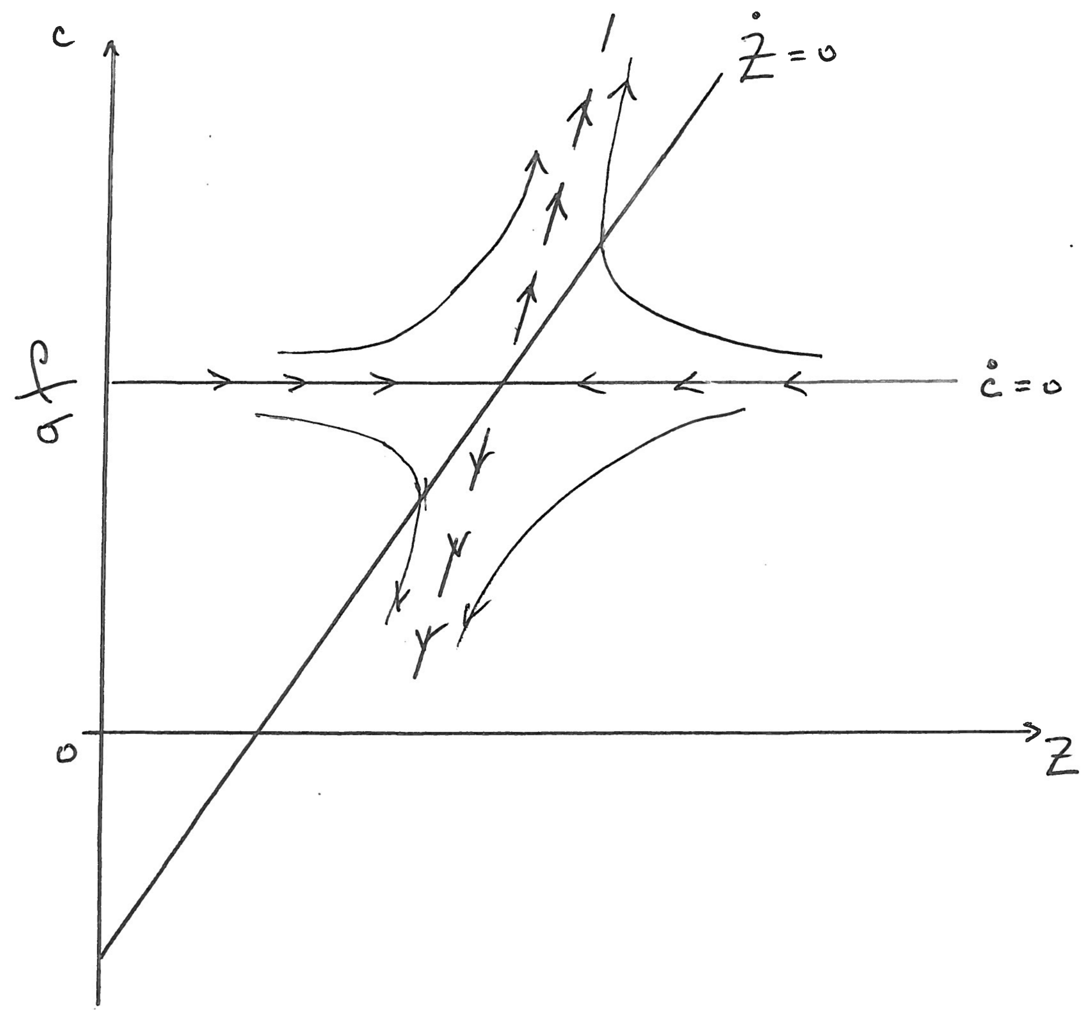
where $Z^* = \gamma/\eta > 0$. For $Z_0 > Z^*$, $\dot{Z} < 0$ and $Z_0 < Z^*$, $\dot{Z} > 0$. Hence Z(t) converges to Z^* from its initial value. Since $\dot{A}/A = (1-a)bN$ and 'a' is set at a^* above, $\dot{Z}/Z$ moves with $\dot{K}/K$. Thus the stability of Z(t) represents the stability of K(t). Thus for the case of $\sigma = \alpha$, the system is stable (a_0 is set at a^* and $\dot{A}/A$ is fixed; then K(t) and C(t) converge lockstep to their steady state (balanced growth) values). We sketch the dynamics of c(t) and Z(t) in Figure 1. (Recall that a_0 is set at a^* and a^* remains unchanged over time.)

Funding Knowledge Production with Knowledge Stock Rentals

The size of the knowledge production sector above is, roughly speaking, determined by (i) equality of wage rates in goods production and (ii) equality of rates of return to capital K and knowledge capital A. We replace (ii) with a condition that indicates that labor in the KPS is funded by rentals accruing to owners of the knowledge stock K. New knowledge is funded by

Case ($\alpha=\sigma$)

Figure 1



income derived from using existing knowledge in goods production. Our price of new knowledge above, ψ/λ, we now indicate by p. Then equality of wages in goods production and in knowledge production is

$$\beta Q/(aN) = pbA.$$

Compare this with (5) above. In place of (7) above we introduce the knowledge funding relation

$$(1-\alpha-\beta)Q = pb(1-a)NA$$

or knowledge rentals, $(1-\alpha-\beta)Q$, equals the value of new knowledge, $p\dot{A}$ ($= pb(1-a)NA$). This latter can be viewed as the wage bill of workers $(1-a)N$ in the KPS. If we substitute for p from one equation in the other, we obtain

$$\frac{(1-a)}{a} = \frac{1-\alpha-\beta}{\beta}$$

which yields $a = \beta/(1-\alpha)$. Given 'a' our dynamic system is characterized by (11) and (12) above, differential equations in c and Z. In balanced growth, we have from (11)

$Ya^{\beta}Z = \frac{\sigma c}{\sigma-\alpha} - \frac{\rho}{\sigma-\alpha}$. Using this in (12) in balanced growth yields

$$c = \frac{1}{\alpha}(\rho+(\sigma-\alpha)g).$$

where $g = \frac{(1-\alpha-\beta)(1-a)Nb}{(1-\alpha)} \left[= \left(\frac{1-\alpha-\beta}{1-\alpha}\right)^2 Nb \text{ given a above} \right]$. Then

$$Z = \frac{(1-\alpha)^{\beta}}{\alpha Y\beta^{\beta}}(\rho+\sigma g).$$

The preference parameter σ and discount rate ρ affect c and Z but not the balanced growth rate g. The rate of balanced growth is technologically determined. This is a consequence of the fraction of the labor force in production being unaffected by σ or ρ (or by population N or R&D efficiency parameter b).

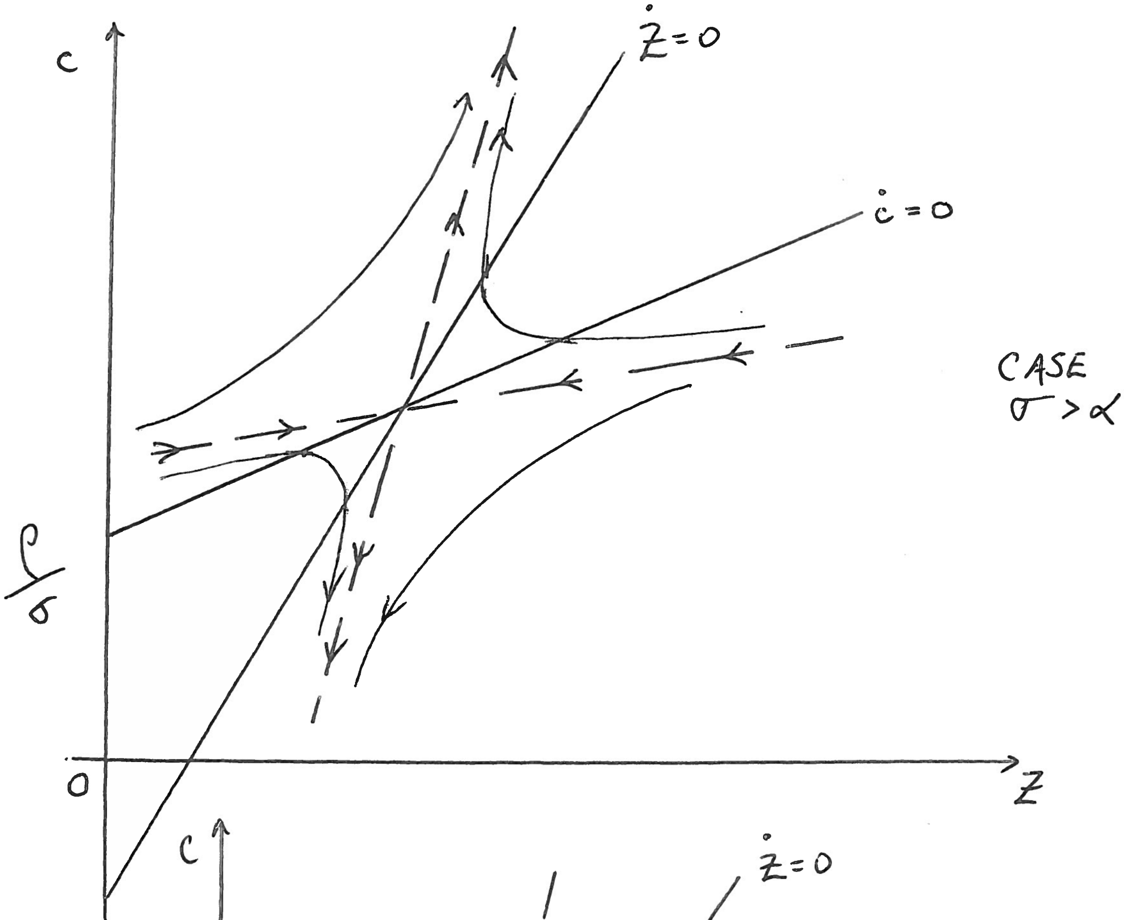
For a particular selection of parameters the optimal balanced growth rate in (14) will coincide with the balanced growth rate under the rule: finance the KPS with knowledge stock rentals. For $\sigma = 1$ and $(1-\alpha)\rho = (1-\alpha-\beta)bN$, (13) reduces to $a = \beta/(1-\alpha)$ and g in (14) reduces to $\left[\frac{1-\alpha-\beta}{1-\alpha}\right]^2 Nb$. Thus g (optimal growth) in (14) will be less than (exceed) g (under invest knowledge stock rentals in R&D) as $(1-\alpha)\rho > (<) (1-\alpha-\beta)bN$ for the case of $\sigma = 1$. Recall the comparative statics on g in (14). Only particular choices of parameters will yield the same balanced growth rate in the two models.

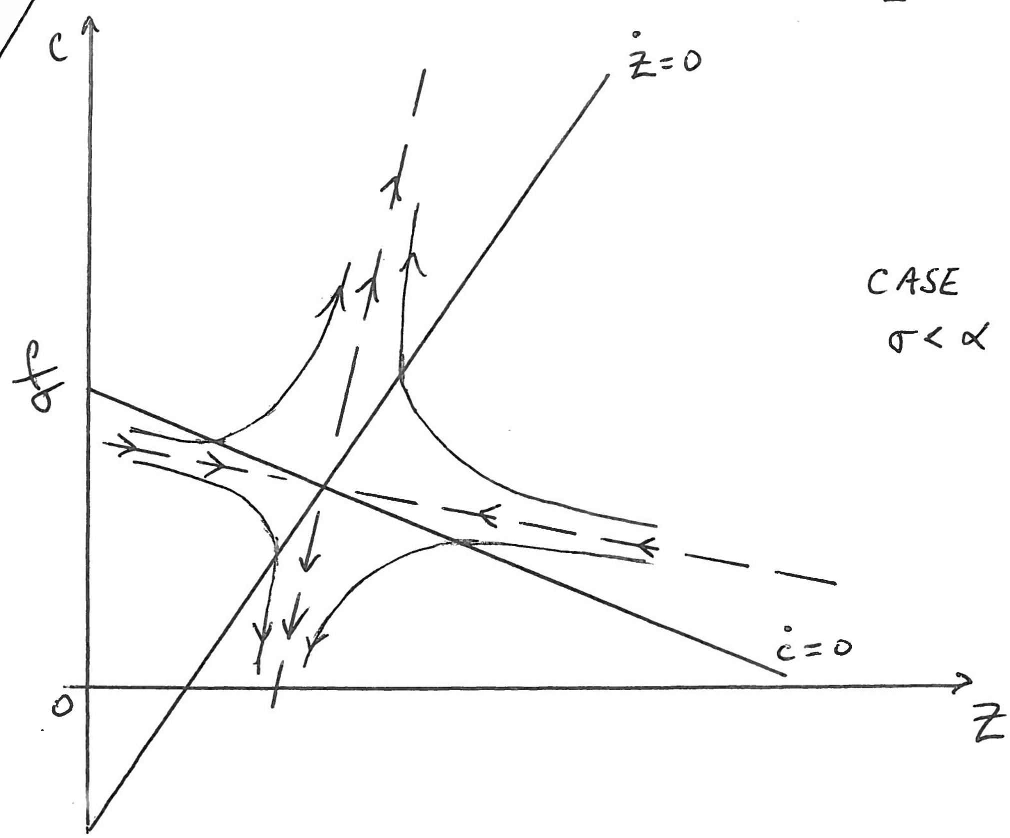
Since 'a' is constant under investing knowledge stock rentals in R&D, the dynamics involve only c(t) and Z(t). That is, we are interested in the dynamics of (11) and (12) given 'a' exogenously fixed. The case of $\alpha = \sigma$ is set out in Figure 1. (Though the dynamics of optimal growth and one case of growth under investing knowledge stock rentals are the same, 'a' is constant in each model for a different reason.) The two remaining cases of dynamics under invest knowledge stock rentals involve $\sigma > \alpha$ (Figure 2) and $\sigma < \alpha$ (Figure 3). In all cases there exists a path which converges asymptotically to the balanced growth path.

Concluding Remarks

We set out an endogenous growth model with the knowledge stock in production a non-public input. There were fundamental similarities between this model and one in which the knowledge stock was a pure public input. The dynamics for optimal growth in each model were similar (Hartwick [1992a]), given the Xie [1992] assumption, and the comparative statics of the balanced growth rate were similar. We investigated sustaining growth by investing knowledge stock rentals in the production of new knowledge. We found that this special case could be "retrieved" from the optimal growth version for a particular choice of parameters. The rate of balanced optimal growth could be above or below the corresponding rate under investing knowledge stock rentals. The preference parameters ρ and σ do not affect the balanced growth rate under the invest knowledge stock rentals regime. Only the goods production parameters and the size of the economy affect the growth rate, a species of technological determinism.

Figure 2 and 3





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