Queen's Economics Department Working Paper No. 875

Capitalization of Productivity Growth in Urban Land Rent

John M. John M. Hartwick

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

1-1993

Discussion Paper #875

Capitalization of Productivity Growth in Urban Land Rent

by

John M. Hartwick
Queen's University

January 1993

John M. Hartwick
Queen's University

Capitalization of Productivity Growth in Urban Land Rent

Abstract

The first order effect of a productivity increment in the output sector of a monocentric city which increases wages in the city is exactly capitalized in the increment in land rent in the city. We observe this result in the "open city" model in which a worker's utility level in city i is determined outside of city i.

John M. Hartwick
Queen's University

Capitalization of Productivity Growth in Urban Land Rent

Introduction

It is a commonplace to observe that the higher wages observed in larger cities[1] must be offset by higher commuting and housing costs for residents. For higher housing costs, one infers higher land rents. It is easy to make this argument precise in the open city model. That is, utility levels are identical for identical workers across cities given free mobility and higher rent (larger) cities must generate higher wages to maintain identical workers on the same utility level. (See Mills and Hamilton [1989; p. 116].) But how can larger cities offer higher wages? A worker must be more productive in a larger city. If this is the case, then higher wages must capitalize a productivity premium and this productivity premium gets transmitted via wage premia to land rent premia in larger cities.[2] Hence higher land rents in larger cities must reflect (labor?) productivity premia in larger cities. We will establish this result formally for the monocentric city model below. There are two distinct cases. First when the city in question is a small

---

[1]See Rosen [1979]. His sample comprises nineteen U.S. cities with a spread of about thirty percent between the lowest wage and highest wage city. He estimated at $48 wage increase for each ten percent increase in population.

[2]Roback [1982] exploits the fact that identical workers will migrate to equalize utility levels and that this equalization is associated with differences in wages and land rents across cities, as long as a location amenity varies across cities. Her key argument is: local rents fail to fully capitalize local amenity differences; wages also respond to amenity differences. She abstracts from the internal structure of a representative city. She finds that "the well-known regional differences in wages ... are found to be explained almost entirely by differences in amenities" (p.1259). Rauch [1991] associates the Roback's "amenity" with a local public good which induces cost savings to producers who locate near the public good. Rauch is linking Roback's generalized "amenity" with an abstract localized productivity advantage. Hence the productiveness of agglomerated activity.

open economy (price taker in its output), marginal improvements in productivity will be fully capitalized in land rent increments. Secondly when the city faces a negatively-sloped demand curve for its product, marginal improvements in productivity will be partially capitalized in land rent increments; some of the productivity increment will show up in output price declines (increments in consumer surplus).

These results are not too surprising since in non-spatial models we expect improvements in productivity to be capitalized in part in the market value of the firm and to show up in part in increments in the consumer surplus of the firm's customers. Of interest in the urban context is how under the open city assumption (the same utility level for identical workers across all cities), productivity increments in one city at the margin are not "eroded" by increases in transportation costs incurred by workers. For the output price-taking city, marginal improvements in productivity are exactly capitalized in land rent increments. The open city represents a mechanism for transmitting improvements in productivity to land owners behind the veil of wage increases. Improvements in productivity are one manifestation of the general productivity growth many observers (eg. J. Jacobs [1969], R. Lucas [1988]) have suggested is bred or spawned in agglomerated social structures (eg. cities).[3] Our capitalization result[4] thus represents a piece of a formal theory of the economic growth of a nation.[5]

---

[3]Brown and Medoff [1989] examine six reasons to explain the observed positive correlation of wages with employer size. They did not draw on the Rosen-Roback urban economics literature which analyzes the positive correlation of wages and city size. It would seem plausible that employer size is positively correlated with city size.

[4]The traditional capitalization issue is relating increases in local taxes on residences ("property taxes") to changes in the price of the residence (lot and structure). See Wildasin [1986; pp. 64-81]. Local taxes are notoriously inexact "measures" of marginal or total benefits accuring to a resident.

[5]Miyao [1977] introduced a neoclassical urban growth model in which the accumulation of producible capital generated growth.

The Model

The model focuses on a representative city and comprises a monocentric city residential submodel and an export good submodel in the CBD (central business district). The two submodels are linked by the requirement that labor in export good production in the CBD must reside in the residential annulus around the CBD. The N workers in production equal the N workers in the residential area.

(a) Export Good Production

Goods Q are produced for export with production function $f(A,N,K)$ where A is a productivity index, N is labor and K is produced capital with rental rate r, parametric to our city. $f(A,0,K) = f(A,N,0) = 0$. First derivatives are positive and second negative. We do not assume homogeneity in $f(\ )$ of degree 1 in N and K, though it is allowed. At any date we have

$$p(t) = w(t)a_N + r\,a_K \qquad (1)$$

where a_N and a_K are equilibrium labor and capital requirements per unit output. a_N and a_k will each in general be functions of input prices w(t) and r, as well as efficiency parameter A and possibly scale index Q. w(t) is the wage in the city and p(t) the price of output exported from the city. Under the small open economy assumption p(t) is constant and changes in A(t) do not affect p(t). Under the alternative assumption, an increase in A causes p(t) to decline since p(t) then depends on city output Q(t). We are interested in the effect on w(t) of an exogenous increment in A, the productivity index. We assume, via some institutional mechanism, workers are able to "soak up" much of the transitory profit accruing to the firm as A increases. What workers do not obtain in wage increases is translated into product price reductions and output increases for the case of a city facing a negatively sloped demand curve for its output. Recall that r is treated as parametric. From (1) we obtain

$$\frac{dw}{dA} = \frac{\frac{dp}{dQ} \cdot \frac{dQ}{dA} - w\left[\frac{da_N}{dQ} \cdot \frac{dQ}{dA} + \frac{da_N}{dA}\right] - r\left[\frac{da_N}{dQ} \cdot \frac{dQ}{dA} + \frac{da_k}{dA}\right]}{\left[a_N + w\frac{da_N}{dw} + r\frac{da_k}{dw}\right]} \qquad (2)$$

Under the not small economy assumption, $dp/dQ < 0$ and $dQ/dA > 0$. If labor productivity is primarily enhanced by an increment in A, then we have $da_N/dA < 0$ and $da_K/dA > 0$. If capital productivity is primarily enhanced by an increment in A, then we have $da_N/dA > 0$ and $da_K/dA < 0$. In either case, capital rental r is constant and wage earners "soak up" the profit increment arising from the increment in A as w rises. However the increment in A impinges on the economy (capital saving or labor saving), the constancy in r arising from the open economy assumption, leaves our city's wage w to rise. An attractive special case has the technology Leontief. Then a_N and a_K are independent of each other. An increment in labor productivity simply reduces a_N and

$$\frac{dw}{dA} = \left[\frac{dp}{dQ} \cdot \frac{dQ}{dA} - w\frac{da_N}{dA}\right]/a_N.$$

In the small open economy case, $dp/dQ = 0$ and we would have the increase in wages absorb all of the increment in the labor productivity index A.

But any productivity increment, labor saving or otherwise, will drive up wages when r and p are fixed by the small open economy assumption. With p responsive to output levels, the division of a productivity increment into changes in w and p is difficult to predict. Moreover, one surmises that the nature of the productivity increment would affect its impact on w and p. This is a topic beyond the scope of our analysis here. We simply infer then in all cases, increments in A show up in some increment in w.

We assume goods production uses no land. (We discuss this in detail below.) This is a technical convenience since it relieves us from tracking changes in the size of the CBD as productivity parameter A changes. It also captures the familiar idea of strong agglomeration economies being captured by geographically concentrated activity. Some form of scale economies (eg.

statistical economies in labor market-fit between labor supplied and labor demanded in a stochastic product demand environment as in Krugman [1991; Appendix C]) is being captured by concentrating export activity in the CBD.

(b) The Residential Annulus

Workers receive historically given wage w and spend it on round trip commuting costs t(x), housing costs h(x)l(x), and other consumption c(x) where x is distance from the CBD.

t(x) is roundtrip commuting costs for distance x from the CBD per wage period.

h(x) is housing (land with an identical structure) per worker at distance x.

l(x) is land rent at x.

c(x) is composite good consumption per worker at x. Composite good price is unity.

Some modellers choose to have identical workers live on identical sized lots for any distance x. Then h(x) and c(x) are independent of x and workers have implicit Leontief utility functions. This convenient special case is compatible with our model. We assume each worker attains utility level $\bar{u}$, determined in the system of cities. $\bar{u}$ is exogenous to each open city, and constant across cities for the same workers. Given the assumed form of u(h(x), c(x)) and the parameters w, and $\bar{u}$, we can solve for the edge of the residential ring $\bar{x}$ and the total number of workers accommodatable in the residential annulus (and available for production in the CBD), N.

$\bar{u}$ constant implies

$$u_h dh + u_c dc = 0 \tag{3}$$

and consumer equilibrium implies

$$\frac{u_h}{u_c} = l(x) \tag{4}$$

Hence

$$l(x)dh + dc = 0$$

for each x where workers reside. Also at each x, $w = t(x) + l(x)\,h(x) + c(x)$ which implies

$$dw = 0 = t_x dx + h dl + dc + l(x)dh \tag{5}$$

Combining (4) and (5) yields the fundamental residential location equilibrium condition:

$$h(x)\,\frac{dl}{dx} = -t_x \tag{6}$$

(3), (4) and (5) define equilibrium h(x) and c(x) at each x. (See Hartwick, Schweizer, and Varaiya [1976].) Then edge $\bar{x}$ is defined as

$$l(\bar{x}) = [w - t(\bar{x}) - c(\bar{x})]/h(\bar{x}) \tag{7}$$

Given $\bar{x}$ in (7), we have labor force size[6]

$$N(\bar{x}) = \int_0^{\bar{x}} \frac{2\pi x}{h(x)}\,dx \tag{8}$$

Given p, r, w, A and the production function, equilibrium Q and K can be solved for.

Land rent in the urban area is defined as

$$R(\bar{x}) = \int_0^{\bar{x}} \frac{w-t(x) - c(x)}{h(x)}\,2\pi x\,dx \tag{9}$$

The Capitalization of Labor Productivity Improvements in Land Rent Increases

Observe in (9)

$$\frac{dR(\bar{x})}{dw} = l(\bar{x})\,\frac{d\bar{x}}{dw} + \int_0^{\bar{x}} \frac{2\pi x}{h(x)}\,dx \tag{10}$$

where $l(\bar{x})$ equals zero because $\frac{w-t(\bar{x})-c(\bar{x})}{h(\bar{x})} = 0$ in (7).[7] (See the Appendix for a worked example with Cobb-Douglas utility functions.) Observe that (8) and

---

[6]The 2π implies a circular city. The model holds for a semi-circular city or for any fraction of 2π less than unity.

[7]This sign of this derivative in (10) was reported in Wheaton [1974; p. 235] but its magnitude was not. Wheaton focused on changes in local land rent, our l(x), not a total land rent. He inferred that "rents rise at all points" from his investigation of local land rent changes.

(10) are identical. This central property of the equilibrium residential annulus yields our central capitalization result:

$$N(\bar{x})\,\frac{dw}{dA} = \frac{dR(\bar{x})}{dw}\cdot\frac{dw}{dA}. \qquad (11)$$

(11) indicates that the marginal improvement in productivity that is captured by wage increments (namely (dw/dA)N) is exactly capitalized by land rent increments in the city. In the special case of output price constant (the small open economy case) then all of the productivity increment will be captured by wage increases and in turn by land rent increases in the city.

After w increases in response to the increase in A, the city will grow as $\bar{x}$ increases and new workers migrate to the city until $\bar{u}$ is re-established. Q and K will change. There will be a once-over increase in land rent in the city. There is no reason to believe that for discrete changes (not first order) in A and w, that labor productivity captured by wage increases is exactly capitalized in the land rent increases. Marginal effects only equal total effects for linear functions and our functions in question are not linear. Discrete changes will show up in changes in $\bar{x}$, $N(\bar{x})$, and the ratios of $c(x)/l(x)$ at each x. Nevertheless there is a prima facie case for viewing land rent increases in cities as reflecting in large part productivity induced wage increments. We discuss non-marginal effects below.

With two classes of workers in a residential annulus, an increase in one wage w_i will lead to rent in its subannulus rising. This will allow workers of the i^{th} type to bid land at the margin away from workers of the other type. Given the open city assumption the number of workers of type i will rise in the city and those of the other type will decline. There will be changes in the outputs in the CBD, given the new mix of workers. Products associated with the higher wage will experience increased output as workers of type i migrate to the city. The induced land rent changes will drive workers of the other type away from the city. In summary, workers

experiencing wage increases will increase in absolute and relative numbers in the city. Products associated with these higher wages will also increase relative to other products. We have then a significant extension of our one worker class capitalization result. There wage increments were totally capitalized in land rent increments. With multiple worker classes, a wage increment is partly capitalized in land rent increments and partly in increases in the numbers of workers who experienced the wage increases.[8]

A CBD with Land

The factory or production site is a sink for the owners of the site and capital to in part capitalize productivity gains. The value of the firm would be capitalized in its site and plant and machines. Hence any productivity advance that workers could not capture in wages and customers could not capture in reduced product prices would be capitalized in the value of the firm, specifically its site. In an equilibrium in the land market, adjacent sites command the same land rent at the margin. So capitalization in plant site is mediated by what rent adjacent sites command. Nevertheless, our argument needs revision. Not all productivity increments show up in wage increments and product price reduction. Some show up in plant site rent increases. But this is a variant of our central point that much of urban land rent is a capitalization of past productivity advances. In the revised case, part of the advance accrues to plant owners, who own the plant site. This would dilute the productivity gain captured by wage earners. We are

---

[8]Matters are quite different in the closed city case examined in Hartwick, Schweizer and Varaiya [1976]. Then an increment in w_i changed all boundaries between classes and all endogenous utility levels. Capitalization effects spilled over more directly than is the case with many worker classes and exogenously set utility levels. In the closed city case, changes in utility levels absorb much of the effect of a parameter change (as in an increase in w_i) whereas in the open city case, changes in worker numbers of various types absorb much of the effect of a parameter change.

spreading the capitalization over land in the CBD and land in the residential annulus. Land rent increments still reflect productivity increments.

It would be absurd not to acknowledge that much land rent in many cities reflects a site advantage such as a natural port. One can view a port in Ricardian terms as an especially fertile farming site. The owner would apply labor in order to maximize profit which would represent land rent for the site. Owners of shoreline in a port area can hire labor in the same way our hypothetical farmer would and the owner could reap substantial rents in the selling price of his or her site. There is no natural mechanism for labor to cut into such site rents. Workers would receive the prevailing wage in the economy at large. Workers with homes near the port would have lower commuting costs and would own houses and plots with higher land rents than owners farther away. Equilibrium could be a monocentric city based on a half disc of land around the port with very high land value at port side and much lower land values in the residential area. An approximate discrete jump down in land rent as one moved in land from the shore is quite conceivable. In this case the land rent accruing to the port shore owners is unrelated to the rent in the residential half annulus. The latter rent is determined by prevailing wages in the hinterland economy. In this case the urban area exists because of the port and the value of the city is capitalized in value of the shore land. There is no productivity premium capitalized in the residential area since land rent there corresponds to the bids of workers earning the equilibrium wage in the economy as a whole. This is a portrait of a city in the pre-industrial age. Primary commodities such as wheat and wine could be exported from the port and fabricated consumer goods imported from places which produced them in a handicraft mode.

Additional rents would be generated by linking a rail head to the port side. Then producers would cluster around the transportation node and

workers in production and in transportation would live around the node. Still workers would be hired at the prevailing wage in the economy. However as those transporters and goods producers experienced technical progress, some of the productivity increment would be captured in local wages which would show up in local rents in the residential area. This is our central observation. Nevertheless owners of land devoted to transportation and goods production could capitalize some of the productivity increment in their own land values. Let us return however to an abstract CBD without land and dominated by goods or service production exported from the urban area.

Urban Wages and Large City Disamenities

Hoch and Drake [1974] and Rosen [1979] attempted to measure the impact of large city disamenities in the large city wage premia. Wage premia are viewed as compensation for residents of large cities being forced to consume disamenities such as congestion, pollution, and crime. In this view our resident's utility function should be expressed as $u(h,c,D)$ where D is a flow of disamenities. The new land rent function in the monocentric city will not be much different from what we had above. Equilibrium values of $h(x)$ and $c(x)$ will be affected by D and D may vary with x but land rent $l(x)$ at x will still be $[w-t(x)-c(x)]/h(x)$ because D is not traded in a market. And even if D changes equilibrium values of $c(x)$ and $h(x)$ via the bid-rent function, the function defining N in (8) will also "adjust" to the revised $h(x)$ function. Thus $dR(\bar{x})/dw$ should still equal $N(\bar{x})$, our key preliminary result on capitalization.

In D, presumably congestion is the most location sensitive disamenity. Suppose congestion were the only disamenity and $D(x)$ was the time cost (disutility) of round trip community from x. Given the equilibrium values of $h(x)$ and $c(x)$, sensitive to $D(x)$, the density of the urban area at various

distances could be quite different from what it would be if D(x) were zero. Again, we note that even with a quite different city our result on $dR(\bar{x})/dw$ equal to $N(\bar{x})$ should hold. Thus wage increments $N(\bar{x})dw$ will show up as particular land rent increments in $dR(\bar{x})/dw$. The implication of this is that one can remain agnostic on the question of whether wage premia represent compensation for disamenities or simply windfalls to workers because a productivity improvement has occurred. Our marginal capitalization result remains unaffected by the assumption one makes. However total (non-marginal) capitalization of wage increments in land rent increments will change as the structure (density and compactness) of the urban area changes in response to different assumptions about how disamenities interact with c(x) and h(x).

If larger cities are more congested, identical individuals in two different sized cities will, with identical housing-composite commodity bundles be on lower utility levels in the larger cities. But by definition the residents of larger cities receive higher wages. Hence the wage premium in a larger city will partly show up as extra land rent and partly show up as compensation in the form of extra income used to buy extra land and composite goods in order to offset the extra congestion. Hence some land rent capitalization will be diluted by disamenities positively correlated with city size. This is not a marginal capitalization argument, rather it deals with non-marginal impacts of wage premia in large cities. The same argument holds in the absence of congestion. The "average resident" of a large city will spend more on commuting than the "average resident" of a smaller city. Thus total capitalization of wage premia in larger cities will be diluted somewhat by extra commuting costs.

Disequilibrium and Stochastic Demand

The demand schedule (or schedules) facing a city's primary export can be subject to shocks possibly pro-cyclical. Variations in demand can show up in factor demands and possibly wages in the city. Real estate prices in cities are not steady over a larger economy's business cycle. We have been dealing with an equilibrium scenario. Actual land prices will be subject to fluctuations caused by output demand shocks hitting the urban area's product demand schedules. In empirical work it could be very difficult to separate observed land prices into an equilibrium component and a disequilibrium part.

There can also be persistent disequilibria premia in real estate prices arising from the fact that the supply of new houses and buildings can lag increases in demand for them arising from a growth in demand for a city's export or exports. The reverse is also true. "Over-building" can cause persistent depression in real estate prices and their associated land rents. These sorts of deviations in prices in cities from equilibrium prices have received much attention in different analyses of urban phenomena and may have diverted attention from our central point, namely in equilibrium much urban land rent can be seen to emerge from growth in productivity in a city's export sector.

Concluding Remarks

Let us leap from marginal to total effects and assume that most of urban land rent is simply capitalized wage premia resulting from past productivity gains. The novelty is presumably not that workers benefit from productivity gains but that the benefits end up capitalized in residential land prices. Remaining benefits appear in lower prices of the output which the workers produce. An urban area is in a sense a sink capturing productivity gains via localized wage premia. A more traditional view is that wages for similar

workers are equal throughout the economy and that productivity gains can be captured by the owners of capital via capital gains in the values of firms enjoying productivity gains. Capital gains plus income constitute profits ex post and define the firm's rate of return. High rates of return attract new investment which in turn drives the realized rates down to the market rates. Capital gains are dissipated by the free entry of new capital. In the long run all units of capital make the market rate of return which is also itself in the long run affected by past productivity increments. We made essential use of the capital rental being constant in an open economy setting and this permitted us to infer urban wage increases arising form exogenous productivity gains. If K were suddenly more productive in a city, we would expect r to rise locally. We have ruled this out by treating r as exogenously set in the open system of cities. This productivity increment is assumed to create excess profits for the firm which are "soaked up" by workers forcing up their wages. We do not want to mull over the large question of historical trends in w and r in response to continuous productivity increments. This is a fascinating issue to reflect on.

We have glossed over a number of important topics. To what extent is an increment in A the result of purposeful investment? This is the old issue that investment in R&D will be reduced if the benefits to the investor cannot be capitalized as profit downstream to him or her. We have implicitly treated the increment in A as manna from heaven. This is not an unusual approach but it neglects the well-known nexus linking investment in new knowledge to the investor's downstream profit. Part of the difficulty for the investor in R&D is the fact that new knowledge is relatively costless to "replicate" and transmit. This difficulty-of-exclusion property of new knowledge makes it easier for urban workers to claim the downstream benefits in their higher wages. The difficulty of establishing property rights on ex

post benefits of a productivity increment makes workers' claim in the form of wage increments as legitimate as any other person's claim. Many observers have emphasized the difficulty to an investor in R&D of completely capturing the downstream benefits. Spillovers are the rule. The degree of spillover of benefits from an R&D investment is the open question.

If investment in R&D is a "public project", then one gets involved in measuring benefits relative to costs. Then attention has to be focused on the precise magnitude of non-marginal capitalization of benefits. (See Starrett [1981].) The issues to be addressed there are quite different from an analysis of first order or marginal effects. Starrett raised the question of the distribution of benefits. To the extent that residents are renters, they do not gain from land rent increases. Thus the nature of home ownership affects the distribution of benefits significantly. Our point is that land prices do capitalize productivity gains and this can be seen clearly in our marginal impact analysis. The welfare economics of productivity growth is a separate large topic.

It is useful to distinguish between larger cities as more productive invention centers (Jacobs [1969] and Lucas [1988]) and more effective innovators or implementers of new productive results from R&D. If an invention lowers product i's average cost curve and there are returns to scale exhibited by the average costs, then a large city is the natural place to exploit the new knowledge. One large production center would be more efficient than two medium sized production centers. It is not outrageous to argue that the industrial revolution not only lowered average costs in many sectors but increased the minimum efficient scale. From this perspective large industrial centers were a consequence of the particular form invention took and in turn much of the productiveness of large scale activity was

capitalized in land prices in the resulting urban form (monocentric city).[9]

Crucial cost-effects should be accounted for. Large cities were not typically monolithic production centers. Economies of scale in a few crucial sectors attracted complementary sectors. A declining average cost curve will start agglomeration but the agglomeration of a range of industries creates a large city. Positively correlated with large size will be high wages which in turn yield high land rents characteristic of large cities.

Large cities without a dominant center or nucleus should not display the land rent Everest characteristic of the monocentric city. Non-centered cities are presumably not experiencing large scale economies in key productive sectors. It is probably correct to infer that each little center in such cities does have a basis in scale economies but no single activity or complex of activities displays the large economies associated with activities in the center of large monocentric cities. Have people not labelled many large cities in the pre-industrial age as collections of villages?

Suburbanization of residential activity and the decentralization of centers of employment in the post-1945 era has made the CBD of many cities less dominant. Perhaps large cities are returning to earlier forms, now as collections of towns rather than villages. We would not be the first to suggest that technical change since 1945 has reduced the minimum efficient scale of many processes. This would suggest that suburbanization may have been technology driven rather than driven by such forces as the automobilization trend or simply the greater-than-unity income elasticity of

---

[9]Henderson [1986] tested whether larger cities are associated with scale advantages in manufacturing. The results were not positive. "Rather, resources in any industry are more productive in places where there is more of similar activities" (p.66). "We expect to find that small and medium size cities are highly specialized in production" (p.66). Note that Henderson was dealing with manufacturing activity and not services such as financial intermediation, insurance, and administration.

demand for residential land. Because a city acquires multiple centers, in no way vitiates the argument that much of urban land rent is a capitalization of past productivity gains in production. The distribution of rent differs in a dispersed city but there will still be peaks and valleys in land rent, peaks indicating places of "extra" productivity. The statistical efficiencies in the labor market, gleanable by locating firm i in a large city compared with a small one, hold at the urban area level, not at the CBD level. Thus modern large cities as collections of towns rather than as monocentric monster areas make much economic sense even if no activity has pervasive declining average costs. Even the gains to the firm in a large city from labor market statistical economies should in part be capitalized in higher land rents around the firms to the extent the employees live near their work place.

The large monocentric city both symbolized the modern industrial era and can be said to be a particular manifestation of the technological changes characteristic of economic life after the industrial revolution. From this perspective, we should not be surprised that the monocentric urban form was in fact a sink for the productivity gains entailed in modern industrial development. Productivity gains were in large part capitalized in urban land prices.

Appendix: Cobb-Douglas Utility and the Capitalization Condition

Each individual has utility $\bar{u} = h^{\alpha}c^{1-\alpha}$. h is actually residential land and sells for l(x) per unit. c has price, unity. Consumer equilibrium $\partial U_h/\partial U_c = 1$ yields

$$c = \gamma r h \tag{A1}$$

where $\gamma \equiv (1-\alpha)/\alpha$. From the budget constraint $w-t(x) = lh+c$, where w is income or the wage and t(x) is roundtrip commuting cost. Using (A1), we obtain

$$lh = \alpha[w-t(x)] \tag{A2}$$

Substitute for c from (A1) in $\bar{u} = h^{\alpha}c^{1-\alpha}$ and for lh from (A2). One obtains

$$l(x) = A[w-t(x)]^{\beta} \tag{A3}$$

where $\beta \equiv 1/\alpha$ and $Z \equiv \{[\alpha\gamma^{1-\alpha}]/\bar{u}\}^{\beta}$. Substitute l(x) from (A3) in (A2) and observe

$$h(x) = [\alpha/Z]\ [w-t(x)]^{-\gamma} \tag{A4}$$

Now $\bar{x}$ is defined as $l(\bar{x}) = 0$ in (A3). Then total residents are

$N(\bar{x}) = \int_0^{\bar{x}} [2\pi x/h(x)]dx = 2\pi\int_0^{\bar{x}} [Zx/\alpha]\ [w-t(x)]^{\gamma}dx$ and residential land rent is

$R(\bar{x}) = \int_0^{\bar{x}} [2\pi x l(x)]dx = 2\pi\int_0^{\bar{x}} Zx[w-t(x)]^{\beta}dx$. Inspection reveals $dR(\bar{x})/dw$ equals $N(\bar{x})$ which is an example of our central result in the text in establishing capitalization of wage increments. ■

References

Brown, Charles and James Medoff [1989], "The Employer Size-Wage Effect", Journal of Political Economy, 97, 5, October, pp. 1027-1059.

Hartwick, J.M., U. Schweizer, and P. Varaiya [1976], "Comparative Statics of a Residential Economy with Several Classes", Journal of Economic Theory, December.

Henderson, V. [1986], "Efficiency of Resource Usage and City Size", Journal of Urban Economics, 19, January, pp. 47-70.

Hoch, I. and J. Drake [1974], "Wages, Climate and the Quality of Life", Journal of Environmental Economics and Management, 1, pp. 368-395.

Jacobs, Jane [1969], The Economy of Cities, New York: Random House.

Krugman, P. [1991], Geography and Trade, Cambridge, Massachusetts: MIT Press.

Lucas, R.E. [1988], "On the Mechanics of Economic Development", Journal of Monetary Economics, 22.

Mills, E.S. and B.W. Hamilton [1989], Urban Economics, Fourth Edition, Glenview, Illinois: Scott, Foresman.

Miyao, T. [1977], "A Long-run Analysis of Urban Growth Over Space", Canadian Journal of Economics, 10, 4, November, pp. 678-685.

Rauch, J.E. [1991], "Productivity Gains from Geographic Concentration of Human Capital: Evidence from the Cities", NBER Working Paper No. 3905.

Roback, J. [1982], "Wages, Rents, and the Quality of Life", Journal of Political Economy, 90, December, pp. 1257-1278.

Rosen, S. [1979], "Wage-Based Indexes of Urban Quality of Life", in P. Mieszkowski and M. Straszheim, eds., Current Issues in Urban Economics, Baltimore: Johns Hopkins Press.

Starrett, D.A. [1981], "Land Value Capitalization in Local Public Finance", Journal of Political Economy, 89, 2 pp. 306-327.

Wheaton, W. [1974], "A Comparative Static Analysis of Urban Spatial Structure", Journal of Economic Theory, 9, pp. 223-237.

Wildasin, D.E. [1986], Urban Public Finance, Chur Switzerland: Harwood Academic.