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Tariffs in a Dynamic Differentiated Duopoly

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ABSTRACT

The effects of an unilateral tariff on prices, quantities, and welfare in the presence of dynamic elements facing imperfectly competitive firms is examined. International duopolists, selling differentiated products, interact strategically in a price setting game under the tariff policy. It is shown that the presence of dynamic elements in the form of adjustment costs to changing sales volume significantly affect the relationship between tariff rates and aggregate domestic welfare even when sales are constant and no adjustment costs are actually paid. The level of optimal tariffs is affected by the size of adjustment costs with lower optimal tariffs associated with higher adjustment costs. Of particular interest is the possibility that autarky is optimal under positive adjustment costs while tariff restricted trade is optimal in the absence of such costs.

I. INTRODUCTION

In the past decade, a number of papers have examined the effects of tariffs and other protectionist policies in a variety of static environments with imperfectly competitive markets. Only a very small number of papers, however, have considered the effects of these policies in environments with dynamic elements. Recent papers incorporating some form of dynamics include Dockner and Lansing(1991), Dockner and Haug(1990) and Grossman and Helpman(1990). This paper examines the effects of unilateral ad valorem tariffs on prices, quantities, and aggregate domestic welfare in the presence of dynamic elements facing price-setting firms selling differentiated products. It is shown that the incorporation of dynamic considerations have a significant effect on the policy implications of tariffs in these environments.

Many of the papers which consider tariffs under imperfect competition with multiple firms fall into one of two categories. One group employs environments incorporating oligopolistic market structures with strategically interacting firms selling a homogeneous product. Firms interact strategically in these models in the sense that the choices of other firms directly affect the choices of a firm and best response functions are used to capture firms' optimal responses to their competitors behavior, (examples include Cournot and Bertrand competition)[1]. A second group employs environments incorporating monopolistically competitive market structures with firms selling differentiated products but not interacting strategically. Firms do not interact strategically in these models as large number assumptions imply constant demand elasticities and, therefore, constant mark-up pricing. In particular, best response functions are not used in these environments (examples include models with constant elasticity of substitution preferences).[2]

Few papers in this area have considered environments in which firms both interact strategically and sell differentiated products. Exceptions include the static models of Schmitt(1990) and Lancaster(1984). These environments are particularly interesting as they provide a framework for analyzing the effects of tariffs when welfare is affected not only by the *strategic* price setting behavior of firms but also by the *range of variety* of products available as consumers desire variety. This paper examines the effects of tariffs on equilibria and the properties of optimal tariffs when strategic behavior, preferences for variety, *and* dynamic elements are present.

A static environment is considered first as the benchmark case. Dynamic elements are then incorporated by introducing adjustment costs to changing market shares. A solution method for solving the dynamic price-setting game between international duopolists is described. The solution method allows numerical calculation of aggregate domestic welfare levels under various tariff rates and, therefore, analyses of optimal tariffs.

It is demonstrated that the presence of adjustment costs significantly affects the relationship between tariff rates and aggregate domestic welfare even when sales are constant and no adjustment costs are actually paid. This result is a consequence of the strategic interaction between firms and would not be present if firms did not price strategically. This illustrates that the presence of dynamic elements in strategic markets significantly alters the properties of optimal tariffs and resulting policy prescriptions.

The remainder of the paper is organized as follows. Section II presents the benchmark case of a static environment. Section III examines the effects of dynamic elements on the impact of tariffs on prices, quantities, and domestic welfare with particular focus on the properties of optimal tariffs in these environments. Section IV concludes.

II. THE STATIC ECONOMY

There are two countries, a domestic country denoted country 1 and a foreign country denoted country 2. In a differentiated product industry, there is a single domestic firm selling its product in the domestic market. The domestic firm may face competition from a foreign producer selling a differentiated product in the domestic market. The analysis presented below considers the effects on prices, quantities, and domestic welfare of an ad valorem tariff imposed on imports of the foreign produced differentiated good.

II.1. The Economy

Commodities

There are three classes of goods: a traded homogeneous good, non-traded goods produced in each country, and differentiated goods produced in a differentiated duopoly composed of a domestic firm, firm 1, and a foreign firm, firm 2. Following Salop(1979), the characteristics space of potentially producible differentiated commodities is the unit-circumference circle. The differentiated goods are located symmetrically around the circle so that the distance around the circle between the domestic good, good 1, and the foreign good, good 2, equals .5. The homogeneous good is an endowment good and a non-traded good is produced in perfectly competitive industries under constant-returns-to scale in each country.

Preferences

Domestic Consumers

Consumers in the domestic country are indexed by $i \in [0,1)$ according to their most preferred commodity in the space of differentiated goods. Consumers are uniformly distributed in the commodity space with unit density, implying that the measure of domestic consumers is one.

The consumption possibilities set for domestic consumers is

$$X_1 = \{m_1 \in \mathbb{R}_+,\ c_1 \in \mathbb{R}_+,\ \forall k=1,2,\ \mu_k \in \{0,1\}:\ \sum_{k=1}^{2} \mu_k \leq 1\}$$

A consumption bundle for domestic consumer i is represented by $(m_{1i}, c_{1i}, \mu_{ki}) \in X_1$. Here m_{1i} denotes consumption of the endowment good, c_{1i} denotes consumption of the domestically produced non-traded good and μ_{ki} denotes consumption of the k^{th} differentiated commodity, k=1,2. The restrictions, $\mu_k \in \{0,1\}$ and $\sum_{k=1}^{2} \mu_k \leq 1$, limits consumers to purchase, at most, one unit of a single differentiated good and zero units of the other differentiated good.

Domestic consumer $i \in [0,1)$ has preferences over consumption bundles ordered by the following utility function:

$$U_i(m_{1i}, c_{1i}, \mu_i) = m_{1i} + c_{1i} + \sum_{k=1}^{2} [\upsilon - d\Lambda(i,k)]\mu_{ki}$$

where $\mu_i \equiv \{\mu_{ki}\}_{k=1}^{2}$. Here υ is the utility associated with consumption of a consumer's ideal good. $\Lambda(i,k)$ denotes the shortest distance around the circle between points i and k, and $d\Lambda(i,k)\mu_{ki}$ is the disutility associated with consuming a good other than the consumer's ideal good. Note that d is a measure of the degree of product substitutability of goods within the differentiated industry with lower d associated with products which are perceived to be highly substitutable.

Foreign Consumers

Foreign consumers are identical and have preferences over the homogeneous good and the foreign non-traded good as follows:

$$U(m_2, c_2) = m_2 + c_2$$

Endowments

Each consumer in each country j=1,2, is endowed each period with ω_j units of the endowment good and one unit of time. Time is supplied inelastically and is used to produce the differentiated good and the non-traded good produced in country j.

Technologies

Non-Traded Goods Production

The technology for producing the non-traded good in country j is:

$$x_{jt} = \theta_{jt} n_{jt} \qquad \forall j=1,2, \quad \forall t$$

Here n_{jt} is labor input into production of the non-traded good in country j and θ_{jt} is a country-specific, time-varying production technology parameter. Distributions for the production technology parameters are specified in Section III.

Differentiated Goods Production

The technology for producing differentiated goods is identical across countries and is linear in labor input, ℓ_{jt}. The technology for differentiated goods production is given by

$$y_{jt} = \left[\frac{1}{\phi}\right]\ell_{jt} \qquad \forall j=1,2, \quad \forall t, \quad 0<\phi<1$$

II.2 Market Arrangements

Domestic Consumers:

The quasi-linear structure of preferences for domestic consumers implies that in an interior equilibrium, the interest rate will be constant. Furthermore, this quasi-linearity implies that any intertemporal consumption smoothing requires varying consumption of the homogeneous good or the non-traded good. However, since utility is linear in these goods, a consumer is indifferent as to when the good is consumed. As consumers receive no benefit from intertemporal consumption smoothing, attention is restricted to interior equilibria with no borrowing or lending. Interior solutions are guaranteed by making endowments of the homogeneous good sufficiently large.

Therefore, domestic consumer i at time t faces the following maximization problem:

$$\max_{m_{1it},\, c_{1it},\, \mu_{it}} \quad m_{1it} + c_{1it} + \sum_{k=1}^{2} [\upsilon - d\Lambda(i,k)]\mu_{kit}$$

$$\text{subject to} \quad m_{1it} + r_{1t}c_{1it} + p_{1t}\mu_{1it} + \tilde{p}_{2t}\mu_{2it} \le \omega_1 + w_{1t} + \Pi_{1t} + TR_t$$

The homogeneous good is the numeraire good. r_{1t} is the price of the domestic non-traded good, p_{1t} is the price of the domestic differentiated good, and $\tilde{p}_{2t}$ is the tariff distorted price of the foreign differentiated good in units of the homogeneous good. That is, letting p_{2t} denote the price set by the foreign producer, then $\tilde{p}_{2t} = p_{2t}(1+\tau)$ under an ad valorem tariff at rate τ.

Consumers receive income each period from four sources: endowment of the homogeneous good, ω_1; wage income, w_{1t}; profits from ownership of the domestic differentiated firm, Π_{jt}; and tariff revenues, TR_t. Note that, because the measure of domestic consumers is one, per capita profits and tariff revenues equal their aggregate levels.

Parameters are restricted so that equilibrium prices induce all domestic consumers to purchase a differentiated product (υ large). Therefore, a domestic consumer's problem can be separated into two stages. In the first stage, consumers choose which differentiated product to consume. In the second stage, consumers allocate their remaining income between the homogeneous good and the non-traded good produced in their country.

Foreign Consumers:

Each foreign consumer faces the following maximization problem at each date t:

$$\max_{m_{2t},c_{2t}} \quad m_{2t} + c_{2t}$$

$$\text{subject to } m_{2t} + r_{2t}c_{2t} \le \omega_2 + w_{2t}$$

where w_{2t} is the foreign wage at time t and r_{2t} is the price of the foreign non-traded good at time t in units of the homogeneous good at time t. Since the homogeneous good and the foreign non-traded good are perfect substitutes, the foreign consumer will consume only the cheaper good in equilibrium. If the goods have the same price, the distribution of a foreign consumer's income across the two goods is indeterminate.

Non-Traded Goods Production

Perfectly competitive firms in country j producing non-traded goods seek to maximize discounted profits. Since there are no dynamic factors affecting firms' profits, allocations resulting from maximizing profits each period will be the same as those resulting from maximizing discounted profits. Therefore, the producers of the non-traded good in country j face the following maximization problem:

$$\max_{x_{jt}} \quad r_{jt}x_{jt} - w_{jt}n_{jt}$$

$$\text{subject to} \quad x_{jt} = \theta_{jt}n_{jt}$$

Preferences and technologies imply that in an interior equilibrium with all goods produced, the following conditions must hold:

$$r_{jt} = 1 \qquad \forall j=1,2, \quad \forall t$$

$$r_{jt}\theta_{jt} = \theta_{jt} = w_{jt} \qquad \forall j=1,2, \quad \forall t$$

Note that equilibrium wages are determined by the technology parameters in non-traded goods production in each country. In particular, the behavior of differentiated firms will have no affect on the equilibrium wage in an interior solution. In what follows, attention is restricted to interior equilibria.

Differentiated Goods Production

The demand functions facing differentiated producers will depend on their own price, their competitor's price, and the ad valorem tariff rate, τ. Define $\tilde{y}_t$ as the shortest distance around the unit circle between the domestic firm and the domestic consumer indifferent between purchasing the domestic good and purchasing the foreign good. Then $\tilde{y}_t$ is implicitly defined by

$$\tilde{y}_t = ((1+\tau)p_{2t} - p_{1t} + .5d)/2d.$$

Note that $\tilde{y}_t$ denotes the measure of domestic consumers in one direction around the circle from the domestic firm who purchase the domestic good at time t. Therefore, the measure of domestic consumers who purchase the domestic product at time t under an ad valorem tariff is

$$y_{1t} = 2\tilde{y}_t = ((1+\tau)p_{2t} - p_{1t})/d + .5.$$

Symmetrically, the measure of domestic consumers who purchase the foreign good at time t under an ad valorem tariff is

$$y_{2t} = 1 - y_{1t} = (p_{1t} - (1+\tau)p_{2t})/d + .5.$$

Given these demand functions, each firm j=1,2, faces the following concave maximization problem at time t:

$$\max_{p_{jt}} \quad (p_{jt} - \phi w_{jt})y_{jt}$$

subject to y_{jt} given above.

Note that the firms' problems are static. The first-order necessary conditions for an interior solution implicitly define each firms best response function: $\hat{p}_{jt}(p_{it}, w_{jt}, \tau)$, $i \neq j$. Solving the system of best response functions gives Nash equilibrium prices, market shares, domestic profits, and tariff revenues as functions of the tariff rate under ad valorem tariffs as follows.

Under Ad Valorem Tariffs:

$$
\begin{aligned}
p^v_{1t}(\tau) &= (2/3)(\phi w_{1t}) + (1/3)(\phi(1+\tau)w_{2t}) + .5d \\
p^v_{2t}(\tau) &= (1/3)(\phi w_{1t}/(1+\tau)) + (2/3)(\phi w_{2t}) + .5d/(1+\tau) \\
y^v_{1t}(\tau) &= (\phi/3d)((1+\tau)w_{2t} - w_{1t}) + .5 \qquad (1) \\
y^v_{2t}(\tau) &= 1 - y^v_{1t}(\tau) \\
\Pi^v_{jt}(\tau) &= [p^v_{jt}(\tau) - \phi w_{jt}]y^v_{jt}(\tau) \qquad \text{for } j=1,2 \\
TR^v_t(\tau) &= \tau p^v_{2t}(\tau) y^v_{2t}(\tau)
\end{aligned}
$$

These equilibrium equations illustrate that increases in the tariff rate which continue to allow foreign competition (foreign profits are non-negative under the tariff) lead to an increase in the domestic firm's price, output, and profits and a decrease in the foreign firm's price, output, and profits.

Setting $\tau=0$ in the above set of equations, gives free trade equilibrium prices, outputs, profits and tariff revenues as follows:

Under Free Trade:

$$
\begin{aligned}
p^f_{1t} &= (2/3)(\phi w_{1t}) + (1/3)(\phi w_{2t}) + .5d \\
p^f_{2t} &= (1/3)(\phi w_{1t}) + (2/3)(\phi w_{2t}) + .5d \\
y^f_{1t} &= (\phi/3d)(w_{2t} - w_{1t}) + .5 \qquad (2) \\
y^f_{2t} &= 1 - y^f_{2t} \\
\Pi^f_{jt} &= [p^f_{jt} - \phi w_{jt}]y^f_{jt} \qquad \text{for } j=1,2
\end{aligned}
$$

$TR^f_t = 0$

Define the prohibitive tariff rate at time t as the tariff rate such that $\Pi^v_{2t}(\tau)=0$. This gives the following prohibitive tariff rates $\forall t$:

$$\tau^p_t = w_{1t}/w_{2t} + (1.5d)/(\phi w_{2t}) - 1 \tag{3}$$

Note that τ^p_t also gives the foreign firm zero market share, i.e. $y^v_{2t}(\tau^p_t)=0$. Equation (3) implies that the prohibitive tariff rate is increasing in domestic wages and in the productivity of labor in differentiated goods production. The prohibitive rate is decreasing in foreign wages and in the degree of product substitutability. Therefore, as expected, the tariff rate at which the foreign firm will not enter will be low when the foreign firm's costs are relatively high and when the firm has a low degree of market power (goods are close substitutes).

If the tariff rate is set at its prohibitive level, then the domestic firm behaves as a domestic monopolist. Assuming that $w_{1t} \leq \upsilon - d$ $\forall t$, guarantees that the monopolist will fully serve the market in every period and will price so as to just capture the consumer who is farthest away from the firm on the unit circle. Therefore, we have

Under Autarky:

$$\begin{aligned} p^a_{1t} &= \upsilon - .5d \\ y^a_{1t} &= 1 \\ y^a_{2t} &= 0 \\ \Pi^a_{1t} &= [p^a_{1t} - \phi w_{1t}] \\ TR^a_t &= 0 \end{aligned} \tag{4}$$

II.3: Welfare and Optimal Tariffs

II.3.1: Aggregate Welfare

Under Free Trade:

Under free trade, each domestic consumer who purchases good j at time t will have remaining income equal to:

$$\omega_1 + w_{1t} + \Pi^f_{1t} - p^f_{jt}.$$

This remaining income is used for consumption of the homogeneous good and the domestic non-traded good at time t. Since the measure of consumers purchasing good j at time t equals y^f_{jt}, aggregate domestic welfare at time t under free trade is given by:

$$W^f_t = y^f_{1t}\left[\omega_1 + w_{1t} + \Pi^f_{1t} - p^f_{1t}\right] + 2\int_0^{.5y^f_{1t}} (\upsilon-xd)dx \quad +$$

$$y^f_{2t}\left[\omega_1 + w_{1t} + \Pi^f_{1t} - p^f_{2t}\right] + 2\int_0^{.5y^f_{2t}} (\upsilon-xd)dx$$

The first two terms in W^f_t are the aggregate utility of consumers who purchase the domestic good and the last two are the aggregate utility of consumers who purchase the foreign good. Integrating allows aggregate domestic welfare at time t under free trade to be written as follows:

$$W^f_t = \omega_1 + \upsilon - .25d + w_{1t}(1-\phi y^f_{1t}) + (.5dy^f_{1t} - p^f_{2t})y^f_{2t}, \tag{5}$$

where p^f_{2t}, y^f_{1t}, and y^f_{2t} are given in equation (2).

Under Tariff Restricted Trade:

By a similar argument to that made under free trade, aggregate domestic welfare at time t under tariffs with foreign production ($\tau<\tau^p_t$) can be shown to equal:

$$W^v_t(\tau)= \omega_1 + \upsilon - .25d + w_{1t}(1-\phi y^v_{1t}(\tau)) + (.5dy^v_{1t}(\tau)- p^v_{2t}(\tau))y^v_{2t}(\tau) \qquad (6)$$

where $p^v_{2t}(\tau)$, $y^v_{1t}(\tau)$, and $y^v_{2t}(\tau)$ are given in equation (1).

Under Autarky:

If the foreign firm does not sell in the domestic market, then the domestic firm prices according to equation (4) and aggregate domestic welfare at time t equals

$$W^a_t = \omega_1 + \upsilon - .25d + w_{1t}(1-\phi) \qquad (7)$$

II.3.2: Optimal Tariffs

Consider the case in which the production technology parameters on non-traded goods production are constant: $\theta_{jt}=\theta_j$ $\forall t$, for j=1,2. This implies that wages are constant: $w_{jt}=w_j$ $\forall t$, for j=1,2. In this case, $W^v_t(\tau) = W^v(\tau)$ and $\tau^p_t=\tau^p$, $\forall t$.

Using equations (1) and (6), it can be shown that:

$$\partial W^v(\tau)/\partial\tau|_{\tau=0} = [\phi^2(w_1-w_2)^2]/9d + \phi w_1/3 + .25d > 0.$$

This implies that if the foreign firm enters under free trade, then a small positive ad valorem tariff will be welfare improving over free trade.

Let $\tilde{\tau}$ be implicitly defined as follows:

$$\partial W^v(\tilde{\tau})/\partial\tau = 0.$$

Using the Nash equilibrium pricing functions and market shares from equation (1), and the definition of $W^v(\tau)$ from equation (6), then $\tilde{\tau}$ is implicitly defined by the following cubic equation:

$$(w_2)^2(\tilde{\tau})^3 + w_2(w_2+2w_1)(\tilde{\tau})^2 + w_2(4w_1-w_2)(\tilde{\tau}) = (w_1-w_2)^2 + (3d/\phi)[w_1+(.75d)/\phi] \qquad (8)$$

For $0\leq\tau\leq\tau^p$, it can be shown that $W^v(\tau)$ is a strictly concave function.

Therefore, if $0<\tilde{\tau}<\tau^P$, then $\tilde{\tau}$ maximizes aggregate domestic welfare in the presence of foreign competition.

Therefore, the optimal ad valorem tariff rate in the constant wage case, τ^*, is given below:

$$\tau^* = \begin{cases} 0 \text{ if } (w_2-w_1)\geq(3/2)(d/\phi) & \text{(i)} \\ \tilde{\tau} \text{ if } \tilde{\tau}<\tau^P \text{ and } (w_2-w_1)<(3/2)(d/\phi) & \text{(ii)} \\ \tau^P \text{ if } \tilde{\tau}\geq\tau^P & \text{(iii)} \end{cases} \qquad (9)$$

If $(w_2-w_1)\geq(3/2)(d/\phi)$, then the foreign firm will not enter even in the absence of tariffs. Therefore, in cases (i) and (iii), the economy is in autarky, the domestic producer sets prices according to equation (4), and aggregate domestic welfare is given by equation (7). In case (ii), the domestic and foreign firm compete and set prices according to equation (1).

It can be shown that a sufficient condition for the optimal tariff rate under duopoly, $\tilde{\tau}$, to be increasing in d and decreasing in ϕ is $4w_1>w_2$. The optimal tariff rate under duopoly may be increasing or decreasing in domestic and foreign wages depending on economy parameters.

Given the implicit definition of the optimal ad valorem tariff rate under foreign competition, $\tilde{\tau}$, given by equation (8), it is difficult to further analyze optimal tariff levels and aggregate welfare under optimal tariffs analytically. Numerical methods are employed to examine the properties of optimal tariffs. Figure 1 illustrates the relationship between economy parameters and optimal tariffs. The welfare function is flat at the autarky level of welfare given by equation (7) for tariff rates above the prohibitive rate, τ^P.

The figure illustrates that it may be optimal to prohibit foreign competition when domestic wages are relatively low, when products are highly substitutable (low d), and when labor is highly productive in production of the differentiated good (low ϕ). In these cases, the gains from trade are negative as the domestic monopoly can produce at low enough cost so that the rent-shifting motive for restricting trade outweighs the consumers' desire for variety motives for allowing trade.

Figures 1.C and 1.D also illustrate that the optimal tariff rate is decreasing in the degree of product substitutability and increasing in the productivity of labor in non-traded goods production (as $4w_1>w_2$). For the economy parameters considered in this experiment, the optimal tariff rate is slightly decreasing in domestic wages and in foreign wages.

If wages are not constant, then prices will perfectly adjust to their new equilibrium levels and the previous results will continue to hold each period. In the next section, dynamic elements are introduced into the differentiated producers' maximization problems. The effects of tariff policy and the structure of optimal tariffs are analyzed in those environments and compared to the static results presented here. It is shown that dynamic considerations significantly alter the optimal tariff structure.

III. THE DYNAMIC ECONOMY

III.1 Introducing Dynamics

In this section, dynamic elements are incorporated into the differentiated firms' maximization problems by introducing adjustment costs to changing market shares. These costs can be thought of as advertising costs, costs of altering the size of the firm's sales and distributional forces, etc.

Suppose that each firm j, j=1,2, faces the following quadratic adjustment costs to changing sales in the domestic market:

$$C(y_{jt}-y_{jt-1}) = .5\alpha(y_{jt}-y_{jt-1})^2, \qquad \forall t,\ \alpha\geq 0.$$

The effects of tariff policy and the properties of optimal tariffs are examined when firms play a dynamic price setting game with $\alpha>0$. The results are compared to those derived in the static case presented in the previous section where $\alpha=0$.

III.2 Determining Equilibrium Pricing Functions

In this section, a solution algorithm for determining time-invariant Nash equilibrium pricing functions as functions of the state (last period's prices), current and future wages, and the constant tariff rate is described. Determination of these pricing functions allows determination of market shares for the domestic and foreign firm and of aggregate domestic welfare at time t as a function of the state, current and future wages, and the tariff rate. As will be discussed below, the complexity of these time-invariant functions necessitates employing numerical methods for analyzing the relationships between economy parameters and optimal tariffs.

Equilibrium pricing functions which maximize firms' discounted stream of profits are determined by solving the dynamic game backward from time T. As demonstrated below, taking the limiting solution of this game as $T\to\infty$ gives the time-invariant pricing functions desired. To illustrate the effects of dynamics alone, firms are assumed to have perfect foresight of future wages. The solution method allows for stochastic wages but the results are not significantly altered by the incorporation of uncertainty.

Consider the problem of the domestic firm at time T. The firm chooses its price at time T to solve the following maximization problem.

$$\max_{p_{1T}} \ (p_{1T}- u_{1T})y_{1T} - .5\alpha(y_{1T}-y_{1T-1})^2$$

$$\text{subject to } y_{1T}= [\tilde{p}_{2T}- p_{1T}]/d + .5$$

where $u_{1t} \equiv \phi w_{1t} = \phi \theta_{1t}$ and $\tilde{p}_{2t} \equiv p_{2t}(1+\tau)$, $\forall t$. A first order necessary condition for a maximum to this problem gives the domestic firm's best response function at time T:

$$p_{1T} = [(d+\alpha)\tilde{p}_{2T} + du_{1T} + \alpha(p_{1T-1} - p_{2T-1}) + .5d^2]/(2d+\alpha) \qquad (10)$$

Consider the problem of the foreign firm at time T. The firm chooses its price at time T to solve the following maximization problem:

$$\max_{p_{2T}} \quad (p_{2T} - u_{2T})y_{2T} - .5\alpha(y_{2T} - y_{2T-1})^2$$

$$\text{subject to } y_{2T} = [p_{1T} - \tilde{p}_{2T}]/d + .5$$

where $u_{2t} \equiv \phi w_{2t} = \phi \theta_{2T}$ $\forall t$. A first order necessary condition for a solution to this problem gives the foreign firm's best response function at time T:

$$\tilde{p}_{2T} = [(d+\tilde{\alpha})p_{1T} + d\tilde{u}_{2T} + \tilde{\alpha}(\tilde{p}_{2T-1} - p_{1T-1}) + .5d^2]/(2d+\tilde{\alpha}) \qquad (11)$$

where $\tilde{\alpha} = \alpha(1+\tau)$ and, $\tilde{u}_{2t} = u_{2t}(1+\tau)$ $\forall t$.

Combining the best response functions given by equations (10) and (11) gives Nash equilibrium prices at time T as a function of prices at time T-1 and wages at time T. Define the following:

$$p_{T-1} \equiv p_{1T-1} - \tilde{p}_{2T-1} \qquad u_T \equiv u_{1T} - \tilde{u}_{2T} \qquad Z_1 \equiv 3d + \alpha + \tilde{\alpha}.$$

Then Nash equilibrium prices for the domestic firm at time T as a function of the state, p_{T-1}, and current wage differences, u_T, can be written as follows:

$$p^*_{1T}(p_{T-1}, u_T) = A^1_{11}p_{T-1} + B^1_{111}u_T + u_{1T} + C^1_{11} + .5d \qquad (12)$$

where $\quad A^1_{11} = (2\alpha - \tilde{\alpha})/Z_1 \qquad B^1_{111} = -(d+\alpha)/Z_1 \qquad C^1_{11} = 0$

Symmetrically, tariff-adjusted Nash equilibrium prices for the foreign firm at time T are written as follows.

$$\tilde{p}^*_{2T}(p_{T-1}, u_T) = -A^2_{11}p_{T-1} - B^2_{111}u_T + \tilde{u}_{2T} + C^2_{11} + .5d \qquad (13)$$

where $\quad A^2_{11}=(2\tilde{\alpha}-\alpha)/Z_1 \qquad B^2_{111}=-(d+\tilde{\alpha})/Z_1 \qquad C^2_{11}=0$

Focusing on the domestic firm, these pricing functions imply the following market share function for the domestic firm at time T:

$$y^*_{1T}(p_{T-1},u_T) = \left[-(A^1_{11}+A^2_{11})p_{T-1} - (1+B^1_{111}+B^2_{111})u_T + (C^2_{11}-C^1_{11})\right]/d + .5$$

In addition, the change in market share for the domestic firm at time T is given by

$$y^*_{1T}(p_{T-1},u_T)-y_{1T-1} = [(1-A^1_{11}-A^2_{11})p_{T-1} - (1+B^1_{111}+B^2_{111})u_T + (C^2_{11}-C^1_{11})]/d$$

Let $V^*_{1T}(p_{T-1},u_T)$ denote Nash equilibrium profits for the domestic firm at time T as a function of the state, p_{T-1}, and current wage differences, u_T. That is,

$$V^*_{1T}(p_{T-1},u_T) = (p^*_{1T}(p_{T-1},u_T)-u_{1T})y^*_{1T}(p_{T-1},u_T) - .5\alpha(y^*_{1T}(p_{T-1},u_T)-y_{1T-1})^2 \tag{14}$$

Then, using the above equations, the derivative of the domestic firm's Nash profits at time T with respect to the domestic firm's prices at time T-1 is given by:

$$\partial V^*_{1T}/\partial p_{1T-1} = \left(1/d\right)\left(\gamma^1_1 p_{T-1} + \psi^1_{11}u_T + \lambda^1_1\right) \tag{15}$$

where

$$\gamma^1_1 = -[2A^1_{11}(A^1_{11}+A^2_{11}) + \alpha(1-A^1_{11}-A^2_{11})^2/d]$$

$$\psi^1_{11} = -A^1_{11}(1+B^1_{111}+B^2_{111}) - B^1_{111}(A^1_{11}+A^2_{11}) + \alpha(1+B^1_{111}+B^2_{111})(1-A^1_{11}-A^2_{11})/d$$

$$\lambda^1_1 = A^1_{11}(C^2_{11}-C^1_{11}) - C^1_{11}(A^1_{11}+A^2_{11}) -.5dA^2_{11} - \alpha(C^2_{11}-C^1_{11})(1-A^1_{11}-A^2_{11})/d$$

Symmetrically, the derivative of the foreign firm's Nash equilibrium profit function at time T with respect to p_{2T-1} is given by:

$$\partial V^*_{2T}/\partial p_{2T-1} = \left(1/d\right)\left(\gamma^2_1 p_{T-1} - \psi^2_{11} u_T + \lambda^2_1\right)$$

where

$$\gamma^2_1(1+\tau) = -[2A^2_{11}(A^1_{11}+A^2_{11}) + \tilde{\alpha}(1-A^1_{11}-A^2_{11})^2/d]$$

$$\psi^2_{11}(1+\tau) = -A^2_{11}(1+B^1_{111}+B^2_{111}) - B^2_{111}(A^1_{11}+A^2_{11}) + \tilde{\alpha}(1+B^1_{111}+B^2_{111})(1-A^1_{11}-A^2_{11})/d$$

$$\lambda^2_1(1+\tau) = A^2_{11}(C^1_{11}-C^2_{11}) - C^2_{11}(A^1_{11}+A^2_{11}) -.5dA^1_{11} - \tilde{\alpha}(C^1_{11}-C^2_{11})(1-A^1_{11}-A^2_{11})/d$$

At time T-1, the domestic firm chooses its price to maximize discounted future Nash equilibrium profits taking into account its current choice on future Nash profits according to equation (14). Therefore, the firm faces the following maximization problem:

$$\max_{p_{1T-1}} \; (p_{1T-1}-u_{1T-1})y_{1T-1} - .5\alpha(y_{1T-1}-y_{1T-2})^2 + \beta V^*_{1T}(p_{T-1},u_T)$$

$$\text{subject to } y_{1T-1} = [\tilde{p}_{2T-1}-p_{1T-1}]/d + .5$$

where $V^*_{1T}(p_{T-1},u_T)$ is defined in equation (14). Substituting in for $\partial V^*_{1T}/\partial p_{1T-1}$ from equation (15) gives the domestic firm's best response function at time T-1:

$$p_{1T-1} = \left[1/(2d + \alpha - d\beta\gamma^1_1)\right] \left[(d+\alpha-d\beta\gamma^1_1)p_{2T-1} + du_{1T-1} + \alpha p_{T-2} + .5d^2 + d\beta(\psi^1_{11}u_T + \lambda^1_1)\right] \quad (16)$$

Symmetrically, the foreign firm's best response function is given by:

$$\tilde{p}_{2T-1} = \left[1/(2d + \tilde{\alpha} - d\beta\gamma^2_1)\right] \left[(d+\tilde{\alpha}-d\beta\gamma^2_1)p_{1T-1} - du_{2T-1} - \alpha p_{T-2} + .5d^2 + d\beta(-\psi^2_{11}u_T + \lambda^1_1)\right] \quad (17)$$

Combining the best response functions given by equations (16) and (17) gives Nash equilibrium prices at time T-1 as a function of the state, p_{T-2}, and of current and future wage differences, u_{T-1}, and u_T. Nash equilibrium prices for the domestic firm at time T-1 are given by:

$$p^*_{1T-1} = A^1_{21}p_{T-2} + B^1_{211}u_{T-1} + B^1_{212}u_T + u_{1T-1} + C^1_{21} + .5d.$$

The definitions of A^1_{21}, B^1_{211}, B^1_{212}, and C^1_{21} are confined to the appendix.

The Nash equilibrium tariff adjusted price for the foreign firm, $\tilde{p}^*_{2T-1}$, is determined symmetrically. As for period T, Nash equilibrium market shares for each firm at time T-1, y^*_{1T-1} and y^*_{2T-1}, as functions of p_{T-2}, u_{T-1}, and u_T can be determined.

In addition, by substituting the Nash equilibrium pricing functions at time T-1 into the Nash equilibrium pricing functions at time T as given by equations (12) and (13), Nash equilibrium prices at time T as a function of p_{T-2}, u_{T-1}, and u_T can be determined.

$$p^*_{1T}(p_{T-2},u_{T-1},u_T) = A^1_{12}p_{T-2} + B^1_{12,-1}u_{T-1} + B^1_{121}u_T + u_{1T} + C^1_{12} + .5d.$$

The definitions of A^1_{12}, $B^1_{12,-1}$, B^1_{121}, and C^1_{12} are confined to the appendix. Nash equilibrium prices for the foreign firm at time T as functions of p_{T-2}, u_{T-1}, and u_T are determined symmetrically.

Market share for the domestic firm at time T as a function of p_{T-2}, u_{T-1}, and u_T is given by

$$y^*_{1T}(p_{T-2},u_{T-1},u_T) = \left[1/d\right] \Big[-(A^1_{12}+A^2_{12})p_{T-2} - (B^1_{12,-1}+ B^2_{12,-1})u_{T-1} - (1+B^1_{121}+B^2_{121})u_T + (C^2_{12}-C^1_{12}) \Big] + .5$$

Let $V^*_{1T-1}(p_{T-2},u_{T-1},u_T)$ denote the sum of the domestic firm's Nash equilibrium profits at time T-1 and discounted Nash equilibrium profits at time T. Then the derivative of V^*_{1T-1} with respect to p_{1T-2} can be written as follows:

$$(\partial V^*_{1T-1}/\partial p_{1T-2}) = [1/d][\gamma^1_2 p_{T-2} + \psi^1_{21} u_{T-1} + \psi^1_{22} u_T + \lambda^1_2]$$

The definitions of γ^1_2, ψ^1_{21}, ψ^1_{22}, and λ^1_2 are confined to the appendix. This derivative is then used to determine best response functions at time T-2.

Continuing in this manner and solving the firms' maximization problems for each time T-t, t≥0, will generate the sequences $\{A^j_{t1}\}$, $\{B^j_{t11}\}$, $\{B^j_{t12}\},\ldots,\{B^j_{t1t+1}\}$ and $\{C^j_{t1}\}$ for j=1,2. These sequences are given in the appendix. Given these sequences, then Nash equilibrium prices at time T-t, t≥0, under ad valorem tariffs as functions of p_{T-t-1}, u_{T-t}, $u_{T-t+1},\ldots,u_T$ are given by:

$$p^*_{1T-t} = (A^1_{t1})p_{T-t-1} + (B^1_{t11})u_{T-t} + \sum_{k=2}^{t+1}(B^1_{t1k})u_{T-t+k-1} + u_{1T-t} + C^1_{t1} + .5d$$

and

$$\tilde{p}^*_{2T-t} = -(A^2_{t1})p_{T-t-1} - (B^2_{t11})u_{T-t} - \sum_{k=2}^{t+1}(B^2_{t1k})u_{T-t+k-1} + u_{2T-t} + C^2_{t1} + .5d$$

Now if each of the sequences, $\{A^j_{t1}\}$, $\{B^j_{t11}\}$, $\{B^j_{t12}\},\ldots,\{B^j_{t1t+1}\}$ and $\{C^j_{t1}\}$, converges as t increases, then time-invariant Nash equilibrium pricing functions will exist. Furthermore, the limits of the sequences will be the coefficients and constant term in those functions. Define the following limits:

$$A_j(\tau) \equiv \lim_{t\to\infty} A^j_{t1}, \quad j=1,2$$

$$B_{jk}(\tau) \equiv \lim_{t\to\infty} B^j_{t1k}, \quad j=1,2;\ k\geq 1$$

and $$C_j(\tau) \equiv \lim_{t\to\infty} C^j_{t1}, \quad j=1,2$$

When these limits exist, time-invariant pricing functions at time t as functions of last period prices, current wages, future wages, and the ad valorem tariff rate are given by

$$p^*_{1t}(p_{t-1},w^t,\tau) = A_1(\tau)(p_{1t-1} - (1+\tau)p_{2t-1}) + \phi\sum_{k=1}^{\infty} B_{1k}(\tau)(w_{1t+k-1} - (1+\tau)w_{2t+k-1}) + \phi w_{1t} + C_1(\tau) + .5d \tag{18}$$

and

$$p^*_{2t}(p_{t-1},w^t,\tau) = A_2(\tau)(p_{2t-1} - p_{1t-1}/(1+\tau)) + \phi\sum_{k=1}^{\infty} B_{2k}(\tau)(w_{2t+k-1} - (w_{1t+k-1})/(1+\tau)) + \phi w_{2t} + (C_2(\tau) + .5d)/(1+\tau)$$

where $w^t \equiv \{w_{1t},w_{2t},w_{1t+1},w_{2t+1},\ldots\}$. These functions are well defined if the infinite sums converge.

When these functions are well defined, sales for each firm can be written as functions of last period prices, current and future wages, and the ad valorem tariff rate as follows.

$$y^*_{1t}(p_{t-1},w^t,\tau) = [(1+\tau)p_{2t}(p_{t-1},w^t,\tau) - p_{1t}(p_{t-1},w^t,\tau)]/d + .5 \tag{19}$$

$$y^*_{2t}(p_{t-1},w^t,\tau) = 1-y^*_{1t}(p_{t-1},w^t,\tau)$$

Because the definitions of these sequences are complex, it is impractical to determine analytically if the sequences converge. Therefore, in the results reported in the next section, the sequences are calculated numerically and their limits are approximated by the values of the sequences when successive iterations do not significantly change their value.

Numerical simulations indicate that if $\beta<1$, then all sequences converge and the infinite sum in the Nash equilibrium pricing equations converges. Therefore, the numerical results indicate the existence of well-defined, time-invariant Nash equilibrium pricing functions for this dynamic game for a wide range of economy parameter values.

III.3.1: <u>Welfare and Optimal Tariffs</u>

Constant Wages

Consider, first the case in which $\theta_{1t}=\theta_1$ and $\theta_{2t}=\theta_2$ $\forall t$ implying constant wages in each country. In this case, prices and market shares will be constant. Hence, adjustment costs are never actually paid in this economy. However, as will be shown below, the presence of adjustment costs changes the Nash equilibrium prices in the dynamic game between the domestic and foreign producer. This result can be explained as follows. In this dynamic environment, a firm's optimal response to its competitors price incorporates both the effects of the competitors price on current market share and the effects on the change in market share from the previous period. The latter effect arises because of the presence of adjustment costs. Therefore, one should not expect best response functions to be the same in the dynamic case when adjustment costs are not paid ($\alpha>0, y_{jt}=y_j$ $\forall t$) as they are in the static case ($\alpha=0$). These differences give rise to different Nash equilibria and different policy conclusions in the presence of dynamics.

<u>Equilibria Prices and Quantities</u>

Under Ad Valorem Tariffs:

Under an ad valorem tariff imposed at rate τ which is expected to last forever, the following Nash equilibrium prices and quantities in the constant wage case as functions of the ad valorem tariff rate arise. Define the following (finite) sums:

$$BS_j(\tau) \equiv \sum_{k=1}^{\infty} B_{jk}(\tau) \qquad \text{for } j=1,2.$$

Then,

$$p_1(\tau) = \left[\frac{1}{1 - A_1(\tau) - A_2(\tau)}\right]\left[\begin{array}{l} \phi(w_1-(1+\tau)w_2)[A_1(\tau)BS_2(\tau) + BS_1(\tau)(1-A_2(\tau))] \\ + \phi w_1(1-A_2(\tau)) - \phi w_2(1+\tau)A_1(\tau) \\ + C_1(\tau)(1-A_2(\tau)) - A_1(\tau)C_2(\tau) \end{array}\right] + .5d \tag{20}$$

$$(1+\tau)p_2(\tau) = \left[\frac{1}{1 - A_1(\tau) - A_2(\tau)}\right]\left[\begin{array}{l} \phi((1+\tau)w_2-w_1)[A_2(\tau)BS_1(\tau) + BS_2(\tau)(1-A_1(\tau))] \\ + \phi(1+\tau)w_2(1-A_1(\tau)) - \phi w_1A_2(\tau) \\ + C_2(\tau)(1-A_1(\tau)) - A_2(\tau)C_1(\tau) \end{array}\right] + .5d$$

$$y_1(\tau) = [(1+\tau)p_2(\tau) - p_1(\tau)]/d + .5$$

$$y_2(\tau) = 1-y_1(\tau)$$

Under Free Trade:

The free trade prices and market shares when the foreign firm enters can be derived by setting $\tau=0$ in equation (20). Let p_1^f, p_2^f, y_1^f, y_2^f denote equilibrium prices and quantities under free trade.

Under Autarky:

If the economy is operating in autarky because the foreign firm does not deem it profitable to enter then, since the monopolist's market share never changes, it can be shown that he will price just as in the static case:

$$p_1^a = \upsilon - .5d$$

$$y_1^a = 1 \qquad (21)$$
$$y_2^a = 0$$

Aggregate Domestic Welfare

In the constant wage case, aggregate domestic welfare will be constant and will equal

$$W^z(\tau) = \omega_1 + \upsilon - .25d + w_1(1-\phi y_1^z) + (.5dy_1^z - p_2^z)y_2^z \qquad (22)$$

for z=v,f,a. Where p_2^z, y_1^z , and y_2^z are defined in equations (19)-(20) above for z=v,f,a.

Let τ^P be the tariff rate such that the foreign firm's price equals its unit cost at that rate under the duopoly prices given by equation (20). Let τ^o be the ad valorem tariff rate such that the foreign firm's market share equals zero under the duopoly pricing functions given by equation (20). Therefore these two tariff rates are implicitly defined by the following equations:

$$\phi(w_2(1+\tau^P)-w_1)[A_2(\tau^P)(BS_1(\tau^P)+1)+BS_2(\tau^P)(1-A_1(\tau^P))] + \qquad (23)$$
$$C_2(\tau^P)(1-A_1(\tau^P)) - C_1(\tau^P)A_2(\tau^P) + .5d(1-A_1(\tau^P)-A_2(\tau^P)) = 0$$

$$\phi(w_2(1+\tau^o)-w_1)[1+BS_1(\tau^o)+BS_2(\tau^o)] + C_2(\tau^o) - C_1(\tau^o) \qquad (23)$$
$$- .5d(1-A_1(\tau^o)-A_2(\tau^o)) = 0$$

If $\tau \geq \tau^P$ or $\tau \geq \tau^o$, then the foreign firm will not sell in the domestic market and the domestic economy will revert to autarky.

As demonstrated in section II, in the static case, $\tau^P = \tau^o$, which implies that the welfare function is continuous at the prohibitive rate. However, in the dynamic case, these tariff rates will not generally be equal. Therefore, when dynamic elements are introduced into the firms' maximization problems, the domestic welfare function may be a discontinuous function of the tariff rate with the discontinuity at the prohibitive tariff rate $\tau^P<\tau^o$.

Figure 2 illustrates the possible forms of aggregate domestic welfare as a function of the tariff rate when the firms face positive adjustment costs. In Figures 2.A and 2.B, the optimal policy is to set the tariff rate at its prohibitive level, τ^P as defined by equation (23), and revert to autarky. In Figures 2.C and 2.D, it is optimal to set tariffs at some positive level which still allows foreign imports. In Figure 2.C, it is optimal to set the tariff rate just below its prohibitive level, allow foreign entry, but give the foreign firm negligible profits. This policy is optimal as the foreign sales satisfy consumers' demand for variety while allowing maximum rent shifting away from the foreign producer to the domestic one. In Figure 2.D, it is optimal to set tariffs well below their prohibitive rate. As will be demonstrated below, which situation results will depend on economy parameters.

Figure 3 provides further explanation of the discontinuity in the welfare function when adjustment costs are present. The diagram depicts prices and market shares as functions of the tariff rate in markets with and without adjustment costs. (The autarky price charged by the domestic firm, $p_1^A=4.5$, is not depicted in Figure 3.A so as to make the scaling between the two figures comparable.) As Figure 3.A demonstrates, both firms set lower prices in the dynamic environment than in the static one. This implies that the tariff rate which sets price equal to unit cost for the foreign firm, τ^P, will be lower in the presence of adjustment costs. It also implies that adjustment costs have less of an impact on the foreign firm's market share than on its price level since *both* firms set lower prices. That is, as the diagram illustrates, the foreign price function shifts down by more than the foreign market share function when adjustment costs are added. Therefore, the foreign firm's price will equal its unit cost at a lower tariff rate than the rate at which its market share equals zero. These effects produce the wedge between τ^p and τ^o and generate the discontinuity in the welfare function.

Figure 4 illustrates the effect of positive adjustment costs on aggregate domestic welfare. For relatively low wages in the domestic country, Figures 4.A and 4.B illustrate that autarky is the optimal policy in both the static case and in the presence of positive adjustment costs. For higher relative wages, tariff restricted trade is optimal. The diagram illustrates that both the prohibitive tariff rate and the optimal tariff rate are lower in the presence of adjustment costs.

Figure 5 presents an interesting case. In the absence of adjustment costs, tariff restricted trade is optimal. With positive adjustment costs, however, it is optimal to revert to autarky and to set the tariff rate at the prohibitive level. This illustrates that the presence of dynamic elements in the firms' objectives can lead to different policy prescriptions even when adjustment costs are never actually paid as in the constant wage case examined here.

Time-Varying Wages

In this section, an economy is examined in which wages, and therefore market shares, are not constant and in which adjustment costs are actually paid. Consider the following policy experiment. Suppose the domestic economy is operating under free trade and the government is considering imposing a constant ad valorem tariff on imports at time t_o which will be in place indefinitely. Depending on the realization of relative wages and the size of the tariff, the economy may be operating in autarky in some (or all) periods in the future. Let $S(\tau)$ be the set of periods in which the economy will be in autarky in the future given the wages at time t_o and the tariff rate under consideration. That is, $S(\tau)$ is defined as follows:

$$S(\tau) = \{s: \ \Pi_{2s}(\tau)\leq 0, \ s\geq t_o\}.$$

The government's objective function is the discounted sum of future aggregate domestic welfare:

$$W(\tau) = \sum_{t\in S(\tau)} \beta^{(t-to)} W_t^A + \sum_{t\notin S(\tau)} \beta^{(t-to)} W_t^v(\tau)$$

where $W_t^A = \omega_1 + \nu - .25d + w_{1t}(1-\phi)$ and

$$W_t^v(\tau) = \omega_1 + \nu - .25d + w_{1t}(1-\phi y_{1t}^v(\tau)) + (.5dy_{1t}^v(\tau) - p_{2t}^v(\tau))y_{2t}^v(\tau).$$

Prices and quantities under the tariff, $p_{2t}^v(\tau)$, $y_{1t}^v(\tau)$, and $y_{2t}^v(\tau)$, are given by equations (18) and (19).

In the constant wage case, it was shown that when $\alpha>0$ the tariff rate which sets the foreign firm's profits equal to zero may differ from the rate which gives the foreign firm zero market share under duopoly pricing. The divergence in these rates and the difference between domestic aggregate welfare under autarky and under free trade and implies that $W(\tau)$ may be a discontinuous function of the tariff rate in the dynamic case with discontinuities at multiple tariff rates. These effects are illustrated in the following computational experiment.

Let the distribution of the technology parameters to non-traded goods production in each country be given by the following Markov process:

$$\theta_{jt} = \rho\theta_{jt-1} + \varepsilon_{jt} \qquad \forall \ j=1,2,$$

where $\varepsilon\sim\log N(\bar{\theta}_j(1-\rho),\sigma^2)$, iid across time and across countries. This distribution was chosen as it generates positive, stationary technology parameters. Recall that in an interior solution, $w_{jt}=\theta_{jt}$, $\forall j, \forall t$.

Figure 6 depicts $W(\tau)$ for different means of domestic wages and for different levels of adjustment costs. The figure represents domestic aggregate welfare for a particular realization of the technology process. The figures illustrate that, for relatively high tariffs, aggregate domestic welfare is a discontinuous function of the tariff rate in the presence of adjustment costs. The general conclusions of the constant wage case are still evident in this case. The presence of dynamic elements affects the optimal tariff level with lower optimal tariffs associated with higher adjustment costs.

IV. CONCLUSIONS

This paper has examined the effects of an unilateral tariff on prices, quantities, and welfare in the presence of dynamic elements facing imperfectly competitive firms. In the environment studied here, firms selling differentiated products interact strategically in a price setting game. It was shown that the presence of adjustment costs to changing sales volume significantly affected the relationship between tariff rates and aggregate domestic welfare even when sales are constant and no adjustment costs are actually paid.

Introducing these dynamic elements into firms objective functions leads to very different best response functions and Nash equilibria than in the static game. When the potential for positive adjustment costs exists, a wedge is driven between the tariff rate which gives the foreign firm zero profits and the rate which gives the foreign firm zero market share. This wedge does not arise in the static case. The difference in these rates leads to a discontinuity in the aggregate domestic welfare function as a function of the tariff rate. This property implies that for some levels of relative wages across countries, it may be optimal to set the tariff rate just below the level which gives the foreign firm zero profits. This policy allows foreign entry with the foreign firm earning negligible profits and satisfies consumers' desire for variety while providing maximum rent-shifting.

The level of optimal tariffs is affected by the size of adjustment costs with lower optimal tariffs associated with higher adjustment costs. Of particular interest is the possibility that autarky is optimal under positive adjustment costs while tariff restricted trade is optimal in the absence of such costs.

A purpose of this paper was to answer the question of whether or not dynamic elements in a game between imperfectly competitive firms would affect the relationship between tariffs and prices, quantities, and welfare. The dynamics introduced were of the simplest possible form which could allow one to address this question. An obvious next step is to introduce more interesting forms of dynamics such as investment technologies or research and development considerations. In such environments questions regarding the effects of tariffs on investment and R&D in imperfectly competitive markets could be addressed.

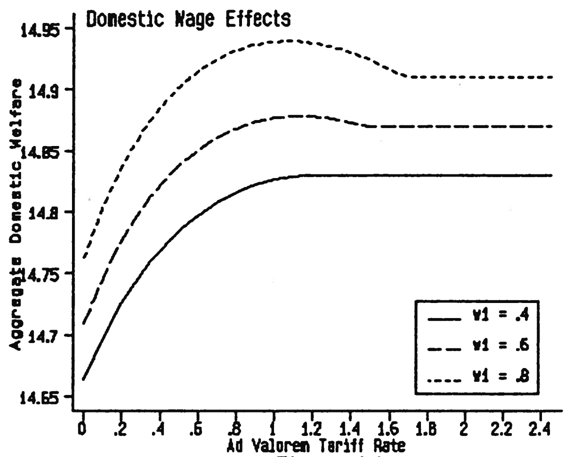

Figure 1.A

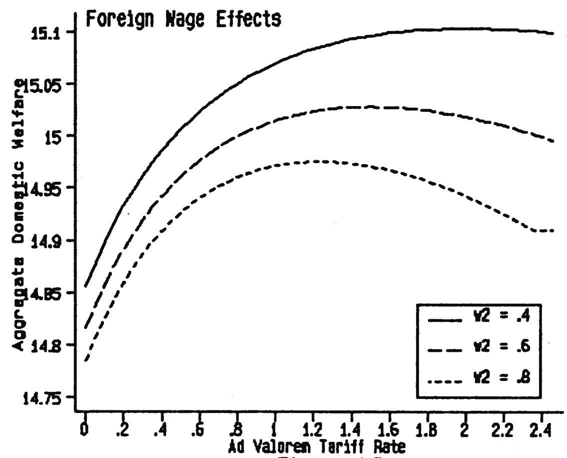

Figure 1.B

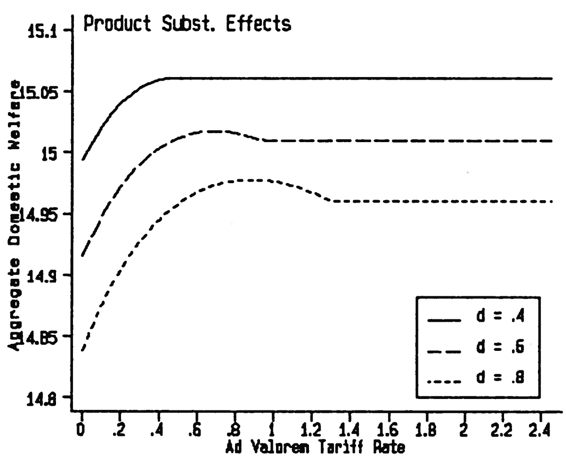

Figure 1.C

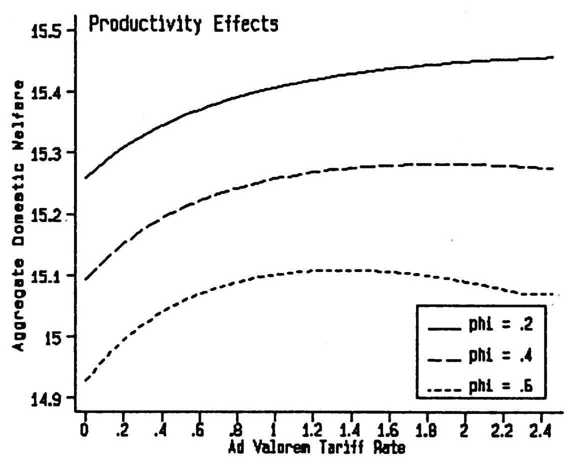

Figure 1.D

Economy Parameters:

$\beta = .98$ $\quad\omega_1 = \omega_2 = 10$ $\quad\upsilon = 5$

$\theta_{1t} = .8 \;\; \forall t$ $\quad\theta_{2t} = 1. \;\; \forall t$ $\quad d = 1.$ $\quad\phi = .8$

(in cases where the parameter does not vary)

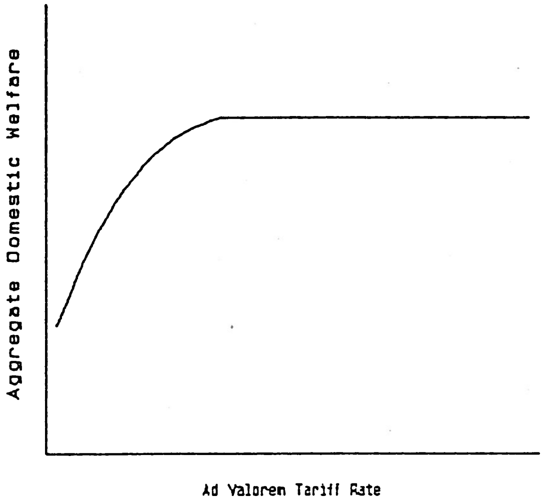

Figure 2.A

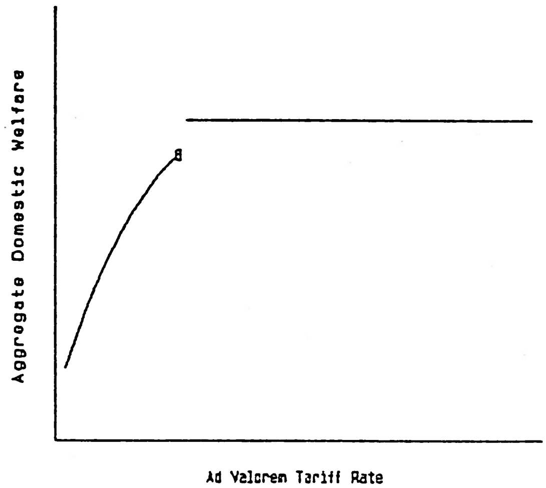

Figure 2.B

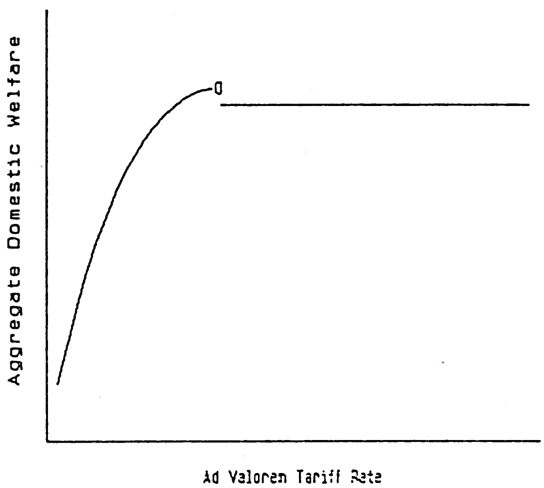

Figure 2.C

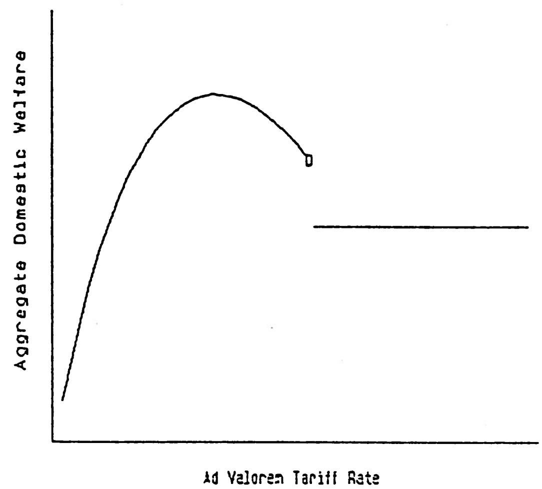

Figure 2.D

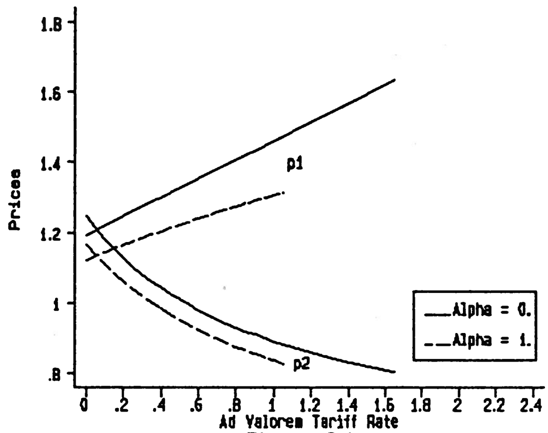

Figure 3.A

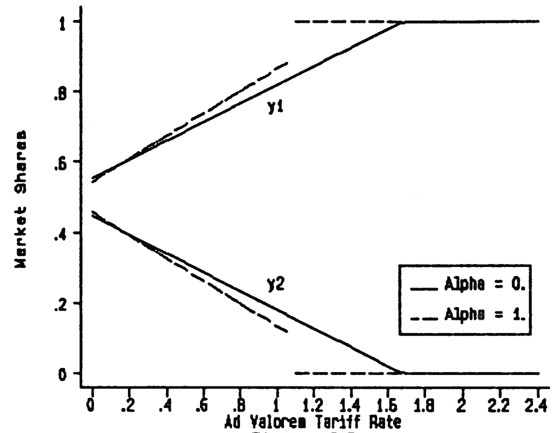

Figure 3.B

Economy Parameters:

$\beta = .98 \qquad \omega_1 = \omega_2 = 10 \qquad \upsilon = 5$

$\theta_{1t} = .8 \quad \forall t \qquad \theta_{2t} = 1. \quad \forall t \qquad d = 1. \qquad \phi = .8$

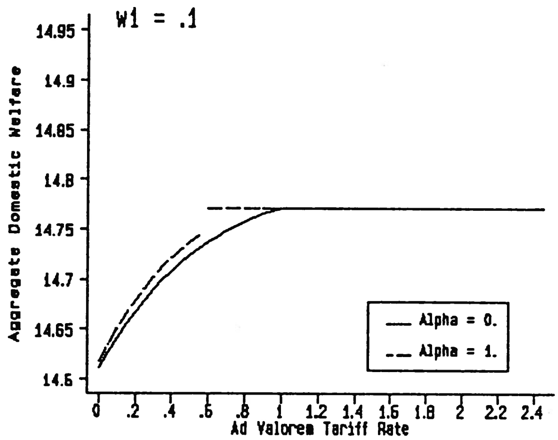

Figure 4.A

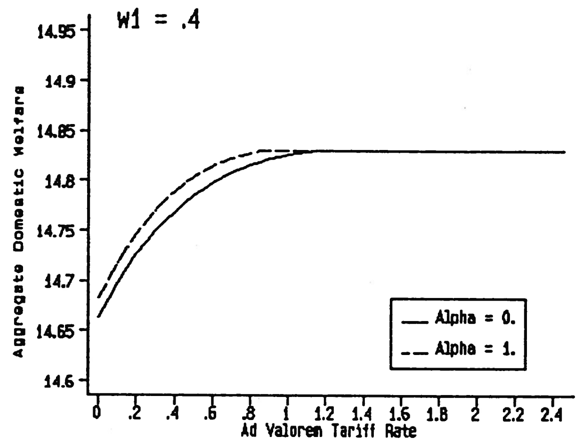

Figure 4.B

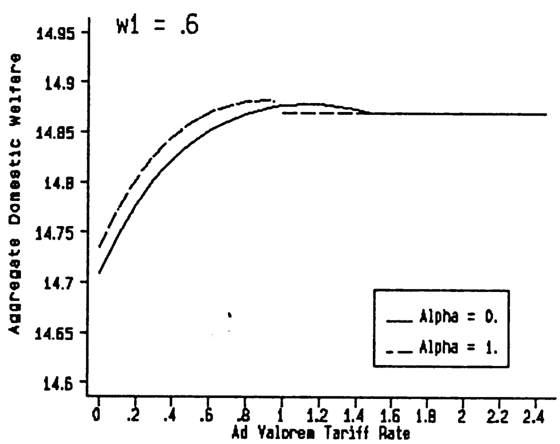

Figure 4.C

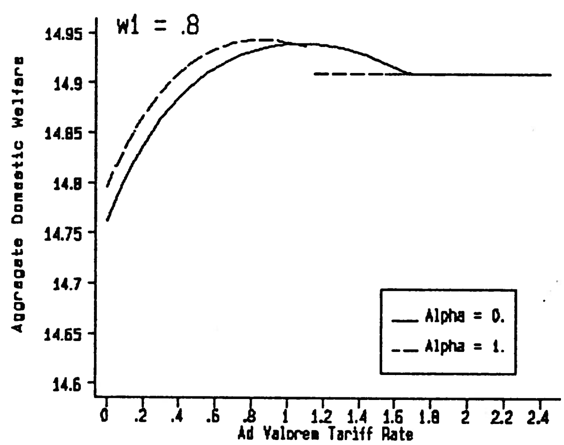

Figure 4.D

Economy Parameters:

$\beta = .98$ $\omega_1 = \omega_2 = 10$ $\upsilon = 5$

$\theta_{2t} = 1. \quad \forall t$ $d = 1.$ $\phi = .8$

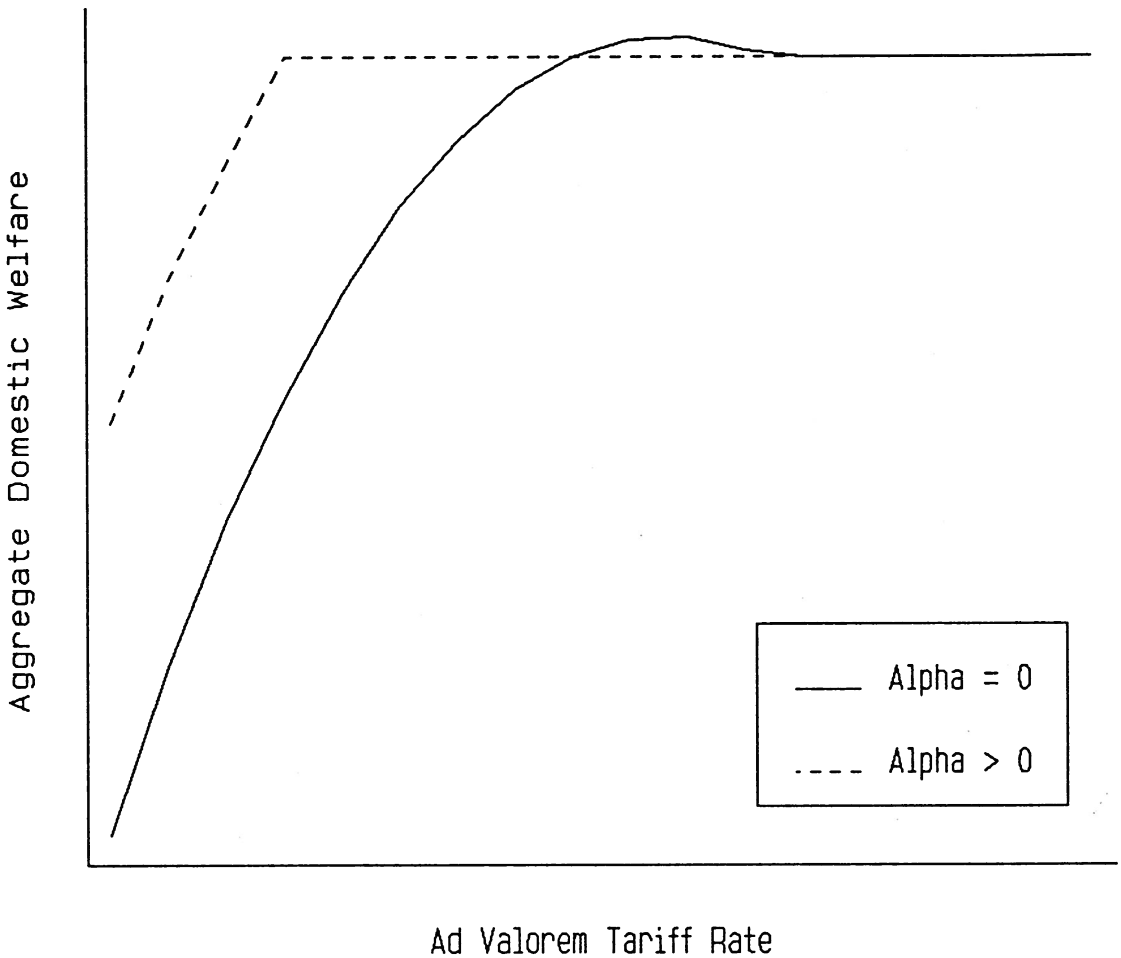
Aggregate Domestic Welfare
Alpha = 0
Alpha > 0
Ad Valorem Tariff Rate

Figure 5

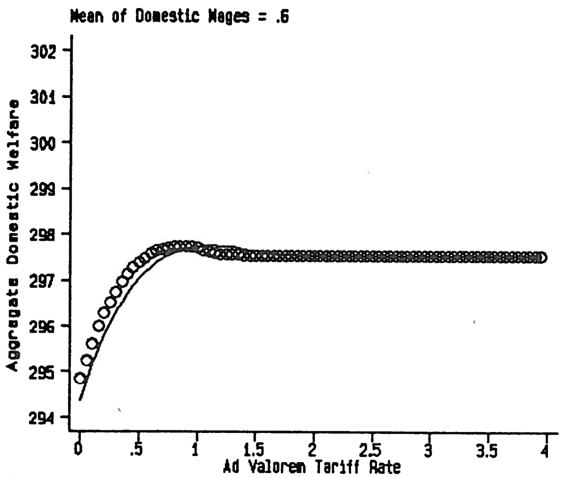

Figure 6.A

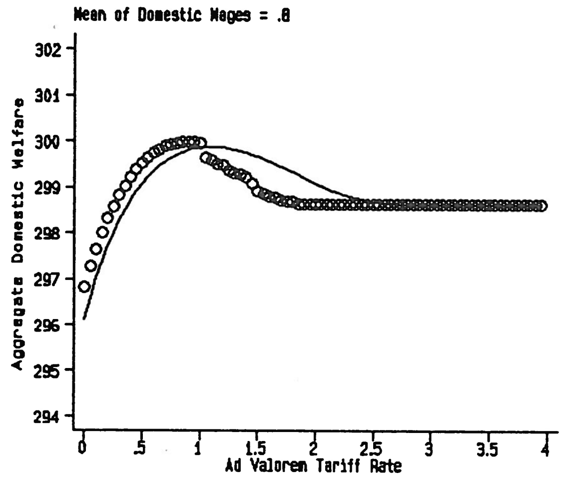

Figure 6.B

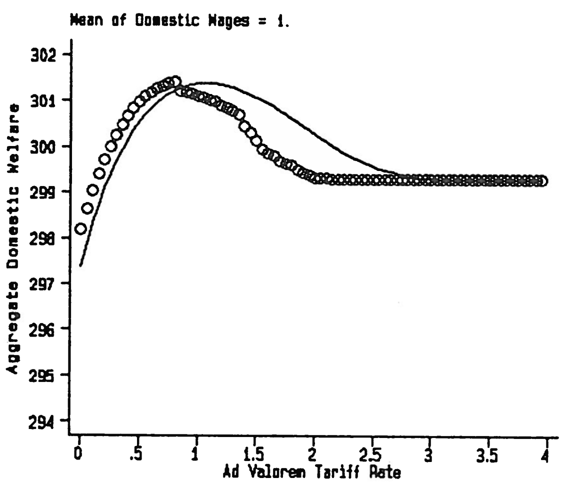

Figure 6.C

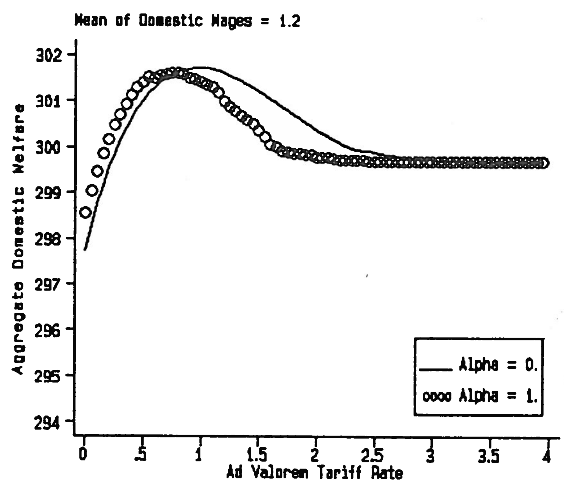

Figure 6.D

Economy Parameters:

$\beta = .98$ $\omega_1 = \omega_2 = 10$ $\upsilon = 5$

$\bar{\theta}_2 = 1.$ $\sigma^2_\varepsilon = .04$ $d = 1.$ $\phi = .8$

NOTES

1. See, for example, Eaton and Grossman(1986), Brander and Spencer(1985), Horstmann and Markusen(1985), Venables(1985), and Dixit(1984).

2. See, for example, Flam and Helpman(1987), Gros(1987), and Venables(1987,1982).

APPENDIX

The coefficients and constant terms in the time-invariant Nash equilibrium pricing functions as functions of last period's prices and current and future wages under an ad valorem tariff structure are presented.

Let $\alpha^1 \equiv \alpha$ and $\alpha^2 \equiv \alpha(1+\tau)$, let $\sigma^1 \equiv 1$ and $\sigma^2 \equiv (1+\tau)$, and let n=1,2 and m=1,2, m≠n. Define the following sequences recursively.

Z_i

$$Z_1 \equiv 3d + \alpha^1 + \alpha^2$$

$$\forall i \geq 2: \quad Z_i \equiv Z_1 - d\beta(\gamma^1_{i-1} - \gamma^2_{i-1})$$

A^1_{ij}, A^2_{ij}

$$A^n_{11} \equiv (1/Z_1)(2\alpha^n - \alpha^m)$$

$$\forall i \geq 2: \quad A^n_{i1} \equiv [1/Z_i][2\alpha^n - \alpha^m + \beta(\alpha^m \gamma^n_{i-1} - \alpha^n \gamma^m_{i-1})]$$

$$\forall i \geq 1: \; \forall j \geq 2: \quad A^n_{ij} \equiv A^n_{ij-1}[A^n_{i+j-1\,1} + A^m_{i+j-1\,1}]$$

B^1_{ijk}, B^2_{ijk}

$$B^n_{111} \equiv -(1/Z_1)(d + \alpha^n)$$

$$\forall i \geq 2: \quad B^n_{i11} \equiv (1/Z_i)(d\beta\gamma^n_{i-1} - d - \alpha^n)$$

$\forall i \geq 2$:
$\forall 2 \leq k \leq i$: $B^n_{i1k} \equiv (\beta/Z_i) \begin{bmatrix} (2d + \alpha^m - d\beta\gamma^m_{i-1})\phi^n_{i-1k-1} \\ -(d + \alpha^n - d\beta\gamma^n_{i-1})\phi^m_{i-1k-1} \end{bmatrix}$

$\forall i \geq 1$:
$\forall j \geq 2$:
$\forall 3-j \leq k \leq i$: $B^n_{ijk} \equiv A^n_{ij-1}(B^n_{i+j-11k+j-1} + B^m_{i+j-11k+j-1}) + B^n_{ij-1k}$

$\forall i \geq 1$:
$\forall j \geq 2$: $B^n_{ij1-j} \equiv A^n_{ij-1}(1 + B^n_{i+j-111} + B^m_{i+j-111})$

<u>C^1_{ij}, C^2_{ij}</u>

$C^n_{11} \equiv 0$

$\forall i \geq 2$: $C^n_{i1} \equiv (\beta/Z_i) \begin{bmatrix} (2d + \alpha^m - d\beta\gamma^m_{i-1})\lambda^n_{i-1} \\ +(d + \alpha^n - d\beta\gamma^n_{i-1})\lambda^m_{i-1} \end{bmatrix}$

$\forall i \geq 1$:
$\forall j \geq 2$: $C^n_{ij} \equiv A^n_{ij-1}(C^n_{i+j-11} - C^m_{i+j-11}) + C^n_{ij-1}$

$\underline{\gamma_i^1,\ \gamma_i^2}$

$$\forall i \geq 1: \quad \gamma_i^n \equiv (1/\sigma^n)\left[-2A^n_{i1}(A^n_{i1} + A^m_{i1}) - (\alpha^n/d)(1 - A^n_{i1} - A^m_{i1})^2\right]$$

$$+ (1/\sigma^n)\sum_{j=1}^{i-1} \beta^j \left[\begin{array}{l} -2A^n_{i-jj+1}(A^n_{i-jj+1} + A^m_{i-jj+1}) \\ -(\alpha^n/d)(A^n_{i-j+1j} + A^m_{i-j+1j} - A^n_{i-jj+1} - A^m_{i-jj+1})^2 \end{array}\right]$$

$\underline{\lambda_i^1,\ \lambda_i^2}$

$$\forall i \geq 1: \lambda_i^n \equiv (1/\sigma^n)\left[\begin{array}{l} A^n_{i1}(C^m_{i1} - C^n_{i1}) - C^n_{i1}(A^n_{i1} + A^m_{i1}) \\ -.5dA^m_{i1} - (\alpha^n/d)(1 - A^n_{i1} - A^m_{i1})^2 \end{array}\right]$$

$$+(1/\sigma^n)\sum_{j=1}^{i-1} \beta^j \left[\begin{array}{l} A^n_{i-jj+1}(C^m_{i-jj+1} - C^n_{i-jj+1}) \\ - C^n_{i-jj+1}(A^n_{i-jj+1} + A^m_{i-jj+1}) - .5dA^m_{i-jj+1} \\ -(\alpha^n/d)(A^n_{i-j+1j} + A^m_{i-j+1j} - A^n_{i-jj+1} - A^m_{i-jj+1}) \\ \quad \cdot \quad (C^n_{i-j+1j} - C^m_{i-j+1j} + C^m_{i-jj+1} - C^n_{i-jj+1}) \end{array}\right]$$

$\underline{\phi^1_{ik},\ \phi^2_{ik}}$

$$\phi^n_{11} \equiv (1/\sigma^n)\left[\begin{array}{l} -A^n_{11}(1 + B^n_{111} + B^m_{111}) - B^n_{111}(A^n_{11} + A^m_{11}) \\ + (\alpha^n/d)(1 + B^n_{111} + B^m_{111})(1 - A^n_{11} - A^m_{11}) \end{array}\right]$$

$$\forall i \geq 2:\quad \phi^n_{i1} \equiv (1/\sigma^n)\left[\begin{array}{l} -A^n_{i1}(1 + B^n_{i1,1} + B^m_{i1,1}) - B^n_{i1,1}(A^n_{i1} + A^m_{i1}) \\ + (\alpha^n/d)(1 + B^n_{i1,1} + B^m_{i1,1})(1 - A^n_{i1} - A^m_{i1}) \end{array}\right]$$

$$+ (\beta/\sigma^n)\left[\begin{array}{l} -A^n_{i-12}(B^n_{i-12,-1} + B^m_{i-12,-1}) - B^n_{i-12,-1}(A^n_{i-12} + A^m_{i-12}) \\ -(\alpha^n/d)(1 + B^n_{i1,1} + B^m_{i1,1} - B^n_{i-12,-1} - B^m_{i-12,-1}) \\ \quad \cdot\ (A^n_{i1} + A^m_{i1} - A^n_{i-12} - A^m_{i-12}) \end{array}\right]$$

$$+ \sum_{j=2}^{i-1} (\beta^j/\sigma^n)\left[\begin{array}{l} -A^n_{i-j\,j+1}(B^n_{i-j\,j+1,-j} + B^m_{i-j\,j+1,-j}) \\ - B^n_{i-j\,j+1,\,j}(A^n_{i-j\,j+1} + A^m_{i-j\,j+1}) \\ -(\alpha^n/d)\left[\begin{array}{ll} B^n_{i-j+1\,j,\,1-j} & + B^m_{i-j+1\,j,\,1-j} \\ - B^n_{i-j\,j+1,-j} & - B^m_{i-j\,j+1,-j} \end{array}\right] \\ \quad\quad \cdot \left[\begin{array}{ll} A^n_{i-j+1\,j} & + A^m_{i-j+1\,j} \\ - A^n_{i-j\,j+1} & - A^m_{i-j\,j+1} \end{array}\right] \end{array}\right]$$

$$\phi^n_{22} \equiv (1/\sigma^n)\left[\begin{array}{l} -A^n_{21}(\,B^n_{212} + B^m_{212}) - B^n_{212}(A^n_{21} + A^m_{21}) \\ + (\alpha^n/d)(B^n_{212} + B^m_{212})(1 - A^n_{21} - A^m_{21}) \end{array}\right]$$

$$+ (\beta/\sigma^n)\left[\begin{array}{l} -A^n_{12}(1 + B^n_{121} + B^m_{121}) - B^n_{121}(A^n_{12} + A^m_{12}) \\ + (\alpha^n/d)(B^n_{212} + B^m_{212} - 1 - B^n_{121} - B^m_{121}) \\ \quad \cdot\, (A^n_{21} + A^m_{21} - A^n_{12} - A^m_{12}\,) \end{array}\right]$$

$\forall i \geq 3$:
$\forall k \geq 2$:

$$\phi^n_{ik} \equiv (1/\sigma^n)\left[\begin{array}{l} -A^n_{i1}(1 + B^n_{i1k} + B^m_{i1k}) - B^n_{i1k}(A^n_{i1} + A^m_{i1}) \\ + (\alpha^n/d)(B^n_{i1k} + B^m_{i1k})(1 - A^n_{i1} - A^m_{i1}) \end{array}\right]$$

$$+ (\beta^{k-1}/\sigma^n)\left[\begin{array}{l} -A^n_{i-k+1k}(1 + B^n_{i-k+1k,1} + B^m_{i-k+1k,1}) \\ - B^n_{i-k+1k,1}(A^n_{i-k+1k} + A^m_{i-k+1k}) \\ -(\alpha^n/d)\left[\begin{array}{l} B^n_{i-k+2k-1,2} + B^m_{i-k+2k-1,2} \\ - 1 - B^n_{i-k+1k,1} - B^m_{i-k+1k,1} \end{array}\right] \\ \qquad \cdot \left[\begin{array}{l} A^n_{i-k+2k-1} + A^m_{i-k+2k-1} \\ - A^n_{i-k+1k} - A^m_{i-k+1k} \end{array}\right] \end{array}\right]$$

$$
+ (\beta^{k}/\sigma^{n}) \left[\begin{array}{l}
-A^{n}_{i-kk+1}\left(B^{n}_{i-kk+1,-1} + B^{m}_{i-kk+1,-1}\right) \\[1ex]
- B^{n}_{i-kk+1,-1}\left(A^{n}_{i-kk+1} + A^{m}_{i-kk+1}\right) \\[1ex]
-(\alpha^{n}/d) \left[\begin{array}{l} 1 + B^{n}_{i-k+1k,1} + B^{m}_{i-k+1k,1} \\ - B^{n}_{i-kk+1,-1} - B^{m}_{i-kk+1,-1} \end{array} \right] \\[2ex]
\qquad \cdot \left[\begin{array}{ll} A^{n}_{i-k+1k} & + A^{m}_{i-k+1k} \\ - A^{n}_{i-kk+1} & - A^{m}_{i-kk+1} \end{array} \right]
\end{array} \right]
$$

$$
+ \sum_{\substack{j=1 \\ j\neq k-1 \\ j\neq k}}^{i-1} (\beta^{j}/\sigma^{n}) \left[\begin{array}{l}
-A^{n}_{i-jj+1}\left(B^{n}_{i-jj+1,k-j} + B^{m}_{i-jj+1,k-j}\right) \\[1ex]
- B^{n}_{i-jj+1,k-j}\left(A^{n}_{i-jj+1} + A^{m}_{i-jj+1}\right) \\[1ex]
-(\alpha^{n}/d) \left[\begin{array}{ll} B^{n}_{i-j+1j,k-j+1} & + B^{m}_{i-j+1j,k-j+1} \\ - B^{n}_{i-jj+1,k-j} & - B^{m}_{i-jj+1,k-j} \end{array} \right] \\[2ex]
\qquad \cdot \left[\begin{array}{ll} A^{n}_{i-j+1j} & + A^{m}_{i-j+1j} \\ - A^{n}_{i-jj+1} & - A^{m}_{i-jj+1} \end{array} \right]
\end{array} \right]
$$

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