



Queen's Economics Department Working Paper No. 846

An International Economy with Country-Specific Money and Productivity Growth Processes

Nicholas Ricketts

Thomas H. McCurdy

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

10-1991

**An international economy with country-specific
money and productivity growth processes***

by

Nicholas Ricketts and Thomas H. McCurdy
Department of Economics
Queen's University
Kingston, Ontario, K7L 3N6
Discussion Paper #846

October 1991

This revision, December 1991

* Our thanks to participants in the macroeconomics workshop at Queen's University and the 1991 Canadian Macroeconomics Study Group at the University of Waterloo, D. Andolfatto, J-O. Cho, M. Devereux and G. Smith for helpful comments. We also wish to thank A. Marcet for providing us with the initial Fortran code for the solution algorithm. We gratefully acknowledge financial support from the Social Sciences and Humanities Research Council of Canada.

Abstract

This paper analyses a stochastic international growth model with money and country-specific forcing processes for productivity and money growth rates. Monies are required due to cash-in-advance constraints for consumption goods but the liquidity constraints need not be binding for all periods. An individual can trade claims on future currency units for both countries through government bond markets. Each country specializes in the production of one of the goods but individual agents can invest, subject to installation costs, in any available technology. Two versions of the model are simulated in order to compare different degrees of international mobility of physical capital.

The moments of the forcing processes are calibrated to a sample of U.S. and Canadian data. A perfectly pooled equilibrium solution is computed numerically, using the Marcet method of parameterized expectations, and the moments of the endogenous variables are compared to those for the actual data. The interdependence implied by the model is illustrated by a series of impulse responses. Particular attention is focused on the implications of capital mobility, the asymmetry of the forcing processes across countries, the implications of the liquidity constraints, and the interaction between the real and nominal components of the model. For example, with endogenous production, we find that monetary fluctuations cause business cycle behavior in consumption and investment while the effect on goods and asset prices can be substantially different from that in endowment models. We also find that the effects of monetary policy are transmitted to other countries via exchange rate and terms of trade adjustments.

1. Introduction

This paper analyses a stochastic international growth model with money. By integrating stochastic growth with the intertemporal pricing of nominal assets in a model in which money has an explicit role, and by allowing country-specific forcing processes for productivity and money growth rates, this international artificial economy extends the existing literature in several ways.

Firstly, we build on recent international real business cycle models. For example, Backus, Kehoe and Kydland (1989) construct an international real business cycle model in which countries have different technology shocks. They also analyze the effect of allowing international borrowing. Baxter and Crucini (1990) focus on various savings/investment correlations. Head (1990) analyses a multi-country real growth model which distinguishes between tradeable and nontradeable goods. Stockman and Tesar (1990) concentrate on the open economy aspects of real business cycles in a two-country model which also incorporates traded and nontraded goods sectors. They find that shocks to tastes are required to replicate some features of the data.

Our international monetary growth model extends this real business cycle literature by incorporating a financial structure and also by introducing steady-state growth with country-specific forcing processes for both money and productivity growth rates. These processes exhibit cross-country correlations but are also allowed to have asymmetric international diffusion rates as well as different degrees of persistence across countries. The productivity shocks are parameterized as being integrated in levels but stationary in growth rates. This structure generates processes for aggregate quantities which are integrated in levels as suggested by historical data.

The endogenous production and growth structure also builds on the international monetary endowment economy models, for example, Bansal (1989), Hodrick (1989), Lucas (1982), Macklem (1991) and Svensson (1985a). As in many closed-economy stochastic general equilibrium monetary models, for example, Coleman (1988), Cooley and Hansen (1988), Giovannini (1989), Labadie (1989), Hodrick, Kocherlakota and Lucas (1991), Lee (1989) and Svensson (1985b), money enters the model through cash-in-advance constraints (CIA).

In our international case, consumption goods produced in a particular country must be purchased with that country's currency. The timing of markets and arrival of new information is that of Svensson which leads to the possibility of nonbinding liquidity constraints. In order to get explicit solutions, most models in this genre impose a strict equality for the liquidity constraints. Using the Marcet (1988) numerical solution technique, we are able to solve the model with the liquidity constraints binding in some periods but not others. As in Hodrick, Kocherlakota and Lucas (1991), these constraints are invariably binding for our baseline parameterization. Therefore, we also solve the model with a counter-factual parameter setting (lower domestic money growth) which leads to a significant number of realizations of nonbinding liquidity constraints. This feature is also asymmetric across countries. These simulations give some indication of the impact of scarce money and the implications of allowing the velocity of money to vary in CIA models.¹

In this version of the model there is no government spending on goods and no taxation so that the governments' budget constraints equate net changes in stocks of government bonds (claims on future currency units)

¹An alternative method of addressing such issues is to extend the CIA structure to incorporate money using a transactions costs structure as in Bansal (1989) and Marshall (1991).

with the exogenous innovations to the respective money supplies. This method of injecting new money into the economies and the opportunity to trade in markets which allow diversification of risk associated with monetary innovations is one way in which our model differs from the recent international monetary growth model by Cho and Roche (1991).²

In our model, each country specializes in the production of one of the goods. This output can either be consumed or re-invested subject to an installation cost. Individual agents can invest in any available technology by setting up their own firm in either or both countries (our substitute for equity markets). We construct two versions of the model which differ with respect to international mobility of physical capital or investment goods. The no-cross-investment (NCI) version requires agents to use output from a particular technology to augment the capital for that technology (no substitutability of country 1 and country 2 goods for investment purposes), whereas the cross-investment (CI) version allows agents to import or export investment goods such that they are allocated to the most productive technology. The assumption that investment goods are perfect substitutes in the CI version implies that purchasing power parity obtains for that case, in contrast to the NCI version.

The moments of the forcing processes are calibrated to a sample of U.S. and Canadian data. The structure of the model is such that we are able to compute a perfectly pooled equilibrium solution for this stochastic international monetary growth model. That solution is computed numerically, using the Marcet (1988) method of parameterized expectations, and the moments of the endogenous variables are compared to those for the actual data.

The interdependence implied by the model is illustrated by a series of

²We also allow diversification of risk associated with productivity shocks and introduce country-specific forcing processes and varying degrees of international mobility of investment goods.

impulse responses. Particular attention is focused on the implications of capital mobility, the liquidity constraints, the asymmetry of the forcing processes across countries, and on the interaction between the real and nominal components of the model. For example, with endogenous production, we find that monetary fluctuations cause business cycle behavior in consumption and investment while the effect on goods and asset prices can be substantially different from that in endowment models (for example, Svensson (1985a,b)). We also find that the effects of monetary policy are transmitted to other countries via exchange rate and terms of trade adjustments.

Section 2 outlines the structure of our international model including the optimization problem faced by private sector agents and section 3 summarizes a stationary equilibrium. Section 4 and tables 1 and 2 summarize the calibration of the preference and technology parameters as well as those for the forcing processes. Appendix B describes the numerical solution procedure while section 5 and table 3 report various summary statistics for the historical sample and for the artificial economy under different assumptions about capital mobility. Section 6 and the figures illustrate some interdependencies implied by the model using impulse response plots. Section 7 concludes with some comments about shortcomings and work to be done.

2. Structure of the International Artificial Economy and Agent Optimization

2.1 Market and information structure

We construct a two-country international growth model with money and bonds. There are two goods. Each country specializes in the production of one of the goods. There is trade in goods, monies and bonds between countries.

Goods can either be consumed or invested in physical capital

accumulation. Allocating output to augment or maintain capital stock involves installation costs. In the cross-investment (CI) version of our model, output from a particular country can be invested in either country. In the no-cross-investment (NCI) version, goods installed as capital stock in a particular country must be produced in that country. That is, in the former version investment goods are perfect substitutes whereas in the latter version the substitutability of goods produced in different countries is zero for investment purposes. In both versions, the goods are imperfect substitutes from the perspective of consumption.

Factor markets are not explicit. Production is by self-employed entrepreneurs such that revenue is imputed to the capital and managerial/labor factors. We abstract from equity market issues by allowing agents to invest in any available technology by setting up their own firms in either or both countries.

Country-specific monies are required in the model by the cash-in-advance (CIA) restrictions that all purchases of the goods for consumption must be paid for in the currency of the producing country. The trading and information structure is the same as that in Svensson (1985b). Goods markets open at the beginning of the period when all information regarding realizations of the stochastic variables during the period is revealed. Asset markets open at the end of the period. In the goods market the agent uses money carried over from the previous period's asset markets to purchase desired levels of the consumption goods. This structure introduces a potential precautionary demand for money since the amount of money available to purchase consumption goods in any period must be determined in the previous period before the uncertainty concerning the state of the world is resolved. The solution technique we use allows the CIA constraints to be binding in some periods but not others.

2.2 A government's problem

The assumption that agents can set up factories in either country allows them to share risk associated with technological shocks by diversification of production. Analogously, we allow agents to share risk associated with the country-specific money growth rates by trading bonds issued by the governments of both countries. These one-period discount bonds pay-off one unit of the country's currency the following period.³

We abstract from fiscal policy and do not distinguish between governments and Central Banks. In this case, the budget constraint for the government of country i is,

$$(M_{i,t+1} - M_{i,t}) + (P_{i,t}^b B_{t+1} - B_t) = 0, \quad i=1,2, \quad (1)$$

where M_i and B_i are the aggregate stocks of money and government bonds, respectively, and $P_{i,t}^b$ is the price of the discount bond. This extreme simplification of the role of governments implies that, given the exogenous forcing processes for the growth rates of country-specific monies, governments will issue debt such that their budget constraints given in (1) obtain.

2.3 A private sector agent's optimization problem

Resident in each country is a continuum of identical, infinitely-lived agents or households which integrate to a representative agent for each country. Where necessary, variables associated with a particular country are subscripted with a 1 or a 2 while variables associated with a particular representative agent are superscripted with a d or an f . This subsection describes the optimization problem faced by the domestic representative agent who is assumed to be resident in country 1.

2.3.1 Consumption

Each domestic agent maximizes expected discounted utility over an

³ Hodrick (1989) introduces state-contingent government bonds.

infinite horizon,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}) \right\}, \quad (2)$$

where c_1 and c_2 are the individual's consumption of domestic and foreign goods respectively. Intraperson utility is specified as a homothetic function of a bundle of domestic and foreign goods. That is,

$$u(c_{1t}, c_{2t}) = \frac{(c_{1t}^\sigma c_{2t}^{1-\sigma})^{1-\gamma}}{1-\gamma}, \quad [\text{if } \gamma=1, \text{ then } \log(c_{1t}^\sigma c_{2t}^{1-\sigma})], \quad (2')$$

in which γ is the coefficient of relative risk aversion, $1/\gamma$ is the elasticity of intertemporal substitution, and σ is the share of country 1's goods in the domestic agent's consumption bundle.

As stipulated above, all purchases of goods for consumption must be paid for in the currency of the producing country. Denoting P_{1t} and P_{2t} as the own-currency prices of domestic and foreign goods respectively, this restriction implies cash-in-advance (CIA) constraints,

$$P_{1t} c_{1t} \leq m_{1t}, \quad (3)$$

$$P_{2t} c_{2t} \leq m_{2t}. \quad (4)$$

In conformity with the market and information structure described above, the m_{it} , $i=1,2$, are the domestic agent's country 1 and country 2 money balances carried over from asset market transactions at the end of period $t-1$ and thus available at the start of period t to purchase desired levels of the consumption goods.

2.3.2 Production

Output produced in country 1 by the domestic agent is denoted y_1 while that produced by the domestic agent in the foreign country is denoted y_2 . The production technologies available in the two countries are given by,

$$y_{1t} = f_1(k_{1t-1}, \theta_{1t}) = \theta_{1t} k_{1t-1}^{\alpha_1}, \quad (5)$$

$$y_{2t} = f_2(k_{2t-1}, \theta_{2t}) = \theta_{2t} k_{2t-1}^{\alpha_2}, \quad (6)$$

where k_{it-1} is the domestic agent's capital stock in place in country i at

the end of period $t-1$ and thus available for production in that country in period t , α_i is the capital share parameter in country i , and θ_i is the level of the productivity shock realized in period t in country i .

New investment becomes productive in the period following the decision to invest. The capital stock depreciates at a rate of $1-\delta$ per period. Gross investment in country i in period t is denoted i_{it} so that

$$i_{1t} = k_{1t} - \delta k_{1t-1} , \quad (7)$$

$$i_{2t} = k_{2t} - \delta k_{2t-1} . \quad (8)$$

Investment is costly to install. In particular, a decision in period t to invest i_t units of capital for use in period $t+1$ requires $(i_t + \phi \frac{i_t^2}{2\delta k_{t-1}})$ units of output in period t . This formulation means that the marginal cost of investment is increasing in the rate of investment. We define output net of installation costs as y^n . That is,

$$y_{it}^n = y_{it} - \phi \frac{i_{it}^2}{2\delta k_{it-1}} , \quad i=1,2 . \quad (9)$$

Output not sold for consumption is invested in the available production technologies. To take advantage of different productivity shocks in the two countries it may be optimal for an agent to import the entire next period capital stock and consume the current domestic output plus depreciated capital, or even to consume some of the imported capital, unless restrictions are placed on such activities.

In this paper, we assume that installed capital cannot be consumed and also that it is immobile between countries. However, in the version of our model which allows cross-investment (CI), output from a particular country can be invested in either country. In this case, there is a resource constraint on investment,

$$q_{1t} + q_{2t} = i_{1t} + i_{2t} , \quad (10)$$

in which q_1 and q_2 are investments goods available from home and foreign production respectively, while i_1 and i_2 are the desired allocations of

those goods.⁴ In contrast to the NCI case, this cross-country arbitrage in goods will ensure that purchasing power parity will obtain in equilibrium.

2.3.3 An agent's budget constraint and choice set

Given initial endowments of physical and financial assets, the domestic representative agent maximizes (1) by choosing,

$$z_t \equiv \{c_{1t}, c_{2t}, q_{1t}, q_{2t}, i_{1t}, i_{2t}, m_{1t+1}, m_{2t+1}, b_{1t+1}, b_{2t+1}\}_{t=1}^{\infty},$$

subject to, for all $t=1, \infty$, the production possibilities given by (5) and (6), the CIA constraints given by (3) and (4), and the investment installation and resource constraints in (7), (8), (9) and (10). In particular, the domestic agent's period t budget constraint can be expressed as:

$$\begin{aligned} & P_{1t}c_{1t} + S_t P_{2t}c_{2t} + P_{1t}q_{1t} + S_t P_{2t}q_{2t} + \\ & \quad m_{1t+1} + S_t m_{2t+1} + P_{1t}^b b_{1t+1} + S_t P_{2t}^b b_{2t+1} \\ & \leq P_{1t} \left(\theta_{1t} k_{1t-1}^{\alpha_1} - \frac{\phi i_{1t}^2}{2\delta k_{1t-1}} \right) + S_t P_{2t} \left(\theta_{2t} k_{2t-1}^{\alpha_2} - \frac{\phi i_{2t}^2}{2\delta k_{2t-1}} \right) \\ & \quad + m_{1t} + S_t m_{2t} + b_{1t} + S_t b_{2t}, \end{aligned} \tag{11}$$

where S_t , the exchange rate, is the price of foreign currency in terms of domestic currency; P_{1t}^b and P_{2t}^b are the own-currency prices of a domestic and foreign government bond, respectively, which promise to pay one unit of the corresponding currency in period $t+1$. Note that b_{it} are the number of units of country i bonds which this agent redeems in period t . Recall that the m_{it} are the country i money balances carried over from period $t-1$ to period t which are available to this agent to purchase goods in period t .

2.4 Structure of the forcing processes and the state of the world economy

Each agent's choices are conditional upon her own past actions and information about the aggregate economy. The state of the economy at the

⁴Recall that in the no-cross-investment (NCI) version of our model, goods installed as capital stock in a particular country must be produced in that country so that there is no distinction between q_{it} and i_{it} in that case.

beginning of time t is described by the beginning of period capital and money stocks, the current period realizations of the technology and money growth rates, and the stochastic processes governing the evolution of the technology and money growth rates.

The stochastic processes for the forcing variables are modelled as AR1 processes with cross-country correlations. Thus letting $\theta_{it+1}/\theta_{it} \equiv \nu_{it+1}$ and $M_{it+1}/M_{it} \equiv \omega_{it}$ be the country i growth factors for technology and money respectively, with $X_\nu \equiv \begin{pmatrix} \ln \nu_1 \\ \ln \nu_2 \end{pmatrix}$ and $X_\omega \equiv \begin{pmatrix} \ln \omega_1 \\ \ln \omega_2 \end{pmatrix}$, then the bivariate vector AR1 processes for productivity growth and money growth are

$$\begin{aligned} X_{\nu t+1} &= A_\nu + B_\nu X_{\nu t} + \varepsilon_{\nu t+1} , \\ X_{\omega t+1} &= A_\omega + B_\omega X_{\omega t} + \varepsilon_{\omega t+1} , \end{aligned} \tag{12}$$

in which the A_j and ε_j , $j=\nu, \omega$, are two-element vectors, and the B_j , $j=\nu, \omega$, are two-by-two non-symmetric matrices. For the moment, we do not allow contemporaneous correlation between the shocks to productivity growth rates and those to money growth rates. On the other hand, productivity shocks may be correlated across countries as may those for money, that is,

$$\begin{pmatrix} \varepsilon_{\nu 1} \\ \varepsilon_{\nu 2} \end{pmatrix} \sim \text{NID} (0, \Sigma_\nu) , \quad \begin{pmatrix} \varepsilon_{\omega 1} \\ \varepsilon_{\omega 2} \end{pmatrix} \sim \text{NID} (0, \Sigma_\omega) ,$$

with non-diagonal Σ_ν and Σ_ω .

2.5 Stationarity inducing transformation

Given that the stochastic processes governing money creation and technological innovation are integrated in levels, the neoclassical model implies that consumption, investment, and output will also be integrated. The approach to solving a model with integrated driving processes is to calibrate the forcing processes in terms of their growth factors, as in (12), and to transform the system to induce stationarity by dividing all the variables of the system by their associated growth components. Goods prices are normalized by a ratio of the appropriate money

supplies and productivity factors while quantity variables are normalized by the latter. All transformed variables are indicated with an overline, for example,⁵ $\bar{P}_{it} = P_{it}\theta_{it}^{1/1-\alpha_i}/M_{it}$ and $\bar{K}_{it} = K_{it}/\theta_{it}^{1/1-\alpha_i}$.

With freely mobile capital⁶, all quantity variables of the two economies will have a common long-run growth rate. Given the exogenous, and potentially asymmetric, processes for θ_1 and θ_2 this means choosing α_1 and α_2 so that the implied long run growth rates for country 1 and country 2 variables are the same, that is, $\nu_{1\infty}^{1/1-\alpha_1} = \nu_{2\infty}^{1/1-\alpha_2}$, where $\nu_{1\infty}$ and $\nu_{2\infty}$ are long-run growth rates of the technological processes for θ_1 and θ_2 respectively.

2.6 Solution to a private sector agent's problem

The vector of information about the state of the economy at time t is

$$\Omega_t = (K_{1t-1}, K_{2t-1}, \theta_{1t-1}, \theta_{2t-1}, \nu_{1t}, \nu_{2t}, M_{1t}, M_{2t}, \omega_{1t}, \omega_{2t}).$$

In terms of variables which have been transformed to obtain stationarity, the value function of an individual agent satisfies:

$$\begin{aligned} V(\bar{k}_{1t-1}, \bar{k}_{2t-1}, \bar{m}_{1t}, \bar{m}_{2t}, \bar{b}_{1t}, \bar{b}_{2t}, \Omega_t) = & \max_{\bar{z}_t} \left\{ \frac{(\bar{c}_{1t}^\sigma, \bar{c}_{2t}^{1-\sigma})^{1-\gamma}}{1-\gamma} \right. \\ & \left. + E_t [\bar{\beta}_{t+1} V(\bar{k}_{1t}, \bar{k}_{2t}, \bar{m}_{1t+1}, \bar{m}_{2t+1}, \bar{b}_{1t+1}, \bar{b}_{2t+1}, \Omega_{t+1}) \mid \Omega_t] \right\}, \end{aligned} \quad (13)$$

subject to transformed versions of the budget constraint, of the CIA constraints and of the resource constraints on domestic and foreign investment. A complete solution of an agent's problem for the cross-investment (CI) version of the model, including definitions of the transformed variables and first-order conditions, appears in Appendix A.1.

⁵Normalization of the consumption variables in the agents' utility functions also requires transforming their subjective discount factors.

$$\text{Thus } \bar{\beta} = \beta (\nu_1^{\frac{\sigma}{1-\alpha_1}} \nu_2^{\frac{1-\sigma}{1-\alpha_2}})^{1-\gamma}.$$

⁶In our CI version of the model cross-investment introduces goods arbitrage and allows capital mobility on the margin even though installed capital is not mobile.

3. A stationary competitive equilibrium

An equilibrium in the transformed variables consists of stochastic processes for the states,

$$\Omega_t = \{\bar{K}_{it-1}, \theta_{it-1}, \nu_{it}, M_{it}, \omega_{it} ; i=1,2 \}, t=1,\infty ; \quad (14)$$

for the choice variables,

$$\{\bar{C}_{it}(\Omega_t), \bar{I}_{it}(\Omega_t), \bar{Q}_{it}(\Omega_t), \bar{M}_{it+1}(\Omega_t), \bar{B}_{it+1}(\Omega_t) ; i=1,2\}, t=1,\infty ; \quad (15)$$

and for the endogenous prices,

$$\{\bar{P}_{it}(\Omega_t), P_{it}^b(\Omega_t), \bar{S}_t ; i=1,2\}, t=1,\infty ; \quad (16)$$

such that:

(i) given (14) and (16), the choices for the domestic and foreign representative agents,

$$\bar{z}_t^j \equiv \{\bar{c}_{it}^j, \bar{i}_{it}^j, \bar{q}_{it}^j, \bar{m}_{it+1}^j, \bar{b}_{it+1}^j ; i=1,2\}, j=d,f \text{ and } t=1,\infty ;$$

satisfy the optimization program described in section 2.6 above;

(ii) the government budget constraints are satisfied for all t ; and

(iii) the markets for money, bonds and goods clear.

The assumptions that the preferences of the representative agents for countries 1 and 2 are identical, that their initial endowments of physical and financial assets are the same, in combination with the assumed market structure that allows diversification of risks⁷, implies that their optimal choices will be identical in a perfectly pooled equilibrium. In this case,

$$\bar{z}_t^d(\Omega_t) = \bar{z}_t^f(\Omega_t) = \bar{Z}(\Omega_t)/2$$

for each z , where z is an individual demand for a commodity or asset and Z is an aggregate demand. Then market clearing conditions can be expressed for money markets as:

$$\bar{M}_{it+1}(\Omega_t) = \frac{M_{it+1}}{M_{it}} = \omega_{it} \quad , \quad i=1,2 ; \quad (17)$$

for bond markets as:

⁷That is, agents have access to the technologies in both countries and can trade claims on future money issued by both countries.

$$\bar{B}_{i,t+1}(\Omega_t) = B_{i,t+1}/M_{i,t} = \bar{B}_{i,t+1}, \quad i=1,2; \quad (18)$$

and for goods markets as:

$$\bar{C}_{i,t}(\Omega_t) + \bar{Q}_{i,t}(\Omega_t) = \bar{Y}_{i,t}^n, \quad i=1,2. \quad (19)$$

The world budget constraint (Walras' Law), the budget constraints for the two governments and the six market equilibrium conditions reduce to five independent equilibrium conditions in the five prices (16).

In the CI version of the model, since aggregate investment goods produced must add up to those allocated across countries, that is,

$$\bar{Q}_{1,t} + \bar{Q}_{2,t}\rho_t = \bar{I}_{1,t} + \bar{I}_{2,t}\rho_t, \quad \rho_t = \theta_{2,t}^{1/1-\alpha_2}/\theta_{1,t}^{1/1-\alpha_1}, \quad (20)$$

combined with the aggregate budget constraints and the market equilibrium conditions, implies that purchasing power parity obtains in goods, that is,

$$\bar{P}_{1,t} = \bar{S}_t \bar{P}_{2,t}. \quad (21)$$

Of course, purchasing power parity will, in general, not obtain in the NCI version of the model.

The simplifications which lead to a perfectly pooled equilibrium will result in the two economies being indistinguishable in some respects. For example, even with the heterogeneity due to country-specific forcing processes, in the CI version of the model aggregate consumption and GNP will be perfectly correlated for the two countries. On the other hand, GDP, capital stocks, investments, prices and interest rates will differ across the countries. The exchange rate and exchange rate risk will, of course, be influenced by these heterogeneities. In equilibrium, the trade and current account balances⁸ will reflect trade in goods and assets between the representative agents' domestic and foreign operations.

The pooled equilibrium allowed us to equate individual demands with equal shares of economy-wide aggregates which we combine with an individual's first-order conditions to obtain the aggregate Euler

⁸The current account balance can be measured by the change in the net asset positions across countries.

conditions. Appendix A.2 does this in detail. Eight of these Euler conditions associated with the transformed system are reproduced below for purposes of interpretation.

$$\lambda_t \bar{P}_{1t} \left[1 + \frac{\phi \bar{I}_{1t} \nu_{1t}^{1/1-\alpha_1}}{\delta \bar{K}_{1t-1}} \right] = E_t \bar{\beta}_{t+1} \lambda_{t+1} \bar{P}_{1t+1} \left[\frac{\alpha_1 \bar{K}_{1t}^{\alpha_1-1}}{\nu_{1t+1}^{\alpha_1/1-\alpha_1}} + \frac{\phi \bar{I}_{1t+1}^2 \nu_{1t+1}^{1/1-\alpha_1}}{2 \delta \bar{K}_{1t}^2} + \frac{\delta}{\nu_{1t+1}^{1/1-\alpha_1}} + \frac{\phi \bar{I}_{1t+1}}{\bar{K}_{1t}} \right], \quad (A19)$$

$$\lambda_t \bar{S}_t \bar{P}_{2t} \left[1 + \frac{\phi \bar{I}_{2t} \nu_{2t}^{1/1-\alpha_2}}{\delta \bar{K}_{2t-1}} \right] = E_t \bar{\beta}_{t+1} \lambda_{t+1} \bar{S}_{t+1} \bar{P}_{2t+1} \left[\frac{\alpha_2 \bar{K}_{2t}^{\alpha_2-1}}{\nu_{2t+1}^{\alpha_2/1-\alpha_2}} + \frac{\phi \bar{I}_{2t+1}^2 \nu_{2t+1}^{1/1-\alpha_2}}{2 \delta \bar{K}_{2t}^2} + \frac{\delta}{\nu_{2t+1}^{1/1-\alpha_2}} + \frac{\phi \bar{I}_{2t+1}}{\bar{K}_{2t}} \right], \quad (A20)$$

$$\lambda_t = E_t \bar{\beta}_{t+1} \frac{(\lambda_{t+1} + \mu_{1t+1})}{\omega_{1t}} = E_t \frac{\bar{\beta}_{t+1} \sigma (\bar{C}_{1t+1}^\sigma \bar{C}_{2t+1}^{1-\sigma})^{1-\gamma}}{\bar{P}_{1t+1} \bar{C}_{1t+1} \omega_{1t}}, \quad (A21)$$

$$\lambda_t \bar{S}_t = E_{t+1} \bar{\beta}_{t+1} \frac{(1-\sigma) (\bar{C}_{1t+1} \bar{C}_{2t+1}^{1-\sigma})^{1-\gamma}}{\bar{P}_{2t+1} \bar{C}_{2t+1} \omega_{2t}}, \quad (A22)$$

$$\lambda_t P_{1t}^b = E_t \frac{\bar{\beta}_{t+1} \lambda_{t+1}}{\omega_{1t}} \quad (A23)$$

$$\lambda_t \bar{S}_t P_{2t}^b = E_t \frac{\bar{\beta}_{t+1} \lambda_{t+1} \bar{S}_{t+1}}{\omega_{2t}}. \quad (A24)$$

$$\lambda_t \bar{P}_{1t} = \min \left(\frac{\lambda_t}{2 \bar{C}_{1t}}, \frac{\sigma (\bar{C}_{1t}^\sigma \bar{C}_{2t}^{1-\sigma})^{1-\gamma}}{\bar{C}_{1t}} \right) \quad (A25)$$

$$\lambda_t \bar{S}_t \bar{P}_{2t} = \min \left(\frac{\lambda_t \bar{S}_t}{2 \bar{C}_{2t}}, \frac{(1-\sigma) (\bar{C}_{1t}^\sigma \bar{C}_{2t}^{1-\sigma})^{1-\gamma}}{\bar{C}_{2t}} \right). \quad (A26)$$

Equation (A19) equates the current marginal cost of investing $\bar{I}_{1t}$ units of capital in the domestic industry with the expected future marginal

return on invested capital. Equation (A20) does the same for investment in the foreign industry. Marginal costs and benefits are measured in terms of utility via the factors $\lambda \bar{P}_1$ and $\lambda \bar{S} \bar{P}_2$ where λ is the marginal utility of nominal wealth, S is the exchange rate and the P_i are the money prices of the two goods. The marginal cost of additional investment reflects the decrease in current consumption and the reduction in output due to installation costs. The expected marginal benefits from investment are composed of the future marginal product of capital and the reduction in future installation costs because of the increase in the level of the depreciated capital stock next period.

Equations (A21) and (A22) equate the marginal utility of increased nominal wealth (adjusted for the increase in the money supply) with the expected discounted marginal utility of consumption next period. Additional money balances carried over to the next period are valued for their purchasing power in next period's goods market. As equations (A25) and (A26) show the marginal utility of consumption will be greater than the marginal utility of wealth when the agent is liquidity constrained. Changes in the growth rate of the money supply have a direct effect on the marginal utility of wealth and thus affect agents' choices of real variables as well. Our simulations show that the volatility of consumption is directly related to the volatility of the money supply. (A23) and (A24) are the Euler conditions associated with government bonds.

The system of eight simultaneous equations together with the resource constraint on world investment capital (20) and the purchasing power parity relation (21) can be solved for λ , $\bar{S}$, $\bar{P}_1$, $\bar{P}_2$, P_1^b , P_2^b , $\bar{I}_1$, $\bar{I}_2$, $\bar{C}_1$, and $\bar{C}_2$ at each point in time. As an analytic solution to the Euler equations and equilibrium conditions is not possible, we employ the Marcet (1988) method of parameterizing expectations and generating approximate solutions via

simulations of the exogenous stochastic processes. This numerical solution procedure is summarized in Appendix B.

4. Calibration

The results of the simulations are dependent upon the values chosen for the underlying preference and technology parameters as well as the parameterizations of the exogenous processes.

The values assigned to the preference and technology parameters are given in Table 1. Although these parameters could have been estimated via calibration to the historical sample, in this paper we set them at values which have been used elsewhere in the literature. Of course, they can be varied across simulations to determine the sensitivity of the output to different settings of the parameters. We have found the homotopy approach described in Marcet (1989) to be useful for this exercise.

The subjective annual discount factor, β , is set at 0.96. The coefficient of relative risk aversion, γ , is initially set at 1.5. The value of α chosen for the U.S. has been used by Baxter and Crucini (1990). In the cross-investment (CI) version of the model, the value of α for Canadian technology was then chosen, as described in section 2.5 above, to support equality in long-run growth rates across countries. In the NCI version, the values of α_i were chosen to be the same as those for the CI version for comparison purposes. Of course, one could also simulate the NCI version with α_i calibrated from historical data for each country.

The parameterizations of the forcing processes are summarized in Table 2. Parameters for the productivity growth processes, were obtained from data in a study by Costello (1989). The availability of data on productivity growth rates limited the use of data to yearly periods. Monetary growth rate data were obtained from the CANSIM and CITIBASE databases using annual data (average of quarterly data) on money supply

(M1) levels.

Notice from Table 2 that the bivariate forcing processes for both productivity growth (X_v) and money growth (X_ω) exhibit contemporaneous correlation (non-diagonal C matrices) as well as asymmetric lagged spillovers (non-diagonal and asymmetric B matrices). The implied long-run growth rates which are reported in Table 2 as $\bar{X}_v$ and $\bar{X}_\omega$ are also not equal across countries.

In summary, for this sample (1957-85) and frequency (annual), the volatility of shocks to productivity growth (v) are similar across countries ($\sigma_{v1} = .0302$, $\sigma_{v2} = .0306$) while the volatility of shocks to money growth are higher in Canada ($\sigma_{\omega1} = .0340$, $\sigma_{\omega2} = .0160$). The contemporaneous correlation across countries is higher for productivity shocks (0.79) than for money shocks (0.44). Lagged spillovers of shocks to productivity are negative for both countries although considerably higher from the U.S. to Canada than the other way around. Money growth rates were much more persistent in the U.S. than in Canada, and although lagged spillovers were fairly large for both countries they had opposite signs.

5. Baseline results

Table 3 contains summary statistics for selected variables. These include the average growth rates for real GDP (Y), private gross fixed capital investment (I), consumption of nondurables and services (C), the GDP deflator (P), and the exchange rate of Canadian for U.S. dollars (S),⁹ as well as the ratios of the trade balance (NX) and the current account balance (CA) to GNP, and the rates of interest (R). The means, standard deviations, cross correlations and first-order auto correlations for these percentage growth rates and ratios are presented in Tables 3.A to 3.D respectively. In all these cases, the summary statistics are reported for

⁹Note that the statistics for P will refer to the rate of inflation and those for S to the rate of depreciation.

the historical sample and for both the NCI and CI versions of the model.

As described in the previous section, the forcing processes for money growth were calibrated to historical data using a VAR process to estimate the parameters. This was also the case indirectly for the productivity growth processes since we used Costello's parameterization of international productivity growth rates for the same sample period. However, the preference and technology parameters of the model were assigned values from other studies rather than being calibrated to a particular sample in order to match historical moments. In other words, our objective was not to match historical data but rather to construct an artificial economy which could be simulated under alternative assumptions about the structure of the international economy in order to analyse some general equilibrium implications of such assumptions. Therefore, any comparisons of the moments of the model with those from historical data should be interpreted with appropriate caution.

Mean growth rates for the artificial economy reported in Table 3.A reflect the long-run equilibrium (the growth rate of quantities is similar across countries and across categories as determined by the real forcing process) and the two-country model (the trade and current account balances of one country must be the negative of that of the other). Nominal interest rates are higher than in the data but the differentials have the correct sign. Relative purchasing power parity holds for both versions of the model, in contrast to the historical sample, but absolute PPP only holds for the CI version of the model as indicated in (21) above.¹⁰

¹⁰ We chose, for comparison purposes, to calibrate the NCI version using the capital share parameters α_1 and α_2 which were consistent with the balanced growth solution for the CI version of the model. We have also solved the NCI model with α_1 and α_2 corresponding to the historical data for each country. This required $\alpha_1 = .38$ and $\alpha_2 = .35$, and resulted in a non-balanced growth solution for the NCI version. This had implications for inflation rates, investment volatilities, and for some cross-correlations.

Table 3.B reports that variability of consumption relative to output is closer to what we observe in the data than is the case for many theoretical models which often generate consumption series which are too smooth. Investment is too variable relative to output although we could correct for this by increasing the cost of installation parameter ϕ . As will be clearer in the next section, the difference in the volatility in investment across countries in the NCI version of the model is due to the asymmetric forcing processes -- in particular, the higher volatility of money growth shocks in Canada. The volatility of the exchange rate matches that in the data quite closely.

One stylized fact for the international economy is that the correlation across countries for consumption is lower than that for output. This stylized fact is difficult to match using theoretical models which allow risk sharing. Indeed, as is clear from Table 3.C, even with the heterogeneity due to asymmetric forcing processes, in the CI version of the model consumption across countries is perfectly correlated. On the other hand, the NCI version generates relative rankings of the cross-country correlations for Y , C and I much closer to that in the data. Also, unlike some international business-cycle models, domestic and foreign output growth is positively correlated in our models and also relatively close to the correlation observed in historical data. The investment-savings correlations (I, Q) within each country are high in both versions of the model.

Another stylized fact discussed in Backus, Kehoe and Kydland (1989) is the countercyclical behaviour of trade balances. The correlations between GDP growth and NX/GNP ratios reported in Table 3.C for the NCI version are negative while those for the CI version are negative for the foreign country as was the case in the historical sample.

First-order auto correlations are reported in Table 3.D. These are generally of the correct sign, except for two investment cases, and in some cases of similar size to those in the historical data. The time-series behaviour of the growth rates can be analysed using impulse responses which we discuss in the next section.

6. Dynamic responses to impulse shocks

The dynamic responses of the endogenous variables to shocks to money and productivity growth rates allow us to better interpret some of the correlations seen in the tables. The figures provide impulse response surfaces for these shocks for cross-investment versus no-cross-investment (CI versus NCI) versions of the model. These figures are for the baseline parameterization of the model for which the liquidity constraints were invariably binding (BL). The shocks are once-off and one standard deviation in size, and the figures plot percent deviations from steady state growth rates.

The response of the economy to monetary shocks is substantially different from the Svensson endowment economy model. The reason is that in the endowment model there can be no shifts between consumption and savings so that any desired shift induces offsetting changes in asset prices. To trace the effect of a positive monetary shock in the steady state, note that the purchasing power of money is a function of the current realization of the state vector Ω . In our case,

$$\pi(\Omega) = \frac{1}{P(\Omega)} = \frac{\theta^{1/1-\alpha}}{\bar{P}(\Omega) M}$$

A high current realization of the growth rate of money, ω_t , means more money will be available in all states next period since $M_{t+1} = \omega_t M_t$. Thus, a high ω means that the purchasing power of money will be low next period. Agents will therefore prefer to increase their current consumption. If agents are not currently liquidity constrained, this will simply bid up the

current price of goods and hence lower the current purchasing power of money and current real balances. If agents are liquidity constrained or become liquidity constrained, the shadow price of liquidity, μ_t , increases while the marginal utility of nominal and of real wealth, λ_t and $\lambda_t P_t$ respectively, decline at given prices.

The decrease in λ_t has implications for production. For fixed current output gross of installation costs, the marginal cost of investment declines relative to its expected future return. Thus saving in the form of new physical capital will be higher. Less output will be available for consumption and since, as described above, agents wish to increase their current consumption, goods prices will increase. If the liquidity constraint is binding, current consumption will fall. This explains why high variability in money growth rates can lead to high variability in consumption and investment relative to output in the model.

The transmission mechanism for monetary shocks between countries is the exchange rate. When liquidity constraints are mostly binding, the rate of depreciation reflects the relative sizes of domestic and foreign money growth rates. In the model with international capital flows, domestic and foreign capital savings will be adjusted until the purchasing powers of the two monies are equalized in each time period. Both countries will therefore experience similar responses to a monetary shock in either one of the countries. It can be seen that the effect of a money shock is more asymmetrical when no cross-investment is allowed. Prices respond in very much the same manner, but investment is more volatile and more unevenly distributed.

Productivity shocks have very little effect on exchange rates. Agents respond to increased productivity by moving investment capital into the more productive technology and by increasing their current consumption. With

international capital mobility price changes must be of the same size and direction in both countries to maintain purchasing power parity. With no cross investment, agents are unable to obtain full advantage of the productivity increase and the effects of the shock are mostly contained to the country in which it originated.

Relaxing the liquidity constraint for the domestic economy 75% of the time¹¹, has a limited effect upon the magnitude of responses to real and nominal shocks and no effect on their direction. The most notable effect is on the volatility of the exchange rate and the forward risk premia. When the domestic liquidity constraint is mostly non-binding, exchange rate volatility is greatly reduced and risk-premia volatility increases. Forward risk premia are negatively correlated with domestic money and foreign productivity shocks while positively correlated with foreign money and domestic productivity shocks. The nominal risk premia responds very little to these shocks.

7. Concluding comments

Although preference and technology parameters were not calibrated to a particular historical sample, our baseline artificial economy is able to generate some of the international stylized facts. The growth model generates aggregate series which are integrated in levels and, for the most part, exhibits volatilities and correlations which are reasonable. In particular, the relative ranking of output, consumption and investment volatilities correspond to those in the historical sample and, at least for one version of the model, the relative ranking of the cross-country correlations for output, consumption and investment also reflects stylized

¹¹ This was implemented by adjusting the intercepts of the VAR for money growth rates such that the long-run rate of growth for domestic money was reduced to $-.04$ and that for foreign money was unchanged. The impulse response figures for this non-binding liquidity (NBL) parameterization are available from the authors on request.

facts.

The financial structure of the model also generates sensible prices. In particular, interest rates are positive (although with higher means and lower variances than in the historical data), relative inflation rates have the correct sign for all moments and, except for the mean rate of depreciation, the exchange rate behaviour is quite close to historical experience. The counter-factual absolute purchasing power parity result for the CI version of the model could be relaxed by modifying the assumption that investment goods are perfect substitutes across countries. The relative purchasing power parity result for the NCI version of the model might be relaxed by applying the CIA to investment goods. It could also be relaxed by introducing trading frictions as in Backus, Kehoe and Kydland (1989).

The effect of the cross-country asymmetric forcing processes for money and productivity growth rates appear to have been useful in reproducing stylized facts. Their effect was also seen by comparing the dynamic responses to impulses to domestic versus foreign forcing processes. These responses also highlight the transmission channels of financial versus real shocks. Introducing contemporaneous correlation between real and nominal shocks would be interesting in this regard. It would also be interesting to further investigate the non-balanced growth solution associated with capital shares calibrated to the historical data. Further investigations of non-binding liquidity constraints or other extensions of the CIA structure might be fruitful.

REFERENCES

- Backus, D.K., P.J. Kehoe and F.E. Kydland, 1989, International borrowing and world business cycles, **Federal Reserve Bank of Minneapolis Working Paper No. 426R**.
- Bansal, R., 1989, Can non-separabilities explain exchange rate movements and risk premia, **manuscript**, Carnegie-Mellon University.
- Baxter, M. and M.J. Crucini, 1990, Explaining saving/investment correlations, **manuscript**, University of Rochester.
- Cho J-O. and M.J. Roche, 1991, An international business cycle model with money, **manuscript**, Queen's University.
- Coleman, W.J., 1988, Money, interest, and capital in a cash-in-advance economy, **International Finance Discussion Paper No. 323**.
- Cooley, T.F. and G.D. Hansen, 1988, The inflation tax in a real business cycle model, **manuscript**.
- Costello, D.M., 1989, A cross-country, cross-industry comparison of the behaviour of Solow residuals, **manuscript**, University of Rochester.
- Giovannini, A., 1989, Uncertainty and liquidity, **Journal of Monetary Economics**, 23, 239-258.
- Head, A.C., 1990, Aggregate fluctuations in a world of heterogeneous countries, **manuscript**, University of Minnesota.
- Hodrick, R.J., 1989, Risk, uncertainty, and exchange rates, **Journal of Monetary Economics**, 23, 433-459.
- Hodrick R.J., N. Kocherlakota and D. Lucas, 1991, The variability of velocity in cash-in-advance models, **Journal of Political Economy**, 358-384
- Labadie, P., 1989, Stochastic inflation and the equity premium, **Journal of Monetary Economics**, 24, 277-298.
- Lee, B.S., 1989, Solving, estimating, and testing a nonlinear stochastic equilibrium model, with an example of the asset returns and inflation relationship, **Journal of Economic Dynamics and Control**, 13, 499-531.
- Lucas, R.E., 1982, Interest rates and currency prices in a two-country world, **Journal of Monetary Economics**, 10, 335-359.
- Marcet, A., 1988, Solving nonlinear models by parameterizing expectations, **manuscript**, GSIA Carnegie-Mellon University.
- Macklem, T., 1991, Forward exchange rates and risk premiums in artificial economies, **Journal of International Finance and Money**, forthcoming.
- Marshall, D.A., 1991, Inflation and asset returns in a monetary economy, **manuscript**, Northwestern University.

Stockman, A.C. and L.L. Tesar, 1990, Tastes and technology in a two-country model of the business cycle: Explaining international comovements, **manuscript**.

Svensson, L.E.O., 1985a, Currency prices, terms of trade, and interest rates: a general equilibrium asset-pricing cash-in-advance approach, **Journal of International Economics**, 18, 17-42.

Svensson, L.E.O., 1985b, Money and asset prices in a cash-in-advance economy, **Journal of Political Economy**, 93, 919-944.

APPENDIX A

Appendix A.1: Solution to an Agent's Problem

Let $\Omega_t = (K_{1t-1}, K_{2t-1}, M_{1t}, M_{2t}, \theta_{1t-1}, \theta_{2t-1}, \omega_{1t}, \omega_{2t}, \nu_{1t}, \nu_{2t})$ denote the vector of information about the state of the economy at time t .

In terms of variables which have been transformed to obtain stationarity, the value function of an individual agent satisfies,

$$V(\bar{k}_{1t-1}, \bar{k}_{2t-1}, \bar{m}_{1t}, \bar{m}_{2t}, \bar{b}_{1t}, \bar{b}_{2t}, \Omega_t) = \max_{\bar{c}_{1t}, \bar{c}_{2t}} \left\{ \frac{(\bar{c}_{1t}^\sigma, \bar{c}_{2t}^{1-\sigma})^{1-\gamma}}{1-\gamma} \right. \\ \left. + E_t [\bar{\beta}_{t+1} V(\bar{k}_{1t}, \bar{k}_{2t}, \bar{m}_{1t+1}, \bar{m}_{2t+1}, \bar{b}_{1t+1}, \bar{b}_{2t+1}, \Omega_{t+1}) | \Omega_t] \right\}$$

subject to the following constraints (Lagrange multipliers in brackets):

(i) Overall Budget Balance Constraint (λ_t)

$$\bar{m}_{1t+1} + \bar{S}_t \bar{m}_{2t+1} + \bar{P}_{1t} \bar{c}_{1t} + \bar{S}_t \bar{P}_{2t} \bar{c}_{2t} + P_{1t}^b \bar{b}_{1t+1} + \bar{S}_t P_{2t}^b \bar{b}_{2t+1} \\ + \bar{P}_{1t} \bar{q}_{1t} + \bar{S}_t \bar{P}_{2t} \bar{q}_{2t} \leq \bar{P}_{1t} \left[\frac{\bar{k}_{1t-1}^{\alpha_1}}{\nu_{1t}^{\alpha_1/1-\alpha_1}} - \frac{\phi \bar{i}_{1t}^2 \nu_{1t}^{1/1-\alpha_1}}{2\delta \bar{k}_{1t-1}} \right] \\ + \bar{S}_t \bar{P}_{2t} \left[\frac{\bar{k}_{2t-1}^{\alpha_2}}{\nu_{2t}^{\alpha_2/1-\alpha_2}} - \frac{\phi \bar{i}_{2t}^2 \nu_{2t}^{1/1-\alpha_2}}{2\delta \bar{k}_{1t-1}} \right] + \frac{\bar{m}_{1t}}{\omega_{1t-1}} + \frac{\bar{S}_t \bar{m}_{2t}}{\omega_{2t-1}} \\ + \frac{\bar{b}_{1t}}{\omega_{1t-1}} + \frac{\bar{S}_t \bar{b}_{2t}}{\omega_{2t-1}}$$

(ii) Cash-In-Advance Constraints on Consumption Purchases (μ_{1t}, μ_{2t})

$$\bar{P}_{1t} \bar{c}_{1t} \leq \frac{\bar{m}_{1t}}{\omega_{1t-1}} \\ \bar{S}_t \bar{P}_{2t} \bar{c}_{2t} \leq \frac{\bar{S}_t \bar{m}_{2t}}{\omega_{2t-1}}$$

(iii) Resource Constraints on Domestic and Foreign Investment ($\eta_{1t}, \eta_{2t}, \eta_{3t}$)

$$\bar{i}_{1t} = \bar{k}_{1t} - \frac{\delta \bar{k}_{1t-1}}{\nu_{1t}^{1/1-\alpha_1}}$$

$$\bar{i}_{2t} = \bar{k}_{2t} - \frac{\delta \bar{k}_{2t-1}}{\nu_{2t}^{1/1-\alpha_2}}$$

$$\bar{q}_{1t} + \rho_t \bar{q}_{2t} = \bar{i}_{1t} + \rho_t \bar{i}_{2t}$$

$$\text{where } \rho_t = \theta_{2t}^{1/1-\alpha_2} / \theta_{1t}^{1/1-\alpha_1}$$

$$\bar{c}_{1t} = c_{1t} / \theta_{1t}^{1/1-\alpha_1}$$

$$\bar{c}_{2t} = c_{2t} / \theta_{2t}^{1/1-\alpha_2}$$

$$\bar{k}_{1t} = k_{1t} / \theta_{1t}^{1/1-\alpha_1}$$

$$\bar{k}_{2t} = k_{2t} / \theta_{2t}^{1/1-\alpha_2}$$

$$\bar{m}_{1t+1} = m_{1t+1} / M_{1t}$$

$$\bar{m}_{2t+1} = m_{2t+1} / M_{2t}$$

$$\bar{b}_{1t+1} = b_{1t+1} / M_{1t}$$

$$\bar{k}_{2t+1} = k_{2t+1} / M_{2t}$$

$$\bar{i}_{1t} = i_{1t} / \theta_{1t}^{1/1-\alpha_1}$$

$$\bar{i}_{2t} = i_{2t} / \theta_{2t}^{1/1-\alpha_2}$$

$$\bar{q}_{1t} = q_{1t} / \theta_{1t}^{1/1-\alpha_1}$$

$$\bar{q}_{2t} = q_{2t} / \theta_{2t}^{1/1-\alpha_2}$$

$$\bar{P}_{1t} = P_{1t} \theta_{1t}^{1/1-\alpha_1} / M_{1t}$$

$$\bar{P}_{2t} = P_{2t} \theta_{2t}^{1/1-\alpha_2} / M_{2t}$$

$$\bar{S}_t = S_t M_{2t} / M_{1t}$$

$$\bar{\beta}_{t+1} = \beta \left(\nu_{1t+1}^{\sigma/1-\alpha_1} \nu_{2t+1}^{1-\sigma/1-\alpha_2} \right)^{1-\gamma}$$

The following are the first order necessary conditions for the value function to be a maximum:

$$(A1) \quad (\bar{c}_{1t}): \quad \frac{\sigma(\bar{c}_{1t}^\sigma \bar{c}_{2t}^{1-\sigma})^{1-\gamma}}{\bar{c}_{1t}} = (\lambda_t + \mu_{1t}) \bar{P}_{1t}$$

$$(A2) \quad (\bar{c}_{2t}): \quad \frac{(1-\sigma)(\bar{c}_{1t}^\sigma \bar{c}_{2t}^{1-\sigma})^{1-\gamma}}{\bar{c}_{2t}} = (\lambda_t + \mu_{2t}) \bar{S}_t \bar{P}_{2t}$$

$$(A3) \quad (\bar{k}_{1t-1}): \quad V_{1t} = \lambda_t \bar{P}_{1t} \left[\frac{\alpha_1 \bar{k}_{1t-1}^{\alpha_1-1}}{\nu_{1t}^{\alpha_1/1-\alpha_1}} + \frac{\phi \bar{i}_{1t}^2 \nu_{1t}^{1/1-\alpha_1}}{2\delta \bar{k}_{1t-1}^2} \right] + \frac{\delta \eta_{1t}}{\nu_{1t}^{1/1-\alpha_1}}$$

$$(A4) \quad (\bar{k}_{1t}): \quad E_t \bar{\beta}_{t+1} V_{1t+1} = \eta_{1t}$$

$$(A5) \quad (\bar{k}_{2t-1}): \quad V_{2t} = \lambda_t \bar{S}_t \bar{P}_{2t} \left[\frac{\alpha_2 \bar{k}_{2t-1}^{\alpha_2-1}}{\nu_{2t}^{\alpha_2/1-\alpha_2}} + \frac{\phi \bar{i}_{2t}^2 \nu_{2t}^{1/1-\alpha_2}}{2\delta \bar{k}_{2t-1}^2} \right] + \frac{\delta \eta_{2t}}{\nu_{2t}^{1/1-\alpha_2}}$$

$$(A6) \quad (\bar{k}_{2t}): \quad E_t \bar{\beta}_{t+1} V_{2t+1} = \eta_{2t}$$

$$(A7) \quad (\bar{I}_{1t}): \quad \eta_{1t} = \eta_{3t} + \frac{\lambda_t \bar{P}_{1t} \phi \bar{I}_{1t} \nu_{1t}^{1/1-\alpha_1}}{\delta \bar{k}_{1t-1}}$$

$$(A8) \quad (\bar{I}_{2t}): \quad \eta_{2t} = \eta_{3t} \rho_t + \frac{\lambda_t \bar{S}_t \bar{P}_{2t} \phi \bar{I}_{2t} \nu_{2t}^{1/1-\alpha_2}}{\delta \bar{k}_{2t-1}}$$

$$(A9) \quad (\bar{q}_{1t}): \quad \eta_{3t} = \lambda_t \bar{P}_{1t}$$

$$(A10) \quad (\bar{q}_{2t}): \quad \eta_{3t} \rho_t = \lambda_t \bar{S}_t \bar{P}_{2t}$$

$$(A11) \quad (\bar{m}_{1t}): \quad V_{3t} = \frac{\lambda_t + \mu_{1t}}{\omega_{1t-1}}$$

$$(A12) \quad (\bar{m}_{1t+1}): \quad E_t \bar{\beta}_{t+1} V_{3t+1} = \lambda_t$$

$$(A13) \quad (\bar{m}_{2t}): \quad V_{4t} = \frac{(\lambda_t + \mu_{2t}) \bar{S}_t}{\omega_{2t-1}}$$

$$(A14) \quad (\bar{m}_{2t+1}): \quad E_t \bar{\beta}_{t+1} V_{4t+1} = \lambda_t \bar{S}_t$$

$$(A15) \quad (\bar{b}_{1t}): \quad V_{5t} = \frac{\lambda_t}{\omega_{1t-1}}$$

$$(A16) \quad (\bar{b}_{1t+1}): \quad E_t \bar{\beta}_{t+1} V_{5t+1} = \lambda_t P_{1t}^b$$

$$(A17) \quad (\bar{b}_{1t+1}): \quad V_{6t} = \frac{\lambda_t \bar{S}_t}{\omega_{2t-1}}$$

$$(A18) \quad (\bar{b}_{2t+1}): \quad E_t \bar{\beta}_{t+1} V_{6t+1} = \lambda_t \bar{S}_t P_{2t}^b$$

Appendix A.2: Deriving Aggregate Euler Conditions

By equating individual demands with equal shares of economy-wide aggregates and using equations (A1) to (A18) we can obtain the Euler equations. From equations (A3), (A4), (A7) and (A9) we obtain,

$$\begin{aligned} \eta_{1t} &= E_t \bar{\beta}_{t+1} \lambda_{t+1} \bar{P}_{1t+1} \left[\frac{\alpha_1 \bar{K}_{1t}^{\alpha_1-1}}{\nu_{1t+1}^{\alpha_1/1-\alpha_1}} + \frac{\phi \bar{I}_{1t+1}^2 \nu_{1t+1}^{1/1-\alpha_1}}{2\delta \bar{K}_{1t}^2} \right] + \frac{\bar{\beta}_{t+1} \delta \eta_{1t+1}}{\nu_{1t+1}^{1/1-\alpha_1}} \\ (A19) \quad \lambda_t \bar{P}_{1t} \left[1 + \frac{\phi \bar{I}_{1t} \nu_{1t}^{1/1-\alpha_1}}{\delta \bar{K}_{1t-1}} \right] &= E_t \bar{\beta}_{t+1} \lambda_{t+1} \bar{P}_{1t+1} \left[\frac{\alpha_1 \bar{K}_{1t}^{\alpha_1-1}}{\nu_{1t+1}^{\alpha_1/1-\alpha_1}} + \frac{\phi \bar{I}_{1t+1}^2 \nu_{1t+1}^{1/1-\alpha_1}}{2\delta \bar{K}_{1t}^2} \right. \\ &\quad \left. + \frac{\delta}{\nu_{1t+1}^{1/1-\alpha_1}} + \frac{\phi \bar{I}_{1t+1}}{\bar{K}_{1t}} \right] . \end{aligned}$$

Substituting $\bar{I}_{1t} = \bar{K}_{1t} - \delta \bar{K}_{1t-1} / \nu_{1t}^{1/1-\alpha_1}$ from (iii) we have,

$$\begin{aligned} \lambda_t \bar{P}_{1t} \left[1 - \phi + \frac{\phi \bar{K}_{1t} \nu_{1t}^{1/1-\alpha_1}}{\delta \bar{K}_{1t-1}} \right] \\ = E_t \bar{\beta}_{t+1} \lambda_{t+1} \bar{P}_{1t+1} \left[\frac{\alpha_1 \bar{K}_{1t}^{\alpha_1-1}}{\nu_{1t+1}^{\alpha_1/1-\alpha_1}} + \frac{\phi \bar{K}_{1t+1}^2 \nu_{1t+1}^{1/1-\alpha_1}}{2\delta \bar{K}_{1t}^2} + \delta \left(\frac{2-\phi}{2\nu_{1t+1}^{1/1-\alpha_1}} \right) \right] . \end{aligned}$$

Similarly equations (A5), (A6), (A8), (A10) and the resource constraint on $\bar{I}_{2t}$ from (iii) give,

$$\begin{aligned} (A20) \quad \lambda_t \bar{S}_t \bar{P}_{2t} \left[1 + \frac{\phi \bar{I}_{2t} \nu_{2t}^{1/1-\alpha_2}}{\delta \bar{K}_{2t-1}} \right] &= E_t \bar{\beta}_{t+1} \lambda_{t+1} \bar{S}_{t+1} \bar{P}_{2t+1} \left[\frac{\alpha_2 \bar{K}_{2t}^{\alpha_2-1}}{\nu_{2t+1}^{\alpha_2/1-\alpha_2}} \right. \\ &\quad \left. + \frac{\phi \bar{I}_{2t+1}^2 \nu_{2t+1}^{1/1-\alpha_2}}{2\delta \bar{K}_{2t}^2} + \frac{\delta}{\nu_{2t+1}^{1/1-\alpha_2}} + \frac{\phi \bar{I}_{2t+1}}{\bar{K}_{2t}} \right] , \end{aligned}$$

Equations (A1), (A11) and (A12) imply that,

$$(A21) \quad \lambda_t = E_t \bar{\beta}_{t+1} \frac{(\lambda_{t+1} + \mu_{1t+1})}{\omega_{1t}} = E_t \frac{\bar{\beta}_{t+1} \sigma (\bar{C}_{1t+1}^{\sigma} \bar{C}_{2t+1}^{1-\sigma})^{1-\gamma}}{\bar{P}_{1t+1} \bar{C}_{1t+1} \omega_{1t}} .$$

Similarly from (A2), (A13) and (A14) we get

$$(A22) \quad \lambda_t \bar{S}_t = E_{t+1} \bar{\beta}_{t+1} \frac{(1-\sigma)(\bar{C}_{1t+1} \bar{C}_{2t+1}^{1-\sigma})^{1-\gamma}}{\bar{P}_{2t+1} \bar{C}_{2t+1} \omega_{2t}} .$$

Equations (A15), (A16), (A17) and (A18) yield,

$$(A23) \lambda_t P_{1t}^b = E_t \frac{\bar{\beta}_{t+1} \lambda_{t+1}}{\omega_{1t}} \quad , \quad \text{and} \quad (A24) \lambda_t \bar{S}_t P_{2t}^b = E_t \frac{\bar{\beta}_{t+1} \lambda_{t+1} \bar{S}_{t+1}}{\omega_{2t}} \quad .$$

In the perfectly pooled equilibrium, money market clearing implies that the cash-in-advance constraints from (ii) reduce to,

$$\bar{P}_{1t} \bar{C}_{1t} \leq \frac{1}{2} \quad \text{and} \quad \bar{P}_{2t} \bar{C}_{2t} \leq \frac{1}{2} \quad .$$

Thus when the liquidity constraints are binding,

$$\bar{P}_{1t} = \frac{1}{2\bar{C}_{1t}} \quad \text{and} \quad \bar{P}_{2t} = \frac{1}{2\bar{C}_{2t}} \quad .$$

When the liquidity constraints are not binding the Lagrange multipliers μ_{1t} and μ_{2t} are zero. Thus in that case, from equations (A1) and (A2), prices are given by,

$$\bar{P}_{1t} = \frac{\sigma (\bar{C}_{1t}^\sigma \bar{C}_{2t}^{1-\sigma})^{1-\gamma}}{\lambda_t \bar{C}_{1t}} \quad \text{and} \quad \bar{P}_{2t} = \frac{(1-\sigma) (\bar{C}_{1t}^\sigma \bar{C}_{2t}^{1-\sigma})^{1-\gamma}}{\lambda_t \bar{S}_t \bar{C}_{2t}} \quad .$$

Combining these yields.

$$(A25) \lambda_t \bar{P}_{1t} = \min \left(\frac{\lambda_t}{2\bar{C}_{1t}}, \frac{\sigma (\bar{C}_{1t}^\sigma \bar{C}_{2t}^{1-\sigma})^{1-\gamma}}{\bar{C}_{1t}} \right) \quad \text{and}$$

$$(A26) \lambda_t \bar{S}_t \bar{P}_{2t} = \min \left(\frac{\lambda_t \bar{S}_t}{2\bar{C}_{2t}}, \frac{(1-\sigma) (\bar{C}_{1t}^\sigma \bar{C}_{2t}^{1-\sigma})^{1-\gamma}}{\bar{C}_{2t}} \right) \quad .$$

Finally, goods market clearing and the resource constraint on total investment in (iii) imply that,

$$\frac{\bar{K}_{1t-1}^{\alpha_1}}{\nu_{1t}^{\alpha_1/1-\alpha_1}} - \frac{\phi \bar{I}_{1t}^2 \nu_{1t}^{1/1-\alpha_1}}{2\delta \bar{K}_{1t-1}} - \bar{C}_{1t} - \bar{I}_{1t} + \rho_t \left[\frac{\bar{K}_{2t-1}^{\alpha_2}}{\nu_{2t}^{\alpha_2/1-\alpha_2}} - \frac{\phi \bar{I}_{2t}^2 \nu_{2t}^{1/1-\alpha_2}}{2\delta \bar{K}_{2t-1}} - \bar{C}_{2t} - \bar{I}_{2t} \right] = 0$$

that is,

$$\begin{aligned}
& \frac{\bar{K}_{1t-1}^{\alpha_1}}{\nu_{1t}^{\alpha_1/1-\alpha_1}} - \frac{\phi \bar{K}_{1t}^2 \nu_{1t}^{1/1-\alpha_1}}{2\delta \bar{K}_{1t-1}} + (\phi-1)\bar{K}_{1t} - \left(\frac{\phi-2}{2}\right) \frac{\delta \bar{K}_{1t-1}}{\nu_{1t}^{1/1-\alpha_1}} - \bar{C}_{1t} \\
& + \rho_t \left[\frac{\bar{K}_{2t-1}^{\alpha_2}}{\nu_{2t}^{\alpha_2/1-\alpha_2}} - \frac{\phi \bar{K}_{2t}^2 \nu_{2t}^{1/1-\alpha_2}}{2\delta \bar{K}_{2t-1}} + (\phi-1)\bar{K}_{2t} - \left(\frac{\phi-2}{2}\right) \frac{\delta \bar{K}_{2t-1}}{\nu_{2t}^{1/1-\alpha_2}} - \bar{C}_{2t} \right] = 0 .
\end{aligned}$$

APPENDIX B: Numerical solution procedure

Solving the system of simultaneous equations consisting of the Euler equations and the equilibrium conditions requires evaluating expectations of future returns conditional on the current values and transition processes of the state variables. The system of Euler equations can be written symbolically as

$$H(Z_t, X_t) = E\{G(Z_{t+1}, X_{t+1}) \mid \Omega_t\} \quad (B1)$$

where X_t is a vector of period t state variables, Z_t is a vector of endogenous variables, Ω_t is the information set of the agent at time t , and E is the conditional expectations operator. Ω_t contains the vector of state variables X .

For our solutions, $X_t = (\bar{K}_{1t-1}, \bar{K}_{2t-1}, \theta_{1t}, \theta_{2t}, v_{1t}, v_{2t}, \omega_{1t}, \omega_{2t})$ and $Z_t = (\bar{C}_{1t}, \bar{C}_{2t}, \bar{I}_{1t}, \bar{I}_{2t}, \lambda_t, S_t, \bar{P}_{1t}, \bar{P}_{2t}, P_{1t}^b, P_{2t}^b)$.

In this application there are eight Euler equations involving conditional expectations. The method of parameterized expectations, developed by Marcet (1988) for solving systems of Euler equations, involves approximating each conditional expectation with an exponential polynomial of X_t , $g(X_t, \psi)$, where ψ is a parameter vector. In the space of positive continuous functions of X_t , of which the conditional expectation is a member, the approximation can be made arbitrarily close by increasing the degree of the polynomial.

Given a sufficiently close approximation of the right hand side of (B1) at each t , solutions can be found for the vector Z_t . The information contained in the calculated series $\{Z_t\}$ is then used to estimate a new parameter vector for $g(X_t, \psi)$. Iterations on this procedure lead to convergence of the vector ψ .

An approximate rational expectations equilibrium consists of:

an expectations rule $g(X, \hat{\psi})$ which approximates the equilibrium

conditional expectations; $g(X_t, \psi)$ is taken to be the non-linear least squares estimator of $E\{G[Z_{t+1}(X_{t+1}(X_t, \rho), \psi), X_{t+1}(X_t, \rho)] \mid X_t\}$;

transition functions $X_{t+1}(X_t, \rho)$ for the exogenous and endogenous state variables where ρ is the vector of parameters for the transition functions;

policy functions $Z(X, \hat{\psi})$ for the endogenous variables. The policy functions are defined implicitly by the equilibrium Euler equations with $g(X, \hat{\psi})$ replacing the conditional expectations. Thus, for each t ,

$$H(Z_t(X_t, \hat{\psi}), X_t) \equiv g(X_t, \hat{\psi}) .$$

The solution procedure involves finding the fixed point, $\hat{\psi}$, of this non-linear mapping and solving for the associated policy functions Z .

The algorithm for the solution of the approximate rational expectations equilibrium is as follows:

(i) Fix n , the dimension of the ψ vector; T , the length of the time series; ψ^0 , an initial guess for the n -dimensional parameter vector; and starting values for the state vector elements ($t=1$ values).

(ii) For each $t = 2, \dots, T$,

compute $g(X_t, \psi^0)$,

compute Z_t^0 from the implicit relation $g(X_t, \psi_t^0) \equiv H(Z_t^0, X_t)$;

(iii) Solve the problem $\psi^1 = \underset{\psi}{\operatorname{argmin}} \sum_{t=1}^T [G(Z_t^0, X_t) - g(X_t, \psi)]^2$;

(iv) Repeat (ii) to (iii) with ψ^1 replacing ψ^0 ;

(v) Iterate until the difference $\psi^1 - \psi^0$ is as small as desired.

TABLE 1
Baseline Parameters for Preferences and Technology

β	=	0.96	:	Annual subjective discount factor
σ	=	0.5	:	Share of domestic consumption in period utility
γ	=	1.5	:	Degree of relative risk aversion
ϕ	=	2.0	:	Installation cost parameter
δ	=	0.97	:	One minus the rate of capital depreciation
$(\alpha_1, \alpha_2) = (0.28, 0.42)$ Capital shares in domestic and foreign production which support balanced growth				

TABLE 2
Specification and Baseline Parameters for Exogenous Stochastic Processes

Vector Autoregressive specifications, that is,

$$X_{t+1} = A + B X_t + C E_{t+1}, \quad E \sim \text{NID}(0, I),$$

and $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ in which X_1 is the log of a domestic (Canadian) growth rate
and X_2 is the log of a foreign (U.S.) growth rate

Productivity Growth Rate Parameters (based on Costello (1989) data):

$$A = \begin{pmatrix} 0.0155 \\ 0.0114 \end{pmatrix} \quad B = \begin{pmatrix} 0.1945 & -0.0428 \\ -0.0060 & 0.1846 \end{pmatrix} \quad C = \begin{pmatrix} 0.0302 & 0.0000 \\ 0.0242 & 0.0188 \end{pmatrix}$$

$$\bar{X}_v = \begin{pmatrix} 0.0185 \\ 0.0139 \end{pmatrix}$$

Monetary (M1) Growth Rate Parameters
(based on CANSIM and CITIBASE data for the period 1957-1985):

$$A = \begin{pmatrix} 0.0384 \\ 0.0218 \end{pmatrix} \quad B = \begin{pmatrix} 0.3394 & 0.0864 \\ -0.0832 & 0.7384 \end{pmatrix} \quad C = \begin{pmatrix} 0.0340 & 0.0000 \\ 0.0070 & 0.0144 \end{pmatrix}$$

$$\bar{X}_w = \begin{pmatrix} 0.0663 \\ 0.0624 \end{pmatrix}$$

TABLE 3: Summary Statistics for Selected Variables

TABLE 3.A

Mean Growth Rates and Ratios* (%)

	HISTORICAL		NCI MODEL**		CI MODEL**	
	DOMESTIC	FOREIGN	DOMESTIC	FOREIGN	DOMESTIC	FOREIGN
Y	4.16	2.97	2.42	2.40	2.42	2.40
I	4.12	3.25	2.42	2.40	2.41	2.39
C	4.11	3.13	2.42	2.41	2.41	2.41
NX/GNP	1.34	-0.53	-0.16	0.16	24.68	-24.68
CA/GNP	-1.29	-0.01	5.21	-5.18	1.03	-1.03
R	7.14	6.64	12.42	12.01	12.41	12.02
P	5.31	4.73	4.04	3.65	4.05	3.66
S	1.30		.40		.40	

* Y - Real Gross Domestic Product; I - Private Gross Fixed Capital Investment; C - Consumption of Non-durables and Services; NX - Net Exports; CA - Current Account Balance; R - Interest Rates on 1-year Treasury Bills and Government Bonds; P - GDP Deflator; S - Exchange Rate of Canadian for U.S. Dollars. Historical data are obtained from the Cansim, Citibase, and IFS databases for the period 1957-1985. Data are first differences of logarithms (except for NX/GNP, CA/GNP and R).

** NCI Model refers to results obtained from a model in which there are no physical capital movements across countries. The two goods are distinct for investment purposes. In the CI Model cross-investment is allowed.

TABLE 3.B

Standard Deviations

	HISTORICAL		NCI MODEL		CI MODEL	
	DOMESTIC	FOREIGN	DOMESTIC	FOREIGN	DOMESTIC	FOREIGN
Y	2.15	2.83	3.02	3.09	2.99	3.02
I	6.15	7.29	16.41	6.08	12.31	14.69
C	2.00	1.22	2.80	3.29	2.24	2.24
NX/GNP	1.43	1.06	1.44	1.44	33.72	33.72
CA/GNP	1.44	0.24	1.04	1.00	5.21	5.21
R	3.10	3.13	1.71	1.85	1.82	1.89
P	4.02	3.11	3.95	3.20	3.61	2.80
S	3.00		3.39		3.39	

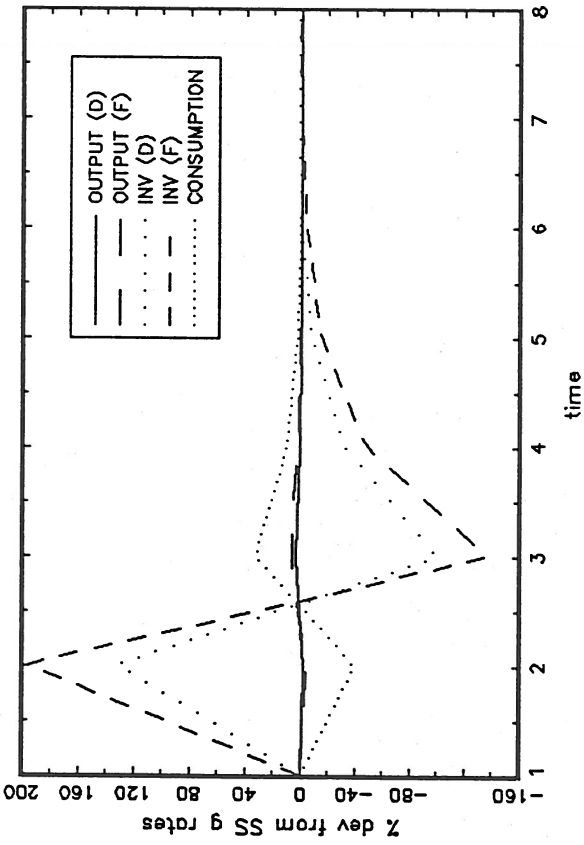
TABLE 3.C
Cross Correlations

	HISTORICAL		NCI MODEL		CI MODEL	
	DOMESTIC	FOREIGN	DOMESTIC	FOREIGN	DOMESTIC	FOREIGN
Y_d , Y_f	0.64		0.78		0.77	
I_d , I_f	0.24		0.45		0.84	
C^d , C^f	0.49		0.66		1.00	
I , Q			0.94	0.86	0.91	0.90
Y , I	0.65	0.80	0.30	0.84	0.74	0.65
Y , C	0.86	0.72	0.97	0.93	0.77	0.80
$Y, NX/GNP$	0.17	-0.47	-0.05	-0.03	-0.01	-0.01
$Y, CA/GNP$	0.02	-0.20	-0.01	-0.16	0.01	-0.03
Y , R	-0.27	-0.22	0.07	0.09	-0.02	0.03
Y , P	-0.33	-0.60	-0.66	-0.78	-0.46	-0.56

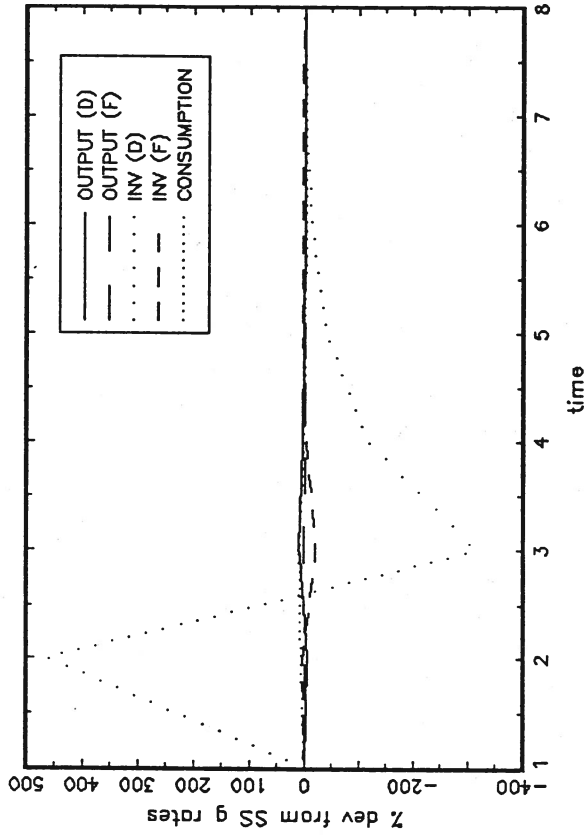
TABLE 3.D
First-Order Auto Correlations

	HISTORICAL		NCI MODEL		CI MODEL	
	DOMESTIC	FOREIGN	DOMESTIC	FOREIGN	DOMESTIC	FOREIGN
Y	0.21	0.11	0.19	0.24	0.21	0.30
I	0.28	0.14	-0.24	0.23	-0.01	-0.06
C	0.38	0.36	0.14	0.00	0.19	0.19
NX/GNP	0.65	0.69	0.44	0.44	1.00	1.00
CA/GNP	0.71	0.77	0.87	0.87	1.00	1.00
P	0.44	0.52	0.43	0.40	0.54	0.55
S	0.38		0.43		0.43	

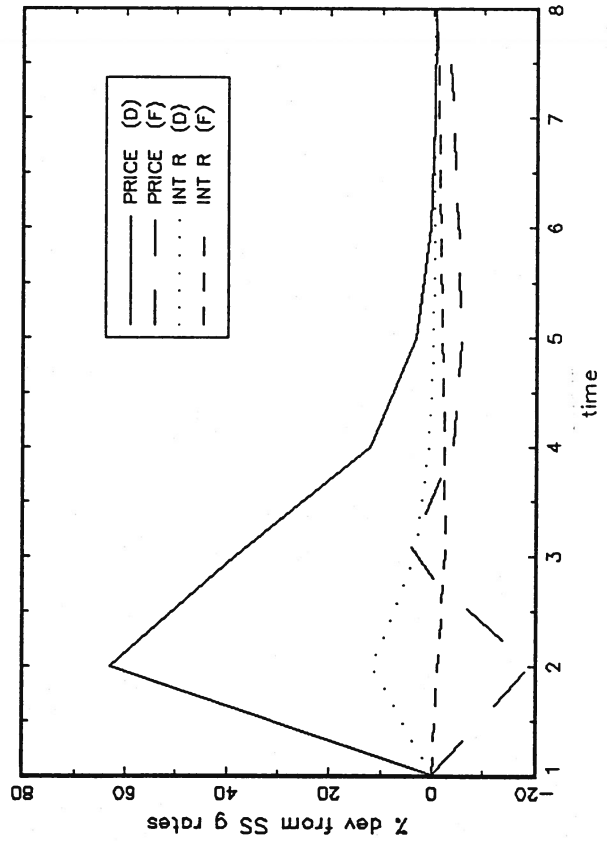
1st domestic money shock: BL_{CI}



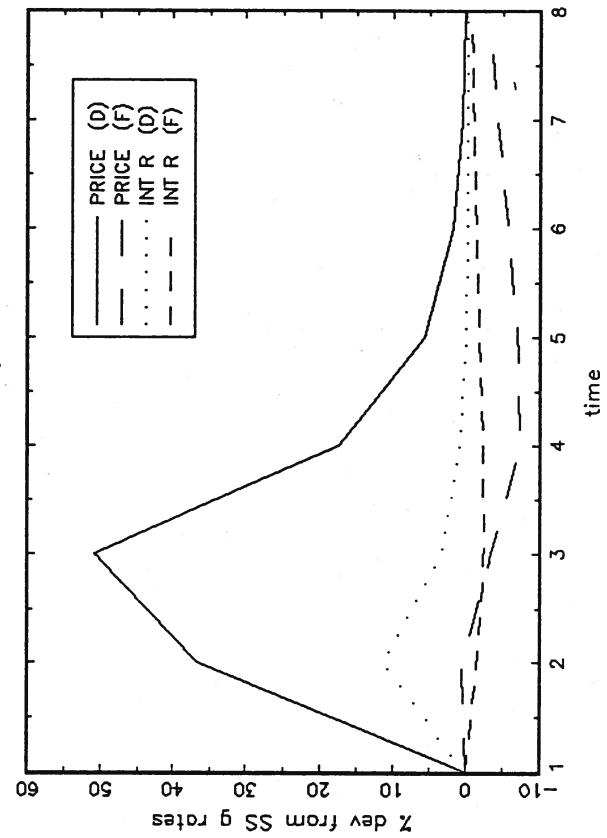
1st domestic money shock: BL_{NCI}



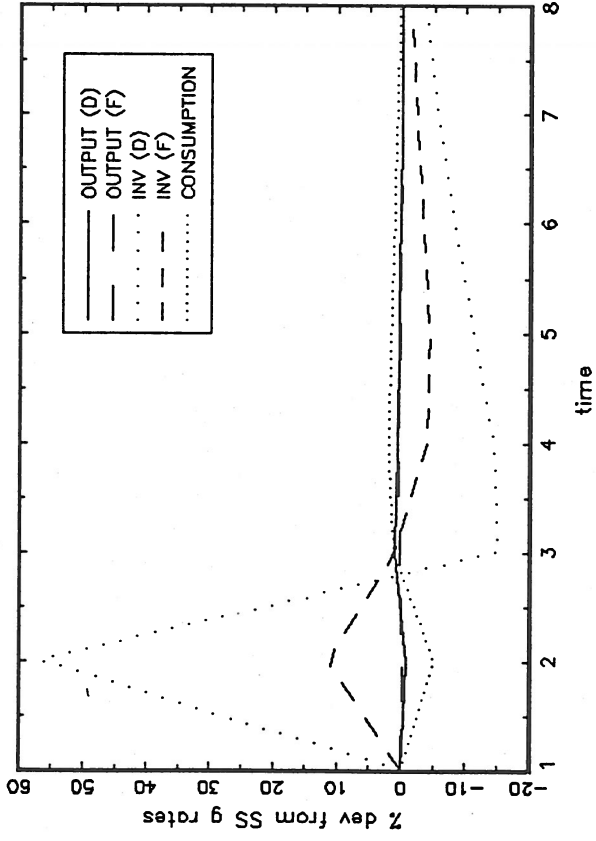
1st domestic money shock: BL_{CI}



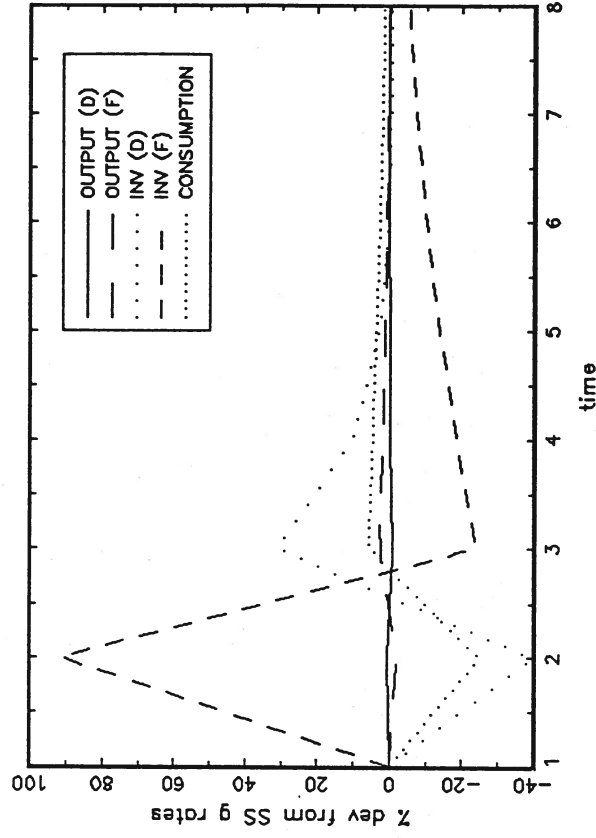
1st domestic money shock: BL_{NCI}



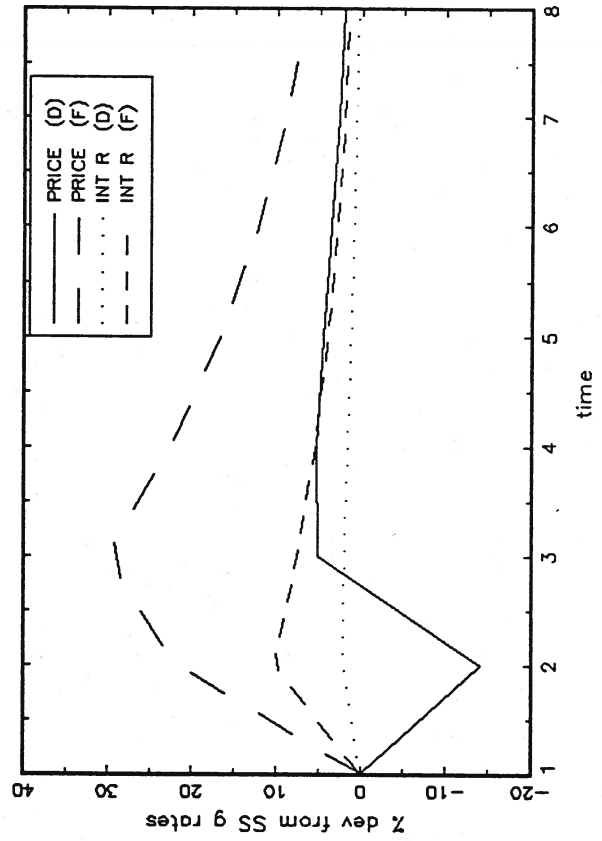
1st foreign money shock: BL,C1



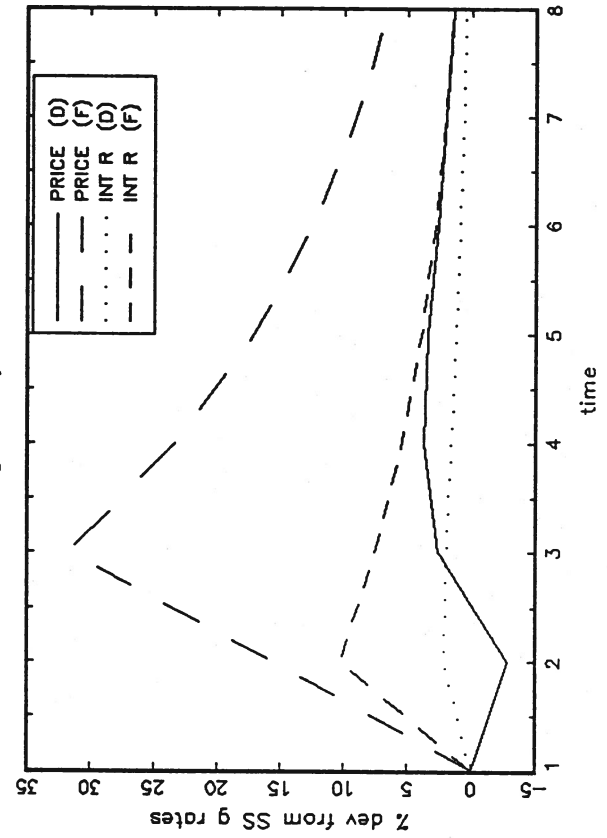
1st foreign money shock: BL,NC1



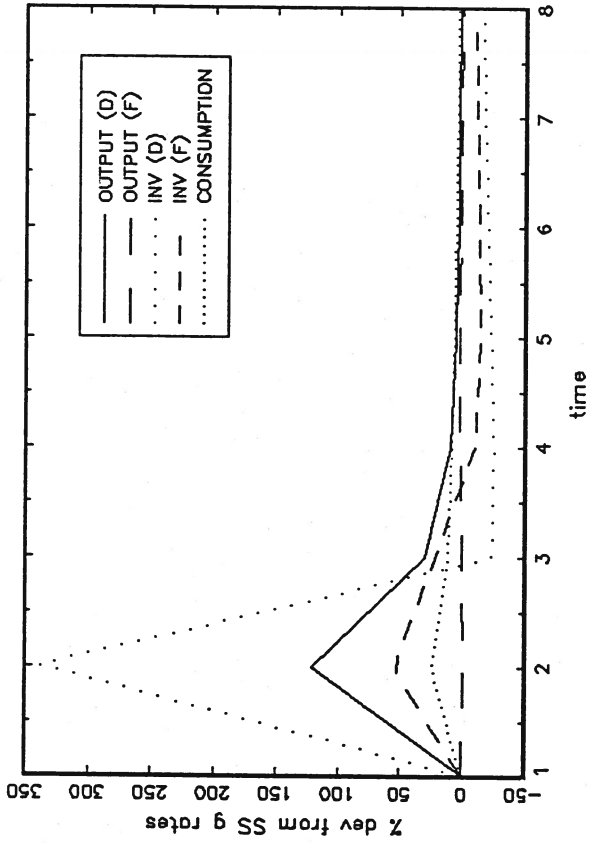
1st foreign money shock: BL,C1



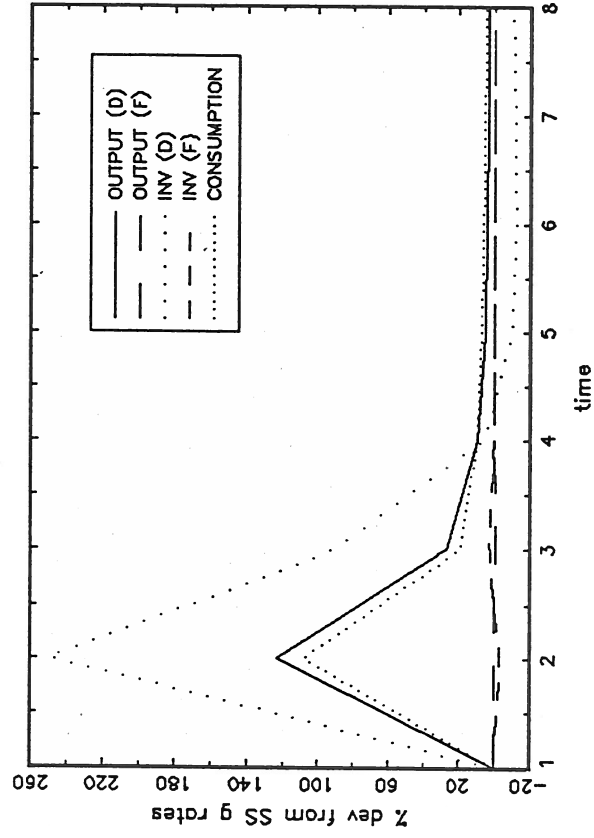
1st foreign money shock: BL,NC1



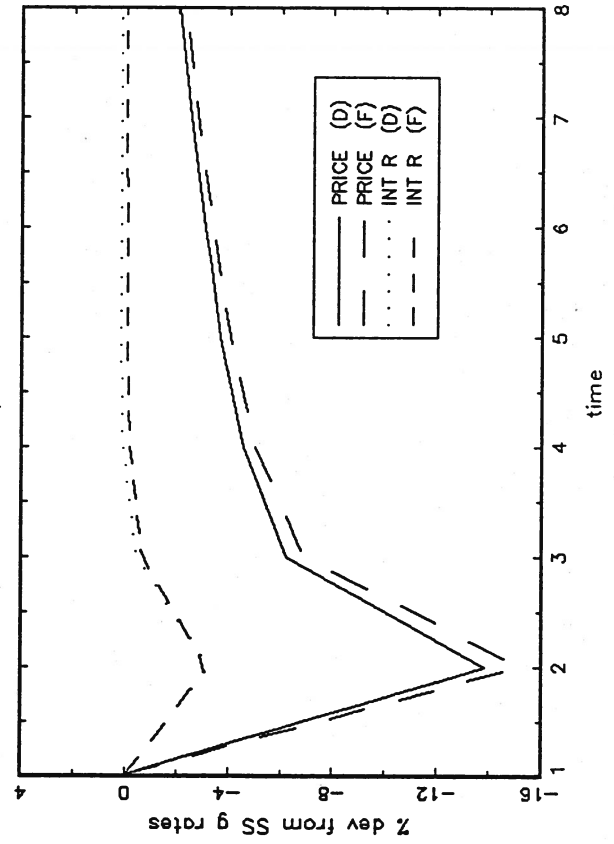
1st domestic prod shock: BL,C1



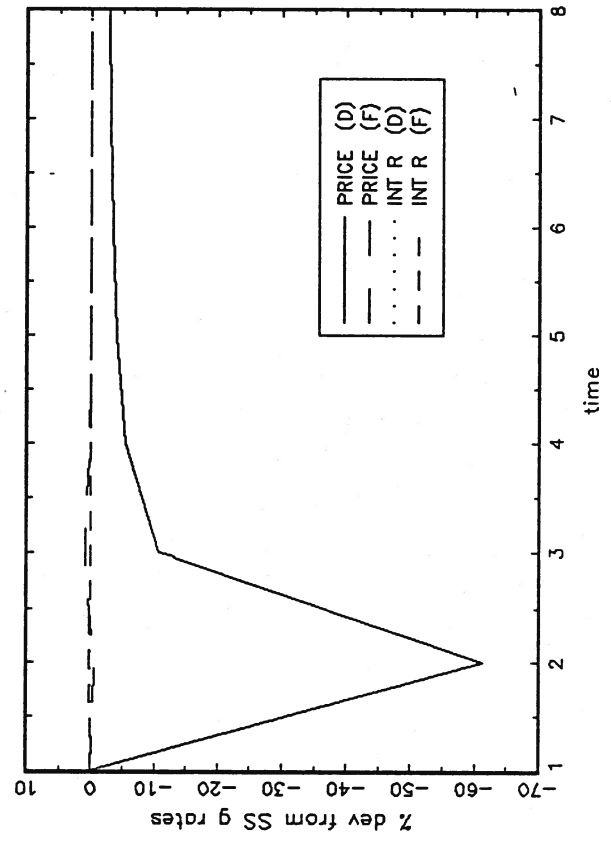
1st domestic prod shock: BL,NC1



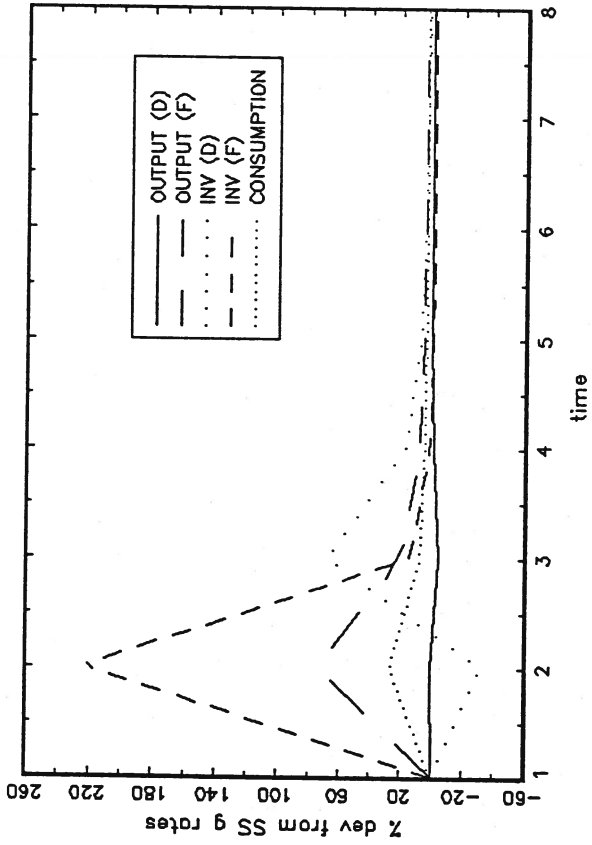
1st domestic prod shock: BL,C1



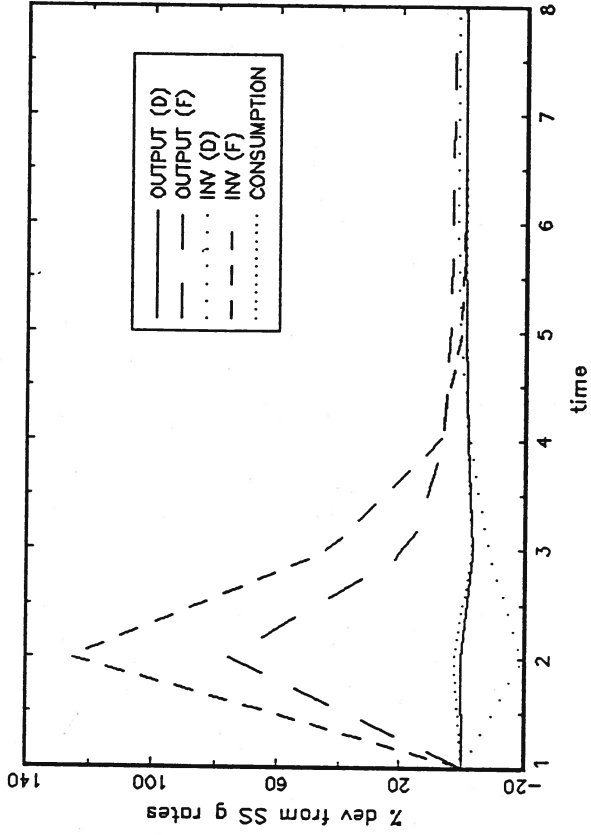
1st domestic prod shock: BL,NC1



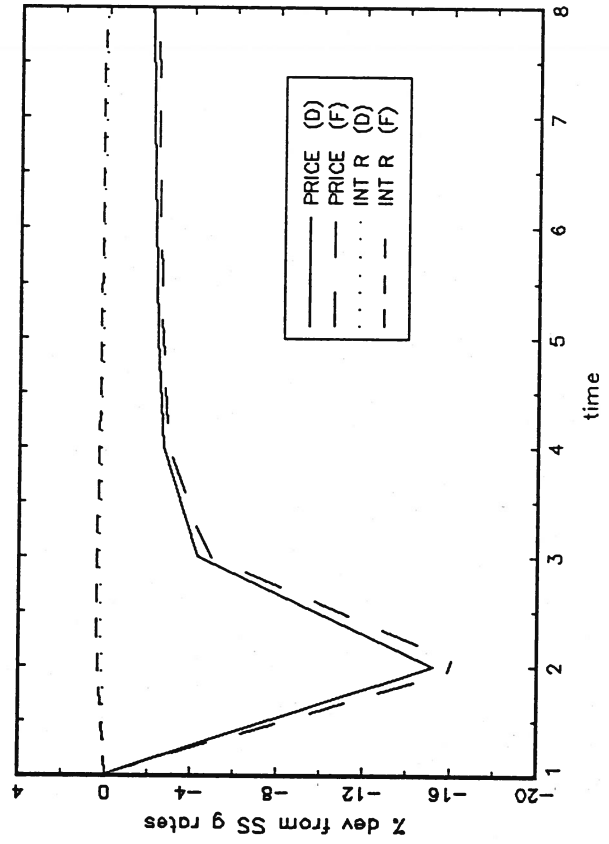
1st foreign prod shock: BL,CI



1st foreign prod shock: BL,NCI



1st foreign prod shock: BL,CI



1st foreign prod shock: BL,NCI

