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PERRY'S MODEL OF WAGE-DETERMINATION WITH STOCHASTIC PARAMETERS

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Empirical studies of wage-determination, within the general framework of the Phillips' curve and its variants, demonstrate remarkable consistency with respect to two particular specifications. They usually involve explanatory variables in the form of simple fourth-order moving averages and a dependent variable which is represented by the sequence of overlapping annual changes in an aggregative index of wages. Explicit justifications for these choices are rare but George Perry [5] does indicate some basis for them and his arguments may have proved persuasive to other economists. These arguments, which are based on institutional features of the labour market, have been discussed in several of our earlier papers, [14], [15], and [16], and we indicate two important implications of them within the context of distinct bargaining groups in the labour force. If the specifications are justified by reference to aggregation over these groups which bargain at different points in time, then a fixed-weight paradox can be illustrated ([14]) and a Yule-Slutsky effect is indicated for the linear statistical model ([15], [16]), which is used as the basis for estimates in the empirical studies of wage-determination.

The fixed-weight paradox is particularly disturbing since it suggests that the assumption of fixed weights in the moving averages of the explanatory variables is extremely restrictive for the potential changes in the aggregative wage index. Further, the restrictions leave no role for economic explanations of these changes. Some alternative approach appears to be needed if the paradox is to be circumvented. The problem is due to aggregation and it is an obvious first step in the search for its solution to look for some appropriate analogy in the existing theory for aggregation. Unfortunately no direct analogy seems to exist although the use of stochastic parameters by Arnold Zellner [19] to resolve the question of aggregation bias, which had earlier been raised by Henri Theil [18], indicates a less restrictive form for the aggregative equation without providing the basis for any simple relationship with its micro-foundations. This aggregate equation is superior to the conventional one since it is not subject to the paradox except where it is restricted to its special case, the conventional equation.

Let $\{y_t\}$ represent the sequence of overlapping annual changes in the aggregative wage index. Then, if this sequence is to be explained in terms of k sequences of four-quarter moving averages of observations for explanatory variables $\{x_{jt}\}$, the linear aggregate equation with stochastic parameters can be written as

(1) $$y_t = x_{1t}b_{1t} + x_{2t}b_{2t} + \cdots + x_{kt}b_{kt} \qquad \text{for } t = 1,2, \ldots , n;$$

where n is the length of the sample for the dependent variable and

(2) $$b_{jt} = \beta_j + v_{jt} \qquad \text{for } j = 1,2, \ldots , k.$$

In these equations, the nk parameters $\{b_{jt}\}$ are assumed to be stochastic with mean values $\{\beta_j\}$ depending only on their first index. Substitution for these parameters in (1) yields

(3) $$y_t = x_{1t}\beta_1 + x_{2t}\beta_2 + \ldots + x_{kt}\beta_k + (x_{1t}v_{1t} + x_{2t}v_{2t} + \ldots + x_{kt}v_{kt}).$$

The temporal-composite error in the third equation has k components in the absence of further restrictions. If the errors $\{v_{2t}, v_{3t}, \ldots , v_{kt}$ for $t = 1,2, \ldots , n\}$ are identically zero and if x_{1t} is a constant over time, then (3) is the conventional aggregate equation. The fixed-weight paradox asserts that if these conditions hold, the final sequence of errors $\{v_{1t}\}$ is non-stochastic and members of their sequence can take only four values which solely depend on the temporal index.

If the equation with stochastic parameters is an appropriate specification, estimates based on the incorrectly-specified conventional equation with fixed parameters may still be adequate. If the $\{x_{jt}\}$ observations are non-stochastic, ordinary least-squares estimates of the k parameters $\{\beta_j\}$ remain unbiased but estimates of their standard errors and Student's t-statistics are based on inappropriate formulae. Whether the latter estimates are misleading is an empirical question and cannot be settled a priori. Some evidence is presented in the five tables given below. It should be remembered that this evidence is based upon the assumption that the length of the sample is sufficient for large-sample theory to be appropriate so that a consistent estimate for the dispersion matrix of the temporal-composite errors (based on supplemental restrictions on the processes generating these errors) can be treated as if it were the actual matrix.

Clifford Hildreth and James Houck [3] have developed a method of estimating both the parameters $\{\beta_j\}$ and variance components of the temporal-composite errors. A full account of their technique is provided in the technical appendix, where it is extended to cases in which the errors are temporally autocorrelated and contemporaneously correlated. Two variants of the technique are available since the variance components can be estimated both by least-squares and by instrumental-variable methods, and these are labelled HH1 and HH2 respectively. Modifications to their technique are indicated by parenthetic additions. For example HH1 (Cov) is used to denote that contemporaneous errors can be correlated but there is no intertemporal correlation, whereas HH1 (ρ) indicates the absence of contemporaneous correlation and the presence, for each of the k temporal sequences of errors, of a first-order autoregressive generating process with known parameter ρ.

Some empirical results are tabulated below for two well-known models which attempt to explain wage-determination in manufacturing industries. Tables One, Three and Four contain new estimates for Perry's model with U.S. data whereas Table Two contains new estimates for the Canadian model of Ronald Bodkin et al., [2]. The latter estimates must be treated with some care since one of the explanatory variables is a lagged dependent variable. The Hildreth-Houck estimators are Aitken estimators with an estimated dispersion matrix for the temporal-composite errors. Hence, notice must be given to the warning provided by G. S. Maddala [4] for estimators of this type when autocorrelation is present. He indicates an asymptotic lack of efficiency and the consequent asymptotic bias in calculations of Student's t-statistics due to differences in limiting distributions. Tables One and Two contrast the conventional least-squares estimates with the two Hildreth-Houck estimates. Table Three illustrates the relative sensitivity for Perry's model of HH1 estimates to changes in lengths of samples and Table Four illustrates the relative sensitivity of these estimates to known autoregressive processes for the errors. Table Five indicates their sensitivity with respect to small changes in the estimated variance components. An account of the data is contained in the second appendix. These are consistent with those used in the original studies and our earlier investigation of the Yule-Slutsky effect.

One important feature of Tables One and Three is the robustness of Perry's estimates. For the period from 1948 to 1969, the differences between the three collections of estimates are small when all variables are included in the fit. Both Hildreth-Houck methods indicate variance components for profits, changes in profits, and the Korean dummy variable. These two technoques yield almost identical estimates for this model (although not for the

Canadian model). This result might be derived from the sensitivity elasticities given in the first two columns of Table Five, which indicate that only substantial changes in the estimates of the variance components for profits and change in profits can affect the final Hildreth-Houck estimates. For examples, first-order changes in the estimates will not exceed 0.163 percent following a ten percent change in the estimated variance component for profits. Even the joint effects of a thirty percent change for each of the two variance components (caused by the omission of both dummy variables) leaves some estimated coefficients changed by less than ten percent. The principal effects of this omission are a decline in the estimated coefficients for unemployment and changes in profits and increases in both estimates of variance components. This robustness is confirmed by the results in Table Three for three different sample periods. Estimated coefficients for profits and change in profits exhibit greater variability than other estimates but this is predictable from the larger sensitivity elasticities in the first two columns of Table Five. Directional changes in estimated coefficients are also in accordance with the signs given in this table and the relative insensitivity of the estimates to the stochastic-parameter specification is especially revealed by their stability despite the substantial changes in the estimated variance component for the change in profits. The only discordant feature concerns this particular variable and the significance of the first two years of Perry's sample. Omission of eight quarterly observations for the years 1948 and 1949 leads to a doubling of the estimated variance component for the change in profits and significant reduction in both the estimated coefficient of this variable and its (asymptotic) t-statistic under the hypothesis that this coefficient is zero.

TABLE 1. PERRY: TWO HILDRETH-HOUCK ESTIMATES

	OLS	HH1	HH2	
Constant	-0.003289 (-0.567)	-0.001520 (-0.309)	-0.001519 (-0.309)	-0.000058 (-0.0105)
Unemployment	0.000809 (5.645)	0.000842 (6.063)	0.000841 (6.062)	0.000606 (5.020)
Prices	1.930 (11.141)	1.994 (11.241)	1.995 (11.243)	2.099 (10.502)
Profits	0.1991 (3.648)	0.1721 (3.099)	0.1722 (3.099)	0.1902 (3.389)
Change in Profits	0.5322 (3.458)	0.5132 (3.348)	0.5133 (3.350)	0.3803 (2.219)
Korea	-0.01321 (-2.619)	-0.0140 (-3.019)	-0.0140 (-3.0167)	
Guideposts	-0.009763 (-4.593)	-0.008868 (-4.242)	-0.008867 (-4.241)	
<u>Variances</u>				
Profits		0.01092	0.01097	0.01304
Change in Profits		0.0937	0.0960	0.1264
Korea		0.000008	0.000009	

Length of sample: 1948 - 1969, quarterly observations.

TABLE 2. BODKIN et al.: TWO HILDRETH-HOUCK ESTIMATES

	OLS	HH1	HH2
Constant	-4.1251 (-2.429)	-6.3152 (-3.205)	-3.4050 (-2.130)
Unemployment	10.4112 (1.550)	3.9977 (0.541)	12.4436 (1.897)
Prices	0.3767 (4.879)	0.3739 (5.316)	0.3813 (4.627)
Profits	0.05265 (2.949)	0.07441 (3.626)	0.04600 (2.7227)
Spillover	0.4321 (3.9485)	0.4105 (3.327)	0.4350 (4.117)
Lagged Wages	-0.09154 (-2.278)	-0.04992 (-1.772)	-0.1166 (-2.489)
<u>Variances</u>			
Constant		0.3609	0.2749
Unemployment		42.826	33.386
Prices		0.1475	0.01117
Profits			0.000003

Length of sample: 1953 I - 1965 II, quarterly observations.

TABLE 3. PERRY: SENSITIVITY OF HILDRETH-HOUCK ESTIMATES TO LENGTH OF SAMPLE

Observations	1948 - 1969	1950 - 1969	1948 - 1959
Constant	-0.001520 (-0.309)	-0.006194 (-1.100)	-0.00758 (-1.063)
Unemployment	0.000842 (6.063)	0.000728 (4.719)	0.001048 (5.749)
Prices	1.9940 (11.241)	1.8914 (7.846)	2.0514 (10.500)
Profits	0.1721 (3.099)	0.2518 (3.542)	0.1761 (2.618)
Change in Profits	0.5132 (3.348)	0.3449 (1.701)	0.5319 (2.986)
Korea	-0.0140 (-3.019)	-0.0120 (-2.514)	-0.01861 (-3.352)
Guideposts	-0.008868 (-4.242)	-0.009832 (-4.543)	
Variances			
Profits	0.01092	0.009434	0.01233
Changes in Profits	0.0937	0.1947	0.7343

TABLE 4. PERRY: HILDRETH-HOUCK ESTIMATES WITH KNOWN AUTOCORRELATION

	OLS	HH1	HH1 (Cov)	HH1 (0.4)	HH1 (0.6)	HH1 (0.8)
Constant	-0.001994	-0.000074	0.000931	-0.002774	-0.007201	-0.01723
	(-0.314)	(-0.013)	(0.183)	(-0.409)	(-0.848)	(-1.489)
Unemployment	0.0005953	0.000606	0.000570	0.0006203	0.0006418	0.0006817
	(4.544)	(5.034)	(4.866)	(4.172)	(3.393)	(2.537)
Prices	2.0636	2.0975	2.1003	2.0230	1.9343	1.8108
	(10.646)	(10.491)	(12.168)	(9.082)	(7.739)	(6.354)
Profits	0.2116	0.1904	0.1909	0.2138	0.2513	0.3344
	(3.718)	(3.391)	(3.717)	(3.097)	(2.888)	(2.744)
Change in Profits	0.4156	0.3803	0.4082	0.3802	0.3735	0.3449
	(2.416)	(2.219)	(2.851)	(2.174)	(2.071)	(1.770)
<u>Variances</u>						
Profits		0.01316	0.00491	0.01512	0.01692	0.02015
Change in Profits		0.1233	0.1368	0.1619	0.1667	0.1275
Covariance			0.02239			

TABLE 5. SENSITIVITY ELASTICITIES

	PERRY		BODKIN et al.	
	Profits	Change in Profits	Prices	Unemployment
Constant	$-1.03\ (10^{-2})$	$-9.45\ (10^{-6})$	$-1.58\ (10^{3})$	$4.77\ (10^{-3})$
Unemployment	$4.67\ (10^{-3})$	$6.13\ (10^{-7})$	$1.44\ (10^{5})$	$2.10\ (10^{-1})$
Prices	$3.03\ (10^{-3})$	$-1.64\ (10^{-7})$	$-6.16\ (10^{1})$	$-8.39\ (10^{-6})$
Profits	$-1.63\ (10^{-2})$	$-1.49\ (10^{-6})$	$2.58\ (10^{-1})$	$-1.98\ (10^{-7})$
Change in Profits	$-3.08\ (10^{-3})$	$-1.00\ (10^{-6})$		
Korea	$6.44\ (10^{-3})$	$5.48\ (10^{-7})$		
Guideposts	$-9.33\ (10^{-3})$	$-5.78\ (10^{-7})$		
Spillover			$-4.71\ (10^{1})$	$-6.47\ (10^{-6})$
Lagged Wages			$-2.01\ (10^{1})$	$-3.64\ (10^{-6})$

For Table Four, the dummy variables are omitted from Perry's model and non-zero variance components are restricted to those indicated when autocorrelation is absent. Thus the only potential cross-correlation between contemporaneous errors associates profits and change in profits. An estimate of the covariance is given in the third column. This small gain in flexibility leaves the Hildreth-Houck estimates relatively unchanged. Allowance for autocorrelation leads to greater changes especially as the autoregressive parameter approaches unity. These changes appear monotonic with the estimated coefficients for unemployment and profits increasing and those for prices and the change in profits decreasing as the autoregressive parameter increases. Similarly, there are consistent increases in the estimated variance components and a decrease in calculated t-statistics. These directional changes cannot be predicted on the basis of the sensitivity elasticities in Table Five since these are based on inappropriate formulae in the presence of autocorrelation.

In Table Two, the estimated coefficients for prices and the international spillover of wages in the model of Bodkin, et al., are reasonably insensitive with respect to changes in the technique of estimation, but those for unemployment, past wages and the constant term are extremely variable. The sensitivity elasticities for this model are contained in the final two columns of Table Five. They suggest that particular point estimates are extremely unreliable in the sense that small changes in the estimated variance component for prices will lead to substantial changes in estimates. The elasticity of the coefficient of the unemployment variable with respect to the variance component for prices is eighty times as large as any of the other elasticities which are reported in the table.

Some Tentative Conclusions

The derivation of Hildreth-Houck estimates for stochastic parameters appears to be both a worthwhile check on the sensitivity of estimates based on classical linear model and a potential source for resolution of the fixed-weight paradox. In the particular studies of wage-determination which are reported above, that of Perry appears to be robust especially with respect to the coefficients for unemployment and prices. The model of Bodkin et al. appears to be very sensitive to the choice of econometric technique and their unusual specification for the unemployment variable may be a principal choice of the variability in the estimates of their coefficients.

APPENDIX 1. The Hildreth-Houck Estimators

The temporal sequence of measurements for a specific variable of economic interest is represented by $\{y_t$ for $t = 1,2, \ldots , n\}$. In the model adopted by Clifford Hildreth and James Houck [3], this sequence can be explained in terms of a linear model of the form

$$(1) \qquad y_t = \sum_{j=1}^{k} x_{jt} b_{jt} \qquad \text{for } t=1,2, \ldots , n ;$$

where $\{x_{jt}$ for $j=1,2, \ldots , k$ and $t=1,2, \ldots , n\}$ are associated measurements for k explanatory variables which are assumed to be non-stochastic. Each of the other terms $\{b_{jt}\}$ in the k convolutions is assumed to represent a stochastic parameter which can be represented as the sum of a temporally-invariant fixed parameter and a random error:

$$(2) \qquad b_{jt} = \beta_j + v_{jt} .$$

Their model is completed by the specifications that all errors have zero means, they are uncorrelated, and their variances depend only on their non-temporal index. In particular, they require

$$(3) \qquad E(v_{jt}) = 0 \qquad \text{for all } j \text{ and } t; \text{ and}$$

$$(4) \qquad E(v_{is}\, v_{jt}) = \alpha_i \quad \text{if both } i=j \text{ and } s=t$$

but zero otherwise for all i, j, s and t.

There are considerable advantages for later manipulations if these specifications are re-stated in matrix notation. Let y, β and u represent column vectors with typical elements y_t, β_j and $\sum_{j=1}^{k} x_{jt} v_{jt}$ respectively. X denotes the matrix with typical element x_{jt}, whereas $\{z_j\}$ and $\{v_j\}$ are two collections of column vectors, of order n, with typical elements $\{z_{jt}\}$ and

$\{v_{jt}\}$ respectively. The Schur, or Hadamard, Product of two matrices with the same order is indicated by the presence between them of an asterisk. (See Rao and Mitra [11], pp. 11-12, for an account of this product which may be unfamiliar to many economists.) Then, we have

$$(5) \qquad y = X\beta + u$$

$$(6) \qquad u = \sum_{j=1}^{k} (x_j * v_j)$$

$$(7) \qquad E(u) = 0$$

and
$$(8) \qquad E(uu') = \sum_{j=1}^{k} \alpha_j \Lambda(x_j \cdot x_j')$$
, where the symbol $\Lambda(B)$ represents the matrix formed from an arbitrary square matrix B by replacing all of its off-diagonal elements by zero values. In a later part of this appendix, the very restrictive specification (8) is relaxed to permit errors to be either contemporaneously correlated or temporally generated by first-order autoregressive processes with known parameters for each of the k temporal sequences. The seminal work of Hildreth and Houck on estimation of the structural means $\{\beta_j\}$ and the variance components $\{\alpha_j\}$ is extended to these two cases.

Notice that the dispersion matrix of the temporal-composite errors, $E(uu')$, in (8) is not a scalar matrix and these errors are heteroscedastic, even though the k constituent temporal sequences of errors are free from this particular form of nonstationarity by assumption. This heteroscedasticity is completely characterized by the k alpha parameters and the principal diagonals of outer products formed from the constituent column vectors of X.

In a series of papers ([7]-[10]), C. Radhakrishna Rao has developed the MINQUE method of estimation for linear models with variance components or heteroscedastic errors. (J.N.K. Rao and Kathleen Subrahmaniam [12] have applied this method to two important but non-economic situations.) The Hildreth-Houck specifications can be considered as indicative of a special case in the more general framework of Rao. In particular, they involve a requirement that the vector of variances for the n temporal-composite errors belongs to the range space of $(X*X)$. That is,

$$(9) \qquad \text{vec}\,\{E(uu')\} = (X*X)\alpha \ ,$$

where α is the column vector with typical element α_j and order k. (The symbol vec {B} represents the vector formed from the principal diagonal of an arbitrary square matrix B so vec {E(uu')} is the vector of variances of temporal-composite errors.) This form of linear restriction is extremely rare in econometrics but it can be identified with the Almon technique, which is used in some studies of distributed lags, although it is free from the arbitrariness indicated for this technique by Shirley Almon [1] and Gordon Sparks [17].

Since the temporal-composite errors are heteroscedastic, application of the principle of ordinary least squares (OLS) in equation (1) to estimate the parametric vector β will yield unbiased but inefficient estimators as compared with Aitken estimators which are based upon true values of the alpha coefficients for the variance components. (See Rowley, [13], Ch. 2, for a description of this alternative approach and the extent of relative efficiencies.) Unfortunately, these coefficients are unknown. Hildreth and Houck suggest two methods whereby consistent estimates of them are used in Aitken's procedure instead of their true values. These methods form the basis for the empirical results tabulated above.

Let $\hat{u}$ represent the vector of least-squares residuals from OLS estimates of β in (5). Then,

(10) $\hat{u} = Pu$ where $P \equiv I - X(X'X)^{-1}X'$.

(11) $\hat{u}*\hat{u} = \text{vec}\{\hat{u}\hat{u}'\}$

$= \text{vec}\{Puu'P\}$ since P is symmetric.

(12) $E(\hat{u}*\hat{u}) = \text{vec}\{P\ E(uu')P\}$

$= \sum_{j=1}^{k} \alpha_j \text{ vec}\{P\Lambda(x_j x_j')P\}$ from (8).

$= \sum_{j=1}^{k} \alpha_j (P * P) \text{ vec}\{\Lambda(x_j x_j')\}$

$= (P * P) \sum_{j=1}^{k} \alpha_j \text{ vec}\{\Lambda(x_j x_j')\}$

$= (P * P) \sum_{j=1}^{k} \alpha_j (x_j * x_j)$

(13) $E(\hat{u}*\hat{u}) = (P * P)(X * X)\alpha$

Let e represent the difference between the vector of squared residuals $(\hat{u}*\hat{u})$ and its mean value. Then, e will have a zero mean and

(14) $\hat{u}*\hat{u} = (P * P)(X * X)\alpha + e.$

This equation provides the basis for the derivation of two collections of consistent estimators of the alpha coefficients; namely, estimators $\hat{\alpha}_j$ derived from use of the OLS principle and those based upon the use of instrumental variables $(X * X)$, α_j^0. These are the solutions of the following two alternative normal equations.

(15) $(X * X)'(P * P)'(\hat{u} * \hat{u}) = (X * X)'(P * P)'(P * P)(X * X)\hat{\alpha}$

(16) $(X * X)'(\hat{u} * \hat{u}) = (X * X)'(P * P)(X * X)\alpha^0$

If the estimated dispersion matrices associated with these alternatives are denoted $\hat{E}$ and E^0, then

$$(17) \quad \hat{E} \equiv \sum_{j=1}^{k} \hat{\alpha}_j \, \Lambda(x_j \, x_j')$$

and

$$(18) \quad E^0 \equiv \sum_{j=1}^{k} \alpha_j^0 \, \Lambda(x_j \, x_j').$$

Clearly these two sums are reduced if any of the alpha coefficients are constrained to be zero. One problem with these estimates is the possibility of their negativity even though the alpha coefficients are known to be non-negative. Hildreth and Houck followed the popular practice of setting negative estimates equal to zero. That is, they based their estimated dispersion matrices upon the rule

$$(19) \quad \tilde{\alpha}_j \equiv \max\,(\hat{\alpha}_j \, , 0)$$

and

$$(20) \quad \tilde{E} \equiv \sum_{j=1}^{k} \tilde{\alpha}_j \, \Lambda(x_j \, x_j')$$

with a similar adjustment for the instrumental-variable approach. We adopted a similar procedure even though it would have been possible to obtain direct OLS estimates subject to the non-negativity constraints with the use of quadratic programming.

The principal Hildreth-Houck estimator (HH1) of β used in our study is based on (20) and has the form

$$(21) \quad \tilde{\beta} \equiv (X' \, \tilde{E}^{-1} \, X)^{-1} \, X' \, \tilde{E}^{-1} y.$$

A simple substitution for $\tilde{E}$ yields the alternative estimator (HH2) based upon instrumental variables. Other substitutions for $\tilde{E}$ are necessary if we attempt to take account of contemporaneous cross-correlation and temporal autocorrelation of errors. If we suppress equation (8), then

$$(22)\quad E(uu') = \sum_{i=1}^{k}\sum_{j=1}^{k} E[(x_i * v_i)(x_j * v_j)']$$

$$= \sum_{i=1}^{k}\sum_{j=1}^{k} E[(x_i\ x_j') * (v_i\ v_j')]$$

or $$(23)\quad E(uu') = \sum_{i=1}^{k}\sum_{j=1}^{k} (x_i x_j') * E(v_i v_j').$$

Thus, contemporaneous cross-correlation of errors implies that

$$(24)\quad E(uu') = \sum_{i=1}^{k}\sum_{j=1}^{k} (x_i x_j') * \alpha_{ij}\ I_n$$

for some variance and covariance components $\{\alpha_{ij}\}$. Suppose contemporaneous errors are free from cross-correlation but generated by first-order autoregressive processes with the same known parameter ρ. That is,

$$(25)\quad v_{jt} = \rho v_{j,t-1} + \varepsilon_{jt} \qquad \text{for all } j,\ t\ ;$$

where $\{\varepsilon_{jt}\}$ is a collection of k temporal sequences of stationary white noise with variances $\alpha_j(1-\rho^2)$. The dispersion matrix of the temporal-collective errors becomes

$$(26)\quad E(uu') = \sum_{j=1}^{k} \alpha_j\ (x_j\ x_j') * \Omega\ ,$$

where Ω is the well-known Laurent, or Toeplitz, matrix with typical element given by the (i-j)th absolute power of ρ. In both cases, the Hildreth-Houck procedures can be followed. The vector of squared residuals $(\hat{u} * \hat{u})$ is expressed in terms of a linear function of the alpha coefficients, which are estimated by OLS or instrumental-variable methods. These estimates provide the bases for estimated dispersion matrices which are subsequently used to obtain feasible Aitken estimates.

Indices for the relative sensitivity of Hildreth-Houck estimates with respect to small changes in the choice of estimated variance components can be obtained by differentiation in equations (20) and (21). Thus,

$$(27) \quad \frac{\partial \tilde{\beta}}{\partial \tilde{\alpha}_j} = (X' \tilde{E}^{-1} X)^{-1} X' \frac{\partial (\tilde{E}^{-1})}{\partial \tilde{\alpha}_j} (y - X\tilde{\beta}) \quad \text{for } j=1, 2, \ldots, k$$

and

$$(28) \quad \frac{\partial \tilde{\beta}}{\partial \tilde{\alpha}_j} = - (X' \tilde{E}^{-1} X)^{-1} X' \tilde{E}^{-1} \Lambda(x_j x_j{}') \tilde{E}^{-1} (y - X\tilde{\beta}).$$

Elements of these vectors are deflated by ratios of the form $(\tilde{\beta}_i / \tilde{\alpha}_j)$ in order to obtain dimension-free elasticities. Note that these expressions are inappropriate when there is contemporaneous cross-correlation between errors or autocorrelation but simple adjustments to them can be made in these situations.

APPENDIX 2. Definitions of Variables

(1) Perry, U.S., manufacturing industry.

Dependent variable. Annual percentage change in straight-time hourly earnings of production workers for total manufacturing.

Prices. Four-quarter moving average of one-quarter percentage changes in the consumer price index, all lagged one quarter.

Unemployment. Reciprocal of the four-quarter moving average of the unemployment rate.

Profits. Four-quarter moving average of the annual profit rate (that is, the ratio of corporate earnings after taxes to stockholders' equity) for total manufacturing, all lagged one quarter.

Korea and Guideposts. Simple (0, 1) dummy variables for appropriate periods of time.

(2) Bodkin, et al., Canada, manufacturing industry.

Dependent variable. Annual percentage change in average hourly earnings of production workers in manufacturing.

Prices. Four-quarter moving average of annual percentage change in the consumer price index.

Unemployment. Squared reciprocal of the four-quarter moving average of a two-quarter moving average of the unemployment rate.

Profits. Four-quarter moving average of the profit markup on output (that is, the index of corporate profits before taxes divided by the index of production in manufacturing) lagged two quarters.

Lagged Wages. Dependent variable lagged four quarters.

Spillover. Four-quarter moving average of the annual percentage change in average hourly earnings in U.S. manufacturing, expressed in U.S. dollars.

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