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# Auctions Versus Posted-Price Selling

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**Abstract**

Two most popular selling methods -- posted-price selling and auctions -- are compared in this paper. We confirm the common belief that auctions are most often used when the distribution of the object's value is widely dispersed. The choice of selling methods usually depends on the costs of displaying, storing and auctioning. In the absence of auctioning costs, auctioning at every instant is optimal. The "dispersion" of a distribution is then formally defined and developed. Using the definition of dispersion, we prove that auctions becomes preferable when a potential buyer's valuation becomes more dispersed. Finally, the optimization of a social planner is studied and we find that the monopoly seller's price can be higher or lower than that of the social optimum.

## 1. Introduction

It is commonly believed that auctions are often used when the seller has difficulty in determining the value of the object to the bidders. One answer to the question of why auctions are used in lieu of other selling methods such as posting a fixed price, is, "perhaps, that some products have no standard value. For example, the price of any catch of fish (at least of fish destined for the fresh fish market) depends on the demand and supply conditions at a specific moment of time, influenced possibly by prospective market developments. For manuscripts and antiques, too, prices must be remade for each transaction. For example, how can one discover the worth of an original copy of Lincoln's Gettysburg Address except by auction method?" (Cassady 1967, also cited in McAfee and McMillan 1987, p.701). According to Milgrom (1989), "When goods are not standardized or when the market clearing prices are highly unstable, posted prices work poorly, and auctions are usually preferred." Intuition tells us that perhaps the more dispersed the object value is, the more preferred the auctions are. There is, however, no existing treatment that offers rigorous explanation for this intuition, which is primarily because the dispersion of a distribution is hard to define.

The literature on optimal auctions (see Riley and Samuelson 1981, Myerson 1981) does not help too much in this matter as it considers problems under static settings and the set of bidders is unaffected by the selling policy. It is easy to see that posted-price selling methods always generate less revenue than auctions, since the seller can always set a reservation price equal to the posted price and be better off. Some dynamic features have recently been introduced into the game by McAfee and McMillan (1988). In their paper, a monopsonist (the seller of a contract) wishing to buy an indivisible object incurs some costs in seeking potential sellers (bidders). The potential sellers vary in their production costs which are identically and independently distributed (i.e. it is a private-value auction). When the number of potential sellers becomes infinite, it is proven that the optimal selling mechanism is posted price: the monopsony buyer offers a price to buy and approaches the potential sellers sequentially. It suggests that posted-price selling is the best selling method (at least under their assumptions) no matter how dispersed the object value to a bidder is.

This counter-intuitive result suggests that there is much to be investigated about this issue. While McAfee and McMillan (1988) are mainly concerned with a monopsony buyer seeking a most favorable contract, in this paper we are concerned with a more traditional situation: a monopoly seller seeks the optimal way to sell an indivisible object. The object is private-valued--each potential buyer's valuation (or willingness to pay) is independent of each other's. Potential buyers arrive at a store randomly, following some stochastic process. If the seller chooses to post a fixed price to sell the object, she incurs a continuing cost of displaying the object until the object is sold. If the seller chooses to auction the object, she can store the object at a lower cost until the auctioning date. The seller also incurs an auctioning cost whenever she holds an auction.

Under our setting, it is difficult to use the Revelation Principle to calculate the optimal selling method as has been done in various previous papers (see Myerson 1981, McAfee and McMillan 1988). This is because the cost structure is different under different selling methods. For this reason, we just compare the two most popular selling methods: posted-price selling and auctions. In posted-price selling, the seller posts a price in the store and potential buyers arrive randomly and sequentially, and decide whether or not to buy at the price. Similar to a search model, potential buyers that turn down the offer are assumed never to come back. (As buyers searching for commodities, they arrive at any one store randomly. The probability of arriving at the same store twice is assumed to be zero.) The game ends the moment that the object is sold. In contrast to posted-price selling, the auctioning seller is assumed to invite buyers that arrive to attend an auction at a future date. The seller can set a reservation price in the auction, and if the object is not sold, another auction is then planned for potential buyers arriving at the store after the auction date. The procedure continues until the object is sold.

We find that if there is no auctioning cost, then the seller would choose to auction the object (with a reservation price) at every instant. This is equivalent to a posted-price selling with the posted price equal to the reservation price and the seller incurring a continuous storage cost. This is consistent with McAfee and McMillan's result that posted-price is the best selling method when the storage cost is equal to the display cost. If the auctioning cost is positive, however, the choice will depend on the magnitude of different costs. Apparently, auctions may be chosen if the cost of auctioning and/or the cost of storage is low enough.

Our primary interest is to investigate how the dispersion of the distribution of a bidder's valuation affects the choice of selling mechanisms. We first provide a precise definition for dispersion. It is related to the J(x) function in various previous papers (McAfee and McMillan 1988, Maskin and Riley 1984, and the c(t) function in Myerson 1981). Various properties of dispersion are discussed and developed. The main result of this paper is that auctions become more attractive as the dispersion of buyers' valuations increases.

The dispersion of a distribution is different from its variance and cannot generally be described by a single number. An increase in dispersion implies an increase in variance, however; and when a distribution is characterized by mean and variance, an increase in variance is equivalent to an increase in its dispersion.

Finally, the optimal reservation price and the time length between auctions are compared with those of the social optimum. It is found that the reservation prices in the optimal auctions can be either higher or lower than the social optimum, and the time length between auctions can be either longer or shorter.

The rest of the paper is organized as follows: Section 2 describes the model; in Subsection 2.1 we discuss the optimal price in posted-price selling; in Subsection 2.2 we investigate the optimal sequence of auctions. In Section 3 we compare these two selling methods and study the effect of the dispersion of the distribution on the choice of selling methods. In Section 4 we discuss social welfare. Section 5 contains some remarks.

## 2. The Model

Suppose that there is a unique indivisible object to be sold by its owner. Potential buyers arrive randomly according to a Poisson process.[1] The probability of exactly k potential buyers arriving within any interval of length t is given by

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[1]This assumption makes the analysis much simpler. If the arrivals are independent but do not follow a Poisson process, the qualitative results are essentially the same. We shall discuss this further at the end of Subsection 2.1 and Section 4.

$$P_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \qquad k = 0, 1, 2, \ldots \tag{1}$$

The object is assumed to be private-valued by the potential buyers. The value of the object to potential buyer i ($V_i$) is assumed to be a random and independent draw from a distribution F(v) with support $[\underline{v}, \bar{v}]$, where $F(v) = Pr\{V_i \leq v\}$ and $\underline{v} \geq 0$. Buyers are thus symmetric and the $V_i$ have i.i.d. distributions. We assume for simplicity that F(v) is differentiable on $[\underline{v}, \bar{v}]$. The object is worth zero to the seller. To make the analysis as simple as possible, we assume that there is no discounting and that a buyer's valuation does not depend on the date the object is obtained.[2]

The seller can choose either to sell the object at auction or by posting a price. In the posted-price selling option, the owner incurs a cost of displaying at rate $\theta_d$ until an arriving buyer agrees to pay the posted price. A buyer who refuses the posted price never returns. This assumption can be justified as consumers shopping randomly. If there are a lot of stores, a consumer will have low probability of coming back to the same store. The owner can change the price or change to the auctioning option at any time before the next potential buyer arrives, but as buyers arrive according to a Poisson process and the object is private-valued, we do not expect the seller to do so. In the auctioning option, the owner incurs a cost of storage at rate $\theta_s$ until he sells the object. In addition to that, the seller incurs also an auctioning cost $\Theta_a$ each time he auctions the object. In this option, the owner chooses a specific future time at which the object is auctioned.

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[2]These assumptions would bias the result in favor of auctions. The date for an auction would be earlier than what is to be calculated. The qualitative results are not expected to change.

Potential buyers who arrive at the store get a notice of such an event; and it is assumed that they will all go to the auction house at that time.[3] This implies that the potential buyers are acting passively and the cost of going to an auction is negligible. The owner can set a reservation price in the auction; and if the object is not sold, the owner will keep the object in storage and plan a future date for another auction. Potential buyers who came to earlier auctions in which the object is unsold are assumed never to return for later auctions for the same reasons as in the posted-price selling option. The seller and all potential buyers are assumed to be risk neutral.

### 2.1 Posted-Price Selling

The seller does not know the valuation (V) of a potential buyer when he arrives, but she sets the price to maximize her overall expected profit. Suppose that the price is set at p. Since the buyer will not come back to check if the price is lower in the future, the object is sold if $V \geq p$. This happens with probability $1-F(p)$. The probability that the object is sold by the time k buyers have arrived is $1-F^k(p)$. Since buyers' arrivals follow a Poisson process, the probability that the object is sold by time t is

$$S(t) \equiv \sum_{k=0}^{\infty} [\ 1 - F^k(p)\ ] \cdot P_k(t) = 1 - e^{-\lambda(1-F(p))t}. \tag{2}$$

Note that $1-S(t)$ is the failure probability of a Poisson process with a parameter of $\lambda(1-F(p))$, which is the arrival rate of a potential buyer with willingness to buy higher than p. Recall that $\theta_d$ is the cost of display. The

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[3] It may be more reasonable to assume that they come back with some positive probability, probably increasing with their valuations. The analysis would become too complicated.

total expected profit for the seller is then

$$\Pi^S(p) = p - \theta d \int_0^{+\infty} t\, S'(t)dt = p - \frac{\theta d}{\lambda(\ 1-F(p)\ )} . \tag{3}$$

The price p that maximizes $\Pi^S(p)$ is given by:

$$\frac{d}{dp}\,\Pi^S(p) = 1 - \frac{\theta d f(p)}{\lambda[1-F(p)]^2} = 0, \tag{4}$$

$$\text{assuming that } \frac{d^2}{dp^2}\,\Pi^S(p) = -\frac{\theta d}{\lambda}\,\frac{d}{dp}\left(\frac{f(p)}{[\ 1-F(p)\ ]^2}\right) \le 0,\ \forall p. \tag{5}$$

Define $J(v) = v - \dfrac{1 - F(v)}{f(v)}$ . From (3) and (4), we have $\Pi^S(p) = J(p)$ for the optimal p.

Since the arrivals follow a Poisson process and buyers' valuations are independent, it is easy to see that the seller will not change the price even if no one arrives for a period of time or if all buyers arrived have rejected the offer. If the arrivals are not Poisson, the optimal prices may change over time, since the history before t can be informative about the future. Even though this affects the calculation of the expected profit generated by posted-price selling and auctions, as we shall see, it does not affect the comparison between the two selling methods.

## 2.2 Auctions

Suppose instead that the owner hands out an invitation to each arriving potential buyer to inform him that an auction will be held to sell the object at time T. All buyers that arrive at, or before, T are assumed to participate in the auction.[4] Since the object is privately valued and bidders are risk neutral, the Revenue Equivalence Theorem applies (cf. Riley and Samuelson (1981), Milgrom and Weber (1982)), and each of the four types of auctions yields the same expected revenue to the seller.

Consider then a second-price, sealed-bid auction. Each bidder bidding his own valuation is a dominant strategy in the auction. Since the winner (the highest bid) will pay the price of the second highest bid, the revenue for the seller is the expected value of the second highest valuation of the bidders.[5] (For a reference on the auction theory literature, see McAfee and McMillan (1987) or Milgrom (1990).)

Suppose that by the specified auction time, there are k arrivals. The probability density function for the second-highest valuation is $k(k-1)(1-F(v))F(v)^{k-2}f(v)$. Let $\tilde{\Pi}$ be the reservation utility of the seller and p be the reservation price in the auction. $\tilde{\Pi}$ is what the object is worth to the seller if it is not sold and it can be obtained by maximizing the seller's profit from later selling of the object. Hypothesizing that auctioning is the optimal way of selling, $\tilde{\Pi}$ represents the expected winning bid from later auctions minus the expected storage and auctioning costs. Suppose that $\tilde{\Pi}$ is given, the expected revenue for the seller calculated from the second-price auction with reservation price p is

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[4]We implicitly assume that the cost of participation for the buyers is zero. If that cost is positive, the rate of participation becomes endogenous. (cf. Harstad (1990) for an analysis when the object is commonly valued.) The profit generated by auctions will then be lower than what is to be calculated. We conjecture that none of the qualitative results changes, however.

[5]If the seller pre-announces the reservation price, bidders with lower valuation will not come to the auction even if the cost is infinitesimal. This does not affect the revenues generated by a second-price and the English auction. But the revenue generated by a first-price and a Dutch auction may be slightly different.

$$\Pi(k;p) = \int_p^{\bar{v}} v\cdot k(k-1)(1-F(v))F^{k-2}(v)f(v)dv + p\cdot k\Big[1-F(p)\Big]F^{k-1}(p) + F^k(p)\tilde{\Pi}$$

$$= \int_p^{\bar{v}} v d\Big[kF^{k-1}(v)-(k-1)F^k(v)\Big] + p\cdot k\Big[1-F(p)\Big]F^{k-1}(p) + F^k(p)\tilde{\Pi}$$

$$= \bar{v} - p\cdot F^k(p) - \int_p^{\bar{v}}\Big[kF^{k-1}(v)-(k-1)F^k(v)\Big]dv + F^k(p)\tilde{\Pi} . \tag{6}$$

In particular, $\Pi(0;p)=\tilde{\Pi}$, $\Pi(1;p)=p(1-F(p))+F(p)\tilde{\Pi}$.

Recalling that $J(v) = v - (1-F(v))/f(v)$ and that

$$\bar{v} - p\cdot F^k(p) = \int_p^{\bar{v}}\Big[F^k(v) + k\cdot vf(v)F^{k-1}(v)\Big]dv ,$$

we can rewrite (6) as

$$\Pi(k;p) = \int_p^{\bar{v}} kF^{k-1}(v)f(v)J(v)dv + F^k(p)\tilde{\Pi} . \tag{7}$$

First, we have the following lemma:

**Lemma 1**

The optimal reservation price p does not depend on k.

**Proof**

$$\frac{\partial\Pi(k;p)}{\partial p} = -kF^{k-1}(p)f(p)\Big[\ J(p) - \tilde{\Pi}\ \Big] \tag{8}$$

From this, we can see that the optimal price does not depend on k. ■

From this lemma, we know that the seller sets a fixed reservation price regardless of the number of participants in the auction, which simplifies the analysis greatly. Notice that $J(p) < p$. Thus, in the optimal auctions, the seller sets a reservation price that is higher than $\tilde{\Pi}$, the reservation value of the seller. Such results are common in the optimal auction literature. Interested readers can refer to, e.g., Riley and Samuelson (1981).

With probability $P_k(T)$, there are k arrivals by time T. The owner incurs a cost of storage $\theta sT$, plus a cost of auctioning $\Theta a$.[6] The expected profit of the owner is then given by:

$$\Pi^A = \sum_{k=0}^{\infty} \Pi(k;p)P_k(T) - \theta sT - \Theta a \ . \tag{9}$$

The following lemma describes the optimization behavior of a seller who chooses to sell the object by auctions:

**Lemma 2**

The optimal auctioning scheme for the owner is given by a pair $(T^*, p^*)$ that maximize

$$\Pi^A(T;p) = \frac{\lambda T \int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)J(v)dv - \theta sT - \Theta a}{1 - e^{-\lambda T(1-F(p))}}, \tag{10}$$

where the owner auctions the object at $T^*$, $2T^*$, .... until it is sold, and in each auction, the reservation price is $p^*$.

**Proof**

Recall that $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$ , and $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!}(ky^{k-1}) = x \cdot e^{xy}$.

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[6] Assume for simplicity that the owner incurs the auctioning cost even if no one show up in the auction.

$$\Pi^A = \sum_{k=0}^{\infty} \left[ \int_p^{\bar{v}} kF^{k-1}(v)f(v)J(v)dv + \tilde{\Pi}F^k(p) \right] \frac{(\lambda T)^k}{k!} e^{-\lambda T} - \theta sT - \Theta a$$

$$= \lambda T \int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)J(v)dv + \tilde{\Pi}e^{-\lambda T(1-F(p))} - \theta sT - \Theta a .$$

Since the arrival of potential buyers follows a Poisson process, and buyers never come back if they fail to obtain the object in an auction, then if the seller fails to sell the object in one auction, he faces the same profit maximization problem as if he were at the beginning of the game. Thus, $\tilde{\Pi}$ must be equal to $\Pi^A$. Defining that value as $\Pi^A(T;p)$, we complete the proof of this lemma. ■

For reasons similar to those in McAfee and McMillan (1988) and others, we need to assume that J(v) is a strictly increasing function of v to simplify the analysis. Lemma 4 gives us some intuition about this assumption. To prove Lemma 4, we need the following lemma:

**Lemma 3**

Let x(v), y(v), z(v) be differentiable functions, with $x(v)>0$, $y(v)>0$, $y'(v)<0$, $z'(v)>0$, $\forall v\in\Psi$. Then

$$\frac{\int_{v\in\Psi} x(v)y(v)z(v)dv}{\int_{v\in\Psi} x(v)y(v)dv} < \frac{\int_{v\in\Psi} x(v)z(v)dv}{\int_{v\in\Psi} x(v)dv} . \qquad (11)$$

**Proof**

$$\int_{v\in\Psi} x(v)y(v)z(v)dv\cdot\int_{v\in\Psi} x(v)dv - \int_{v\in\Psi} x(v)z(v)dv\cdot\int_{v\in\Psi} x(v)y(v)dv$$

$$= \int_{v\in\Psi}\int_{\tilde{v}\in\Psi}\left[x(v)y(v)z(v)x(\tilde{v}) - x(v)z(v)x(\tilde{v})y(\tilde{v})\right]dvd\tilde{v}$$

$$= \int\int_{\substack{v\in\Psi\\ \tilde{v}\in\Psi}} x(v)x(\tilde{v})z(v)\left[y(v) - y(\tilde{v})\right]dvd\tilde{v}$$

$$= \int\int_{v>\tilde{v}} x(v)x(\tilde{v})\left[y(v) - y(\tilde{v})\right]\left[z(v) - z(\tilde{v})\right] dvd\tilde{v} < 0$$ ■

This lemma is quite intuitive. As $z(v)$ is increasing, putting a decreasing weight on it decreases its weighted average. We shall make use of this lemma in the proofs of Lemma 4 and others.

**Lemma 4**

Given p, $J(v)$ increasing implies that the average selling price from the auction increases with T.

**Proof**

The average price from the auction is given by

$$R(T;p) \equiv \frac{\lambda T\int_p^{\bar{v}} e^{-\lambda T(1-F(v))}f(v)J(v)dv}{1 - e^{-\lambda T(1-F(p))}} = \frac{\int_p^{\bar{v}} e^{-\lambda T(1-F(v))}f(v)J(v)dv}{\int_p^{\bar{v}} f(v)e^{-\lambda T(1-F(v))}dv} \equiv \frac{N}{D}$$

$$\frac{\partial R(T;p)}{\partial T} = \frac{D_T(T;p)}{D(T;p)} \left[ \frac{N_T(T;p)}{D_T(T;p)} - \frac{N(T;p)}{D(T;p)} \right]$$

$$= \frac{D_T(T;p)}{D(T;p)} \left\{ \frac{\int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)[-\lambda(1-F(v)]J(v)dv}{\int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)[-\lambda(1-F(v)]dv} - \frac{\int_p^{\bar{v}} f(v)e^{-\lambda T(1-F(v))} J(v)dv}{\int_p^{\bar{v}} f(v)e^{-\lambda T(1-F(v))} dv} \right\}$$

Cancel out $(-\lambda)$ in $D_T$ and $N_T$. Because $J(v)$ is increasing and $1-F(v)$ is decreasing, from Lemma 3, the expression within the brackets is negative. Since $D_T<0$, $\partial R/\partial T>0$. ■

As T increases, the number of bidders in an auction tends to be larger. J(v) increasing insures that as more bidders attend the auction, the average price is higher. This is considered as a regularity condition in auctions, as is in Maskin and Riley (1984), Myerson (1981), McAfee and McMillan (1988). As we shall demonstrate later in the examples, this requirement is satisfied by most commonly-used continuous distributions.

The following lemma describes how T* and p* change when some of the parameters change in the model.

**Lemma 5**

The optimal auction interval T* is increasing in $\Theta_a$ and decreasing in $\theta_s$; the optimal reservation price p* is decreasing in both $\Theta_a$ and $\theta_s$; if the distribution F(v) shifts to the right by $\delta$, T* is unchanged and p* is increased by $\delta$.

**Proof**

Suppose that $T(\Theta_a, \theta_s)$ and $p(\Theta_a, \theta_s)$ maximize $\Pi^A(T;p)$. We have

$$\frac{\partial \Pi^A(T;p)}{\partial p} = \frac{D_p(T;p)}{D(T;p)} \left[ J(p) - \Pi^A(T;p) \right] = 0 \ ,$$

$$\frac{\partial^2 \Pi^A(T;p)}{\partial p^2} < 0 \ , \qquad \frac{\partial \Pi^A(T;p)}{\partial T} = 0, \qquad \frac{\partial^2 \Pi^A(T;p)}{\partial T^2} < 0,$$

where D(T;p) denotes the denominator of $\Pi^A(T;p)$.

Taking the derivative of $\partial\Pi^A/\partial T=0$ with respect to $\Theta_a$, we have

$$\frac{\partial^2\Pi^A}{\partial T^2}\cdot\frac{\partial T}{\partial\Theta_a} + \frac{\partial^2\Pi^A}{\partial T\partial p}\cdot\frac{\partial p}{\partial\Theta_a} + \frac{\partial^2\Pi^A}{\partial T\partial\Theta_a} = 0 \ .$$

Simple calculations show that $\partial^2\Pi^A/\partial T\partial\Theta_a > 0$. Making use of the first order conditions, it is easy to see that $\partial^2\Pi^A/\partial T\partial p = 0$. Thus, we conclude that $\partial T/\partial\Theta_a > 0$.

Similarly, because $\partial^2\Pi^A/\partial T\partial\theta_s < 0$, we conclude that $\partial T/\partial\theta_s < 0$.

Using the same method, and making use of the following inequality: $\partial^2\Pi^A/\partial p\partial\Theta_a < 0$, $\partial^2\Pi^A/\partial p\partial\theta_s < 0$, we conclude that $\partial p/\partial\Theta_a<0$, $\partial p/\partial\theta_s<0$.

Now we consider the case when the distribution for bidders' private valuation F(v) is shifted to the right by $\delta$, i.e., $\tilde{F}(v) = F(v-\delta)$.

$$\tilde{\Pi}^A(\tilde{T};\tilde{p}) = \frac{\lambda\tilde{T}\int_{\tilde{p}}^{\tilde{\bar{v}}} e^{-\lambda\tilde{T}(1-\tilde{F}(\tilde{v}))}\tilde{f}(\tilde{v})\tilde{J}(\tilde{v})d\tilde{v} - \theta_s\tilde{T} - \Theta_a}{1 - e^{-\lambda\tilde{T}(1-\tilde{F}(\tilde{p}))}}$$

Let $v=\tilde{v}-\delta$ and $p=\tilde{p}-\delta$ in the above expression and recall that

$$\tilde{J}(\tilde{v}) = \tilde{v} - \frac{1-\tilde{F}(\tilde{v})}{\tilde{f}(\tilde{v})} = \delta + (\tilde{v}-\delta) - \frac{1-F(\tilde{v}-\delta)}{f(\tilde{v}-\delta)} = \delta + J(\tilde{v}-\delta).$$

We have

$$\tilde{\Pi}^A(\tilde{T};\tilde{p}) = \frac{\lambda\tilde{T}\int_p^{\bar{v}} e^{-\lambda\tilde{T}(1-F(v))}f(v)(J(v)+\delta)dv - \theta_s\tilde{T} - \Theta_a}{1 - e^{-\lambda T(1-F(p))}}$$

$$= \delta + \Pi^A(\tilde{T};p) \ .$$

If $T^*$, $p^*$ maximize $\Pi^A(T;p)$, then $\tilde{T}=T^*$, $\tilde{p}=p^*+\delta$ must maximize $\Pi^A(\tilde{T};\tilde{p})$. ■

As $\Theta_a$ increases, it is more expensive to hold an auction. Thus, the seller would rather wait longer to have more bidders. As $\theta_s$ increases, the cost of waiting is increased, and the seller would like to have an auction more quickly. A shift in the distribution affects only the profit of the seller. It has no effect on the timing of the auction. The reservation price, of course, will shift up by the same amount in response. As we shall see, this result will be needed in the next section.

## 3. Comparison between Auctions and Post-Price Selling

We are now ready to compare the revenues generated by the two selling methods. It is generally true that the cost of storage is lower than that of displaying (since the latter is a special case of the former). If they are the same, the following lemma implies that auctions will never be used.

**Lemma 6**

If $\Theta_a=0$, then $T^* = 0$ in the optimal sequence of auctions.

**Proof**

Since $1 - e^{-\lambda T(1-F(p))} = \lambda T \int_p^{\bar{v}} e^{-\lambda T(1-F(v))}f(v)dv$ , we have

$$\Pi^A(T;p) = \frac{\int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)J(v)dv - \theta s/\lambda}{\int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)dv} \equiv \frac{N(T;p)}{D(T;p)} . \qquad (12)$$

Taking the derivative with respect to p, we have

$$\frac{\partial \Pi^A(T;p)}{\partial p} = \frac{D_p(T;p)}{D(T;p)} \left[ J(p) - \frac{N(T;p)}{D(T;p)} \right] = 0.$$

So $\Pi^A(T;p) = J(p)$.[7]

Take the derivative with respect to T, we have

$$\frac{\partial \Pi^A(T;p)}{\partial T} = \frac{D_T(T;p)}{D(T;p)} \left[ \frac{N_T(T;p)}{D_T(T;p)} - \frac{N(T;p)}{D(T;p)} \right] .$$

Since J(v) is an increasing function of v, making use of the condition above, we have

$$\frac{N_T(T;p)}{D_T(T;p)} - \frac{N(T;p)}{D(T;p)} = \frac{\int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)J(v)(1-F(v))dv}{\int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)(1-F(v))dv} - J(p) \geq 0 , \qquad (13)$$

which together with $D_T(T;p) < 0$ imply that $T = 0$ is optimal.■

In the absence of an auctioning cost, the owner will choose to auction the object at every instant, which is essentially the same as posted-price

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[7] It is easy to see that $p < \bar{v}$ in the optimal auction. If $p=\underline{v}$ is optimal, however, $\frac{\partial \Pi^A(T;p)}{\partial p} \leq 0$. This implies that $\Pi^A(T;p) \geq J(p)$. Inequality (13) still holds.

selling. More significantly, it means that the optimal posted-price selling is equivalent to the optimal sequence of auctions with storage cost $\theta_d$ and no auctioning cost. If the displaying costs and the storage costs are the same and there is some auctioning cost, then posting a fixed price would be the optimal way to sell the object. This result is consistent with McAfee and McMillan's (1988) finding that if there are infinitely many potential sellers (buyers in our case), then offering a fixed price and approaching the sellers sequentially (in our case, offering each arriving buyer the fixed price) is optimal.

From this lemma, we see that the advantage of using auctions periodically derives from the hypothesis that storing is cheaper than displaying. Certain objects that are usually auctioned like agricultural produce, fresh fish, cut flowers, are expensive to display, relative to keeping them in fields or in water. Other frequently auctioned objects, like antiques and art works that have high values have high relative display costs due to security considerations.

Since posted-price selling is equivalent to the optimal auction with storage costs equal to $\theta_d$ and no auctioning cost, we can compare it with the optimal auctions in an easy way. We have the following theorem:

**Theorem 1**

There exists a function $H(\theta_s,\Theta_a)$, such that the optimal sequence of auctions described in Lemma 2 is preferred to the optimal posted-price selling if and only if $H(\theta_s,\Theta_a)\leq\theta_d$, where $H(\theta_s,\Theta_a)$ is increasing in its arguments, with $H(\theta_s,0)=\theta_s$, and $H(\theta_s,\Theta_a)>\theta_s$, $\forall\Theta_a>0$.

**Proof**

Denote the profit from the optimal posted-price selling as $S(\theta_d)$, which depends on the cost of display $\theta_d$; the profit from selling by the optimal auctions is denoted $A(\theta_s,\Theta_a)$.

It is easy to see that $A(\theta_s,\Theta_a)$ must be decreasing in $\theta_s$ and $\Theta_a$, and $S(\theta_d)$ must be decreasing in $\theta_d$. If $\Theta_a$=0, from Lemma 2, $A(\theta_s,0)=S(\theta_s)$. If $\Theta_a > 0$, however, $A(\theta_s,\Theta_a) < S(\theta_s)$. Let $H(\theta_s,\Theta_a) = S^{-1}(A(\theta_s,\Theta_a))$. It is easy to see that H is increasing in both $\theta_s$ and $\Theta_a$ with the following properties: $H(\theta_s,0)=\theta_s$, $H(\theta_s,\Theta_a) > \theta_s$, $\forall\Theta_a>0$; the optimal sequence of auctions yields a higher profit if and only if $H(\theta_s,\Theta_a) \leq \theta_d$. ■

The profit generated by the optimal sequence of auctions with storage cost $\theta_s$ and auctioning cost $\Theta_a$ is the same as the profit generated by the posted-price selling with display cost $H(\theta_s,\Theta_a)$. From (10), it is easy to see that

$$\left.\frac{d\theta_s}{d\Theta_a}\right|_{H=\text{constant}} = -\frac{1}{T(\theta_s,\Theta_a)} . \qquad (14)$$

Given any $\theta_d$, a representative curve can be drawn that divides the $(\Theta_a,\theta_s)$ space into an auction-favorable region and a posted-price-favorable region. (See Figure 1 on the next page).

Since $H(\theta_d,0)=\theta_d$, given $\theta_d$, $H(\theta_d,\Theta_a)=\theta_d$ is indirectly but completely determined by $T(\theta_s,\Theta_a)$. This is because the curve always intercepts the $\theta_s$-axis at $\theta_d$ and its slope is determined by (14). If there is a uniform shift in F(v), from Lemma 5 $T(\theta_s,\Theta_a)$ is unchanged, and thus the comparison between auctions and posted-price selling is unchanged.

In what follows, we shall explore the effect of dispersion on the H function in Theorem 1. First of all, we need to define the dispersion of a distribution. As different distributions may have different supports, a definition that depends on the support of a distribution is difficult. It becomes easier, however, if we consider the cumulated distribution function of

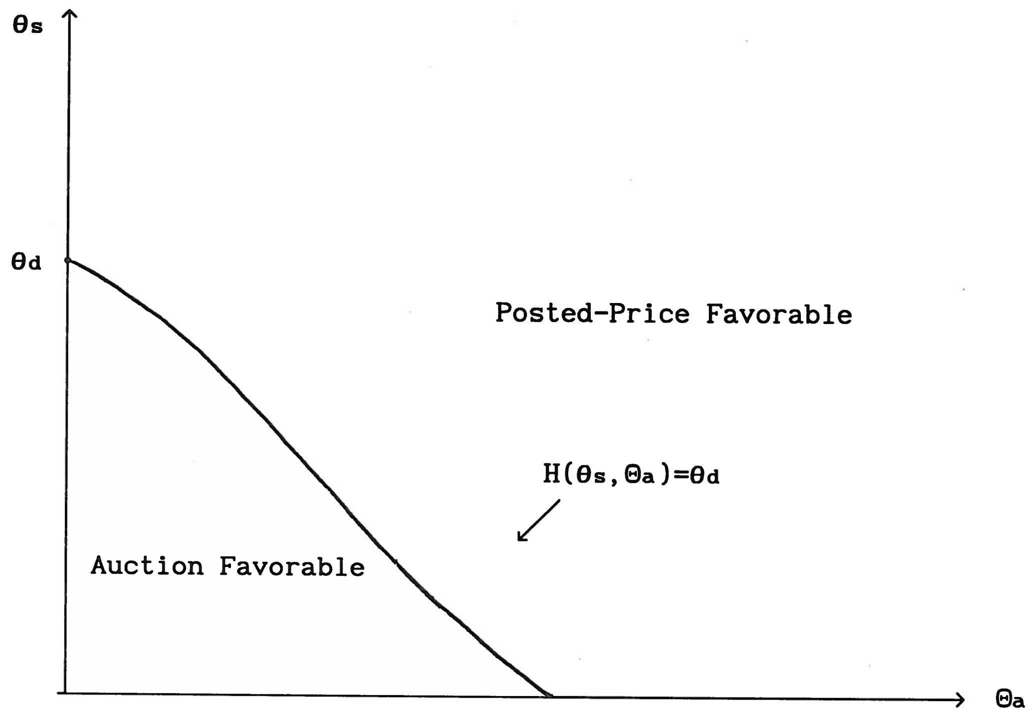


**Figure 1** Comparison between Posted-Price and Auctions

a random variable. Since a c.d.f. is always between 0 and 1, a definition based on it offers a common domain for comparing different distributions.

Assume that $f(v)>0$, $\forall v\in[\underline{v},\bar{v}]$, and thus $F(\cdot)$ is strictly increasing on its support. Define $g \equiv F^{-1}$. The domain of g is [0,1], while the range is the support of the distribution $[\underline{v},\bar{v}]$. Since $g'=1/f$, we have

$$J\circ g = g - \frac{1-F(g)}{f} = g - (1-F(g))g'. \tag{15}$$

Let $\Phi\in[0,1]$. The optimal auction problem we have studied can also be translated in to the $\Phi$ language. By change of variable $v=g(\Phi)$ and letting $F=F(p)$, $\Pi^A(T;p)$ can be expressed as

$$\Pi^A(T;F) = \frac{\lambda T \int_F^1 e^{-\lambda T(1-\Phi)} J(g(\Phi))d\Phi - \theta sT - \Theta a}{1 - e^{-\lambda T(1-F)}} . \tag{16}$$

Denote $J \circ g \equiv \mathfrak{J}$. The dispersion of a distribution is measured by $\mathfrak{J}$:

**Definition:** $F_1$ is more dispersed than $F_2$ if and only if

$$\mathfrak{J}_1'(\Phi) > \mathfrak{J}_2'(\Phi), \quad \forall \Phi \in [0,1].$$

This definition is consistent with our intuition about the dispersion of commonly used distributions. Here are some examples:

1) Exponential Distribution: $F(v) = 1 - e^{-kv}$, $v \in [0, +\infty)$.

In this case, $J(v) = v - \frac{1}{k}$, which is increasing,

$$g(\Phi) = \frac{1}{k}\ln\frac{1}{1-\Phi} , \text{ and } \mathfrak{J}'(\Phi) = \frac{1}{k(1-\Phi)} .$$

And as is generally agreed, the smaller the k, the more dispersed the distribution, which can be confirmed by $\mathfrak{J}'(\Phi)$.

2) Uniform Distribution: $F(v) = \dfrac{v - \underline{v}}{\bar{v} - \underline{v}}$, $v \in [\underline{v}, \bar{v}]$.

In this case, $J(v) = 2v - \bar{v}$, which is increasing. And

$g(\Phi) = (\bar{v}-\underline{v})\Phi + \underline{v}$ , and $\mathfrak{J}'(\Phi) = 2(\bar{v}-\underline{v})$.

The larger the $(\bar{v}-\underline{v})$, the more dispersed the distribution is.

3) Normal Distribution: $F(v) = \int_{-\infty}^{v} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\{- \frac{(x-\mu)^2}{2\sigma^2}\} \, dx$ , $v \in (-\infty, +\infty)$.

It is easy to prove that J(v) is increasing.[8] Define G(v) as the c.d.f. for the standard Normal distribution (i.e. $\mu=0$, $\sigma=1$). We have $F(v)=G(\frac{v-\mu}{\sigma})$. Thus, $g(\Phi)=\mu+\sigma G^{-1}(\Phi)$. And $\mathfrak{J}'(\Phi) = \sigma \cdot dJ(G^{-1}(\Phi))/d\Phi > 0$, since $G^{-1}$ is an increasing function. As $\sigma$ increases, the distribution becomes more dispersed.[9]

Let $V_1$, $V_2$ denote the random variables with differentiable c.d.f. The following lemma helps us establish some intuitions about the definition of dispersion.

**Lemma 7**

(i) $\mathfrak{J}_1'(\Phi) > \mathfrak{J}_2'(\Phi) \Rightarrow g_1'(\Phi) > g_2'(\Phi)$;

(ii) $g_1'(\Phi) > g_2'(\Phi) \Rightarrow Var(V_1) > Var(V_2)$.

**Proof**

(i) $\mathfrak{J}_i'(\Phi) = 2g_i'(\Phi) - (1-\Phi)g_i''(\Phi)$, $i=1,2$.

$\mathfrak{J}_1'(\Phi) > \mathfrak{J}_2'(\Phi)$ implies that $2(g_1'-g_2') - (1-\Phi)(g_1'-g_2')' > 0$.

Multiplying the above inequality by $-(1-\Phi)$, we have

$$\frac{d}{d\Phi}\left[(1-\Phi)^2(g_1'-g_2')\right] < 0 .$$

Since the expression within the square brackets is equal to zero when

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[8] $J(v) = v - \frac{1-F(v)}{f(v)}$, and $J'(v) = 2 + \frac{1-F(v)}{f^2(v)} f'(v)$. $f'(v) = -(v-\mu)f(v)/\sigma^2$, and $(v-\mu)(1-F(v)) = \int_v^{+\infty}(v-\mu)f(x)dx < \int_v^{+\infty}(x-\mu)f(x)dx = \sigma^2 f(v)$. Thus, $J'(v)>1$.

[9] In fact, from the above calculation we can conclude that for any distribution that is characterized by mean and variance only, with $J'(v)>0$, the higher the variance, the more dispersed the distribution is.

$\Phi=1$, we have, for $\Phi<1$, $(1-\Phi)^2(g_1'-g_2') > 0$, i.e. $g_1'(\Phi) > g_2'(\Phi)$. It is easy to see that $\mathfrak{J}_1'(1) > \mathfrak{J}_2'(1)$ implies $g_1'(1) > g_2'(1)$.

(ii) Let $\mu_i = \int_{\underline{v}_i}^{\overline{v}_i} v dF_i(v) = \int_0^1 g_i(\Phi)d\Phi$ denote the mean of $V_i$ and define

$$\tilde{g}_i(\Phi) = g_i(\Phi) - \mu_i .$$

Thus, $\quad \mathrm{Var}(V_i) = \int_{\underline{v}_i}^{\overline{v}_i} (v-\mu_i)^2 dF_i(v) = \int_0^1 \tilde{g}_i^2(\Phi)d\Phi.$ Therefore,

$$\mathrm{Var}(V_1) - \mathrm{Var}(V_2) = \int_0^1 \left[\tilde{g}_1(\Phi) + \tilde{g}_2(\Phi)\right]\left[\tilde{g}_1(\Phi) - \tilde{g}_2(\Phi)\right]d\Phi .$$

The expressions in the square brackets are both strictly increasing, since $\tilde{g}_i'(\Phi)=g_i'(\Phi)>0$, $i=1,2$, and by assumption, $g_1'(\Phi)>g_2'(\Phi)$. Since

$$\int_0^1 \left[\tilde{g}_1(\Phi) - \tilde{g}_2(\Phi)\right]d\Phi = 0 , \tag{17}$$

there exists $\Phi^*\in[0,1]$, such that

$$\tilde{g}_1(\Phi) - \tilde{g}_2(\Phi) > 0, \text{ if } \Phi > \Phi^*, \text{ and } \tilde{g}_1(\Phi) - \tilde{g}_2(\Phi) < 0, \text{ if } \Phi < \Phi^* .$$

Thus,

$$\int_{\Phi^*}^1 \left[\tilde{g}_1(\Phi)+\tilde{g}_2(\Phi)\right]\left[\tilde{g}_1(\Phi)-\tilde{g}_2(\Phi)\right]d\Phi > \int_{\Phi^*}^1 \left[\tilde{g}_1(\Phi^*)+\tilde{g}_2(\Phi^*)\right]\left[\tilde{g}_1(\Phi)-\tilde{g}_2(\Phi)\right]d\Phi \tag{18}$$

$$\int_0^{\Phi^*} \left[\tilde{g}_1(\Phi)+\tilde{g}_2(\Phi)\right]\left[\tilde{g}_1(\Phi)-\tilde{g}_2(\Phi)\right]d\Phi > \int_0^{\Phi^*} \left[\tilde{g}_1(\Phi^*)+\tilde{g}_2(\Phi^*)\right]\left[\tilde{g}_1(\Phi)-\tilde{g}_2(\Phi)\right]d\Phi \tag{19}$$

Adding (18) and (19), and making use of (17), we complete the proof of (ii).■

It it easy to see that the converse to (i) and (ii) is not true. Making use of this lemma, the dispersion of a distribution becomes more understandable with the help of a graph. As $g_1(\Phi)$ is steeper than $g_2(\Phi)$ in

$(\Phi, g)$ space, $F_1(v)$ is flatter than $F_2(v)$ in $(v,F)$ space, which means that $V_1$ is not distributed as concentrated as $V_2$. If we integrate $g_1'(\Phi) > g_2'(\Phi)$ both sides from 0 to 1, we have $\bar{v}_1 - \underline{v}_1 > \bar{v}_2 - \underline{v}_2$, i.e., the support of $V_1$ must be wider than $V_2$. These properties are shown in Figure 2.

The definition of dispersion here imposes stronger restrictions on the distribution than many other definitions, like those in Rothschild and Stiglitz (1970). Assume that $V_1$ is more dispersed than $V_2$. It is easy to see that it implies $V_1$ has more weight in the tails than $V_2$, which is one of the definitions in Rothschild and Stiglitz. The definition is also more restricted than those depending mainly on the cumulated distribution function, like risk dominance of higher orders.

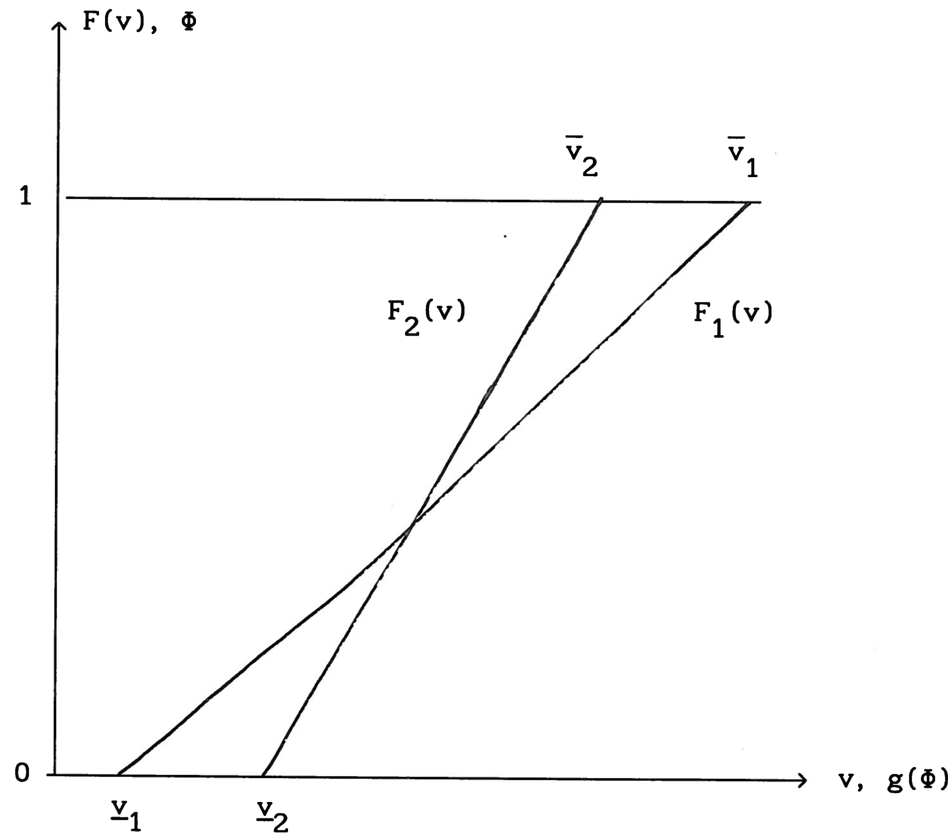


**Figure 2** The Dispersion of Distributions

The following theorem outlines the effect on the choice of selling methods when the distribution of a buyer's valuation becomes more dispersed.

**Theorem 2**

Suppose $\mathfrak{J}_1'(\Phi) > \mathfrak{J}_2'(\Phi)$, $\forall\Phi\in[0,1]$. Then $T_1(\Theta s,\Theta a) > T_2(\Theta s,\Theta a)$, $\forall\Theta s>0$, $\Theta a>0$; auctions are more attractive for $F_1(v)$ than $F_2(v)$.

**Proof**

Let $\mathfrak{J}_\xi(\Phi) = \xi\cdot\mathfrak{J}_1(\Phi)) + (1-\xi)\mathfrak{J}_2(\Phi)$, $\xi\in[0,1]$. Thus,

$$\Pi_\xi^A(T;F) \equiv \frac{\lambda T\int_F^1 e^{-\lambda T(1-\Phi)}\mathfrak{J}_\xi(\Phi)d\Phi - \Theta sT - \Theta a}{1 - e^{-\lambda T(1-F)}}$$

$$= \frac{\int_F^1 e^{-\lambda T(1-\Phi)}\mathfrak{J}_\xi(\Phi)d\Phi - \frac{\Theta s}{\lambda} - \frac{\Theta a}{\lambda T}}{\int_F^1 e^{-\lambda T(1-\Phi)}d\Phi} \equiv \frac{N(T;F)}{D(T;F)}\ .$$

Maximizing $\Pi_\xi^A(T;F)$ with respect to T and F, we have

$$\frac{\partial\Pi_\xi^A}{\partial T} = \frac{D_T}{D}\left[\frac{N_T}{D_T} - \frac{N}{D}\right] = \frac{D_T}{D}\left[\frac{\int_F^1 e^{-\lambda T(1-\Phi)}\mathfrak{J}_\xi(\Phi)(1-\Phi)d\Phi - \frac{\Theta a}{\lambda T^2}}{\int_F^1 e^{-\lambda T(1-\Phi)}(1-\Phi)d\Phi} - \frac{N}{D}\right] = 0$$

$$\frac{\partial\Pi_\xi^A}{\partial F} = 0, \qquad \frac{\partial^2\Pi_\xi^A}{\partial T\partial F} = 0, \quad \text{and} \quad \frac{\partial^2\Pi_\xi^A}{\partial T\partial T} < 0.$$

$$\frac{\partial^2\Pi_\xi^A}{\partial T\partial\xi} = \frac{\partial}{\partial\xi}\left(\frac{D_T}{D}\right)\left[\frac{N_T}{D_T} - \frac{N}{D}\right] + \frac{D_T}{D}\frac{\partial}{\partial\xi}\left[\frac{N_T}{D_T} - \frac{N}{D}\right]$$

$$= \frac{D_T}{D}\left[\frac{\int_F^1 e^{-\lambda T(1-\Phi)}\Delta\mathfrak{J}(\Phi)(1-\Phi)d\Phi}{\int_F^1 e^{-\lambda T(1-\Phi)}(1-\Phi)d\Phi} - \frac{\int_F^1 e^{-\lambda T(1-\Phi)}\Delta\mathfrak{J}(\Phi)d\Phi}{\int_F^1 e^{-\lambda T(1-\Phi)}d\Phi}\right] > 0, \qquad (20)$$

where $\Delta\mathfrak{I}(\Phi) \equiv \mathfrak{I}_1(\Phi) - \mathfrak{I}_2(\Phi)$. By assumption, $\Delta\mathfrak{I}'(\Phi)>0$. From Lemma 3 and noticing that $D_T<0$, equality (20) is positive.

Since

$$\frac{d}{d\xi}\left(\frac{\partial\Pi_\xi^A}{\partial T}\right) = \frac{\partial^2\Pi_\xi^A}{\partial T\partial T} \cdot \frac{dT}{d\xi} + \frac{\partial^2\Pi_\xi^A}{\partial T\partial F} \cdot \frac{dF}{d\xi} + \frac{\partial^2\Pi_\xi^A}{\partial T\partial\xi} = 0,$$

we conclude that $dT/d\xi > 0$. That is, $T_1(\theta s,\Theta a) > T_2(\theta s,\Theta a)$, $\forall\theta s,\Theta a\geq 0$.

Consider the iso-profit lines ($H_1$ and $H_2$ for distributions $F_1(v)$ and $F_2(v)$) that pass $(0,\theta d)$ in Figure 1. The slope for $H_1$ is flatter than that of $H_2$ at that point, since $T_1>T_2$. It is easy to see that $H_1$ does not cross $H_2$ from above, since otherwise $H_1$ is steeper than $H_2$ at the crossing point and thus $T_1>T_2$ is violated. Thus, $H_1$ must lie above $H_2$ entirely (see Figure 3). That is, auctions are attractive in more occasions for $F_1$ (a more dispersed distribution) than $F_2$. ■

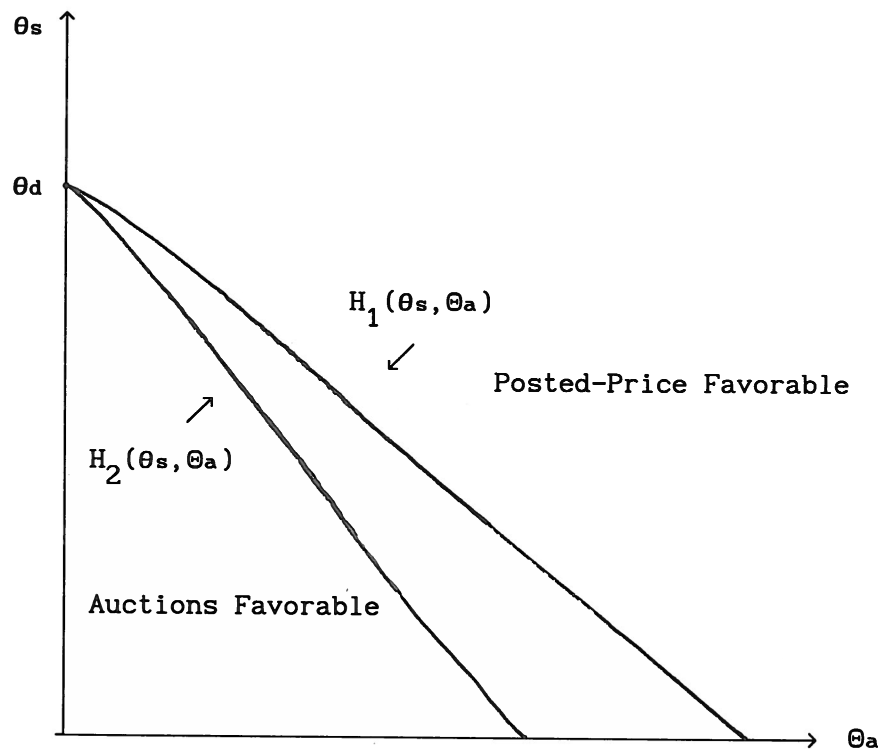


**Figure 3** Auctions Favorable Regions and the Dispersion of Distributions

Theorem 2 says that objects with larger values are more likely to be auctioned, given that they have more dispersed distributions. A more dispersed distribution provides more incentive for the seller to wait longer, since the distribution has a steeper profile of $\mathfrak{J}(\Phi)$ than a less dispersed distribution. As T increases, the distribution of winning bid shifts up, and the higher portions of $\mathfrak{J}(\Phi)$ receive more weight. Thus, the seller would rather set a longer T in order to have more bidders in the auction and the profits from such auctions are increased. Therefore, the position for the optimal sequence of auctions is improved, and the posted-price selling becomes less attractive.

### 3. The Welfare Effect of Searching

Assume that a social plan has exactly the same information as the seller. Since a social planner evaluates the sale by the willingness to pay of the successful buyer instead of the price, we may expect that he should set a lower price and wait less and thus the cost of waiting is reduced. The following theorem shows that this is not always the case. The intuition is that as one waits longer, a buyer with higher valuation may come. Therefore, the total effect of cutting price or waiting less between auctions is ambiguous.

Let $\psi(v) \equiv \frac{1-F(v)}{f(v)}$ denote the reciprocal of the hazard rate of the distribution. Thus, $J(v) = v - \psi(v)$. We have the following theorem:

**Theorem 3**

If $\psi(v)$ is monotone increasing (decreasing), then the price is lower (higher) than the socially desirable level, and T is shorter (longer) than the socially desirable level.

**Proof**

The expected social welfare is defined as the expected willingness to pay of a successful buyer minus the expected cost of selling or auctioning the object. For a given price p and time interval T, the expected social welfare

$$W(T;p) = \frac{\lambda T \int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)vdv - \theta_s T - \theta_a}{1 - e^{-\lambda T(1-F(p))}} \tag{21}$$

$$= \Pi^A(T;p) + \frac{\int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)\psi(v)dv}{\int_p^{\bar{v}} e^{-\lambda T(1-F(v))} f(v)dv}$$

At the T and p that maximize $\Pi^A(T;p)$, $\frac{\partial \Pi^A(T;p)}{\partial T} = 0$, $\frac{\partial \Pi^A(T;p)}{\partial p} = 0$. Let $h(v) \equiv e^{-\lambda T(1-F(v))} f(v)$, and denote the numerator and denominator in (21) as N(T;p) and D(T;p). We have

$$\frac{\partial W}{\partial p} = \frac{D_p(T;p)}{D(T;p)} \left[ \frac{N_p(T;P)}{D_p(T;P)} - \frac{N(T;p)}{D(T;p)} \right]$$

$$= \frac{-h(p)}{D(T;p)} \left[ \frac{-h(p)\ g(p)}{-h(p)} - \frac{\int_p h(v)\psi(v)dv}{\int_p h(v)dv} \right]$$

$$= \frac{h(p)}{D(T;p)} \left[ \frac{\int_p h(v)\psi(v)dv}{\int_p h(v)dv} - g(p) \right] \tag{22}$$

If $\psi(v)$ is increasing, then $\frac{\partial W}{\partial p} > 0$. Thus, a marginal increase at the optimal price increases social welfare. This implies that the price set by the owner is too low compared to the socially desirable level. If $\psi(v)$ is decreasing, then $\frac{\partial W}{\partial p} < 0$. Thus, a marginal decrease at the optimal price increase social welfare. This implies that the price is too high.

$$\frac{\partial W}{\partial T} = \frac{D_T(T;p)}{D(T;p)} \left[ \frac{N_T(T;P)}{D_T(T;P)} - \frac{N(T;p)}{D(T;p)} \right]$$

$$= \frac{-\lambda \int h(v)(1-F(v))dv}{D(T;p)} \left[ \frac{\int h(v)\psi(v)(1-F(v))dv}{\int h(v)(1-F(v))dv} - \frac{\int h(v)\psi(v)dv}{\int h(v)dv} \right]$$

Since $F(v)$ is increasing, if $\psi(v)$ is increasing, from Lemma 3, the above is negative and thus $\frac{\partial W}{\partial T} > 0$. This implies that a marginal increase at the optimal T will increase the social welfare. Thus, the optimal T chosen by the owner is smaller than the socially desirable level. If $\psi(v)$ is decreasing, then $-\psi(v)$ is increasing. Similarly, from Lemma 3, $\frac{\partial W}{\partial T} < 0$. A decrease in T improves social welfare. Thus, the optimal T chosen by the owner exceeds the socially desirable level. ■

A social planner cares about the winning buyer's valuation (v) instead of his expected actual payment (J(v)). The difference between those two values is $\psi(v)$. If $\psi(v)$ is increasing, the social planner tends to have a higher reservation price than the seller's. Since the social planner has a steeper profile for the valuation in this case, he would rather wait longer for more high valuation bidders to come. Thus, the social planner would choose a larger T. When $\psi(v)$ is decreasing, it is easy to see that the effects work in the opposite direction. As posted-price selling is a special case of a sequence of auctions, the prices chosen by the social planner have the same tendency as above.

## 4. Concluding Remarks

In this paper, a stochastic dynamic model is presented to address the problem of optimal selling mechanisms. Given a proper definition of dispersion, we prove that auctions are more often used when the distribution of the object's value is more dispersed. This measure of dispersion is found to be closely related to the J(x) function in various previous papers. A few examples are provided to illustrate the fact that the definition coincides with our intuition. We also prove that dispersion and variance are closely related--a more dispersed distribution also has a higher variance. It is found that the optimal reservation price (the fixed price in posted-price selling) and the optimal time between auctions may be different from those of a social planner who maximizes the social welfare.

We have made a few assumptions in the paper. Those assumptions are meant to simplify the analysis. The assumption that potential buyers arrive following a Poisson process is not important to the qualitative results in the paper. If arrivals follow other processes, the optimal reservation price and the optimal time length may vary over time. The comparison between auctions and posted-price selling, however, is expected to have the same result. We also assume that buyers' valuations are distributed independently. If their valuations are correlated, the revenue equivalence among different forms of auctions will not hold. In this case, if buyers in a posted-price selling method do not know their positions in the arriving order, then the comparison will favor a sequence of auctions (assuming, for example, that English auctions are used), since more information is revealed in an auction, and thus the revenue of the seller is improved (cf. Milgrom and Weber 1982). The assumption on the risk-neutrality of buyers is not important. As buyers become more risk averse, their valuation (or willingness to pay) is higher, since the uncertain outcome of searching further has less utility. This, however, should not affect the comparison between auctions and posted-price selling, since a uniform shift in a buyer's valuation does not affect the comparison.

The results obtained in the paper are supported by the observation that auctions are often used to gather potential buyers (or sellers). Buyers getting a good bargain in the auction is certainly not the objective of the auctioneer. As it is usually true that displaying an object is more expensive than storing it, cost saving considerations are probably the main reason for holding an auction.

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