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The Role of Limit Pricing in Sequential Entry Models

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Abstract: In this paper we establish a complete characterization of the strategic interaction of firms in sequential entry models. The limit price plays an important coordinating role in non-cooperative sequential entry models. We show that for many firms in a large range of sequential entry equilibria, the limit price is effectively parametric, so that firms make investment decisions in a quasi-competitive manner. Entry deterrence is only pursued by firms at the beginning of the sequence if it is profitable; otherwise it is delegated to the last firms to enter.

"Despite the great progress that has been made in oligopoly theory and entry deterrence, relatively little is known about the interactions between strategic entry deterrence and market structure".

Richard Gilbert (1989,p. 516)

1. Introduction

The modern literature on strategic entry deterrence can be seen as a reaction to and a refinement of the Bain-Sylos-Modigliani limit price model of *monopoly* entry deterrence. Numerous contributions, surveyed in Gilbert (1989) and Ware (1991) have combined Schelling's (1962) concepts of commitment and strategic behaviour with Selten's (1975) subgame perfection to explain monopoly entry deterrence as an equilibrium phenomenon. However, the one incumbent-one entrant framework framework typically used suppresses interesting questions regarding the ability of multiple incumbents to non-cooperatively deter entry.

In the non-cooperative oligopoly case, there is a fundamental tension not found in the monopoly case. On the one hand, investment in entry deterrence has attributes of a public good. The entire set of incumbents benefit from the investment in entry deterrence made by each firm. If these investments are costly firms may attempt to delegate the responsibility for entry deterrence to rivals. On the other hand, investment in entry deterrence may confer a strategic advantage on the firm vis-a-vis other incumbents, implying a larger market share in the (entry deterring) perfect equilibrium.

In this paper we construct an equilibrium model of market structure with sequential entry in which these opposing forces can be analyzed. We describe the conditions under which the costs of entry deterrence dominate equilibrium behavior and those under which market share considerations are paramount. We determine the circumstances under which delegation occurs and distinguish between the circumstances under which delegation is profitable and those in which delegation imposes costs on later entrants. In particular we highlight the role of the Limit Price in summarizing the investment decisions of incumbent firms. Although the actual strategy of each firm considers deviations from equilibrium in the normal way, many of the firms in an equilibrium entry sequence to our model choose quantities as if the limit price were parametric, in much the same way as would a competitive firm.

Game theoretic models of sequential entry have provided a rich framework for the study of the interaction of strategic investment and market structure. The framework involves firms entering sequentially, making an irreversible commitment to produce by incurring sunk costs. Each firm's sunk investment provides the means to behave strategically with respect to future entrants. Firms recognize that their investment choice will induce an equilibrium market structure: they will induce the one most profitable to themselves. A perfect equlibrium of this game is equivalent to perfect foresight on the part of each entrant about the number and size of future entrants. In addition, an equilibrium market structure has the property that the profitability of additional entry is non-positive.

The richness of the framework arises from considerations of the potential market structures that a firm can induce by its investment. For example, the first firm must consider whether the second firm will deter entry or allow the third firm to enter. The second firm will have to consider whether the third firm will deter the fourth an so on. Within a modelling framework of this type, the work of Eaton and Ware (1987), Schwartz and Baumann (1988), Vives (1988) and McLean and Riordan (1989) has addressed the issue of non-cooperative entry deterrence.

The specification of the cost function in Eaton and Ware, Vives, and McLean and Riordan is not general enough to capture the costs and benefits of the provision of entry deterrence. Either market share considerations always dominate cost considerations so that entry deterrence is never delegated (Eaton and Ware(1987); Vives (1988); Gilbert and Vives(1986)) or the cost considerations always dominate the market share implications and it is always delegated (McLean and Riordan). The simulation work of Schwartz and Baumann is suggestive, but by its very nature it provides little analytical or intuitive explanation for the observed equilibrium market structures.

In our model firms have U-shaped average cost curves, the marginal cost of production is increasing and firms can commit directly to quantities. As a result the marginal cost of providing entry deterrence is also increasing. This leads to a natural trade-off between the market share benefits of investing in greater entry deterrence and the increasing cost of providing additional entry deterrence.

Two concepts play an important role in our analysis of the equilibrium market structure. The first is the limit pricing output (LPO). A firm is producing at its limit pricing output when it chooses the quantity at which marginal cost equals the limit price. Secondly, a firm is producing its Stackelberg quantity when it takes as given the quantity choices of earlier entrants, and anticipates unconstrained (by the threat of entry) optimal chioces by later incumbents.

We find that there are two possible equilibrium configurations. In the first the extent of economies of scale and strategic behaviour are such that any delegation which takes place is profitable for later entrants. In this case there are typically three goups of firms. The first entrants produce at the LPO. After these firms there is one firm which produces more than the Stackelberg output but less than the LPO. The final group of entrants produce their Stackelberg quantities. In this equilibrium configuration firms find it profitable to increase their investment in entry deterrence, if it is required to deter entry, since marginal profit is positive when output is less than the LPO.

In the second equilibrium configuration, delegation imposes costs on later entrants. In this configuration there are typically two groups of firms. Early entrants produce at the LPO and delegate responsibility for additional entry deterrence to later entrants. Later entrants produce above the LPO and hence the marginal profit for firms in this group is negative. Early entrants in the sequence of investment have an incentive not to bear an equal share of the costs of entry deterrene in the knowledge that later entrants will be forced to pick up the slack. The option for later entrants of allowing entry is not credible unless in the equilibrium to the accommodation subgame the later entrants earn higher profit. If this is not the case, early entrants delegate costly investments in entry deterrence to later entrants with the knowledge that it is optimal for the later entrants to deter entry.

As the minimum-efficient cost and scale decrease, the free-entry number of firms increases, the limit price approaches the competitive price and the extent of strategic behaviour diminishes in perfect equilibrium. A greater percentage of firms produce at the LPO and treat the limit price as parametric. Provided the minimum efficient cost and scale are bounded away from zero, the perfect equilibirium is characterized by strategic behaviour.

Robson (1990) has attempted a more general description of sequential investment models. He established that in the limit as the minimum efficient scale goes to zero, the equilibrium of a sequential entry model with U-shaped costs and quantity commitment is the competitive equilibrium.

We also comment on the definition of underinvestment in entry dettence by incumbent firms and the possibility that the delegation of entry deterrence will result in underinvestment. We find that compared to a multiplant monopolist, the non-cooperative solution always involves fewer firms. Despite delegation, the non-cooperative equilibrium involves overinvestment.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3, the analytical core of the paper, characterizes the equilibria into blockaded and strategic, and within the strategic category, shows that exactly five sub-categories of equilibrium will always occur. Section 4 discusses the underinvestment issue, and Section 5 draws conclusions.

2. The Model

Consider a market for a homogenous product with linear demand given by $P(X) = A - X$, where A is a positive constant and X is total output. The cost function for firm i is given by $c(x_i) = x_i^2 + F$, where x_i is the output for firm i and F is the fixed cost of entry. Though simple, two features of this cost function distinguish our analysis from previous work: marginal cost is increasing and hence average cost is U-shaped. Denote average cost of firm i by $a(x_i)$, and the vector of quantities for n firms as $x = (x_1, x_2, \ldots, x_n)$. We assume that there is a set N of potential entrants, $N = \{1,\ldots,n\}$, where n is such that $P(x) < a(x_i)$ for at least one $i = 1, \ldots, n$ for any x where $x_i > 0 \; \forall$ i. That is, not all n firms are viable at strictly positive outputs.

The sequential entry game we analyze is one of perfect information and it consists of n stages. In stage i firm i decides whether or not to enter the market and if it enters, it commits to an output level. When selecting its output firm i takes as fixed the choices of firms 1 through $i - 1$ and it recognizes that there are $n - i$ firms to follow. Firm i's behaviour is Cournot with respect to its predecessors; Stackelberg with respect to its successors. If we let $Y_i = \sum_{j=1}^{i-1} x_j$, ie the cumulative output of the firms preceding firm i, then a strategy for firm $i (i \neq 1)$ is a function R_i which assigns a level of production for firm i to every possible Y_i. The strategy of firm 1 is simply an output level.[1] Given a strategy for each firm 2 through n and a quantity for firm 1 we can recursively determine the implied output vector, x, since $Y_2 = x_1$ and $Y_{i+1} = Y_i + x_i = Y_i + R_i(Y_i)\ \forall\ i \geq 2$. Hence x =

$$\{x_1, x_2 = R_2(x_1) = R_2(Y_2), x_3 = R_3(Y_2 + x_2) = R_3(Y_3), \cdots x_n = R_n(Y_{n-1} + x_{n-1}) = R_n(Y_n)\}$$

The profits of firm i are defined as $\pi_i(x) = P(x)x_i - c(x_i)$.

A subgame perfect equilibrium to this game is a set of strategies R = $\{x_1, R_2, R_3, \ldots, R_n\}$ such that the following two conditions are satisfied:

$$(i) \quad x_1 = argmax\ \pi_1(x_1, \sum_{i=2}^{n} R_i(Y_i(x_1))) = P(x_1 + \sum_{i=2}^{n} R_i(Y_i(x_1)))x_1 - c(x_1)$$

$$(ii) \quad R_i(Y_i) = argmax\ \pi_1(Y_i, x_i, \sum_{j=i+1}^{n} R_j(Y_j(x_i))) = P(Y_i + x_i + \sum_{j=i+1}^{n} R_j(Y_j(x_i)))x_i - c(x_i)$$

$\forall i = 2, \ldots, n$.

Condition (i) requires that the output choice of firm 1 be profit maximizing, given the responses of firms 2 through n. Condition (ii) states that R_i must maximize the profits of firm i for any Y_i, given the responses of firms $i + 1$ through n.

In general for sequential entry games with U-shaped average cost and downward-sloping demand, Robson (1990) has established the following two intuitive properties, and in what follows we will make use of them.[2]

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[1] In any sequential game of perfect information, once the strategies of all but the first mover are specified, the first move determines the outcome.

[2] Both of these results are also derived in Eaton and Ware (1987) for the case of Leontief costs.

(P1) A necessary and sufficient condition for entry by firm i is that its profits, conditioned on Y_i, but ignoring the subsequent equilibrium output choices by firms $i + 1$ through n, be positive (Robson, 1990, Lemma 3, p. 74)

(P2) Active firms are first. If m firms produce with positive output in the subgame perfect equilibrium, these will be firms 1 through m. Firms $m + 1$ through n will be inactive (Robson, 1990, Lemma 4, p. 75) .

Thus, in our subsequent analysis, whenever we refer to an m firm equilibrium, (P2) implies that these are the first m firms.

3. Equilibrium Strategies

In this section we characterize the equilibrium strategies of firms, and the corresponding equilibrium market structures. It is useful to define two types of subgame perfect equilibria. A *blockaded equilibrium* is defined to be an equilibrium to the game in which $m<n$ firms are active, such that if the game is redefined by setting $n'=m$ (*i.e.* by restricting the total number of potential entrants to m), the equilibrium to the redefined game coincides with the original n firm equilibrium. In a blockaded equilibrium firms 1 through m do not engage in strategic entry deterrence. In a manner to be made precise shortly, this means that they do not expand their output with the intent of deterring subsequent entrants. All other equilibria not satisfying this restriction are described as *strategic equilibria.*[3]

The *limit output* (LO) is defined by the smallest value of Y_i which solves the equation $\max_{x_i} \pi_i(Y_i, x_i) = 0$. That is, the limit output is the cumulative output by firms 1 through i -

[3]This terminology was introduced by Eaton and Ware (1987)

1, Y_i, such that if firm i were to enter and select its profit maximizing quantity without regard to firms $i+1$ through n, the profits of firm i would be zero. We assume that if Y_i = LO (implying post-entry profits of at most zero), entry of firm i is deterred, and by extension, entry of firms $i + 1$ through n will also be deterred. Define the *limit price* (LP) by LP=P(LO). It is a simple matter to derive explicit expressions in our linear-quadratic model for these values; $LO = A - 2\sqrt{2F}$ and $LP = 2\sqrt{2F}$. The LO and the LP are both functions of the fixed costs of entry: LO(F) is decreasing in F; LP(F) increasing. Decreases in F reduce the minimum market share a firm requires to earn profits post-entry. Hence to continue to deter entry the incumbent firms must expand their output when F decreases.

With these definitions in mind proposition 1 is straightforward.

Proposition 1: Industry output in equilibrium is at least as large as the limit output; industry price is no larger than the limit price.[4]

Proof. Suppose that the equilibrium output vector is such that firms 1 through m are active and cumulative output, X, is less than the limit output. Then by definition firm $m+1$ could enter and earn positive profits, regardless of the equilibrium behaviour of firms $m+2$ through n, by (P1); which is a contradiction. *QED.*

Blockaded Equilibrium[5]

An m firm blockaded equilibrium corresponds to the familiar game of Stackelberg with m firms and no fixed costs. The Stackelberg reaction or best response functions for firms 2 through m are defined, by

[4] See Eaton and Ware (1987) Corollary 2, p.8 for a similar result.

[5] We have recently become aware of Economides (1991) which offers an analysis of the blockaded case which is similar to our own.

$$S_i(Y_i) = argmax\, \pi_i(Y_i, x_i, \sum_{j=i+1}^{m} S_j(Y_j(x_i))) = P(Y_i + x_i + \sum_{j=i+1}^{m} S_j(Y_j(x_i)))x_i - c(x_i) \qquad (1)$$

Strictly speaking, firm 1 does not have a reaction function, rather it selects

$$x_1^s = argmax\, \pi_1(x_1, \sum_{j=2}^{m} S_j(Y_j(x_1))) = P(x_1 + \sum_{j=2}^{m} S_j(Y_j(x_1)))x_1 - c(x_1).$$

However, if we recognize that $Y_1 = 0$, the analysis for firms 2 through m can be used for firm 1. The first-order condition for firm i, from (1) is

$$P + P'\left[1 + \sum_{j=1}^{m-i} S'_{i+j} \frac{\partial Y_{i+j}}{\partial x_i}\right] x_i - c'(x_i) = 0 \qquad (2)$$

where we have for the sake of clarity suppressed the functional arguments and used the prime notation to indicate a first derivative.

Lemma 1: The first-order condition for firm i can be written as

$$P + P'\left[\prod_{j=1}^{m-i} [1 + S'_{i+j}]\right] x_i - c'(x_i) = 0 \qquad (3)$$

The second order condition for firm i in the linear-quadratic case is

$$-2\left[\prod_{j=1}^{m-i} (1 + S'_{i+j})\right] - 2 < 0$$

Proof: The term $1 + \sum_{j=i}^{m-i} S'_{i+j} \frac{\partial Y_{i+j}}{\partial x_i}$ in (2) is the effect on total output from a unit increase in the output of firm i. The definition of Y_{i+j} and recursive manipulation yields

$$1 + \sum_{j=1}^{m-i} S'_{i+j} \frac{\partial Y_{i+j}}{\partial x_i} = \prod_{j=1}^{m-i} [1 + S'_{i+j}]$$

which when substituted into (2) gives (3). Differentiation of (3), noting that $S''(\cdot) = c''(x_i) = 0$ for the linear quadratic case, yields the second order condition. *QED.*

Recall that in the linear-quadratic model, $P(X) = A - X$ and $c(x_i) = x_i^2 + F$. In the following series of Lemmas, we solve for and characterize the reaction function of firm i in the linear-quadratic case.

Lemma 2: The m firm blockaded reaction function for firm i ($i \neq m$) in the linear-quadratic model is

$$S_i(Y_i) = (A - Y_i) \frac{\prod_{j=1}^{m-1} (1 + S'_{i+j})}{2\left(\prod_{j=i}^{m-i} (1 + S'_{i+j}) + 1\right)} \tag{4}$$

Proof. The blockaded reaction function for firm m is

$$S_m(Y_m) = \frac{(A - Y_m)}{4} \tag{4a}$$

Using (3), (4a) and backwards recursion we obtain (4) *QED*

From (4) it is immediate that the reaction function for firm i ($i \neq 1$) is linear.

Lemma 3. $-1 < S'_i(Y_i) < 0$

Proof. Follows from backward recursion on (4) and (4a). *QED*

Note that Lemma 3 implies that the second order condition is always satisfied.

Lemma 4: The effect on total output from a unit expansion in output by firm i is greater than zero, but less than one, ie

$$0 < \prod_{j=i}^{m-i} [1 + S'_{i+j}] < 1.$$

Proof. Follows immediately from Lemma 3. QED.

Define $x_i^s(Y_i,m) = \{S_i(Y_i), S_{i+1}(Y_{i+1}),..., S_m(Y_m)\}$ as the vector of outputs for firms i through m, contingent on Y_i, when they each follow the blockaded reaction functions, (4). Let the aggregate output produced by firms i through m associated with this vector be $X_i^s(Y_i,m)$, *i.e.*

$$X_i^s(Y_i,m) = \sum_{j=0}^{m-i} S_{i+j}(Y_{i+j})$$

and thus

$$X_1^s(m) = x_1^s + \sum_{j=2}^{m} S_j(Y_j)$$

Lemma 5: (First-Mover Advantages) In the m firm blockaded equilibrium,

$$x_1^s > x_2 > \ldots > x_m \text{ and } \pi_1 > \pi_2 > \ldots > \pi_m.$$

Proof. In equilibrium, each firm is producing at an output level where its marginal revenue equals its marginal cost:

$$P - \left[\prod_{j=1}^{m-i} [1 + S'_{i+j}]\right] x_i = c'(x_i) \quad \forall i \tag{5}$$

Marginal revenue is a decreasing function of the firm index i since from Lemma 3, $\left[\prod_{j=1}^{m-i} [1 + S'_{i+j}]\right]$ is increasing in the firm index i and firms face a common price. As marginal cost is increasing in output, equilibrium firm output is monotone decreasing in the index i. The profit ordering follows naturally from the output ordering. *QED*

Lemma 6: The LPO is strictly greater than the output of firm i ($x_i = S(Y_i)$) when the outputs of firms j > i are determined by S_j.

Proof: When a firm is producing at the LPO, price equals marginal cost. For all i, marginal revenue is less than price when firms j > i follow strategy S_j. See equation (5).

In the following section, we determine the strategies firms will follow in a strategic equilibrium. It turns out that in our linear-quadratic model, the Stackelberg reaction functions for firms $m - 2$, $m - 1$, and m play a leading role. The reaction function for firm m is (4a); using (4) and (4a), the reaction functions for firms $m - 1$ and $m - 2$ are

$$S_{m-1}(Y_{m-1}) = \frac{3(A - Y_{m-1})}{14} \tag{4b}$$

and

$$S_{m-2}(Y_{m-2}) = \frac{33(A - Y_{m-2})}{178} \tag{4c}$$

Strategic Equilibrium

In this section we begin our discussion of strategic equilibrium in a descriptive manner for the monopoly, duopoly and triopoly cases. The intuition and principles developed here will be established rigorously following this discussion.

Figure 1 shows the demand curve, the marginal revenue curve for a monopolist, the marginal cost curve, the limit pricing output and the limit output. The monopoly price and output are P_m and x_m respectively. For values of $F \geq F_1(1)$, the equilibrium is a blockaded monopoly. For values of $F < F_1(1)$ further entry will result if the monopolist does not increase its output. Proposition 1 indicates that in the most profitable market structure firm 1 can induce, the highest price will be the limit price. By setting its output equal to the limit output, the monopolist can deter entry. This will be more profitable than any other alternative provided marginal cost is less than the limit price, since relative to any other alternative, the monopolist makes a positive marginal profit on each unit between x_m and the LO.

At $F_2(1)$, the monopolist is producing where the LPO equals the LO and hence the limit price equals marginal cost. For values of $F < F_2(1)$, continues deterrence of additional firms by the monopolist will require that the monopolist produce units with a negative marginal profit, i.e. marginal cost greater than the limit price. The alternative to deterence is to choose an optimal accommodation strategy, recognizing that the choice by potential entrants must eventually deter further entry. In order to evaluate the payoff to allowing entry, the incumbent monopolist must know the post entry equilibium market structure.

The most profitable two firm market structure for the monopolist would be blockaded duopoly. It involves both firm 1 and firm 2 producing according to S_i i = 1,2. We establish in Lemma 7 that for the relevant region that this will also be an equilibrium market structure since aggregate output will exceed the limit output. The optimal accommodation equilibrium for the incumbent monpolist involves both a reduction in market share (Lemma 5), and a reduction in price since blockaded duopoly output exceeds the limit output (Lemma 6).

Consequently, it will be profit maximizing for firm 1 to continue to deter entry by producing above the LPO as long as the costs from continued entry deterrence are less than the reduction in profits from allowing entry. Decreases in F will require increasing output further and further above the LPO, and since blockaded duopoly profits are independent of F, eventually the costs of entry deterrence exceed the benefits, and firm 1 allows firm 2 to enter.[6] Denote this value of F by $F_3(1)$.

Figure 2 shows the equilibrium outputs in the monopoly and duopoly cases. Figure 3 shows the marginal cost of firm 1 and firm 2 in the monopoly and doupoly cases. As F continues to decline, the LO rises, and eventually at $F_1(2)$ the blockaded duopoly output equals the limit output. Continued deterrence of firm 3 will require the two incumbents to

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[6]Of course, for this to be an equilibrium, we must check that for this value of F, blockaded duopoly still deters entry.

expand their output. Once again, both firms know that the most profitable market structure will involve price equal to the limit price. At the blockaded output levels, limit price exceeds marginal cost. Hence, if given the opportunity, each is willing to expand output to keep aggregate output equal to the limit output. By virtue of the order of movement, firm 1 has this opportunity first, and expands output such that given the reaction of firm 2, S_2, aggregate output will equal the limit output. Firm 2 produces optimally according to S_2; with such a response, entry will be deterred.

Further decreases in F increase x_1: firm 2 responds optimally by decreasing x_2. This will continue to be its most profitable strategy as F declines until it is producing at the LPO. Thereafter for firm 1 to continue to deter entry, marginal profit will be negative since marginal costs will exceed the limit price. However, it knows that the optimal response of firm 2 if firm 1 produces at the LPO is to deter entry by producing the difference between the LO and the LPO, since the limit price is greater than marginal cost. This will continue to be the case until F is such that both firms producing at the LPO just deters entry ($F_2(2)$). In this region, output of firm 2 will increase up to the LPO; the output of firm 1 tracks the LPO and since the LPO is declining in F, so is the output of firm 1.

For $F < F_2(2)$, continued deterrence by firms 1 and 2 of firm 3 will require at least one of them to produce above the LPO. Continued entry deterrence will now be costly, in the sense that marginal units will be produced at a marginal cost which exceeds the equilibrium price. Firm 1 will delegate this costly entry deterrence to firm 2. Firm 2 then faces the choice between producing the additional units necessary to attain the LO or allowing entry. By the same reasoning as we used in the monopoly case, firm 2 will prefer to continue to deter for some interval of F, because further entry will entail at the very least a reduction in output and possibly a reduction in price as well.

For firm 2, the profits from this continued deterrence strategy are decreasing as F decreases. Allowing entry, given that firm 1 has chosen the LPO(F), becomes more profitable as F decreases, since the output of firm 1 will be smaller. Thus at $F_3(2)$ firm 2 will be just indifferent between deterring and permitting entry, by choosing a blockaded duopoly quantity, *conditioned* on the choice of firm 1 at x_1 = LPO(F).

For smaller values of F, firm 1 must also produce above the LPO, or entry will occur.

It is willing to do this for some range of F, because allowing entry will give it a smaller market, but the same price, LP(F).[7] In order to maintain deterrence of firm 3 in equilibrium, firm 1 must produce enough so that firm 2 is kept just indifferent between deterrence and accommodation itself, with firm 1 choosing a quantity which makes up the difference between firm 2's production (given the indifference constraint) and the limit output. Figure 4 shows why firm 1 still has an incentive to engage in costly entry deterrence, even though the equilibrium price is the same (the limit price) whether it allows entry or not. By producing more to keep firm 3 out, firm 1 produces units at a marginal cost exceeding the limit price, incurring a loss on these marginal units of the shaded area A in Figure 4. By allowing entry, although the price will be the same, it will lose market share, and produce below the LPO(F) in equilibrium. The loss on marginal units, relative to producing at the LPO is given by the cross-hatched area B. At the value of F at which area A exceeds area B, firm 1 gives up its strategy of deterrence, and a new three firm equilibrium will result with firms 2 and 3 producing on their Stackelberg reaction functions, and firm 1 producing the residual necessary to make up the limit output.

With three incumbent firms it turns out that the types of equilibrium which occur encompass all possible equilibria which occur for the general case of m firms. We will briefly describe the three firm case, and then present a series of propositions, lemmas and corollaries which establish these equilibrium types for the m firm case.

Type I strategic equilibrium

Firm 1 now has an incentive to expand output, as F decreases, up to the LPO. As in the one and two firm equilibria above, entry deterrence is profitable in this range because profit is being earned on marginal production (given the equilibrium limit price). Firms 2 and 3 continue to produce on their Stackelberg reaction functions.

[7]When firm 1 accomodates entry, the price stays at the limit price. This is an application of Lemma 8 (see below).

Type II strategic equilibrium

As F falls firm 1 will eventually produce at the LPO(F). It has no incentive to expand output further, since in any equilibrium with further entry, it could guarantee itself both the limit price and output equal to the LPO(F). Hence, firm 1 continues to produce at the LPO(F) as F is decreased further. The equilibrium behaviour of firms 2 and 3 is exactly as described by the duopoly case above, conditioned on a production of LPO(F) by firm 1. Firm 2 expands output up to the LPO(F), and firm 3 remains on its Stackelberg reaction function.

Type III strategic equilibrium

In this region firm 3 expands to the LPO(F) as F falls. Firms 1 and 2 continue to produce LPO(F). Again, the equilibrium behaviour of firms 2 and 3 is the same as the duopoly case, given x_1 = LPO(F).

Type IV strategic equilibrium

When firm 3 gets to the LPO(F), we have a symmetric three firm equilibrium with $x_1 = x_2 = x_3 =$ LPO(F). Exactly as in the duopoly case, for smaller values of F, firms 1 and 2 *delegate* continued entry deterrence to firm 3, who has an incentive to produce above the LPO(F). The lower bound of this region occurs where firm 3 is indifferent between allowing entry and continuing its costly entry deterrence.

Type V strategic equilibrium

As in the duopoly case, firm 2 has an incentive to expand output above LPO(F), maintaining firm 3's indifference between deterrence and allowing entry, and producing just sufficient output to continue to deter firm 4. Firm 1 continues to produce LPO(F).

The lower bound for the type V equilibrium occurs with indifference for firm 2 between continued deterrence and allowing entry, exactly as described above for the duopoly case, and illustrated in Figure 4. When entry occurs, a Type I equilibrium with 4 incumbent firms results, and Equilibrium Types I through V re-occur in sequence, followed by a Type

I equlibrium with 5 firms, and so on.

A complete description of strategic equilibrium with m firms

Having described, somewhat loosely, the sequence of equilibria, we can now characterize them more formally, for the general case of m firms, in the following propositions, lemmas, and corollaries. Define F_1 as the level of fixed costs such that $X_1^s(m) = LO(F_1)$. For this value of the fixed costs the aggregate output in the m firm blockaded equilibrium is just sufficient to deter firm $m+1$. For any $F < F_1$, the aggregate output of the m firm blockaded equilibrium is less than the limit output and at the very least we know by (P1) that firm $m+1$ could enter and earn positive profits. We also know that the effect of further entry on the profits of the m incumbent firms will be adverse. The profit of each of the m incumbent firms will decline if there is further entry, due to a reduction in both market share and the equilibium price. The alternative is that the cumulative output of the m incumbent firms expands to the limit output; in this section we determine the circumstances under which the m incumbent firms will non-cooperatively expand their output to deter entry and how the required expansion in output is allocated among the m firms.

In equilibrium, given the output choices of its predecessors, each firm will invest in an output level which induces the most favourable market structure. Proposition 1 indicated that the equilibrium market structure involves aggregate output equal t o or exceeding the limit output. Thus, the various alternative market structures that a firm can induce can be summarized by the limit price. This considerably reduces the complexity of the analysis.

Define F_2 as the level of fixed costs such that m LPO(F) = $LO(F_2)$: it is the level of fixed costs such that when all of the m incumbent firms produce the limit pricing output, the aggregate output equals the limit output. In $X_1^s(m)$ each of the m incumbents is producing at an output level where marginal revenue equals marginal cost[8]. Marginal cost

[8]Marginal revenue is defined here in the Stackelberg sense as

$$MR_i(x_i) = P - [\prod_{j=1}^{m-i} [1 + S'_{i+j}]]x_i$$

for each firm in the blockaded solution is less than the limit price, thus

$$m\ LPO(F_2) = LO(F_2) > X_1^s(m) = LO(F_1).$$

and hence $F_1 > F_2$

Proposition 2: For $F_2 \leq F \leq F_1$, the subgame perfect equilibrium strategy for firm *i* is as follows:

(i) If $Y_i + X_i^s(Y_i,m) \geq LO(F)$, then $x_i = S_i(Y_i)$

(ii) If $Y_i + X_i^s(Y_i,m) < LO(F)$, then $x_i = \min\{LO(F) - X_{i+1}^s(Y_i + x_i,m) - Y_i, LPO(F)\}$

Proof: See Appendix A.

Proposition 2 rests on the fact that, given that aggregate output will be at least the limit output, it is profitable for an incumebent firm to expand output such that total output equals the limit output, provided the required expansion does not exceed the limit pricing output. Firm i knows that if it expands in such a fashion, firm j > i will profit maximize by setting x_j by S_j since entry will be deterred. By doing so firm i deters entry and is assured a positive margin on each unit up to the LPO. It is not profitable for firm i to expand beyond the level required to induce aggregate output equal to the limit output, given that its followers are seting output according to their Stackelberg reaction functions. If firms j > i are setting output by S_j, then the profit maximizing choice for i is S_i and any deviations from this reduce its profits. Expanding beyond the amount necessary to induce the limit output represents an unnecessary reduction in profit from the Stackelberg level. Moreover, firm i need not expand beyond the LPO to deter entry, since it knows that firms j > i will expand output up to the LPO to deter entry (marginal profit is positive). For this range of F, it is advantageious to provide entry deterrence, given that some firm must, and delegation is profitable. The early mover advantage is summarized in the following corollary.

Corollary 1. In the subgame perfect equilibrium for $F_2 \leq F \leq F_1$, the equilibrium outputs will be such that for $0 \leq j \leq i \leq m$, x_j = LPO(F); provided i < m, then for j = i + 1, x_j = LO(F) - $X^s_{i+2}(Y_{i+2})$ - Y_{i+1} and for $i + 2 \leq j \leq m$, $x_j = S_j(Y_j)$. Thus $x_1 = x_2 = x_i \geq x_{i+1} > x_{i+2} > x_m$ and consequently $\pi_1 = \pi_2 = \pi_i \geq \pi_{i+1} > \pi_{i+2} > \pi_m$.

Proof. Follows immediately from the proof of proposition 2 and lemma 5.

Corollary 1 states that, in general, when $F_2 \leq F \leq F_1$, the first i firms will be producing at the limit pricing output, where the equilibrium limit price equals marginal cost; firm $i + 1$ will be producing above its blockaded reaction function, but below the limit pricing output, and its marginal cost is greater than its marginal revenue but less than the limit price; firms $i + 2$ through m will be producing on their blockaded reaction functions, where marginal revenue (incorporating the response of subsequent entrants) equals marginal cost. Moreover the output and profit of the first i firms will exceed the output and profits of firm $i + 1$, which in turn exceed the output and profit of the firms which are still on their blockaded reaction functions.

For values of $F < F_2$, continued deterrence by the m incumbent firms will entail one or more firms producing at an output level where marginal cost exceeds the limit price. Provided another firm is willing to invest in the required entry deterrence, a firm will not find it profitable to expand its output beyond the limit output. In effect early entrants have an incentive to *delegate* the burden of entry deterrence to firms entering later in the sequence[9].

In the analysis of the remaining types of equilibrium the following two lemmas will prove to be useful.

Lemma 7: The market structure x = {LPO,LPO, . . . , LPO, $S_m(Y_m)$, $S_{m+1}(Y_{m+1})$} is entry deterring. That is (m - 1)LPO + X^s_m((m - 1)LPO,m + 1) > LO, where $m = \dfrac{LO}{LPO}$.

[9]The concept of delegation in sequential entry models appears first in McLean and Riordan (1989). Because our model has a richer strategy space for firms, the paterns of delegation are correspondingly richer.

Proof. See Appendix A.

The importance of lemma 7 is that the market structure where firm m optimally accommodates entry by firm $m + 1$, given $Y_m = (m - 1)$ LPO, is entry deterring.

Lemma 8: The market structure $x = \{LPO, LPO, \ldots, S_{m-1}(Y_{m-1}), S_m(Y_m), S_{m+1}(Y_{m+1})\}$ is not entry deterring. That is $(m - 2)\,LPO + X^s_{m-1}((m - 2)\,LPO, m + 1) < LO$, where $m = \frac{LO}{LPO}$.

Proof. See Appendix A.

The import of lemma 8 is that the blockaded alternative for firm $m - 1$, when it optimally accommodates entry by firm $m + 1$, invites further entry.

Proposition 3: For $F_3 \leq F < F_2$ the equilibrium strategy for firm i is

(i) for $1 \leq i \leq m - 1$, $x_i = LPO(F)$

(ii) for firm m, $x_m(F) = LO(F) - (m - 1)\,LPO(F)$

F_3 is implictedly defined by $\pi_m(Y_m,m,F_3) = \pi^s_m(Y_m,m+1,F_3)$ where $\pi_m(Y_m,m,F) = LP(F)\, x_m(F) - c(x_m(F))$ are the profits of firm m if it deters firm $m + 1$ and $\pi^s_m(Y_m, m + 1, F) = P(Y_m + X^s_m(Y_m,m+1))\, S_m(Y_m) - c(S_m(Y_m))$ are the profits of firm m if it optimally accommodates firm $m + 1$.

Proof: We begin with part (ii). We first show that the market structure $x^s_m(Y_m,m+1)$ deters further entry, where $Y_m = LPO\,(m - 1)$. That is, the most profitable alternative for firm m, when it elects to let firm $m + 1$ enter, deters entry. This is true by lemma 7.

The second step in the proof of the proposition is to show that firm m is willing to continue to deter entry for some $F < F_2$. Figure 5 shows the blockaded marginal revenue function of firm m when there are $m + 1$ firms where Y_m = (m - 1) LPO, the marginal cost of firm m, and the limit price. To deter entry at F_2, firm m produces at the LPO, where the limit price equals marginal cost. If firm m optimally accommodates firm $m + 1$, firm m will produce $x_m^s(m+1)$. From Lemma 6 we know that this is less than the LPO. If price was to remain unchanged, firm m would forgo profits equal to the shaded area: ie units of output for which the limit price exceeded marginal cost. However, the diminution in the profits of firm m is greater than this area. In lemma 6 we established that the aggregate output when firm m optimally accommodates $m + 1$ exceeds the limit output and hence the price is less than the limit price. For $F < F_2$ firm m must expand beyond the LPO to deter entry and this obviously involves a loss in profits. However this loss, at least for values of F close to F_2 is less than the loss involved in accommodation. Compare the shaded area with the lined area in Figure 5.

However the willingness of firm m to continue to increase its share of the costs of entry deterrence is limited. As F decreases, the LPO decreases, reducing Y_m = (m - 1) LPO. This contraction in output by firms 1 through $m - 1$ has two effects. On the one hand it shifts the blockaded marginal revenue of firm m up, increasing x_m^s. This combined with the reduction in the LP decrease the loss in profits from optimally accommodating $m + 1$. On the other hand, firm m must increase its output if it continues to deter entry. Combined with the decrease in the LP, this means that it is more costly for firm m to continue to deter entry. Not only must it expand its output, but the difference between the limit price and marginal cost is increasing. Eventually F_3 is reached where firm m finds it more profitable to allow entry than continue deterrence.

Part (i) follows from part (ii). Entry deterrence is profitable up to the point where marginal cost equals the limit price. Thereafter it involves expanding output to levels where marginal cost exceeds the limit price. Hence if firm m is willing to deter entry, none of its predecessors have an incentive to expand output beyond the LPO. *QED*

For this range of the fixed costs, the last firm has been delegated unprofitable or costly entry deterrence. Earlier firms need not bear the costs of entry deterrence since they know that its provision is optimal for firm m. The implicit threat by firm m not to deter further entry is not credible since its profits from the best market structure in which firther entry is accommodated are less than those from detering further entry. The consequences for firm size and profits are summarized in the following corollary.

Corollary 2. In the subgame perfect equilibrium for $F_3 \leq F \leq F_2$, $x_m > x_i = x_j$ and $\pi_i = \pi_j > \pi_m$ for $i,j = 1,2,...,m - 1$.

Proof. Follows immediately from proposition 3

For values of F less than F_3, firms 1 through $m - 1$ know that firm m cannot be delegated any more costs of entry deterrence. However, in the next proposition, we show that firms 1 through $m - 2$ can delegate the cost of further entry deterrence to firm $m - 1$.

Proposition 4: For $F_4 \leq F < F_3$, the equilibrium strategy for firm i is

(i) for $1 \leq i \leq m - 2$, $x_i = LPO(F)$

(ii) for firm $m - 1$, $x_{m-1}(F) = LO(F) - (m - 2)\, LPO(F) - x_m(F)$

(iii) for firm m,

$$x_m(F) = \frac{LP(F) + \sqrt{(LP(F))^2 - 4\pi^s_m(Y_m, m + 1, F)}}{2}$$

F_4 is implictedly defined by $\pi_{m-1}(Y_{m-1}, m, F_4) = \pi_{m-1}(Y_{m-1}, m + 1, F_4)$ where $\pi_{m-1}(Y_{m-1}, m, F)$ are the profits of firm $m - 1$ if firm $m + 1$ is deterred from entry and $\pi_{m-1}(Y_{m-1}, m + 1, F)$ are the profits of firm $m - 1$ if it optimally accommodates the entry of firm $m + 1$, and further entry is deterred. In both cases the price is LP(F).

Proof: See Appendix A.

Proposition 4 shows that firm m - 1 is delegated the costs of entry deterrence within a range of F. Firm m - 1 will expand ouput where marginal cost exceeds LPO(F), so as to just ensure the production of the LO(F), where firm m is being kept just indifferent between deterrence and accommodation. The critical value F_4 occurs when firm m - 1 itself becomes indifferent between deterrence and allowing entry.

From proposition 4, part (iii), the equilibrium output of firm *m* declines as F decreases over the range $F_4 \leq F < F_3$. For these values of F in equilibrium, firm *m* is just indifferent between deterring entry and accommodating entry. However its accommodation profits are increasing as F falls since a decrease in F reduces the LPO and hence Y_m. On the other hand firm m's profits from entry deterrence decrease as F decreases since the LP decreases as F decreases. Consequently, to remain indifferent the output of firm *m* must decline. Corollary 3 summarizes the relative size and profitability of the *m* incumbent firms for this range of the fixed costs.

Corollary 3: In the subgame perfect equilibrium for $F_4 \leq F < F_3$, $x_m > x_{m-1} > x_i = x_j$ and $\pi_i = \pi_j > \pi_{m-1} > \pi_m$ for i,j = 1,2,...,m - 2.

Proof. See Appendix A.

An implication of Corollaries 2 and 3 is that the size of a firm and the profitability of a firm are not necessarily positiveley correlated.[10] When delegation is profitable, there will be a positive correlation between the size and profits of a firm. When delegation imposes costs, there will be a negative correlation between the size and profits of a firm. When delegation is costly, our model predicts that later entrants will be larger than early entrants and less profitable.

[10]Schwartz and Baumann (1988) observe this pattern in their simulation results.

Proposition 5: For $F < F_4$, the m incumbent firms allow entry by firm $m+1$.

Proof. From propositions 4 and 5, neither firm m or $m-1$ are willing to continue to expand output to deter firm $m+1$. It is easy to establish that none of firms 1 through $m-2$ will expand to deter the entry of $m+1$. Consider firm $m-2$. Lemma 7 established that the market structure $x = \{LPO, LPO, \ldots, S_{m-1}(Y_{m-1}), S_m(Y_m), S_{m+1}(Y_{m+1})\}$ is not entry deterring, ie $(m-2)\,LPO + X_m^s((m-2)\,LPO, m+1) < LO$. From proposition 4, if firm $m-2$ were to continue to produce the LPO, firm $m-1$ would permit the entry of firm $m+1$, but not firm $m+2$. Thus in the most profitable $m+1$ firm market structure that firm $m-2$ can induce has $x_{m-2} = LPO$. Continued deterrence of firm $m+1$ would entail $x_{m-2} > LPO$. This is clearly less profitable. A similar analysis applies to firms 1 through $m-3$.

QED.

Proposition 5 can be intuitively understood by recognizing that unlike firms m and $m-1$, firms 1 through $m-2$ would not suffer a decrease in output if they allow further entry. Whether they choose quantity so as to deter firm m + 1 or so as to allow that firm's entry, the price will be LP(F). Hence their is no incentive engage in costly entry deterrence (by producing where marginal cost exceeds LP(F)). With regard to figures 1 and 2, the shaded area does not exist, since the post-entry output is still the LPO. There are no units for which the limit price exceeds the marginal cost which will not be produced if firm $m+1$ enters. Hence these firms are not willing to incur costs to avoid losing these units: the rationale which led firms m and $m-1$ to expand their outputs above the LPO.

Corollary 4: The equilibrium outputs when firm $m+1$ enters are

(i) for $i \leq m-2$, $x_i = LPO$

(ii) for $i = m, m+1$, $x_i = S_i(Y_i)$

(iii) for $i = m-1$, $x_i = LO - (m-2)\,LPO - S_m(Y_m) - S_{m+1}(Y_{m+1})$

Proof. Follows immediately from propositions 5 and 6.

Propositions 3, 4, 5 and 6 and their corollaries characterize completely the possible outcomes in strategic equilibria to the model. The following corollary is immediate.

Corollary 5: The equilibrium output in a strategic equilibrium equals the limit ouput and the equilibrium price in a strategic equilibrium is the limit price.

Proof: Propositions 3, 4, 5, and 6 characterize all possible strategic equilibria; In all cases output equals the limit output, and price equals the limit price.

At F_4 when the m incumbent firms independently allow the entry of firm $m + 1$, the aggregate output of the $m + 1$ firms exceeds the blockaded output ($X_1^s(m + 1)$), since only the last two firms are on their blockaded reaction functions and firms 1 through $m - 1$ are producing above their blockaded reaction functions. Indeed the implication of propositions 3, 4, 5, and 6 is that for all values of $F < \bar{F}$, where $X_1^s(2) = LO(\bar{F})$, blockaded equilibria do not exist and the equilibrium price is the limit price. In the linear-quadratic model, there are only blockaded monopoly and blockaded duopoly solutions.

We can specify the critical values of the fixed costs, F_i, $i = 1,2,3,4$, as a function of the number of firms m, $F_i(m)$. Suppose that $m = 1$. Then the equilibrium market structure, for the following values of the fixed cost are

$F_1(1) \leq F$	Blockaded Monopoly.
$F_2(1) \leq F < F_1(1)$	Strategic Monopoly, $x_1 \leq LPO$
$F_3(1) \leq F < F_2(1)$	Strategic Monopoly, $x_1 > LPO$

For $F < F_3(1)$, firm 1 allows entry by firm 2 and the equilibrium market structure for the following values of fixed costs are

$F_1(2) \leq F < F_3(1)$	Blockaded Duopoly

$F_2(2) \leq F < F_1(2)$ Strategic Duopoly, $x_i \leq$ LPO $i = 1,2$
$F_3(2) \leq F < F_2(2)$ Strategic Duopoly, $x_1 =$ LPO, $x_2 >$ LPO
$F_4(2) \leq F < F_3(2)$ Strategic Duopoly, $x_2 > x_1 >$ LPO

For $F < F_4(2)$, the two incumbent firms would permit the third firm to enter. Depending on the value of F, the m firm strategic equilibrium will be one of the five types sketched at the beginning of this section. When $m > 2$, the behaviour of firms 1 through $m - 3$ is invariant to the type of strategic equilibrium. In all five types, these firms always produce at the LPO. The strategic interaction is only between the last three firms and the next potential entrant. As F declines, the number of firms in the industry increases and the limit price approaches the competitive price. While the relative importance of the strategic behaviour diminishes it does not disappear, since these five phases always exist.The five types of strategic equilibrium are completely characterized in Appendix B, and the important features are summarized in Table 1 below[11].

[Table 1 should be placed approximately here.]

4. Under- and Over-investment in entry deterrence

Non-cooperative models of commitment and entry deterrence may involve inefficient levels of investment in entry deterrence, from the perspective of the group as a whole. The outcome is the result of two well understood opposing forces: first, the incentive not to invest in *costly* entry deterrence, which leads to free-riding and a tendency towards underinvestment in equilibrium. Second, a larger quantity committed to implies a larger market share in equilibrium, so that shouldering the burden of entry deterrence may be advantageous up to a point.[12]

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[11]The characterization of strategic equilibria draws on lemma 5 and corollaries 1 through 5.

[12] See Gilbert and Vives (1986), McLean and Riordan (1989), Waldman (1987) and (1988) and Appelbaum and Weber (1990) for some clarification of these issues.

Several precise definitions of under and over investment in entry deterrence can be considered. We consider the following:

Definition: An m firm perfect equilibrium displays underinvestment in entry deterrence if, for some range of fixed costs where m is the equilibrium number of firms, a *multiplant monopolist* with m-1 plants, would not have added the mth plant. Overinvestment is defined analogously.

We are now in a position to state proposition 7 on overinvestment.

Proposition 7: The perfect equilibrium of the model involves overinvestment for all m.

Proof: Consider an m-plant monopolist producing the limit output so as to deter entry. It will be efficient to switch to m+1 plants where

$$C(m) > C(m+1)$$

and

$$C(m) = m\left(\frac{LO}{m}\right)^2 + mF$$

Simplifying, the switch-point can be summarized in terms of the level of fixed costs as

$$F^* = \frac{A^2}{(2\sqrt{2} + \sqrt{m(m+1)}\,)^2}$$

It remains to show that at F^* m firms producing the LPO would deter entry, and thus F^* implies a perfect equilibrium of *at most* m firms. The F value at which m firms producing the LPO just deters is given by the solution to

$$LO(F) = mLPO(F)$$

Substituting and solving for F, we obtain

$$F^{**} = \frac{A^2}{2(m + 2)^2}$$

It is easy to show that $F^{*} > F^{**}$ which completes the proof. *QED*

The intuition for these results follows from the rationale for firms m and m-1 producing above the LPO in the non-cooperative equilibrium. In the collusive solution there are obviously no adverse market share consequences from reducing output below the LPO. That is if the m collusive plants each reduce their output below the LPO, the market share of the profit unit (the m-plant monopolist) does not suffer, because another plant is added. When the standard of comparison is the collusive solution, it is market share considerations which drive the overinvestment result.

It is instructive to compare our definition of under and over-investment with that adopted by McClean and Riordan (1989) and Waldman (1991). These authors define underinvestment as existing when some subset of the firms that actually enter in perfect equilibrium could make themselves better off had they invested more in entry deterrence. Consider a value of F just below $F_4(2)$, at which firm 1 just prefers to allow entry of firm 3, rather than produce more so that entry continues to be deterred. At this point, $x_1 < x_2$ (see Table 1), and if firms 1 and 2 were to *collude* and continue to produce LO, they could clearly lower joint costs by producing equal quantities. Moreover, their joint profits would exceed the sum of profits of firms 1 and 2 in the three firm perfect equilibrium following entry. Thus, this parameter value of F satisfies the above definition of *underinvestment*. However, it is easy to explain the apparent paradox. We can show that a cartel of firms 1 and 2 would not in fact choose to deter firm 3 at $F_4(2)$, but would *accommodate* the entry of firm 3. This observation is then perfectly consistent with Proposition 7 above.[13]

[13] In the light of this observation, however, we question whether the McClean and Riordan and Waldman definition of underinvestment is the most appropriate one for models of sequential entry.

5. Conclusions

In some previous work on models of sequential entry, the possibility of odd patterns of investment among the incumbent firms has been observed. Schwartz and Baumann (1988) for example, note from their simulation results based on a Cobb-Douglas technology, that equilibrium investment for an individual firm may first rise, then fall, then rise again as the fixed costs of entry decrease. Robson (1990) also solves a simulation model, in which the last firm produces more than all of the earlier entrants, all of whom choose identical outputs. Our analysis in this paper offers a complete explanation for these observations. In the first case the firm is expanding quantity up to the limit pricing output, then contracting quantity as the limit pricing output falls with fixed costs, and then finally increasing output above the limit pricing output when it is required in order to deter entry. In the Robson case we have an equilibrium in which all firms produce at the limit pricing output except the last firm, which expands output to deter entry.

Our paper offers for the first time a complete characterization of the strategic interaction of firms in sequential entry models. We show how the limit price summarizes the equilibrium behaviour of firms in an intuitively appealing way. Delegation of entry deterrence is shown to be pervasive, and to have a natural sequential property that matches the structure of the game (the last firm bears all the costs, then the last but one, etc.).

Our results are derived for a model with linear demand and quadratic costs. This allowed us to characterize all possible types of equilibrium explicitly and completely. We believe that the qualitative nature of our results would extend to more general demand and cost formulations, provided that average costs are U- shaped. In particular, the concept and significance of the limit pricing output would seem perfectly general, as would the pattern of delegation.

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Appendix A: Proofs of Propositions, Lemmas and Corollaries

(i) proof of proposition 2

Given that Y_i is such that, $Y_i + X_i^s(Y_i, m) \geq LO(F)$, then firm *i* will profit maximize and deter further entry by utilizing its blockaded reaction function, $x_i = S_i(Yi)$, since *i* knows that the profit maximizing response of firms *i + 1* through *m* is to produce according to their blockaded reaction functions. This establishes (i).

Consider firm *i* where $\hat{x}_i < LO(F) - X_{i+1}^s(Y_{i+1}, m) - Y_i < LPO(F)$ and let $\hat{X}_i(Y_i)$ be the subgame perfect equilibrium output that corresponds to $\hat{x}_i$. From propositions 2 and 3 there are two possible cases: (a) $\hat{X}_i(Y_i) > LO(F)$; (b) $\hat{X}_i = LO(F)$. In either case $\hat{x}_i$ cannot be profit maximizing. Consider (a). In this case by producing $x_i = LO(F) - X_{i+1}^s(Y_{i+1}, m) - Y_i$ the equilibrium price will rise to the limit price and the profits of firm *i* will increase due to two effects. First since the equilibrium price rises, the margin on each of the $\hat{x}_i$ units increases; secondly, for $\hat{x}_i < x_i \leq LPO(F)$, the limit price exceeds the marginal cost of production and hence marginal profit is positive. For case (b), the profits of firm *i* will increase if it expands its output to $x_i = LO(F) - X_{i+1}^s(Y_{i+1}, m) - Y_i$ due to the second of the preceding two effects .

Suppose that $\hat{x}_i > LO(F) - X_{i+1}^s(Y_{i+1}, m) - Y_i$. Then firm *i* could increase its profits by reducing its output to $x_i = LO(F) - X_{i+1}^s(Y_{i+1}, m) - Y_i$. By definition such an x_i would still deter entry. The profits of firm *i* would increase since the deviations from its profit maximizing output $S_i(Y_i)$, given that firms *i + 1* through *m* select output according to $S_j(Y_j)$, would be minimized. This proves (ii).

By assumption m $LPO(F) \geq LO(F)$, thus from (i) and (ii), entry will be deterred by the *m* incumbent firms and the equilibrium price will be the limit price. Thus no firm will be willing to produce more than the limit pricing output since for units of output greater than LPO(F), marginal cost is greater than the limit price and such an expansion in output is not required to deter entry. This establishes (iii). *QED.*

(ii) *proof of lemma 7*

We require the market structure x = {LPO, LPO, . . . , LPO, $S_m(Y_m)$, $S_{m+1}(Y_{m+1})$} to deter entry at the F value at which firm m is indifferent between continuing to deter entry and switching to an accommodation strategy. The latter would consist of choosing the profit maximizing quantity conditioned on the previous choices, and anticipating an optimal choice of the entrant, firm $m + 1$.

If firm m allows entry in this way, total industry output can be computed as:

$$\frac{23}{56}A + (m - 1)\frac{33}{56}LPO$$

Setting the above expression equal to LO(F) and solving for F, we obtain

$$\sqrt{2F} > \left(\frac{33}{(m - 1)33 + 112}\right)A \qquad \text{(A1)}$$

as the inequality defining the minimum bound for which the market structure x = {LPO, LPO, . . . , LPO, $S_m(Y_m)$, $S_{m+1}(Y_{m+1})$} deters further entry.

It remains to show that the switch-over point for firm m exceeds this minimum bound. Rather than solve for the switch-over F value directly, we proceed by a more indirect route, and show that at the value of F given above in (A1), entry is the preferred strategy. Since we know that deterrence profits are increasing in F, and profits from allowing entry are (weakly) decreasing in F, it follows that the switch over F value lies in the region of inequality (A1) i.e. that when firm m defects from deterrence to accommodation, its preferred accommodation strategy will deter further entry.

<u>Profits from continued deterrence</u> by firm m, holding the outputs of firms 1 to m - 1 constant at LPO(F), can be derived as

$$\pi^s_m = ((m + 3)\sqrt{2F} - A)(A - (m + 1)\sqrt{2F})$$

<u>Profits from allowing entry</u> in the market structure x = {LPO, LPO, . . . , LPO, $S_m(Y_m)$, $S_{m+1}(Y_{m+1})$} are equal to

$$\pi^B_m = \frac{9}{112}(A - (m - 1)\sqrt{2F})^2$$

By evaluating these two expressions at the critical F value given in (A1) we obtain $\pi^B_m > \pi^S_m$. We can easily show that $\pi^B_m - \pi^S_m$ is decreasing in F, so the switch over value of F lies in the region given in (A1). *QED.*

(iii) proof of lemma 8

For x = {LPO, LPO, ... ,$S_{m-1}(Y_{m-1})$,$S_m(Y_m)$,$S_{m+1}(Y_{m+1})$} to be entry deterring,

$$(m - 2)\ \text{LPO} + S_{m-1}(Y_{m-1}) + S_m(Y_m) + S_{m+1}(Y_{m+1}) < \text{LO} \tag{A2}$$

For the linear-quadratic model,

$$S_{m-1}(Y_{m-1}) + S_m(Y_m) + S_{m+1}(Y_{m+1}) = (5183/9968)(A - (m - 2)\ \text{LPO})$$

by (4a), (4b), and (4c); Substituting the values of LO and LPO into (A2) establishes the inequality.

(iv) proof of proposition 4

A consequence of proposition 3, is that for F < F_3, firm *m* would optimally accommodate firm *m + 1* if its predecessors continued to produce at the LPO. Given part (ii), that firm $m - 1$ will increase its output to deter entry, firms 1 through $m - 2$ have no incentive to expand their output beyond the LPO. Since the limit price will be the equilibrium price, they have every incentive to produce at the LPO. This establishes (i).

In order to establish (ii) we need first show that the market structure $x^s_{m-1}(Y_{m-1}, m + 1)$, where $Y_{m-1} = LPO\ (m - 2)$ does not deter further entry. That is, the most profitable alternative for firm m, when it elects to let firm $m + 1$ enter deters entry. This is true by lemma 7.

However from lemma 6 the market structure $x = \{LPO, LPO, \ldots, LPO, S_m(Y_m), S_{m+1}(Y_{m+1})\}$ has aggregate output in excess of the limit output. Thus by proposition 2, the most profitable market structure for firm $m - 1$ if firm $m + 1$ enters is strategic with vector of outputs: $x = \{LPO, LPO, \ldots, x_{m-1}(Y_{m-1}, m + 1), S_m(Y_m), S_{m+1}(Y_{m+1})\}$, where $x_{m-1}(Y_{m-1}, m + 1)$is implictedly defined by

$$Y_{m-1} + x_{m-1}(Y_{m-1}, m + 1) + S_m(Y_m) + S_{m+1}(Y_{m+1}) = LO$$

In the linear-quadratic model

$$x_{m-1}(Y_{m-1}, m + 1) = \frac{33(A - (m - 2)LPO) - 112\sqrt{2F}}{33}$$

Having identified the most profitable market structure firm $m - 1$ can induce if it does not deter firm $m + 1$, we now establish that for some $F < F_3$, firm $m - 1$ will deter firm $m + 1$. The willingness of firm $m - 1$ to deter the entry of firm m when $F < F_3$ arises since the most profitable alternative for firm $m - 1$ reduces its output below the LPO. Figure 4 illustrates the limit pricing output, the equilibrium output for firm $m - 1$ when firm $m + 1$ enters, and the limit pricing output. At F_3 firm $m - 1$ is producing at the LPO and relative to $x_{m-1}(Y_{m-1}, m + 1)$its profits are greater by the amount of the shaded area. As F falls, for firm $m + 1$ to continue to be deterred, firm $m - 1$ must increase its output to levels where marginal cost is greater than the limit price. The cost of the deterrence strategy is indicated by the lined area in Figure 4. For values of F below and close to F_3 firm *m-1* is clearly willing to incur these costs in order to continue to earn the profit indicated by the shaded area.

As with firm *m* however, as F continues to fall, the profits protected by deterring entry (the shaded area) decrease as both the limit price and $Y_{m-1} = (m - 2)$ LPO decrease. Moreover the same two effects increase the cost of continued entry deterrence (the lined area increases). When $F = F_4$, firm *m - 1* is indifferent between the strategic equilibrium with *m* firms and the strategic equilibrium with *m + 1* firms. This proves part (ii).

Part iii) follows from proposition 3. Entry deterrence by firm *m - 1* above the LPO is costly. Hence firm *m - 1* will minimize the necessary amount of the expansion. It will expand such that firm *m* is just indifferent between optimally accommodating firm *m + 1* and continuing to deter firm *m + 1*. The most profitable alternative market structure for firm *m*, if it optimally accommodates firm *m + 1*,continues to be $x = \{LPO, LPO, \ldots, x_{m-1}, S_m(Y_m), S_{m+1}(Y_{m+1})\}$. Lemma 6 established that $x = \{LPO, LPO, \ldots, LPO, S_m(Y_m), S_{m+1}(Y_{m+1})\}$ deterred entry. From lemma 4 and $x_{m-1} > LPO$, $x = \{LPO, LPO, \ldots, x_{m-1}, S_m(Y_m), S_{m+1}(Y_{m+1})\}$ is also entry deterring. Setting $\pi_m(Y_m, m, F) = LP(F)\, x_m(F) - c(x_m(F))$, the profits of firm *m* if it deters firm *m + 1*, equal to $\pi^s_m(Y_m, m + 1, F) = P(Y_m + X^s_m(Y_m, m + 1))\, S_m(Y_m) - c(S_m(Y_m))$ the profits from accommodation and solving for x_m, gives (iii).

QED.

(v) *proof of Corollary 4*

We need only show that $x_m > x_{m-1}$. The rest follows from proposition 4. For this range of F, both x_m and x_{m-1} exceed the LPO. The extent to which they are willing to go above the LPO is a function of their output in the most profitable market structure they can induce if they allow entry. Whichever has the smallest output in this alternative market structure will be willing to incur the greatest cost of entry deterrence and hence have the larger equilibrium market share.

The alternative for firm $m - 1$, is from proposition 4, defined by

$$LO - (m - 2)\,LPO - S_m((m - 2)\,LPO + \hat{x}_{m-1}) - S_{m+1}((m - 2)\,LPO + \hat{x}_{m-1} + S_m = \hat{x}_{m-1}$$

The alternative for firm m, is from proposition 3, defined by $S_m(x_{m-1} + (m - 2)\,LPO) = x_m$, where x_{m-1} is the equilibrium output for firm $m - 1$ and it is greater than $S_{m-1}(Y_{m-1})$ but less than LPO. If

$$\begin{aligned} LO \quad > \quad & (m - 2)\,LPO + S_m((m - 2)\,LPO + \hat{x}_{m-1}) \\ & + S_{m+1}((m - 2)\,LPO + \hat{x}_{m-1} + S_m) + S_m(x_{m-1} + (m - 2)\,LPO) \end{aligned}$$

then $\hat{x}_{m-1} > \hat{x}_m$. From lemma 7, $LO > (m - 2)\,LPO + S_{m-1}(Y_{m-1}) + S_m(Y_m) + S_{m+1}(Y_{m+1})$. Using lemma 3 and comparing terms shows that the inequality is satisfied. *QED.*

Appendix B: The Five Types of Strategic Equilibrium.

If $F_2(m) < F \leq F_4(m - 1)$, then either

Type I (i) for $1 \leq i \leq m - 3$, $x_i = LPO(F)$

(ii) for firm m - 2, $x_{m-2} = LO(F) - (m - 2)\, LPO(F) - S_{m-1}(Y_{m-1}) - S_m(Y_m) \leq LPO$

(iii) for firm m - 1, $x_{m-1} = S_{m-1}(Y_{m-1})$

(iv) for firm m, $x_m = S_m(Y_m)$

or

Type II (i) for $1 \leq i \leq m - 2$, $x_i = LPO(F)$

(ii) for firm m - 1, $x_{m-1} = LO(F) - (m - 2)\, LPO(F) - S_m(Y_m)$

(iii) for firm m, $x_m = S_m(Y_m)$

or

Type III (i) for $1 \leq i \leq m - 1$, $x_i = LPO(F)$

(ii) for firm m, $x_m = LO(F) - (m - 1)\, LPO(F) \leq LPO$

depending on the value of F. This follows from proposition 2.

If $F_3(m) \leq F \leq F_2(m)$, then from proposition 3,

Type IV (i) for $1 \leq i \leq m - 1$, $x_i = LPO(F)$

(ii) for firm m, $x_m = LO(F) - (m - 1)\, LPO(F) > LPO$

If $F_4(m) \leq F \leq F_3(m)$, then from proposition 4,

Type V (i) for $1 \leq i \leq m - 2$, $x_i = LPO(F)$

(ii) $x_{m-1} = LO(F) - (m - 2)\, LPO(F) - x_m > LPO$

(iii) $$x_m = \frac{LP(F) + \sqrt{(LP(F))^2 - 4\pi^s_m(Y_m, m + 1, F)}}{2} > LPO$$

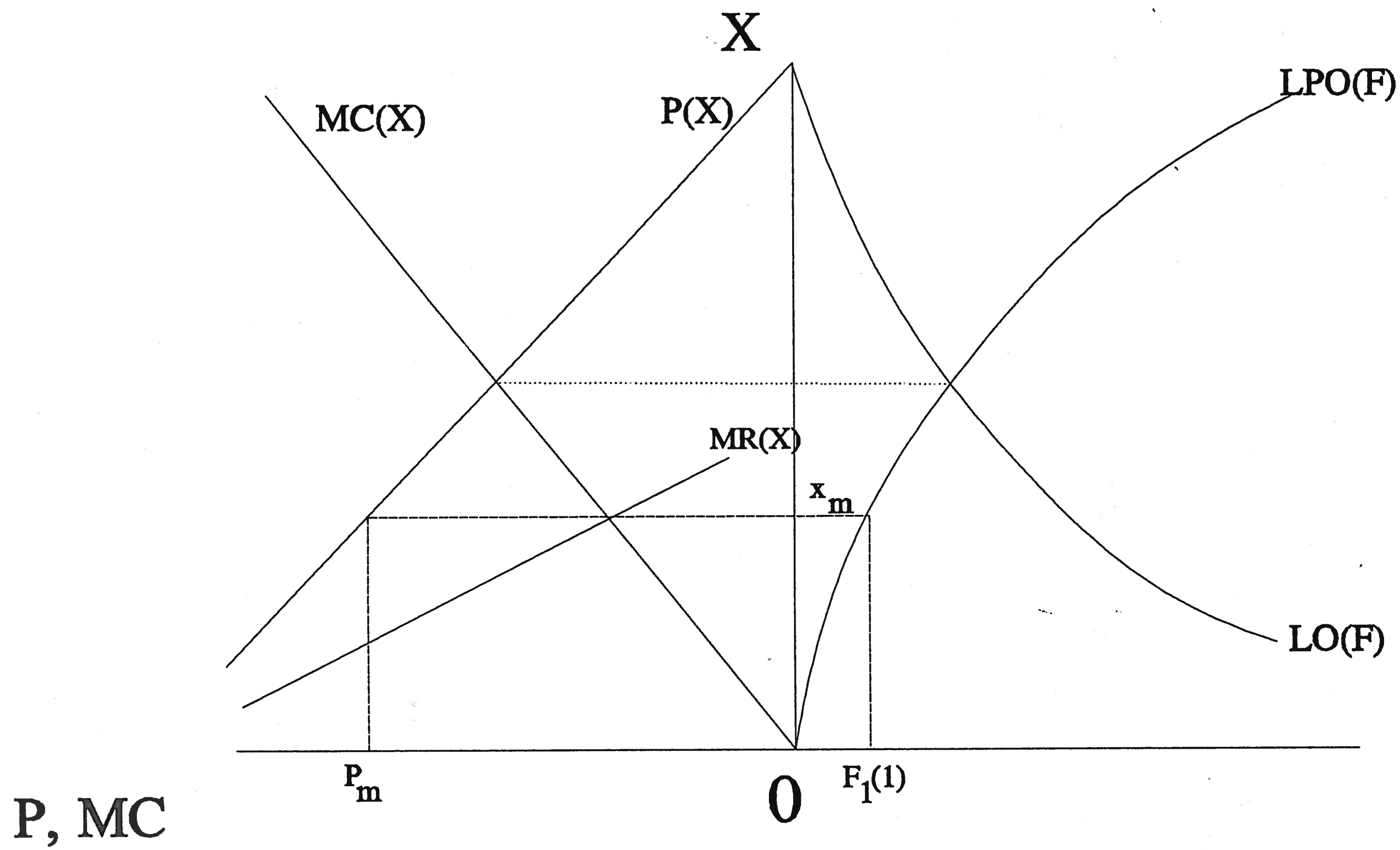

Figure 1: the relationship between LO(F), LPO(F), price, and marginal cost

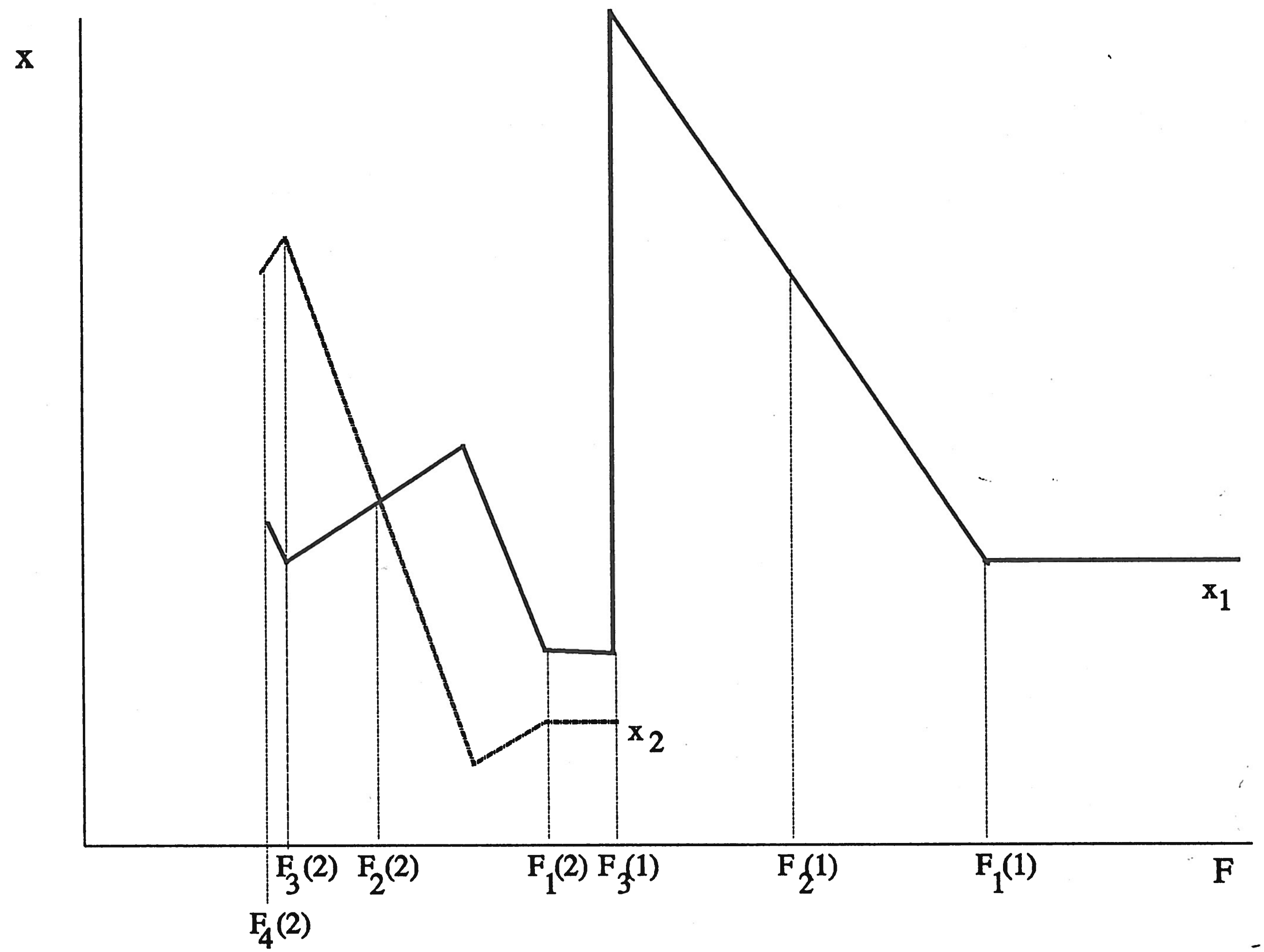

Figure 2: Equilibrium Quanities in the Monopoly and Duopoly Cases

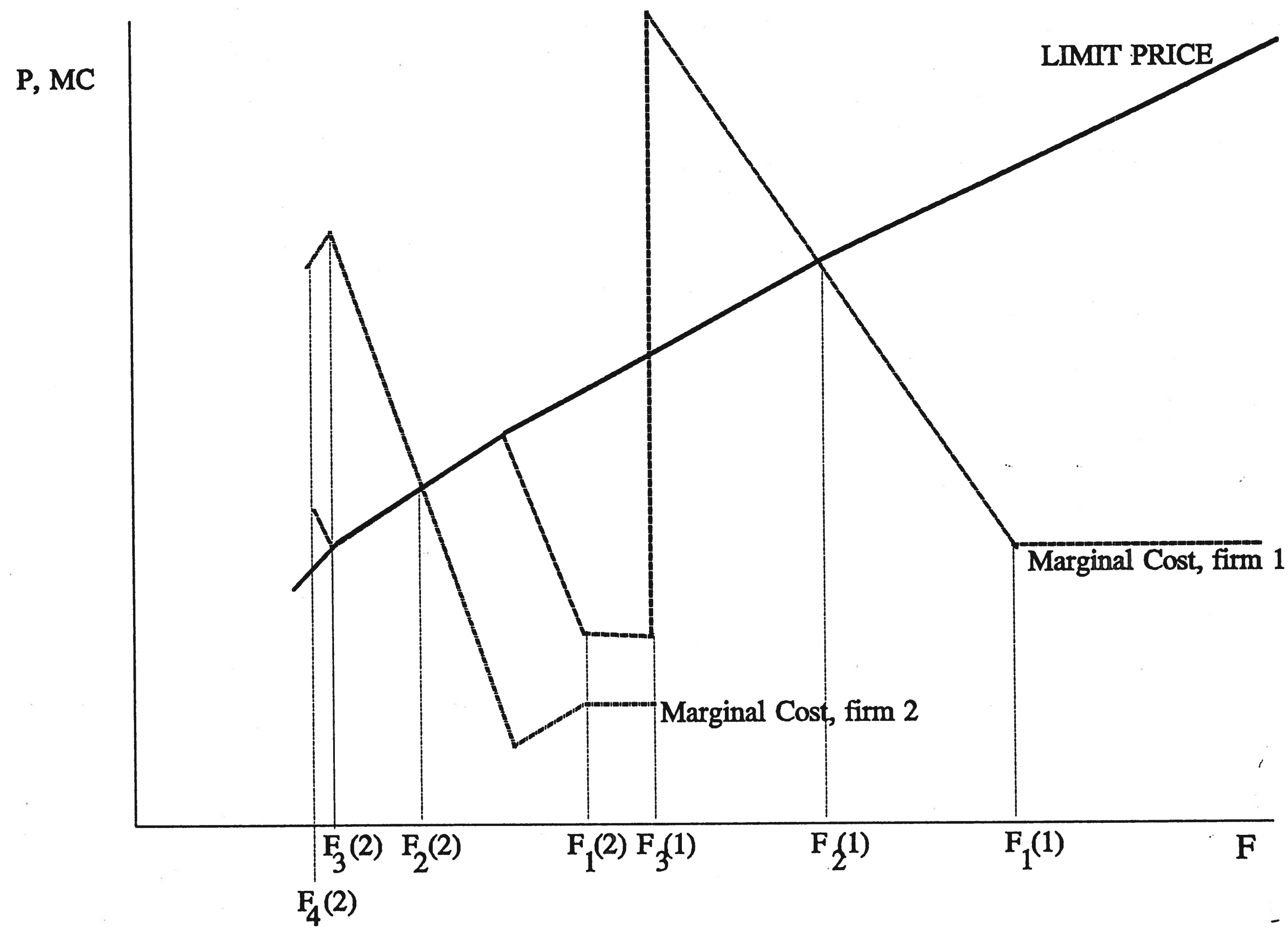

Figure 3: Limit Price and Marginal Costs in the Duopoly and Monopoly Cases

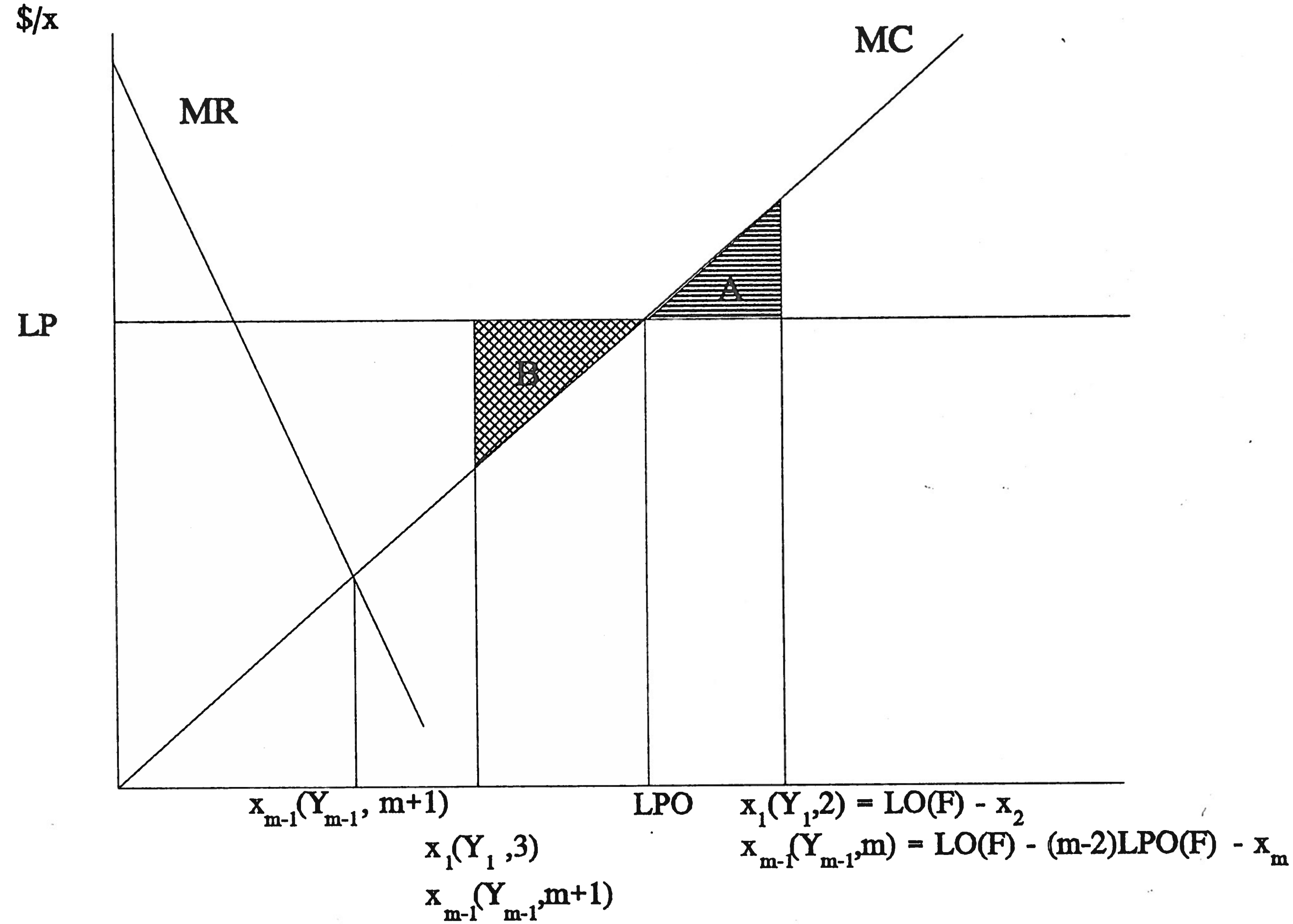
$/x
MC
MR
LP
A
B
$x_{m-1}(Y_{m-1}, m+1)$
$x_1(Y_1, 3)$
$x_{m-1}(Y_{m-1}, m+1)$
LPO
$x_1(Y_1, 2) = LO(F) - x_2$
$x_{m-1}(Y_{m-1}, m) = LO(F) - (m-2)LPO(F) - x_m$

Figure 4

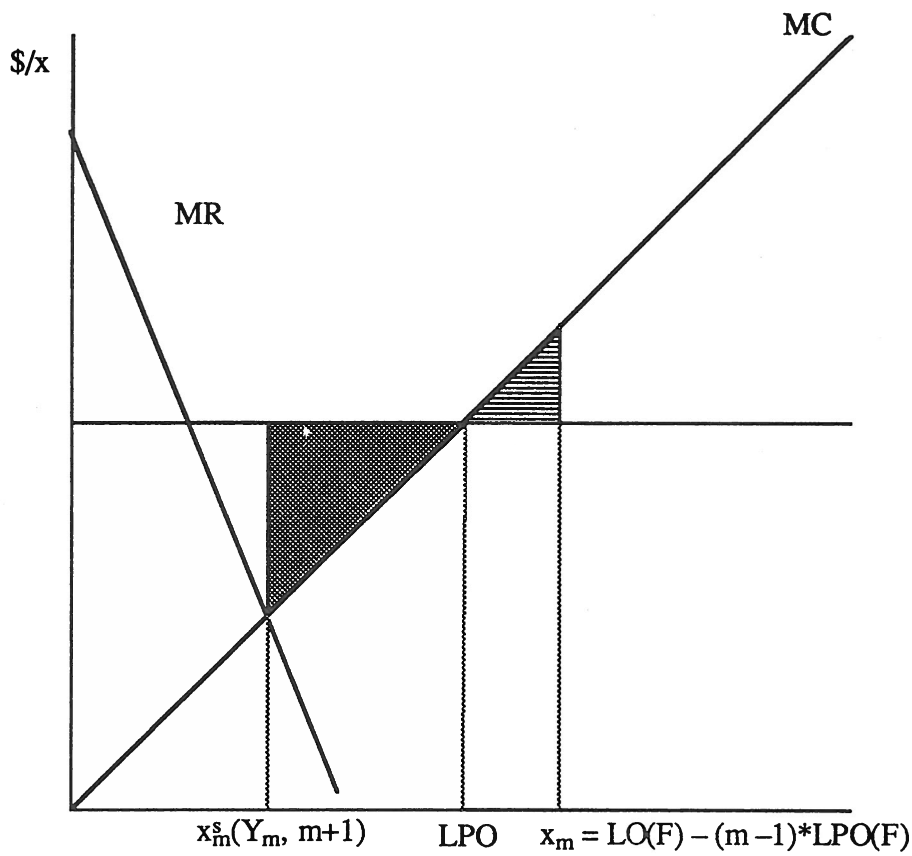

Figure 5

Table 1: Summary Characteristics of the Equilibrium

Type of Equilibrium	Quantity Rankings	$x_i <=> LPO(F)$	Profit Rankings
Blockaded	$x_1 > x_2 > \ldots > x_m$	$x_i < LPO\ \forall i$	$\pi_1 > \pi_2 > \ldots > \pi_m$
Strategic Type I	$x_1 = x_2 = \ldots = x_{m-3}$ $\geq x_{m-2} > x_{m-1} > x_m$	$x_i = LPO\ \forall 1, m-3$ $x_{m-2} \leq LPO$ $x_i < LPO\ \forall m-1, m$	$\pi_1 = \pi_2 = \ldots = \pi_{m-3}$ $\geq \pi_{m-2} > \pi_{m-1} > \pi_m$
Strategic Type II	$x_1 = x_2 = \ldots = x_{m-2}$ $\geq x_{m-1} > x_m$	$x_i = LPO\ \forall 1, m-2$ $x_{m-1} \leq LPO$ $x_m < LPO$	$\pi_1 = \pi_2 = \ldots = \pi_{m-2}$ $\geq \pi_{m-1} > \pi_m$
Strategic Type III	$x_1 = x_2 = \ldots = x_{m-1}$ $\geq x_m$	$x_i = LPO\ \forall 1, m-1$ $x_m \leq LPO$	$\pi_1 = \pi_2 = \ldots = \pi_{m-1}$ $\geq \pi_m$
Strategic Type IV	$x_1 = x_2 = \ldots = x_{m-1}$ $< x_m$	$x_i = LPO\ \forall 1, m-1$ $x_m > LPO$	$\pi_1 = \pi_2 = \ldots = \pi_{m-1}$ $> \pi_m$
Strategic Type V	$x_1 = x_2 = \ldots = x_{m-2}$ $< x_{m-1} < x_m$	$x_i = LPO\ \forall 1, m-2$ $x_i > LPO\ \forall m-1, m$	$\pi_1 = \pi_2 = \ldots = \pi_{m-2}$ $> \pi_{m-1} > \pi_m$