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EFFICIENT RESOURCE ALLOCATION IN A MULTINUCLEATED CITY

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DISCUSSION PAPER NO. 86

Abstract

Efficient Resource Allocation in a Multinucleated City with Intermediate Goods

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Mills' programming model of efficient resource allocation in an urban area is extended to include multiple nuclei and intermediate goods. The new model is a decomposable linear program in which the primary resource value of the city plus transportation costs are minimized subject to a prescribed bill of final demands for the city and to land constraints consisting of cells in a areal grid. Shadow prices on land emerge in the dual problem.

The second large class of spatial models being developed are those developed from estimating parameters in equations plausibly characterizing urban processes. Lowry's 1964 model was one of the early examples of models in this stream of analysis and recent developments are taking place at such places as the National Bureau of Economic Research.[2] These models are designed to be planning instruments to aid decision-makers in allocating resources within cities to improve some notion of urban quality.

> "Such models derive the urban spatial patterns from conditions of supply and demand, profit and utility maximization, and the market equilibrium. Most models involve simultaneous relationships and significant non-linearities. As with the planning models, numerical solution is often difficult and expensive. Whereas the optimization models have no market interpretation, the market oriented models contain no optimality criteria. Most are either too complex or inadequately articulated to permit determination of whether market solutions are efficient and, if not, what kind of intervention would be best to ensure efficiency. The situation is paradoxical because most of the market oriented models are formulated to guide public sector planning for future urban growth." (Mills (1972); p. 2).

Mills defined a model which was designed to bridge the gap between these two classes of models. Thus his model replicates resource allocation in a spatial context and urban area, and has an explicit notion of efficiency in the sense of welfare economics. In this paper we outline a reformulation of the model presented by Mills and we extend Mills' model in such a way that the urban area being simulated could have multiple foci or nuclei. Our reformulation contains certain similarities in conception between the inter-regional model of Moses (1960) and a model of allocation within a city.

Mills examines an urban area as being composed of a grid of squares, each square of a fixed equal area. At the arbitrarily specified origin of the grid an export facility exists through which all production in the city except housing and transportation is exported to meet exogenously given demands. The particular nature of the central square causes an essentially circular set of identical squares to be generated in annuluses around the focus. Such a result is similar to that in Mills' theoretical model (1967).

In this reformulation, we allow for multiple export (or import) nodes in contrast to Mills' single node formulation. This extension will permit one to simulate cities by market mechanisms or with linear programming which have multiple nodes and multiply peaked land rent functions. The key to this reformulation is the treatment of each square in the land grid as a member of five square contiguous group. The demands which transportation makes on land are determined for any square by the flows of commodities crossing each of the four sides of the square in question, to or from its four neighbours. Each square in this reformulation has the potential for being uniquely treated by the market forces in an equilibrium, whereas in Mills' formulation a family of squares of equal distance from the unique export node received like treatment from the market forces in equilibrium. In this reformulation intermediate goods are presented whereas in Mills' formulation they were not.

The linearity of the model is preserved and so, like Mills, we can simulate cities with the aid of a linear program. The reformulation differs from Moses' interregional model in that we treat regions as squares, have transportation and production as land using, and treat inter-square flow relationships in the five-square grouping arrangement outlined above. In addition, like Mills, we allow for substitutability of production techniques and for congestion costs.

Where possible we use the same symbols for the analogous variables employed by Mills.

2. The Geography of the Urban Area.

The space which our hypothetical city occupies before the erection of buildings and roads etc. consists of an unbounded almost homogeneous plane divided into a set of squares of equal area. For convenience we say that the size of the square is 1 urban unit; it could be a square mile or a square yard or what have we. In reality the choice of units in which to measure the squares would depend on the capacity of the computer to handle the particular problem in hand. All squares are homogeneous in the sense that the technology available for production and the transportation within any square is identical before a city has been erected. However, a certain subset of the available squares will be indicated *a priori* as transportation nodes. A transportation node has the property that imports or exports to or from the city as a unit move from that or to that node.

It does not matter how many transportation nodes we have within a system as long as they form a subset of the total number of squares available for the city to be developed on. Note that we consider the landscape to be unbounded so that there is not the need to pack activities into a finite area because of the scarcity of land in general as in Forrester's urban model. Land is only scarce in the sense that activities require to be near transportation nodes; near in the sense to be defined precisely in the general equilibrium model of the city.

The allocation problem is to organize production within the city on the landscape in order to meet given exogenous demands for physical quantities of commodities potentially producible in the city imposed on the city from outside the particular city. These demands will be separated into transportation - node specific subsets of demands by the city-wide optimization process. More microscopically, the allocation problem consists of dividing land within a square between use for the transportation of commodities within the city and use for the production of various commodities within the city. It will, of course, be assumed that the transportation of all commodities including workers requires land for transportation and also the production of all commodities requires land in order to have spaces for the buildings and activities taking place within the city. The solution to our allocation problem does not yield a particular location-allocation of space within squares but it simply indicates the amount of each square to be occupied by transportation and production. A sub-allocation problem, not considered in this model, would be to allocate space within squares to transportation and to production.[3]

It does not matter how we number our squares. That is the origin in our grid is arbitrary. In fact it suffices to analyze only one representative square and its four immediate neighbours in order to see the analytical characteristics of the model. We shall label this one representative square, square ij. That is the square is i units from the vertical axis and j units above the horizontal axis. This is illustrated in Figure 2.1 below.

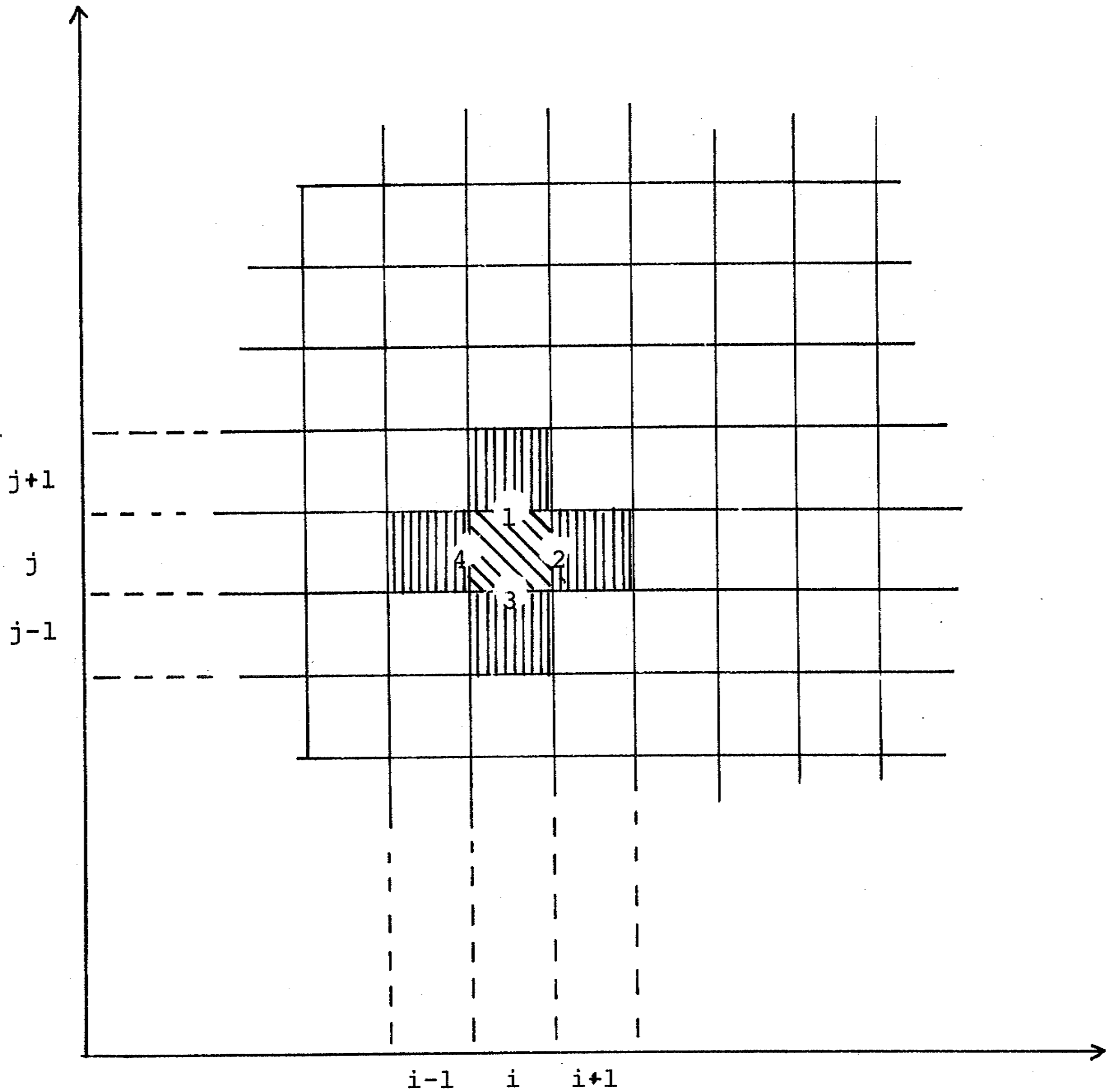

Figure 2.1

In treating the transportation of commodities, we must know the volume of flow crossing each of the four sides of a square. We shall refer to the sides of square i,j as 1, 2, 3 and 4 corresponding to the north, east, south, west directions in Figure 2.1 respectively.

Production in the urban area is devoted to the production of $\bar{r}$ goods of which the $1,2,\ldots,\bar{r}-1$ commodities are available for export and all $\bar{r}$ as inputs into the production processes of one another and of housing, the $\bar{r}^{th}$ production activity. Transportation within the urban area is also produced internally, and could be a consumer of goods $1,2,\ldots,\bar{r}-1$; however we have not formulated it in that manner.

There is an external demand for quantities $\bar{x}_r$, of the export goods $r=1,2,\ldots,\bar{r}-1$.

Following Mills, we define a set of $\bar{s}$ production activities $1,2,\ldots,\bar{s}$ as being the number of stories in the building employed in the production of a commodity. We specify a set of input-output coefficients a_{qrs} to be the amount of input q required per unit production of good r by production activity s where:-

$$q = \left| \begin{array}{ll} \left.\begin{array}{l} 1 \\ 2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{r}-1 \end{array}\right\} & \text{goods 1 to } \bar{r}-1 \\ \bar{r} & \text{- labour (labour as an input in production can be equated to housing} \\ \bar{r}+1 & \text{- land} \quad \text{production; see discussion below)} \\ \bar{r}+2 & \text{- capital} \end{array} \right.$$

Markets of labour and capital are competitive hence demanders in the urban area for these commodities face fixed prices of w and R respectively. In equilibrium, the price of land will vary throughout the city. R_A is the opportunity cost or the price of a unit when agriculture is worked on it. Household consumption of the goods 1,2,..., $\bar{r}$-1 is included in the inputs into housing production.

Since we will assume that each unit of labour requires one unit of some type of housing, then we can regard the labour input in production, as an output of housing. It is possible to extend the model by introducing different classes of workers, by having workers distinguished by different wage rates and housing preferences. Adjusting the model along this line is simple and would provide an effective way of making social interpretations of the market allocation of housing type. If one wishes to identify certain classes of workers with a particular housing type then consumption patterns can also be associated with them, in the form of inputs in housing production. However, we assume that all workers are identical, hence inputs into housing production in this case are to be interpreted only as requirements in the physical production process.

The housing/labour commodity differs from any other commodity in the model in the respect that there can be no net export or import from or to the city of this commodity in equilibrium. Actually there would be no difficulty in having part of the cities labour force commute to and from the city through transport nodes in this model but we shall not incorporate this extension.

3. Flow Equilibrium for Square ij.

We have two distinct cases for flow equilibrium for square ij; first is when square ij is not a transportation node and second is when square ij is a transportation node. In the first case for commodity r we have the following sets of flows: i) there are gross flows to square ij potentially from the four adjacent squares, ii) there is net output of commodity r in square ij after intermediate uses within square ij have been deducted, and iii) there is the export flow of commodity r from square ij to the four adjacent squares. A square can be a transshipment point if, for commodity r, gross flows in equal gross flows out. These various sets of flows are indicated in equation 3.1.

$$(3.1)\quad T_r^1\ (i,\ j-1) + T_r^2\ (i-1,\ j) + T_r^3\ (i,\ j+1) + T_r^4\ (i+1,\ j) + \sum_s x_{rs}\ (i,j)$$

$$-\sum_{q=1}^{\bar{r}} \sum_s a_{rqs}\ x_{qs}\ (i,j) - \sum_d^4 T_r^d\ (i,j) = 0$$

$$(r=1,\ldots,\bar{r})$$

where the first four terms indicate flows of commodity r to square i,j from each of the four contiguous squares (super scripts indicate the appropriate boundary of squares and location indices refer to square of origin) the fifth term is gross output of commodity r in square i,j, the sixth term is total intermediate use of r by other activities in square i, j and the seventh term indicates total flows of commodity r from square i,j (all of which must go to a contiguous square, possibly for trans-shipment of course).

T_r^3 (i,j + 1) is the flow of commodity r from the square to the north of i,j to square i,j.

x_{rs} (i,j) is the gross output of commodity r in an s storey building in square i,j. Included in x_{rs} (i,j) are exports to other sources generated within the square (i,j) plus exports to the outside of the city when square (i,j) is a node.

T_r^d (i,j) is the flow of commodity r from square i,j to the contiguous square across the d th boundary or side of square i,j.

If square i,j is a transportation node then equation (3.1) must be adjusted by adding x_r (i,j) to the left hand side as an eighth term. x_r (i,j) is then the flow of commodity r from node i,j to the world outside of the city. As we mentioned above x_r (i,j) is a variable to be determined in the optimization. It could _a priori_ be assigned a negative sign if the city is a net importer of commodity r.

As we noted above we designated a priori certain squares within the city to be transportation nodes where commodities flow into or out of the city. In the objective function we assign relative values to the various flows meeting at any particular node. These relative values can be looked upon as indices of the cost of transporting the particular commodity from (or to) the node to (or from) the outside of the city. Different commodities then have different relative attractivenesses to the various nodes in the city depending on the location of the node and the nature of the transport facility. A node could be for example either a railhead, a water shipping head, a road network head, a teletype exchange, or some combination of these facilities. City wide, we must have a balance equation which indicates that of all nodes either importing or exporting commodity r the overall imports or exports meet the given exogenous demands on the city.

We noted that the flows of commodities choose their particular nodes according to the prices of exporting or importing the commodity at the particular node in question. Let ℓ be the set of all squares which are transportation nodes. Then for commodity r

$$\sum_{i}\sum_{j} x_r(i,j) = \bar{x}_r \qquad (r=1,\ldots,\bar{r}-1) \tag{3.2}$$

$$(i,j)\,\varepsilon\,\ell$$

or the sum of exports (imports) from (to) all transportation nodes must equal the exogenous requirements $\bar{x}_r$ imposed on the city.

4. Transportation Costs and Land Requirements for square (i,j)

Transportation costs take two distinct forms in the objective function, we have the flow crossing the boundary of a square to be composed of an element varying non-linearly (in general in an increasing way) with the volume of flow. The non-linearity is introduced to permit congestion costs to be operative in the model. Finally, flows across the boundaries of a square are land and capital using.

The non-linear component of transportation costs is developed from an exogenously given cost step-function designed to approximate a smooth non-linear function. In Figure 4.1 we have one such step-function.

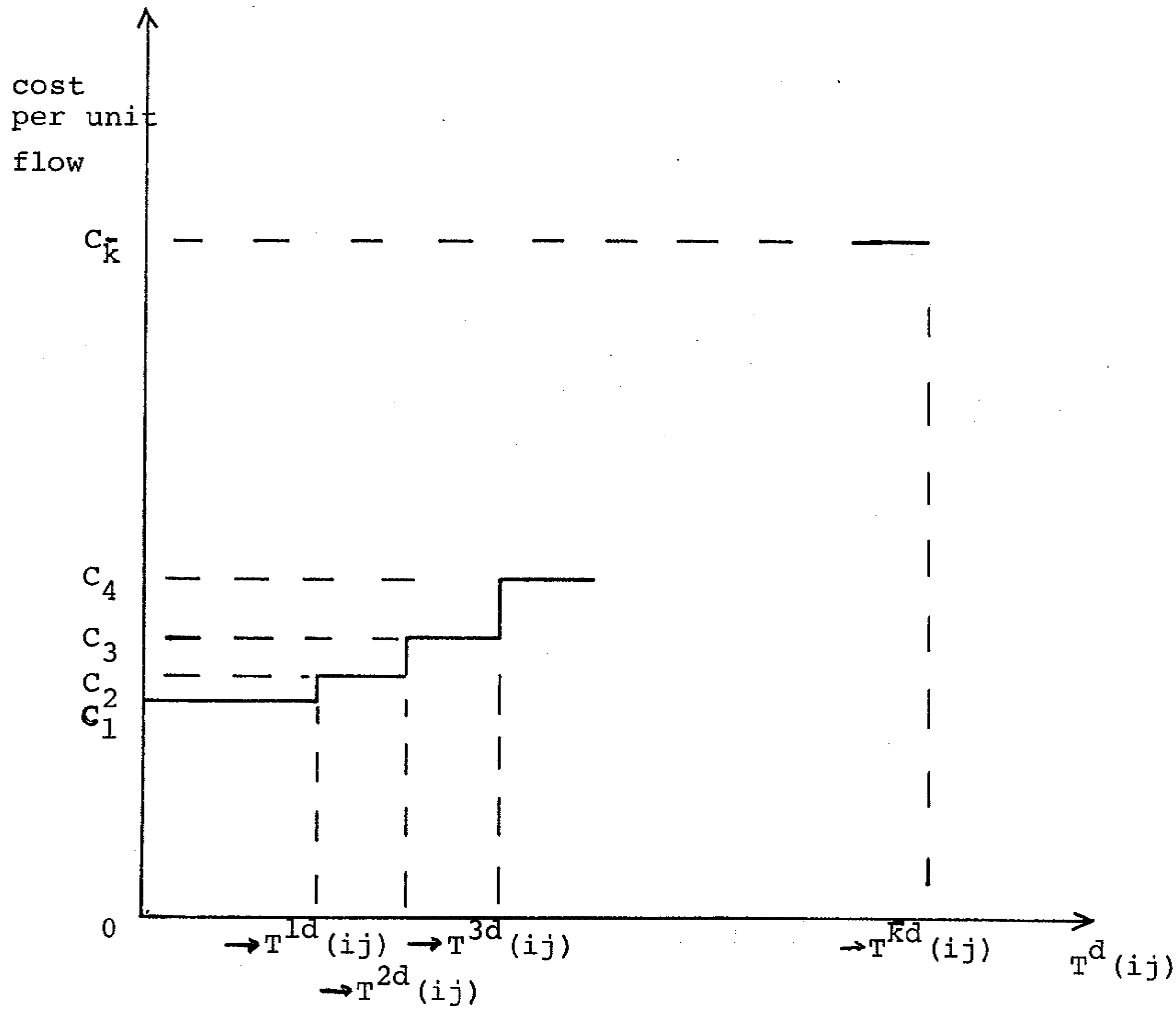
cost
per unit
flow
$C_{\bar{k}}$
C_4
C_3
C_2
C_1
0
$\rightarrow T^{1d}(ij)$
$\rightarrow T^{3d}(ij)$
$\rightarrow T^{2d}(ij)$
$\rightarrow T^{kd}(ij)$
$T^{d}(ij)$

Figure 4.1

In Figure 4.1, we note the aggregated flows crossing a boundary of square i,j, namely T^{d} (i,j) is decomposed into a set of flows moving at various "congestion levels"; for example T^{kd} (i,j) flowing at cost C_k per unit flow.[4]

The aggregate flow for a single boundary is developed in the following constraint:

$$\sum_k T^{k1}(i,j) - \sum_r t_r \left(T_r^1(i,j) + T_r^3(i,j+1)\right) \geq 0$$

where t_r is a coefficient relating flows of commodity r to aggregate flows over a boundary. For the three other boundaries, we have analogously:

$$\sum_k T^{k2}(i,j) - \sum_r t_r \left(T_r^2(i,j) + T_r^4(i+1,j)\right) \geq 0$$

$$\sum_k T^{k3}(i,j) - \sum_r t_r \left(T_r^3(i,j) + T_r^1(i,j-1)\right) \geq 0$$

$$\sum_k T^{k4}(i,j) - \sum_r t_r \left(T_r^4(i,j) + T_r^2(i-1,j)\right) \geq 0$$

The step-function illustrated in Figure 4.1 is defined to approximate a smooth quasi-convex function. Thus $C_{k+1} \geq C_k$ for $k > 0$ and

$$T^{1d}(i,j) \geq T^{kd}(i,j) \quad k \geq 2$$

This implies that though in general we expect the smallest flow to take place at the highest congestion level (the familiar always increasing average - marginal cost curve), we admit sets of flows which do not always satisfy this condition as we move from level k to k+1. Note that by setting $C_{k+1} = C_k$ for all k, we have the familiar case of total transportation costs varying linearly with the volume of flow. Marginal equals average cost.[5]

The total cost of transportation in the urban area, excluding the land and capital inputs, is

$$\sum_{ij} \sum_{k} C_k \sum_{d} T^{kd}(i,j)$$

The transportation system requires inputs of capital and land and we assume these inputs to vary linearly with the size of the flow at the first congestion level, b_1 and b_2, units of land, and capital respectively are assumed required for each unit of flow.

Finally we constrain the model so that the available land in a square i,j is not exceeded by use requirements. Thus

$$\sum_{q} \sum_{s} a_{\bar{r}+1,q,s} \; x_{q,s}(i,j) + b_1 \sum_{d} T^{1d}(i,j) \leq 1$$

5. The Objective Function and the Model's Decomposable Structure.

The objective function is

$$\text{Min } Z = \sum_{i,j} \Big[\sum_{d} \Big\{ (C_1 + Rb_2 + R_A b_1)\, T^{1d}(i,j) + \sum_{k=2}^{\bar{k}} C_k\, T^{kd}(i,j) + \underbrace{\sum_{r} z_r(i,j)\, x_r(i,j)}_{(i,j)\,\varepsilon\,\ell}$$

$$+ \sum_{s} (Ra_{\bar{r}+2,r,s,} + R_A . a_{\bar{r}+1,r,s} + wa_{\bar{r},r,s})\, x_{rs}(i,j) \Big\} \Big]$$

where the variables for square i,j are:

$T^{1d}(i,j)$ the flow of commodities crossing boundary d at congestion level 1,

$T^{kd}(i,j)$ the flow of commodities crossing boundary d at congestion level k,

$x_r(i,j)$ the export of commodity r to the out side of the city; takes value of zero for $(i,j) \not\in \ell$,

$x_{rs}(i,j)$ the gross output of commodity r with technology s.

These variables must be non-negative. The coefficient $z_r(i,j)$ is an index of the relative attractiveness of node (i,j) for exporting commodity r from the city.

Though the city was assumed to develop around transportation nodes on an unbounded grid, we treat the problem as being composed of n (finite) squares. Thus in equilibrium the open boundary of the city must have a band of unexploited squares (a subset of the n) between the exploited squares and the open space not covered by the grid of n squares. Thus land is only a constraint to the city as a unit in the sense that it must be bid away from agriculture at the exogenously given price of R_A. Unexploited squares are assumed left in agriculture.

The linear program whose solution yields an efficiently structured city in equilibrium consists of two general sets of constraints. The first set is nearly decomposable square-wise and the second set is decomposable square-wise. Hence the Dantzig-Wolfe decomposition algorithm could be used to solve the linear program. However, the decomposable part of the system has relatively small, sparse matrices which cause the Dantzig-Wolfe algorithm to be relatively slow to converge to a final solution.[6] We have not used the decomposition algorithm in solving example problems. Figure 5.1 contains a schematic representation of the constraint matrix for the linear program.

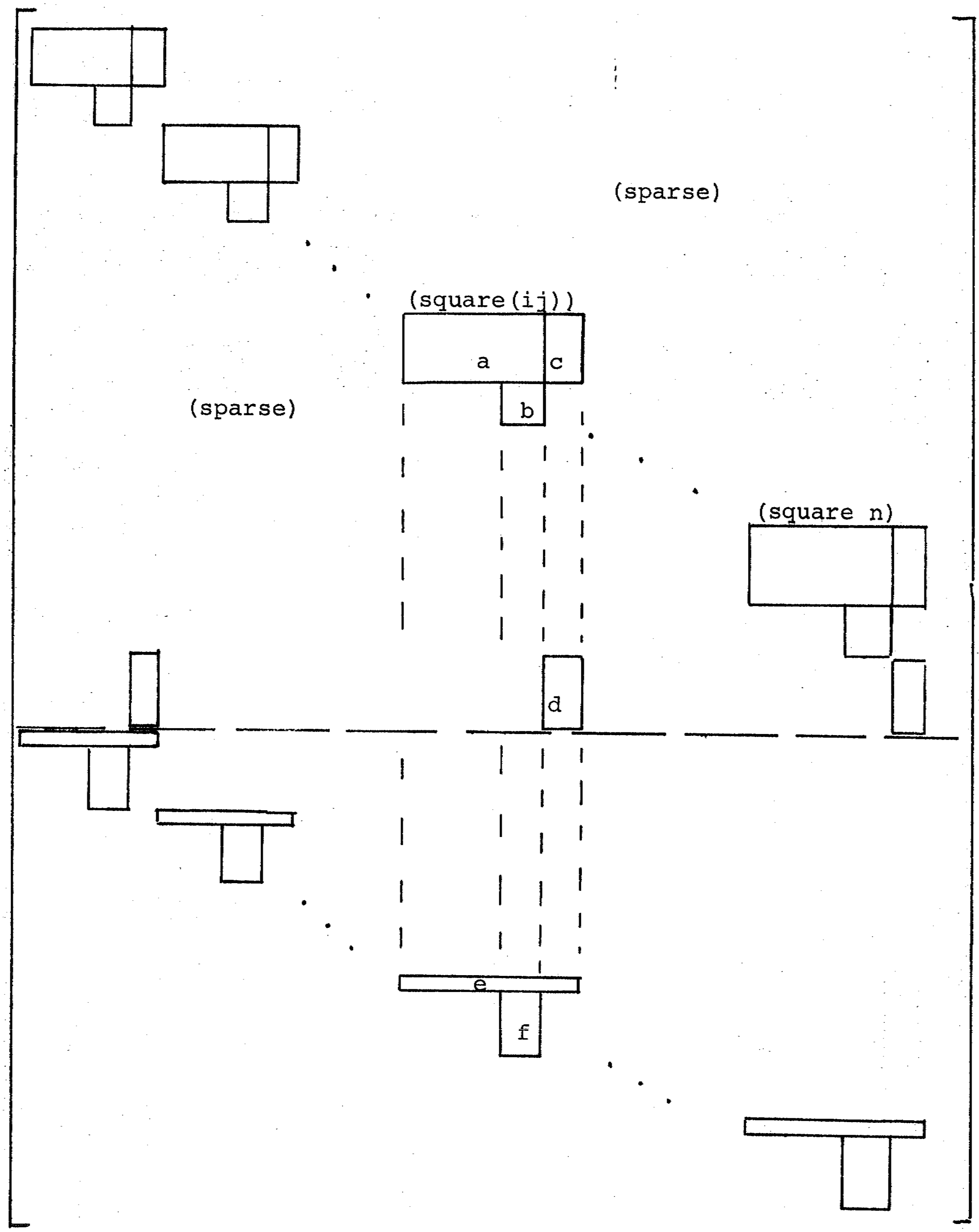

Figure 5.1

For representative square (i,j) in Figure 5.1, matrix a contains the technical coefficients of production, matrix b contains part of the aggregation operation for transported flows, matrix c contains the transfer of exports from transportation node (i,j) to the balance identity of which matrix d is a part, vector e contains the land constraints elements, and matrix f contains the elements of the congestion cost function. In the upper part of Figure 5.1, there are off-diagonal elements, in the area indicated as sparse, which are acting to transfer flows from one square to or from the four adjacent squares. It is these inter-square flows which link the system together and prevent the problem from being completely decomposable and hence trivial.

It is well-known that, for a decomposable linear program, the decomposition principle (Dantzig-Wolfe algorithm) can be interpreted as a pricing mechanism operating when externalities are present.[7] This quality has relevance to the urban model in this paper. The land owner of square (i,j) operates a "divisional program". The "executive program" (the upper part in Figure 5.1) is essentially one of co-ordinating flows between blocks in a city and the "divisional program" is one in which the owner of land in a block attempts to maximize profits on his particular block. As Baumol and Fabian have indicated, no decentralized price system will solve a decomposable linear program; a centralized authority must operate the "executive program". The executive in our case will be the planning authority which must allocate land to transportation in the city at the prevailing price for land and charge marginal costs for users of the transportation system.

6. Land Prices and Examples

The dual problem to minimizing the cost of a city is a linear program in which the value of urban output exported and the value of urban land is maximized. Shadow prices on land or land rent on each square will emerge in the dual problem.

Several actual numerical runs of the model have been completed at this time. The largest problem to date is composed of a grid of 49 squares (7x7), with a production technology for three export goods plus housing. Technologies with from one to five storeys are permitted for each good. The transportation system is composed of three congestion levels and cities with one, two and three export nodes have been simulated. The linear program generated by these runs has 754 constraints and 2959 variables (including slacks) and has density of .55. Using MPSX (Mathematical Programming System Extended) on an IBM 360 Model 85 optimal solutions to these linear programs were generated in less than 8 minutes.

Cities of 170 squares, and production in buildings up to 10 storeys will involve programs with 2700 constraints and 13,000 variables.

Outputs from a 7 x 7 square, two node example are presented in Figures 6.1 to 6.3. Coefficients are presented in the Appendix.

In Figure 6.1 we have a presentation of commodities produced in each square, the intersquare flows of commodities, and the storeys of building used in production. On the left hand side of each square we have outputs of commodities listed in decreasing orders of magnitude of land occupied by the production (top, down). For example, only agriculture was produced in the three squares located center-top in Figure 6.1. Note the zones of relative specialization in various commodities. For example, immediately around the two export nodes, housing occupies most land. The symbols on the right-hand side of each square indicate flows from each square. For example, good 3 having relatively low transportation costs and relatively high labour (housing) inputs is exported from all squares across the third row of squares. Finally, we observe that in this example all production took place in single storey buildings except that in the nodes. There good 1 was produced in structures of two different heights.

In Figure 6.2 we have a presentation of the land rents sustaining the allocation presented in Figure 6.1. The open squares indicate the lowest land rents, prevailing in agricultural areas where rent is $.8 per acre. The highest rents obtain around the export node, being $5.8 per acre or about 7 times the basic land price in agriculture. The four intervening land values are $3.597, $1.999, $.842, and $.821 per acre.

Figure 6.3 contains a road system which permits efficient resource allocation. Recall that the program yields the amounts of land in a square required for transportation plus the direction of flow. The program does not produce a road system. Only four areas for roads are indicated in the diagram and yet more than four were generated by the optimization. We have grouped areas for roads into four classes. Corresponding to the lines in <u>decreasing order of width</u> we have the following fractions of the relevant square used for roads.

Line 1	Land occupied	.01	- .09
2		.001	- .009
3		.0001	- .0009
4		.00001	- .00009

Observe that in certain cases a thick line feeds onto a thinner line in the next square. This occurs because the system has substituted congested flow for less congested flow on relatively cheaper land.

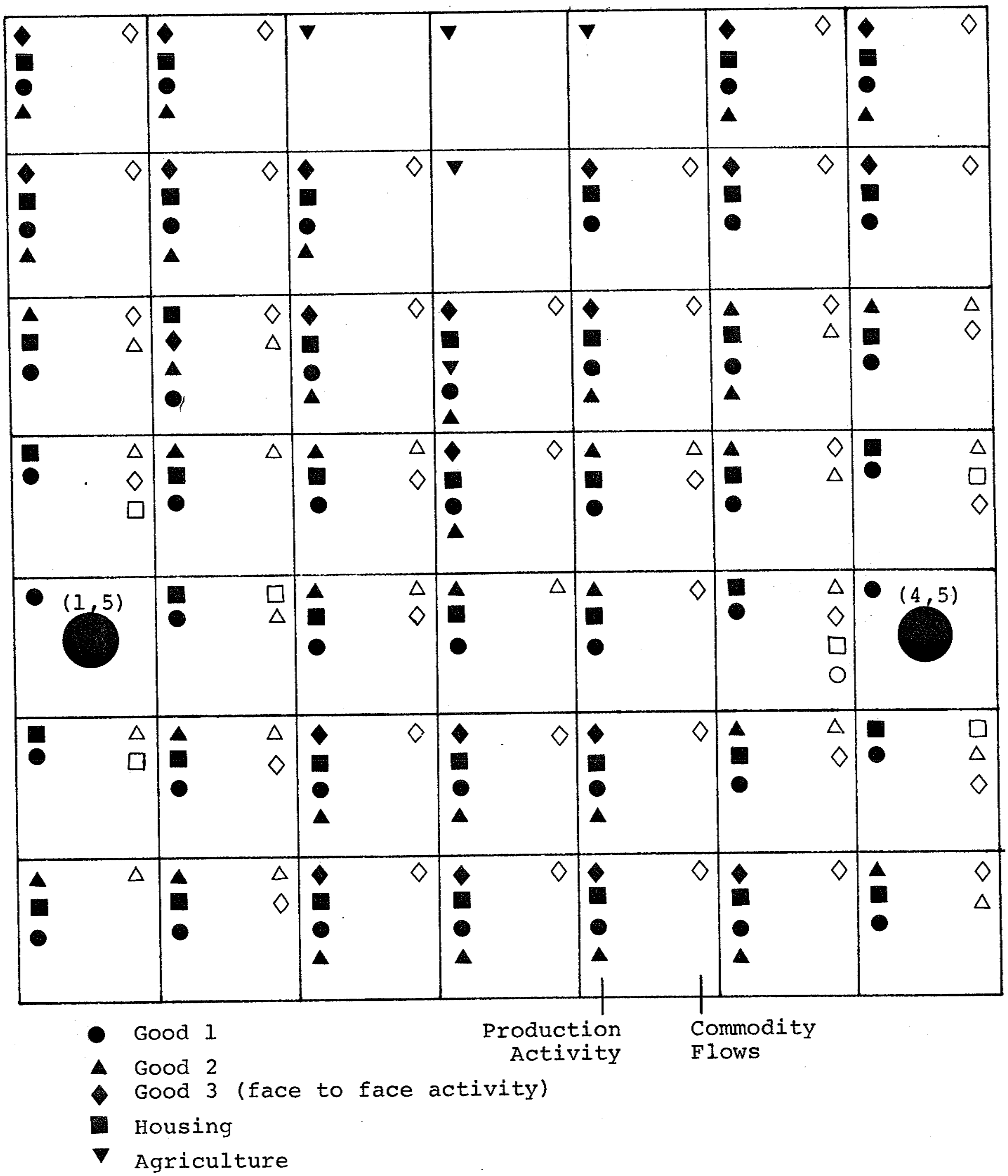
(1,5)
(4,5)
Production Activity
Commodity Flows
Good 1
Good 2
Good 3 (face to face activity)
Housing
Agriculture

Figure 6.1

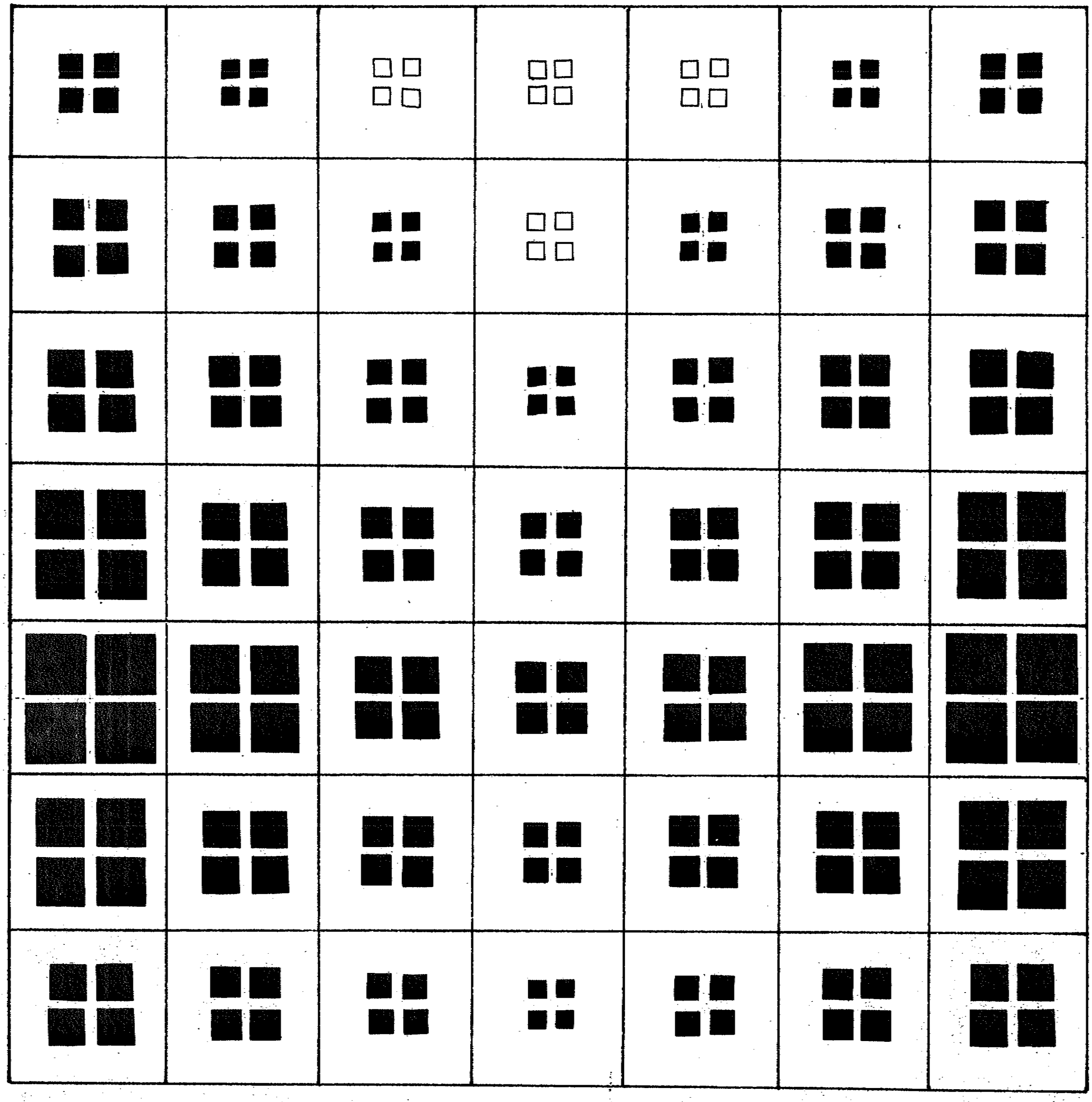

Figure 6.2

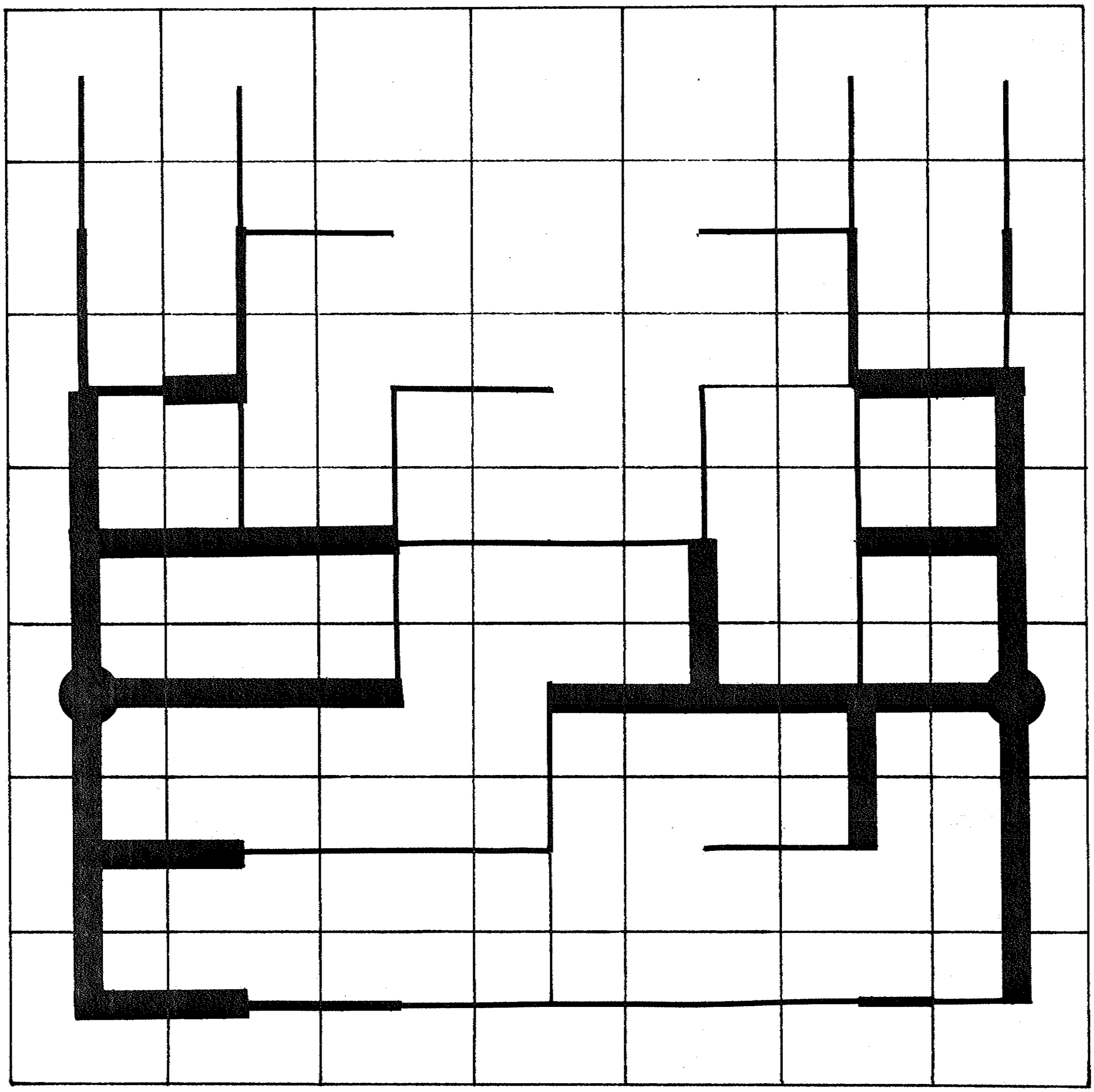

Figure 6.3

FOOTNOTES

1. See for example the survey, Scott (1970).

2. See for example the survey, Killbridge, O'Block, and Teplitz (1969).

3. Aguilar and Hand (1968) have considered problems of constructing the most profitable building on a given parcel of land facing given factor prices and given demand for services functions.

4. We have followed Mills in this formulation of congestion costs. For any given road and a given time phasing of transportation in a daily cycle, the C_k's can be looked on as technical coefficients or technically given impedence measures. However, the time phasing of transportation will be a function of the C_k's in a more realistic model and efficient resource allocation in an urban area will be simultaneously a time allocation problem constrained within a daily cycle and a timeless resource allocation problem. Like most model builders, we choose to abstract from the optimal time phasing problem.

5. An amendment in the formulation of the resource using aspects of transportation would be in order for the case $C_{k+1} > C_k$ for all k. That is rather than have land and capital use as a function of the flow at the first congestion level, we would make these uses as a function of total flow at the now constant cost per unit flow. The economic aspects of the decomposable nature of the model discussed in Section 6 still hold for the non-congestion cost case.

6. See Orchard - Hays (1968; p. 263).

7. See for example Dantzig (1964) or Baumol and Fabian (1964).

8. An alternate way of looking at a system of cities is to consider general supply and demand schedules defined at m points in space, to be designated as cities. Each city then has to produce a set of final demands (possibly negative) which satisfy the excess demand schedules at the point in question. A fixed point theorem is required to prove that a bill of final demands and a set of prices exist to satisfy the excess demand functions. See for example Dorfman, Samuelson, and Solow (1958; Chapt. 13).

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APPENDIX

SIMULATION STATISTICS

M= 7 N= 7

R-BAR= 4 S-BAR = 5

PRODUCTION TECHNOLOGY

	R=1	R=2	R=3	LABOUR	LAND	CAPITAL
S= 1						
R=1	0.010	0.050	0.300	0.400	0.420	1.100
R=2	0.100	0.010	0.300	0.400	0.440	0.570
R=3	0.050	0.050	0.010	0.500	0.500	0.100
R=4	0.250	0.050	0.250	0.010	0.660	0.940
S= 2						
R=1	0.010	0.050	0.300	0.400	0.340	1.200
R=2	0.100	0.010	0.300	0.400	0.380	0.690
R=3	0.050	0.050	0.010	0.500	0.480	0.150
R=4	0.250	0.050	0.250	0.010	0.520	1.080
S= 3						
R=1	0.010	0.050	0.300	0.400	0.260	1.300
R=2	0.100	0.010	0.300	0.400	0.320	0.810
R=3	0.050	0.050	0.010	0.500	0.460	0.200
R=4	0.250	0.050	0.250	0.010	0.480	1.220
S= 4						
R=1	0.010	0.050	0.300	0.400	0.180	1.400
R=2	0.100	0.010	0.300	0.400	0.260	0.930
R=3	0.050	0.050	0.010	0.500	0.440	0.250
R=4	0.250	0.050	0.250	0.010	0.440	1.360
S= 5						
R=1	0.010	0.050	0.300	0.400	0.100	1.500
R=2	0.100	0.010	0.300	0.400	0.200	1.050
R=3	0.050	0.050	0.010	0.500	0.420	0.300
R=4	0.250	0.050	0.250	0.010	0.400	1.500

EXPORT DEMANDS

COMMODITY: 1	QUANTITY DEMANDED:	15.000
COMMODITY: 2	QUANTITY DEMANDED:	15.000
COMMODITY: 3	QUANTITY DEMANDED:	0.0

FACTOR PRICES: RENT ON CAPITAL= 4.000 AGRIC. LAND RENT= 0.800 WAGE RATE= 1.000

EXPORT NODES	EXPORT COSTS		
SQUARE	GOOD 1	GOOD 2	GOOD 3
(3. 1)	1.000	1.000	1.000
(3. 7)	1.000	1.000	1.000

TRANSPORTATION:

FACTOR INPUT COEFFICIENTS: LAND= 0.100 CAPITAL= 0.120

CONGESTION LEVELS:
CK(1)= 1.000
CK(2)= 1.500
CK(3)= 1.750

TRANSPORTATION WEIGHTS:
TR(1)= 0.500
TR(2)= 0.350
TR(3)= 0.010
TR(4)= 0.500

Where M and N are row and column indices indicating the size of the grid under analysis

S indicates number of storeys (S = 1, ..., 5)

R indicates number of the commodity (R = 1, ..., 3)

Export Costs indicate relative attractiveness of the nodes for the various commodities (written as z in text)

CK's are the terms C_k's in the text.

TR's are the terms T_r's in the text.