The Existence of Equilibrium and the Objective Function of the Firm

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Abstract We consider an economy in which firms' decisions are
made by a collective decision of the shareholders. The main
result shows that the simultaneous existence of an exchange
equilibrium in the market for shares and a voting equilibrium in
the internal decisions of firms. We present our results in a
general framework, with a measure space of agents. Our framework
covers the cases of incomplete markets and externalities between
firms and shareholders. We show that a voting rule due to
Kramer is a special case.

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Keywords Shareholder voting, incomplete markets, sophisticated
voting, objective function of the firm.

JEL Classification D52, D70, L20.
1 INTRODUCTION

This paper proves the existence of equilibrium when firms' production decisions are made by a collective choice of shareholders. In addition to the usual simultaneity problems in general equilibrium, the attractiveness of shares to individuals and hence the demand for shares depends upon production decisions, while firms' production decisions depend upon the outcome of shareholder voting, which is in turn determined by the demand for shares. Thus we need to look for a simultaneous exchange equilibrium in markets and a voting equilibrium in firms' internal decisions.

1.1 Background

When markets are complete and economic agents are price-takers, conflict between the interests of shareholders cannot arise. The Fisher Separation Theorem (see for instance Milne (1974)) shows that all shareholders will desire the firm to maximise its value with respect to any given contingent price system. With a complete set of contingent markets, individuals can allocate consumption across states of nature in any desired way. Hence the only effect of changes in the firm's production decision is to induce parallel shifts in shareholders' budget constraints. Naturally all shareholders desire their budget sets to be as large as possible, hence no disagreement arises.

This argument does not hold in general when markets are incomplete. A change in the firm's production plan causes shareholders' wealth to be reallocated between states of nature. In the absence of a complete set of insurance markets this would force individuals to reallocate their consumption across states of nature. If different shareholders have different subjective probabilities or different risk attitudes then they would have different preferences over production plans. In other words,
REFERENCES


production plan which the first individual regards as inferior to the status quo.

Notes on the text
1. A complete measure space satisfies the property that every subset of a set of measure zero is measurable.

1.2 Brief Description of Model

The model has one physical commodity. There are two time periods. In the second period there are $S$ possible states of nature. Firms can produce $S$ contingent commodities. The firm has to commit itself to a production plan in period 0. The firm's production plan is constrained to lie in a production possibilities set which is assumed to be compact and convex. We shall model the firm's decision by an abstract set of preferences. These preferences are not assumed to be complete or transitive but merely to satisfy weak continuity and convexity assumptions. Individuals have fixed endowments of shares in the firms and the consumption good, which is not storable. These they may exchange in a market in the first period. In the second period the true state of nature is revealed. Firms produce according to their previously decided production plans and allocate their output to individuals in proportion to their shareholdings. There is no trade in the second period.

We shall look for an equilibrium in the following sense. Firstly there is a competitive equilibrium in the market for shares and first period consumption. Each consumer maximises his or her utility given the prices prevailing and there is no excess demand, both for goods and shares. Secondly no firm has a production plan, which is preferred by the firm's collective choice rule to the status quo.

One might envisage this equilibrium could arise as result of the following kind of process. There is an auctioneer who announces prices. People then respond by stating demands at those prices. Assuming that everybody has their desired holdings of shares, a provisional vote is conducted on the production plans of the firms. (Note that the sum of desired shareholdings over all individuals may differ from the actual
This process could be implemented to test the majority of the negotiation. In this case, the pattern of shares ownership might change. In this setup, there would be a new majority among the participants, and the pattern could be adapted to the new owners, with a new pattern of shares ownership. This process could be repeated, with new patterns of shares ownership, until the desired result is achieved.

The second approach is also difficult to apply to the second layer. This approach is difficult to apply to the second layer because the patterns of shares ownership are very strong.

However, there is not sufficient evidence to conclude that these approaches could potentially be applied to the shareholders for further study of these phenomena.

In order to test the effectiveness of the patterns of shares ownership, we plan future research to investigate this. We cannot test the patterns of shares ownership to test the effectiveness of the patterns of shares ownership in order to test the effectiveness of the patterns of shares ownership. We assume the patterns of shares ownership to test the effectiveness of the patterns of shares ownership.

After providing our general framework, we will present some results of our experiments. We will present some results of our experiments in the following sections. We will present some results of our experiments in the following sections. We will present some results of our experiments in the following sections.
price-taking behaviour, honest voting and the assumption that individuals take production decisions as given when deciding share purchases, are best justified when each individual has measure zero. The previous literature has almost exclusively focused on incomplete markets, while we consider both externalities and incomplete markets. In addition we prove existence with a large class of procedures for the firm's internal decisions while previously only specific rules have been considered. A general model is advantageous, since we have little detailed analysis of firms' internal decisions.

4.2 Possible Extensions

It would be possible to extend our model to an economy with many goods along the lines of Radner (1972), who proceeds by placing bounds on the trades of agents. The disadvantage of this is that the bounds are arbitrary and the equilibrium which results may depend on them. If no such bounds are imposed it is not possible to prove existence for all economies since the budget correspondence may fail to be continuous (see Hart (1975)). It is likely, however, that existence could be proved for the generic case, using differential topology, as in Duffie and Shafer (1985).

An advantage of including more goods in the model is that it would be possible to consider decision rules for the firm which were based on other economic variables in addition to shareholdings. In particular if our model was extended to identify labour as a separate good it would be possible to include labour managed firms. (Dreze (1989) considers firms which are jointly managed by workers and shareholders.) If we model the firm's decisions by majority voting constrained by a veto to "the board of directors", in a multi-good model it would be possible to give board membership to those who supply

Unless a good argument can be found to the contrary it does not seem desirable to impose such restrictions in the shareholder's problem. An example of this approach can be found in Caplin and Nalebuff (1991), who show that an equilibrium will exist under 64% majority rule provided certain restrictions are imposed, both on the nature of preferences and on their distribution. So far we have not been able to apply this result to decision-making within firms. Caplin and Nalebuff require all voters to have the same utility function, apart from a term which is linear in a characteristic of the individual. However a shareholder's preferences will depend on the size of his or her shareholding. This relation is unlikely to be linear except in special cases. We hope in future research to be able to relax these restrictions, so that this voting system may be applied to firms' decisions. The other possibilities may all be applied to the shareholder problem. We shall consider each of them in turn.

Kramer (1972) has demonstrated the existence of equilibrium in a voting game, in which a vector in \( \mathbb{R}^n \) is determined by voting on each coordinate in turn. This is in effect putting a restriction on the directions in which changes may be made. He shows that provided preferences are additively separable, the game so defined has an iterative dominance equilibrium. In section 3, we show that if firms' decisions are made by Kramer's voting rule then a simultaneous voting and exchange equilibrium will exist. We also demonstrate the existence of equilibrium when a subset of the shareholders have vetoes.

It should be emphasised that while the examples we use are mainly concerned with shareholder voting, there is no need in our general model to restrict attention to voting, instead an abstract collective choice rule or mechanism could be used. The participants in such a mechanism need not be restricted to
This is important since many of the typical assumptions such as continuity and well-defined derivative of the primitive function apply to the following results. We extend these and other results to the exchange and exchange-guarantee, the exchange and exchange-guarantee, we extend these and other results to the exchange and exchange-guarantee, the exchange and exchange-guarantee, we extend these and other results to the exchange and exchange-guarantee.

In the present paper, we prove the existence of a second fundamental theorem. In the present paper, we prove the existence of a second fundamental theorem. In the present paper, we prove the existence of a second fundamental theorem.

In section 2 of the present paper, we present a general model of an exchange-guarantee, which includes both individual and aggregate decisions. In section 2 of the present paper, we present a general model of an exchange-guarantee, which includes both individual and aggregate decisions. In section 2 of the present paper, we present a general model of an exchange-guarantee, which includes both individual and aggregate decisions.

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Definition 3.5 A Kramer voting and exchange equilibrium is a state $z$ of the economy such that no firm wishes to change its production plan in an allowable direction,

$$P^*_s(z) \cap \beta^*_s(z) = \emptyset, \ 1 \leq s \leq S, \ 1 \leq f \leq F$$

all consumers are maximising their utility subject to their budget constraints,

$$\int 0^{x^*_f} da \leq 1 \text{ for } 0 \leq f \leq F, \text{ and } \int x^* da \leq \int x^* da.$$

Corollary 3.2 A Kramer voting equilibrium exists.

Proof This can be shown by adapting Theorem 2.1 along the lines indicated in the proof of Theorem 2.2 and using Propositions 3.2 and 3.3.

It is possible to generalise the analysis of this subsection since it would be possible to define a similar voting rule for any other (not necessarily linear) coordinate system for $\mathbb{R}^n$. However, it would be necessary to ensure that preferences are quasi-concave in each coordinate.

4 CONCLUSION

4.1 Related Literature

Models of shareholder voting in general equilibrium have been studied previously by Dreze (1985), (1989), De Marzo (1990) and Sadanand and Williamson (1991).

De Marzo (1990) has shown that in an equilibrium the firm's decisions will coincide with the preferences of the dominant shareholder, the individual with the largest shareholding. He does not prove existence of equilibrium. If, equilibrium does not typically exist the dominant shareholder property may fail to be robust. De Marzo's proof relies upon Plott's (1967) symmetry conditions which imply that voters' preferences are highly restricted.

2. EQUILIBRIUM IN AN ABSTRACT ECONOMY

In this section we present a general model, which allows simultaneously for the possibility of incomplete markets and externalities between firms and individuals. We shall model firms' decisions as being made by an abstract set of preferences. In section 3 we shall give examples of decision rules for firms, which fit this framework.

Market Structure There are two time periods $t = 0, 1$. There are $S$ states of nature $1 \leq s \leq S$. In period 0, the state of nature is unknown. The true state is revealed before the beginning of period 1. There is one physical commodity which is non-storable. There are $F$ firms $1 \leq f \leq F$. At time $t = 0$ there are markets in the physical commodity and the shares of the firms. Let $q^*_f, 1 \leq f \leq F$ denote the price of shares in firm $f$ and $q^*$ the price of the physical commodity. Let $q$ denote the vector $(q^*_1, q^*_2, \ldots, q^*_F) \in \mathbb{R}^{F-1}$. We shall normalise prices so that they lie in the unit simplex $\Delta \subseteq \mathbb{R}^{F-1}$.

Consumers There is a complete finite measure space $(\Lambda, \Omega, \mu)$ of consumers. We shall denote a generic consumer by $a \in \Lambda$. It is assumed that $L^1(\mu)$ is separable. Note that this includes the possibilities of a discrete set of consumers, a continuum of atomless consumers or some combination of the two. This gives useful generality when modelling the stock market, since one could think of private investors as a continuum of agents while institutional investors could be modelled as point masses.

Individual $a$ is assumed to have a utility function of the form $u(x^*_a, x^{y}_a, \ldots, x^{y}_a, y_1, \ldots, y_F) = u(x^*_a, y)$, wherever $x^*_a$ (resp. $x^*_a$) denotes the consumption of individual $a$ in period 0 (resp. period 1 state $s$), $x^*$ denotes the vector $(x^{y}_a, \ldots, x^{y}_a)$, $y_1$ denotes the
Proposition 3.3. The graph $\text{graph} \, f$ is an open set.

Remark. The conditions given in subsection 3.2 are sufficient for $f$ to be strictly quasi-convex in $\mathbb{R}^n$. The result follows.

Contradiction. The result follows.

There is a compact convex subset $Y$ of $\mathbb{R}^n$, definitively $\mathbb{R}^n$, the vector $\mathbf{y}$ is constrained to lie in a compact set, $\mathbb{R}^k$, with $k \leq n$. Each time $t$, the sum is a constant.

There are $\langle d, g \rangle \in \mathbb{R}^n$, $\langle d, g \rangle \in \mathbb{R}^n$, and $\langle d, g \rangle \in \mathbb{R}^n$, hence the function $\langle d, g \rangle$ is a constant. Consider an open and dense set $I$, the function $\langle d, g \rangle$ is not measurable.

Inductively, $\mathbb{R}^n$'s budget is constant. Sufficiently large $\mathbb{R}^n$ is non-empty for every $\langle d, g \rangle \in \mathbb{R}^n$, where $0 < u \leq r$ and $x$ is some $\mathbb{R}^n$'s interior, is constrained to lie in the interior of the set $\mathbb{R}^n$. If $\langle d, g \rangle$ is assumed to be measurable, both components are measurable. Therefore, $\mathbb{R}^n$'s boundary is measurable. Inductively, $\mathbb{R}^n$ is assumed to have endomorphisms $X'$ and $X''$.

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\( P^*_f(z) - (y^* \in \beta^*_f(y^*): v(\Pi(y^*_f), z) > V_0) \) where \( v \) denotes the measure on \( \Lambda \) defined by \( v(B) = \int_B \theta^* d\alpha/\int_{\Lambda} \theta^* d\alpha \).

**Definition 2.4** A state \( z \) of the economy is a Kramer voting equilibrium for firm \( f \) if \( P^*_f(z) \cap \beta^*_f(z) = \emptyset \), for \( 1 \leq s \leq S \).

The Kramer voting rule is not defined when almost all individuals have zero demand for the shares. As a first step we can ensure this does not occur by placing lower bounds on shareholdings in the sets of allowable shareholdings \( \Theta^* \). This assumption could be dispensed with by considering a sequence of economies such that this restriction does not hold in the limit. (For details of this argument see Buzez (1989) p. 129).

**Proposition 3.2** Assume that all individuals induced preferences over \( y^*_f \) are strictly quasi-concave. If \( z = \langle x, y, \theta, q \rangle \) then \( y^*_f \notin \text{co}(P^*_f(z)) \).

**Proof** By continuity and strict quasi-concavity of the utility function each individual \( s \) has a unique optimal production plan \( y^* \in \beta^*_s(y^*) \). By definition of \( \beta^*_s(y^*) \) we may write

\[ y^* = y^* + \zeta^* e_s + \xi^* e_o \]

where \( e_s \) (resp. \( e_o \)) denotes the unit vector in direction \( s \) (resp. \( o \)).

Suppose, if possible, that the result is false. Then there exist \( y^* = y^* + \zeta^* e_s + \xi^* e_o \) for \( j = 0, 1 \). Without loss of generality we may assume that \( \zeta^*_t < 0, \xi^*_t > 0 \).

**Definition 2.1** We define a state of the economy to be a four-tuple \( z = \langle x, y, \theta, q \rangle \), where \( x \in X, y \in Y, \theta \in \Theta, q \in Q \).

Note, the term "state" is being used in two distinct ways, to denote a state of nature and to denote a state of the economy. This should not cause any confusion.

**Definition 2.2** A Simultaneous Voting and Exchange Equilibrium is a state of the economy \( z^* = \langle x^*, y^*, \theta^*, q^* \rangle \) such that,

\[ x^* \notin P^*_f(z^*) \]

for \( 1 \leq f \leq F \), for almost all \( \omega \in \Lambda, x^*, \theta^* \)

maximises \( u^* \) subject to individual \( s \)'s budget constraint, \( \int \theta^* d\alpha \leq 1 \) for \( 1 \leq f \leq F \), and \( \int x^* d\alpha \leq \int w^* d\alpha \).

To prove the existence of equilibrium we shall use the following result on the existence of equilibrium in an abstract economy, which is proved in Kahn-Vohra (1984).

**Theorem 2.1** (Kahn-Vohra) Let an abstract economy satisfy the following assumptions.

1. \( \langle W, \Omega, \theta \rangle \) is a complete finite measure space such that \( L^1(\omega) \)

is separable.

2. \( X \) is an integrably bounded measurable correspondence such that for all \( \omega \in W \), \( X(\omega) \) is nonempty, convex and compact.

3. \( B \) is a correspondence such that

a. for all \( x \) in \( L^1(\mu, \chi(\cdot)) \), the graph of \( B(\cdot, x) \) belongs to \( \Omega \times \beta^*(W) \).

b. for all \( \omega \in W \) and for all \( x \) in \( L^1(\mu, \chi(\cdot)) \), \( B(\omega, x) \) is a nonempty closed and convex subset of \( X(\omega) \).

\[ 16 \]

9
Theorem of the correspondence.

Each of these points is given measure \( \lambda \) by the Kuhn-Yoneda.

Let \( \varphi \) be the measure space derived from \( (\mathcal{A}', \mathcal{B}', \mathcal{C}' \times \mathcal{D}') \).

Let \( \pi \) be the projection space corresponding to the auctioneer and the firm.

Then there exists a Nash equilibrium.2

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By adding the property of agent 0, the preferences of agent 0, etc.

Each of these points is given measure \( \lambda \) by the Kuhn-Yoneda.
individuals with positive shareholdings. This criterion has been studied previously by Dreze (1974) and Grossman and Hart (1979).

3.1.2 Sufficient Conditions for Strict Quasi-Concavity

The above analysis relies upon the assumption that at least one of the individuals with a veto has preferences which are strictly quasi-concave in $y'$. If there are no externalities between firms and individuals this would be implied by strict quasi-concavity of the utility function. In the presence of externalities induced preferences will be strictly quasi-concave in $y'$ provided that the utility function takes the additively separable form $u^*(x^*, y) = v^*(x^*) + w^*(y)$, where $v^*$ and $w^*$ are strictly concave.

3.2 The Kramer Voting Rule

The Kramer voting rule enables a group of individuals to choose a vector in $\mathbb{R}^r$ by voting over each component in turn. The outcome of the Kramer voting rule is independent of the order in which decisions are taken, however, it does depend upon the choice of bases for the coordinate system of $\mathbb{R}^r$.

Sadanand and Williamson (1991) have analysed the existence of equilibrium when firm's decisions are made by this rule. They assume that at each stage in the voting process the output of one firm in one state of nature is determined. We shall continue to make this assumption. One implication of this is that it is not possible to make coordinated changes in the production plans of more than one firm. This seems reasonable and is in keeping with the spirit of non-co-operative models of firms. A second implication of this assumption is that it is not possible for a single firm to simultaneously introduce changes in more than one state of nature. This implication seems less easy to defend and possible that outputs in some states are negative, then extra restrictions must be placed on the choice of production plan to ensure that no shareholder is bankrupt in any state of nature. The correspondence $B(f, \cdot)$ will still be closed and convex with these restrictions. For details see Sadanand and Williamson (1991) p.14.

The Kahn-Vohra theorem implies that in state $z^*$, all firms are maximising their preferences and almost all individuals are maximising utility. It is now sufficient to prove that excess demand is non-positive. Since individuals' utility is strictly increasing in first period consumption its price will be strictly positive. Therefore all individuals' budget constraints will hold with equality. Substituting from the budget constraint into the auctioneer's preferences we obtain,

$$P^0(z) = \{q' \in \Delta : \sum_{x} q' \left[ t(x^* - x^*_a) \right] da + \sum_{x} q' \left[ \int_{0}^{t} da - 1 \right] > 0 \}. $$

Therefore $P^0(x) \cap \Delta - \emptyset$ implies that $\int_{0}^{t} da \leq 1, \ 1 \leq f \leq F$. The result follows.

3 THE FIRMS' DECISION RULE

In this section we give some examples of decision rules for the firm which satisfy the assumptions of our general model.

3.1 Veto Based Decision Procedures

Suppose that there is a group of individuals $G^f(z)$ who have veto power over the decisions of firm $f$. A veto implies that the production plan of firm $f$ cannot be changed if any individual $g \in G^f(z)$ objects.
Proposition 3.1 was previously proved by Breze (1963). By Proposition 3.1, we have a certain extension of the concept of production plans and the set of production plans. In other words, if a certain extension of the concept of production plans and the set of production plans is assumed, then the set of production plans is an extension of the concept of production plans and the set of production plans.

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Theorem 3.1 is a corollary of Theorem 2.1 and Proposition 3.1. We assume that the graph of a certain extension of the concept of production plans and the set of production plans is an extension of the concept of production plans and the set of production plans. In other words, if a certain extension of the concept of production plans and the set of production plans is assumed, then the set of production plans is an extension of the concept of production plans and the set of production plans.

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