Queen's Economics Department Working Paper No. 858

Limited Rationality and Endogenously Incomplete Contracts

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10-1992

Discussion Paper #858

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Current Draft, October 1992

* I wish to thank Debra Holt and seminar participants at Carnegie Mellon, Western Ontario, Penn, the 1990 Winter Econometric Society meetings, Washington University, Queen's, LSE, Bonn, Dortmund, the 1991 International Conference on Game Theory in Florence, and Summer in Tel Aviv 1991 for numerous useful suggestions. Financial support from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged. Of course, I alone am responsible for any errors.

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ABSTRACT

The purpose of this paper is to provide a simple model in which limited rationality endogenously generates incomplete contracts. I model limited rationality as in Lipman [1991, 1992], focusing on the idea that boundedly rational agents do not necessarily know every implication of their knowledge, but may, at a cost, compute these implications. This assumption implies the existence of subjective uncertainty in addition to the objective uncertainty the agents are contracting over. The presence of noncontractable subjective uncertainty can lead to contracts which are incomplete with respect to the contractable objective uncertainty. The most surprising result is that strategic bargaining over contracts can lead to incomplete contracts even with infinitesimal computation costs.

ABSTRACT

The purpose of this paper is to provide a simple model in which limited rationality endogenously generates incomplete contracts. I model limited rationality as in Lipman [1991, 1992], focusing on the idea that boundedly rational agents do not necessarily know every implication of their knowledge, but may, at a cost, compute these implications. This assumption implies the existence of subjective uncertainty in addition to the objective uncertainty the agents are contracting over. The presence of noncontractable subjective uncertainty can lead to contracts which are incomplete with respect to the contractable objective uncertainty. The most surprising result is that strategic bargaining over contracts can lead to incomplete contracts even with infinitesimal computation costs.

I. Introduction.

The existence of incomplete contracts — that is, contracts which do not specify an agreement for every relevant contingency — has long posed a quandary in economics. Despite the fact that such contracts are generally Pareto inefficient, they certainly exist. In fact, it seems likely that *most* real contracts are incomplete! The possibility that incompleteness is due to bounded rationality has long been noted. Many economists (see Williamson [1975], for example) have argued that bounded rationality makes it impossible for agents to completely foresee all relevant contingencies and therefore contracts must be incomplete. However, this, by itself, is not an adequate explanation. It is, in fact, trivial to write a complete contract: on the last line, one simply writes "... and if none of the contingencies named above occurs, the outcome we agree to is x." This contract specifies an outcome in every possible contingency and so is complete. Of course, the only problem is that it may be difficult to know what a good choice of x is. In other words, writing a complete contract is easy, but writing a *good* complete contract may be very hard.[1] As a result, agents may agree to an incomplete contract knowing that there is a complete contract which strictly Pareto dominates it because it is too hard to find the better contract.

The purpose of this paper is to provide a simple model in which limited rationality endogenously generates incomplete contracts. I model limited rationality as in Lipman [1991, 1992],[2] taking the view that limited rationality means that agents do not necessarily know every implication of their knowledge, but may, at a cost, compute these implications. It is these costs which make writing a "good" complete contract hard. This

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[1] The same observation is made by Hart [1987].

[2] Many alternative approaches have been proposed recently. Examples include Abreu and Rubinstein [1988], Binmore [1987], Fudenberg and Kreps [1988], Geanakoplos [1989], Kalai and Stanford [1988], Radner and Van Zandt [1992], and Rubinstein [1992].

assumption implies the existence of subjective uncertainty in addition to the objective uncertainty the agents are contracting over. It is easy to show that if computation costs are high, this noncontractable subjective uncertainty can lead to contracts which are incomplete with respect to the contractable objective uncertainty. In this situation, the agents agree to an incomplete contract though they know it is suboptimal because it is too costly to find a better complete contract. The more intriguing result is that strategic bargaining over contracts can lead to incomplete contracts even with infinitesimal computation costs. The intuition behind this result is also straightforward: in a strategic setting, getting better information can be harmful since it causes opponents to behave differently. As I show by example, one player's bargaining position may be harmed if he becomes better informed, thus endogenously generating a cost of acquiring information even when computation costs are negligible. To create this effect in the simplest possible way, I assume that computation or information acquisition is observable. As I explain in the conclusion, one obtains the same result when computation is not an observable action if the number of contingencies is large enough relative to the cost of computation.

While I stress the limited rationality interpretation of the model, this interpretation is not necessary. An alternative story is that contracts cannot be contingent on court rulings regarding provisions of the contract, but that the parties can hire lawyers who can predict these rulings. The key to the model, under either interpretation, is that there is some uncertainty which is not contractable but can be resolved at a cost and that this uncertainty leads the parties to write contracts that are incomplete with respect to the contractable uncertainty.

As mentioned above, I will say that a contract is incomplete if it includes fewer contingencies than are relevant to the parties to the contract — either because the relationship between the parties is expected to last longer than the contract or because there are some relevant possible events for which the contract does not specify anything. As I use the term, then, with an incomplete contract, the parties must either renegotiate or abandon the relationship in situations not covered by the contract. Of course, a contract which covers every relevant state of affairs can always imitate a contract covering fewer contingencies. Hence for any incomplete contract, we can always find a complete contract which weakly Pareto dominates it. Incomplete contracts can be strictly Pareto inferior to complete ones for many reasons. First, risk sharing with incomplete contracts is generally suboptimal since renegotiation takes place after the uncertainty is resolved and hence at a point at which risk sharing considerations are no longer relevant. Second, as noted by Williamson [1979, 1983], Tirole [1986], Crawford [1988], and others, incomplete contracts can lead to underinvestment in relationship–specific capital. On the other hand, Hart and Moore [1988], Huberman and Kahn [1988], Malcomson and Spinnewyn [1988], and Fudenberg, Holmstrom, and Milgrom [1990] show that in some settings, renegotiated incomplete contracts are as efficient as complete contracts.

There are other ways to define incompleteness. Anderlini and Felli [1992]'s definition is that a contract is incomplete if the parties could be made better off by conditioning more finely on the contingencies. For example, suppose there are only two possible events and that the optimal contract calls for different outcomes in these two contingencies. Anderlini and Felli would say that a contract which stipulates a single outcome in either event is incomplete, while my definition would call this a complete contract. With their approach, an incomplete contract is inefficient by definition.

There are very few models which predict incomplete contracts under either definition, so that in most of the literature on incomplete contracts, the nature of the incompleteness is assumed, rather than derived. As a result, predictions are necessarily conditional on the validity of assumptions about variables that are actually endogenous. By contrast, a model which predicts incomplete contracts can generate predictions about what kinds of contingencies will be omitted and how this incompleteness affects the specification of the contract for the contingencies which are included.

One well–known approach to this problem (see, for example, Grossman and Hart [1986] or Hart and Moore [1988, 1990]) is to assume that all limitations on the completeness of contracts are limitations of verifiability. That is, the parties observe certain variables which they would like to write contracts based on, but they are unable to do so since it is impossible to verify these observations to the court for enforcement. These limitations on verifiability may be due to the difficulty of specifying the exact state of nature in enough detail for the court to unambiguously recognize the intent of the parties or simply because the parties cannot prove to an outsider what they have observed.

A few authors have put forth alternative explanations of incompleteness. The first to do so was Dye [1985], who studied optimal contracts when there is a fixed cost per contract contingency. With high enough costs, optimal contracts are incomplete. Dye's model is similar to mine in that my model also includes a cost per contingency, but in my model, it is the cost of determining the best way to include the contingency. Also, I assume that the contingency can be included without paying this cost, though only at the risk of specifying an inefficient agreement for that contingency. Furthermore, since Dye's costs are purely exogenous, optimal contracts converge to complete contracts as these costs go to zero. In my model, there is an endogenous component to these costs which may not converge to zero.

More recent papers have tried to explain incompleteness without recourse to exogenous contracting costs — in particular, Hermalin [1990], Allen and Gale [1992], Busch and Horstmann [1992], and Anderlini and Felli [1992]. Hermalin [1990] and Allen and Gale [1992] consider signaling models in which offering a complete contract is a negative signal about the offerer. Hence incomplete contracts may be chosen instead. I assume that information acquisition or computation is observable, but my results do not change qualitatively if computation is unobservable. In this case, my results become analogous to those of Hermalin or Allen and Gale: offering a complete contract signals that computation has been carried out and this signal hurts the proposer.

Busch and Horstmann [1992] analyze a bargaining model in which incomplete contracts can emerge in equilibrium. As in my model, incomplete contracts are signed because they are preferred by one party as a way of making the "disagreement payoffs" favor him. A crucial assumption in their model, however, is that the parties must decide at the outset whether to bargain over complete contracts or incomplete contracts and this decision is irrevocable. Hence in equilibrium, the parties may sign an inefficient incomplete contract even though the identity of a complete contract which is strictly Pareto superior is common knowledge. In my model, the parties agree to an incomplete contract knowing that there is a complete contract which Pareto dominates it, but they do so because the identity of the preferred complete contract is not common knowledge.

Anderlini and Felli [1992] analyze contracting where both the contract and the choice criteria for the contract are required to be computable. They show that the optimal contract under these constraints may be incomplete (according to their definition of incompleteness). While their motivation is similar to mine in that both models focus on a form of bounded rationality and its effect on contracting, their approach is quite different.[3]

In the next section, I present the model and the intuition behind it. In the basic model, I do not specify the procedure by which renegotiation takes place in contingencies which were not included in the contract. Instead, in Section III.A., I treat the outcome of renegotiation in omitted contingencies as exogenous and characterize equilibrium contracts as a function of these outcomes. In Section III.B., I specify a simple and plausible renegotiation

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[3] In a related vein, see Holm [1992] for an analysis of the computational complexity of writing contracts.

subgame and characterize the equilibrium renegotiation outcomes. I then use this to show in the context of an example that incomplete contracts are possible. As I also show, renegotiation outcomes with infinitesimal computation costs can differ dramatically from the outcomes with zero computation costs, causing important differences in equilibrium contracts. In particular, with the renegotiation game I analyze, every equilibrium with zero computation costs has complete contracts, while, for some parameter values, every equilibrium with infinitesimal computation costs has incomplete contracts. In Section IV, I discuss the roles of some key simplifying assumptions. All proofs are contained in the Appendix.

II. Contracting with Limited Rationality.

As argued above, it is easy for agents to write complete contracts; what is hard is writing good complete contracts — that is, contracts which specify "acceptable" or "good" outcomes for each possible contingency. Implicit in this statement is the view that an agent must compute or think in order to know what he would like in a given contingency. Paraphrasing, the agent has subjective uncertainty about his preferences as a function of the contingency.

To be more concrete, I assume there are two agents, 1 and 2, also called the seller and the buyer respectively. The seller can produce one unit of an indivisible good which the buyer can consume. S is the set of possible *contingencies.* A contingency should be interpretted as a complete specification of all objective variables of relevance to the contract — the prices and availability of each of the seller's inputs, the prices of substitutes for the seller's product, etc. Such a complete specification determines the value of the object to the buyer and its cost to the seller. A *contract* is a mapping from some subset of S into agreements. That is, for contingencies included in the contract, the contract specifies whether or not the seller produces the object for the buyer and the amount the buyer pays the seller. More formally, a contract is a triple $c = (\hat{S}, \hat{a}(\cdot), \hat{p}(\cdot))$ where $\hat{S} \subseteq S$, $\hat{a} : \hat{S} \rightarrow$

$\{0,1\}$ gives the number of units sold to the buyer, and $\hat{p} : \hat{S} \to \mathbf{R}$ gives the price paid by the buyer. Since a contingency is a complete specification of all relevant factors, it determines the utility buyer and seller obtain from any given agreement in that contingency.

However, knowing the description of a contingency and how to use that to compute the utility of an agreement in that contingency does not imply that the utility of the agreement in that contingency is known. If computation is costly, one may not wish to compute this quantity. More formally, let $\xi_i(c, s)$ be the utility of i under contract c in contingency s. While s may completely determine the quantity $\xi_i(c, s)$, this does not mean that the agents immediately know this number for any given s. If they do not know it, presumably, they have beliefs about it. That is, they have subjective uncertainty regarding this objective quantity. I model this subjective uncertainty by supposing that there is a set of *subjective states* (states for short), Θ, and that the utility of a contract c in contingency s to agent i depends on which state obtains — *i.e.*, utility is a function of c, s, and θ.[4] Each agent can learn about the subjective state by computation, but computation is costly and so will not always be optimal.[5]

Lipman [1991, 1992] develops in more detail the argument that a crucial aspect of bounded rationality is the fact that real people are not *logically omniscient* — that is, a real person can know a fact without knowing all

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4 Kreps [1988] derives a representation of preferences in the presence of unforeseen contingencies which is quite similar to this. See Lipman [1992] for a different axiomatic treatment.

5 Subjective uncertainty could enter into the model in other ways as well. For example, one could suppose that the utility associated with a given contract in a given contingency is known, but the agent is unsure of the utility of the contract prior to knowing the contingency. Intuitively, the agent may find it hard to aggregate this information — *i.e.*, hard to integrate utility as a function of s to obtain expected utility.

its logical implications. Under this view, people often have subjective uncertainty about things which objectively cannot be stochastic. Objectively, there is a single fixed number which, when cubed, yields 1343. Subjectively, however, there may be several numbers which one gives a positive probability of possessing this property. I do not wish to claim that this is all of what bounded rationality is; however, it is the aspect of bounded rationality I will focus on. Hence for my purposes, the difference between boundedly rational agents and perfectly rational agents is that the latter find it costless to learn the logical implications of their information.

It is easy to see intuitively that this formulation could generate incomplete contracts. If there are many contingencies, it may be very costly to compute a Pareto optimal contract. Hence the parties may prefer to observe the realization of s and then compute an optimal agreement for that contingency alone, as this presumably involves less computation. This is true even if there are advantages to contracting in advance, such as risk sharing. Intuitively, the parties trade off the advantages of contracting in advance against the costs of complete computation and this tradeoff may lead to an interior optimum — *i.e.*, an incomplete contract. I will show that with a simple model of strategic bargaining over contracts, incomplete contracts may emerge even with infinitesimal computation costs.

Remark 1. One may object to this formulation on the grounds that truly complete contracts have been precluded by assumption. In a sense, the true set of contingencies is not S but $S \times \Theta$. Hence a truly complete contract would specify trades and prices as a function of (s, θ), not just s. There are two counterarguments to this view. First, one could argue that contracts which condition on the resolution of purely subjective uncertainty are neither sensible nor enforceable. Recall that θ captures the subjective uncertainty generated by the need to compute $\xi_i(a, s)$ to know it. Hence this information need not be verifiable and certainly is not costlessly observed even *ex post.* Intuitively, writing a contract over such uncertainty is like including clauses like, "If contract 1 would give the seller profits of

$100, then we agree to contract 1. Otherwise, we agree to contract 2." However, even if one views such contracts as feasible, there is a deeper problem. To be concrete, let C_0 denote the set of possible contracts as defined above. Let Θ_0 denote the set of subjective states relative to this set of contracts. That is, the subjective uncertainty the agents have about contracts in C_0 is modelled by assuming that the utility of such a contract depends on $\theta \in \Theta_0$. If we enlarge the set of contracts to allow contracts to depend on both s and θ, we must recognize that we have not yet modelled the subjective uncertainty associated with the contracts we have added into the choice set. More concretely, let C_1 denote the set of contracts which specify a trades and prices for each (s, θ) in some subset of $S \times \Theta_0$. Just as above, the value of an agreement in (s, θ) is completely determined by (s, θ), but this does not mean that the agents know this value. Hence we need to specify the subjective uncertainty associated with these contracts, leading us to construct a larger set of subjective uncertainty, say Θ_1. Obviously, this procedure could be repeated *ad infinitum.* No matter where one tries to truncate this construction (even if one carries it out infinitely or transfinitely), in general, every contract will have the property that agents have subjective uncertainty about that contract which is not covered by the contract itself. (This is straightforward to show formally; see Remark 4 of Lipman [1991] for an analogous result.) In this sense, noncontractable subjective uncertainty is unavoidable.

In the rest of this section, I will add more structure to the problem so that I can provide a more concrete characterization of equilibrium contracts. I assume that there are $T \geq 1$ different types of contingencies, with a continuum of each type. Only the type of the contingency is directly payoff relevant. As will be seen, the continuum assumption is analytically very convenient. The assumption that only the type of the contingency is payoff relevant is a common one in the literature with a continuum of states. More specifically, $S = \{1, \ldots, T\} \times [0, 1]$. If $s = (t, x)$ for some $x \in [0, 1]$, I will say that s is of type t, or $t = t(s)$. The probability of this event is ϕ_t. For any t, the conditional distribution of s given that $t(s) = t$ is uniform on $\{t\} \times [0,1]$. An alternative interpretation is that there are at most T periods to the relationship and the contingency in period t is uniformly distributed on [0,1] for each t. Under this interpretation, ϕ_t can be thought of as discounting period t returns to the present.

For each possible s, there is a random variable, $\tilde{\theta}_s$, which describes the subjective uncertainty about contingency s. The $\tilde{\theta}_s$'s are iid and take on values in the set $\bar{\Theta} = \{1, \ldots, M\}$. That is, $\Theta = \bar{\Theta}^S$. The iid assumption implies[6] that exactly the fraction $\Pr(\tilde{\theta}_s = 1)$ of the type t contingencies have a realization $\theta_s = 1$,[7] exactly the fraction $\Pr(\tilde{\theta}_s = 2)$ have $\theta_s = 2$, etc. Thus there is no "aggregate" uncertainty; the only thing which is uncertain is precisely which of the type t contingencies have $\theta_s = 1$, not how many of them do.

The value of the object to the buyer in contingency s if the realization of $\tilde{\theta}_s$ is θ_s is given by $v_{t(s)}(\theta_s)$ while the cost to the seller is $c_{t(s)}(\theta_s)$. To simplify some expressions, I assume $\mathrm{E}_\theta v_t > \mathrm{E}_\theta c_t$ for all t. (It is straightforward to modify the results for the case where this does not hold.) I do not require v_t and c_t to be independent or to have any particular form of correlation.[8]

The seller is risk neutral, while the buyer is risk averse. Because of this, they have an incentive to contract prior to the realization of the contingency: the seller can insure the buyer. It is useful to describe the bargaining game and the information of the players before discussing preferences

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6 As Judd [1985] shows, the iid assumption does not literally imply this statement, but is not inconsistent with it.

7 Throughout, a θ with a tilde will denote a random variable, while one without a tilde will denote a typical realization.

8 As Vincent [1989] has shown, the existence of correlation between v and c has important consequences in standard bargaining models.

in more detail. The game is divided into two stages. In the first stage, the players do not know s. I also assume that player 1 (the seller) does not know any of the θ_s's, but player 2 (the buyer) knows the realization of $\tilde{\theta}_s$ for every $s \in S$. (The results generalize in a simple but notationally tedious way to the case where the players are symmetrically informed at the outset.) Player 1 moves first at stage 1. First, he can compute $\tilde{\theta}_s$ for whatever values of s he likes. Computing $\tilde{\theta}_s$ means that he learns the exact realization of the random variable $\tilde{\theta}_s$. The interpretation is that the seller uses his information about s to deduce the value of the good to the buyer and its cost of production for contingency s. For simplicity, I assume that the seller's computation choices are observable. As discussed in the conclusion, one can obtain similar results if computation is unobservable.

Next, the seller makes an offer to the buyer. An offer is a triple $(\hat{S}, \hat{a}(\cdot), p)$, where, as in the definition of a contract, $\hat{S} \subseteq S$ and $\hat{a} : \hat{S} \to \{0, 1\}$. However, unlike the way a contract was defined, p is a number. Player 2 observes the computation choices of player 1 and his offer and then can either accept or reject the offer. If he accepts, he chooses any $\hat{p}(\cdot)$ function such that $\mathrm{E}_s[\hat{p}(s) \mid s \in \hat{S}] = p$. (Since player 1 is risk neutral, he cares only about the expected price. Hence it seems only reasonable to allow player 2 complete latitude in determining the prices in the different contingencies given a fixed mean. The role this assumption plays in the analysis is discussed in Section IV.) In this case, the players have agreed to the contract $(\hat{S}, \hat{a}(\cdot), \hat{p}(\cdot))$. If player 2 rejects the offer, there is no contract.

At the beginning of stage 2, the players learn the true contingency, say s. If the players agreed to a contract $(\hat{S}, \hat{a}(\cdot), \hat{p}(\cdot))$ and $s \in \hat{S}$, then the buyer pays the seller $\hat{p}(s)$ and, if $a(s) = 1$, the seller provides the good. If $s \notin \hat{S}$ or if they did not reach an agreement on a contract, then they renegotiate. In what follows, I will derive the equilibrium contract as a function of the outcome of this renegotiation under some innocuous assumptions. In Section III.B., I illustrate these results with a very simple and reasonably natural specification of the renegotiation subgame — namely, where player 2 makes a take–it–or–leave–it offer to player 1. As I show, there are parameter values for which every equilibrium in this case has incomplete contracts if computation costs are infinitesimal.

The payoffs of the two players depend on whether or not trade occurs, the price, the value of s, the value of θ_s, and the computations performed. Suppose player 2 pays player 1 p, a is the number of units traded (either 0 or 1), the contingency is s where s is of type t, and the realization of $\tilde{\theta}_s$ is θ. Then the payoff to player 2 is $u(av_t(\theta) - p)$. I assume that $u(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly concave and that $u(0) = 0$. I consider two alternative specifications of player 1's payoffs, which I refer to as costless computation and almost costless computation. As we will see, the difference between these two specifications is relevant only in renegotiation; consequently, it will only appear in the analysis when a specific renegotiation subgame is discussed in Section III.B. However, for the sake of completeness, I will define these terms here. With costless computation, player 1's payoff in this event is $p - ac_t(\theta)$, regardless of the set of s such that he computed $\tilde{\theta}_s$. With almost costless computation, player 1's preferences are lexicographic. In comparing any two strategies given some strategy for player 2, player 1 strictly prefers the strategy yielding the higher expected payoff based on costless computation. If the two strategies yield equal costless computation payoffs, he strictly prefers that strategy which involves less computation. Here the notion of "less computation" is defined in the weakest possible way — strategy 1 computes less than strategy 2 if the set of s such that strategy 1 computes $\tilde{\theta}_s$ is a strict subset of the set of s such that strategy 2 computes $\tilde{\theta}_s$.[9]

Recall that the difference between completely rational and boundedly rational agents in this framework is that completely rational agents have zero computation costs. Thus I interpret the costless computation case as a model of completely rational agents and the almost costless computation

[9] More precisely, strategy 1 computes less if this is true at every information set.

case as a model of agents who are arbitrarily close to completely rational. As we will see in Section III.B., these two cases can be quite different.

Remark 2. The informational assumptions and the assumptions on the bargaining game are certainly not the most general possible ones. However, my purpose is to illustrate how incomplete contracts can be generated from bounded rationality, not to provide a definitive analysis.

I consider the set of pure strategy sequential equilibria which satisfy two conditions. First, no player ever uses a weakly dominated strategy. (This requirement only plays a role in the analysis of Section III.B.) Second, I impose a certain stationarity on strategies; namely, I focus on equilibria in which the renegotiation strategies for a contingency s not included in the contract only depend on whether or not the seller computed $\tilde{\theta}_s$, on $t(s)$, and, if known, on θ_s, not on the seller's first period contract offer nor s (except for its type).[10]

In light of this stationarity assumption, the outcome of renegotiation in contingency s can be summarized by four functions for each t, two giving the outcome if the seller did compute $\tilde{\theta}_s$ in the first stage and two giving the outcome if he did not. If s is of type t and the seller did compute $\tilde{\theta}_s$, $d_t^c : \bar{\Theta} \to \{0,1\}$ gives the trade decision from renegotiation and $p_t^c : \bar{\Theta} \to \mathbf{R}$ the price, where both are functions of the realization of $\tilde{\theta}_s$. Similarly, d_t^{nc} and p_t^{nc} will give the renegotiated trade decision and price as a function of the realization of $\tilde{\theta}_s$ if the seller did not compute $\tilde{\theta}_s$ in the first stage. Throughout the analysis, I assume

$$\text{(A1)} \qquad d_t^c(\theta) = 1 \implies v_t(\theta) \geq p_t^c(\theta) \geq c_t(\theta).$$

10 Implicitly, this assumption requires that the renegotiation game itself — so far left unspecified — satisfies a certain stationarity. If, for example, the rules for renegotiation were to vary with the first period contract offer, then it might be impossible for this stationarity condition to be satisfied.

In words, (A1) simply says that if the seller computed $\tilde{\theta}_s$ in stage 1 and second stage negotiations do lead 1 and 2 to trade in contingency s, then they must trade at a mutually beneficial price. It seems clear that any reasonable bargaining game will have this property. Also, I assume that for each t, the gamble $d_t^c(v_t - p_t^c)$ is nondegenerate. That is, for each t, there exists θ, θ' such that

$$\text{(A2)} \qquad d_t^c(\theta)[v_t(\theta) - p_t^c(\theta)] \neq d_t^c(\theta')[v_t(\theta') - p_t^c(\theta')].$$

Intuitively, (A2) simply says that renegotiation, by itself, does not perfectly insure the buyer. Hence it guarantees that contracting in advance has the advantage of allowing for better risk–sharing. As explained above, I derive the equilibrium contract as a function of the renegotiation outcome. In Section III.B., I will illustrate these results under a specific renegotiation game in which (A1) and (A2) are satisfied and show that incomplete contracts can emerge.

III. Results.

A. Equilibrium Contracts Given Renegotiation Outcomes.

In this subsection, I derive equilibrium contracts as a function of the renegotiation outcome functions. The difference between costless computation and almost costless computation plays no role at all in this section; consequently, it is simpler to focus on the costless case. In the next subsection, I illustrate these results by making specific assumptions about the renegotiation subgame and characterizing the equilibrium renegotiation outcome functions. There, the difference between costless and almost costless computation will be crucial.

To derive the equilibrium contract, let u_t^c denote player 2's expected payoff in a contingency of type t which is not included in the contract given that player 1 computes $\tilde{\theta}$ for that contingency in stage 1. That is,

$$u_t^c = \mathrm{E}_\theta[u(d_t^c(v_t - p_t^c))].$$

(Throughout, I will include the θ argument of functions only when necessary.) Similarly, let u_t^{nc} denote 2's expected payoff when player 1 does not compute $\tilde{\theta}_s$ in stage 1, *i.e.*,

$$u_t^{nc} = \mathrm{E}_\theta[u(d_t^{nc}(v_t - p_t^{nc}))].$$

Also, let ψ_t give the socially optimal trading rule — that is, $\psi_t(\theta) = 1$ if $v_t(\theta) \geq c_t(\theta)$ and $\psi_t(\theta) = 0$ otherwise.

To understand the way the equilibrium contract is determined, notice that player 1's computation has two effects. First, of course, it gives player 1 more information to use in determining his contract offer. In particular, for those contingencies for which he computes $\tilde{\theta}_s$, he learns which have the buyer's valuation larger than his productions costs and hence he learns how to specify $\hat{a}(\cdot)$ efficiently for these contingencies. Notice that this is the only useful information computation gives him, however. By the iid assumption, there is no aggregate uncertainty in the sense that the fraction of type t contingencies that have a given realization of $\tilde{\theta}_s$ is known. Since the optimal choice of the set of contingencies to include in the contract and the optimal price are both functions of aggregate information, player 1 knows enough to determine these without computation.

The second effect of player 1's computation is to alter the renegotiation subgame, at least potentially. This changes the disagreement payoffs and, as will be seen starkly in the next subsection, can improve player 2's bargaining position in the first stage. If so, first stage computation has an endogenous cost to player 1: it forces him to offer player 2 a larger share of the gains from trade.

These facts have two important implications. First, we can determine the optimal set of contingencies for player 1 to include and the optimal price for him to offer as a function of the set of computations he performs, without regard to the outcome of those computations. Second, if player 1 computes $\tilde{\theta}_s$, he should include contingency s in the contract, regardless of what he learns from the computation. The cost of computing will be sunk (both the exogenous and endogenous portions) and so there is no point in not reaping the benefits of including s in the contract. Note, though, that the reverse need not be true, however — player 1 may include s in a contract offer without computing $\tilde{\theta}_s$.

To simplify further, notice that for any given t, player 1 has no reason to favor some contingencies of type t over others. Hence we can simply reduce player 1's choice of a contract to a choice of how many type t contingencies to include, how many of these to compute $\tilde{\theta}_s$ for, and what price to charge. Of the contingencies for which he computes $\tilde{\theta}_s$, the contract he proposes specifies trade iff it is efficient — that is, iff $\psi_{t(s)}(\theta_s) = 1$. It is not hard to show that the assumption $\mathrm{E}_\theta(v_t - c_t) > 0$ implies that the seller should propose trade in every s included in the contract for which he has not computed $\tilde{\theta}_s$. Summarizing, then, player 1 chooses numbers α_t and β_t for $t = 1, \ldots, T$ satisfying $0 \leq \alpha_t \leq \beta_t \leq 1$. He then computes $\tilde{\theta}_s$ for each $s = (t, x)$ such that $x \leq \alpha_t$. Letting $x(s)$ give the second component of s, we can write this as $x(s) \leq \alpha_{t(s)}$. Less formally, α_t gives the fraction of the type t contingencies for which he computes $\tilde{\theta}_s$. The contract he proposes has $\hat{a}(s) = 1$ if $x(s) \leq \alpha_{t(s)}$ and $\psi_{t(s)}(\theta_s) = 1$ or if $x(s) \in (\alpha_{t(s)}, \beta_{t(s)}]$. The contract includes all those contingencies s with $x(s) \leq \beta_{t(s)}$ — that is, β_t is the fraction of the type t contingencies included in the contract. The price is the largest price player 2 will accept given player 1's choice of the α_t's and β_t's. These simplifications provide a very straightforward way to characterize equilibrium contracts.

Given a choice of $(\alpha, \beta) = (\alpha_1, \ldots, \alpha_T, \beta_1, \ldots, \beta_T)$ and p, if player 2 accepts the contract offer, he chooses $\hat{p}(s)$ so that his utility is constant across contingencies included in the contract. Hence his expected payoff from accepting the contract offer is

$$\gamma u \left(\frac{1}{\gamma} \sum_t \phi_t [\alpha_t \mathrm{E}_\theta(\psi_t v_t) + (\beta_t - \alpha_t) \mathrm{E}_\theta(v_t)] - p \right) + \sum_t \phi_t (1 - \beta_t) u_t^{nc}$$

where $\gamma = \sum_t \phi_t \beta_t$ is the probability that the contingency is one which is included in the contract. Player 2's expected payoff from rejecting the offer is

$$\sum_t \phi_t[\alpha_t u_t^c + (1-\alpha_t)u_t^{nc}].$$

Clearly, given (α, β), player 1 sets p so that 2 is indifferent between accepting and rejecting the contract. Solving, we see that

$$p(\alpha,\beta) = \frac{1}{\gamma}\sum_t \phi_t[\alpha_t \mathrm{E}_\theta(\psi_t v_t) + (\beta_t - \alpha_t)\mathrm{E}_\theta(v_t)]$$
$$-u^{-1}\left(\frac{1}{\gamma}\sum_t \phi_t[\alpha_t u_t^c + (\beta_t - \alpha_t)u_t^{nc}]\right).$$

For convenience, let $U(\cdot) = u^{-1}(\cdot)$. Finally, $\pi(\alpha,\beta)$, player 1's expected profit given that he chooses (α,β) and 2 accepts the contract offer, is

$$\pi(\alpha,\beta) = \gamma p(\alpha,\beta) - \sum_t \phi_t[\alpha_t \mathrm{E}_\theta(\psi_t c_t) + (\beta_t - \alpha_t)\mathrm{E}_\theta(c_t)]$$
$$+\sum_t \phi_t(1-\beta_t)\mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)].$$

Substituting for p and rearranging yields:

$$\pi(\alpha,\beta) = \sum_t \phi_t\Big\{\alpha_t \mathrm{E}_\theta[\psi_t(v_t - c_t)]$$
$$+(\beta_t - \alpha_t)\mathrm{E}_\theta(v_t - c_t) + (1-\beta_t)\mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)]\Big\}$$
$$-\gamma U\left(\frac{1}{\gamma}\sum_t \phi_t[\alpha_t u_t^c + (\beta_t - \alpha_t)u_t^{nc}]\right).$$

Theorem. *In equilibrium, player 1 chooses (α,β) to maximize $\pi(\alpha,\beta)$ subject to the contraints that $0 \le \alpha_t \le \beta_t \le 1$ for all t. He computes $\tilde{\theta}_s$ for every s such that $x(s) \le \alpha_{t(s)}$ and offers the contract $(\hat{S}, \hat{a}(\cdot), p)$ satisfying:*

$$s \in \hat{S} \iff x(s) \le \beta_{t(s)},$$

$$p = p(\alpha,\beta),$$

and

$$\hat{a}(s) = \begin{cases} 1, & \textit{if } x(s) \le \alpha_{t(s)} \textit{ and } \psi_{t(s)}(\theta_s) = 1; \\ 1, & \textit{if } \alpha_{t(s)} < x(s) \le \beta_{t(s)}; \\ 0, & \textit{otherwise.} \end{cases}$$

Player 2 accepts the offer.

Let α_t^* and β_t^* denote optimal values of α_t and β_t. (These will not necessarily be unique.) As noted above, in his choice of α_t, the seller has to trade off the worsening of his bargaining position with the ability to offer a better contract. Not surprisingly, that tradeoff can lead the seller to choose $\alpha_t^* < \beta_t^*$. It is also easy to see that this might lead to $\beta_t^* < 1$. When $\alpha_t^* < \beta_t^*$, the seller is not able to perfectly specify which type t contingencies are the appropriate ones to trade in. Hence if he includes many type t contingencies in the contract, there are many contingencies in which inefficient trade occurs. Obviously, this is costly to the seller.

Another way of looking at why incomplete contracts may emerge is given in Proposition 1. Let r_t^c and r_t^{nc} denote the risk premia for the buyer for the gambles $\psi_t(v_t - c_t)$ and $d_t(v_t - p_t)$ respectively. That is,

$$u(\mathrm{E}_\theta[\psi_t(v_t - c_t)] - r_t^c) = u_t^c$$

and

$$u(\mathrm{E}_\theta[d_t(v_t - p_t)] - r_t^{nc}) = u_t^{nc}.$$

By (A2) and the strict concavity of u, $r_t^c > 0$. Also, the concavity of u implies $r_t^{nc} \ge 0$.

Proposition 1. *If $\beta_t^* < 1$ for some t such that $r_t^{nc} > 0$, then it is common knowledge that there is a complete contract which strictly Pareto dominates the seller's offer. That is, given the true $\underline{\theta} \in \bar{\Theta}^S$, both parties would be strictly better off. However, the identity of that contract is not common knowledge.*

In other words, the seller makes an offer he knows to be Pareto suboptimal because he does not know what contract would be Pareto superior. He does not wish to learn what contract would be Pareto superior because this strengthens the buyer's bargaining position so much that the buyer would no longer accept this contract. (Note that if $r_t^{nc} = 0$, then the anticipated renegotiation itself insures the buyer, so an incomplete contract could be Pareto optimal.)

In the remainder of this subsection, I develop conditions which are sufficient for either $\beta_t^* = 0$ or $\beta_t^* = 1$. These conditions do not characterize the equilibrium contract for all parameter values. The proposition below gives two simple conditions, either of which is sufficient for all type t contingencies to be included in the contract.

Proposition 2. *If*

(1) $$r_t^c > \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] + \mathrm{E}_\theta[d_t^c(v_t - p_t^c) - \psi_t(v_t - c_t)],$$

or

(2) $$r_t^{nc} > \mathrm{E}_\theta[(1 - d_t^{nc})(c_t - v_t)]$$

then $\beta_t^* = 1$.

Intuitively, both conditions say that the buyer is willing to pay a lot to insure against risk on the type t contingencies. Recall that it is the possibility of such insurance that makes complete contracts optimal. Proposition 2 has two very intuitive corollaries.

Corollary 1. *If* $\psi_t \equiv 1$ *and* $r_t^{nc} > 0$, *then* $\beta_t^* = 1$.

Proof: Suppose $\psi_t \equiv 1$. For any θ such that $d_t^{nc}(\theta) = 0$, we have $(1 - d_t^{nc}(\theta))(c_t(\theta) - v_t(\theta)) < 0$. Hence $\mathrm{E}_\theta[(1 - d_t^{nc})(c_t - v_t)] \leq 0$, so $r_t^{nc} > 0$ implies (2). ▌

If $\psi_t \equiv 1$, then there is no need for player 1 to compute to choose an efficient contract; he knows he should offer $\hat{a}(s) = 1$ for all s. Furthermore, the fact that $r_t^{nc} > 0$ means that he cannot insure the buyer through renegotiation — he can only provide insurance with a complete contract. Therefore, there is no tradeoff between efficiency and his bargaining position. Put differently, if $\psi_t \equiv 1$ and $r_t^{nc} > 0$ for all t, an incomplete contract cannot be an equilibrium contract since it is common knowledge what the strictly Pareto dominating complete contract is. When $\psi_t \not\equiv 1$, the existence of such a contract is known, but not its identity.

Corollary 2. *If $p_t^{nc} \equiv c_t$, then $\beta_t^* = 1$.*

Proof: By (A1), for any θ such that $d_t^c(\theta) = 1$, we must have $\psi_t(\theta) = 1$ and $p_t^c(\theta) \geq c_t(\theta)$. Hence for such a θ,

$$d_t^c(\theta)[v_t(\theta) - p_t^c(\theta)] - \psi_t(\theta)[v_t(\theta) - c_t(\theta)] = c_t(\theta) - p_t^c(\theta) \leq 0.$$

For any θ such that $d_t^c(\theta) = 0$,

$$d_t^c(\theta)[v_t(\theta) - p_t^c(\theta)] - \psi_t(\theta)[v_t(\theta) - c_t(\theta)] = -\psi_t(\theta)[v_t(\theta) - c_t(\theta)] \leq 0.$$

Hence the second term on the right–hand side of (1) is nonpositive. If $p_t^{nc} \equiv c_t$, the first term on the right–hand side of (1) is zero. Hence $r_t^c > 0$ implies (1). ▌

When $p_t^{nc} \equiv c_t$, player 1 has no bargaining advantage to lose by computing. Hence he has no reason not to find the optimal complete contract and offer it.

In light of Proposition 2, a necessary condition for $\beta_t^* < 1$ is

(3) $$\mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] > \mathrm{E}_\theta[\psi_t(v_t - c_t)] - \mathrm{E}_\theta[d_t^c(v_t - p_t^c)].$$

Also, if $r_t^{nc} > 0$, then $\beta_t^* < 1$ also requires

(4) $$\mathrm{E}_\theta[d_t^{nc}(v_t - c_t)] > \mathrm{E}_\theta[v_t - c_t].$$

(If $r_t^{nc} = 0$, then (4) modified by replacing the strict inequality with a weak one is necessary.) The following proposition shows that if (3), (4), and one other condition hold, then $\beta_t^* = 0$ if the buyer is not too risk averse. Let

$$\bar{w} = \max_{\theta,t} v_t(\theta) - c_t(\theta).$$

Proposition 3. *If (3) and (4) hold and if* $u_t^c \geq u_t^{nc}$*, then there exists a* $\bar{\epsilon}_t > 0$ *such that if*

$$1 - \frac{u'(\bar{w})}{u'(0)} < \epsilon_t,$$

then $\beta_t^* = 0$.

Since $u(\cdot)$ is strictly concave, we must have $u'(0) > u'(\bar{w})$ or $1 - [u'(\bar{w})/u'(0)] > 0$. However, the closer $u(\cdot)$ is to a linear function, the smaller $1 - [u'(\bar{w})/u'(0)]$ will be. Hence, intuitively, $1 - [u'(\bar{w})/u'(0)] < \bar{\epsilon}_t$ says that the buyer is "close" to risk neutral.[11] Just as with Proposition 2, this result says that the key tradeoff for the seller is the bargaining advantage of incomplete computation versus what he can earn by insuring the buyer.

Note that this proposition also assumes $u_t^c \geq u_t^{nc}$ — that is, that first stage computation by player 1 improves player 2's bargaining position, at least weakly. Clearly, if player 1's computations hurt 2's bargaining position, then player 1 has no disincentive to compute. Hence he would certainly compute all the $\tilde{\theta}_s$'s and offer a complete contract.

B. Results for a Specific Renegotiation Game.

It is certainly not obvious that player 1's first stage computation would affect renegotiation. Intuitively, perhaps 1 is better off being uninformed since this might force player 2 to try harder to induce him to accept an agreement. On the other hand, when computation costs are zero or infinitesimal, it is hardly clear that this would be true.

In this subsection, I consider a very simple and reasonably natural specification of the renegotiation subgame and show that it implies (A1),

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11 Given the normalization that $u(0) = 0$, u is determined up to a linear transformation. It is easy to see that $u'(\bar{w})/u'(0)$ is invariant with respect to such transformations.

(A2), and $u_t^c \geq u_t^{nc}$. Furthermore, I give a simple parametric example in which conditions (3) and (4) hold for that renegotiation game. In light of Proposition 3, then, if the buyer is sufficiently close to risk neutral, the equilibrium contract must be incomplete in this example. As we will see, the distinction between almost costless computation and costless computation will be crucial in the renegotiation stage. In particular, with costless computation, the equilibrium contract is complete, while, as noted, with almost costless computation, it may be incomplete.

The renegotiation game is very simple: in a contingency which is not included in the contract, player 2 offers a price, player 1 can compute if he likes, and then player 1 accepts or rejects the price. If he accepts, then trade occurs at the offered price; otherwise, no trade occurs. I also make some additional assumptions on v_t and c_t for the analysis of this game. Let V_t (resp. C_t) denote the set of realizations of $v_t(\tilde{\theta}_s)$ (resp. $c_t(\tilde{\theta}_s)$), where I assume $\#V_t \geq 2$ and $\#C_t \geq 2$. I also assume that for each t, the set of realizations of $(v_t(\tilde{\theta}_s), c_t(\tilde{\theta}_s))$ is just $V_t \times C_t$ — that is, the support of v_t is independent of c_t and vice versa. For simplicity, I assume that $V_t \cap C_t = \emptyset$ for all t. Let $\bar{v}_t$ (resp. $\bar{c}_t$) denote the largest element of V_t (resp. C_t). Finally, I assume that $\bar{v}_t > \bar{c}_t$.

To analyze this subgame, suppose that the contingency is s and that s is not covered by contract. Let t denote the type of contingency s. First, suppose that if the seller has already computed $\tilde{\theta}_s$. In this case, of course, computation costs are irrelevant. Clearly, we are effectively in a complete information game. Hence if $v_t(\theta) \geq c_t(\theta)$, the buyer will offer a price of $c_t(\theta)$ and the seller will accept. Otherwise, there will be no trade. Therefore, with this renegotiation game, $d_t^c \equiv \psi_t$ and $p_t^c \equiv c_t$, implying that (A1) is satisfied. Also, to see that (A2) holds, note that the fact that the support of v_t is independent of c_t together with $\bar{v}_t > \bar{c}_t$ implies that there exists a θ such that

$$\psi_t(\theta)[v_t(\theta) - c_t(\theta)] = \bar{v}_t - \bar{c}_t.$$

But since $\#C_t \geq 2$, the support independence also implies that there exists

$c' < \bar{c}_t$ and θ' such that

$$\psi_t(\theta')[v_t(\theta') - c_t(\theta')] = \bar{v}_t - c' \neq \bar{v}_t - \bar{c}_t,$$

implying (A2).

Next consider the case where the seller has not computed $\tilde{\theta}_s$, first under the assumption of costless computation. No matter what offer the buyer makes, any strategy for player 1 which does not involve computing $\tilde{\theta}_s$ is dominated.[12] Hence player 2 knows that player 1 will learn θ_s. In effect, then, we are in a complete information world again, so the outcome is the same. By Corollary 2 to Proposition 2, we see that we must have complete contracts in equilibrium if computation is costless.[13]

Finally, suppose the seller has not computed $\tilde{\theta}_s$ and computation is almost costless. To characterize the equilibrium outcome in this case requires some additional notation. Let $\hat{c}_t$ be the smallest $c \in C_t$ such that for all $v \in V_t$ with $v > c$, it is true that $v > \bar{c}_t$. Since $\bar{v}_t > \bar{c}_t$ by assumption, $\hat{c}_t$ is well–defined. More intuitively, if there are values of v between the two highest possible values of c, then $\hat{c}_t = \bar{c}_t$. If not but there are values of v between the second and third highest values of c, then $\hat{c}_t$ is the second highest $c \in C_t$, etc. Finally, let

$$\Theta_t = \{\theta \in \bar{\Theta} \mid v_t(\theta) > \bar{c}_t \;\text{ and }\; c_t(\theta) \geq \hat{c}_t\}.$$

The following lemma is an extension of results in Lipman [1990].

Lemma. *p_t^{nc} and d_t^{nc} must satisfy*

(L1) $$\forall \theta \in \Theta_t, \;\; d_t^{nc}(\theta) = 1 \text{ and } p_t^{nc}(\theta) = \bar{c}_t$$

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12 More precisely, this is true when the price offered by player 2 is between the largest and smallest elements of C_t.

13 It is worth noting that this is the only result which uses the elimination of dominated strategies.

and

(L2) $$\forall\theta,\ \ d_t^{nc}(\theta)[p_t^{nc}(\theta) - c_t(\theta)] \geq 0.$$

Combining (L1) and (L2), the seller's expected profits must be strictly positive if there are values of $\theta \in \Theta_t$ such that $c_t(\theta) < \bar{c}_t$.

To see the intuition behind this result, return for a moment to the case where computation is costless. For expositional ease, suppose that $v_t(\theta) > c_t(\theta)$ for all θ. As explained above, the seller must compute $\tilde{\theta}_s$ in stage 2 when computation is costless. This leads the buyer to offer a price equal to the seller's costs. Hence in equilibrium, the seller carries out the computation necessary to learn his costs, even though this information is revealed to him by the buyer's offer. When computation is costless, the redundant computation does not hurt the seller, so his strategy is optimal.

However, when computation is costly, even infinitesimally so, this is not optimal for the seller. Hence we cannot have an equilibrium in which for each realization of $\tilde{\theta}_s$, the buyer offers a price equal to the seller's costs. To see why, suppose this is the buyer's strategy. If computation is costly, the seller's best reply is to accept the offer without computation. But the buyer's best reply to this strategy is to offer a price equal to the lowest possible value of the seller's costs, regardless of the true value of the seller's costs. Consequently, in equilibrium, there must be some realizations of $\tilde{\theta}_s$ for which either no trade occurs or trade occurs at a price strictly above the seller's costs.[14] As the Lemma indicates, depending on the relationship between the possible values of v_t and c_t, it may be true that the latter *must* occur in equilibrium.

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14 As this discussion suggests, there may be mixed strategy equilibria as well. See Fishman [1992] for an analysis of the mixed strategy equilibria of this game. Also, the results of Cremer and Khalil [1992] in a different context seem to be driven by very similar reasoning.

It is easy to see that we must have $u_t^c \geq u_t^{nc}$ with this specification of renegotiation. Using the definition of u_t^c and the characterization of d_t^c and p_t^c above,

$$u_t^c = \mathrm{E}_\theta[u(\psi_t(v_t - c_t))] = \sum_{\theta \in \bar{\Theta}} \Pr[\tilde{\theta}_s = \theta] u[\psi_t(\theta)(v_t(\theta) - c_t(\theta))].$$

Since the buyer would not accept a price above his valuation, (L2) implies that trade never occurs in states where $v_t < c_t$. However, d_t^{nc} may not specify trade for some θ's for which trade would be efficient. Hence the fact that $u(z) > 0$ for all $z > 0$ implies

$$u_t^c \geq \sum_{\theta \in \bar{\Theta}} \Pr[\tilde{\theta}_s = \theta] u[d_t^{nc}(\theta)(v_t(\theta) - c_t(\theta))].$$

Finally, using $u' > 0$ and (L2),

$$u_t^c \geq \mathrm{E}_\theta[u(d_t^{nc}(v_t - p_t^c))] = u_t^{nc}.$$

I now give a simple example in which conditions (3) and (4) hold. Fix any integer k^* strictly between 1 and M. Choose any $c_t(\theta)$ function such that $c_t(k^*) < \bar{c}_t$ and let $\underline{c}_t = \min_\theta c_t(\theta)$. Choose any v_t function such that

$$v_t(\theta) \begin{cases} < \underline{c}_t, & \text{if } \theta < k^*; \\ > \bar{c}_t, & \text{otherwise.} \end{cases}$$

In other words, every possible v lies either below all possible values of c or above all possible values. Note, then, that for any value of $c \in C_t$, every $v > c$ must exceed $\bar{c}$. Hence $\hat{c}_t = \underline{c}_t$, so that $c_t(\theta) \geq \hat{c}_t$ for all θ. Also, $v_t(\theta) > \bar{c}_t$ iff $\theta \geq k^*$. Therefore,

$$\Theta_t = \{k^*, k^* + 1, \ldots, M\}.$$

It is easy to see then that the lemma implies

$$d_t^{nc}(\theta) = \begin{cases} 1, & \text{if } \theta \geq k^*; \\ 0, & \text{otherwise,} \end{cases}$$

and $p_t^{nc}(\theta) = \bar{c}_t$ for $\theta \geq k^*$.

Since this bargaining game implies that $d_t^c \equiv \psi_t$ and $p_t^c \equiv c_t$, (3) holds iff

$$\mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] > 0.$$

Substituting, we see that this holds iff

$$\sum_{k \geq k^*} \Pr[\tilde{\theta}_s = \theta][\bar{c}_t - c_t(\theta)] > 0.$$

Since $\bar{c}_t > c_t(k^*)$, this inequality is satisfied, so (3) holds. Substituting into (4), we see that it holds iff

$$\sum_{\theta \geq k^*} \Pr[\tilde{\theta}_s = \theta][v_t(\theta) - c_t(\theta)] > \sum_{\theta} \Pr[\tilde{\theta}_s = \theta][v_t(\theta) - c_t(\theta)]$$

or

$$\sum_{\theta < k^*} \Pr[\tilde{\theta}_s = \theta][v_t(\theta) - c_t(\theta)] < 0.$$

This inequality holds by the assumption that $v_t(\theta) < \underline{c}_t \leq c_t(\theta)$ for all $\theta < k^*$. Hence with the v_t and c_t functions specified above and almost costless computation, all the conditions of Proposition 3 are satisfied. Therefore, if the buyer is sufficiently close to risk neutral, we must have $\beta_t^* = 0$. Since this renegotiation game implies complete contracts with costless computation, we see that almost costless computation and costless computation yield dramatically different conclusions.

IV. Conclusion.

I have provided a simple model of how limited rationality can lead to incomplete contracts. While it is clear that this might occur with high computation costs, the surprising result is that strategic bargaining can lead to incomplete contracts even with infinitesimal computation costs.

The model is quite simplistic in many respects. In this section, I wish to discuss the roles played by some of the simplifications. One important assumption is that computation is observable. Under certain conditions, the main conclusions still hold without this assumption as is shown in earlier versions of this paper. The key to the analysis with unobservable computation is the credibility of incomplete computation. In particular, suppose we have very low computation costs and unobservable computation. Imagine that we have an equilibrium in which the seller does incomplete computation and offers an incomplete contract. Suppose he deviates to complete computation and uses the following strategy. He "admits" his deviation and offers a complete contract if he finds that he can do better this way. Otherwise, he pretends to be ignorant and sticks to the incomplete contract. If computation is unobservable, no deviation is observed by the buyer if the seller sticks with the incomplete contract. Hence the seller may gain from this deviation if the cost of the extra computation is outweighed by the expected gain. With a continuum of contingencies, one can show that the probability of gain this way is zero. Hence for any computation cost, even infinitesimal, the cost outweighs the gain with a continuum of contingencies. More generally, as the number of contingencies goes to infinity, the probability of a gain goes to zero. Hence if the number of contingencies is sufficiently large relative to the cost of computation, incomplete computation will be credible and incomplete contracts possible.

Another important assumption is that the seller proposes an expected price rather than a price function. Without this assumption, the contracts the seller can offer with incomplete computation would be worse for the buyer than the contracts analyzed here since the seller would not know how to choose $\hat{p}(s)$ to insure the buyer. It is worth noting that if the seller could offer either an expected price or a price function, he would always choose the former since it enables him to offer the buyer a better contract (and hence enables him to charge a higher price) without computation which would erode his bargaining position.

Notice that an analogous device for $\hat{a}(s)$ would not satisfy this condition. That is, suppose the seller offered the buyer the right to choose $\hat{a}(s)$ subject to a constraint regarding the fraction of the contingencies in which trade occurs. Since the seller knows exactly how many contingencies he should trade in, he can set the level of the constraint appropriately without computation. However, the buyer would specify trade in those contingencies in which his valuation is highest, without regard to whether his valuation exceeds the seller's costs. Hence, in general, the seller would not wish to offer such a choice to the buyer.

This should also clarify the role of the seller's risk neutrality. In part, of course, this simplifies the determination of the optimal complete contract. In addition, risk neutrality makes the seller willing to allow the buyer to determine how risk will be shared. I expect that if the seller were risk averse and the definition of an offer modified so that the seller offers a price function, then it would be more difficult to get incomplete contracts (since contracts without computation would be less desirable) but not impossible. The analysis would certainly be more complex, however.

Finally, the structure of the bargaining game and the informational assumptions used are very simplistic. The key to the analysis is that the seller may make himself worse off if he reveals that he has acquired information. While the particular renegotiation game in Section III.B. generates this strategic disadvantage to information in a simple way, it is hardly necessary for this effect.

Appendix

Proof of Theorem.

Let y denote the set of s such that player 1 computes $\tilde{\theta}_s$. The stationarity restriction implies that, along the equilibrium path, second stage strategies in a contingency not covered by contract depend only on $t(s)$, whether or not $s \in y$, and θ_s if known. Let d_t^c, d_t^{nc}, p_t^c, and p_t^{nc} be defined as in the text.

An important point to note is that the buyer's first stage response to the seller's offer necessarily reveals no information about contingencies not included in the offer. To see this, recall that, except for a set of measure zero, every realization $\underline{\theta} \in \bar{\Theta}^S$ has exactly proportion $\Pr[\tilde{\theta}_s = 1]$ of the type t contingencies with $\theta_s = 1$, etc. Let L denote this set of realizations. Without loss of generality, we can restrict attention to $\underline{\theta} \in L$. Therefore, there is no aggregate uncertainty. Hence the only way the buyer's response could signal information about the contingencies not covered would be if it revealed which contingencies are associated with which possible values of θ. Since the buyer's payoff only depends on aggregates, this cannot happen in equilibrium.

So suppose the seller has computed $\tilde{\theta}_s$ for every $s \in y$ and that the payoffs in the second stage in a contingency not included in an offer are determined by d_t^{nc}, p_t^{nc}, d_t^c, and p_t^c. What is the optimal offer for the seller to make as a function of the realizations of $\tilde{\theta}_s$ for $s \in y$? Let $\hat{S}$ denote the set of contingencies included in the optimal offer and let $\hat{a}(\cdot)$ be the trading rule specified. Clearly, given $\hat{S}$ and $\hat{a}(\cdot)$, the seller sets the expected price to leave the buyer indifferent between accepting and rejecting. This is true because, without loss of generality, we can assume that the seller is best off if the buyer accepts his offer. If the seller wanted the buyer to reject, he could simply set $\hat{S} = \emptyset$.

Next, consider the optimal choice of $\hat{a}(\cdot)$ given $\hat{S}$, y, $\{\theta_s \mid s \in y\}$, and the way p is determined. Suppose that there is a set of strictly positive measure included in $y \cap \hat{S}$ such that the optimal offer has $\hat{a}(s) \neq \psi_{t(s)}(\theta_s)$. Call this set I. By the finiteness of T and $\bar{\Theta}$, there must be a $(v, c) \in V_t \times C_t$ such that I has a subset, I', with strictly positive measure such that every $s \in I'$ is of type t and has $(v_t(\theta_s), c_t(\theta_s)) = (v, c)$. Suppose $v > c$ and $\hat{a}(s) = 0$ for $s \in I'$. Let γ denote the measure of $\hat{S}$ and γ' the measure of I'. Let p be the price offered. Consider the alternative offer which is identical to this one except that $\hat{a}(s) = 1$ for all $s \in I'$ and the price is changed to $\bar{p}$ where $\bar{p}$ leaves the buyer indifferent between accepting and rejecting the offer. It is easy to see that $\bar{p} - p = \gamma' v/\gamma$. Hence the increase in the seller's expected revenue from the contract, $\gamma(\bar{p} - p)$, is $\gamma' v$. The seller's expected costs increase by $\gamma' c$, so $v > c$ implies that the seller's profits increase. A similar argument shows that the seller can do better if $v < c$ but $\hat{a}(s) = 1$ for all $s \in I'$. Hence, up to a set of measure zero, $\hat{a}(s) = \psi_{t(s)}(\theta_s)$ for all $s \in y \cap \hat{S}$. A slight variation on this argument shows that $\mathrm{E}_\theta(v_t - c_t) > 0$ implies that, up to a set of measure zero, $\hat{a}(s) = 1$ for all $s \in \hat{S} \setminus y$.

Now let us consider the optimal $\hat{S}$ given y, $\{\theta_s \mid s \in y\}$, and the way $\hat{a}(\cdot)$ and p are chosen. It is easy to see that this solution must be independent of the realizations — that is, it is only a function of y itself. To see this, note that the fact that there is no aggregate uncertainty makes the objective function for the seller independent of these realizations. In other words, given the choice of $\hat{a}(\cdot)$, the seller's expected profits depend only on aggregates and hence is independent on the exact realizations $\{\theta_s \mid s \in y\}$. Hence given y, we can compute the optimal set of contingencies to include, the optimal $\hat{a}(\cdot)$ given this and the realizations, and the optimal p given all this.

A convenient fact about the optimal $\hat{S}$ is that it necessarily contains y (up to a set of measure zero). To see this, let η_{ci}^t denote the measure of type t contingencies in $\hat{S} \cap y$, η_{cn}^t the measure of type t contingencies in $y \setminus \hat{S}$, η_{ni}^t

the measure of type t contingencies in $\hat{S} \setminus y$, and $\eta^t_{nn} = 1 - \eta^t_{ci} - \eta^t_{cn} - \eta^t_{ni}$. As in the text, let α_t denote the measure of type t contingencies in y. Of course, $\eta^t_{cn} = \alpha_t - \eta^t_{ci}$. It is easy to see that p must solve

$$u\left(\frac{1}{\gamma}\sum_t \phi_t[\eta^t_{ci}\mathrm{E}_\theta(\psi_t v_t) + \eta^t_{ni}\mathrm{E}_\theta(v_t)] - p\right) = \frac{1}{\gamma}\sum_t \phi_t[\eta^t_{ci}u^c_t + \eta^t_{ni}u^{nc}_t]$$

where $\gamma = \sum_t \phi_t(\eta^t_{ni} + \eta^t_{ci})$. Solving for p and computing the seller's profits gives:

$$\pi = \sum_t \phi_t\Big\{\eta^t_{ci}\mathrm{E}_\theta[\psi_t(v_t - c_t)] + \eta^t_{ni}\mathrm{E}_\theta(v_t - c_t) + \eta^t_{cn}\mathrm{E}_\theta[d^c_t(p^c_t - c_t)]$$
$$+\eta^t_{nn}\mathrm{E}_\theta[d^{nc}_t(p^{nc}_t - c_t)]\Big\} - \gamma U\left(\frac{1}{\gamma}\sum_t \phi_t[\eta^t_{ci}u^c_t + \eta^t_{ni}u^{nc}_t]\right).$$

I now show that, holding α_t fixed, the seller's profits are strictly increasing in η^t_{ci}. The derivative is

$$\frac{\partial \pi}{\partial \eta^t_{ci}} = \phi_t\Big\{\mathrm{E}_\theta[\psi_t(v_t - c_t)] - \mathrm{E}_\theta[d^c_t(p^c_t - c_t)] + (y - u^c_t)U'(y) - U(y)\Big\}$$

where

$$y = \frac{1}{\gamma}\sum_t \phi_t[\eta^t_{ci}u^c_t + \eta^t_{ni}u^{nc}_t].$$

Viewing $\partial\pi/\partial\eta^t_{ci}$ as a function of y, we see that the derivative with respect to y is $(y - u^c_t)U''(y)$. Since u is strictly concave, U is strictly convex, so $U'' > 0$. Hence this is negative for $y < u^c_t$ and positive for $y > u^c_t$, so that $\partial\pi/\partial\eta^t_{ci}$ is minimized at $y = u^c_t$. Therefore, if the derivative is positive when we substitute u^c_t for y, the derivative is necessarily positive at the correct value of y. Hence a sufficient condition for $\partial\pi/\partial\eta^t_{ci} > 0$ is

$$\mathrm{E}_\theta[\psi_t(v_t - c_t)] - \mathrm{E}_\theta[d^c_t(p^c_t - c_t)] - U(u^c_t) > 0.$$

By definition,

$$U(u^c_t) = \mathrm{E}_\theta[d^c_t(v_t - p^c_t)] - r^c_t,$$

so this condition reduces to

$$r^c_t > \mathrm{E}_\theta[(d^c_t - \psi_t)(v_t - c_t)]. \tag{5}$$

For any θ such that $d_t^c(\theta) = 1$, assumption (A1) implies $\psi_t(\theta) = 1$ so that $d_t^c(\theta) - \psi_t(\theta) = 0$. For any θ such that $d_t^c(\theta) = 0$, we must have

$$-\psi_t(\theta)[v_t(\theta) - c_t(\theta)] \leq 0.$$

Hence

$$\mathrm{E}_\theta[(d_t^c - \psi_t)(v_t - c_t)] \leq 0.$$

As mentioned in the text, (A2) and the strict concavity of u imply $r_t^c > 0$, so (5) must hold. Hence, given y, the optimal η_{ci}^t is α_t. Therefore, $y \subseteq \hat{S}$.

All that remains is characterizing the optimal y. As noted in the text, the symmetry of the contingencies of a given type implies that we can characterize y by focusing only on the α_t's, where α_t denotes the measure of the type t contingencies in y. We now see that we can reduce the seller's choice of $\hat{S}$ to a choice of β_t for $t = 1, \ldots, T$, where, as in the text, β_t is the measure of type t contingencies in $\hat{S}$ and where $\beta_t \geq \alpha_t$ for all t. Given the results shown above regarding the optimal $\hat{a}(\cdot)$ and the optimal p, it is easy to see that the seller's profits are given by $\pi(\alpha, \beta)$ as in the text, so that his maximization problem is as described there. ▌

Proof of Proposition 1.

Suppose $\beta_t^* < 1$ for some t. Consider the following alternative contract. The set of contingencies included in the contract is all those with $x(s) \leq \hat{\beta}_{t(s)}$ where $\hat{\beta}_t \in [\beta_t^*, 1]$ for all t. Also, $\hat{a}(s)$ is unchanged for the contingencies included in the seller's offer and is given by $\hat{a}(s) = \psi_{t(s)}(\theta_s)$ for any additional contingencies included in this contract. Finally, $\hat{p}(s)$ perfectly insures the buyer over the contract contingencies with an expected price of p where p solves

$$\hat{\gamma} u\left(\frac{1}{\hat{\gamma}} \sum_t \phi_t[(\hat{\beta}_t - \beta_t^* + \alpha_t^*)\mathrm{E}_\theta(\psi_t v_t) + (\beta_t^* - \alpha_t^*)\mathrm{E}_\theta(v_t)] - p\right) + \sum_t \phi_t(1 - \hat{\beta}_t)u_t^{nc} = \sum_t \phi_t[\alpha_t^* u_t^c + (1 - \alpha_t^*)u_t^{nc}]$$

where $\hat{\gamma} = \sum_t \phi_t \hat{\beta}_t$. The right–hand side gives player 2's expected payoff in the equilibrium, while the left–hand side gives his payoff under this alternative contract. Hence the alternative contract does not change his payoff.

The seller's payoff under this contract is

$$\sum_t \phi_t \Big\{ (\hat{\beta}_t - \beta_t^* + \alpha_t^*) \mathrm{E}_\theta[\psi_t(v_t - c_t)] + (\beta_t^* - \alpha_t^*) \mathrm{E}_\theta(v_t - c_t)$$
$$+ (1 - \hat{\beta}_t) \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] \Big\} - \hat{\gamma} U\Big(\frac{1}{\hat{\gamma}} \sum_t \phi_t [\alpha_t^* u_t^c + (\hat{\beta}_t - \alpha_t^*) u_t^{nc}] \Big).$$

If $\hat{\beta}_t = \beta_t^*$ for all t, then this contract is identical to the seller's equilibrium offer. I now show that for each t such that $\beta_t^* < 1$, the seller's profits are strictly increasing in $\hat{\beta}_t$ throughout the range $[\beta_t^*, 1]$, establishing that the alternative contract with $\hat{\beta}_t = 1$ for all t yields strictly higher profits. Note that

$$\frac{\partial \pi}{\partial \hat{\beta}_t} = \phi_t \left\{ \mathrm{E}_\theta[\psi_t(v_t - c_t)] - \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] + (\hat{y} - u_t^{nc}) U'(\hat{y}) - U(\hat{y}) \right\}$$

where

$$\hat{y} = \frac{1}{\hat{\gamma}} \sum_t \phi_t [\alpha_t^* + (\hat{\beta}_t - \alpha_t^*) u_t^{nc}].$$

Analogously to the reasoning used in the proof of Theorem 2, this derivative is strictly positive if it is positive when we substitute u_t^{nc} for $\hat{y}$. Hence a sufficient condition for $\partial \pi / \partial \hat{\beta}_t > 0$ is

$$\mathrm{E}_\theta[\psi_t(v_t - c_t)] - \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] - U(u_t^{nc}) > 0.$$

But $U(u_t^{nc}) = \mathrm{E}_\theta[d_t^{nc}(v_t - p_t^{nc})] - r_t^{nc}$. Substituting and rearranging yields:

$$r_t^{nc} + \mathrm{E}_\theta[(\psi_t - d_t^{nc})(v_t - c_t)] > 0. \tag{6}$$

For any θ such that $\psi_t(\theta) = 0$, $v_t(\theta) < c_t(\theta)$, so

$$[\psi_t(\theta) - d_t^{nc}(\theta)][v_t(\theta) - c_t(\theta)] = -d_t^{nc}(\theta)[v_t(\theta) - c_t(\theta)] \geq 0.$$

For any θ with $\psi_t(\theta) = 1$,

$$[\psi_t(\theta) - d_t^{nc}(\theta)][v_t(\theta) - c_t(\theta)] = [1 - d_t^{nc}(\theta)][v_t(\theta) - c_t(\theta)] \geq 0.$$

Hence

$$\mathrm{E}_\theta[(\psi_t - d_t^{nc})(v_t - c_t)] \geq 0,$$

so $r_t^{nc} > 0$ implies (6).

Therefore, a complete contract exists which, given the true $\underline{\theta} \in \bar{\Theta}^S$, makes the seller strictly better off and buyer weakly better off than the seller's equilibrium offer.[15] Furthermore, the fact that such a contract exists is independent of $\underline{\theta}$ and hence is common knowledge. However, it is easy to see that if, without additional computation, the seller could identify a strictly Pareto preferred contract, he could strictly increase his profits by offering it. Hence if $\beta_t^* < 1$ for some t, he must not know what contract dominates his offer. ▌

Proof of Proposition 2.

Player 1's choice of a contract must satisfy $\alpha_t^* = \beta_t^*$ if $\partial\pi/\partial\alpha_t > 0$ and $\alpha_t^* = 0$ if $\partial\pi/\partial\alpha_t < 0$. Similarly, $\beta_t^* = 1$ if

$$\max\left\{\frac{\partial\pi}{\partial\beta_t}, \ \frac{\partial\pi}{\partial\beta_t} + \frac{\partial\pi}{\partial\alpha_t}\right\} > 0 \tag{7}$$

and $\beta_t^* = 0$ if this is strictly negative at the optimum. Computing the derivatives:

$$\frac{\partial\pi}{\partial\alpha_t} = \phi_t\left\{\mathrm{E}_\theta[(1-\psi_t)(c_t - v_t)] - U'(y)(u_t^c - u_t^{nc})\right\}$$

and

$$\frac{\partial\pi}{\partial\beta_t} = \phi_t\left\{\mathrm{E}_\theta[v_t - c_t] - \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] + (y - u_t^{nc})U'(y) - U(y)\right\},$$

[15] Since payoffs are continuous in p, it is obviously possible to construct a complete contract that makes both parties strictly better off.

where

$$y = \frac{1}{\gamma^*} \sum_t \phi_t[\alpha_t^* u_t^c + (\beta_t^* - \alpha_t^*) u_t^{nc}]$$

and $\gamma^* = \sum_t \phi_t \beta_t^*$.

Analogously to the proof of Theorem 2, $\partial\pi/\partial\beta_t$, viewed as a function of y, is minimized at $y = u_t^{nc}$. Hence if this derivative is positive at $y = u_t^{nc}$, (7) holds at the optimum, so $\beta_t^* = 1$. Hence a sufficient condition for $\beta_t^* = 1$ is

$$\text{(8)} \qquad \mathrm{E}_\theta[v_t - c_t] - \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] - U(u_t^{nc}) > 0.$$

By definition of r_t^{nc},

$$U(u_t^{nc}) = \mathrm{E}_\theta[d_t^{nc}(v_t - p_t^{nc})] - r_t^{nc}.$$

Substituting into (8) and rearranging yields (2).

Similarly,

$$\frac{\partial\pi}{\partial\beta_t} + \frac{\partial\pi}{\partial\alpha_t} = \phi_t \Big\{ \mathrm{E}_\theta[\psi_t(v_t - c_t)] - \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] + (y - u_t^c)U'(y) - U(y) \Big\}.$$

If this is strictly positive at $y = u_t^c$, (7) holds, implying $\beta_t^* = 1$. Substitution yields

$$\mathrm{E}_\theta[\psi_t(v_t - c_t)] - \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] - \{\mathrm{E}_\theta[d_t^c(v_t - p_t^c)] - r_t^c\} > 0.$$

Rearranging yields (1). ▌

Proof of Proposition 3.

Let $\delta = u'(0) - u'(\bar{w})$. Since u is strictly concave, $\delta > 0$. Let $k = u'(0)$. Clearly, for all $w \in [0, \bar{w}]$,

$$\text{(9)} \qquad u(w) \in [(k - \delta)w, \quad kw]$$

and

$$(10) \qquad u'(w) \in [k - \delta, \quad k].$$

Rearranging (9) and (10) gives

$$(11) \qquad U(y) \in [\frac{y}{k}, \quad \frac{y}{k-\delta}]$$

and

$$(12) \qquad U'(y) \in [\frac{1}{k}, \quad \frac{1}{k-\delta}]$$

for all $y \in [u(0), \ u(\bar{w})]$. Recall that

$$\frac{\partial \pi}{\partial \alpha_t} = \phi_t \left\{ \mathrm{E}_\theta[(1-\psi_t)(c_t - v_t)] - U'(y)(u_t^c - u_t^{nc}) \right\}.$$

By assumption, $u_t^c \geq u_t^{nc}$. Using (9) and (12), it is not difficult to show

$$U'(y)(u_t^c - u_t^{nc}) \geq \mathrm{E}_\theta[d_t^c(v_t - p_t^c)] - \mathrm{E}_\theta[d_t^{nc}(v_t - p_t^{nc})] - \frac{\delta}{k}\mathrm{E}_\theta[d_t^c(v_t - p_t^c)].$$

Hence

$$\frac{\partial \pi}{\partial \alpha_t} \leq \phi_t \Bigg\{ \mathrm{E}_\theta[(1-\psi_t)(c_t - v_t)] - \mathrm{E}_\theta[d_t^c(v_t - p_t^c)] + \mathrm{E}_\theta[d_t^{nc}(v_t - p_t^{nc})] + \frac{\delta}{k}\mathrm{E}_\theta[d_t^c(v_t - p_t^c)] \Bigg\}.$$

Similarly, recall that

$$(13) \quad \frac{\partial \pi}{\partial \beta_t} = \phi_t \left\{ \mathrm{E}_\theta[v_t - c_t] - \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] + (y - u_t^{nc})U'(y) - U(y) \right\}.$$

Using (9) and (12),

$$(14) \qquad u_t^{nc}U'(y) \geq \left(1 - \frac{\delta}{k}\right) \mathrm{E}_\theta[d_t^{nc}(v_t - p_t^{nc})].$$

Also, note that $yU'(y) - U(y)$ is strictly increasing in y, so

$$yU'(y) - U(y) \leq u(\bar{w})U'(u(\bar{w})) - U(u(\bar{w})) = u(\bar{w})U'(u(\bar{w})) - \bar{w}.$$

Using (9) and (13),

$$yU'(y) - U(y) \leq k\bar{w}\left(\frac{1}{k-\delta}\right) - \bar{w} = \frac{\delta}{k}\,\frac{\bar{w}}{1-(\delta/k)}.$$

Substituting for $yU'(y) - U(y)$ and from (14) into (13) and rearranging yields

$$\frac{\partial\pi}{\partial\beta_t} \leq \phi_t\Big\{ \mathrm{E}_\theta(v_t - c_t) - \mathrm{E}_\theta[d_t^{nc}(v_t - c_t)] + \frac{\delta}{k}\mathrm{E}_\theta[d_t^{nc}(v_t - p_t^{nc})] + \frac{\delta}{k}\,\frac{\bar{w}}{1-(\delta/k)}\Big\}.$$

Hence

$$\max\left\{\frac{\partial\pi}{\partial\beta_t}, \frac{\partial\pi}{\partial\beta_t} + \frac{\partial\pi}{\partial\alpha_t}\right\}$$
$$\leq \phi_t \max\{\mathrm{E}_\theta(v_t - c_t) - \mathrm{E}_\theta[d_t^{nc}(v_t - c_t)] + \frac{\delta}{k}K_1(\delta, k),$$
$$\mathrm{E}_\theta[\psi_t(v_t - c_t)] - \mathrm{E}_\theta[d_t^c(v_t - p_t^c)] - \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)] + \frac{\delta}{k}K_2(\delta, k)\}$$

where K_1 and K_2 are bounded from above as $\delta/k \to 0$. Hence if

$$\max\Big\{\mathrm{E}_\theta(v_t - c_t) - \mathrm{E}_\theta[d_t^{nc}(v_t - c_t)], \quad \mathrm{E}_\theta[\psi_t(v_t - c_t)] - \mathrm{E}_\theta[d_t^c(v_t - p_t^c)] - \mathrm{E}_\theta[d_t^{nc}(p_t^{nc} - c_t)]\Big\} < 0, \tag{15}$$

then

$$\max\left\{\frac{\partial\pi}{\partial\beta_t}, \frac{\partial\pi}{\partial\beta_t} + \frac{\partial\pi}{\partial\alpha_t}\right\} < 0$$

whenever δ/k is sufficiently close to zero. Hence $\beta_t^* = 0$ whenever δ/k is sufficiently close to zero. But

$$\frac{\delta}{k} = \frac{u'(0) - u'(\bar{w})}{u'(0)} = 1 - \frac{u'(\bar{w})}{u'(0)}.$$

Clearly, (15) holds if (3) and (4) hold. ▌

Proof of Lemma.

Fix a contingency s not included in a contract where $\tilde{\theta}_s$ is not computed by the seller at stage 1 in equilibrium and fix any equilibrium strategies for this event. I will say that a price is automatically rejected (accepted) if the seller rejects (accepts) the offer without computing $\tilde{\theta}_s$. Let $p^1 > \ldots > p^k$ denote the prices offered by the buyer in equilibrium which are not automatically rejected. It is easy to see that at most one of these prices is automatically accepted — if two were automatically accepted, the buyer would never offer the higher of the two. Similarly, if any of these is automatically accepted, it must be p^1. If the price which is automatically accepted is lower, then p^1 would never be offered.

Suppose that none of these prices is automatically accepted. Since, by hypothesis, none are automatically rejected, this means that the seller's response to each is to compute $\tilde{\theta}_s$. Since this is costly, for each p^i, there must be some θ' such that the buyer offers p^i when $\theta = \theta'$ but $p^i < c_t(\theta')$. Otherwise, the seller would not compute $\tilde{\theta}_s$ in response to p^i. This implies $p^1 < \bar{c}_t$, so the buyer's payoff is zero for any θ such that $c_t(\theta) = \bar{c}_t$. Recall, though, that $\bar{v}_t > \bar{c}_t$. Hence for any θ such that $(v_t(\theta), c_t(\theta)) = (\bar{v}_t, \bar{c}_t)$, the buyer would be strictly better off offering a price of $\bar{c}_t + \epsilon$, as this is necessarily accepted. This contradiction implies that p^1 must be accepted automatically.

Since p^2 is not automatically accepted, we must have $\bar{c}_t \geq p^2$. Suppose that $p^2 \geq \hat{c}_t$. By the definition of $\hat{c}_t$, then, for all $v \in V_t$ such that $v > p^2$, we must have $v > \bar{c}_t$. Since it is a dominated strategy for the buyer to offer a price above his valuation, this implies that whenever the buyer offers p^2, we must have $v_t(\theta) > \bar{c}_t$. Suppose, then, that there is a θ such that the buyer offers p^2 but $c_t(\theta) > p^2$. Since p^2 cannot be automatically accepted, the seller's response is to compute $\tilde{\theta}_s$. Hence for such a θ, the seller rejects p^2, yielding a payoff of zero to the buyer. But since $v_t(\theta) > \bar{c}_t$, this cannot be optimal as offering $\bar{c}_t + \epsilon$ is better. Therefore, for any θ such that p^2 is offered, $c_t(\theta) \leq p^2$. But since computation is not costless, this implies that it is not optimal for the seller to learn $\tilde{\theta}_s$ in response to this offer, a contradiction. Therefore, $p^2 < \hat{c}_t$. Hence for every θ such that $c_t(\theta) \geq \hat{c}_t$, the buyer's offer is rejected unless he offers p^1. Thus for every $\theta \in \Theta_t$, the buyer offers p^1. Clearly, $p^1 \leq \bar{c}_t$.

Suppose there is a θ such that $d_t^{nc}(\theta) = 1$ and $p_t^{nc}(\theta) = p^i < c_t(\theta)$. Player 1 could compute $\tilde{\theta}_s$ whenever p^i is offered and reject the offer for such a θ. Since computation is almost costless, he would be strictly better off, a contradiction. Hence (L2) holds. This implies $p^1 = \bar{c}_t$, so (L1) holds. ▮

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