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Regulatory Auditing and Ramsey Pricing

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by

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ABSTRACT

Truthful revelation mechanisms with auditing have the undesirable property that audits are not actually performed in equilibrium because all inference problems have been solved. A model is proposed in which the inference problem is preserved by separating the regulatory and auditing functions and transfers are costly. The auditor designs a Bayesian audit procedure and the regulator credibly commits to using this procedure in the regulatory mechanism. The auditor is conservative, that is, he does not like to make mistakes for his client. Second-best allocation is achieved over a well-defined auditing region of the regulator's prior beliefs about firm type. The auditing region is increasing in the precision of the auditing technology and is decreasing in the strength of the regulator's prior beliefs.

Regulatory Auditing and Ramsey Pricing

§1. Introduction

This paper examines auditing as a regulatory strategy to reduce rents required to induce a privately informed monopolist to reveal its type when raising and transferring funds is costly. A Bayesian regulatory game is proposed that separates the regulatory and auditing functions. The regulator can credibly commit to employing a conservative auditor at the end of the production period to determine whether the firm truthfully reported costs. The auditor uses an imperfect auditing technology and is risk averse in making mistakes for his client, the regulator. A set of conditions are proposed under which auditing reduces the welfare loss attributable to ex ante information asymmetries.

Various directly implementable delegation and revelation mechanisms have been proposed in the literature as a solution to the problem of regulating a monopoly with unknown costs. Loeb and Magat (1979) propose a mechanism where the firm is offered a transfer function equal to consumer surplus and sets price by picking a point on this function. Although first-best allocation is achievable using this delegation mechanism, the least-cost solution entails transferring the entire net social surplus to the firm. In a comprehensive literature review, Caillaud, Guesnerie, Rey and Tirole (1988) survey departures from the Loeb and Magat solution. Alternative allocation schemes are examined wherein information rents are reduced by introducing either distributional considerations or a cost of public funds. A general reference for revelation mechanisms with distributional considerations is Baron (1989).

More recently, attention has shifted to reducing distortions arising from informational asymmetries in contracting relationships by making contracts contingent on information which becomes available ex post.[1] Regulation, itself a learning process, is an obvious candidate for application of contingent or conditional rules. The mechanism design literature implicitly recognizes this fact by treating the regulator as a Bayesian statistician. The regulator forms a prior assessment on the firm's true type on the basis of acquired knowledge of the industry and past performance of the firm. Using this source of information, the regulator is able to design an efficient mechanism to induce the firm to report truthfully its costs.

Curiously, the design of an auditing procedure has not been treated as a Bayesian statistical decision problem in the regulation literature. Baron and Besanko (1984) and Baron (1989), for instance, treat the audit as a classical hypothesis test rather than as a mechanism design problem.[2] This approach is unsatisfactory and inefficient in terms of information usage. Discarding the prior information will always result in an inefficient auditing procedure, unless the regulator's prior is noninformative. The Bayesian approach adopted in this paper explicitly recognizes that audit information is a complement rather than a substitute for the regulator's prior information and that all decisions must be made within a formal framework. The approach provides a basis for optimally combining three sources of information - prior, audit and loss.

An efficient Bayesian auditing procedure is constructed by minimizing the expected risk of the audit. The auditing procedure is endogenized in two different ways.[3] First, due to the

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1. See Demougin and Garvie (1991), McAfee and Reny (1989), Riordan and Sappington (1988), Baron and Besanko (1984) and Laffont and Tirole (1986).

2. Baron and Besanko (1984) examine the impact of making regulatory contracts contingent on costly information which becomes available ex post. The model is characterized by distributional considerations and a bound on the penalty. Under certain conditions, a globally incentive compatible mechanism with auditing strictly dominates the no auditing solution.

3. Baron (1984), (1989) and Baron and Besanko (1984) examine regulatory strategies with an exogenous auditing technology.

client relationship between the regulator and the auditor, the audit decision rules are decisions against the regulator's prior on the distribution of types of firms. Second, the quality of the audit information is a function of the heterogeneity in firm types. The audit decision rules are assumed to be common knowledge so that expected penalties can be calculated using these rules.

Surprisingly, the ex ante value of costless auditing is not always positive. Three regions of the prior distribution on firm's costs result from the auditing decision rules. The auditing solution Pareto dominates the no auditing solution only in the mid-region of the prior, that is, when the regulator is relatively poorly informed ex ante or the auditing technology is relatively precise. This result holds because errors due to imperfections in the auditing technology are costly to the auditor and these costs are particularly high in the outer regions. The second-best solution of Ramsey prices is implementable only in the auditing region of the regulator's prior. Modified Ramsey rules with an additional marginal information cost term are third-best in the no auditing regions.

The paper proceeds as follows. In section 2 the nature and timing of the problem are outlined. A benchmark incentive compatible mechanism without auditing is characterized in section 3. Mechanism design with auditing is discussed in section 4. A Bayesian auditing procedure is constructed in section 4.1. An optimal regulatory policy with auditing is characterized in section 4.2. Conclusions are presented in section 5.

§2. The Nature of the Problem

Regulation of the firm is considered socially desirable to prevent monopoly pricing. The regulatory relationship is characterized by an information asymmetry regarding industry technology. This asymmetry directly affects the setting of socially efficient pricing rules. The cost technology is assumed to have the functional form $C(q, c(\theta)) = c(\theta)q(p) + K$, where $q(p)$ is the demand function, $c(\theta) = 1/\theta$ is marginal costs and K is overhead costs. K is assumed to be known to the regulator and is suppressed in the subsequent analysis since overhead costs are generally considered to be verifiable. The efficiency level or type of firm, θ, is likely to be unobservable by the regulator. To capture this asymmetry, we assume that the firm's marginal cost or type is private information. Costs are assumed to be high, $c(\theta_h)$, or low, $c(\theta_l)$, where $0 < \theta_h < \theta_l < 1$. The support of the distribution of θ, the cost and demand functions are common knowledge.

The regulator requires pricing rules to be a function of true costs for social efficiency. Two alternative mechanisms have been proposed as solutions to this problem of regulating a natural monopoly with unknown costs: a delegation mechanism in which the pricing decision is delegated to the firm or a revelation mechanism in which the firm 'reports' its true costs by choosing a point on a schedule of pricing rules designed by the regulator.

The revelation approach is adopted in this paper. Given that the firm's type is private information at the time of reporting, a low-cost firm has an incentive to report high costs to obtain a higher price and greater profit. In the absence of a directly implementable truth-telling mechanism, the best the regulator can do is to set price equal to the higher marginal cost level.[4]

The timing of the regulatory problem is as follows. Nature chooses the type of firm, θ, where $\theta \in \{\theta_h, \theta_l\}$. The regulator designs a mechanism or pair of contracts $\{(p_l, T_l), (p_h, T_h)\}$, each contract consisting of a two-part pricing rule: a per unit price p_i and a fixed fee or transfer payment T_i given to the firm reporting cost level i, where i = h, l. The subscripts l and

4. It is assumed that the market must be served, otherwise the regulator might wish to set price equal to the lower marginal cost level, and forgo service in the event of a high cost firm.

h denote that the rules are functions of θ_l and θ_h, respectively. The firm 'reports' its type by choosing a contract from this set. Production occurs and the lump sum payment is transferred after the goods have been sold to consumers. If the regulator has access to an auditing technology, the regulator can costlessly audit the firm to verify its report at the end of the production period.

§3. Mechanism Design with No Auditing

By the Revelation Principle, we may restrict our attention to finding a truth-telling regulatory policy.[5] The regulator's task is to find a mechanism $M = \{(p_l, T_l), (p_h, T_h)\}$ that will maximize expected social welfare, ensure the firm's participation and induce the firm to truthfully report its costs.[6] In a revelation game, transfer payments are the regulator's primary instrument for inducing truth-telling. There are, however, real resource costs attached to raising and transferring funds: administrative costs and tax distortions can be particularly costly. A positive cost of funds, λ, is introduced to take account of these resource costs.[7]

The social welfare function W is assumed to be the sum of the indirect utility function V(p) of a representative consumer and producer surplus or profit, π, less the resource cost of the transfer payment λT: $W = V + \pi - \lambda T$. The indirect utility function is twice

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5. The Revelation Principle states that the regulator can construct a mechanism consisting of a two-part pricing schedule $M^* = \{p^*(c,c), T^*(c,c)\}$ that induces truth-telling which does at least as well as any other pricing schedule $M = \{p(r,c), T(r,c)\}$ with lying, that is, where the report r does not equal true costs c. Thus, the Revelation Principle eliminates the need to consider pricing schedules with strategic manipulation of private information.

6. The participation constraint on the regulator's design problem is known as the individual rationality constraint while the truth-telling constraint is known as the incentive compatibility constraint.

7. The incomplete information problem of regulating a multitype monopolist with a cost of funds condition bears marked similarities to the complete information problem of a multiproduct monopolist subject to break-even constraints. In the optimal solution to the latter problem, revenue is raised to meet the break-even constraints by taxing products relative to demand inelasticity. The prices emerging from such a mechanism are Ramsey prices.

differentiable, with $-V'(p)=q(p)$ by Roy's Identity.[8] To find expected social welfare, EW, the regulator forms a prior $\alpha(\theta)$ on the distribution of the unknown efficiency parameter θ. The prior is a two-point distribution, where

$$\alpha(\theta) = \begin{cases} \alpha & \text{if } \theta = \theta_l \\ 1 - \alpha & \text{if } \theta = \theta_h \end{cases}$$

The regulator's design problem can be written as

$$\max_{\{(p_l,T_l),(p_h,T_h)\}} EW = \alpha\left[V(p_l) + (p_l - c_l)q(p_l) - \lambda T_l\right] + (1 - \alpha)\left[V(p_h) + (p_h - c_h)q(p_h) - \lambda T_h\right]$$

subject to

$$(p_l - c_l)q(p_l) + T_l \geq (p_h - c_l)q(p_h) + T_h \tag{1}$$

$$(p_l - c_l)q(p_l) + T_l \geq 0 \tag{2}$$

$$(p_h - c_h)q(p_h) + T_h \geq (p_l - c_h)q(p_l) + T_l \tag{3}$$

$$(p_h - c_h)q(p_h) + T_h \geq 0 \tag{4}$$

Equations (1) and (3) are the incentive compatibility constraints for type θ_l and θ_h firms, respectively. The optimal policy must also satisfy the individual rationality constraints (2) and (4), since the regulator requires production from both types and the firm will not produce if the regulatory policy results in negative profits.

Lemma 1: *In the optimal regulatory policy, price is nondecreasing with marginal cost.*

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8. Strictly $V(p,y) \equiv V(p) + y$ but we will suppress the income argument as it has no effect on our subsequent (partial equilibrium) analysis.

Proof: See Appendix.

***Proposition 1:** The solution to the regulator's design problem with no auditing is characterized by the following set of equations:*

$$p_l^* = c_l - \frac{\lambda}{1+\lambda}\frac{q(p_l^*)}{q'(p_l^*)} \tag{5}$$

$$T_l^* = \frac{\lambda}{1+\lambda}\frac{q(p_l^*)^2}{q'(p_l^*)} + (c_h - c_l)q(p_h^*) \tag{6}$$

$$p_h^* = c_h - \frac{\lambda}{1+\lambda}\frac{q(p_h^*)}{q'(p_h^*)} + \frac{\lambda}{1+\lambda}\frac{\alpha}{1-\alpha}(c_h - c_l) \tag{7}$$

$$T_h^* = \frac{\lambda}{1+\lambda}\frac{q(p_h^*)^2}{q'(p_h^*)} - \frac{\lambda}{1+\lambda}\frac{\alpha}{1-\alpha}(c_h - c_l)q(p_h^*) \tag{8}$$

The proof is omitted for brevity.[9]

Note first that the per unit pricing rules given by equations (5) and (7) are variants on Ramsey prices for a multiproduct monopolist subject to break-even constraints. The traditional rules are modified somewhat for regulating a multitype monopolist when the true type is known only to the firm and transfers are costly. Recall that once a shadow cost of public funds is introduced, the firm effectively becomes a taxation instrument. Distortionary taxes are used for raising funds for both the firm and the government itself. Consumers are taxed using Ramsey

9. The characteristics of the solution are the same for a continuum of types.

prices and these taxes are collected from the firm using lump sum transfers.

The contract offered to a low-cost firm is characterized by efficient production and information rents. The incentive compatible price p_l^* is a standard Ramsey pricing rule and is independent of the regulator's prior beliefs about type. A low-cost firm must be 'bribed' to reveal its type and produce efficiently as this private information is valuable: a bribe payment or information rent equal to $(c_h - c_l)q(p_h^*)$ is required to induce the low-cost firm to truthfully reveal its type.

The contract offered to a high-cost firm is characterized by production distortion but no information rent. Production is distorted using a third-best modified Ramsey pricing rule. The transfer is a tax: bribe payments required to induce truth-telling by the efficient firm are raised by taxing an inefficient firm. For any given λ, the price and transfer adjust as the prior odds on the firm being low-cost, $\alpha/ 1 - \alpha$, and/or the heterogeneity in firm types, $(c_h - c_l)$, varies. The adjustment occurs through the last term on the right hand side of equation (7). This is a marginal information cost term, weighted by $\lambda/ 1 + \lambda$. Note that as marginal information costs increase, p_h^* and T_h^* increase while T_l^* falls.[10]

One can easily verify that the mechanism $M^* = \{(p_h^*, T_h^*), (p_l^*, T_l^*)\}$ is globally optimal. The first-best solution of marginal cost pricing is not achievable due to the cost of public funds. Referring to Figure 1, if the regulator offered the firm two Ramsey prices p_h^r and p_l^*, then a low-cost firm will choose p_h^r and receive information rents AB. However, the regulator can raise welfare by distorting p_h^r since the information rents required to induce a low-cost firm to report truthfully adjusts as p_h changes. Using a standard envelope theorem argument, the first-order

10. The transfer to the low-cost firm switches from a subsidy to a tax when the marginal information cost term is sufficiently high.

efficiency cost of distorting production at the margin is zero whereas the corresponding first-order reduction in information rents is strictly positive. In Figure 1, this entails sliding the incentive compatibility constraint for the low-cost firm (IC-L) from point A, down the individual rationality constraint or zero profit curve of the high-cost firm (IR-H), to point C. At the optimum C, the price offered to a high-cost firm is $p_h^* > p_h^r$ and information rents are DE < AB.

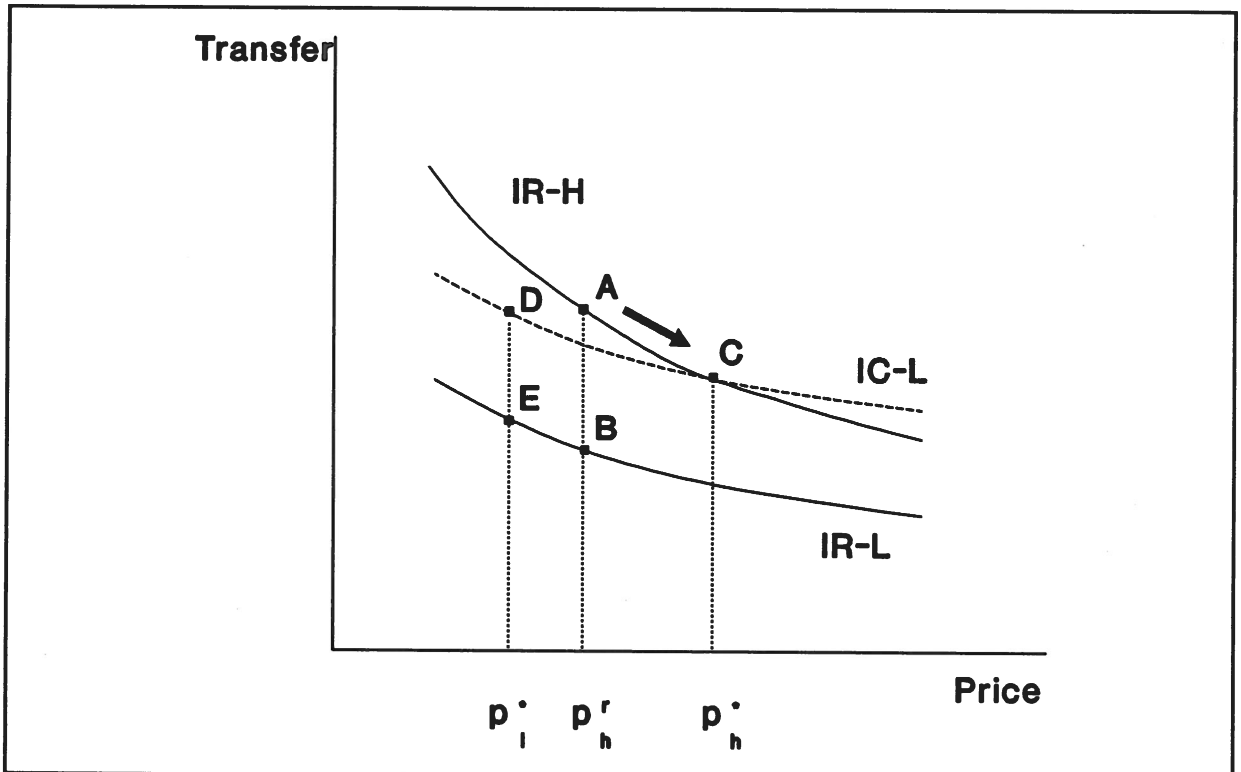

Figure 1. No Auditing Model

§4. Mechanism Design with Auditing

The regulator requires additional information to reduce the rents accruing to an efficient firm from information asymmetries. Rent reduction can be achieved by improving the quality of the regulator's prior information or aquisition of additional information ex post. This paper examines the latter option. One obvious source of ex post information is an audit of the firm's costs after production has occurred to determine where the firm's true type lies in the parameter space. The probabilistic outcome of an auditing procedure can be incorporated into the ex ante agreement provided that the regulator can credibly commit to using the procedure.

In this section, a Bayesian Nash regulatory game with auditing is modelled by separating the regulatory and auditing functions. The auditor is assumed to be a non-strategic but risk-averse player in the game, either employed within the regulatory agency or hired by the regulator from an outside agency. This amounts to assuming that there is no agency problem in the auditor-regulator relationship or collusion in the auditor-firm relationship.[10] This is not an unreasonable assumption given that the accounting industry is characterized by standardized procedures and reputation effects place an upper bound on collusive behaviour of auditors.[11] Hence, while the regulator's and auditor's objective may or may not be coincident, it is assumed that their objectives are not harmful to the other.

This section proceeds as follows. Standard accounting industry practice is modelled as a Bayesian statistical decision problem in section 4.1. The audit procedure is then incorporated into the mechanism design problem in section 4.2.

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10. Dye (1987) addresses the issue of agency problems between auditors and employers. Kofman and Lawaree (1989) examine the problem of collusive behaviour between internal auditors and managers and the role of 'honest' external auditors.

11. Where annual audits of corporations are required by law, the accounting firm is chosen by a voting procedure among shareholders at the beginning of the fiscal year. Hence, accounting firms do have a vested interest in securing a repeat customer relationship. However, the discretionary power of accounting firms is limited by reputation effects. In particular, discretion is generally limited to a "true and fair region" which ranges from 5% to 10% of reported costs.

§4.1 A Bayesian Auditing Procedure

The auditing procedure design problem is treated as a Bayesian statistical decision problem for several reasons. A Bayesian auditing procedure captures the conservative nature of standard procedures currently in practice. It provides a formal framework for combining three types of information used in standard procedures - subjective assessment (prior), accounting records (sample) and the auditor's objective (loss). Further, the auditing procedure can be made endogenous to the economic problem by linking the sources of information. Finally, Bayes Theorem has several attractive properties which are desirable in an auditing technology. The preceding is expanded upon below and then a Bayesian auditing procedure is constructed.

In practice, an audit begins with a planning stage. An auditor examines the nature of the business, the audit report from the previous year, reviews information from newspapers and other published sources to discover whether there have been any problems in the past and so on. On the basis of these findings, a conservative auditor forms a subjective assessment or prior on the distribution of types and determines the risky areas of the audit. An auditing plan is devised to minimize the expected risk of the audit and carried forward to completion.

As a statistical decision problem, the audit is defined by a parameter space θ, a decision space D, a loss function $L(\theta, d)$ and a family of conditional probability density functions $p(\cdot|\theta)$ of an observation X, a signal of costs. The value of X is learned during the course of the audit. The auditing procedure, like the regulatory mechanism, is designed ex ante. However, the former, unlike the latter, incorporates off-the-equilibrium path occurrences. Clearly, it is necessary that a credible threat of auditing be posed ex ante to deter cheating.[12]

The auditing procedure is linked to the economic model by incorporating the regulator's prior and making the audit information a function of firm type. Given that the regulatory agency is likely to be in a superior position to form a prior assessment on firm type, the auditor is assumed to adopt the regulator's prior distribution $\alpha(\theta)$ on the unknown efficiency parameter θ. The audit itself is an observation of a signal X correlated with the firm's costs. The signal is assumed to be a binomial random variable defined on an independent Bernoulli trial. The terminology adopted is that X=1 (success) is a signal of low-cost and X=0 (failure) is a signal of high-cost. The audit results in a success with probability θ.[13]

The auditor believes that the random variable X has the following conditional distributions:

$$p(X = 0|\theta = \theta_l) = (1 - \theta_l) \qquad p(X = 1|\theta = \theta_l) = \theta_l$$

$$p(X = 0|\theta = \theta_h) = (1 - \theta_h) \qquad p(X = 1|\theta = \theta_h) = \theta_h$$

In words, the auditor assigns θ_l as the probability that the signal indicates low costs when the firm is low-cost. Similarly, $(1 - \theta_l)$ is the probability that the signal indicates high costs when the firm is low-cost. Note that the closer together are θ_l and θ_h on the unit interval, the noisier is the signal X.

After observing X, the auditor updates the prior using Bayes Law. The posterior on the

12. Although auditing is not ex post rational in a revelation game, the commitment to audit is desirable because it reduces the ex ante cost of inducing the firm to report truthfully.

13. Probability is a subjective measure in a Bayesian context, that is, it is defined in terms of the degree of belief in an event occurring. In this model, the auditor knows the support of the distribution of types, $\{\theta_h, \theta_l\}$, and believes that a higher value of θ_l (lower costs) results in a higher probability of a correct signal being observed when the firm's true type is θ_l. Analogously, a lower value of θ_h (higher costs) results in a higher probability of a correct signal being observed when the firm has high costs.

distribution of a low-cost firm conditional on observing the signal x is denoted by

$$\alpha(x) = p(\theta = \theta_l | X = x) = \frac{p(X = x | \theta = \theta_l) p(\theta = \theta_l)}{p(X = x)}$$

Analogously, the posterior on the distribution of a high-cost firm conditional on observing the signal x is denoted by $\bar{\alpha}(x)$.

Three intuitively appealing properties of Bayesian updating procedures characterize the imperfect auditing technology. These properties are summarized in Lemma 2.

Lemma 2: *(i) With an imperfect auditing technology, a signal of low costs leads to an upward revision in the probability assessment of low-cost firm and a downward revision in the probability assessment of high-cost firm. An analogous result holds for a signal of high costs.*
(ii) The degree of revision in beliefs about firm types is nonincreasing in the regulator's prior beliefs. The more firmly the regulator believes the firm to be a particular type, the lower the impact of the additional information obtained through the audit.
(iii) The degree of revision in beliefs about firm types is nondecreasing in the heterogeneity of firm types. The greater the distance between firm types, the less noisy is the signal and the higher the impact of the additional information obtained through the audit.
Proof: *See Appendix.*

After combining the audit information with the regulator's prior information, the auditor must choose an optimal decision rule for determining the firm's type. The optimal decision rule is a function $\delta(x)$ which maps the sample space into the decision space, that is, $\delta(x)$: $\{0,1\} \rightarrow \{d_h, d_l\}$. For any decision $\delta(x) \in \Delta$, where Δ denotes the class of all decision functions, the expected loss or risk, $\rho(\alpha, \delta)$ is defined by

$$\rho(\alpha,\delta) = \sum_{x=0}^{1} \Big(\alpha\big[L(\theta_l,\delta(x))p(X = x|\theta_l)\big] + (1 - \alpha)\big[L(\theta_h,\delta(x))p(X = x|\theta_h)\big]\Big)$$

Given an imperfect auditing technology, there is a non-zero probability that the auditor will make a mistake leading the regulator to either penalize a high-cost firm for truthfully reporting costs or fail to penalize a low-cost firm for cheating. The auditor is risk averse in making mistakes for the client and chooses a decision $\delta(x)$ which minimizes expected posterior loss or risk of the audit. For the distribution $\alpha(\theta)$ and for any decision function $\delta \in \Delta$, the Bayes risk $\rho(\alpha, \delta^*)$ is defined as the greatest lower bound for the risks $\rho(\alpha, \delta)$ for all the decisions $\delta \in \Delta$:

$$\rho(\alpha,\delta^*) = \inf_{\delta\in\Delta} \rho(\alpha,\delta) \tag{9}$$

Equation (9) states that δ^* is a Bayes decision function against α.

If the regulator's and auditor's objectives are coincident, then the loss function would simply be the negative of the social welfare function. A more general loss function, $L(\theta, d)$, is specified below to capture the notion that the preferences of the auditor are exogenous to the economic problem at hand. The loss function is assumed to be a generalized form of the "0-1" loss function.[14] For each outcome $\theta \in \{\theta_h, \theta_l\}$ and each decision $D \in \{d_h, d_l\}$, where d_i is the decision that the firm has costs i, i=h,l, the loss is defined by the table:

14. The "0-1" loss function is a common choice for the binary decision problem, where the loss is zero if a correct decision is made and one if an incorrect decision is made. Generalizing, loss could be an increasing function of the distance from the true type of firm. This explicitly recognizes that the loss is a function of the severity of the auditor's mistake.

Loss Table

	d_l	d_h
$\theta = \theta_l$	0	b
$\theta = \theta_h$	a	0

where b ≥ a. A Type I error is committed if the firm's true type is low-cost and the auditor decides it is high-cost and losses are some constant b. Analogously, a Type II error is committed if the firm's true type is high-cost and the auditor decides it is low-cost and losses are some constant a.

Proposition 2: *The auditor's Bayes decision function, $\delta^*(x)$, given $L(\theta, \delta(x))$, $\alpha(\theta)$ and $p(\cdot|\theta)$ is:*

$$\delta^*(1) = \begin{cases} d_l \text{ iff } \alpha > A \text{ where } A = \dfrac{a\theta_h}{b\theta_l + a\theta_h} \\ d_h \text{ otherwise} \end{cases} \tag{10}$$

$$\delta^*(0) = \begin{cases} d_l \text{ iff } \alpha > B \text{ where } B = \dfrac{a(1 - \theta_h)}{b(1 - \theta_l) + a(1 - \theta_h)} \\ d_h \text{ otherwise} \end{cases} \tag{11}$$

and $B > A$ since $\theta_l > \theta_h$.

Proof: See Appendix.

The auditor's decision function over the regulator's prior is depicted in Figure 2. The more heterogeneous the firm types, the larger is the region [A, B]. A higher degree of heterogeneity in firm types is equivalent to reducing the error probabilities or increasing the accuracy of the auditing technology. The region [A, B] also increases as the losses attached to failing to detect cheating by a low-cost firm increase. However, as the losses from incorrectly identifying a high-cost firm as low-cost increase, this region decreases.

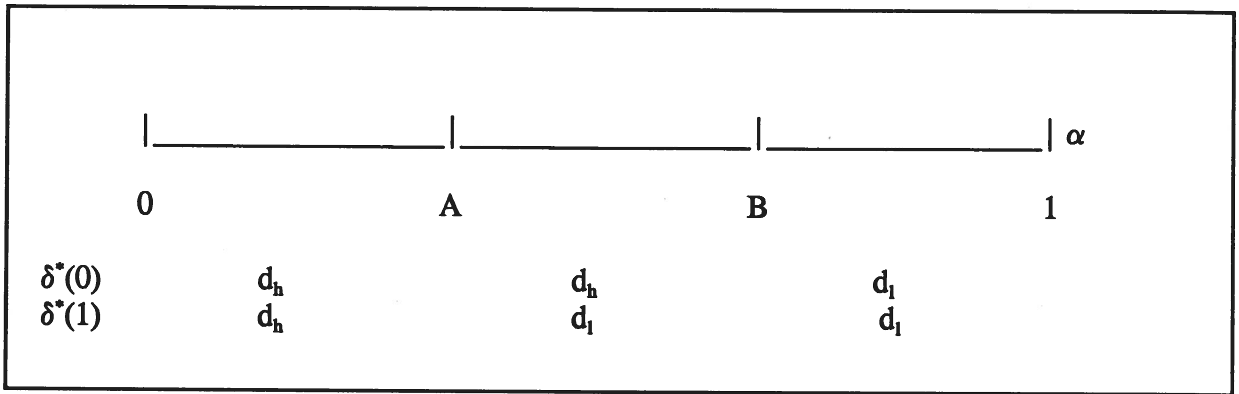

Figure 2. Auditing Region

§4.2 Optimal Regulatory Policy with Auditing

In the regulatory game, the regulator and firm take the Bayesian auditing procedure as given. The regulator, acting as a Stackelberg leader, designs an incentive compatible and individually rational regulatory mechanism $\{(p_l, T_l), (p_h, T_h), Z\}$ to maximize expected social welfare where Z is a penalty for cheating. The firm computes its expected penalty and picks a point on the regulatory schedule to maximize expected profits.

The introduction of a penalty for cheating, Z, creates an additional transfer instrument. Given the firm's risk neutrality, the fact that the instrument is stochastic does not affect its role as a transfer. We assume that the regulator adopts a pure strategy of auditing whenever a firm reports high costs and never to audit when a firm reports low-cost. A penalty is imposed on a firm if, after an audit, the auditor's decision is that the firm did not report its true cost.

Proposition 3: *The probability of a firm being penalized for inaccurately reporting costs, $\xi(\theta)$, is nondecreasing in firm type θ and is given by*

$$\xi(\theta) = \begin{cases} 0 & 0 < \alpha < A \\ \theta & A < \alpha < B \\ 1 & B < \alpha < 1 \end{cases}$$

Proof: See Appendix.

The regulator wishes to set the penalty so that the expected penalties, $E(Z|\theta_i) = \xi(\theta_i)Z$ for i=h,l, reduce the information rents to the low-cost firm. Note that in the mid-region of the regulator's prior [A,B], the expected penalty for a low-cost firm reporting high-cost is higher than the expected penalty a high-cost firm will incur. We now examine the mechanisms designed for the three regions of the regulator's prior.[15]

Proposition 4: *If $\alpha < A$ or $\alpha > B$, then the regulator cannot improve upon the standard incentive compatible mechanism by accessing a costless but imperfect auditing technology and will do strictly worse if auditing is costly.*

Proof: Simply note that the constrained design problems correspond to the model in section 3.

Q.E.D.

In the region $\alpha < A$, the regulator assesses a relatively low prior probability on the firm being low-cost. Note that A is always located to the left of the midpoint of the interval containing the regulator's prior, regardless of the heterogeneity in firm type. This occurs because the auditor attaches a relatively high loss to failing to detect a low-cost firm cheating. The auditor's Bayes decision against the regulator's beliefs α is $\delta^*(x) = d_h$ for x=0, 1. As a

15. The auditing region in Baron and Besanko (1984) is defined in terms of the cost parameter and is independent of the regulator's prior, reflecting the non-Bayesian nature of the model.

result, a low-cost firm computes the expected penalty for reporting high costs to be zero. Since the threat of auditing is no longer credible, the regulator effectively loses the use of the penalty instrument in this region.

In the region $\alpha > B$, the regulator's prior odds ratio on the firm being low-cost may or may not be greater than one. As a result, the boundary point B can be located to the left or right of the midpoint of the [0, 1] interval.[16] The auditor's Bayes decision against α is $\delta^*(x) = d_l$ for $x=0, 1$. If the firm reports high costs it will be penalized Z with probability one. As in the case $\alpha < A$, the regulator's design problem and solution correspond to the no auditing solution except that $(T_h - Z)$ is now equal to the transfer T_h defined by equation (8).

An important consequence of the auditor's decisions in the outer regions of the regulator's prior is that the second-best solution of Ramsey prices is not achievable. We can contrast this result with a well known result from the correlated signal literature (see Demougin and Garvie (1991), McAfee and Reny (1989) or Riordan and Sappington (1988)). If transfers are not costly, then the first-best solution is implementable with a direct revelation mechanism provided that the following four conditions hold: (i) all parties to the game are risk neutral; (ii) the signal is costless; (iii) transfer payments are unbounded; and (iv) payments can be made contingent on the observable signal. Condition (i) is relaxed when a conservative or risk averse auditor is introduced into the game. The social value of auditing is not always positive ex ante, regardless of the pecuniary costs of the audit, because the auditing procedure is imperfect and these imperfections are costly to the auditor.

16. For instance, if the firm types are very close together, then the experimental odds $(1 - \theta_l)/(1 - \theta_h)$ approach one and B will be to the left of the midpoint, regardless of the signal. B will be to the right of the midpoint whenever the ratio of losses from a Type II error to a Type I error is greater than the experimental odds in favour of the firm being low-cost.

In the region A < α < B, the auditor's decision depends upon whether a signal of high-cost or low-cost is observed. The regulator's design problem becomes[17]

$$\max_{\{(p_l, T_l), (p_h, T_h)\}} EW = \alpha\left[V(p_l) + (p_l - c_l)q(p_l) - \lambda T_l\right] + (1 - \alpha)\left[V(p_h) + (p_h - c_h)q(p_h) - \lambda(T_h - \theta_h Z)\right]$$

subject to

$$(p_l - c_l)q(p_l) + T_l \geq (p_h - c_l)q(p_h) + T_h - \theta_l Z \tag{12}$$

$$(p_l - c_l)q(p_l) + T_l \geq 0 \tag{13}$$

$$(p_h - c_h)q(p_h) + T_h - \theta_h Z \geq (p_l - c_h)q(p_l) + T_l \tag{14}$$

$$(p_h - c_h)q(p_h) + T_h - \theta_h Z \geq 0 \tag{15}$$

Proposition 5: *The solution to the regulator's design problem with auditing in the mid-region of the regulator's prior, A < α < B, can be characterized by the following set of equations:*

$$p_l^* = c_l - \frac{\lambda}{1 + \lambda}\frac{q(p_l^*)}{q'(p_l^*)} \tag{16}$$

$$T_l^* = \frac{\lambda}{1 + \lambda}\frac{q(p_l^*)^2}{q'(p_l^*)} \tag{17}$$

17. The regulator's design problem with auditing incorporates the restriction that a firm will be audited only if it reports high costs. Further, an equilibrium property of the incentive compatible mechanism is that only a high cost firm can be penalized. Any penalty actually imposed is, however, a result of the auditor's error.

$$p_h^* = c_h - \frac{\lambda}{1+\lambda}\frac{q(p_h^*)}{q'(p_h^*)} \tag{18}$$

$$T_h^* = \frac{\lambda}{1+\lambda}\frac{q(p_h^*)^2}{q'(p_h^*)} + \frac{\theta_h}{\theta_l - \theta_h}(c_h - c_l)q(p_h^*) \tag{19}$$

$$Z^* = \frac{1}{\theta_l - \theta_h}(c_h - c_l)q(p_h^*) \tag{20}$$

Proof: See Appendix.

Several characteristics of the optimal mechanism with auditing are noteworthy. First, second-best Ramsey prices are implementable in the mid-region of the regulator's prior provided that the regulator can credibly commit to employing the auditor after production has occurred. Second, there are no information rents in equilibrium. The subsidy required to induce a low-cost firm to truthfully report in the no auditing model is a pure Ramsey tax in the auditing model. Indeed, the transfer to a low-cost firm T_l^* and the expected transfer to a high-cost firm $T_h^* - \theta_h Z^*$ are both Ramsey taxes: the taxes are proportional to the inverse of the point elasticity of the demand curve. Further, the expected lump sum transfers from the two types of firm to the regulator are exactly equal to the profit differential between marginal cost pricing and Ramsey prices.[18] Third, the regulator sets the penalty Z^* such that expected profits from being incorrectly penalized are zero. Fourth, the optimal mechanism $\{(p_l^*, T_l^*), (p_h^*, T_h^*), Z^*\}$ is independent of the regulator's prior in this region. Finally, the regulator sets the penalty, transfers, and prices such that the individual rationality and incentive compatibility constraints

18. As discussed earlier, distortionary taxation of consumers through producers is optimal given the cost of public funds.

are both binding on the low-cost firm.

Proposition 6: *The auditing solution to the regulatory problem Pareto dominates the no auditing solution in the mid-region of the regulator's prior $B < \alpha < A$ ex ante.*

Proof: See Appendix.

The regulator will choose to audit in the region $B < \alpha < A$ because auditing enables the regulator to distort the transfer rather than the price to a high-cost firm.[19] Recall that in the equilibrium of the no auditing model, the regulator distorts the price for the high-cost firm above the Ramsay price in an effort to reduce the information rent to the low-cost firm. In contrast, in the auditing model, the price to the high-cost firm is fixed at the optimal Ramsey price while the transfer to the high-cost firm and the penalty adjust with the marginal information cost term $(c_h - c_l)/(\theta_l - \theta_h)$. Referring back to Figure 1, auditing enables the regulator to push the incentive compatibility constraint for the low-cost firm back down to the individual rationality constraint or zero profit level thereby eliminating the information rents.

Proposition 7 summarizes the key results obtained in the auditing region.[20]

Proposition 7: *(i) The region over which auditing is welfare-improving is nondecreasing in the heterogeneity of firm type. Alternatively, the more accurate the auditing technology, the greater the impact of the audit information.*

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19. Ex ante expected welfare is a function of price only, that is, it is independent of the transfers and penalty.

20. The existence of an auditing region and no auditing regions of the regulator's prior is a result of the auditor's binary decision rule <u>not</u> the binary support of types. Thus, the characteristics of the regulatory model with auditing are preserved in a model with a continuum of types.

(ii) Given the regulator's prior, the expected welfare gains from auditing are nondecreasing in the heterogeneity of firm type.

(iii) Given firm types, the expected welfare gains from auditing are nondecreasing in the regulator's prior on the firm being low-cost.

Proof: See Appendix.

Proposition 7 (i) states an appealing feature of the auditing model: the size of the auditing region is a function of the quality of the audit information. For instance, if there is a high degree of cost dispersion among firms, then identification of firm type is less fraught with error and the auditing region can be quite large. This follows from a key characteristic of the auditing technology itself - the precision of the auditing technology increases in the heterogeneity in firm type. In the limit, the procedure approaches a perfect auditing technology and the second-best solution can be obtained over the entire prior parameter space.[21]

Proposition 7 (ii) and (iii) follow from Lemma 2 (ii) and (iii). The former states that the value of the information obtained through an audit is a function of its quality. For instance, if the cost differential is very small, then auditing is valuable if and only if the regulator has virtually no opinion on the distribution of firm types. The latter emphasizes the role of the regulator's beliefs. If the regulator is very well informed, then an audit will add little new information and consequently will have a low value. Note that Proposition 7 (ii) and (iii) highlight the source of relative inefficiency of the standard incentive compatible mechanism with

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21. If transfers are costless and the auditing technology is perfect and costless, then the first-best or symmetric information solution can be obtained. The regulator will set the penalty for cheating at least equal to the information rents and price equal to marginal cost for both types of firms. Firms reporting high cost will be audited with probability one and the information rents to the low cost firm will be reduced to zero.

no auditing - the marginal information cost term.[22]

§6. Conclusions

The paper examines two alternative mechanisms for regulating a single firm with unknown costs when raising and transferring public funds is costly. The mechanisms are differentiated on the basis of the information sets upon which the ex ante pricing rules are conditioned. The first mechanism, a standard incentive compatible solution to the regulatory problem, relies solely upon information available to the regulator ex ante. The regulator can incorporate ex post information in the design of the second mechanism: namely, the probabilistic outcome of a conservative auditor's decision regarding the unknown efficiency parameter or firm type. The characteristics of the solutions are compared and contrasted.

Several aspects of the solutions are of interest. First, the assumption of costly public funds effectively transform the firm into a taxation instrument and, as such, Ramsey prices emerge as the optimal pricing rules. The Ramsey prices are, however, modified somewhat to correct for asymmetric information. In the standard mechanism with no auditing, the firm is a vehicle for raising and transferring funds to induce truth-telling as well as to finance the government. To achieve these ends, the two-part Ramsey pricing contract offered to a high-cost firm has an additional marginal information cost term. As marginal information costs increase, the price offered to an inefficient firm is increasingly distorted. The optimal tradeoff between

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22. The relative inefficiency of the no auditing mechanism derives from the method of reducing the information rents to the low-cost firm - increasing price distortion. Recall that the modified Ramsey price offered to a high-cost firm has a built-in marginal information cost term which increases in the prior. As the regulator becomes less uncertain about firm type, this price is increasingly distorted above the Ramsey price until it reaches the choke price.

production efficiency losses and reduction in information rents required to induce truth-telling results in a standard mechanism with 'no production distortion at the top' and 'no profit distortion at the bottom'.

Second, although the auditing and regulatory functions are separated, the Bayesian auditing procedure is endogenous to the economic model: a client relationship between the regulator and auditor is established by having the auditor adopt the regulator's prior beliefs about firm type and the precision of the auditing signal depends upon the degree of cost dispersion. The Bayesian auditing procedure is a risk minimizing strategy for the auditor and as such results in auditing and no auditing regions of the regulator's prior.

Third, the value of auditing is not always positive ex ante, regardless of its costs. The regulator cannot reduce the information asymmetries in the outer regions of his prior by ordering an audit of costs after production has occurred. This result derives from the conservative nature of the auditor which leads to a deterministic auditing procedure in these regions.

Finally, second-best is implementable in the auditing region of the regulator's prior. Incorporating the probabilistic outcome of an audit allows the regulator to discriminate successfully between the two types in the mid-region of his prior. As the auditing technology approaches perfect accuracy, the second-best solution can be implemented over the entire range of the regulator's prior.

Thus, the choice of mechanism for maximizing expected welfare depends upon the strength of the regulator's prior beliefs about firm type and auditing precision. In general, if the regulator has fairly strong prior beliefs about the firm's type or the auditing technology is relatively noisy, then additional information obtained by auditing can have little or no value.

We can logically extend this result to conclude that if auditing is costly, then the regulator should choose a standard incentive compatible mechanism to maximize expected welfare. However, if the regulator is relatively poorly informed ex ante or the auditing technology is relatively precise, then employing an auditor can provide large potential gains in welfare.

In a revelation game, auditing is essentially a commitment to procedural rationality. In other words, by announcing an auditing strategy the regulator commits to an ex ante rational procedure. It is a method of obtaining truth-telling at a lower cost. However, a question still arises as to the appropriateness of examining auditing in a revelation game when the identity of the firm is known by all the players at the time the audit is performed. In this sense, auditing is not ex post rational. An important extension of this paper would be to examine the impact of conservative auditing procedures in a structural model which allows cheating in equilibrium.

Appendix

Proof of Lemma 1: Rearranging equations (1) and (3) in the text,

$$[q(p_h) - q(p_l)]c_h \leq [p_h q(p_h) + T_h] - [p_l q(p_l) + T_l] \leq [q(p_h) - q(p_l)]c_l$$

Since $c_h > c_l$, it follows that $[q(p_h) - q(p_l)] < 0$. Inverting the demand function yields the required result. **Q.E.D.**

Proof of Lemma 2: (i) The following pair of inequalities hold for an imperfect auditing technology

$$p(X = 1|\theta_l) > p(X = 1) > p(X = 1|\theta_h) \qquad \textbf{(A1)}$$

$$p(X = 0|\theta_l) < p(X = 0) < p(X = 0|\theta_h) \qquad \textbf{(A2)}$$

where (A1) $\Leftrightarrow$ (A2). Given (A1) and (A2), it follows that the relative likelihood ratio $p(X=1|\theta_l)/p(X=1)$ is greater than one. Applying Bayes Theorem yields the required result that $p(\theta_l|X=1) > p(\theta_l)$ and $p(\theta_h|X=1) < p(\theta_h)$.
(ii) The probability of observing a signal of low-cost, $p(X=1)$, is a weighted average of $p(X=1|\theta_l)$ and $p(X=1|\theta_h)$,

where the weights are given by the regulator's beliefs. For any given pair of conditional probabilities where $p(X=1|\theta_l) > p(X=1|\theta_h)$, a stronger prior belief that the firm is low-cost leads to a higher probability of observing a signal of low-cost. But a stronger prior belief that the firm is low-cost also results in a smaller relative likelihood ratio $p(X=1|\theta_l)/p(X=1)$. From Bayes Theorem it follows that the stronger is the prior belief, the smaller the distance between posterior and prior beliefs.

(iii) For any given pair of conditional probabilities $p(X=1|\theta_l)$ and $p(X=1|\theta_h)$, increasing heterogeneity in firm types results in a larger relative likelihood ratio $p(X=1|\theta_l)/p(X=1)$. The required result follows from Bayes Theorem. **Q.E.D.**

Proof of Proposition 2: Suppose the auditor observes a signal of high costs. The auditor will decide the firm is low-cost if and only if the expected losses from this decision are less than the expected losses from deciding the firm is high-cost. Algebraically, $\delta^*(0) = d_l$ if and only if $a(1 - \alpha)(1 - \theta_h) < b\alpha(1 - \theta_l)$ or, by rearranging, $\alpha > B$. Similarly, if the auditor observes a signal of low-cost, then $\delta^*(1) = d_l$ if and only if $a(1 - \alpha)\theta_h < b\alpha\theta_l$ or, by rearranging, $\alpha > A$. **Q.E.D.**

Proof of Proposition 3: With knowledge of its type and the auditing procedure, the firm is able to compute the expected penalty as

$$\xi(\theta_i) = \sum_{x=0}^{1} p(X = x|\theta_i)p(\delta^*(x) = d_l)$$

where $i=h, l$ and $p(\delta^*(x) = d_l)$ is the probability the auditor will decide that the firm is low-cost. Equations (10) and (11) in the text are used to compute $p(\delta^*(x) = d_l)$. **Q.E.D.**

Proof of Proposition 4: The Lagrangian for this design problem may be written as[23]

23. Using a standard technique, we assume that constraints (12) and (15) are binding and that constraints (13) and (14) are slack and then check that the solution is globally incentive compatible. Note that this gives the standard solution of no production distortion at the top (second-best production for the most efficient type) and no profit distortion at the bottom (zero information rents for the least efficient type).

$$\max_{\{(p_l,T_l),(p_h,T_h),Z\}} \mathcal{L} = \alpha\left[V(p_l) + (1+\lambda)(p_l - c_l)q(p_l)\right] + (1-\alpha)\left[V(p_h) + (1+\lambda)(p_h - c_h)q(p_h)\right]$$
$$+ (\alpha\lambda + \mu)\left[\left[\frac{\theta_l}{\theta_h} - 1\right]\left(T_h + (p_h - c_h)q(p_h) - (c_h - c_l)q(p_h)\right)\right]$$

where μ is the Kuhn-Tucker multiplier on the individual rationality constraint for a low-cost firm, equation (13) in the text. The first order conditions for the constrained design problem are

$$p_h = c_h - \frac{\lambda(1-\alpha) + (\alpha\lambda + \mu)\left[\dfrac{\theta_l}{\theta_h} - 1\right]}{(1+\lambda)(1-\alpha) + (\alpha\lambda+\mu)\left[\dfrac{\theta_l}{\theta_h} - 1\right]}\frac{q(p_h)}{q'(p_h)} + \frac{(\alpha\lambda+\mu)(c_h - c_l)}{(1+\lambda)(1-\alpha) + (\alpha\lambda+\mu)\left[\dfrac{\theta_l}{\theta_h} - 1\right]} \quad \text{(A3)}$$

$$(\alpha\lambda + \mu)\left[\frac{\theta_l}{\theta_h} - 1\right] = 0 \quad \text{(A4)}$$

$$\mu\left[-T_l - (p_l - c_l)q(p_l)\right] = 0,\ \mu \leq 0 \quad \text{(A5)}$$

and equation (5) in the text defines p_l^*. Since $\theta_l > \theta_h$, we have from (A4) that the individual rationality constraint for a low-cost firm is binding in equilibrium. Hence, p_h reduces to the standard Ramsey pricing rule. **Q.E.D.**

Proof of Proposition 6: Expected welfare for both models can be written as

$$EW = \alpha\left[V(p_l^*) - \lambda\frac{q(p_l^*)^2}{q'(p_l^*)}\right] + (1-\alpha)\left[V(p_h^*) - \lambda\frac{q(p_h^*)^2}{q'(p_h^*)}\right]$$

Since the equilibrium price offered to a high-cost firm is strictly less in the auditing model, expected welfare is strictly greater. **Q.E.D.**

Proof of Proposition 7: (i) Follows from Lemma 4.

(ii) While the equilibrium price for a high-cost firm is fixed in the auditing model, it is nondecreasing in cost differences in the no auditing model.

(iii) While the equilibrium price for a high-cost firm is fixed in the auditing model, it is nondecreasing in the regulator's prior in the no auditing model. **Q.E.D.**

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