Queen's Economics Department Working Paper No. 853

Towards a Theory of the Direct-Indirect Tax Mix

Robin Boadway   Maurice Marchand   Pierre Pestieau

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

3-1992

DISCUSSION PAPER #853

Towards a Theory of the Direct-Indirect Tax Mix

by

Robin Boadway,
Maurice Marchand
and Pierre Pestieau

March 1992

Towards a Theory of the Direct-Indirect Tax Mix

by

Robin Boadway*,
Maurice Marchand**
and Pierre Pestieau***

March 1992

ABSTRACT

Despite the fact that all developed economies levy broadly-based indirect taxes alongside direct taxes, little theory is devoted to explaining the direct-indirect tax mix. Our purpose is to show that if different taxes have different evasion characteristics, some optimal tax mix emerges naturally. Assuming that only income tax can be evaded and focusing on a two-class economy, we analyze the case for supplementing optimal (nonlinear) income taxation with commodity taxation, and we develop conditions under which the latter should or should not be uniform.

J.E.L. Numbers: H21 – H24 – H26

This work was financed, in part, by contract No 26 of the programme "Pôle d'attraction interuniversitaire" of the Belgian government, and by the Social Sciences and Humanities Research Council of Canada. We thank Serge Wibaut for his remarks on an earlier draft.

---

* Queen's University, Canada.
** CORE and I.A.G., Université Catholique de Louvain.
*** Université de Liège and CORE.

ISSN-0771 3894

I. Introduction

Virtually all developed economies levy broadly-based indirect taxes alongside direct taxes. In many cases, the revenue generated by the indirect taxes are of a similar order of magnitude to that of direct taxes. Despite this, there is very little theory devoted to explaining the direct-indirect tax mix. Most of the theory of indirect taxes in the optimal tax literature deals with the structure of commodity tax rates rather than the revenue mix. The classic result is that of Ramsey (1927) as developed further by, among others, Hotelling (1938), Samuelson (1986), and Diamond and Mirrlees (1971).

Much of the early literature was presented in the context of a single consumer facing a set of commodity taxes. Even in this context, the issue of direct versus indirect taxation is an interesting one. In a well-known result, Atkinson and Stiglitz (1972) show that if the household's preferences are separable between goods and leisure and homothetic in goods, the optimal commodity tax structure is uniform. If this condition is not satisfied, a differential commodity tax structure is optimal. However, in both cases, the optimal tax structure is achievable by an infinite number of combinations of differential indirect taxes and uniform income tax since the latter is equivalent to a proportional sales tax. Thus, the Ramsey theory constitutes a normative theory of tax *structure* but not tax *mix*.

A similar conclusion emerges in the multi-consumer context where individuals differ in labour productivity and redistributive taxation is used. In another well-known result, Atkinson and Stiglitz (1976) demonstrate that if the fiscal authorities could use a non-linear income tax as well as differential commodity taxes, the commodity tax structure would be uniform if individual preferences were identical across households and weakly separable between goods and leisure. In this case, commodity taxation could be dispensed with entirely since the effect of proportional commodity taxation could be replicated by adjusting the level of the income tax. If the authorities had been restricted to a linear progressive income tax alongside commodity taxes, the conditions required for proportionality would be more restrictive. General results have not been achieved, but Deaton (1979) shows that proportionality of commodity taxes would result if the utility functions were of the Stone-Geary type in goods (and separable in leisure). Atkinson and Stiglitz (1976) also show that proportionality would result if the utility of goods was quadratic.

In any case, whether the optimal commodity tax structure is uniform or not, the existing theory says nothing about the mix of direct and indirect taxes. The same overall tax structure can be attained by varying combinations of those taxes. In this paper we attempt to develop a rationale for a mix of both kinds of taxes. There are various ways of approaching this. One is to argue that the reason for a mix of taxes is to reduce the tax rate coming from each source and thereby improve incentives. Though this may reflect a popular perception, it is not satisfactory conceptually since it relies on the fact that taxpayers do not realize that the rates of both types of broadly-based taxes impinge simultaneously (additively) on incentives. Thus, rational taxpayers should see through this argument.

Our analysis is based instead on the notion that different taxes have different evasion characteristics, and that the incentive to evade may depend on marginal tax rates.[1] For example, the ability to evade income taxes may be positively correlated to one's earning ability, whereas the ability to evade indirect taxes may not. In this case, indirect taxes may be a useful instrument for reducing evasion of taxes by high income persons, albeit at the expense of some redistributive equity. Our theory is based on that notion. However, to make matters very simple, we assume that evasion is possible only for direct taxes.[2] The government is assumed to be able to levy a general non-linear income tax as well as differential commodity taxation. We analyze the case for supplementing direct taxation with indirect taxation, and we develop conditions under which the latter should or should not be proportional. For the particular model we use, the conditions are more restrictive than those of Atkinson and Stiglitz (1976). Our analysis, however, can be viewed as exploratory since we use a fairly specific form for the costs of tax evasion (or income concealment). Those costs depend on the amount of income evaded, but we assume that once they have been incurred, the tax evader is certain to escape detection by tax authorities. Our model thus focuses on the costs of income concealment borne by tax payers and ignores the audit costs supported by tax authorities.

The paper is organized as follows. In the next section the consumer-taxpayer problem is analyzed. Section III then presents the planner's Pareto-optimal tax problem and derives the first-order conditions for optimal income taxation (for any given commodity taxes). In Section IV, these conditions are discussed in the

[1]This point is made by Slemrod (1990).

[2]Note that allowing evasion on commodity taxes would first require deciding whether consumers or producers benefit from it. See Cremer and Gahvari (1991). As well, the use of multi-stage commodity taxes is generally regarded as providing significant shelter from evasion.

REFERENCES

Atkinson, A.B. and J.E. Stiglitz (1972), "The structure of indirect taxation and economic efficiency," *Journal of Public Economics*, 1, 97–119.

Atkinson, A.B. and J.E. Stiglitz (1976), "The design of tax structure: Direct versus indirect taxation," *Journal of Public Economics*, 6, 55–75.

Cremer, H. and F. Gahvari (1991), "Tax evasion and optimal commodity taxation," mimeo, Virginia Polytechnic Institute.

Deaton, A. (1979), "Optimality uniform commodity taxes," *Economics Letters*, 2(4), 357–361.

Diamond, P.A. and J.A . Mirrlees (1971), "Optimal taxation and public production. Part I: Production efficiency," and "Part II: Tax rules," *American Economic Review*, 61, 8–27 and 261–278.

Hotelling, H. (1938), "The general welfare in relation to problems of taxation and of railway utility rates," *Econometrica*, 6, 242–269.

Layard, P.R. and A.A. Walters (1978), *Microeconomic Theory*. McGraw Hill.

Mirrlees, J.A. (1976), "Optimal tax theory: A synthesis," *Journal of Public Economics*, 6, 327–358.

Ramsey, F.P. (1927), "A contribution to the theory of taxation," *Economic Journal*, 47–61.

Samuelson, P.A. (1986), Theory of optimal taxation," *Journal of Public Economics*, 30, 137–143.

Slemrod, J. (1990), "Optimal taxation and optimal tax systems," *Journal of Economic Perspectives*, 4, 157–178.

Stern, N. (1982), "Optimum taxation with errors in administration," *Journal of Public Economics*, 17, 181–211.

Tuomala, M. (1990), *Optimal Income Tax and Redistribution*. Clarendon Press, Oxford.

and allowed us to focus more on the question of the mix. The optimal solution is however rather trivial in this case. To minimize evasion costs, the marginal income tax should be nil, and redistribution would be made using commodity taxes along with the lump-sum part of the linear income tax. Furthermore, our purpose was to present the case for commodity taxation with the most stringent income tax (given the informational constraint). We show that in presence of tax evasion the self-selection constraint plays a major role in the design of the tax system.

Further research is needed to incorporate within our setting tax evasion on both direct and indirect taxes. Clearly, some optimal balance between income and commodity taxes would emerge, and it would satisfy conditions somewhat different from those found in the present paper. To have a realistic model, we should however recognize that both consumers and producers are commodity-tax evaders, which would require significant changes to our model.

Moreover, it was earlier emphasized that our model of tax evasion is highly simplified our analysis can only be viewed as a first step towards better understanding the optimal direct-indirect tax mix in presence of tax evasion. A natural extension would be to account for audit costs alongside evasion costs with some probability for tax evaders of being detected and punished.

no-commodity-tax case and shown to be similar to the standard ones despite the presence of tax evasion. Section V looks at the possibility of improving welfare by imposing uniform commodity taxes and presents the main results of the paper. In our setting of tax evasion it is optimal to supplement income taxes by commodity taxes. In Section VI the issue of uniform versus differentiated commodity taxation is examined, and sufficient conditions for uniformity are derived. The last section draws the conclusions of the paper.

II. A Simple Model of an Economy with Income Tax Evasion

The model we construct allows for the following features in the simplest conceivable way — a role for redistributive taxation, the possibility of differential commodity taxation, and the occurrence of tax evasion. It is an extension of the two-person self-selection model used by Stiglitz (1982) and Stern (1982) to analyze optimal income taxation. In this model, there are two types of persons, 1 and 2, who differ only in their labour productivity w_i ($i = 1,2$). This is defined as the number of efficiency units of labour they provide per unit of time spent working. By convention, $w_2 > w_1$ so person 2 is the high-ability person. There are N_i persons of type i. Labour is the only input used in the economy, and two consumption goods, a and b, are produced. The technology is linear and transforms one efficiency unit of labour into one unit of either good a and b.[3] Persons have the same strictly concave utility functions $u(c_i^a, c_i^b, \ell_i)$ where c_i^a is consumption of good a by person i and ℓ_i is his labour supply. Throughout the paper, we assume non-inferiority of both leisure and goods. Using an efficiency unit of labour as numeraire and assuming competitive markets, the producer prices of c^a and c^b are equal to unity. Our notational convention is to use subscripts to denote households and superscripts to denote commodities wherever possible.

Household i earns an amount of income $w_i\ell_i$, of which a proportion σ_i is reported for income tax purposes. As the rates of marginal income taxation may be either positive or negative, we allow for both under- and overreporting of income ($\sigma_i \leq 1$ and $\sigma_i \geq 1$). With a positive marginal income tax, the benefit of underreporting is the reduction in tax liabilities whereas with a negative one, that of

[3]These assumptions are all drawn from the well-known optimal income tax model of Mirrlees (1971). Our use of the Stiglitz–Stern two-person version simplifies matters considerably as well as allowing us to take advantage of the self-selection constraint in a useful way.

overreporting comes from the rise in income transfer. This will be discussed below in more detail. The cost of not reporting income honestly is modeled in a very simple way. It is determined by the following expression:

$$w_i \ell_i g(1-\sigma_i)$$

where the function $g(\cdot)$ is assumed to be nonnegative, strictly convex, and to satisfy $g(0)=0$ and $g'(0)=0$. Therefore, the marginal cost of income concealment rises with the proportion of income under- or overreported (and so with the absolute value of $1-\sigma_i$ if it is negative). Note that costs of concealment are proportional to gross income $w_i \ell_i$.

We have chosen not to model explicitly the decision of how much income to conceal as a decision under uncertainty with a probability of getting caught and a penalty for being caught. As will be seen, the problem is already complex enough, and would become exceedingly so were we to make the concealment decision more explicit. The hypothesis that concealment costs are linear in income is made to facilitate the generating of qualitative results, as will become obvious below. Of course, the results would be expected to differ if concealment costs were, say, decreasing more than proportionally with income.

Individuals face two types of taxes — a nonlinear income tax and an indirect tax system. The latter is assumed to be a set of per unit taxes at the rates (q^a-1) and (q^b-1), where q^a and q^b are the consumer prices, identical for both households (recall that producer prices are equal to unity). The income tax system is a general nonlinear function of reported income. Through labour supply and income reporting, each household will choose a point on the tax schedule as part of his optimizing behaviour. At that point, we can define a *virtual budget constraint* by linearizing the after-tax budget constraint. That virtual budget constraint will have associated with it a marginal tax rate and a lump-sum component, both unique to the household. It turns out to simplify our analysis to work with this virtual budget constraint and allow the planner to choose individual-specific marginal and lump-sum tax rates. Denote these by $(1-\tau_i)$ and T_i respectively for household i. The household's budget constraint can then be written:

$$q^a c_i^a + q^b c_i^b = w_i \ell_i (1-\sigma_i+\sigma_i \tau_i) - T_i - w_i \ell_i g(1-\sigma_i). \quad (1)$$

Household i chooses consumption of the two goods, labour supply and the amount of income to report by solving the following optimization problem:

$$\max_{c_i^a, c_i^b, \ell_i, \sigma_i} u(c_i^a, c_i^b, \ell_i) \quad (2)$$

VI. Conclusion

The combination of direct and indirect taxation raises a challenging question for public finance. On the one hand, it is observed almost everywhere and is often justified by tax practitioners on grounds of incentives and compliance. On the other hand, the direct-indirect tax mix cannot be based on results available in the modern theory of optimal commodity and income taxation. This theory sends two alternative messages: either income taxation suffices or, when the tax mix is desirable, it is rather indeterminate since a given tax structure can be attained by an arbitrary combination of levels of direct and indirect taxation.

In this paper, we have incorporated the possibility of tax evasion within a simple two-ability-two-commodity model of direct and indirect taxation. To reflect the empirical observation that problems of compliance concern income taxation to a larger extent than commodity taxation, we have adopted the simplifying assumption that only income taxes could be evaded. Within this particular setting, we show that without commodity taxation, the introduction of tax evasion does not affect the standard results of zero marginal tax rate on the higher ability person and positive marginal tax rate on the lower ability person. When uniform commodity taxation is allowed for, we indicate that a mix of income taxation with negative marginal rates on one and possibly both types of households and of commodity taxation with positive rates is likely to result as the optimal solution. Sufficient conditions leading to optimal uniform rather than differentiated commodity taxes are also derived.

Without evasion, there is an equivalence between a uniform commodity tax and an income (wage) tax. Thus, when a uniform commodity tax is combined with an income tax, all or part or the former can be incorporated into the latter. However, with tax evasion, there is no longer equivalence between reported earnings and consumption. This provides a strong case for some well-defined commodity taxation. Sufficient conditions leading to optimal uniform rather than differentiated commodity taxes are also derived. It is shown that the usual condition of separability between leisure and commodities is not sufficient. This must be supplemented with quasi-homotheticity of indifference curves in the commodity space.

It might be suggested that, within our setting, it would have been much simpler analytically to restrict the planner to linear income taxation alongside the indirect tax system. This would have avoided the complexities of the nonlinear tax where m_i^a is the marginal propensity to consume c_i^a out of income, m_i^b is the same for c_i^b, and K_i does not depend on good a or b. From (35) we obtain:

$$\frac{\tilde{\ell}_{ia}}{\tilde{\ell}_{ib}} = \frac{m_i^a}{m_i^b}, \qquad i = 1, 2.$$

Using this, equation (34) becomes:

$$\frac{c_1^a - \bar{c}_2^a + \frac{m_1^a}{m_1^b} x\tilde{\ell}_{1b}}{c_1^b - \bar{c}_2^b + x\tilde{\ell}_{1b}} = \frac{\frac{m_1^a}{m_1^b} z_1\tilde{\ell}_{1b} + \frac{m_2^a}{m_2^b} z_2\tilde{\ell}_{2b}}{z_1\tilde{\ell}_{1b} + z_2\tilde{\ell}_{2b}}. \tag{36}$$

A condition for (36) to be satisfied is that:

$$\frac{c_1^a - \bar{c}_2^a}{c_1^b - \bar{c}_2^b} = \frac{m_1^a}{m_1^b} = \frac{m_2^a}{m_2^b}.$$

That is, the ratio of the marginal propensities to consume are independent of income. This will be the case if and only if the consumer's utility function is not only separable but also quasi-homothetic in goods. This is summarized as follows:

Proposition 4. *If preferences are separable and quasi-homothetic in goods, the optimal commodity tax structure will be uniform ($q^a = q^b = q$).*

In other words, a sufficient condition for the commodity tax structure to be uniform is that the utility function take the form $u\left(h(c_i^a, c_i^b), l_i\right)$ where the sub-utility function $h(\cdot)$ is homothetic to some arbitrary point, which may or may not be the origin. Notice that this condition is similar to that required to be able to aggregate demand functions over households. It is less restrictive than the condition required for uniformity of commodity tax rates when there is no income tax (see Atkinson and Stiglitz (1972)), but more restrictive than that required when non-linear income taxation is possible in the absence of evasion (see Atkinson and Stiglitz (1976)). A special case is the Stone–Geary utility function which is both quasi-homothetic and has unit elasticity of substitution among goods.

When those conditions are satisfied and thus commodity taxation is uniform, the results derived in the previous section apply readily.[10]

[10] When they are not satisfied, we are unable to obtain general conditions under which, e.g., $q^a > q^b$. For example, it is not sufficient that the ratio of marginal propensities to consume rise with income (e.g., $\frac{m_2^a}{m_2^b} > \frac{m_1^a}{m_1^b}$). That is, we cannot say that luxuries ought to be taxed more heavily than necessities. The ability to relax the self-selection constraint apparently does not depend on that alone. The structure of commodity taxes which satisfies (32) and (33) depends upon all households' characteristics in a very complicated way. Also, we cannot guarantee that one of the tax rates will not be driven to infinity.

subject to budget constraint (1). The first-order conditions are:

$$u_{ia} - \alpha_i q^a = 0 \tag{3}$$

$$u_{ib} - \alpha_i q^b = 0 \tag{4}$$

$$u_{i\ell} + \alpha_i[w_i(1 - \sigma_i + \sigma_i\tau_i) - w_i g_i] = 0 \tag{5}$$

$$g_i' = 1 - \tau_i \tag{6}$$

where α_i stands for the marginal utility of income, $g_i' = g'(1-\sigma_i)$, u_{ia} is the derivative of u_i with respect to c_i^a and so on. These conditions solve for $(c_i^a, c_i^b, \ell_i, \sigma_i)$ in terms of (τ_i, T_i, q^a, q^b). Note that by (6) σ_i depends only on τ_i and that the strict convexity of g_i implies that $\sigma_{i\tau} > 0$. Furthermore, $\sigma_i >, =, < 1$ as $\tau_i >, =, < 1$. That is, the individual over reports if the marginal tax rate is negative, underreports if it is positive, and is honest if the marginal tax rate is zero. These will be relevant for the results derived below. We need an additional assumption to guarantee that a global maximum is reached once σ_i satisfies (6). It is required that $(1-\sigma_i)(1-\tau_i) > g_i$, i.e., the benefit of under- or overreporting is larger than its cost. Using (6), a sufficient condition for this is readily seen to be that the elasticity of g_i with respect to $(1-\sigma_i)$, that is $(1-\sigma_i)g_i'/g_i$ be larger than one for any σ_i. Substituting the behavioural relationships back into the household utility relationships yields the indirect utility function $v(\tau_i, T_i, q^a, q^b)$. From the envelope theorem, the indirect utility function satisfies the following Roy's identities:

$$v_{\tau_i} = \alpha_i w_i \ell_i \sigma_i \tag{7}$$

$$v_{T_i} = -\alpha_i \tag{8}$$

$$v_{iq^a} = -\alpha_i c_i^a \tag{9}$$

$$v_{iq^b} = -\alpha_i c_i^b. \tag{10}$$

In the self-selection model of optimal income taxation, the planner is precluded from attaining a first-best allocation because it is not possible to base taxes directly on ability; only reported income can be observed. This gives rise to the possibility that the high-ability person can reduce tax liabilities by mimicking the reported income of the low-ability person. Therefore, a self-selection constraint facing the planner will be that the high-ability person must be at least as well off when not mimicking. Denote by an 'overbar' the mimicking person. Then, the mimicking of person 1's reported income by person 2 implies:

$$\sigma_1 w_1 \ell_1 = \bar{\sigma}_2 w_2 \bar{\ell}_2. \tag{11}$$

Note that the mimicker's labour supply ($\bar{\ell}_2$) follows directly from his choice of $\bar{\sigma}_2$. The budget constraint of the mimicker may be written:

$$q^a\bar{c}_2^a + q^b\bar{c}_2^b = w_2\bar{\ell}_2(1-\bar{\sigma}_2+\bar{\sigma}_2\tau_1) - T_1 - w_2\bar{\ell}_2 g(1-\bar{\sigma}_2).$$

Using (11), this becomes:

$$q^a\bar{c}_2^a + q^b\bar{c}_2^b = w_1\ell_1\left(\frac{\sigma_1}{\bar{\sigma}_2} - \sigma_1 + \sigma_1\tau_1\right) - T_1 - w_1\ell_1\frac{\sigma_1}{\bar{\sigma}_2}g(1-\bar{\sigma}_2). \tag{12}$$

The mimicker chooses the proportion of income he reports and his consumption bundle by solving the following problem:

$$\max_{\bar{c}_2^a,\bar{c}_2^b,\bar{\sigma}_2} u\left(\bar{c}_2^a, \bar{c}_2^b, \frac{w_1\sigma_1}{w_2\bar{\sigma}_2}\ell_1\right)$$

subject to budget constraint (12). The first-order conditions reduce to:

$$\bar{u}_{2a} - \bar{\alpha}_2 q^a = 0 \tag{13}$$
$$\bar{u}_{2b} - \bar{\alpha}_2 q^b = 0 \tag{14}$$
$$\bar{u}_{2\ell} + \bar{\alpha}_2 w_2(1-\bar{g}_2-\bar{\sigma}_2\bar{g}_2') = 0. \tag{15}$$

where $\bar{\alpha}_2$ is the mimicker's marginal utility of income. Note that an expression similar to (15) is obtained by combining (5) and (6); however, the mimicker does not equate $\bar{g}_2'$ with $1-\tau_1$. This is because $\bar{\ell}_2$ and $\bar{\sigma}_2$ cannot be chosen independently of each other. To reach the same level of reported income as person 1, the mimicker can either work many hours and conceal a large part of his income or work few hours and conceal a small part of his income. Condition (15) characterizes the optimal tradeoff between the disutility of labour supply and the cost of income concealment.

Equations (13), (14), and (15) can be solved for $\bar{c}_2^a$, $\bar{c}_2^b$ and $\bar{\sigma}_2$, and yield the indirect utility function $\bar{v}(\tau_1, T_1, q^a, q^b)$. From the envelope theorem, the indirect utility function satisfies the following properties (after some simplification using (13), (14), and (15)):

$$\bar{v}_{\tau_1} = \bar{\alpha}_2\left[w_1\ell_1\sigma_1 + w_1(\bar{g}_2' - (1-\tau_1))(\sigma_1\ell_{1\tau} + \ell_1\sigma_{1\tau})\right] \tag{16}$$
$$\bar{v}_{T_1} = \bar{\alpha}_2\left[-1 + w_1\sigma_1(\bar{g}_2' - (1-\tau_1))\ell_{1T}\right] \tag{17}$$
$$\bar{v}_{q^a} = \bar{\alpha}_2\left[-\bar{c}_2^a + w_1\sigma_1(\bar{g}_2' - (1-\tau_1))\ell_{1a}\right] \tag{18}$$
$$\bar{v}_{q^b} = \bar{\alpha}_2\left[-\bar{c}_2^b + w_1\sigma_1(\bar{g}_2' - (1-\tau_1))\ell_{1b}\right]. \tag{19}$$

and

$$-\mu\bar{\alpha}_2\left[c_1^b - \bar{c}_2^b + (\bar{g}_2' - (1-\tau_1))w_1\sigma_1\tilde{\ell}_{1b}\right]$$
$$+\gamma\Big[N_1((1-\tau_1)w_1\sigma_1\tilde{\ell}_{1b} + (q^a-1)\tilde{c}_{1b}^a + (q^b-1)\tilde{c}_{1b}^b) \tag{33}$$
$$+N_2((1-\tau_2)w_2\sigma_2\tilde{\ell}_{2b} + (q^a-1)\tilde{c}_{2b}^a + (q^b-1)\tilde{c}_{2b}^b)\Big] = 0.$$

These are obviously complicated expressions to evaluate so we will not be able to obtain a full characterization of the commodity tax optimum. However, assuming that the solution to (32) and (33) is a global optimum we derive a set of conditions under which the optimal commodity tax structure is uniform so the planner would not want to differentiate q^a from q^b. To do so involves showing the circumstances in which (32) and (33) are satisfied with $q^a = q^b$. To start with, it is useful to rewrite them in another way. First, note that the homogeneity property of consumer demand implies that at $q^a = q^b$ $(= q)$:

$$\tilde{c}_{ia}^a + \tilde{c}_{ib}^b = \frac{w_i}{q}(1-\sigma_i+\sigma_i\tau_i - g(1-\sigma_i))\tilde{\ell}_{ia}.$$

Substituting this into (32) and (33), we obtain:

$$-\mu\bar{\alpha}_2\left(c_1^a - \bar{c}_2^a + x\tilde{\ell}_{1a}\right) + \gamma\left(z_1\tilde{\ell}_{1a} + z_2\tilde{\ell}_{2a}\right) = 0 \tag{32'}$$
$$-\mu\bar{\alpha}_2\left(c_1^b - \bar{c}_2^b + x\tilde{\ell}_{1b}\right) + \gamma\left(z_1\tilde{\ell}_{1b} + z_2\tilde{\ell}_{2b}\right) = 0 \tag{33'}$$

where

$$x \equiv (\bar{g}_2' - (1-\tau_1))w_1\sigma_1$$
$$z_1 \equiv N_1\frac{w_1}{q}(\sigma_1(1-\tau_1) + (q-1)(1-g(1-\sigma_1)))$$
$$z_2 \equiv N_2\frac{w_2}{q}(\sigma_2(1-\tau_2) + (q-1)(1-g(1-\sigma_2))).$$

Dividing (32′) by (33′) we obtain:

$$\frac{c_1^a - \bar{c}_2^a + x\tilde{\ell}_{1a}}{c_1^b - \bar{c}_2^b + x\tilde{\ell}_{1b}} = \frac{z_1\tilde{\ell}_{1a} + z_2\tilde{\ell}_{2a}}{z_1\tilde{\ell}_{1b} + z_2\tilde{\ell}_{2b}}. \tag{34}$$

Next, assume that goods and leisure are separable in the utility function. Then, it can be shown (e.g., Layard and Walters (1978), p. 166) that:

$$\tilde{\ell}_{1a} = m_i^a \cdot K_i \quad \text{and} \quad \tilde{\ell}_{1b} = m_i^b \cdot K_i \tag{35}$$

that is larger than one and finite. It is clear that the marginal income tax rate for the high-ability person must be negative at the optimum[9], and that this person must overreport income. (This is also apparent by setting $q^a = q^b = q > 1$ in equation (26).) Furthermore, we cannot rule out the possibility that the marginal tax rate of the low-ability person is negative as well since the general implication of an increase in q is to increase τ_i. This is summarized in the following proposition.

Proposition 3. *When the commodity tax structure is proportional and q is set at its optimal level, there is a local (likely global) optimum such that the marginal income tax rate for the high-ability person will be negative, and that for the low-ability person is either positive or negative, but less than in the absence of commodity taxation.*

Note that if overreporting were not allowed, the term involving σ_{2r} would not appear in (31) for $q \geq 1$ (since $1 - \tau_2 \leq 0$) and that involving σ_{1r} would disappear as soon as q is large enough to make $1 - \tau_1$ negative. Without the possibility of overreporting, we can therefore expect q to be infinite at the optimum (since $\bar{\sigma}_2 < \sigma_1$ even when $\sigma_1 = 1$).

VI. Finding Conditions for Uniform Commodity Taxes

Suppose we now allow for the possibility of differential commodity tax rates. Differentiating the planner's Lagrangian function (20) with respect to commodity prices q^a and q^b and using the optimal tax conditions yield the following first-order conditions:

$$\begin{aligned} -\mu\bar{\alpha}_2 &\left[c_1^a - \bar{c}_2^a + (\bar{g}_2' - (1-\tau_1))w_1\sigma_1\bar{\ell}_{1a}\right] \\ &+ \gamma\left[N_1((1-\tau_1)w_1\sigma_1\bar{\ell}_{1a} + (q^a-1)\bar{c}_{1a}^a + (q^b-1)\bar{c}_{1a}^b)\right. \qquad (32) \\ &\left. + N_2((1-\tau_2)w_2\sigma_2\bar{\ell}_{2a} + (q^a-1)\bar{c}_{2a}^a + (q^b-1)\bar{c}_{2a}^b)\right] = 0 \end{aligned}$$

[9]It should be pointed out that the overall (direct and indirect) marginal tax rate of person 2, defined as $1-(1-\sigma_2+\sigma_2\tau_2-g_2)/q$, is strictly positive for $q > 1$. Substituting from $q(\bar{c}_{2r}^a + \bar{c}_{2r}^b) = w_2(1 - \sigma_2 + \sigma_2\tau_2 - g_2)\bar{\ell}_{2r}$ into (26), we obtain after rearranging: $(1-\tau_2)w_2\ell_2\sigma_{2r} + [(1-g_2) - (1-\sigma_2+\sigma_2\tau_2-g_2)/q]\bar{\ell}_{2r} = 0$. We infer from this expression that when $q > 1$, the overall marginal tax rate of person 2 is larger than g_2 and so strictly positive. This differs from the result obtained in the standard model without tax evasion where the overall marginal tax rate of the top-end individual is zero. See Tuomala (1990), p. 175.

Note that in comparison to Roy's identities (7) through (10), additional terms appear in these derivatives. They are related to the gap between the marginal cost of concealment and the marginal tax rate that results from the optimal tradeoff just mentioned between the disutility of work and the cost of concealment. From these conditions we immediately deduce the following result which will be of use later:[4]

Lemma 1. *With goods and leisure being noninferior, $\bar{\sigma}_2 < \sigma_1$.*

In other words, when person 1 underreports his income ($\sigma_1 < 1$ with $\tau_1 < 1$), the mimicker evades a larger proportion of income than person 1, which implies $\bar{g}_2' > 1 - \tau_1 = g_1'$. Alternatively, where person 1 overreports his income ($\sigma_1 > 1$ with $\tau_1 > 1$), the mimicker either evades some income ($\bar{\sigma}_2 \leq 1$) or overreports a lower proportion of his income than person $1 (1 < \bar{\sigma}_2 < \sigma_1)$. In all those cases, the available income of the mimicker will exceed that of person one, since their reported incomes and their tax liabilities are the same. Therefore, the value of consumption is higher for the mimicker than for person 1, that is, $q^a\bar{c}_2^a + q^b\bar{c}_2^b > q^a c_1^a + q^b c_1^b$.

III. The Planner's Optimal Tax Problem

The planner chooses the income tax function and the indirect tax rates. Following Stiglitz (1982), we model the planner's problem as a Pareto-optimizing one. The solution to this problem will yield a point on the feasibility frontier of the economy which traces out the second-best utility combinations of the households. We will restrict ourselves to that part of the feasibility locus in which the planner redistributes income from the high- to the low-ability person. This implies that the planner need only be concerned with the self-selection constraint which prevents the high-ability person from preferring to mimic the low-ability one ($v_2 \geq \bar{v}_2$). To trace out the entire feasibility locus, we would need to consider also the self-selection constraint in which the low-ability person would not prefer to mimic the high. As long as we are redistributing from person 2 to person 1, the

[4]Proof: Suppose $\sigma_1 = \bar{\sigma}_2$. From (11), since $w_2 > w_1$, $\bar{\ell}_2 < \ell_1$, then $-(\bar{u}_{2l}/\bar{\alpha}_2) < -(u_{1l}/\alpha_1)$. Consequently, using (5), $-\bar{u}_{2l}/(\bar{\alpha}_2 w_2) < -u_{1l}/(\alpha_1 w_1) = (1 - g_1 - \sigma_1 g_1') = 1 - \bar{g}_2 - \bar{\sigma}_2\bar{g}_2'$ at $\bar{\sigma}_2 = \sigma_1$. To have (15) hold, we must increase $\bar{\ell}_2$ and reduce $\bar{\sigma}_2$, which causes $-\bar{u}_{2l}/(\bar{\alpha}_2 w_2)$ to increase and $(1-\bar{g}_2-\bar{\sigma}_2\bar{g}_2')$ to decrease. We conclude, therefore, that $\bar{\sigma}_2 < \sigma_1$ and $\bar{g}_2' > 1 - \tau_1$. This proof uses noninferiority of both leisure and consumption in the utility function, which we assume throughout.

latter will not bind. The planner also faces a budget constraint or, equivalently, the resource constraint of the economy.[5] We suppose that a given amount of revenue R must be raised. The Lagrangian expression for the planner's problem is as follows:

$$\begin{aligned}\mathcal{L}(\tau_i, T_i, q^a, q^b, \mu, \lambda, \gamma) &= v(\tau_1, T_1, q^a, {}^b) \\ &+ \lambda[v(\tau_2, T_2, q^a, q^b) - V_2] + \mu\left[v(\tau_2, T_2, q^a, q^b)\right. \\ &\left.-\bar{v}(\tau_1, T_1, q^a, q^b)\right] + \gamma\left[N_1((1-\tau_1)w_1 l_1\sigma_1 + T_1 + (q^a-1)c_1^a + (q^b-1)c_1^b)\right. \\ &\left.+N_2((1-\tau_2)w_2 l_2\sigma_2 + T_2 + (q^a-1)c_2^a + (q^b-1)c_2^b) - R\right]\end{aligned} \quad (20)$$

where V_2 is the utility level to be reached by person 2.

Consider initially the first-order conditions for the income tax parameters, taking as given the commodity taxes for a moment. They are as follows:

$$\begin{aligned}&\alpha_1 w_1\ell_1\sigma_1 - \mu\bar{\alpha}_2\left[w_1\ell_1\sigma_1 + w_1(\bar{g}_2' - (1-\tau_1))(\sigma_1\ell_{1\tau} + \ell_1\sigma_{1\tau})\right] \\ &\quad + \gamma N_1\left[-w_1\ell_1\sigma_1 + (1-\tau_1)w_1(\sigma_1\ell_{1\tau} + \ell_1\sigma_{1\tau})\right. \\ &\quad\quad \left.+(q^a-1)c_{1\tau}^a + (q^b-1)c_{1\tau}^b\right] = 0 \end{aligned} \quad (21)$$

$$\begin{aligned}&-\alpha_1 - \mu\bar{\alpha}_2\left[-1 + w_1\sigma_1(\bar{g}_2' - (1-\tau_1))\ell_{1T}\right] \\ &\quad + \gamma N_1\left[1 + (1-\tau_1)w_1\sigma_1\ell_{1T} + (q^a-1)c_{1T}^a + (q^b-1)c_{1T}^b\right] = 0\end{aligned} \quad (22)$$

$$\begin{aligned}&(\lambda+\mu)\alpha_2 w_2\ell_2\sigma_2 + \gamma N_2\left[-w_2 l_2\sigma_2 + (1-\tau_2)w_2(\sigma_2\ell_{2\tau} + \ell_2\sigma_{2\tau})\right. \\ &\quad\quad \left.+(q^a-1)c_{2\tau}^a + (q^b-1)c_{2\tau}^b\right] = 0\end{aligned} \quad (23)$$

$$\begin{aligned}&-(\lambda+\mu)\alpha_2 + \gamma N_2\left[1 + (1-\tau_2)w_2\sigma_2\ell_{2T}\right. \\ &\quad\quad \left.+(q^a-1)c_{2T}^a + (q^b-1)c_{2T}^b\right] = 0.\end{aligned} \quad (24)$$

Note that those conditions must be satisfied for any choice of q_a and q_b. It will be useful for interpretive purposes and further reference to combine these equations in the following way. If we multiply (22) by $w_1\ell_1\sigma_1$ and add the result to (21) and

[5]This resource constraint is: $N_1 w_1\ell_1 + N_2 w_2\ell_2 = N_1(c_1^a + c_1^b) + N_2(c_2^a + c_2^b) + N_1 w_1\ell_1 g_1(1-\sigma_1) + N_2 w_2\ell_2 g_2(1-\sigma_2) + R$.

there is thus an indeterminacy as to the optimal choice of direct and indirect taxes. Effectively, uniform indirect taxes become redundant.

When some income is under- or overreported, proportional commodity taxation cannot be replicated by adjustments in the income tax structure. Three differences arise when true income is not reported for tax purposes, and these are reflected in the terms of equation (31).

First, as already pointed out, a rise in q will reduce the benefit that the mimicker draws from reporting less income than person 1 ($\bar{\sigma}_2 < \sigma_1$),[8] and this relaxes the self-selection constraint. This is taken into account in the first term of (31) which tends to favour an increase in q.

Second, a rise in q induces increases in τ_1 and τ_2, and so in σ_1 and σ_2. In other words, this rise in the commodity tax rate allows for reduction in the marginal income tax rates since the full marginal tax rate from earning income now includes both the direct and the indirect marginal tax rates. The reduction in the marginal income tax rate accompanying an increase in the indirect tax rate affects the proportion of incomes reported. This entails changes in the deadweight concealment costs, which explains the two terms involving γ in (31), one for each type of person. Note that multiplier γ transforms into welfare terms the change in deadweight loss due to the cost of income concealment. (γ is the multiplier of the government budget constraint or alternatively of the resource constraint of the economy). Those terms have the same sign as those of $(1-\tau_2)$ and $(1-\tau_1)$, respectively. For values of q above 1 but not too high, $1-\tau_2 < 0$ and $1-\tau_1 > 0$. In this range, a rise in q causes person 2's concealment costs to go up (from 0 at $q=1$ for which $\tau_2 = 1$ and then $\sigma_2 = 1$) and those of person 1 to fall (since $1-\tau_1 = g_1'$ decreases). For values of q larger enough, both $(1-\tau_1)$ and $(1-\tau_2)$ will be negative, implying that the two terms involving γ in (31) will push q downwards.

Third, there is also a term involving μ in the bracketed expression of (31). By referring to (16), it can be interpreted as the rise in the mimicker's utility caused by the increase of τ_1 that follows a rise in q. This term definitively pushes q downwards because reducing indirect taxes makes the self-selection constraint less stringent.

From the above discussion, we can expect that q will reach a (local) optimum

[8]In addition to the explanation given earlier, one can note first that the difference between the net incomes of the mimicker and person 1, $\Delta y = w_1\ell_1\bar{\sigma}_2^{-1}\left[(\sigma_1 - \bar{\sigma}_2) - (\sigma_1\bar{g}_2 - \bar{\sigma}_2 g_1)\right]$, is not proportional to τ_1 (it must be positive for the mimicker to benefit from $\bar{\sigma}_2 < \sigma_1$). So we have $\bar{c}_2 = q^{-1}\Delta y + c_1$. Consequently, a rise in q causes the mimicker's utility to change by $-\bar{u}_{ia}'(\Delta y/q^2) = -\bar{\alpha}_2(\bar{c}_2 - c_1) < 0$.

the net income of the mimicking high-ability person is higher than that of the low-ability person. Therefore, $\bar{c}_2 > c_1$ and:

Proposition 2. *With tax evasion, starting from $q = 1$ a Pareto-improving change in welfare can be attained by imposing a proportional commodity tax.*

The intuition behind this result is as follows. At $q = 1$, an incremental increase in the commodity tax rate dq accompanied by a lump-sum tax payment to person 1 of $dT_1 = -(c_1^a + c_1^b)dq$ and to person 2 of $dT_2 = -(c_2^a + c_2^b)dq$ will leave their welfare, as well as total tax revenue unchanged. However, the amount $c_1 dq$ is not enough to compensate the mimicker for the increment in q; that would require $\bar{c}_2 dq$. Thus, the mimicker is made worse off by an amount of income equal to $(\bar{c}_2 - c_1)dq$. Since this reduces the welfare of the mimicker, it relaxes the self-selection constraint, and the value of the Lagrangian changes by $\mu\bar{\alpha}_2(\bar{c}_2 - c_1)$, which is the value of the reduction in the mimicker's welfare evaluated at the shadow price of the self-selection constraint. Note that a rise in the commodity tax rate also triggers income-compensated changes in τ_1 and τ_2 (see below) and so in σ_1 and σ_2. However, at $q = 1$ these changes do not affect social welfare at the margin.

Proposition 2 also implies that when allowing for uniform commodity taxes, at least a local optimum will involve a mix of an income tax and a positive commodity tax rate. Unfortunately, because of the possibility that this problem, like all second best ones, is not convex, we cannot rule out there being other local optima with negative tax rates. Let us adopt the standard procedure of ignoring this non-convexity problem and analyze the form of the tax structure which satisfies equation (31) being set to zero.

Condition (31′) and Proposition 2 apply only to an incremental change in q starting at the no-commodity-tax allocation (with the income tax being kept optimal). When q is discretely different than unity , the additional terms of (31) must be taken into account. For the sake of interpreting this expression, let us first look at its counterpart when no under- or overreporting of income occurs (i.e., all σ's are kept equal to 1 whatever the tax system). It is straightforward[7] to infer from (31) that in this case $\partial\mathcal{L}/\partial q = 0$ not only at $q = 1$ but also at $q \neq 1$. This is easy to understand. Any change in q is matched by proportionate changes in τ_1, τ_2, T_1, and T_2 so that the optimal allocation is not affected. As mentioned earlier,

[7] Since $\bar{c}_2 = c_1$ and $\sigma_{i\tau} = 0$.

we do the same with (24) and (23), we obtain after some manipulations:

$$-\mu\bar{\alpha}_2(\bar{g}_2' - (1-\tau_1))\left(w_1\sigma_1\tilde{\ell}_{1\tau} + w_1\ell_1\sigma_{1\tau}\right) + \gamma N_1\left[(1-\tau_1)\left(w_1\sigma_1\tilde{\ell}_{1\tau} + w_1\ell_1\sigma_{1\tau}\right) + (q^a - 1)\tilde{c}_{1\tau}^a + (q^b - 1)\tilde{c}_{1\tau}^b\right] = 0 \tag{25}$$

$$(1-\tau_2)(w_2\sigma_2(\tilde{\ell}_{2\tau} + w_2\ell_2\sigma_{2\tau}) + (q^a - 1)\tilde{c}_{2\tau}^a + (q^b - 1)\tilde{c}_{2\tau}^b = 0 \tag{26}$$

where a 'tilde' over a variable indicates that it is a compensated effect (e.g., $\tilde{\ell}_{1\tau}$ is the compensated effect of a change in τ_1 on person 1's labour supply, and so on).

The procedure we follow in the next section is to consider cases in order of increasing complexity. We begin by characterizing the optimal income tax when commodity taxes are not in place. Next, we consider introducing a uniform commodity tax and, then, we allow for differentiated commodity tax rates.

IV. No Commodity Taxes $(q^a = q^b = 1)$

This is the standard optimal income tax problem except that evasion is now a possibility. When no commodity taxes are present so $q^a = q^b = 1$, equations (25) and (26) reduce to:

$$-\mu\bar{\alpha}_2(\bar{g}_2' - (1-\tau_1)) + \gamma N_1(1-\tau_1) = 0 \tag{25′}$$

$$1 - \tau_2 = 0. \tag{26′}$$

The latter yields the usual result that the marginal tax rate on the high-ability person is zero. It continues to apply here despite the presence of income tax evasion. Using this, equation (24) reduces to:

$$-(\lambda + \mu)\alpha_2 + \gamma N_2 = 0 \quad \text{if} \quad q_a = q_b = 1. \tag{27}$$

This equation will prove useful below.

As to (25′) it determines the value of τ_1. Since $\bar{g}_2' > 1 - \tau_1$ from Lemma 1 and $\mu, \gamma > 0$, this equation implies that $1 - \tau_1 > 0$. That is, the marginal tax rate on the low-ability person is positive which is also a standard result. Substituting (25′) into (22) yields:

$$-\alpha_1 + \mu\bar{\alpha}_2 + \gamma N_1 = 0 \quad \text{if} \quad q_a = q_b = 1. \tag{28}$$

Again, this will be useful below.

The following proposition summarizes the results in the no-commodity-tax case. This proposition as well as all those that follow are stated for the specification of tax evasion we have adopted in the paper.

Proposition 1. *Without commodity taxes, the marginal tax rate on the high-ability persons is zero* ($\tau_2 = 1$) *and that on the low-ability persons is positive* ($\tau_1 < 1$).

So the standard results of optimal income taxation carry over to our model despite the presence of tax evasion.

V. Allowing for Uniform Commodity Taxes

We now consider the possibility of introducing a commodity tax of uniform rate $(q-1)$ on both commodities. The analysis here is analogous to the single-commodity case by the composite commodity theorem. In the absence of tax evasion, such a tax would be redundant since its effects could be replicated by appropriate changes in the non-linear income tax function. Here, however, such a tax may be a useful adjunct to the income tax since the latter can be evaded by the individual while we assume that the commodity tax cannot be.

Consider first the effect on welfare of increasing the commodity tax rate incrementally starting at $q = 1$. Differentiating (20) with respect to q where $q^a = q^b = q$ yields:[6]

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial q} =& -\alpha_1(c_1^a + c_1^b) + (\lambda+\mu)\alpha_2(c_2^a + c_2^b)\\ &-\mu\bar{\alpha}_2\left[-(\bar{c}_2^a + \bar{c}_2^b) + w_1\sigma_1(\bar{g}_2' - (1-\tau_1))(\ell_{1a} + \ell_{1b})\right]\\ &+\gamma\left[N_1(c_1^a + c_1^b + (1-\tau_1)w_1\sigma_1(\ell_{1a}+\ell_{1b})\right.\\ &\qquad + (q-1)(c_{1a}^a + c_{1b}^a + c_{1a}^b + c_{1b}^b))\\ &+ N_2(c_2^a + c_2^b + (1-\tau_2)w_2\sigma_2(\ell_{2a} + \ell_{2b})\\ &\qquad \left. +(q-1)(c_{2a}^a + c_{2b}^a + c_{2a}^b + c_{2b}^b))\right].\end{aligned} \tag{29}$$

[6]In deriving this equation, we have used (from (9) and (10)):
$$v_{iq} = -\alpha_i(c_i^a + c_i^b)$$
and (from (18) and (19)):
$$\bar{v}_q = \bar{\alpha}_2\left[-(\bar{c}_2^a + \bar{c}_2^b) + w_1\sigma_1(\bar{g}_2' - (1-\tau_1))(\ell_{1a} + \ell_{1b}\right].$$

This expression can be simplified by using the optimal conditions on the tax income parameters. If we multiply (22) by $(c_1^a + c_1^b)$ and (24) by $(c_2^a + c_2^b)$ and then subtract both of these from (29) we obtain:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial q} =& -\mu\bar{\alpha}_2\left[c_1^a - \bar{c}_2^a + c_1^b - \bar{c}_2^b + (\bar{g}_2' - (1-\tau_1))w_1\sigma_1(\tilde{\ell}_{1a} + \tilde{\ell}_{1b})\right]\\ &+\gamma\left[N_1((1-\tau_1)w_1\sigma_1(\tilde{\ell}_{1a} + \tilde{\ell}_{1b}) + (q-1)(\tilde{c}_{1a}^a + \tilde{c}_{1b}^a + \tilde{c}_{1a}^b + \tilde{c}_{1b}^b))\right.\\ &\left. + N_2((1-\tau_2)w_2\sigma_2(\tilde{\ell}_{2a} + \tilde{\ell}_{2b}) + (q-1)(\tilde{c}_{2a}^a + \tilde{c}_{2b}^a + \tilde{c}_{2a}^b + \tilde{c}_{2b}^b))\right].\end{aligned} \tag{30}$$

This can be further simplified by observing that, by (1), the relative price of labour income in terms of consumption is $(1 - \sigma_i + \sigma_i\tau_i - g_i)/q)$. The change in τ_i which has the same effect on the relative price of labour income as a change in q is given by: $d\tau_i = -(\sigma_i q)^{-1}(1 - \sigma_i + \sigma_i\tau_i - g_i)dq$. As a consequence, for compensated changes, we have:

$$\frac{\partial\tilde{\ell}_i}{\partial q} = -(\sigma_i q)^{-1}(1 - \sigma_i + \sigma_i\tau_i - g_i)\frac{\partial\tilde{\ell}_i}{\partial\tau_i},$$

and likewise for c_{ia} and c_{ib}. Using this result and adding to condition (30) conditions (25) and (26) weighted by $(\sigma_i q)^{-1}(1 - \sigma_i + \tau_i\sigma_i - g_i)$ with $i = 1$ and 2 respectively, we obtain:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial q} =& \mu\bar{\alpha}_2(\bar{c}_2 - c_1)\\ &+\gamma N_2 g_2' w_2\ell_2\sigma_{2r}\frac{1 - \sigma_2 + \sigma_2\tau_2 - g_2}{\sigma_2 q}\\ &+\left[\gamma N_1 g_1' - \mu\bar{\alpha}_2(g_2' - (1-\tau_1))\right]w_1\ell_1\sigma_{1r}\frac{1 - \sigma_1 + \sigma_1\tau_1 - g_1}{\sigma_1 q}.\end{aligned} \tag{31}$$

This expression applies for all values of q. Consider first the case where there are no commodity taxes in effect. Using (25′) and $\tau_2 = 1$, and recalling that $g_2'(0) = 0$ equation (31) reduces to the following at $q = 1$:

$$\frac{\partial\mathcal{L}}{\partial q} = \mu\bar{\alpha}_2(\bar{c}_2 - c_1) \tag{31'}$$

where $\bar{c}_2 = \bar{c}_2^a + \bar{c}_2^b$ and $c_1 = c_1^a + c_1^b$. Thus, equation (31′) states that increasing the commodity tax starting at $q = 1$ will yield a Pareto improvement if aggregate consumption of the mimicking person exceeds that of the low-ability person. In the absence of evasion, $\bar{c}_2 = c_1$ so, as expected, no Pareto improvement is possible. However, as noted at the end of Section II, a consequence of Lemma 1 is that