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The Stability of Economics Integration and Endogenous Growth

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Abstract

This paper examines the transitional dynamics of economic integration in the two country endogenous growth model of Rivera-Batiz and Romer (1991) and in an extension by Rivera-Batiz and Xie (1992). It is shown that, in the absence of knowledge flows across countries, economic integration will generically lead to a corner solution where only one country does all the R&D and the other specializes in manufactures. When countries are symmetric, the world growth rate in this equilibrium will always be higher than in autarky. When countries differ in their human capital endowment, the world growth rate with trade is always greater than the autarky growth rate of the 'low-growth' country, but may or may not be greater than the autarky growth rate of the 'high-growth' country.

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Introduction

In an influential paper, Rivera-Batiz and Romer (1991) (henceforth RBR) analyze the effects of economic integration between two countries in a model of endogenous growth. In their model, economic agents make purposeful investments in R&D in return for the monopoly rights to the sales of new intermediate products. The possibility for sustained growth comes from the public good nature of technological knowledge on the productivity of R&D[1]. Comparing across balanced growth paths before and after integration, they show that economic integration may have both *level* effects and *growth* effects, depending upon the nature of the R&D process and on the possibility for free flow of technological information across borders. Specifically, when the R&D process uses only skilled labour and the available stock of technological knowledge, they argue that free trade between identical countries has no effect on world growth rates if the flow of ideas across borders is inoperative[2]. Although growth rates will be temporarily affected as producers in each country respond to a larger market, in a new balanced growth path the resources that each country devotes to R&D will remain unchanged, and growth will be the same as if both economies were in autarky.

RBR examine a model where countries are structurally identical. In an extension of this basic structure, Rivera-Batiz and Xie (1992) (henceforth RBX) evaluate the effects of integration when countries have differences in their endowments of the skilled human capital that is required for R&D. They argue that trade liberalization, while unifying world growth rates, will tend to *reduce* growth rates for high (autarky) growth countries and *raise* growth rates for low-growth countries.

This note re-examines the model of RBR and a two country version of the extension by RBX. Our specific intent is to focus on the stability of the transitional dynamics associated with a trade liberalization. We report two principal findings. First, we show that the results of RBR and RBX stated above will only hold in a knife-edge case. In the model of RBR, their results require that immediately after trade liberalization, each country's

[1] Grossman and Helpman (1991) survey these types of models in great detail.

[2] This is what they refer to as the 'knowledge-driven' R&D specification. With a free flow of ideas, they show that economic integration will raise world growth rates. This may even happen without trade, if producers in different countries avoid duplication of research findings.

stock of useful research knowledge be exactly half that of the world stock. In the model of RBX, their results require that the *low growth* country begin, immediately after trade liberalization, with *greater than half* of the world's stock of useful research knowledge.

Our analysis implies that the equilibria focused on by RBR and RBX are unstable. Our second finding relates to the implications of this. In the model of RBR, we show that if the condition on country shares fails to hold, then the country with the higher stock of technological knowledge will increase its share of skilled labour in R&D, while the other country will systematically reduce its share in R&D. The end result is that all R&D will be concentrated in the country with the initial advantage, with the other country devoting all of its skilled labour to the production of manufacturing goods. We show, however, that this must lead to a *rise* in world growth rates of income and consumption. Thus, even in the environment with no international flow of ideas, free trade in commodities alone will increase world growth rates, as long as there are even slight differences in the levels of national income between the countries.

In the model of RBX, a similar result applies. If the low growth (in autarky) country begins with a share of world research knowledge below the critical share, it will progressively lose its share and will eventually specialize in manufacturing. In that case, the world growth rate rises above that of the high growth country's autarky rate. If, on the other hand, the low growth country has an initial share above the critical share, then it will continue to raise its share. In that case, the new steady state will involve the initial high growth country specializing in manufactures. World growth in a new steady state then exceeds that of the initial low-growth country, but may fall below that of the initial high-growth country.

An Economy with Knowledge Driven Innovation

The basic model we follow is identical to that of Romer (1990), RBR, and RBX. Take a single economy. In the economy, production is carried out in three sectors: consumption goods, capital goods, and R&D. The consumption goods sector uses unskilled labour, skilled labour (or human capital), and a set of specific capital goods to produce a homogeneous output. The specific capital goods sector has a production technology identical to that for consumption goods. All manufacturers produce either capital goods or consumption goods taking input and output prices as given. Thus, the two sectors may be aggregated into one manufacturing sector.

The right to supply each specific capital good is owned by a single patent holder with an infinitely lived title. Patent holders will engage manufacturing firms to produce their specific capital good as desired and then rent the goods out at the monopoly profit maximizing price. The production function for manufacturing is given as

$$Y = L^{(1-\alpha-\beta)} H_y^{\beta} \int^{A} x(i)^{\alpha} di \tag{1}$$

Here L is unskilled labour, held fixed throughout (and for simplicity we set L=1 henceforth), H_y is the employment of skilled labour in manufacturing, while x(i) is the use of capital of type i in production of manufactures. 'A' represents the total measure of available specific capital goods at a given time. The variable $K = \int^{A} x(i) di$ then represents the economy's total capital stock, valued at cost of production.

The infinite patent on a new type of capital good can be acquired by engaging in basic research and producing a design. If one unit of skilled labour is applied for Δt units of time, $\delta A \Delta t$ new designs for capital goods are produced. Thus, the rate of change of new designs in the economy is

$$\dot{A}/A = \delta(\bar{H} - H_y) \tag{2}$$

where $\bar{H}$ is the economy's fixed stock of skilled labour.

The representative agent in this economy has iso-elastic preferences defined over consumption paths as follows

$$U = \int_0^{\infty} e^{-\rho t} \left[\frac{C(t)^{1-\sigma}}{(1-\sigma)} \right] dt \tag{3}$$

Households receive income from wages in both skilled and unskilled occupations, as well as rental payments from their ownership of patents. To satisfy aggregate feasibility, manufacturing output must add up to consumption plus the production of new capital goods

$$C + \dot{K} = H_y^{\beta} \int^{A} x(i)^{\alpha} di \tag{4}$$

The derivation of the balanced growth path for a single economy is done in Romer (1990) and RBR. Assuming that output in the R&D sector is always positive, then the wage paid to skilled labour in manufacturing and R&D should be equated, so that, setting the price of the manufactured good equal to unity, we have

$$\beta H_y^{\beta-1} \int^A x(i)^\alpha di = \delta P_A A \tag{5}$$

where P_A is the market price of a new design. The manufacturing sector demand for each specific capital input is implicitly given by

$$\alpha H_y^\beta x(i)^{\alpha-1} = p(i) \tag{6}$$

where $p(i)$ is the price of the i^{th} intermediate good.

The patent holder of each specific capital good i will choose the rental price $p(i)$ to maximize profits, taking H_y and r as given, where r is the real rate of interest on a bond (in equilibrium, bonds will be in zero net supply). This leads to the markup pricing rule $p = r/\alpha$. Thus, all capital goods will be priced identically, and from (6), all will be used in the same proportion in manufacturing. Thus, we may write $x(i) = x$, and from (5)

$$H_y = \psi(P_A x^{-\alpha})^{-1/(1-\beta)} \tag{7}$$

where $\psi \equiv (\delta/\beta)^{-1/(1-\beta)}$.

Looking at the behaviour of households, it must be the case that an optimal intertemporal consumption path satisfies

$$\sigma \frac{\dot{C}}{C} = r - \rho \tag{8}$$

With free entry into R&D, the price of a new design must be

$$P_A = \int_t^\infty e^{-\int_t^\tau r(s)ds}(px(1-\alpha))d\tau$$

where $px(1-\alpha)$ represents the per period profits accruing to the owner of the design, beginning with zero capital stock[3]. Differentiating this, we have the condition

$$\dot{P}_A = -px(1-\alpha) + rP_A. \tag{9}$$

This is the standard arbitrage condition, which must hold if agents are to hold both bonds and designs as assets.

Finally, from (6), (7), and the markup rule, we may write the real interest rate implied by equilibrium in the market for skilled labour and specific capital goods as

$$r = \alpha^2 \psi^\beta P_A^{-\beta/(1-\beta)} x^{-(1-\alpha-\beta)/(1-\beta)} \tag{10}$$

Substituting (7) into (2), we derive the economy's growth rate of R&D as

$$\dot{A}/A = \delta\big[\bar{H} - \psi(P_A x^{-\alpha})^{-1/(1-\beta)}\big]. \tag{11}$$

We may use (4), (8), (9), (10), and (11) to derive three differential equations in C/K, x, and P_A. They will determine the path of the economy both on a balanced growth path and away from it. At any point in time, both x and A are predetermined. Equivalently, the aggregate capital stock, $K = Ax$, and the stock of patents, A, are predetermined. Adjustment towards a balanced growth path will take the form of changes in the ratio of K to A (i.e. adjustments in x). If the economy begins with a very low K/A, then there will be a period during which manufacturing output grows faster than R&D [4]. Along a balanced growth path, consumption, K, and A will all grow at the same rate, and P_A will be constant at $x(1-\alpha)/\alpha$. The balanced growth rate, g, and the balanced growth value

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[3] Given the institutional structure assumed, where patent holders purchase capital directly and rent it out to manufacturers, we derive this in the following way. P_A must be equal to the present value of a new patent holder's maximized cash flow, i.e.

$$P_A(t) = max \int_t^\infty e^{-\int_t^\tau r(s)ds}(p(x,t)x - i)d\tau, \ subject\ to\ \dot{x} = i,\ and\ x(t) = 0$$

(since the patent holder starts out with zero capital stock). Here $p(x,t)$ denotes the demand schedule given by (6). It is easy to see that the optimal pricing policy is $p = r/\alpha$, and along the optimal price path, the integrand is as written above.

[4] The stability properties of this system are analyzed in Devereux and Lapham (1992).

for x are [5]

$$g = \frac{\alpha(1-\alpha)\delta\bar{H} - \rho\beta}{\sigma\beta + \alpha(1-\alpha)} \tag{12}$$

$$x = \left[\frac{\alpha^2\phi^\beta + \sigma\delta\phi}{\rho + \sigma\delta\bar{H}}\right]^{(1-\beta)/(1-\alpha)} \tag{13}$$

where $\phi = \psi(\alpha/(1-\alpha))^{1/(1-\beta)}$.

Two Country World Economy

We now extend this single economy structure to examine the effect of two economies that suddenly open to trade in goods and assets with one another at date $t = 0$ [6]. We assume that these economies have identical preferences and technology and are both initially in the balanced growth equilibrium described above.

If the two countries have identical stocks of skilled human capital, then autarky levels of g, x, and P_A are identical across countries. Even in this case, however, it is quite possible to have differences in the stock of patents across countries. This amounts to differences in the *level* of GNP. In an endogenous growth environment, persistent level differences in GNP arise from varying initial conditions across countries. In autarky, there are no forces that work to eliminate these difference over time, even though the *rates of growth* will be equalized along a balanced growth path if the structure of preferences, technology and endowments of skilled labour are the same.

We assume that while there is free trade in commodities, there is no flow of ideas across countries. Thus, the productivity of basic R&D in a country is unaffected by the stock of technological knowledge derived from the other country's past innovations.

A separate issue is the degree of overlap between the designs at the time of liberalization. To maintain simplicity, we assume zero overlap between the innovations. Thus, the home and foreign economy are assumed to have invested in an entirely different range of

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[5] To establish (12) and (13), use (8), (9), (10), and (11), setting $\dot{P}_A = 0$ and $\frac{\dot{C}}{C} = \frac{\dot{A}}{A} = g$.

[6] We also implicitly assume that this liberalization is not anticipated. The anticipation of open trade would have two quite separate effects in this environment. First, it would lead to the standard rational expectations response of investment in light of future changes in its rate of return. Secondly, it would lead to an avoidance of the duplication of R&D, as firms in the domestic economy would attempt to preempt the possibility of any foreign competition in the post trade situation.

designs before trade liberalization. Thus, letting $\tilde{A}$ denote the world stock of technological knowledge, we have $\tilde{A} = A + A^*$[7].

We proceed under the assumption that both countries engage in positive amounts of R&D activity initially. The aim of the analysis is to determine the conditions under which this will be true in a new balanced growth path. For the home and foreign economies, H_y and H_y^* are determined by

$$\beta H_y^{\beta-1}\left[\int^{A} x^{\alpha}(i)di + \int^{A^*} x^{\alpha}(i^*)di^*\right] = P_A A\delta \tag{14}$$

$$\beta H_y^{*\beta-1}\left[\int^{A} x^{*\alpha}(i)di + \int^{A^*} x^{*\alpha}(i^*)di^*\right] = P_A A^*\delta. \tag{15}$$

Here, $x(i)$ and $x(i^*)$ denote domestic demand for home and foreign capital goods, and $x^*(i)$ and $x^*(i^*)$ give analogous demands for the foreign country. For the home country, these demands may be written as

$$x(i) = p(i)^{-1/(1-\alpha)}(\alpha H_y^{\beta})^{1/(1-\alpha)} \tag{16}$$

$$x(i^*) = p(i^*)^{-1/(1-\alpha)}(\alpha H_y^{\beta})^{1/(1-\alpha)}, \tag{17}$$

where $p(i)$ ($p(i^*)$) is the price of the home (foreign) good i (i^*). $x^*(i)$ and $x^*(i^*)$ are derived analogously. Patent holder i at home supplies $x(i) + x^*(i)$ and foreign patent holder i^* supplies $x^*(i) + x^*(i^*)$. As before, both home and foreign firms will price as a constant markup over the common world interest rate – all capital goods are sold at price $p = r/\alpha$. Therefore, the home manufacturers use of capital goods will be the same for each good and analogously for foreign manufacturers. Thus, all patent holders supply equal amounts of capital goods, irrespective of the country they are in. Instead of selling x to the home manufacturing sector, however, they will now sell the amount νx to the home country manufacturers and $(1-\nu)$ to the foreign manufacturers, where

$$\nu = \frac{H_y^{\beta/(1-\alpha)}}{H_y^{\beta/(1-\alpha)} + H_y^{*\beta/(1-\alpha)}}.$$

[7] This assumption is innocuous. We could allow for some overlap as long as only one firm is allowed to retain the patent right following the liberalization.

Since, across countries, the interest rate is the same, sales per firm are the same, and costs of production are the same, the value of a new patent must be the same [8]. Thus, P_A is common across countries.

Let x be the common stock of specific capital per patent holder in either country. Then using $x = x(i) + x^*(i) = x(i^*) + x^*(i^*)$ and (14)-(17), we derive an expression for the interest rate implied by factor market equilibrium as

$$r = \alpha^2 x^{-(1-\alpha-\beta)/(1-\beta)} P_A^{-\beta/(1-\beta)} \psi^\beta \Gamma, \tag{18}$$

where $\Gamma = \left[\theta^{-\beta/(1-\alpha-\beta)} + (1-\theta)^{-\beta/(1-\alpha-\beta)}\right]^{(1-\alpha-\beta)/(1-\beta)}$ and $\theta = A/(A+A^*)$ is the share of the home country in the world stock of designs. This affects the interest rate because from (14) (or (15)), the home country share of intermediates affects the domestic (foreign) wage and, therefore, domestic (foreign) employment in manufacturing. Substitute (18) into (16)-(17) and then back into (14)-(15) to derive H_y and H_y^* as

$$H_y = x^{\alpha/(1-\beta)} P_A^{-1/(1-\beta)} \psi \left[\Gamma^\alpha \theta^{(1-\alpha)}\right]^{-1/(1-\alpha-\beta)} \tag{19}$$

$$H_y^* = x^{\alpha/(1-\beta)} P_A^{-1/(1-\beta)} \psi \left[\Gamma^\alpha (1-\theta)^{(1-\alpha)}\right]^{-1/(1-\alpha-\beta)}. \tag{20}$$

Note that for $A^* > A$, we have $\theta < \frac{1}{2}$ and $H_y > H_y^*$. This is due to the fact that, for a common value of P_A, the wage will be higher in the foreign country. This is because with both countries diversifying, the wage is equal to the marginal product of skilled labour in R&D in each country.

Households in each country will face a common interest rate. Thus, consumption growth for each country and for world consumption must be defined as in equation (8). Define the aggregate world consumption and capital stock as $\tilde{C} = C + C^*$ and $\tilde{K} = (A + A^*)x$, respectively. Then using (19), (20) and world commodity market clearing, $\tilde{C} + \dot{\tilde{K}} = Y + Y^*$, we may derive

$$\tilde{C} + \dot{\tilde{K}} = \tilde{K} x^{-(1-\alpha-\beta)/(1-\beta)} P_A^{-\beta/(1-\beta)} \psi^\beta \Gamma. \tag{21}$$

[8] To see this more clearly, note that for the home country, P_A is determined by $\dot{P}_A + ((1-\alpha)/\alpha) r x = r P_A$. The expression for the foreign country is $\dot{P}_A^* + ((1-\alpha)/\alpha) r x = r P_A^*$. Since only the P_A terms differ in these expressions, if values are determined by fundamentals, then $P_A = P_A^*$ must hold.

Substituting (19) and (20) into the R&D production functions for each country and adding, we derive the growth rate of world technology as

$$\frac{\dot{A}+\dot{A}^*}{A+A^*} = \delta[\theta\bar{H} + (1-\theta)\bar{H}^*] - \delta x^{\alpha/(1-\beta)} P_A^{-1/(1-\beta)} \psi\Gamma. \tag{22}$$

Now, collecting all the pieces, the two country economy may be described as a dynamic system in the variables $\tilde{C}$, A, A^*, x, and P_A. By transformation of variables, we may rewrite this in stationary form as the system $\mathcal{S}$ of four differential equations in the variables $c = \tilde{C}/\tilde{K}$, x, P_A, and θ:

$$\frac{\dot{c}}{c} = [\frac{\alpha^2-\sigma}{\sigma}]\left[x^{-(1-\alpha-\beta)}P_A^{-\beta}\right]^{1/(1-\beta)}\psi^\beta\Gamma + c - \frac{\rho}{\sigma} \tag{$\mathcal{S}1$}$$

$$\begin{aligned}\frac{\dot{x}}{x} = {} & \left[x^{-(1-\alpha-\beta)}P_A^{-\beta}\right]^{1/(1-\beta)}\psi^\beta\Gamma - c - \delta\left[\theta\bar{H} + (1-\theta)\bar{H}^*\right] \\ & +\delta\left[x^{-\alpha}P_A\right]^{-1/(1-\beta)}\psi\Gamma\end{aligned} \tag{$\mathcal{S}2$}$$

$$\frac{\dot{P}_A}{P_A} = \alpha^2\left[x^{-(1-\alpha-\beta)}P_A^{-1}\right]^{1/(1-\beta)}\psi^\beta\Gamma\left[P_A - ((1-\alpha)/\alpha)x\right] \tag{$\mathcal{S}3$}$$

$$\frac{\dot{\theta}}{\theta} = \delta(1-\theta)\left[\bar{H} - \bar{H}^* - H_y + H_y^*\right]. \tag{$\mathcal{S}4$}$$

where Γ is the function of θ described above. In this system, x and θ are predetermined by initial conditions. Let these be given by $x(0)$ and $\theta(0)$. If the system satisfies saddle point stability, the variables c and P_A can adjust to ensure terminal constraints.

Before we analyze the issue of stability, for comparison we take note of the *interior balanced growth path* (IBGP) implied by $\mathcal{S}$. This is defined by a balanced growth path in which $0 < \theta < 1$. The IBGP solutions, $\hat{x}$, $\hat{g}$, and $\hat{\theta}$ are implicitly described by

$$\hat{x} = \left[\frac{(\alpha^2\phi^\beta + \sigma\delta\phi)\Gamma}{\rho + \sigma\delta(\hat{\theta}\bar{H} + (1-\hat{\theta})\bar{H}^*}\right)\right]^{(1-\beta)/(1-\alpha)} \tag{23}$$

$$\hat{g} = \frac{\alpha(1-\alpha)\delta[\hat{\theta}\bar{H} + (1-\hat{\theta})\bar{H}^*] - \rho\beta}{\sigma\beta + \alpha(1-\alpha)} \tag{24}$$

$$\bar{H} - \bar{H}^* = \hat{x}^{-(1-\alpha)/(1-\beta)}\phi\Gamma^{-\alpha/(1-\alpha-\beta)}\left[\hat{\theta}^{-\eta} - (1-\hat{\theta})^{-\eta}\right], \tag{25}$$

where $\eta = (1-\alpha)/(1-\alpha-\beta)$.

First take the RBR case, where $\bar{H} = \bar{H}^*$. From (25), it must be the case that $\hat{\theta} = \frac{1}{2}$. Therefore, $\Gamma = 2^{(1-\alpha)/(1-\beta)}$, and (23) and (24) indicate that, comparing autarky with the IBGP, trade liberalization without the free flow of technological knowledge will leave the growth rate unaffected, but will lead to a *doubling* of the output of each specific capital good.

For the more general case when $\bar{H} \neq \bar{H}^*$, as in RBX, the growth rate in the IBGP will differ from each country's autarky growth rate. From (25), it follows that $\hat{\theta} > \frac{1}{2}$ when $\bar{H}^* > \bar{H}$. The country with the relatively smaller stock of skilled human capital must have a *larger* share of the world's useful technological knowledge in an IBGP [9]. Looking at (24), this implies that the growth rate in an IBGP must exceed the autarky growth rate of the low growth country, but will be less than the autarky rate of the high-growth country.

The Symmetric Case: $\bar{H} = \bar{H}^*$

Our first set of results focus on the symmetric case where $\bar{H} = \bar{H}^*$ as in RBR. Using ($\mathcal{S}4$), we may state our first result as

Proposition 1:

For $\theta(0) \neq \frac{1}{2}$, the (symmetric) IBGP is never attained. Then $\theta \rightarrow 0$ for $\theta(0) < \frac{1}{2}$, and $\theta \rightarrow 1$ for $\theta(0) > \frac{1}{2}$.

Proof:

Rewrite ($\mathcal{S}4$) for the case $\bar{H} = \bar{H}^*$ as

$$\frac{\dot{\theta}}{\theta} = \delta(1-\theta)x^{\alpha/(1-\beta)}P_A^{-1/(1-\beta)}\psi\Gamma^{-\alpha/(1-\alpha-\beta)}\left[(1-\theta)^{-\eta} - \theta^{-\eta}\right] \qquad (\mathcal{S}4').$$

Since all the terms outside the square brackets are positive by definition, $\dot{\theta} < 0$ $(= 0, > 0)$ as $\theta < \frac{1}{2}$ $(= \frac{1}{2}, > \frac{1}{2})$. Thus, if the home country's share of designs begins at a level below that of the foreign country, it will continue to lose its share.

This result is established independently of the rest of the system ($\mathcal{S}1$)-($\mathcal{S}3$). Therefore, the system cannot be saddle point stable around the IBGP. So if $\theta(0) < \frac{1}{2}$, then the home country will continually lose its share in the world stock of designs. This will imply a

[9] Intuitively, this is necessary because for $\bar{H} < \bar{H}^*$, the interior balanced growth path requires that $H_y < H_y^*$. From (25) above, this can only happen when $\theta > \frac{1}{2}$.

progressive movement of skilled labour out of R&D and into manufacturing. Eventually this will violate the constraint $H_y \leq \bar{H}$, and from then on the home country specializes entirely in manufacturing production.

In the non-interior balanced growth outcome with $\theta = 0$, the foreign country will diversify between manufacturing and R&D. Domestic patent holders will still produce specific capital goods and price in the same way as before, but, in relation to the world's stock of designs, they will be of measure zero.

With $\theta = 0$ and $H_y = \bar{H}$, equilibrium in the market for each specific capital good is

$$x = p^{-1/(1-\alpha)}\left[(\alpha \bar{H}^{\beta})^{1/(1-\alpha)} + (\alpha H_y^{*\beta})^{1/(1-\alpha)}\right]. \tag{26}$$

Using (15), however, in a (non-interior) balanced growth path (where $P_A = x(1-\alpha)/\alpha$), we have

$$H_y^* = \phi(x/x^{*\alpha})^{-1/(1-\beta)}$$

where x^*, as before, denotes the share of each specific capital good that is used by the foreign firm. Note that $x^* < x$ will hold, since for $\bar{H} = \bar{H}^*$, the home country will employ more skilled labour when it specializes in manufacturing. Therefore, for a given level of x, H_y^* is lower than in the closed economy (i.e. $\phi x^{-(1-\alpha)/(1-\beta)}$). Then substitute for x^* (from the foreign version of (16)) to derive the following solution for H_y^*:

$$H_y^* = \left[\phi^{-(1-\alpha)(1-\beta)} x^{-(1-\alpha)} (r/\alpha^2)^{-\alpha}\right]^{1/(1-\alpha-\beta)}.$$

Substituting in (26) and rearranging gives

$$r = \left[\phi^{\beta(1-\beta)} \alpha^{2(1-\beta)} \chi^{-(1-\alpha)}\right]^{1/(1-\alpha-\beta)} + \left[\alpha^2 \bar{H}^{\beta}\right]^{1/(1-\alpha)} \chi^{-1} \tag{27}$$

where χ is a composite variable defined by $\chi = x r^{\alpha/(1-\alpha)}$. This gives us one balanced growth relationship between χ and r. To derive the other one, note that the growth rate of consumption must equal that of new designs in the foreign economy. Using the solution for H_y^* in the analog of (2) for the foreign economy, we derive

$$r = \sigma\delta\left[\bar{H} - \left(\phi^{-(1-\beta)} \alpha^{2\alpha/(1-\alpha)} \chi\right)^{-(1-\alpha)/(1-\alpha-\beta)}\right] + \rho. \tag{28}$$

With the aid of (27) and (28), we may state

Proposition 2:

The balanced growth rate with trade liberalization is higher than in autarky when $\theta(0) \neq \frac{1}{2}$.

Proof:

The pair of equations (27) and (28) describe the non-interior balanced growth rate solution for χ and r (and therefore x and r), when the home economy undertakes no R&D, and the foreign economy diversifies between R&D and manufacturing. Although (27) and (28) do not have an analytical solution, we may use a simple diagrammatic technique to establish that the world growth rate is higher than under autarky. First write the autarky versions of equation (27) and (28). These are

$$r = \left[\phi^{\beta(1-\beta)}\alpha^{2(1-\beta)}\chi^{-(1-\alpha)}\right]^{1/(1-\alpha-\beta)} \tag{27'}$$

$$r = \sigma\delta\left[\bar{H} - (\phi^{-(1-\beta)}\alpha^{2\alpha(1-\alpha)}\chi)^{-(1-\alpha)/(1-\alpha-\beta)}\right] + \rho \tag{28'}$$

Figure 1 illustrates this system. The first equation is a downward sloping parabola in r–χ space, represented by RR in the figure while the second equation, represented by GG, is upward sloping. Together they determine r and χ. Now comparing this with equations (27) and (28), we see that the position of the GG locus is unchanged. The RR locus however, must shift upwards, requiring a higher r for any given χ. Thus the new balanced growth values for r and χ must be higher after opening markets. Since the growth rate is increasing in r, given σ and ρ, it follows that the new balanced growth rate is higher.

This proof is based on $\theta(0) < \frac{1}{2}$, but the case with $\theta(0) > \frac{1}{2}$ is entirely symmetric. Then the foreign country will specialize in manufactures and the home country will diversify. In either case, the world growth rate rises.

The intuition behind this result is straightforward. As discussed above, when $\theta < \frac{1}{2}$, the home country will devote more skilled labour to manufacturing and less to R&D, since the wage is lower than in the foreign country. As a result, its share of the world stock of designs falls even further, and, therefore, it will devote a still smaller fraction of skilled labour to R&D. At the same time, because an increased variety of inputs raises productivity, the higher growth in foreign-supplied new intermediate products will continually raise the marginal product of skilled labour in manufactures relative to the wage rate – the marginal product of skilled labour in R&D. Thus, H_y will continue to grow until the constraint $H_y \leq \bar{H}$ is met, and R&D terminates.

On the other hand, the foreign country experiences the opposite — its wage is higher, as it has a greater stock of designs. Thus, it devotes less skilled labour to manufacturing and more to R&D relative to the home country. This raises its relative wage even more, and H_y^* falls still further. This process will end when the home stock of designs becomes arbitrarily small relative to the foreign stock. Then the marginal product of skilled labour in manufacturing and in R&D will be rising at the same rate in the foreign country. But at this point, its share of skilled labour in R&D will exceed that under autarky. As a consequence, the growth rate of new designs in the foreign country exceeds that in autarky. Therefore, the world growth rate is higher.

The Non-Symmetric Case

Now let us turn our attention to the non-symmetric case. In this more general case, equation ($\mathcal{S}4$) is

$$\frac{\dot{\theta}}{\theta} = (1-\theta)\big[\bar{H} - \bar{H}^* - x^{-(1-\alpha)/(1-\beta)}\phi\Gamma^{\alpha/(1-\alpha-\beta)}(\theta^{-\eta} - (1-\theta)^{-\eta})\big] \qquad (\mathcal{S}4'')$$

The proof of Proposition 1 no longer applies here because $\bar{H} - \bar{H}^* \neq 0$. Instead, we take a more direct approach to establishing the instability of the IBGP. $\mathcal{S}$ is linearized around the IBGP, and the roots are numerically calculated for a range of parameter values. Table 1 reports these results. In all cases, the roots contain only one negative element. Since the system $\mathcal{S}$ contains two predetermined and two non-predetermined variables, this rules out the possibility that $\mathcal{S}$ is locally stable around the IBGP[10]. Intuitively, this is clear from ($\mathcal{S}4''$). In the neighborhood of the steady state, $\partial\dot{\theta}/\partial\theta > 0$, so if $\theta < \hat{\theta}$, the home country's share of the total stock of useful technology will be falling over time, while if $\theta > \hat{\theta}$, θ will be rising over time.

Since the IBGP is unstable, then unless $\theta(0)$ is exactly $\hat{\theta}$, the only feasible balanced growth path with free trade will again imply either $\theta = 0$ or $\theta = 1$, depending upon initial

[10] Of course this is conditional upon the parameter values. However, extensive searching over the eligible parameter space failed to change the sign structure of the roots.

conditions. The same sort of reasoning applies as in the symmetric case. Now, however, the implications for world growth are different. Without loss of generality, let $\bar{H} < \bar{H}^*$, so the foreign country is the high-growth country in autarky.

The balanced growth path with $\theta = 0$ is described by the conditions

$$r = \left[\phi^{\beta(1-\beta)}\alpha^{2(1-\beta)}\chi^{-(1-\alpha)}\right]^{1/(1-\alpha-\beta)} + \left[\alpha^2\bar{H}^{\beta}\right]^{1/(1-\alpha)}\chi^{-1} \tag{29}$$

$$r = \sigma\delta\left[\bar{H}^* - (\phi^{-(1-\beta)}\alpha^{2\alpha(1-\alpha)}\chi)^{-(1-\alpha)/(1-\alpha-\beta)}\right] + \rho. \tag{30}$$

Note that (29) depends upon $\bar{H}$ as all skilled labour is devoted to manufacturing in the home economy, while (30) depends on $\bar{H}^*$, since that is what determines the magnitude of skilled labour in R&D, which is done only in the foreign economy.

Using the same logic as in Figure 1, it is straightforward to see that the growth rate implied by (29) and (30) will exceed *both* the home *and* foreign autarky rates. (30) is equivalent to the autarky GG curve for the foreign economy, so the same proof shows that the growth rate must exceed the foreign autarky rate.

In the case $\theta = 1$, the variables $\bar{H}$ and $\bar{H}^*$ are switched in equations (29) and (30). Then for the home country, growth must rise unambiguously relative to the autarky rate. Now, however, the relationship between the world growth rate and the foreign country autarky rate is ambiguous. Relative to the autarky situation in the foreign country, the RR curve shifts up, while the GG curve shifts down, because the level of skilled labour of the home country is lower than $\bar{H}^*$. As a result, the new world growth rate must be above the home autarky rate, but might fall below the foreign autarky rate.

While either case is possible, the case $\theta(0) < \hat{\theta}$ seems far more likely. Otherwise we would require the low-growth country to begin immediately after trade with the *higher* stock of useful technology. Therefore, for empirical purposes, the $\theta = 0$ balanced growth path seems the most relevant. In that case, we again conclude that economic integration will lead to a rise in world growth rates.

Conclusion

The instability of the IBGP is particular to the economy without the free flow of technological information. In an environment where R&D productivity is affected equally by domestic and foreign information, it can be shown that the analog to the differential equation system $\mathcal{S}$ satisfies at least local saddle point stability, and more specifically, the analog to equation ($\mathcal{S}4$) is globally stable, holding the values of x and P_A constant[11]. Thus, the comments in this paper pertain only to a subset of the issues addressed by RBR and RBX.

[11] For these results, see Devereux and Lapham (1992).

Table 1: Eigenvalues for Linearization of $\mathcal{S}$ Around IBGP

Case A: $\alpha = .35$ $\beta = .55$ $\delta = .90$ $\rho = .05$ $\bar{H} = .15$ $\bar{H}^* = .165$

	$\sigma = 0.5$	$\sigma = 1.0$	$\sigma = 1.5$
	($\hat{g} = .057$, $\hat{\theta} = .51$)	($\hat{g} = .037$, $\hat{\theta} = .51$)	($\hat{g} = .028$, $\hat{\theta} = .51$)
	-.22	-.24	-.26
Eigenvalues	.48	.59	.64
	.20	.26	.29
	23.7	29.3	31.9

Case B: $\alpha = .30$ $\beta = .55$ $\delta = .90$ $\rho = .05$ $\bar{H} = .15$ $\bar{H}^* = .165$

	$\sigma = 0.5$	$\sigma = 1.0$	$\sigma = 1.5$
	($\hat{g} = .05$, $\hat{\theta} = .51$)	($\hat{g} = .032$, $\hat{\theta} = .51$)	($\hat{g} = .023$, $\hat{\theta} = .51$)
	-.26	-.28	-.29
Eigenvalues	.41	.49	.52
	.24	.31	.34
	24.7	29.7	31.9

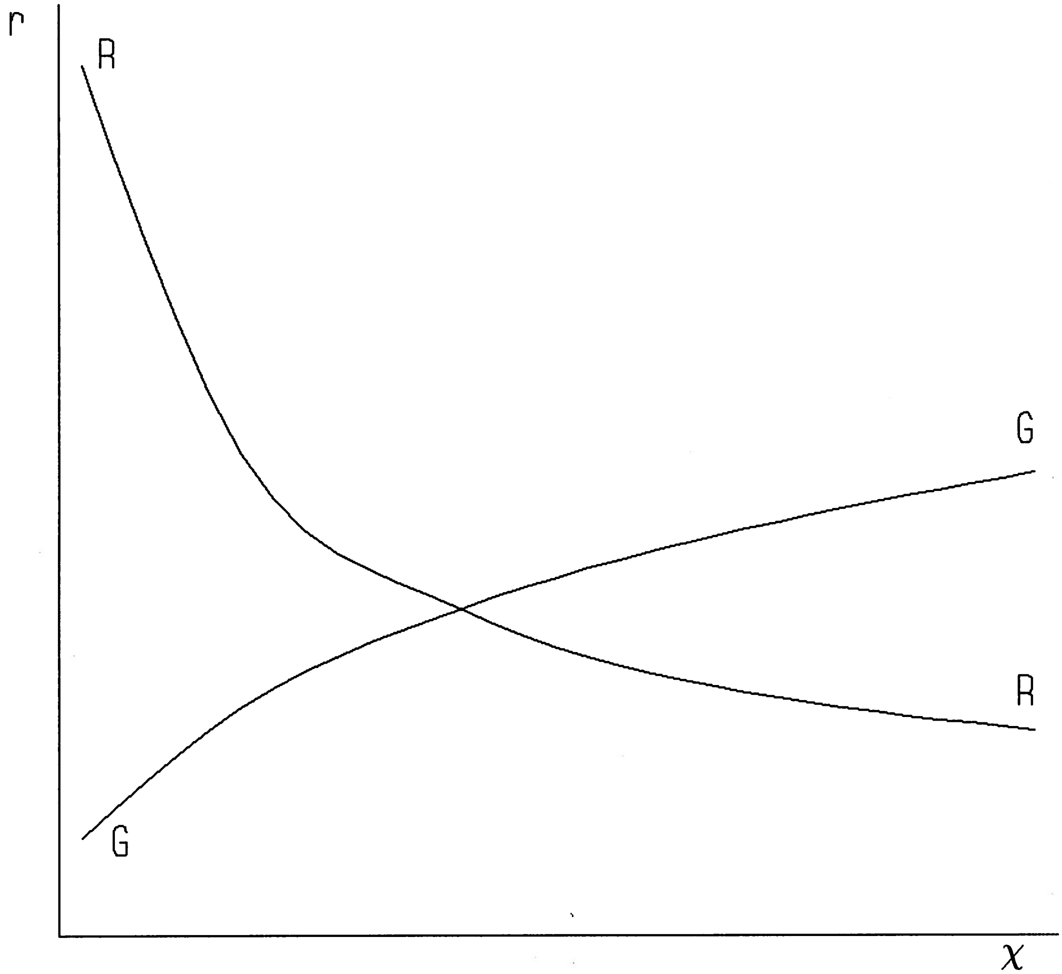

Figure 1

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