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Funding Self-Management

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ABSTRACT

The paper briefly summarises those arguments which explain the paucity of workers' cooperatives under capitalism in terms of their financing arrangements. The possibility is considered that a wage-earners' investment fund, of Scandinavian type, could act as an external funding agency for a 'cooperative sector' within a capitalist economy. A simple model is developed of such an economy with a fund in place. The dynamics of the model are analysed and a number of simulation results reported. These results indicate the economic costs and consequences of various policies which such a fund might pursue.

Funding Self-Management

by Donald A.R. George, Dept. of Economics, Queen's University, Canada and Dept. of Economics, University of Edinburgh.

1. Introduction

There has been an enormous revival of interest in self-management and worker participation in the last twenty years (George, 1982 reviews some of the issues). Allowing workers a greater degree of control over production has been advocated as a means of improving the quality of working life and of increasing productivity. Codetermination has been developed in Western Europe and there has been a renaissance of interest in workers' cooperatives. However, despite all the advantages that self-management is supposed to bring, its most obvious variety, the workers' cooperative, is a relatively rare phenomenon under capitalism. There is a substantial literature which explains this paucity of cooperatives in terms of their financing arrangements. Vanek's (1977) paper argues that it is the tendency of cooperatives to finance their capital accumulation internally (i.e. from retained earnings) that lies at the root of their failure to spread under capitalism. This position is supported by the well-known Furubotn-Pejovich analysis (Furubotn, 1976; Furubotn and Pejovich, 1970). There are strong microeconomic arguments in favour of external financing arrangements for workers' cooperatives, and successful systems of cooperatives do indeed seem to have some measure of external finance. The Mondragon system, for example, is financed partially via a bank, the Caja Laboral Popular (see Thomas and Logan, 1982) while Danish cooperatives are, to some extent, financed via trade union and labour movement institutions (see George, 1982b).

The obvious approach to the external financing of cooperatives would be the issuing of bonds, though this approach has its limits. Firstly bond

financing imposes all the risks of an investment project on the cooperative and none on the lender. Even under risk-neutrality there remains the problem of "increasing risk". Even though bonds carry a fixed and certain rate of interest, there remains the possibility of a default, and the probability of the lender suffering a loss increases as the proportion of equity capital in the financial structure decreases. The lender therefore demands a higher rate of return, and the cost of bond-financed capital rises at the margin. In addition, a fixed commitment to interest and principal repayments "gears" the risk to residual claimants (who, in a cooperative, are its members) thus reducing their willingness to undertake further borrowing.

The capitalist corporation has an obvious way out of these problems, namely the issuing of shares. This option is not open to cooperatives for the obvious reason that shareholders would require some measure of control over the firm, which would therefore cease to be self-managed. Cooperatives might instead opt for the use of "performance bonds". A performance bond is simply a title to a dividend, the amount of which varies with the firm's performance. Vanek (1977b) has suggested that the dividend should be a fixed proportion of value-added, while McCain (1977) suggests a fixed proportion of workers' income. Performance bonds would of course carry no voting rights or other power to influence the firm's management.

McCain (1977) shows that, under uncertainty, when the option of performance bonds[1] is available to the cooperative, it will choose not to finance itself out of retained earnings but rather to issue a mixture of performance and ordinary bonds. The Vanek and Furubotn-Pejovich effects disappear and the cooperative will achieve at least as efficient an allocation of resources as its conventional twin. McCain speculates on a possible extension of his argument:

"Suppose that the (performance) bonds are bought by some social agency such as a state or regional bank. The optimality of external finance with (performance) bonds means that the cooperative will willingly choose the norm of "social property", zero enterprise net worth."

A type of "social agency" which might wish to hold performance bonds is the wage-earner investment fund of the kind widely discussed in Europe during the 1970s and 1980s, and set up in Sweden in 1984. A wage-earner investment fund would raise income from taxation and use it to acquire shares on behalf of wager-earners. The fund's earnings could be returned to workers in a variety of different ways, ranging from redemption of individual stakes after some minimum period, to payments into the pension system. (George, 1985a discusses the various institutional arrangements.) At the time of writing (1991) only the Swedish 1983 plan has been implemented (in 1984). The policy debate on wage-earner funds covered a wide variety of issues (see George, 1990 for a discussion), one of which was the idea that changes in the **ownership** of capital should go along with changes in its **control**. That is, there should be a move towards greater worker influence **within** firms. For example, the Danish 1973 Bill on wage-earner funds was placed before the Folketing (Danish Parliament) in tandem with a Bill on codetermination. The latter passed into law while the wage-earner funds Bill was rejected. Wage-earners funds can increase worker influence on production via the various rights inherent in share ownership (see George, 1990 for a discussion of this point). An alternative approach for such a fund would be to direct resources towards firms, such as workers' cooperatives, which adopt a participatory form of management. The Swedish LO (federation of unions), for example, have expressed the view (see Jansson and Hellmark, 1986) that "There

should, in fact, be considerable mutual advantage in cooperation between a workers' cooperative and a wage-earner fund....wage-earner funds should be a good source of new risk capital for businesses run as cooperatives which are incorporated as joint stock companies". If the fund were permitted to hold performance bonds as well as shares it could also supply capital to those cooperatives not incorporated as joint stock companies, and could play the role of McCain's "social agency". Section 2 of the paper develops a model of an economy with a wage-earner fund established and charged with the responsibility of promoting a self-managed sector. Section 3 analyses the dynamics of the model and section 4 reports the results of several simulation runs (using the model) which demonstrate the consequences and costs of various policies open to the fund. Section 5 concludes.

2. A Simple Model

Consider then a capitalist economy with a wage-earner fund in place and suppose that the fund is charged with the responsibility of financing an emerging self-managed sector. We will suppose that no "capital maintenance rule" holds in the economy, as it might do under a socialist system such as that of Yugoslavia (see Uvalic, 1986 for discussion). For simplicity suppose that the self-managed sector is wholly owned by the fund. This is consistent with the McCain (1977) result that, with performance bonds available, cooperatives would not opt for any internal financing. In general, of course, the "private sector" of the capital market would also have the option of holding performance bonds and ordinary bonds issued by cooperatives. We abstract from this option for the sake of simplicity, perhaps by appealing to putative legislation forbidding the "private" holding of cooperative capital.

The fund then will receive income from a tax levied on both private and

cooperative sectors and from its return on capital held in both the sectors. It will make payments in the form of redeemed savings. Redemption arrangements might take a variety of forms ranging from payments into the pension system to the holding of individual certificates, redeemable after a given minimum time period. (See George, 1990 for a discussion of the various possibilities). The fund would then allocate its net income between investment in the private sector and in the cooperative sector.

Thus the fund will be taking investment and redemption decisions in pursuit of various policy objectives. It will clearly be concerned with its overall rate of return since this will affect its income. It might also be charged, perhaps via legislation, with a responsibility for employment promotion in the economy as a whole and/or promoting the growth of the self-managed sector.[2] Conflicts might well emerge between these objectives, particularly between the first and the other two.

To analyse the dynamics of an economy such as this and the likely implications of the various policy options, a model is clearly required. The starting point for this analysis is provided by the models of George, 1985b and 1987. Those papers dealt with the fund's share in the capital stock and its development over time, and with the effects of varying certain key policy parameters. In this paper it is proposed to relax the full-employment assumption and allow the fund's decisions to affect the <u>level of investment</u> as well as the structure of capital <u>ownership</u>. Wage and profit rates cannot therefore be determined by the interaction of a factor price frontier and a macroeconomic equilibrium condition as they are in the models referred to above.

Velupillai (1982) gives explicit attention to the issues raised by Kaldor (1955-6) and Pasinetti (1974), pointing out that:

> "if workers own a part of the total stock of capital, then not only should they receive the ruling rate of interest on that part of the capital stock which they own - but also should be allowed to influence the direction of future developments in the scale and composition of the capital stock".

He then proceeds to analyse a model in which "both workers and capitalists invest - albeit for achieving different goals". The "workers" in his model are however individual workers, not engaging in concerted or organised action. One may question whether workers' "investment" in this sense is really significant in a modern capitalist economy. Firstly, in such economies, workers' savings are usually made in forms which yield very low real rates of return and carry little or no power over investment decisions. Examples would be savings banks, building societies[3] and pension funds. Moreover even when individual workers do get control over investment it is hard to see whey they should pursue substantially different goals from those of capitalists. Velupillai's analysis would seem, in fact, to be more relevant to collective forms of workers' savings and investment such as wage-earner funds.

In this section a model is developed with which to analyse the dynamics of a fund operating in a capitalist economy but charged with the responsibility of promoting an emerging self-managed sector. Consider a one-good, two-sector economy. Sector 1 (the "private sector") consists of conventional firms while Sector 2 (the "cooperative sector") consists of self-managed firms. It will be assumed that all cooperative capital is externally financed and held by the fund. Adopt the following notation:

K_1 = capital in private sector

K_2 = capital in cooperative sector

$K_1 + K_2 = K$ = total capital in the economy

L_1 = labour employed in private sector (1)

L_2 = labour employed[4] in cooperative sector

$L_1 + L_2 = L$ = total employed labour

Let:

Q_1 = output of private sector

Q_2 = output of cooperative sector (2)

$Q_1 + Q_2 = Q$ = total output of the economy

Taking the single good as numeraire define:

w = pre-tax real wage rate (paid in the private sector)

Assume fixed capital/output ratios in both sectors:

$$k_1 = \frac{K_1}{Q_1}, \quad k_2 = \frac{K_2}{Q_2} \tag{3}$$

and fixed labour productivity:

$$q_1 = Q_1/L_1 \qquad q_2 = Q_2/L_2 \tag{4}$$

There is a considerable body of literature, which makes empirical comparisons between cooperatives and conventional firms in the same industry with respect to variables such as productivity and factor intensity. On the whole this literature deals with comparisons <u>within</u> an industry and does not necessarily justify extrapolation to an entire cooperative <u>sector</u>, since self-managed firms may well be unevenly distributed across industries. These issues are obviously difficult to deal with within the framework of a one good model. We therefore adopt assumptions broadly consistent with the empirical literature. Specifically, on labour productivity we assume superiority of the private sector:

$$q_1 > q_2 \qquad (5)$$

and on capital productivity, the superiority of the cooperative sector:

$$k_1 > k_2 \qquad (6)$$

Together, assumptions (5) and (6) imply a lower capital/labour ratio in the cooperative sector. These assumptions are consistent with many empirical findings, (see, for example, Jackall and Levin, 1984). Some doubt has been cast on assumption (5) in unpublished work by Smith et. al. (1988) which considers both turnover and value-added measures of output for Italian cooperatives. Smith et. al. apparently discover that, using value-added definitions of output, cooperatives have superior capital productivity and superior labour productivity. On the turnover definition, assumptions (5) and (6) re-assert themselves, and the relative factor intensity result holds for both definitions. Presumably cooperatives are economising on raw materials and intermediate inputs to a greater extent than capitalist firms. To deal satisfactorily with this point requires at least a three-factor production function rather than the two-factor production function used here. Nonetheless, brief consideration is given below to the implications of assuming cooperatives to have superior capital productivity and superior labour productivity.

Now consider the wage-profit frontiers for the two sectors. Note that these are determined by the amount of labour actually employed (L_1 or L_2) rather than the amount available. Thus there is no implicit full-employment assumption lying behind the wage-profit frontiers. It is assumed throughout that there is always some unemployed labour in the economy (perhaps only a very small proportion of the available labour force) and that the availability of labour is never a constraint on growth. For sector 1 we have

$$Q_1 = wL_1 + rK_1$$

$$\Rightarrow \quad 1 = \frac{w}{q_1} + rk_1 \qquad (7)$$

where r = profit rate in sector 1 and w = wage rate in sector 1.

For sector 2 we have:

$$Q_2 = y_2L_2 + dK_2$$

$$\Rightarrow \quad 1 = \frac{y_2}{q_2} + dk_2 \qquad (8)$$

where d = profit rate in sector 2 and y_2 = income per worker in sector 2.

The wage-profit frontiers are illustrated in Fig. 1, utilising the assumptions of (5) and (6) above.

Figure 1 near here

We assume that labour is perfectly mobile between the two sectors and thus that the wage rate in sector 1 must equal income per worker in sector 2. We assume also that capital is perfectly mobile between sectors and that the fund does not wish to reduce its overall rate of return by favouring one sector over the other. Thus profit rates in the two sectors must be equal. Given $y_2 = w_2$ and r = d, the only possible equilibrium is the point of intersection of the two factor price frontiers (where $r = r^* = d^* = d$ and $w = w^* = y^*_2 = y_2$). It is easy to show that:

$$w^* = \frac{k_2 - k_1}{k_2/q_1 - k_1/q_2}, \qquad r^* = \frac{q_2 - q_1}{k_2q_2 - k_1q_1} \qquad (9)$$

If one adopts the assumptions, discussed above, that cooperatives have higher capital productivity <u>and</u> higher labour productivity but a lower capital/labour ratio, the wage-profit frontiers for the two sectors are as

depicted in figure 2. If both factors were completely mobile they would presumably both move instantaneously out of the capitalist sector into the cooperative sector, thus rending the fund unnecessary as a device to promote self-management. A more probable scenario would be that one factor is mobile while the other is not. Under capitalism it is plausible to assume capital mobile, so suppose the rate of profit in the two sectors is equalised at r*. Then the cooperative sector will pay income per worker y_2^*, substantially higher than w*, the wage paid in the private sector. Presumably labour would want to move but would be frustrated by the employment restricting tendencies of cooperatives. The situation would persist if these workers were unable to start new cooperatives. This is consistent with the model presented here, in which the speed at which new cooperatives can be formed is dictated by the speed at which the fund supplies capital to the cooperative sector. An alternative, but less likely scenario, is that labour is completely mobile between sectors but that the capital market is willing to tolerate much lower rates of profit in the private sector than in the cooperative sector. This is also illustrated in figure 2 where labour earnings of y_2^{**} (=w**) lead to a profit rate r** in the private sector substantially lower than the profit rate (d**) in the cooperative sector.

Figure 2 near here

Suppose that all performance bonds are held by the fund and also that the fund holds some private sector capital in the form of shares. We may therefore write:

$$K_F = K_{1F} + K_2$$

where K_F = total fund capital

K_{1F} = fund capital in private sector (10)

K_2 = fund capital in cooperative sector

The remainder of private sector capital will be held by the private sector of the capital market. This part of the capital stock we denote K_{1p}. Thus:

$$K = K_F + K_{1p} = K_{1F} + K_2 + K_{1p} \tag{11}$$

We now suppose that individual workers do not undertake any voluntary saving, and that saving in the fund is compulsory. Suppose that the fund pays out in year t a fraction (1 - s(t)) of its income $Y_F(t)$ in the form of redemption payments R(t). Thus:

$$R(t) = (1-s(t))\ Y_F(t) \tag{12}$$

Under the Danish 1973 arrangements (discussed in George, 1985a), redemptions would be made directly to individual workers and would be calculated as the fully accumulated value of the original contribution, that is (supposing all fund certificates redeemed at the earliest possible date):

$$R(t) = B(t-T)\ e^{A(t)} \tag{13}$$

where B(t) = total fund contribution at date t

T = redemption period

$$A(t) = \int_{t-T}^{t} i(u)du \tag{14}$$

Clearly i(u) could be related to r(u) and in George (1985b and 1987) the two rates were taken as equal. However we assume here that the time path of i, and hence of s, is a policy variable available to the fund. Naturally the legal basis of the fund may place restrictions on the values of s. In the 1983 Swedish arrangements, for example, the fund is obliged to make annual payments into the pension fund system (see George, 1991 for a discussion). This obligation clearly imposes an upper limit for s.

3. Dynamics of the Model

The fund receives income from a tax on labour income in both sectors. Suppose the tax is levied at a constant rate (γ) (see George, 1987 for an analysis of the 'short run' during which the tax rate γ can vary). The fund also makes profits on the capital it owns. Accepting assumptions (5) and (6) and their implications for the wage and profit rates (equations 9), it is easy to see that fund income (Y_F) must amount to:

$$Y_F = \gamma \left[\frac{w^*K_1}{k_1q_1} + \frac{w^*K_2}{k_2q_2} \right] + r^*[K_{1F} + K_2] \qquad (15)$$

Suppose that the fund invests a proportion θ of its net income (i.e. income after paying out redemptions) in the cooperative sector and the remainder in the private sector. Suppose that all capital depreciates at a rate of δ and that the private sector invests all its profits. Then:

$$\dot{K}_{1P} = (r^* - \delta)K_{1P} \qquad (16)$$

$$\dot{K}_{1F} = (1 - \theta)sY_F - \delta K_{1F} \qquad (17)$$

$$\dot{K}_2 = \theta sY_F - \delta K_2 \qquad (18)$$

Substituting (15) into (11) and (18) and rearranging gives:

$$\dot{K}_{1F} = \left[(1-\theta)s\left[\frac{\gamma w^*}{k_1q_1} + r^*\right] - \delta\right] K_{1F} + (1-\theta)s \left[\frac{\gamma w^*}{k_2q_2} + r^*\right] K_2 + \frac{(1-\theta)s\gamma w^*}{k_1q_1} K_{1P} \qquad (19)$$

$$\dot{K}_2 = \theta s \left[\frac{\gamma w^*}{k_1q_1} + r^*\right]K_{1F} + \left[\theta s\left(\frac{\gamma w^*}{k_2q_2} + r^*\right) - \delta\right]K_2 + \frac{\theta s\gamma w^*}{k_1q_1} K_{1P} \qquad (20)$$

Equations (16), (19) and (20) together constitute a linear dynamic system in K_{1P}, K_{1F} and K_2. It is not fully coupled since (16) can be solved

independently of (19) and (20) to give:

$$K_{1P}(t) = K_{1P}(0)e^{(r^*-\delta)t} \tag{21}$$

Define the following shares in the capital stock:

$$x = K_2/K\ , \qquad y = K_{1F}/K\ , \qquad z = K_{1P}/K \tag{22}$$

It is proved in Appendix 1 that the following condition (23) is necessary and sufficient for the above system to converge to a steady state in which $z > 0$.

$$\frac{s\gamma w^*}{(1-s)}\left[\frac{1-\theta}{k_1 q_1} + \frac{\theta}{k_2 q_2}\right] \leq r^* \tag{23}$$

It is also proved in Appendix 1 that, in this steady state, the growth rate of the economy is:

$$g = r^* - \delta \tag{24}$$

which is clearly independent of the fund's policy parameters. We shall assume throughout that $g > 0$.

Furthermore it is shown in Appendix 1 that, if condition (23) is violated, the steady state growth rate of the economy is given by:

$$\lambda_2 = s\gamma w^*\left[\frac{1-\theta}{k_1 q_1} + \frac{\theta}{k_2 q_2}\right] + sr^* - \delta \tag{25}$$

which clearly does depend on the fund's policy parameters. It increases with γ, s and θ. In this steady state $z = 0$, i.e. the pure private sector disappears in the long run.

The next section reports some numerical simulations for a variety of values of the policy parameters. Some of these satisfy condition (23) while others do not. Note that condition (23) is more likely to be violated the higher are s, γ and θ.

4. Numerical Simulations

In order to solve the model numerically it was re-expressed in discrete time form. A Fortran program was written to generate and graph time paths of the following variables:

(a) ratio of fund capital in cooperative sector to the total capital stock ($x = K_2/K$)

(b) ratio of fund capital in private sector to the total capital stock ($y = K_{1F}/K$)

(c) ratio of private capital to the total capital stock ($z = K_{1p}/K$)

(d) post tax income per employed worker ($y_w = (1-\gamma)w^* + (1-s)Y_F/L$)

(e) growth rate of output

(f) growth rate of employment

It should be noted that output and employment can grow at slightly different rates though these must converge as $t \to \infty$, provided the model's parameters are held constant.

Initial conditions and parameters were set as follows:

δ (rate of capital depreciation) = 0.02

$k_1 = 3.5$

$k_2 = 2.5$

$q_1 = 10.5$

$q_2 = 10.0$

$K_{1p}(o) = 95.0$

$K_{1F}(o) = 0.0$

$K_2(o) = 5.0$

The program initially solves out for the equilibrium wage and profit rates. They are respectively:

$w^* = 8.936$

$r^* = 0.043$

The levels of the 'policy parameters' s, θ and γ were varied between simulation runs. Eight runs are reported here corresponding to the following parameter values:

Runs satisfying condition (23)

$s = 0.2$

$\gamma = 0.05,\ 0.20$

$\theta = 0.3,\ 0.9$

Runs violating condition (23)

$s = 0.8$

$\gamma = 0.05,\ 0.20$

$\theta = 0.3,\ 0.9$

The results are presented in graphical form in Appendix 2.

The general pattern is similar for all the runs. The share of private capital tends to fall over time while the share of fund capital in both sectors tends to rise. The rates of decline and increase are heavily dependent on the values of the policy parameters. Post-tax income per employed worker tends to rise very slightly over time and it is largely unaffected by changes in policy parameters. Growth rates (of employment and output) tend to fall slightly over time. Their level is influenced considerably by changes in the policy parameters, though their rate of change is hardly influenced at all. Changing θ has a large impact on the division of the fund between private and cooperative sectors (K_{1F}/K_2). This ratio tends towards $(1-\theta)/\theta$ as time tends to infinity. Raising θ slightly increases growth rates but has little impact on post-tax income per employed worker. Raising γ or s significantly lowers the share of private capital

(K_{1P}/K) and raises both fundshares (K_{1F}/K and K_2/K). The effect of increasing s is noticeably greater than the effect of increasing γ. This is illustrated in Tables 1 and 2. Such policies have little cost in terms of lowered post-tax income per employed worker and they tend, in fact, to raise growth rates. This is illustrated in Tables 3 and 4. The impact of s on growth rates is noticeably greater than the impact of γ. Raising s will, of course, also reduce the rate of return that the fund can pay on redemptions. This may, in practice, impose an upper limit on s.

5. Conclusions

Many of the difficulties of self-managed firms under capitalism could be resolved by the introduction of performance bonds. An ideal institution to hold such bonds would be a wage-earners' investment fund of Scandinavian type, which could own capital in both private and cooperative sectors. Such an arrangement would generate a new type of ownership different from 'private' or 'public' ownership as at present understood. Thus the possibility emerges of a new type of 'mixed economy' with three sectors, private, public and cooperative.

The model presented here analyses the possible development over time of a wage-earners' investment fund holding a mixed portfolio of the type described above. The impact of the various policy parameters is analysed. The fund can promote the cooperative sector by directing investment income towards it, at little or no cost to the economy. A high savings ratio (s) of the fund and a high contributions tax (γ) accelerate the fund's development dramatically. Both policies tend to generate high growth rates which might lead to 'overheating' problems, but neither has much cost in terms of reduced post-tax worker income. Raising s will, however, lower the rate of return

that the fund can pay on redemptions. This might, in practice, impose an upper limit on s.

APPENDIX 1

Proposition 1 The dynamical system consisting of equations (16), (19) and (20) converges to a steady state in which $z > 0$ and the rate of growth is given by $g = r^* - \delta$ if and only if condition (23) (of section 3) is satisfied.

Proof The dynamical system clearly has g as one eigenvalue. Let the other two eigenvalues by λ_1 and λ_2. The determinant of the dynamical system is the product of the eigenvalues and the trace is their sum. Thus we may write:

$$\lambda_1\lambda_2 = \delta\ (\delta - (1-\theta)a_2 - \theta a_3)$$

$$\lambda_1 + \lambda_2 = (1-\theta)a_2 + \theta a_3 - 2\delta \qquad \text{(A1)}$$

where

$$a_2 = s\left(\frac{\gamma w^*}{k_1 q_1} + r^*\right), \qquad a_3 = s\left(\frac{\gamma w^*}{k_2 q_2} + r^*\right) \qquad \text{(A2)}$$

The solution of these equations (which are symmetric in λ_1 and λ_2) is:

$$\lambda_1 = -\delta$$

$$\lambda_2 = (1-\theta)\ s\left[\frac{\gamma w^*}{k_1 q_1} + r^*\right] + \theta s\left[\frac{\gamma w^*}{k_2 q_2} + r^*\right] - \delta \qquad \text{(A3)}$$

$$= s\gamma w^*\left(\frac{1-\theta}{k_1 q_1} + \frac{\theta}{k_2 q_2}\right) + sr^* - \delta$$

Solutions of the dynamical system take the form:

$$K_{1P}(t) = k_{1P}(o)e^{gt}$$

$$K_{1F}(t) = b_1 e^{\lambda_1 t} + b_2 e^{\lambda_2 t} + b_3 e^{gt} \qquad \text{(A4)}$$

$$K_2(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + c_3 e^{gt}$$

It follows that the system will tend to steady state growth at a rate g if and only if $\lambda_2 \leq g$ (since $\lambda_1 < 0$). We have

$$g = r^* - \delta$$

so that $\lambda_2 \leq g$ if and only if:

$$(1-\theta)s \left[\frac{\gamma w^*}{k_1 q_1} + r^* \right] + \theta s \left[\frac{\gamma w^*}{k_2 q_2} + r^* \right] \leq r^*$$

$$\Leftrightarrow s\gamma w^* \left[\frac{1-\theta}{k_1 q_1} + \frac{\theta}{k_2 q_2} \right] \leq (1-s)r^*$$

$$\Leftrightarrow \frac{s\gamma w^*}{(1-s)} \left[\frac{1-\theta}{k_1 q_1} + \frac{\theta}{k_2 q_2} \right] \leq r^*$$

which is condition (23) as required.

From (A4) it is clear that, if $\lambda_2 \leq g$,

$$z = K_{1P}/K \to K_{1P}(o)/(b_3 + c_3 + K_{1P}(o)) > 0 \quad \text{as } t \to \infty$$

That is $z > 0$ in the steady state, as required.

Proposition 2 The dynamical system consisting of equations (16), (19) and (20) converges to a steady state in which $z = 0$ and the rate of growth is given by:

$$\lambda_2 = s\gamma w^* \left(\frac{1-\theta}{k_1 q_1} + \frac{\theta}{k_2 q_2} \right) + sr^* - \delta$$

if and only if condition (23) is violated.

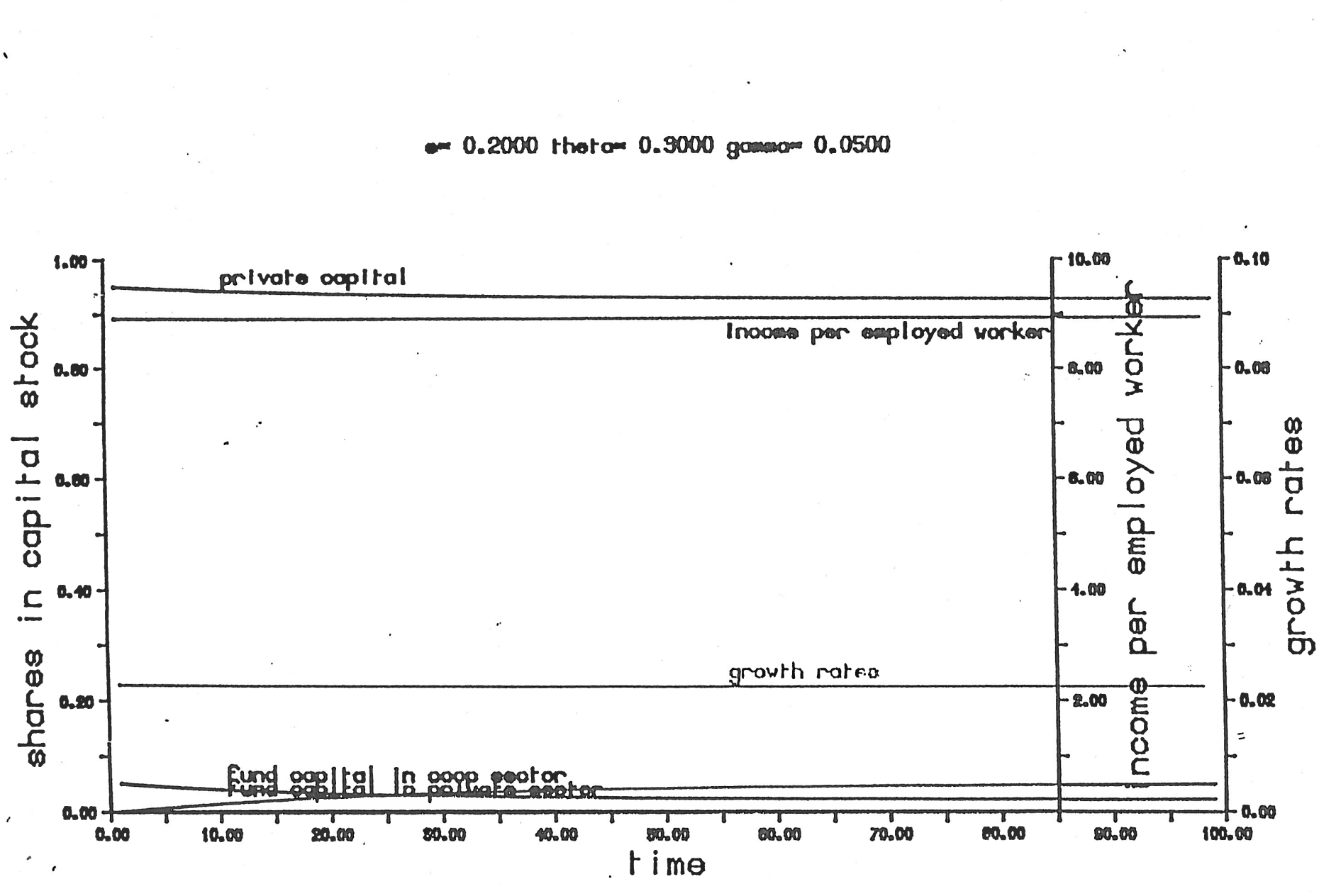
Proof By the argument of proposition 1, $\lambda_2 > g$ if and only if condition (23) is violated. Then, from (A4), the steady state growth rate must be equal to λ_2. Moreover, since $\lambda_2 > g$ and $\lambda_1 < 0$,

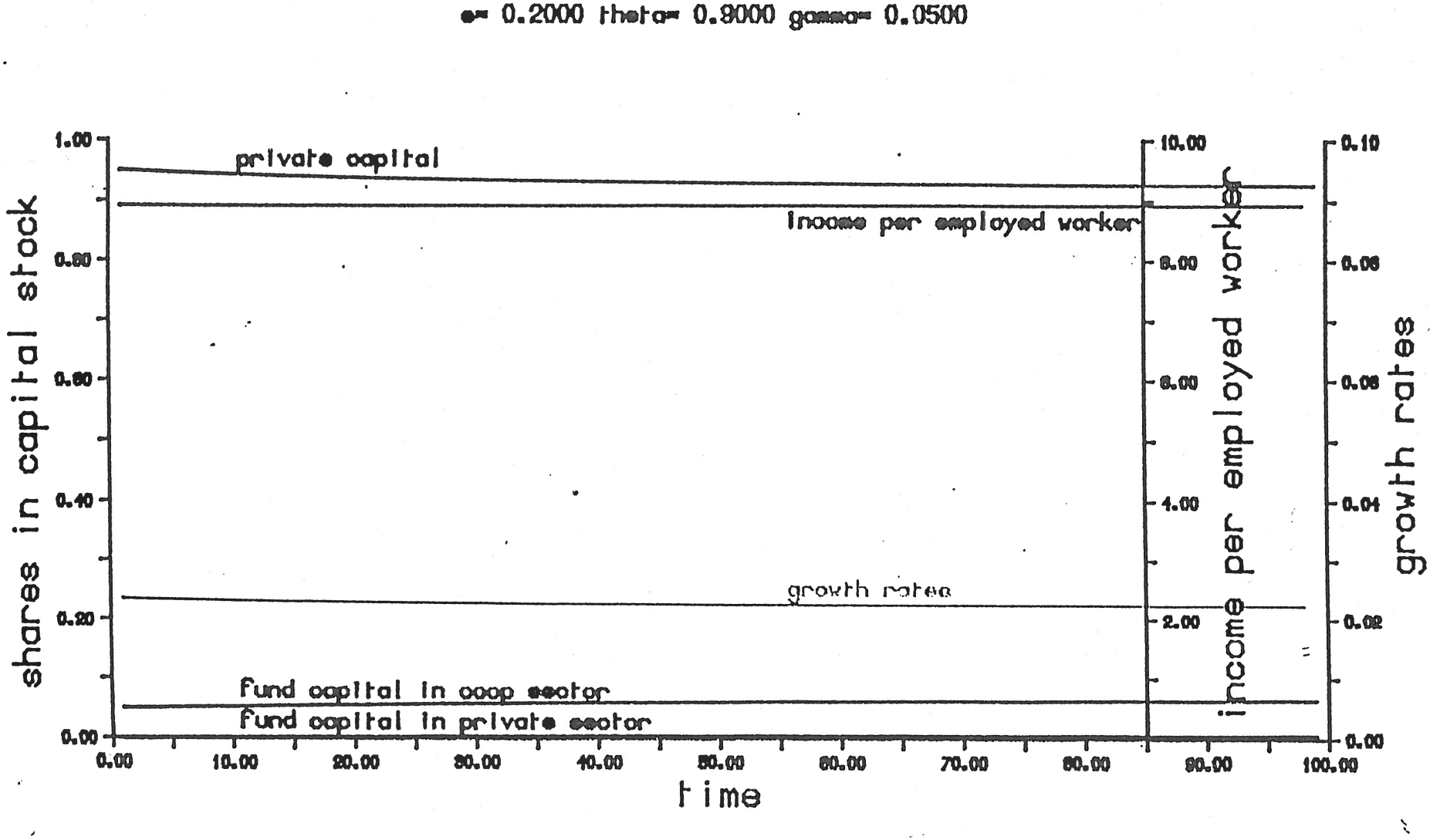
$$z = K_{1P}/K \to 0 \text{ as } t \to \infty$$

That is $z = 0$ in the steady state, as required.

APPENDIX 2

Graphical output of the simulation runs.


a= 0.2000 theta= 0.3000 gamma= 0.0500
private capital
income per employed worker
growth rates
fund capital in coop sector
fund capital in private sector
shares in capital stock
income per employed worker
growth rates
time
1.00
0.80
0.60
0.40
0.20
0.00
10.00
8.00
6.00
4.00
2.00
0.10
0.08
0.06
0.04
0.02
0.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00


e= 0.2000 theta= 0.9000 gamma= 0.0500
private capital
income per employed worker
growth rates
Fund capital in coop sector
Fund capital in private sector
shares in capital stock
income per employed worker
growth rates
time
1.00
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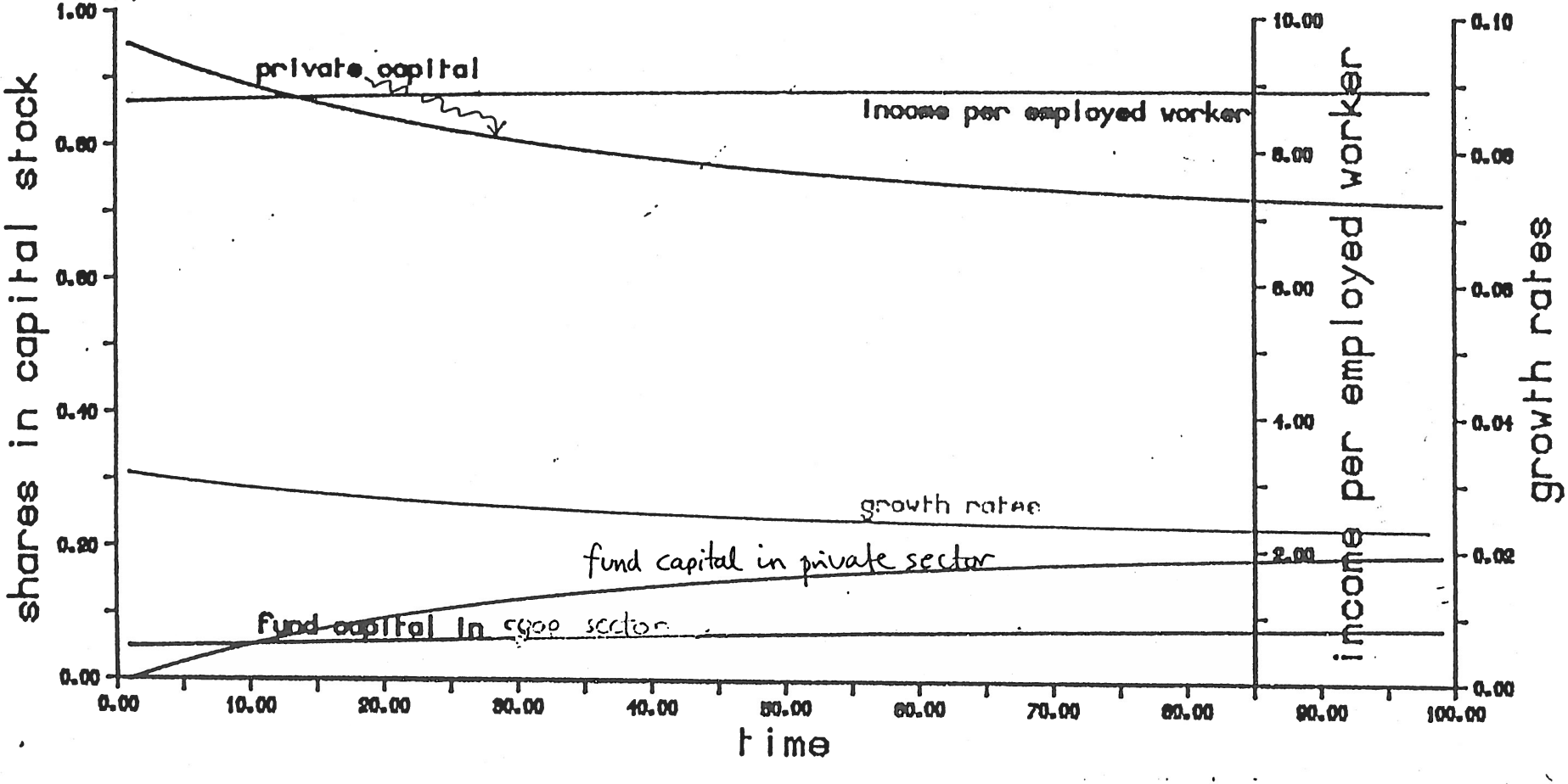
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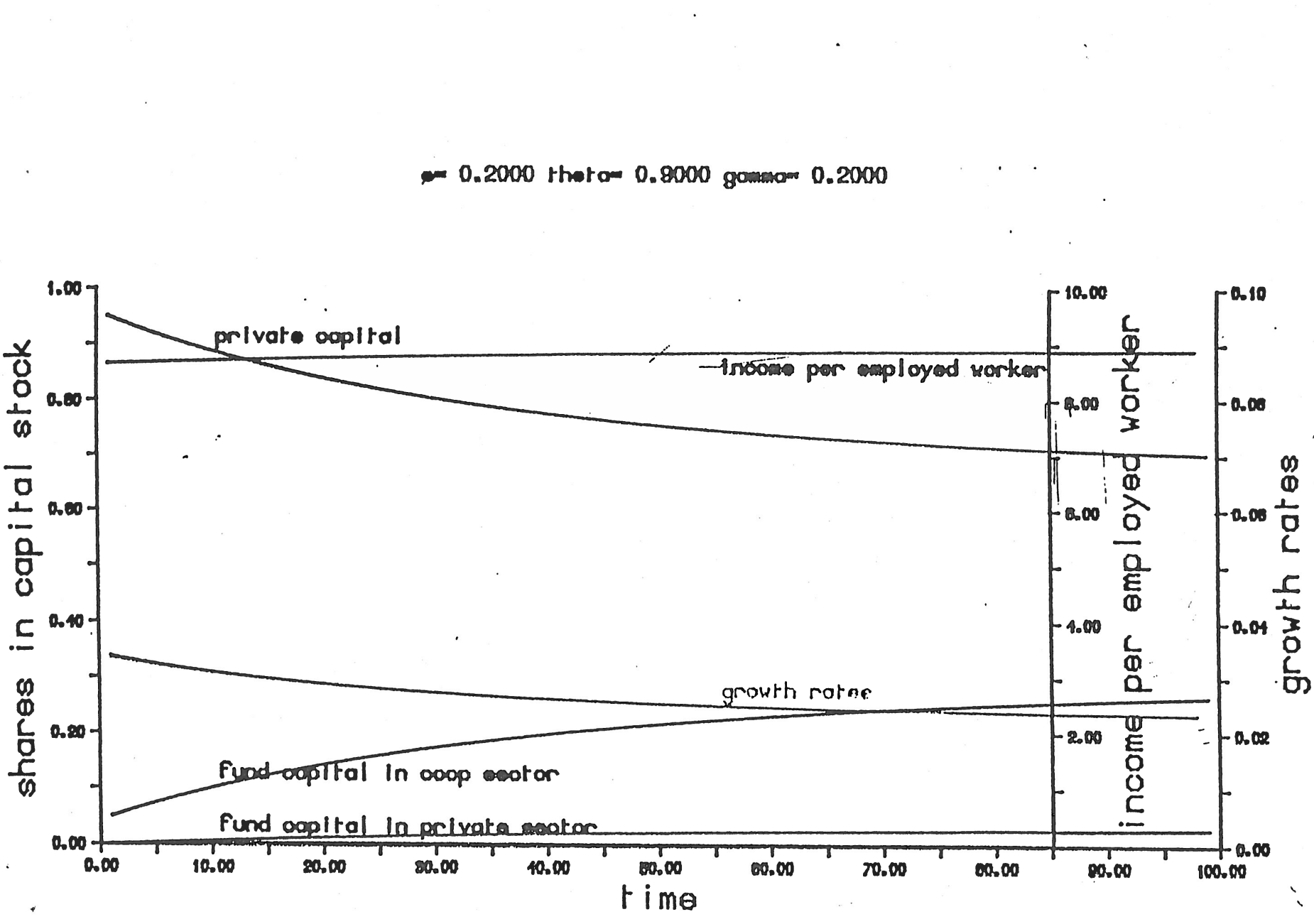
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growth rates
Fund capital in coop sector
Fund capital in private sector
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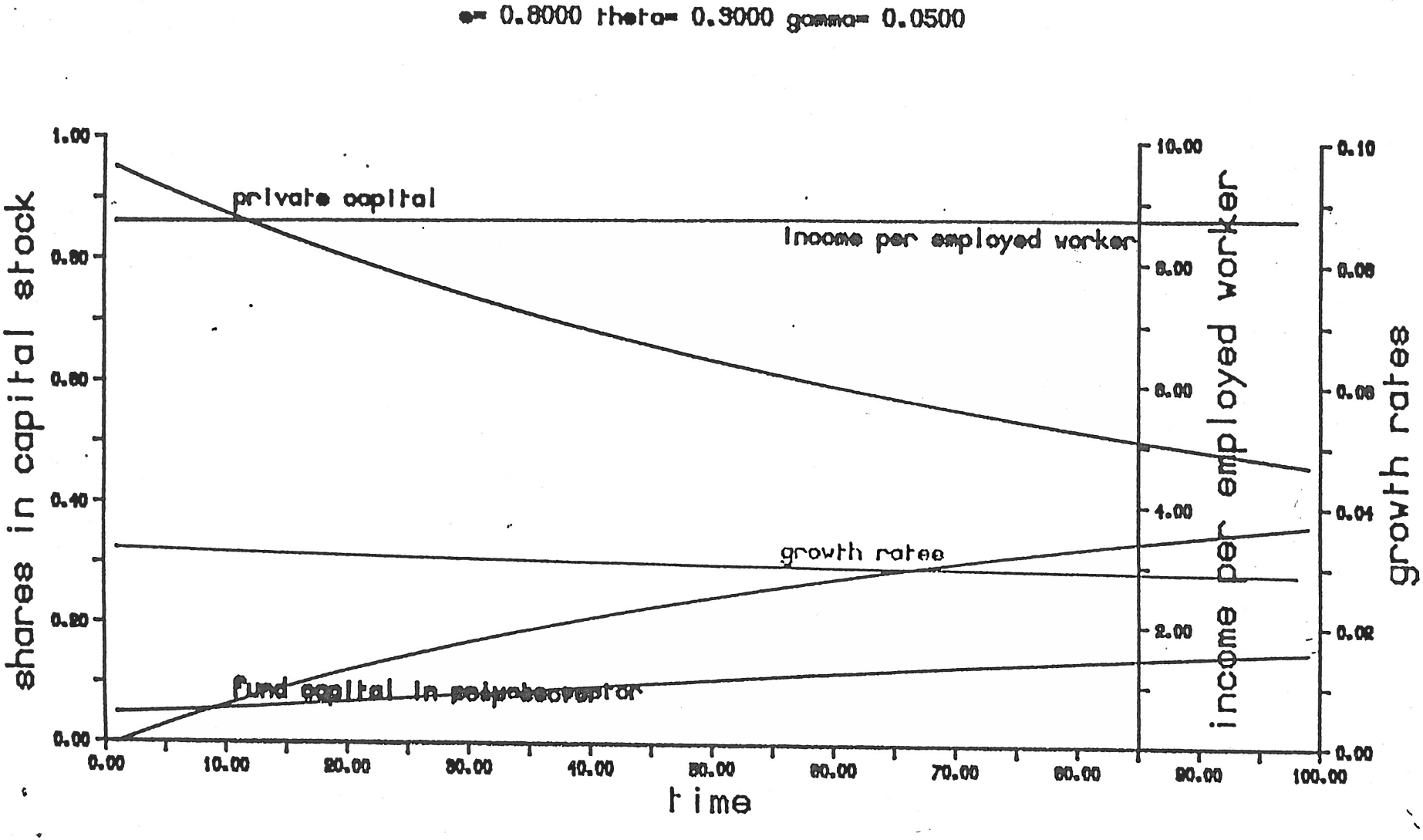
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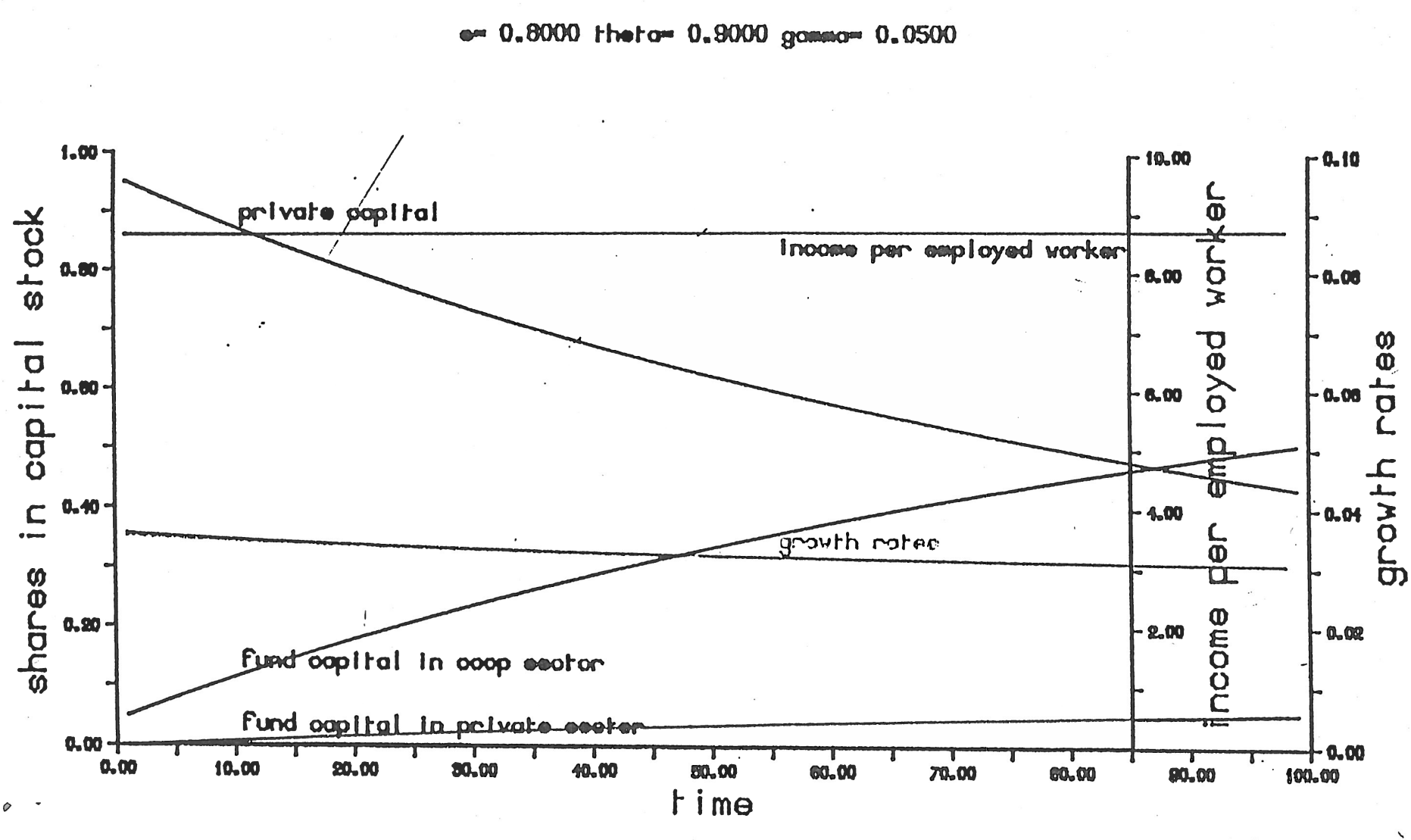
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growth rates
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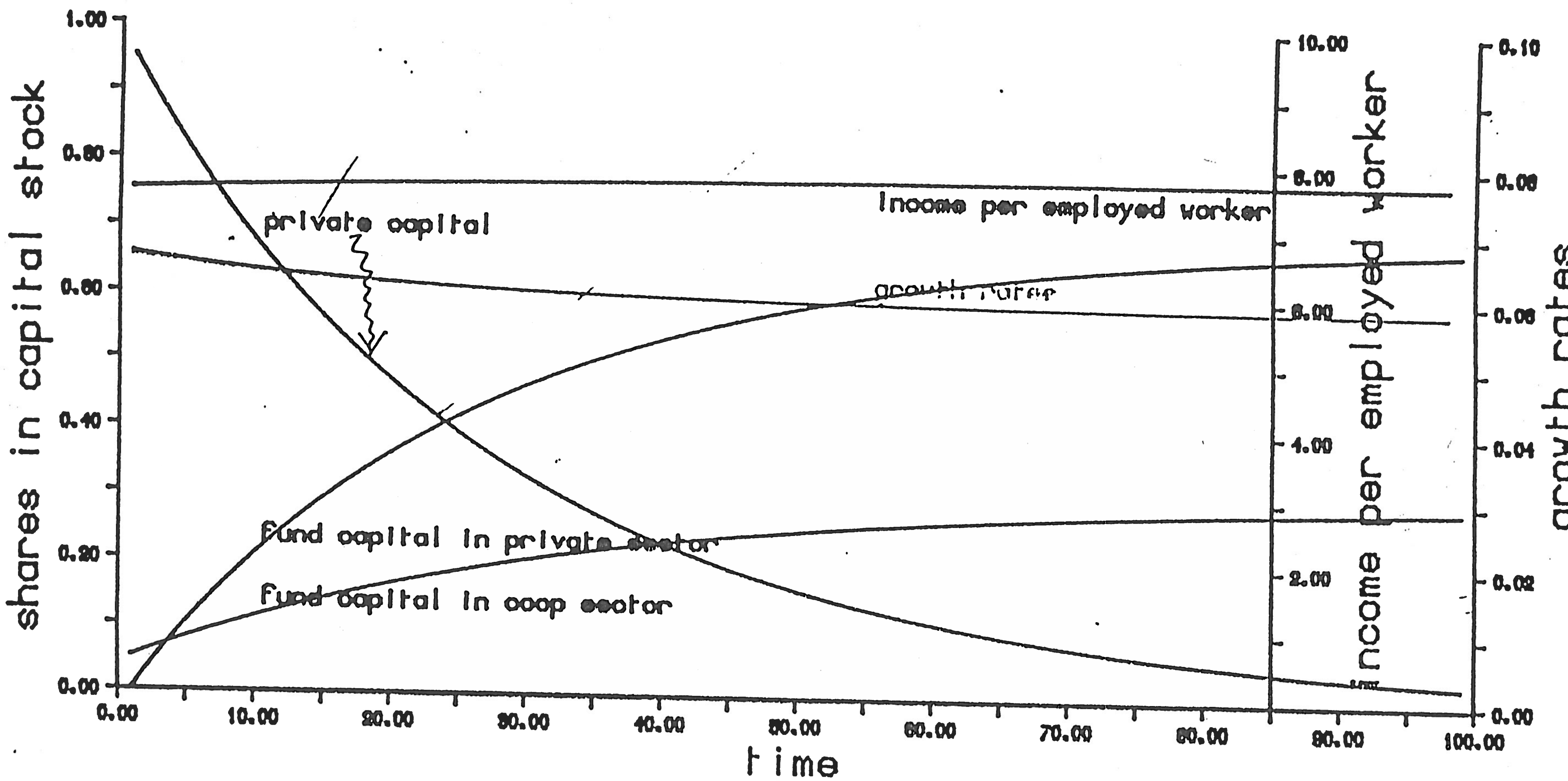

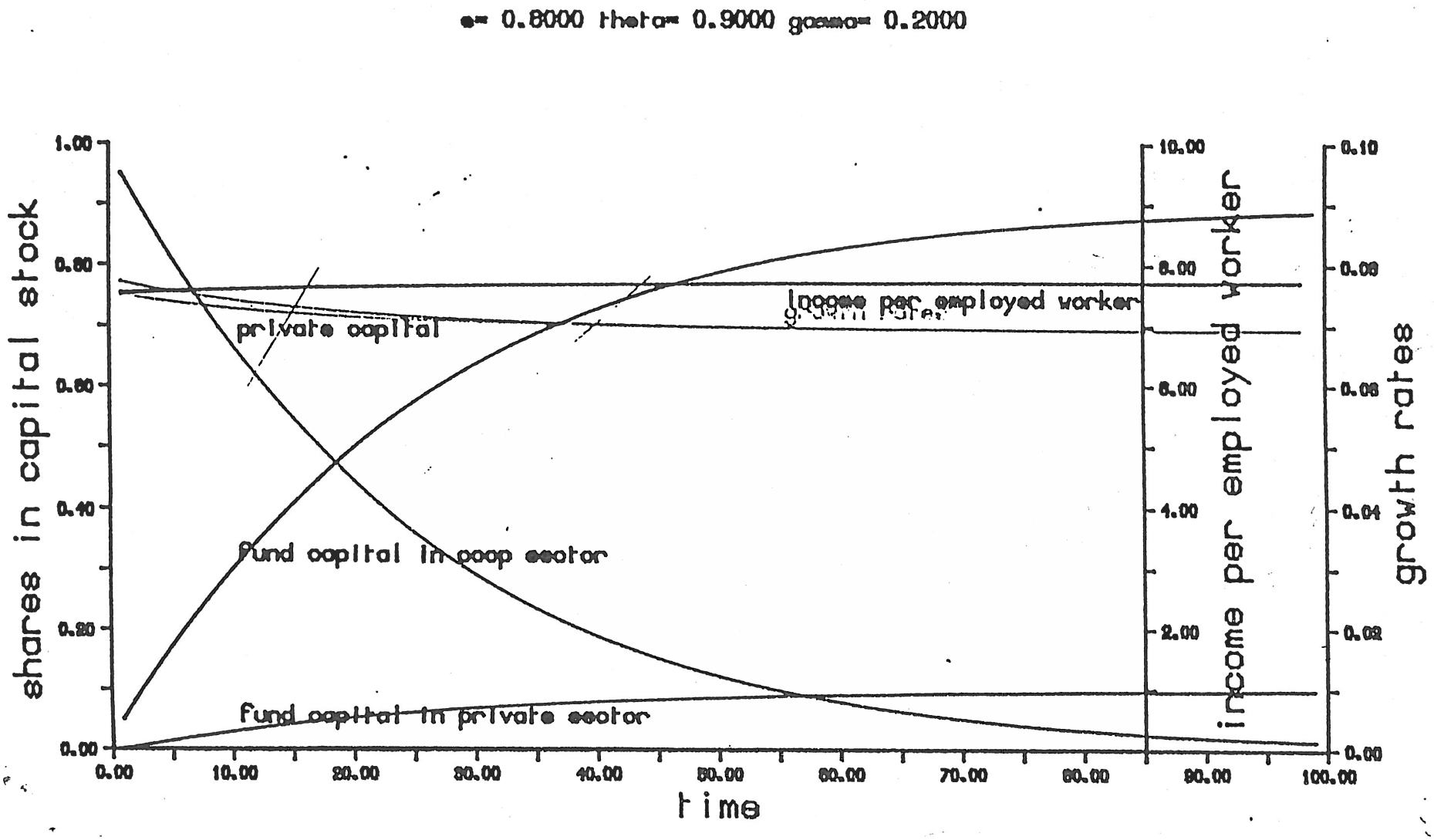
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time

FOOTNOTES

1. McCain calls them "participation bonds".
2. And/or perhaps the promotion of certain industries or geographical regions.
3. During the early 1980s the real rate of return on British Building Society deposits was often negative.
4. Strictly speaking there is no employment relation in cooperatives since they entail "free association of labour". However we adopt the term "employment" for convenience.

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Table 1:

Values of z (= K_{1p}/K) at t=20 (for θ = 0.5)

$\gamma \backslash s$	0.2	0.5	0.8
0.05	0.940	0.876	0.804
0.10	0.907	0.792	0.672
0.20	0.843	0.643	0.464

Table 2:

Values of x (= K_2/K) at t=20 (for θ=0.5)

$\gamma \backslash s$	0.2	0.5	0.8
0.05	0.041	0.072	0.108
0.10	0.057	0.113	0.172
0.20	0.088	0.186	0.274

Table 3:

Post tax income per employed worker at t=20 (for $\theta = 0.5$)

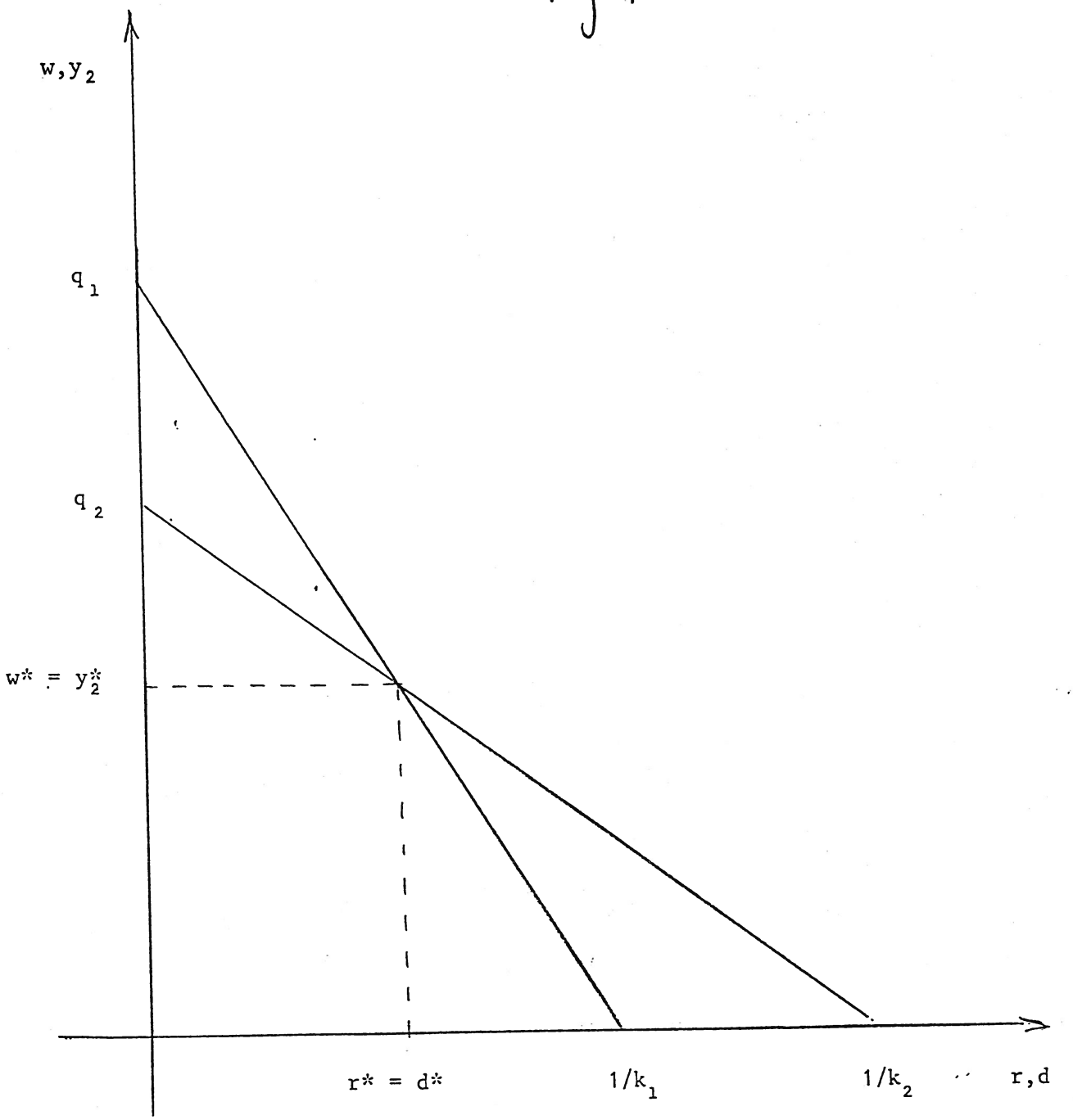
γ\s	0.2	0.5	0.8
0.05	8.921	8.807	8.639
0.10	8.871	8.648	8.316
0.20	8.767	8.299	7.655

Table 4:

Growth rate of output at t=20 (for $\theta = 0.5$)

γ\s	0.2	0.5	0.8
0.05	0.023	0.027	0.032
0.10	0.025	0.032	0.043
0.20	0.028	0.043	0.065

Fig. 1



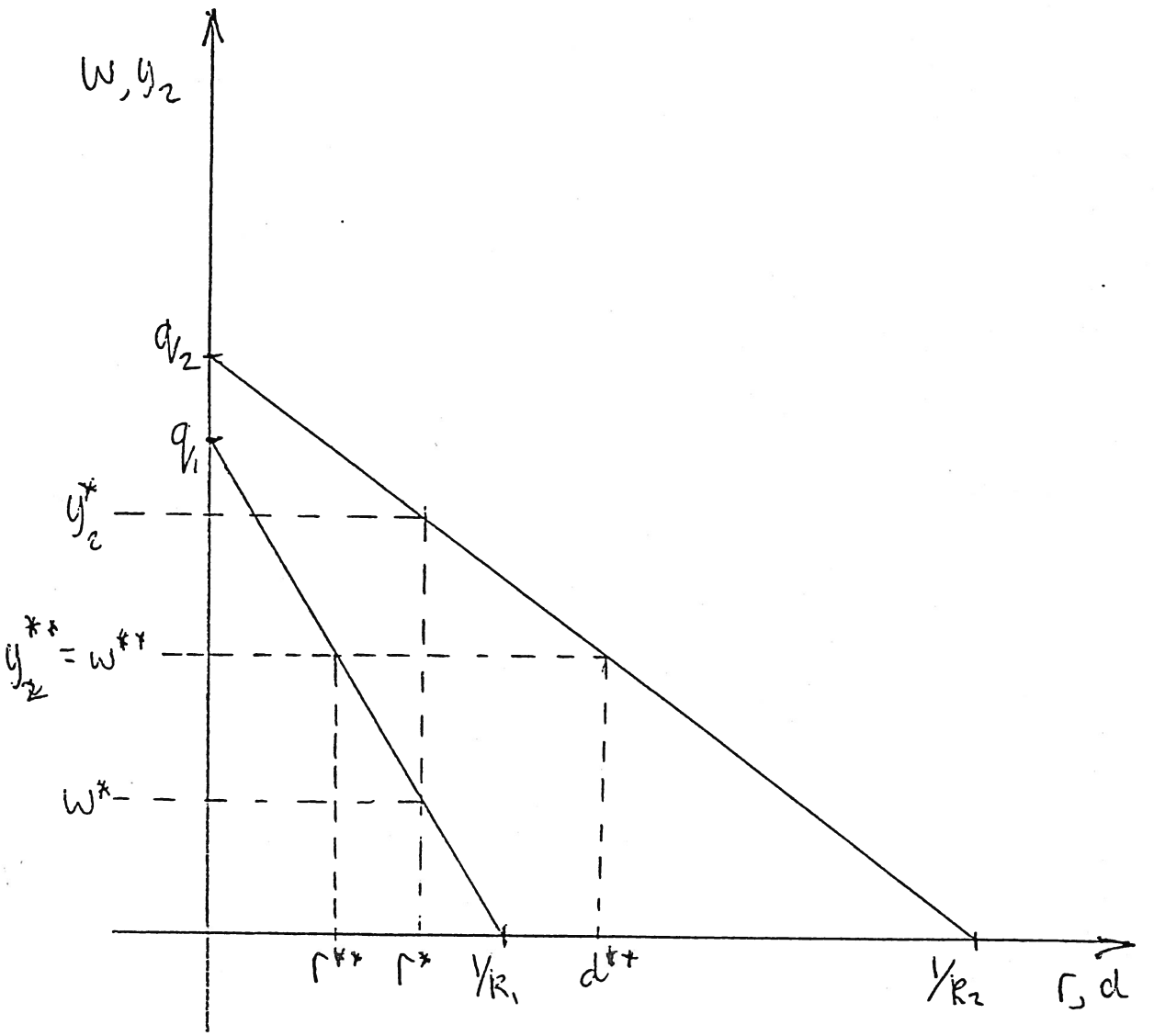


Fig. 2