Queen's Economics Department Working Paper No. 833

Unemployment Insurance and Market Structure

Nicolas Marceau

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

9-1991

DISCUSSION PAPER #833

UNEMPLOYMENT INSURANCE AND MARKET STRUCTURE

by

Nicolas Marceau
Department of Economics
Queen's University

August 1991

Abstract

This paper examines the impact of unemployment insurance (UI) on employment and unemployment in an industry in which the prices can vary due to some market power or general equilibrium (GE) effects. Some non-conventional results are obtained. First, it is shown that in an industry in which firms have some market power [Cournot competition], average industrial employment may be a decreasing function of the experience-rating because the number of firms in the industry is itself a decreasing function of the experience-rating. This contradicts the conventional view according to which employment should be an increasing function of the experience-rating. Second, the case of an industry characterised by perfect competition and general equilibrium effects on prices is examined. It is shown that the GE effects mitigate the conventional results as well as the one obtained by Burdett and Wright (1989b) [which contradict the conventional view] because those results were obtained in a partial equilibrium framework [fixed prices]. The general conclusion is that for industries with different degrees of market power, the same UI scheme has different impacts on employment and unemployment.

I am indebted to Robin Boadway for valuable comments and suggestions. Helpful comments were also received from Huw Lloyd-Ellis and participants at the Queen's University Ph.D. Seminar. Financial support from the Fonds du Québec pour la Formation de Chercheurs et l'Aide à la Recherche is gratefully acknowledged.

Present address: Nicolas Marceau, Department of Economics, Queen's University, Kingston, Ontario, Canada, K7L 3N6.

1. Introduction

This paper examines the impact of unemployment insurance (UI) on employment and unemployment under various industrial market structures. The first part of the paper considers the case of an industry characterized by Cournot competition in the output market. Both the no entry and the free entry cases are considered. It is shown that free entry in the industry might lead to unconventional results. The second part of the paper is devoted to the analysis of UI in the case of an industry characterized by perfect competition in the output market. Again, both the no entry and the free entry cases are examined. It is shown that in both cases, taking into account the general equilibrium (GE) effects of firms' behavior on industry prices may lead to results that contradict the conventional view.

The analysis is performed within an implicit contract theory framework similar to that developed in the mid 1970s by Baily (1974), Gordon (1974), and Azariadis (1975). Since implicit contract theory is used here, attention is restricted to the impact of unemployment insurance on *temporary* layoffs. Feldstein's (1976) demonstration that temporary layoffs represented some 75% of the layoffs in the manufacturing industry for the period 1965-1975 should convince the reader of the importance of this type of layoffs.

Several positive analyses of UI have been made using implicit contract models. These include the classic papers by Feldstein (1976) and Baily (1977) but also, more recent work by Topel and Welch (1980), Burdett and Hool (1983), Mortensen (1983), Burdett and Wright (1989a), and Burdett and Wright (1989b) [for now on, BW]. In general, those papers argue that UI has an adverse impact on employment.[1] More precisely, the *conventional view* is that increasing the experience-rating parameter, by increasing employment in bad states of the world [the *standard* effect], should decrease the number of layoffs in those states and thus increase average employment. Many then advocate that an efficient UI scheme would be characterized by full experience-rating, and argue that, since the UI schemes in place in the US are not fully experience-rated, UI generates harmful unemployment.

The goal of this paper is not to argue in favor of a fully or partially experience-rated UI scheme but rather to show that the conventional view according to which an increase in experience-rating should increase average employment does not necessarily hold when one pays attention to market structure and to entry. In fact the conventional view has been arrived at using models in which there was perfect competition in the product market but no GE effects on prices. It is shown here that:

[1] This is not true for BW. See below for a detailed examination of their argument.

- if the industry is characterized by Cournot competition in the output market and there is free entry, or
- if the industry is characterized by perfect competition in the output market [under both no entry and free entry] and there are GE effects on prices,

it is possible that the average employment level of the industry will fall as a consequence of a higher experience-rating parameter. However, it is also shown that for the case with GE effects, the conventional view can be reestablished.

It should be noted that the conventional view has also been recently challenged by BW. They show that the above conventional prediction no longer holds if one allows for the *firm size* effect, i.e. the fact that the size of the firms, and thus employment and unemployment, is not independent of the UI scheme in place. The current analysis distinguishes two further effects. The first is here referred to as the *free entry* effect and is understood to be the impact of a UI scheme on the number of firms that ultimately decide to enter an industry. The second effect is the *GE* effect which is observed under perfect competition in the output market, i.e. when firms take prices as given. In this case, the behavior of firms will have an impact on the equilibrium prices that is not taken into account by them when making their decisions. It should be noted that the two effects are obtained *independently*[2] of the one identified by BW. They may therefore can thus reinforce or contradict the firm size effect.

The above discussion has centered exclusively on the impact of the extent of experience-rating used by a UI scheme. The US systems[3] are effectively financed through more or less experience-rated taxes. However, elsewhere, payroll taxes are also used to finance those schemes. This is particularly true for Canada where the UI scheme is financed almost exclusively[4] through a payroll tax [i.e. no experience-rating]. The impact of a payroll tax used to finance the UI scheme will also be considered. Again, some interesting results are obtained when one takes into account the impact of free entry or of GE effects.

---

[2] This should not be interpreted to mean that, given an industry market size, the number of firms and the size of the firms in the industry are independent but rather that the free entry effect would be observed even in the absence of the firm size effect i.e. if the firm size is exogenous. The same interpretation of "independent" should be understood when it is said that the GE effect is independent of the firm size or the free entry effects.

[3] In the US, UI is a state jurisdiction. This is in contrast with Canada where it is a federal jurisdiction.

[4] A very small fraction of the financing comes from the government's general revenues.

The plan of this paper is as follows. In the next section, the basic model and the results are presented for the Cournot competition case. Both the no entry and the free entry cases are analyzed. It should be noted that the firm size effect described in BW is not incorporated into the analysis.[5,6] In section 3, the perfect competition case is considered again for both no entry and free entry. The concluding remarks follow.

2. Cournot competition

Consider the following economy. There are $m > 1$ identical, risk-neutral firms in an industry and each has an exogenous number of workers under contract, N. There is uncertainty in this economy which takes the form of two states[7] of demand [1 and 2] for the good produced by the firms. The good state, 1, occurs with probability λ, and the bad state, 2, with probability $1 - \lambda$. The demand[8] in state s is:

$$p_s = k_s - \sum_{i=1}^{m} q_{si}$$

where p_s is the price of output in state s, k_s is a state-dependent constant, and q_{si} is the output of firm i in state s. The good state is called *good* because $k_1 > k_2$. Thus, the main difference with the previous analyses is that, here, the pair of prices $\{p_1, p_2\}$ is not given to the firms as with perfect competition but rather determined

[5] It is not clear whether the firm size effect has to be present when the free entry effect is, and vice versa. In fact, to answer that question one has to judge whether a firm reevaluates its size decision more frequently than its decision to supply some output on the particular market under consideration. For example, if a firm enters into a long-term legally binding contract with its workers, one might argue that that the only thing the firm could do after the UI scheme is modified is to *stay* or to *exit* the market. On the other hand, short term contracts might give some flexibility to the firm which implies that the decision to exit would only be considered as an extreme option.

[6] The same analysis, but with the firm size effect is performed in Appendix C. The qualitative results obtained in section 2 are not changed by the introduction of the firm size effect.

[7] It is solely for simplicity that only two states of the world are assumed; the qualitative results of the analysis would still be obtained with more possible states.

[8] The demand is assumed to be linear so that clear results are obtained. A more general state-dependent demand might well lead to ambiguous results. My concern here is however not to show that the conventional view is never correct but rather that there exists situations in which it is incorrect.

by their behavior.[9]

The technology used by the firms to produce output is linear in labour, the single input[10]:

$$q_{si} = F(n_{si}) = n_{si}$$

where q_{si} and $n_{si} \leq N$ are respectively the output and the number of employed workers in firm i and state s. Obviously, $n_{si} \leq N$ because it might be in the interest of firm i to layoff $(N - n_{si})$ workers in state s. However, when a worker is laid-off, a publicly provided UI benefit b is provided to him. To finance the cost of the UI program, the government imposes a UI tax. The tax bill TB_{si} of firm i in state s given n_{si} is:

$$TB_{si} = eb(N - n_{si}) + \delta N$$

where e is the experience-rating parameter and δ is the rate of the payroll tax. Note that the UI scheme is said to be fully experience-rated if $e = 1$ and partially experience-rated if $e < 1$.[11] Typically, in the US, $e > 0$ while for Canada, $e = 0$ and $\delta > 0$. It should be noted that no rationale for the *public* provision of UI is given here and it is assumed that no severance pay is provided by the firms to their laid-off workers. This is acceptable since my intention is only to perform a positive analysis of the existing UI schemes.

Denote by w_{si} the income paid by firm i in state s to its employed workers and assume that there is a fixed cost c for production to take place. It is assumed that the firms in this economy compete à la Cournot-Nash so that firm i, when making its decisions, takes as given the decisions made by the other firms in the industry. Denote by a "hat" those variables that are taken as given by a firm. The expected profits of firm i are, given the decisions $\{n_{si}, w_{si};\ s = 1, 2\}$:

---

[9] It should be noted that the analysis of implicit contracts in the presence of imperfect competition, but without UI, is the subject of a growing program of research. On this, see Chari, Jones, and Manuelli (1989) or Cooper (1990).

[10] The same remarks apply for the choice of the technology and for the choice of the demand [see the previous footnote].

[11] It should be noted that the UI scheme is not required here to be fully-funded [self-financed].

$$
\begin{aligned}
E\Pi^i =& \lambda \left[n_{1i} \left(k_1 - n_{1i} - \sum_{j \neq i}^{m} \hat{n}_{1j} \right) - n_{1i}w_{1i} - eb(N - n_{1i}) \right] \\
&+ (1-\lambda) \left[n_{2i} \left(k_2 - n_{2i} - \sum_{j \neq i}^{m} \hat{n}_{2j} \right) - n_{2i}w_{2i} - eb(N - n_{2i}) \right] \\
&- \delta N - c.
\end{aligned}
$$

Now consider the workers in this economy. All the workers are assumed to be identical and risk-averse. Each has utility given by $u(I,h)$ where I is the worker's income and h his labour supply. It is assumed that labour is indivisible and that it can take only two values: $h \in \{0,1\}$ where $h=0$ for an unemployed worker and $h=1$ for an employed worker. The utility function is strictly concave and has the following properties:

$$
u_1 > 0; u_{11} < 0; \text{ and } \; u_2 < 0
$$

When a worker joins a firm, he knows that he may face the possibility of a layoff. It is here assumed that the laid-off workers of a firm are *chosen* randomly among the N attached workers. Consequently, a worker joining firm i, given the set of decisions $\{n_{si}, w_{si};\; s=1,2\}$, will have expected utility:

$$
\begin{aligned}
EU^i =& \lambda \left[\frac{n_{1i}}{N} u(w_{1i},1) + \frac{(N-n_{1i})}{N} u(b,0) \right] \\
& (1-\lambda) \left[\frac{n_{2i}}{N} u(w_{2i},1) + \frac{(N-n_{2i})}{N} u(b,0) \right].
\end{aligned}
$$

Assuming that the workers could obtain reservation utility RU elsewhere in the economy, an optimal contract between firm i and its workers is a set of decisions $\{n_{si}, w_{si};\; s=1,2\}$ that solves the following problem:[12]

$$
\max_{\{n_{si}, w_{si};\; s=1,2\}} E\Pi^i \tag{$P1$}
$$

subject to

[12] As noted by Baily (1977), Burdett and Hool (1983), or BW, the dual of the problem [i.e. max EU subject to $E\Pi > K$] would yield the same contract curve. However, since I want to examine the entry [exit] decision of the firms, the version of the problem examined in the text has some heuristic advantages.

$$EU^i \geq RU$$

$$n_{si} \leq N \quad s = 1, 2.$$

The Lagrangian of problem $P1$ is written, using γ_i, β_{1i}, and β_{2i} as Lagrange multipliers:

$$\Omega = E\Pi^i + \gamma_i(EU^i - RU) + \beta_{1i}(N - n_{1i}) + \beta_{2i}(N - n_{2i}).$$

The first order conditions of this maximization problem are[13]:

$$-\lambda n_{1i} + \gamma_i \lambda \frac{n_{1i}}{N} u_1(w_{1i}, 1) = 0 \tag{1}$$

$$-(1-\lambda) n_{2i} + \gamma_i (1-\lambda) \frac{n_{2i}}{N} u_1(w_{2i}, 1) = 0 \tag{2}$$

$$\lambda \left[k_1 - 2n_{1i} - \sum_{j \neq i}^{m} \hat{n}_{1j} - w_{1i} + eb + \frac{\gamma_i}{N}[u(w_{1i}, 1) - u(b, 0)] \right] - \beta_{1i} = 0 \tag{3}$$

$$(1-\lambda) \left[k_2 - 2n_{2i} - \sum_{j \neq i}^{m} \hat{n}_{2j} - w_{2i} + eb + \frac{\gamma_i}{N}[u(w_{2i}, 1) - u(b, 0)] \right] - \beta_{2i} = 0 \tag{4}$$

$$EU^i - RU = 0 \tag{5}$$

$$\begin{aligned} N - n_{si} &\geq 0 \quad s = 1, 2 \\ \beta_{si} &\geq 0 \quad s = 1, 2 \\ \beta_{si}(N - n_{si}) &= 0 \quad s = 1, 2. \end{aligned} \tag{6}$$

Using now equations (1) and (2), the following is obtained:

[13] The second order conditions of this problem are shown to be satisfied in Appendix A.

$$w_{1i} = w_{2i} = w_i \qquad \text{and} \qquad \gamma_i u_1(w_i, 1) - N = 0. \tag{7}$$

Thus, the income offered by firm i to its employed workers is state-independent. This is a standard result of the implicit contract theory. Now consider equations (3) and (4) and let:

$$\beta_{1i} = \beta_{2i} = 0$$

i.e. the constraints on the employment levels are not binding; they then imply, respectively, that:

$$k_1 - 2n_{1i} - \sum_{j \neq i}^{m} \hat{n}_{1j} - w_i + eb + Z = 0 \tag{8}$$

$$k_2 - 2n_{2i} - \sum_{j \neq i}^{m} \hat{n}_{2j} - w_i + eb + Z = 0 \tag{9}$$

where

$$Z = \frac{u(w_i, 1) - u(b, 0)}{u_1(w_i, 1)}.$$

It will be said, using Burdett and Hool (1983) terminology, that a contract is a *labour contract* if $Z > 0$ but that it is a *leisure contract* if $Z < 0$.

Using the first order conditions, one can get well-defined reaction curves for the endogenous variables $\{w_i, n_{1i}, n_{2i}, \gamma_i, \beta_{1i}, \beta_{2i}\}$ as functions of the other firms' employment level in the industry: $\{\hat{n}_{1j}, \hat{n}_{2j};\ j \neq i\}$. In particular, it can be shown that employment is a strategic substitute[14]:

$$\frac{\partial n_{si}}{\partial n_{sj}} < 0 \quad s = 1, 2 \quad i \neq j.$$

Because the reaction curves are well-defined, it would be possible to show that there exists a Nash equilibrium.[15] Now, because all firms in the industry are identical,

[14] This is shown in Appendix B.

[15] A Nash equilibrium exists if [a sufficient condition]: 1) the number of players is finite; 2) the strategy sets are compact [closed and bounded] and convex; and 3) the payoffs of each player are continuous, bounded and strictly quasi-concave in their own strategies. None of these three conditions seems to pose a particular problem although it might be necessary to make some further assumptions to satisfy 2).

the equilibrium obtained here is a *symmetric Nash equilibrium*:

$$n_{1i} = n_{1j} = n_1 \quad \forall i,j$$
$$n_{2i} = n_{2j} = n_2 \quad \forall i,j$$
$$w_i = w_j = w \quad \forall i,j$$
$$\gamma_i = \gamma_j = \gamma \quad \forall i,j$$
$$\beta_{si} = \beta_{sj} = \beta_s \quad \forall i,j;\ s = 1,2.$$

Then, using equations (8) and (9), it is possible to obtain:

$$k_1 - (m+1)n_1 = k_2 - (m+1)n_2.$$

This implies that for both m endogenous or m exogenous, $n_1 > n_2$ since $k_1 > k_2$. Furthermore, this implies that if there are layoffs in state 2, the number of layoffs will be greater than in state 1. For simplicity, I will, for now on, concentrate on the case where $n_1 = N$ and $n_2 = n < N$. It could be demonstrated that a pair of $\{k_1, k_2\}$ exists such that it is the case[16]. That simplification implies that equation (3) is no longer relevant and that we can set $\beta_{2i} = 0$. Thus, from equations (1)–(9), only three equations remain that determine the endogenous variables $\{w, n, \gamma\}$ as functions of the exogenous variables of interest[17] here $\{e, \delta\}$. I rewrite those equations for convenience:

$$\gamma u_1(w,1) - N = 0 \tag{10}$$

$$k_2 - (m+1)n - w + eb + Z = 0 \tag{11}$$

$$\lambda u(w,1) + (1-\lambda)\left[\frac{n}{N}u(w,1) + \frac{(N-n)}{N}u(b,0)\right] - RU = 0. \tag{12}$$

For compactness, equation (10) is substituted in equations (11) and (12). The following two equations now determine the endogenous variables $\{w, n\}$ as functions of the exogenous variables $\{e, \delta\}$:

16 See Burdett and Hool (1983) or BW for a similar simplifying assumption. Note that they assume that the prices [rather than state-dependent constants] are such that $n_1 = N$ and $n_2 = n < N$.

17 I pay no attention in this paper to the magnitude of the UI benefit b. In fact, most of the results that can be obtained with respect to b are usually ambiguous. On this, see Burdett and Hool (1983) or BW.

$$k_2 - (m+1)n - w + eb + \left[\frac{u(w,1) - u(b,0)}{u_1(w,1)}\right] = 0 \tag{13}$$

$$\lambda u(w,1) + (1-\lambda)\left[\frac{n}{N}u(w,1) + \frac{(N-n)}{N}u(b,0)\right] - RU = 0. \tag{14}$$

Let us now consider the two cases previously mentioned: no entry and free entry. It will be shown that in the no entry case, the conventional results hold i.e. an increase in experience-rating leads to less layoffs and to an increase in average employment. However, it will be shown that in the case of free entry, an increase in experience-rating has a negative impact on the number of firms that enter the industry. This effect, the *free entry effect*, will then lead to a decrease in *average industrial employment* [even if layoffs have decreased] under a rather weak sufficient condition. It should be noted that the free entry effect would be present even in the presence of the firm size effect[18]. As to the effect of a payroll tax, it is shown that average industrial employment decreases as a consequence of an increase of the payroll tax when there is free entry. This result is the same spirit as the one obtained for the experience-rating parameter.

2.1. *Cournot competition and no entry*[19]

If there is no entry, m is parametric and comparative static results can be obtained by simply differentiating equations (13) and (14). The endogenous variables being $\{n, w\}$, the following matrix system is obtained:

$$\begin{bmatrix} -(m+1) & rZ \\ (1-\lambda)Z & H \end{bmatrix} \begin{bmatrix} dn \\ dw \end{bmatrix} = \begin{bmatrix} -bde \\ 0 \end{bmatrix} \tag{15}$$

where the following notation has been introduced:

$$H = \lambda N + (1-\lambda)n > 0 \quad \text{and} \quad r = -\frac{u_{11}(w,1)}{u_1(w,1)} > 0$$

i.e. H is the average employment per firm and r is the coefficient of absolute risk aversion of the workers.

Denote by D the 2×2 matrix on the left-hand side of equation (15). Its determinant is:

[18] See footnote 6.

[19] Since there is no entry, the monopoly case is a special case of this section. To see the outcome in this situation, the reader simply has to go through the present section keeping in mind that the number of firms m is equal to one.

$$\left|D\right| = -H(m+1) - (1-\lambda)rZ^2 < 0.$$

Let us now consider the impact of a change in the experience-rating parameter e on the endogenous variables. The following comparative static results can be obtained:

$$\frac{\partial n}{\partial e} = -\frac{Hb}{\left|D\right|} > 0$$

$$\frac{\partial w}{\partial e} = \frac{(1-\lambda)bZ}{\left|D\right|} \quad \begin{matrix} < \\ > \end{matrix} \quad 0 \quad \text{as} \quad Z \quad \begin{matrix} > \\ < \end{matrix} \quad 0.$$

The results here obtained are conventional. The *standard* effect $\partial n/\partial e > 0$, i.e. the firm decreases the number of layoffs it makes in bad states of nature as it bears a heavier fraction of the cost of a layoff, has been recognized by Feldstein (1976) and Baily (1977) among others. Using that result, it has been argued previously that increasing e would then increase employment. Indeed, holding m (and N) constant, and defining the average level of layoffs in the industry by L and the average level of employment in the industry by M,

$$L = (1-\lambda)m(N-n)$$

$$M = \lambda mN + (1-\lambda)mn$$

it is then possible to obtain that:

$$\frac{\partial L}{\partial e} < 0 \quad \text{and} \quad \frac{\partial M}{\partial e} > 0.$$

Thus, average employment M increases because average layoffs L have decreased. As will be seen, this conventional view is misleading because it is based on a fixed level of m (and N) which, when $\partial n/\partial e > 0$, implies that if L is decreasing in e, then M is necessarily increasing in e.

Let us now turn to the impact of the payroll tax δ. Since δ appears nowhere in equation (15), it is then clear that:

$$\frac{\partial n}{\partial \delta} = \frac{\partial w}{\partial \delta} = 0.$$

This is again a conventional result. It will be seen that allowing for free entry leads to very different results.

Before turning to the case of free entry in the industry, it is interesting to note that the expected profits of a firm in this industry, for a fixed number of firms, will vary as the parameters of the UI scheme will be modified. To see this, denote by $E\Pi(e, \delta, m)$ the [expected] profit function. Then, by use of the envelope theorem:

$$\left.\frac{\partial E\Pi(e,\delta,m)}{\partial e}\right|_{m=\text{cst}} = -(1-\lambda)\left[(m-1)n\frac{\partial n}{\partial e} + (N-n)b\right] < 0$$

$$\left.\frac{\partial E\Pi(e,\delta,m)}{\partial \delta}\right|_{m=\text{cst}} = -N < 0.$$

Consequently, and since the entry decision of a firm is made on the basis of the expected profits, the number of firms that will enter [exit] the industry will vary as the tax parameters are changed. This is a motivation for the exercise performed in the following sub-section.

2.2. *Cournot competition and free entry*

If there is free entry in the industry, the number of firms m will be endogenously determined so that the expected profits of entering firms are zero:[20]

$$\begin{aligned} E\Pi =& \lambda\left[N(k_1 - mN) - Nw\right] \\ &+ (1-\lambda)\left[n(k_2 - mn) - nw - (N-n)eb\right] - \delta N - c = 0. \end{aligned} \tag{16}$$

Equation (16), along with equations (13) and (14), now determine the endogenous variables $\{n, m, w\}$. Differentiating those equations, the following matrix system is obtained:

$$E \begin{bmatrix} dn \\ dm \\ dw \end{bmatrix} = \begin{bmatrix} -bde \\ 0 \\ -(1-\lambda)(N-n)bde - Nd\delta \end{bmatrix} \tag{17}$$

where E, a 3×3 matrix, is the following:

[20] Note that this assumes a continuous number of firms. If the number of firms is a discrete number, the so-called "integer problem" may arise. This problem has been dealt with, for the case of Cournot-Nash equilibria, by Mankiw and Whinston (1986) and Novshek (1980). In this case, the equilibrium number of firms m^* will be given by the following two conditions rather than equation (16): $E\Pi(m^*) \geq 0$ and $E\Pi(m^*+1) < 0$.

$$\begin{bmatrix} -(m+1) & -n & rZ \\ (1-\lambda)Z & 0 & H \\ (1-\lambda)[Z+(m-1)n] & J & H \end{bmatrix}$$

and where $J = \lambda N^2 + (1-\lambda)n^2 > 0$.

The determinant of E is:

$$|E| = (1-\lambda)JrZ^2 + \lambda HN^2(m+1) + 2(1-\lambda)Hn^2 > 0.$$

The impact of a change in the experience-rating parameter on the endogenous variables is then given by the following:

$$\frac{\partial n}{\partial e} = \frac{H^2 Nb}{|E|} > 0$$

$$\frac{\partial m}{\partial e} = -\frac{(1-\lambda)b}{|E|}\left[(1-\lambda)(N-n)rZ^2 + H(N-n)(m+1) + Hn(m-1)\right] < 0$$

$$\frac{\partial w}{\partial e} = -\frac{(1-\lambda)HNbZ}{|E|} \quad \begin{matrix} < \\ > \end{matrix} \quad 0 \quad \text{as} \quad Z \quad \begin{matrix} > \\ < \end{matrix} \quad 0.$$

Thus, as in the no entry case, the standard effect $\partial n/\partial e > 0$ is obtained. But now, a free entry effect $\partial m/\partial e < 0$ is also present. Thus, an increase in the experience-rating parameter will decrease the number of layoffs per firm but will as well decrease the number of firms. This last effect is understood to be a consequence of the decrease in profits associated with the increase in experience-rating. It can be seen that, once again, the average level of layoffs in the industry, L, will decrease as a result of a higher level of experience-rating:

$$\frac{\partial L}{\partial e} = (1-\lambda)(N-n)\underbrace{\frac{\partial m}{\partial e}}_{<0} - (1-\lambda)m\underbrace{\frac{\partial n}{\partial e}}_{>0} < 0.$$

But now, the overall impact on average employment M is ambiguous:

$$\frac{\partial M}{\partial e} = \frac{(1-\lambda)Hb}{|E|}\left[-(1-\lambda)(N-n)rZ^2 - HN + 2Hn\right] \quad \gtrless 0$$

Moreover a [weak] *sufficient* condition for $\partial M/\partial e < 0$ is simply that n is sufficiently *small* i.e. $n < \frac{N}{2}$. This sufficient condition will be satisfied if k_2 is itself *small* enough. To see this, simply note that, from equations (13), (14), and (16):

$$\frac{\partial n}{\partial k_2} = \frac{JH + (1-\lambda)Hn^2}{|E|} > 0.$$

Thus, for smaller and smaller k_2, the chances are that the sufficient condition will be satisfied. Obviously, as long as k_2 is not bounded from below, there exists a k_2 such that the sufficient condition is satisfied since ultimately, n goes to 0 as k_2 goes to 0.

It has thus been demonstrated that because of the free entry effect, increasing the experience-rating parameter can be harmful to employment. This results is in the same vein as the one obtained by BW when they considered the firm size effect. Note that the fact that there is exit from the industry when the experience-rating increases means that, at least in the short run, the newly unattached workers will join the permanently unemployed. In the short run, thus, the unemployment rate will increase. In the long run, those workers will reallocate themselves in other industries so that the adverse effects of the experience-rating , described above, will vanish.

Now consider the impact of a change in the payroll tax on the endogenous variables:

$$\frac{\partial n}{\partial \delta} = \frac{HNn}{|E|} > 0$$

$$\frac{\partial m}{\partial \delta} = \frac{-N[(m+1)H + (1-\lambda)rZ^2]}{|E|} < 0$$

$$\frac{\partial w}{\partial \delta} = -\frac{(1-\lambda)NnZ}{|E|} \quad \begin{matrix} < \\ > \end{matrix} \quad 0 \quad \text{as} \quad Z \quad \begin{matrix} > \\ < \end{matrix} \quad 0.$$

The first result obtained is that $\partial n/\partial \delta > 0$. This is explained by the fact that, in this framework, the size of the firms is identified with their capacity and it becomes more and more costly to have unused capacity when the payroll tax increases. Consequently, layoffs will decrease in bad times. A second result obtained here is that, as for the experience-rating parameter, the number of firms decreases as the payroll tax is increased: $\partial m/\partial \delta < 0$. Consequently, even if average layoffs decrease, average industrial employment falls:

$$\frac{\partial L}{\partial \delta} = (1-\lambda)(N-n)\underbrace{\frac{\partial m}{\partial \delta}}_{<0} - (1-\lambda)m\underbrace{\frac{\partial n}{\partial \delta}}_{>0} < 0$$

$$\frac{\partial M}{\partial \delta} = -\frac{(1-\lambda)HNn + \lambda HN^2(m+1) + (1-\lambda)HNrZ^2}{|E|} < 0.$$

Note that these results are in contrast with the previous literature. The payroll taxes here have an effect that could not be obtained with an exogenous labour force N and a fixed number of firms m. The free entry effect is then a mechanism through which making firms bear more of their unemployment costs can lead to a decrease in average industrial employment.

3. Perfect competition

In this section, a model similar to the one of section 2 is presented but, here, it is assumed that firms are price-takers on the output market and that their size N is endogenous. Because the firms are price-takers, there will be GE effects on prices[21] that will mitigate the results obtained in previous work on the subject.

Consider the following model. Assume that the demand for output in state s is:[22]

$$Q_s = Q_s(p_s)$$

where $Q_s' < 0$ and $Q_1(p) > Q_2(p)$, i.e. state 1 is the good one.

Also assume that the technology[23] exhibits decreasing return to scales so that the output of firm i, q_{si}, is given by:

$$q_{si} = F(n_{si})$$

[21] This is in contrast with previous work where the price level was determined *outside* the model. In other words, previous work has considered the impact of a local UI scheme on the local employment [and unemployment] level when the observed prices are *international* prices.

[22] The demand is more general than the one used in the previous section because here, there would be no benefits [as will be seen later] to a simple functional form.

[23] Decreasing return to scales are here necessary to insure that the objective function is concave.

where $F' > 0$, $F'' < 0$. As before, there are m identical firms in an industry and both the cases of no entry [m exogenous] and free entry [m endogenous] will be considered. For simplicity, it is assumed that the observed prices $\{p_1, p_2\}$ are such that $n_{1i} = N_i$ and $n_{2i} = n_i < N_i$. Since the firms are identical [and there are no interactions between the firms other than at the general equilibrium level], the subscripts i are now dropped. The workers are exactly the same.those described in the previous section. Finally, although it could be done, no attention is paid, in the following, to the payroll tax as a mean of financing the UI scheme.

An optimal contract is a set $\{N, n, w_1, w_2\}$ that solves the following problem, where the pair of prices $\{p_1, p_2\}$ is taken as given:

$$\max_{\{N, n, w_1, w_2\}} E\Pi \tag{P2}$$

subject to

$$EU \geq RU$$

where:

$$E\Pi = \lambda \left[p_1 F(N) - N w_1\right] + (1-\lambda)\left[p_2 F(n) - n w_2 - eb(N-n)\right] - c$$

and

$$EU = \lambda u(w_1, 1) + (1-\lambda)\left[\frac{n}{N} u(w_2, 1) + \frac{(N-n)}{N} u(b, 0)\right].$$

The first order conditions of problem $P2$ are[24]:

$$-\lambda N + \gamma \lambda u_1(w_1, 1) = 0 \tag{17}$$

$$-(1-\lambda) n + \gamma (1-\lambda) \frac{n}{N} u_1(w_2, 1) = 0 \tag{18}$$

$$\lambda p_1 F'(N) - \lambda w_1 - (1-\lambda) eb - \frac{\gamma (1-\lambda) n}{N^2}\left[u(w_2, 1) - u(b, 0)\right] = 0 \tag{19}$$

$$p_2 F'(n) - w_2 + eb + \frac{\gamma}{N}\left[u(w_2, 1) - u(b, 0)\right] = 0 \tag{20}$$

[24] The second order conditions of this problem are satisfied. See footnote 13.

$$EU - RU = 0. \tag{21}$$

As in section 2, by use of equations (17) and (18), it can be shown that $w_1 = w_2 = w$. Thus, manipulations of the above equations yield the following three equations determining the endogenous variables $\{N, n, w\}$:

$$\lambda p_1 F'(N) - \lambda w - (1-\lambda)eb - \frac{(1-\lambda)n}{N}\left[\frac{u(w,1)-u(b,0)}{u_1(w,1)}\right] = 0 \tag{22}$$

$$p_2 F'(n) - w + eb + \left[\frac{u(w,1)-u(b,0)}{u_1(w,1)}\right] = 0 \tag{23}$$

$$\lambda u(w,1) + (1-\lambda)\left[\frac{n}{N}u(w,1) + \frac{(N-n)}{N}u(b,0)\right] - RU = 0. \tag{24}$$

Totally differentiating equations (22)–(24), the following matrix system is obtained:

$$\begin{bmatrix} \lambda p_1 F''(N) + \frac{(1-\lambda)nZ}{N^2} & -\frac{(1-\lambda)Z}{N} & -\frac{H+(1-\lambda)nrZ}{N} \\ 0 & p_2 F''(n) & rZ \\ -\frac{(1-\lambda)nZ}{N} & (1-\lambda)Z & H \end{bmatrix} \begin{bmatrix} dN \\ dn \\ dw \end{bmatrix}$$

$$= \begin{bmatrix} (1-\lambda)bde - \lambda F'(N)dp_1 \\ -bde - F'(n)dp_2 \\ 0 \end{bmatrix}. \tag{25}$$

Denote by R the 3×3 matrix on the left-hand side of equation (25). Its determinant is:

$$|R| = \lambda p_1 F''(N) p_2 F''(n) H - \frac{(1-\lambda)rZ^2}{N^2}\left[\lambda p_1 F''(N) N^2 + (1-\lambda) p_2 F''(n) n^2\right] > 0.$$

It is then possible to obtain the following functions:

$$N = N(p_1, p_2, e) \tag{26}$$

$$n = n(p_1, p_2, e) \tag{27}$$

$$w = w(p_1, p_2, e) \tag{28}$$

where[25]:

$$\frac{\partial N}{\partial p_1} > 0; \ \frac{\partial N}{\partial p_2} > 0; \ \frac{\partial N}{\partial e} < 0$$

$$\frac{\partial n}{\partial p_1} > 0; \ \frac{\partial n}{\partial p_2} > 0; \ \frac{\partial n}{\partial e} \gtrless 0$$

$$\frac{\partial w}{\partial p_1} \gtrless 0; \ \frac{\partial w}{\partial p_2} \gtrless 0; \ \frac{\partial w}{\partial e} \gtrless 0$$

Note that the BW results are here reproduced:

$$\frac{\partial n}{\partial e} \gtrless 0 \quad \text{and} \quad \frac{\partial N}{\partial e} < 0.$$

The conclusion one could reach with those partial equilibrium results can however be misleading as will now be shown. Keeping in mind equations (26)–(28), let us now investigate the no entry equilibrium.

3.1 Perfect competition and no entry

Let the number of firms in the industry above described be fixed at m. Then, the pair of prices $\{p_1, p_2\}$ will adjust so that in equilibrium:

$$mq_1 = Q_1(p_1) \quad \text{and} \quad mq_2 = Q_2(p_2).$$

That is, for each state of nature, supply must equal demand in equilibrium. Substituting for q_1 and q_2 yields the following two equilibrium conditions:

$$mF(N(p_1, p_2, e)) = Q_1(p_1) \tag{29}$$

$$mF(n(p_1, p_2, e)) = Q_2(p_2). \tag{30}$$

The impact of a change in the experience-rating parameter on the pair of prices can now be obtained by differentiation of (29) and (30) leading to the following matrix system:

$$\begin{bmatrix} mF'(N)\frac{\partial N}{\partial p_1} - Q_1' & mF'(N)\frac{\partial N}{\partial p_2} \\ mF'(n)\frac{\partial n}{\partial p_1} & mF'(n)\frac{\partial n}{\partial p_2} - Q_2' \end{bmatrix} \begin{bmatrix} dp_1 \\ dp_2 \end{bmatrix} = \begin{bmatrix} -mF'(N)\frac{\partial N}{\partial e}de \\ -mF'(n)\frac{\partial n}{\partial e}de \end{bmatrix}. \tag{31}$$

[25] See Appendix D where those expressions are fully written.

Denote by S the determinant of the 2×2 matrix on the left-hand side of equation (31):

$$\begin{aligned}\left|S\right| =& m^2 F'(N)F'(n)\left[\frac{\partial N}{\partial p_1}\frac{\partial n}{\partial p_2} - \frac{\partial N}{\partial p_2}\frac{\partial n}{\partial p_1}\right] \\ &- mF'(N)\frac{\partial N}{\partial p_1}Q_2' - mF'(n)\frac{\partial n}{\partial p_2}Q_1' + Q_1'Q_2' > 0\end{aligned}$$

where[26]:

$$\left[\frac{\partial N}{\partial p_1}\frac{\partial n}{\partial p_2} - \frac{\partial N}{\partial p_2}\frac{\partial n}{\partial p_1}\right] > 0.$$

The following *GE effects* on prices can then be obtained:

$$\frac{\partial p_1}{\partial e} = \frac{m^2 F'(N)F'(n)\left[\frac{\partial n}{\partial e}\frac{\partial N}{\partial p_2} - \frac{\partial N}{\partial e}\frac{\partial n}{\partial p_2}\right] + mF'(N)\frac{\partial N}{\partial e}Q_2'}{\left|S\right|} > 0$$

$$\frac{\partial p_2}{\partial e} = \frac{m^2 F'(N)F'(n)\left[\frac{\partial n}{\partial p_1}\frac{\partial N}{\partial e} - \frac{\partial N}{\partial p_1}\frac{\partial n}{\partial e}\right] + mF'(n)\frac{\partial n}{\partial e}Q_1'}{\left|S\right|} \gtrless 0$$

where the use has been made of equations (26) and (27) to obtain that[27]:

$$\left[\frac{\partial n}{\partial e}\frac{\partial N}{\partial p_2} - \frac{\partial N}{\partial e}\frac{\partial n}{\partial p_2}\right] > 0$$

$$\left[\frac{\partial n}{\partial p_1}\frac{\partial N}{\partial e} - \frac{\partial N}{\partial p_1}\frac{\partial n}{\partial e}\right] < 0.$$

The above GE effects of prices are to be combined to the direct effects to obtain the *final* impact of an increase of e on both N and n:

$$\frac{dN}{de} = \frac{\partial N}{\partial p_1}\frac{\partial p_1}{\partial e} + \frac{\partial N}{\partial p_2}\frac{\partial p_2}{\partial e} + \frac{\partial N}{\partial e} \quad \gtrless 0$$

$$\frac{dn}{de} = \frac{\partial n}{\partial p_1}\frac{\partial p_1}{\partial e} + \frac{\partial n}{\partial p_2}\frac{\partial p_2}{\partial e} + \frac{\partial n}{\partial e} \quad \gtrless 0$$

26 Again, see Appendix D.
27 See Appendix D.

Thus, because of the GE effects, the overall effects are ambiguous[28] even if the direct effects are well-known. Consequently, arguing that an increase in the experience-rating would decrease average layoffs L, increase M as in the traditional view, or decrease M as in BW, is, in this context, impossible. To argue such things, one has to abstract from the GE effects and thus to assume that the observed prices are exogenous.

Let us now turn to the case of free entry. The analysis will lead to results similar in spirit to that described in this sub-section.

3.2 Perfect competition and free entry

Using equations (26)–(28), it is possible to write an [expected] profit function:

$$\begin{aligned} E\Pi(p_1, p_2, e) = & \lambda \left[p_1 F(N(\cdot)) - N(\cdot)w(\cdot)\right] \\ & + (1-\lambda)\left[p_2 F(n(\cdot)) - n(\cdot)w(\cdot) - eb(N(\cdot) - n(\cdot))\right] - c. \end{aligned}$$

If there is free entry, expected profits will be driven down [brought up] to zero by the entry of new firms [exit of old firms]. An equilibrium condition under free entry is thus:

$$E\Pi(p_1, p_2, e) = 0. \tag{32}$$

Also, as the number of firms adjusts, the pair of prices $\{p_1, p_2\}$ will vary. Let m denote the free entry equilibrium number of firms, then the two following equations must hold simultaneously:

$$mF(N(p_1, p_2, e)) = Q_1(p_1) \tag{33}$$

$$mF(n(p_1, p_2, e)) = Q_2(p_2). \tag{34}$$

This is equivalent to saying that the prices have to vary according to the following equation:

$$\frac{F(N(p_1, p_2, e))}{F(n(p_1, p_2, e))} = \frac{Q_1(p_1)}{Q_2(p_2)}. \tag{35}$$

[28] This is even true for very simple functional forms for the demand and the technology.

Thus, equations (32) and (35) characterize the free entry equilibrium prices. Given the optimal pair of prices, it is then possible to obtain the equilibrium levels of N and n via the functions described in equations (26) and (27). Then, using (33) or (34), the equilibrium number of firms can be obtained.

It is now shown that, as in the previous sub-section, the GE effects make it impossible to assess the sign of dN/de or dn/de even if $\partial N/\partial e$ or $\partial n/\partial e$ are known.

Total differentiation of equations (32) and (35) yield the following matrix system:

$$\begin{bmatrix} \lambda F(N) & (1-\lambda)F(n) \\ C_1 & C_2 \end{bmatrix} \begin{bmatrix} dp_1 \\ dp_2 \end{bmatrix} = \begin{bmatrix} (1-\lambda)(N-n)bde \\ \left(Q_1 F'(n)\frac{\partial n}{\partial e} + Q_2 F'(N)\frac{\partial N}{\partial e}\right) de \end{bmatrix} \quad (36)$$

where both C_1 and C_2 are of ambiguous signs:

$$C_1 = Q_2 F'(N)\frac{\partial N}{\partial p_1} - Q_1 F'(n)\frac{\partial n}{\partial p_1} - Q_1' F'(n) \quad \gtrless 0$$

$$C_2 = Q_2 F'(N)\frac{\partial N}{\partial p_2} - Q_1 F'(n)\frac{\partial n}{\partial p_2} + Q_2' F'(N) \quad \gtrless 0$$

Then, because of the ambiguity of C_1 and C_2, the 2×2 matrix on the left-hand side of equation (36) has a determinant of ambiguous sign. This implies that the signs of the GE effects are also ambiguous: $\partial p_1/\partial e \gtrless 0$ and $\partial p_2/\partial e \gtrless 0$. Obviously then:

$$\frac{dN}{de} = \frac{\partial N}{\partial p_1}\frac{\partial p_1}{\partial e} + \frac{\partial N}{\partial p_2}\frac{\partial p_2}{\partial e} + \frac{\partial N}{\partial e} \quad \gtrless 0$$

$$\frac{dn}{de} = \frac{\partial n}{\partial p_1}\frac{\partial p_1}{\partial e} + \frac{\partial n}{\partial p_2}\frac{\partial p_2}{\partial e} + \frac{\partial n}{\partial e} \quad \gtrless 0$$

This implies that no conclusion can be reached as to what happen to the equilibrium number of firms and consequently, to the average level of layoffs or of average employment.

4. Conclusion

This paper has examined the impact of UI on employment and unemployment. The main contribution of this paper has been to consider the case where the prices in the economy can vary.

It has been shown that under Cournot competition and free entry, increasing the taxes used to finance the UI scheme would decrease average industrial employment under a weak sufficient condition. This result is in the same spirit as the one obtained by BW when they considered the firm size effect. In fact, combining the results of BW's paper with those described here, it can be said that if the experience-rating [or the payroll tax] is increased, the number as well as the size of the firms might be decreased. There are thus mechanisms through which employment can fall if the firms have to bear a heavier fraction of their unemployment costs. This conclusion contradicts the conventional view in that a decrease in unemployment is not necessarily associated with an increase in employment.

Considering perfect competition with GE effects on prices [rather than sticky international prices as in the rest of the literature], it has been shown that almost any results could be obtained even in the presence of both the free entry and the firm size effects. This kind of result is not surprising —it is common to see partial equilibrium results mitigated by the introduction of GE effects.

In conclusion, this paper has demonstrated that as soon as prices are determined endogenously, the UI scheme has an impact on the determination of the prices and the profitability of the firms. Thus, for industries or economies with different degrees of market power, the same UI scheme may well have drastically different impacts.

References

Azariadis, C. (1975), "Implicit Contracts and Underemployment Equilibria", *Journal of Political Economy* **83**, 1183–1202.

Baily, M.N. (1974), "Wages and Employment Under Uncertain Demand", *Review of Economic Studies* **41**, 35–50.

Baily, M.N. (1977), "On the Theory of Layoffs and Unemployment", *Econometrica* **45**, 1043–1063.

Burdett, K. and B. Hool (1983), "Layoffs, Wages, and Unemployment Insurance", *Journal of Public Economics* **21**, 325–357.

Burdett, K. and R. Wright (1989a), "Unemployment Insurance and Short-Time Compensation: The Effects on Layoffs, Hours per Worker, and Wages", *Journal of Political Economy* **97**, 1479–1496.

Burdett, K. and R. Wright (1989b), "Optimal Firm Size, Taxes, and Unemployment", *Journal of Public Economics* **39**, 275–287.

Chari, V.V., L.E. Jones, and R.E. Manuelli (1989), "Labor Contracts in a Model of Imperfect Competition", *American Economic Review: Papers and Proceedings* **79**, 358–363.

Cooper, R. (1990), "Optimal Labour Contracts and Imperfect Competition: A Framework for Analysis", *Canadian Journal of Economics* **23**, 509–522.

Feldstein, M. (1976), "Temporary Layoffs in the Theory of Unemployment", *Journal of Political Economy* **84**, 937–957.

Gordon, D.F. (1974), "A Neoclassical Theory of Keynesian Unemployment", *Economic Inquiry* **12**, 431–459.

Mankiw, N.G. and M.D. Whinston (1986), "Free Entry and Social Inefficiency", *Rand Journal of Economics* **17**, 48–58.

Mortensen D.T. (1983), "A Welfare Analysis of Unemployment Insurance: Variations on Second-Best Themes", *Carnegie-Rochester Series on Public Policy* **19**, 67–98.

Novshek, W. (1980), "Cournot Equilibrium with Free Entry", *Review of Economic Studies* **47**, 473–486.

Topel, R. and F. Welch (1980), "Unemployment Insurance: Survey and Extension", *Economica* **47**, 351–379.

Appendix A: Second order conditions

The second order [sufficient] conditions of problem $P1$ are now derived. By use of the FOCs given by equations (1)–(6) and from the definition of the Lagrangian Ω and of the constraints, the following bordered Hessian matrix is now obtained:

$$\overline{H} = \begin{bmatrix} 0 & 0 & 0 & -\beta_{1i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\beta_{2i} & 0 & 0 \\ 0 & 0 & 0 & EU_1 & EU_2 & EU_3 & EU_4 \\ -\beta_{1i} & 0 & EU_1 & \Omega_{11} & 0 & 0 & 0 \\ 0 & -\beta_{2i} & EU_2 & 0 & \Omega_{22} & 0 & 0 \\ 0 & 0 & EU_3 & 0 & 0 & \Omega_{33} & 0 \\ 0 & 0 & EU_4 & 0 & 0 & 0 & \Omega_{44} \end{bmatrix}$$

where:

$$\Omega_{11} = -2\lambda < 0$$

$$\Omega_{22} = -2(1-\lambda) < 0$$

$$\Omega_{33} = \gamma_i \lambda \frac{n_{1i}}{N} u_{11}(w_{1i}, 1) < 0$$

$$\Omega_{44} = \gamma_i (1-\lambda) \frac{n_{2i}}{N} u_{11}(w_{2i}, 1) < 0$$

$$EU_1 = \frac{\lambda}{N}[u(w_{1i}, 1) - u(b, 0)]$$

$$EU_2 = \frac{(1-\lambda)}{N}[u(w_{2i}, 1) - u(b, 0)]$$

$$EU_3 = \lambda \frac{n_{1i}}{N} u_1(w_{1i}, 1) > 0$$

$$EU_4 = (1-\lambda) \frac{n_{2i}}{N} u_1(w_{2i}, 1) > 0.$$

The [sufficient] second order condition for a maximum is that $\overline{H}$ be negative definite[29]. This is equivalent to requiring that the sign of the bordered principal minors vary in sign according to:

[29] For a detailed discussion of second order conditions, see CHIANG, A.C. (1984), *Fundamental Methods of Mathematical Economics*, 3rd ed., USA, McGraw Hill, or TAKAYAMA, A. (1985), *Mathematical Economics*, 2nd ed., New York, Cambridge University Press.

$$
|\overline{H}_2| = \begin{vmatrix} 0 & 0 & 0 & -\beta_{1i} & 0 \\ 0 & 0 & 0 & 0 & -\beta_{2i} \\ 0 & 0 & 0 & EU_1 & EU_2 \\ -\beta_{1i} & 0 & EU_1 & \Omega_{11} & 0 \\ 0 & -\beta_{2i} & EU_2 & 0 & \Omega_{22} \end{vmatrix} \geq 0
$$

$$
|\overline{H}_3| = \begin{vmatrix} 0 & 0 & 0 & -\beta_{1i} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\beta_{2i} & 0 \\ 0 & 0 & 0 & EU_1 & EU_2 & EU_3 \\ -\beta_{1i} & 0 & EU_1 & \Omega_{11} & 0 & 0 \\ 0 & -\beta_{2i} & EU_2 & 0 & \Omega_{22} & 0 \\ 0 & 0 & EU_3 & 0 & 0 & \Omega_{33} \end{vmatrix} \leq 0
$$

$$
|\overline{H}_4| = |\overline{H}| \geq 0.
$$

Routine calculations yield that:

$$
|\overline{H}_2| = 0 \geq 0
$$

$$
|\overline{H}_3| = -\beta_{1i}^2 \beta_{2i}^2 EU_3^2 \leq 0
$$

$$
|\overline{H}_4| = -\beta_{1i}^2 \beta_{2i}^2 [\Omega_{44} EU_3^2 + \Omega_{33} EU_4^2] \geq 0.
$$

Thus, the second order conditions are satisfied.

Appendix B: Employment is a strategic substitute

It is here shown that for problem $P1$ described in part 2 of the paper, employment is a strategic substitute.

Recall that it has been assumed that $\beta_{1i} = \beta_{2i} = 0$. The following equations are thus obtained from the FOCs (1)–(6):

$$k_1 - 2n_{1i} - \sum_{j \neq i}^{m} \hat{n}_{1j} - w_i + eb + \left[\frac{u(w_i, 1) - u(b, 0)}{u_1(w_i, 1)} \right] = 0 \qquad (B1)$$

$$k_2 - 2n_{2i} - \sum_{j \neq i}^{m} \hat{n}_{2j} - w_i + eb + \left[\frac{u(w_i, 1) - u(b, 0)}{u_1(w_i, 1)} \right] = 0 \qquad (B2)$$

$$\begin{aligned} &\lambda \left[\frac{n_{1i}}{N} u(w_i, 1) + \frac{(N - n_{1i})}{N} u(b, 0) \right] \\ &+ (1 - \lambda) \left[\frac{n_{2i}}{N} u(w_i, 1) + \frac{(N - n_{2i})}{N} u(b, 0) \right] - RU = 0. \end{aligned} \qquad (B3)$$

Totally differentiating equations $(B1)$–$(B3)$ and rearranging, the following is obtained:

$$D^* \begin{bmatrix} dn_{1i} \\ dn_{2i} \\ dw_i \end{bmatrix} = \begin{bmatrix} \sum_{j \neq i}^{m} d\hat{n}_{1j} \\ \sum_{j \neq i}^{m} d\hat{n}_{2j} \\ 0 \end{bmatrix} \qquad (B4)$$

where:

$$D^* = \begin{bmatrix} -2 & 0 & rZ \\ 0 & -2 & rZ \\ \lambda Z & (1 - \lambda) Z & H^* \end{bmatrix}$$

and

$$r = -\frac{u_{11}(w_i, 1)}{u_1(w_i, 1)} > 0 \quad \text{and} \quad H^* = \frac{\lambda n_{1i} + (1 - \lambda) n_{2i}}{N} > 0.$$

The determinant of D^* is positive:

$$|D^*| = 4H^* + 2rZ^2 > 0.$$

Then, using $(B4)$, the following is obtained:

$$\frac{\partial n_{1i}}{\partial n_{1j}} = -\frac{2H^* + (1-\lambda)rZ^2}{|D^*|} < 0 \quad \forall i \neq j$$

$$\frac{\partial n_{2i}}{\partial n_{2j}} = -\frac{2H^* + \lambda rZ^2}{|D^*|} < 0 \quad \forall i \neq j.$$

And thus, employment is a strategic substitute.

Appendix C: Cournot competition and endogenous firm size

Consider the following economy, almost identical to the one described in part 2 of this paper. There are $m > 1$ identical, risk-neutral firms in an industry but here, each has an endogenous number of workers under contract, N. The notation, nature [uncertainty], demand, and technology are the same as before.

The expected profits of firm i are, given the decisions $\{N_i, n_{si}, w_{si};\ s = 1,2\}$:

$$
\begin{aligned}
E\Pi^i =& \lambda \left[n_{1i} \left(k_1 - n_{1i} - \sum_{j \neq i}^{m} \hat{n}_{1j} \right) - n_{1i} w_{1i} - eb(N_i - n_{1i}) \right] \\
&+ (1-\lambda) \left[n_{2i} \left(k_2 - n_{2i} - \sum_{j \neq i}^{m} \hat{n}_{2j} \right) - n_{2i} w_{2i} - eb(N_i - n_{2i}) \right] \\
&- \delta N_i - c.
\end{aligned}
$$

A worker joining firm i, given the set of decisions $\{N_i, n_{si}, w_{si};\ s = 1,2\}$, will have expected utility:

$$
\begin{aligned}
EU^i =& \lambda \left[\frac{n_{1i}}{N_i} u(w_{1i}, 1) + \frac{(N_i - n_{1i})}{N_i} u(b, 0) \right] \\
& (1-\lambda) \left[\frac{n_{2i}}{N_i} u(w_{2i}, 1) + \frac{(N_i - n_{2i})}{N_i} u(b, 0) \right].
\end{aligned}
$$

Assuming that the workers could obtain reservation utility RU elsewhere in the economy, an optimal contract between firm i and its workers is a set of decisions $\{N_i, n_{si}, w_{si};\ s = 1,2\}$ that solves the following problem:

$$
\max_{\{N_i, n_{si}, w_{si}\}_{s=1,2}} E\Pi^i \qquad (P3)
$$

subject to

$$
EU^i \geq RU
$$

$$
n_{si} \leq N_i \quad s = 1,2.
$$

The Lagrangian of problem $P3$ is written, using γ_i, β_{1i}, and β_{2i} as Lagrange multipliers:

$$
\Omega = E\Pi^i + \gamma_i(EU^i - RU) + \beta_{1i}(N_i - n_{1i}) + \beta_{2i}(N_i - n_{2i}).
$$

The first order conditions of this maximization problem are:

$$-\lambda n_{1i} + \gamma_i \lambda \frac{n_{1i}}{N_i} u_1(w_{1i}, 1) = 0 \tag{$C1$}$$

$$-(1-\lambda) n_{2i} + \gamma_i (1-\lambda) \frac{n_{2i}}{N_i} u_1(w_{2i}, 1) = 0 \tag{$C2$}$$

$$-eb - \delta - \frac{\gamma_i \lambda n_{1i}}{N_i^2}[u(w_{1i}, 1) - u(b, 0)] - \frac{\gamma_i (1-\lambda) n_{2i}}{N_i^2}[u(w_{2i}, 1) - u(b, 0)] + \beta_{1i} + \beta_{2i} = 0 \tag{$C3$}$$

$$\lambda \left[k_1 - 2n_{1i} - \sum_{j \neq i}^{m} \hat{n}_{1j} - w_{1i} + eb + \frac{\gamma_i}{N_i}[u(w_{1i}, 1) - u(b, 0)] \right] - \beta_{1i} = 0 \tag{$C4$}$$

$$(1-\lambda) \left[k_2 - 2n_{2i} - \sum_{j \neq i}^{m} \hat{n}_{2j} - w_{2i} + eb + \frac{\gamma_i}{N_i}[u(w_{2i}, 1) - u(b, 0)] \right] - \beta_{2i} = 0 \tag{$C5$}$$

$$EU^i - RU = 0 \tag{$C6$}$$

$$\begin{aligned} N_i - n_{si} &\geq 0 \quad s = 1, 2 \\ \beta_{si} &\geq 0 \quad s = 1, 2 \\ \beta_{si}(N_i - n_{si}) &= 0 \quad s = 1, 2. \end{aligned} \tag{$C7$}$$

Again, the second order conditions of this problem are satisfied. See Appendix A for the derivations of these for problem $P1$ which is similar to this problem.

Using equations ($C1$) and ($C2$), the following is obtained:

$$w_{1i} = w_{2i} = w_i \qquad \text{and} \qquad \gamma_i u_1(w_i, 1) - N_i = 0. \tag{$C8$}$$

Thus, the income offered by firm i to its employed workers is state-independent. Now consider equations ($C4$) and ($C5$) and let $\beta_{1i} = \beta_{2i} = 0$, i.e. the constraints on the employment levels are not binding; they then imply, respectively, that:

$$k_1 - 2n_{1i} - \sum_{j \neq i}^{m} \hat{n}_{1j} - w_i + eb + Z = 0 \tag{$C9$}$$

$$k_2 - 2n_{2i} - \sum_{j \neq i}^{m} \hat{n}_{2j} - w_i + eb + Z = 0. \tag{$C10$}$$

Using the first order conditions, one can get well-defined reaction curves for the endogenous variables $\{w_i, N_i, n_{1i}, n_{2i}, \gamma_i, \beta_{1i}, \beta_{2i}\}$ as functions of the other firms' employment level in the industry: $\{\hat{n}_{1j}, \hat{n}_{2j};\ j \neq i\}$. Now, because all firms in the industry are identical, the equilibrium obtained here is a *symmetric Nash equilibrium*:

$$\begin{gathered}
N_i = N_j = N \quad \forall i, j \\
n_{1i} = n_{1j} = n_1 \quad \forall i, j \\
n_{2i} = n_{2j} = n_2 \quad \forall i, j \\
w_i = w_j = w \quad \forall i, j \\
\gamma_i = \gamma_j = \gamma \quad \forall i, j \\
\beta_{si} = \beta_{sj} = \beta_s \quad \forall i, j;\ s = 1, 2.
\end{gathered}$$

Then, using equations ($C9$) and ($C10$), it is possible to obtain:

$$k_1 - (m+1)n_1 = k_2 - (m+1)n_2.$$

This implies that $n_1 > n_2$ since $k_1 > k_2$. For simplicity, I will, for now on, concentrate on the case where $n_1 = N$ and $n_2 = n < N$. That simplification implies that equations ($C3$) and ($C4$) can be combined through β_{1i} and that we can set $\beta_{2i} = 0$. Thus, from equations ($C1$)–($C10$), only four equations remain that determine the endogenous variables $\{w, N, n, \gamma\}$ as functions of the exogenous variables of interest here $\{e, \delta\}$. I rewrite those equations for convenience:

$$-(1-\lambda)eb - \delta + \lambda[k_1 - (m+1)N] - \lambda w - \frac{(1-\lambda)\gamma n}{N^2}[u(w,1) - u(b,0)] = 0 \tag{$C11$}$$

$$k_2 - (m+1)n - w + eb + \frac{\gamma}{N}[u(w,1) - u(b,0)] = 0 \tag{$C12$}$$

$$\gamma u_1(w,1) - N = 0 \tag{C13}$$

$$\lambda u(w,1) + (1-\lambda)\left[\frac{n}{N}u(w,1) + \frac{(N-n)}{N}u(b,0)\right] - RU = 0. \tag{C14}$$

For compactness, equation $(C13)$ is substituted in equations $(C11)$, $(C12)$, and $(C14)$. The following three equations now determine $\{w, N, n\}$ as functions of $\{e, \delta\}$:

$$-(1-\lambda)eb - \delta + \lambda[k_1 - (m+1)N] - \lambda w - \frac{(1-\lambda)n}{N}\left[\frac{u(w,1)-u(b,0)}{u_1(w,1)}\right] = 0 \tag{C15}$$

$$k_2 - (m+1)n - w + eb + \left[\frac{u(w,1)-u(b,0)}{u_1(w,1)}\right] = 0 \tag{C16}$$

$$\lambda u(w,1) + (1-\lambda)\left[\frac{n}{N}u(w,1) + \frac{(N-n)}{N}u(b,0)\right] - RU = 0. \tag{C17}$$

C.1. *No entry*

If there is no entry, m is parametric and comparative static results can be obtained by differentiating equations $(C15)$–$(C17)$. The endogenous variables being $\{N, n, w\}$, the following matrix system is obtained:

$$\begin{bmatrix} -\lambda(m+1) + \frac{(1-\lambda)nZ}{N^2} & -\frac{(1-\lambda)Z}{N} & -\frac{H+(1-\lambda)nrZ}{N} \\ 0 & -(m+1) & rZ \\ -\frac{(1-\lambda)nZ}{N} & (1-\lambda)Z & H \end{bmatrix} \begin{bmatrix} dN \\ dn \\ dw \end{bmatrix} = \begin{bmatrix} (1-\lambda)bde + d\delta \\ -bde \\ 0 \end{bmatrix}. \tag{C18}$$

Let F denote the 3×3 matrix on the left-hand side of equation $(C18)$. Its determinant is:

$$|F| = \lambda H(m+1)^2 + \frac{(1-\lambda)J(m+1)rZ^2}{N^2} > 0.$$

Let us now consider the impact of a change in the experience-rating parameter e on the endogenous variables. The following comparative static results can be obtained:

$$\frac{\partial N}{\partial e} = -(1-\lambda)b\left[\frac{H(m+1)+(1-\lambda)(1-\frac{n}{N})rZ^2}{|F|}\right] < 0$$

$$\frac{\partial n}{\partial e} = b\left[\frac{\lambda H(m+1) - \frac{(1-\lambda)^2(N-n)nrZ^2}{N^2}}{|F|}\right] \gtrless 0$$

$$\frac{\partial w}{\partial e} = -\frac{(1-\lambda)H(m+1)bZ}{N|F|} \quad \begin{matrix}<\\>\end{matrix} \quad 0 \quad \text{as} \quad Z \quad \begin{matrix}>\\<\end{matrix} \quad 0.$$

These are the BW results; they imply that:

$$\frac{\partial L}{\partial e} = -(1-\lambda)mb\left[\frac{(m+1)H+(1-\lambda)^2(1-\frac{n}{N})^2rZ^2}{|F|}\right] < 0$$

$$\frac{\partial M}{\partial e} = -\frac{(1-\lambda)^2H(N-n)mbrZ^2}{N^2|F|} < 0.$$

Thus, average employment M decreases even if average layoffs L have also decreased. This result is not a surprise as BW showed [for the case of endogenous firm size and no entry, as it is the case here] that average employment is decreasing in the experience-rating parameter if $R_1''(n) \geq R_2''(n)$ where $R_s(n)$ is the revenue of the firm given n workers are employed in state s. Here, $R_s(n_i) = n_i[k_s - n_i - (m-1)\hat{n}_j]$ so that $R_1''(n_i) = R_2''(n_i) = 0$. Thus, BW's sufficient condition is satisfied.

Let us now turn to the impact of the payroll tax δ. Using the matrix system $(C18)$, the following results are obtained:

$$\frac{\partial N}{\partial \delta} = -\frac{[(1-\lambda)rZ^2+H(m+1)]}{|F|} < 0$$

$$\frac{\partial n}{\partial \delta} = -\frac{(1-\lambda)nrZ^2}{N|F|} < 0$$

$$\frac{\partial w}{\partial \delta} = -\frac{(1-\lambda)n(m+1)Z}{N|F|} \quad \begin{matrix}<\\>\end{matrix} \quad 0 \quad \text{as} \quad Z \quad \begin{matrix}>\\<\end{matrix} \quad 0.$$

Which imply that average industrial employment falls even if average layoffs are also falling:

$$\frac{\partial L}{\partial \delta} = -\frac{(1-\lambda)m}{|F|}\left[H(m+1) + \frac{(1-\lambda)(N-n)rZ^2}{N}\right] < 0$$

$$\frac{\partial M}{\partial \delta} = m\left[\lambda \underbrace{\frac{\partial N}{\partial \delta}}_{<0} +(1-\lambda) \underbrace{\frac{\partial n}{\partial \delta}}_{<0}\right] < 0.$$

Before turning to the case of free entry in the industry, it is interesting to note that, for a fixed number of firms, the expected profits of a firm in this industry will vary as the parameters of the UI scheme will be modified. To see that, denote by $E\Pi(e,\delta,m)$ the [expected] profit function. Then, by use of the envelope theorem:

$$\left.\frac{\partial E\Pi(e,\delta,m)}{\partial e}\right|_{m=\text{cst}} = \frac{-2(1-\lambda)(N-n)}{|F|}\left[\frac{(1-\lambda)JbrZ^2}{N^2} + \lambda H(m+1)b\right] < 0$$

$$\left.\frac{\partial E\Pi(e,\delta,m)}{\partial \delta}\right|_{m=\text{cst}} = \frac{-2}{|F|}\left[\frac{(1-\lambda)JrZ^2}{N} + \lambda HN(m+1)\right] < 0.$$

C.2. *Free entry*

If there is free entry in the industry, the number of firms m will be endogenously determined so that the expected profits of entering firms are zero:

$$\begin{aligned} E\Pi =& \lambda\,[N(k_1 - mN) - Nw] \\ &+ (1-\lambda)\,[n(k_2 - mn) - nw - (N-n)eb] - \delta N - c = 0 \end{aligned} \qquad (C19)$$

Differentiating equations $(C15)$–$(C17)$ and $(C19)$, the following matrix system is obtained:

$$G\begin{bmatrix} dN \\ dn \\ dm \\ dw \end{bmatrix} = \begin{bmatrix} (1-\lambda)bde + d\delta \\ -bde \\ 0 \\ -(1-\lambda)(N-n)bde - Nd\delta \end{bmatrix} \qquad (C20)$$

where G, a 4×4 matrix, is the following:

$$\begin{bmatrix} -\lambda(m+1) + \frac{(1-\lambda)nZ}{N^2} & -\frac{(1-\lambda)Z}{N} & -\lambda N & -\frac{H+(1-\lambda)nrZ}{N} \\ 0 & -(m+1) & -n & rZ \\ -\frac{(1-\lambda)nZ}{N} & (1-\lambda)Z & 0 & H \\ \lambda(m-1)N - \frac{(1-\lambda)nZ}{N} & (1-\lambda)[Z+(m-1)n] & J & H \end{bmatrix}.$$

The determinant of G is obtained after tedious manipulations:

$$|G| = -2\lambda(1-\lambda)JrZ^2 - \frac{2(1-\lambda)^2 Jn^2 rZ^2}{N} - 2\lambda HJ(m+1) < 0.$$

The impact of a change in the experience-rating parameter on the endogenous variables is given by the following:

$$\frac{\partial N}{\partial e} = \frac{2(1-\lambda)H^2 nb}{|G|} < 0$$

$$\frac{\partial n}{\partial e} = -\frac{2\lambda H^2 Nb}{|G|} > 0$$

$$\frac{\partial m}{\partial e} = \frac{2(1-\lambda)(N-n)b}{|G|}\left[\lambda H(m+1) + \frac{(1-\lambda)JrZ^2}{N^2}\right] < 0$$

$$\frac{\partial w}{\partial e} = \frac{2(1-\lambda)JHbZ}{N|G|} \quad \begin{matrix} < \\ > \end{matrix} \quad 0 \quad \text{as} \quad Z \quad \begin{matrix} > \\ < \end{matrix} \quad 0.$$

As can be seen, the free entry effect is present even if the firm size effect is allowed for. Then, the average level of layoffs in the industry, L, and the average level of employment in the industry M will decrease as a result of a higher level of experience-rating:

$$\frac{\partial L}{\partial e} = (1-\lambda)(N-n)\underbrace{\frac{\partial m}{\partial e}}_{<0} + (1-\lambda)m\underbrace{\frac{\partial N}{\partial e}}_{<0} - (1-\lambda)m\underbrace{\frac{\partial n}{\partial e}}_{>0} < 0$$

$$\frac{\partial M}{\partial e} = \frac{1}{|G|}\left[2\lambda(1-\lambda)H^2(N-n)b + \frac{2(1-\lambda)^2 J(N-n)brZ^2}{N^2}\right] < 0.$$

Now consider the impact of a change in the payroll tax on the endogenous variables:

$$\frac{\partial N}{\partial \delta} = \frac{2(1-\lambda)Hn^2}{|G|} < 0$$

$$\frac{\partial n}{\partial \delta} = \frac{-2\lambda HNn}{|G|} > 0$$

$$\frac{\partial m}{\partial \delta} = \frac{1}{|G|}\left[2\lambda HN(m+1) + \frac{2(1-\lambda)JrZ^2}{N}\right] < 0$$

$$\frac{\partial w}{\partial \delta} = \frac{2(1-\lambda)JnZ}{N|G|} \quad \begin{smallmatrix} < \\ > \end{smallmatrix} \quad 0 \quad \text{as} \quad Z \quad \begin{smallmatrix} > \\ < \end{smallmatrix} \quad 0.$$

Note that with no entry, the result was that $\partial n/\partial \delta < 0$ while here, $\partial n/\partial \delta > 0$. Again, this is explained by the fact that it becomes more and more expensive to have unused capacity. Also note that, again, the free entry effect is present. The following is obtained:

$$\frac{\partial L}{\partial \delta} = (1-\lambda)(N-n)\underbrace{\frac{\partial m}{\partial \delta}}_{<0} + (1-\lambda)m\underbrace{\frac{\partial N}{\partial \delta}}_{<0} - (1-\lambda)m\underbrace{\frac{\partial n}{\partial \delta}}_{>0} < 0$$

$$\frac{\partial M}{\partial \delta} = \frac{2}{|G|}\left[\frac{(1-\lambda)JHrZ^2}{N} + \lambda JHm + \lambda H^2 N\right] < 0.$$

Thus, again, both average layoffs and average employment are decreased by a higher payroll tax.

This appendix has thus shown that the combination of the firm size and free entry effects can lead to the result that an increase in the experience-rating parameter or in the payroll tax has a negative impact on employment.

Appendix D: Various expressions of section 3

Some of the expressions for which only the signs were given in section 3 are fully written in this appendix.

Consider first equations (26)-(28). The following are obtained:

$$\frac{\partial N}{\partial p_1} = \frac{-\lambda p_2 F'(N)F''(n)H + \lambda(1-\lambda)F'(N)rZ^2}{|R|} > 0$$

$$\frac{\partial N}{\partial p_2} = \frac{(1-\lambda)^2 F'(n)nrZ^2}{N|R|} > 0$$

$$\frac{\partial N}{\partial e} = \frac{(1-\lambda)p_2 F''(n)HNb - (1-\lambda)^2(N-n)brZ^2}{N|R|} < 0$$

$$\frac{\partial n}{\partial p_1} = \frac{\lambda(1-\lambda)F'(N)nrZ^2}{N|R|} > 0$$

$$\frac{\partial n}{\partial p_2} = \frac{-\lambda p_1 F''(N)F'(n)HN^2 + (1-\lambda)^2 F'(n)n^2 rZ^2}{N^2|R|} > 0$$

$$\frac{\partial n}{\partial e} = \frac{-\lambda p_1 F''(N)HN^2 b - (1-\lambda)^2(N-n)nbrZ^2}{N^2|R|} \gtrless 0$$

$$\frac{\partial w}{\partial p_1} = \frac{-\lambda(1-\lambda)p_2 F'(N)F''(n)nZ}{N|R|} \quad \lessgtr \quad 0 \quad \text{as} \quad Z \quad \lessgtr \quad 0$$

$$\frac{\partial w}{\partial p_2} = \frac{\lambda(1-\lambda)p_1 F''(N)F'(n)Z}{|R|} \quad \lessgtr \quad 0 \quad \text{as} \quad Z \quad \gtrless \quad 0$$

$$\frac{\partial w}{\partial e} = \frac{\lambda(1-\lambda)p_1 F''(N)NbZ + (1-\lambda)^2 p_2 F''(n)nbZ}{N|R|} \quad \lessgtr \quad 0 \quad \text{as} \quad Z \quad \gtrless \quad 0.$$

Now, in sub-section 3.1, the sign of matrix S was said to be positive because:

$$\begin{aligned}\left[\frac{\partial N}{\partial p_1}\frac{\partial n}{\partial p_2} - \frac{\partial N}{\partial p_2}\frac{\partial n}{\partial p_1}\right] = & \frac{1}{|R|}\Bigg[\lambda^2 p_1 p_2 F'(N)F''(N)F'(n)F''(n)H^2 \\ & - \frac{\lambda(1-\lambda)^2 p_2 F'(N)F'(n)F''(n)Hn^2 rZ^2}{N^2} \\ & - \lambda^2(1-\lambda)p_1 F'(N)F''(N)F'(n)HrZ^2\Bigg] > 0.\end{aligned}$$

Finally, in the same sub-section, the signs of $\partial p_1/\partial e$ and $\partial p_2/\partial e$ were obtained using the fact that:

$$\left[\frac{\partial n}{\partial e}\frac{\partial N}{\partial p_2} - \frac{\partial N}{\partial e}\frac{\partial n}{\partial p_2}\right] = \frac{1}{|R|}\Bigg[-\frac{\lambda(1-\lambda)^2 p_1 F''(N)F'(n)HnbrZ^2}{N}$$

$$-\Bigg(-\lambda(1-\lambda)p_1p_2F''(N)F'(n)F''(n)H^2b + \frac{(1-\lambda)^3 p_2 F'(n)F''(n)Hn^2brZ^2}{N^2}$$

$$+\frac{\lambda(1-\lambda)^2 p_1 F''(N)F'(n)H(N-n)brZ^2}{N}\Bigg)\Bigg] > 0$$

$$\left[\frac{\partial n}{\partial p_1}\frac{\partial N}{\partial e} - \frac{\partial N}{\partial p_1}\frac{\partial n}{\partial e}\right] = \frac{1}{|R|}\Bigg[\frac{\lambda(1-\lambda)^2 p_2 F'(N)F''(n)Hn^2brZ^2}{N^2}$$

$$-\Big(\lambda^2 p_1p_2F'(N)F''(N)F''(n)H^2b - \lambda^2(1-\lambda)p_1F'(N)F''(N)HbrZ^2\Big)\Bigg] < 0.$$