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The Redistributive Role of Minimum Wage Legislation and Unemployment Insurance

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Abstract

The use of unemployment insurance and minimum wages as instruments for redistributing income are analyzed. The government is assumed to be able to implement an optimal income tax in an economy consisting of two ability-types of persons. The effect of introducing a minimum wage which induces involuntary unemployment combined with unemployment insurance is considered. Social welfare can be improved despite the possible revenue costs to the government if the policy causes a self-selection constraint to be weakened by enough. Sufficiency conditions are derived for this to be the case.

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1. Introduction

Arguments in favour of minimum wage (MW) legislation or unemployment insurance (UI) are seldom seen in the literature. A major reason for this is that a well-known consequence of those policies is an adverse effect on employment, and hence on the efficiency of the economy.[1] We argue in this paper that, in the context of the optimal non-linear income taxation literature, the combination of MW and UI can be part of a social-welfare maximizing policy if the social planner has redistributive objectives and faces a realistic informational constraint.

Recently, the analysis of optimal redistributive policy has been extended to include instruments other than non-linear taxes. The seminal paper is that of Guesnerie and Roberts (1984) who showed that, in general, quantity controls could improve social welfare even when optimal commodity and non-linear income taxes were in place. In Guesnerie and Roberts (1987), they applied their analysis to the case of minimum wages. They assumed the effect of minimum wages was to cause underemployment (reduced hours of work) of low income persons. When the planner was restricted to a linear income tax, they showed that MW policy could improve welfare. However, once non-linear income taxes were allowed, the case for MW was diminished considerably. In fact, if households have concave and identical utility functions (but different wage rates) and if the social welfare function is utilitarian, MW cannot be welfare-improving. Allen (1987) considered the usefulness of a minimum wage in the context of non-linear income taxation with general equilibrium effects on wages. He also argued that minimum wages would

[1] Most of the literature on UI has emphasized its impact on employment. Classic studies of this include Baily (1977), Burdett (1979), Feldstein (1976), and Mortensen (1970). Some recent work is found in Burdett and Hool (1983) and Burdett and Wright (1989a, 1989b). On the normative side, analyses of the optimal UI scheme are found in Baily (1977, 1978). Other work providing a rationale for publicly-provided UI include Boadway and Oswald (1983) and Pissarides (1983). We are not aware of any studies of the welfare analysis of UI in the framework of the optimal non-linear income taxation literature. On MW, the classic paper is by Stigler (1946). A survey of more recent work may be found in Brown, Gilroy and Kohen (1982).

generally not be welfare-improving in this context. While Guesnerie and Roberts conducted their analysis under the assumption that MW caused underemployment, they suggested in their discussion that it may, in some circumstances, be welfare-improving to induce unemployment in which the unemployed are equally as well off as the employed. However, in this case, the unemployment would not involve true rationing; that is, it would not be involuntary.

In this paper, we allow not only MW but also UI policy at the same time. The MW is assumed to cause involuntary unemployment. Although, by itself, MW may not be a useful redistributive device when the optimal non-linear income tax is in place, the combination of the two policies may well be welfare-improving. We derive a set of sufficient conditions for this to be the case. The analysis can be interpreted as investigating whether or not UI and MW are *efficient* instruments for redistribution, alongside non-linear income taxes. Thus, the analysis does not rely on a specific formulation for the social welfare function. In the spirit of Stiglitz (1982), the analysis investigates whether the use of UI and MW can yield a Pareto improvement, given that the optimal non-linear income tax is in place.

Following the optimal income tax literature (see Mirrlees (1971)), we will consider the problem of a government that is able to observe the incomes but not the skill levels (wages) of the workers in a two-skill-level economy. This observability problem rules out any first-best allocation where lump-sum taxation based on skill levels is imposed. Even in the case where there is no observability problem, there are limits to the amount of redistribution that can be undertaken by the government. The observability problem limits further the potential for redistribution. The government will be able to use a non-linear tax schedule to redistribute income but, for observability reasons, the tax rates will depend on incomes, not on the skill levels. Consequently, some workers of type i may try to mimic the income of the workers of type j (by providing less units of labour) if, for example, the tax rates are such that it is in their interest to do so. The government, in its attempt to redistribute income, will thus face a self-selection constraint. The reason why MW and UI policies could be useful here will emerge from the possibility of relaxing the self-selection constraint which, in turn, will imply that a Pareto-improving redistribution is possible. This argument is similar to that of Boadway and Marchand (1990) who argued that public expenditures on education and pension may be useful redistributive devices by relaxing the self-selection constraint.

The model we use will be a decentralized version of the one used by Stiglitz (1982).[2] However, here, wages will be endogenously determined so that the choice of

[2] Stiglitz solves the optimal income tax problem as a centralized problem of the planner by using the revelation principle. A decentralized version of a similar problem was used by Boadway and Marchand (1990). We use the latter version

the tax rates will have a general equilibrium effect on wages[3] and the implementation of a MW will induce some unemployment. The planner will be assumed to be able to implement the optimal non-linear income tax and to choose MW and UI. The latter instruments will apply only to the lower skilled workers, and to those higher skilled workers who choose to mimic the low skilled ones.

This paper is organized as follows. In the next section, we present a model of a decentralized economy in which the wages are endogenously determined and in which there is some unemployment. Following that, the social planner's problem is examined for a given level of unemployment and UI benefit. Then, we consider the potential for welfare improvement associated with the implementation of a MW beginning from an initial situation of no unemployment. The conclusion follows.

2. The Model

Consider an economy in which there are two types of workers: low-skilled workers and high-skilled workers respectively denoted by 1 and 2. There are N_1 low-skilled workers and N_2 high-skilled workers. We denote by U the number of low-skilled workers that may be made unemployed. There is a single consumption good Q in this economy which is produced according to the following technology:

$$Q = F((N_1 - U)L_1) + G(N_2 L_2) \tag{1}$$

where L_1 and L_2 are the labour supplies of low-skilled (when working) and high-skilled workers respectively. The production functions satisfy the following properties:[4]

$$F' > 0; \quad F'' < 0; \quad G' > 0; \quad G'' < 0; \quad F'(NL) < G'(NL).$$

We assume perfect competition in the product and the labour markets and

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here. The results are the same in the two models, but the decentralized version has some heuristic advantages for our purposes.

[3] Feldstein (1973) and Allen (1982) examined the optimal linear income tax schedules in this context. Stiglitz (1982, 1985) and Stern (1982) did the same for non-linear income tax schedules.

[4] Note that there will be positive profits. Those profits will be taken into account later. In particular, they will be fully taxed. An alternative procedure for making wage rates endogenous would have been to follow Stiglitz (1982) and Allen (1987) and allow both types of labour to enter into a linear homogeneous production function as imperfect substitutes. In this case, there are no pure profits to worry about. For our purposes, the technology summarized in (1) was slightly easier to work with.

take the price of Q to be unity so that the workers are paid a wage equal to their marginal productivity:

$$w_1 = F'\left((N_1 - U)L_1\right) \tag{2}$$

$$w_2 = G'\left(N_2 L_2\right). \tag{3}$$

Note that the property of F and G such that $F'(NL) < G'(NL)$ reveals what we mean by low-skilled and high-skilled workers. It implies that, in general, the higher the skills, the higher the wage: $w_1 < w_2$.

It will be useful for later purposes to develop some simple comparative static results for wage rates and profits. As will be seen later, the labour supplies of households can be written as:

$$L_1 = L^1\left(\tau_1, T_1, w_1\right) \tag{4}$$

$$L_2 = L^2\left(\tau_2, T_2, w_2\right) \tag{5}$$

where τ_i is one minus the implicit marginal tax rate on income of a worker of type i, and T_i is the implicit lump-sum tax component of the income tax schedule of a worker of type i. The exact interpretation of these tax rates will be discussed later when we take up the problem of the household. Denote by L^i_τ the partial derivative of L_i with respect to the argument τ_i, and similarly for T_i and w_i. We assume throughout the paper that the following conditions will hold:[5]

$$L^i_\tau, L^i_T, L^i_w \geq 0, \quad i = 1, 2.$$

Thus, consider for now the unemployment level to be exogenous. Differentiating equations (2) and (3) and using (4) and (5) yield the following:

$$\frac{\partial w_1}{\partial \tau_1} = \Phi L^1_\tau \leq 0; \qquad \frac{\partial w_1}{\partial T_1} = \Phi L^1_T \leq 0 \tag{6}$$

$$\frac{\partial w_1}{\partial U} = \frac{-F'' L_1}{1 - (N_1 - U)F'' L^1_w} \geq 0 \tag{7}$$

$$\frac{\partial w_2}{\partial \tau_2} = \Theta L^2_\tau \leq 0; \qquad \frac{\partial w_2}{\partial T_2} = \Theta L^2_T \leq 0 \tag{8}$$

where

$$\Phi = \frac{(N_1 - U)F''}{1 - (N_1 - U)F'' L^1_w}$$

[5] This assumes that the substitution effect is at least as great as the income effect.

and

$$\Theta = \frac{N_2 G''}{1 - N_2 G'' L_w^2}.$$

Since both $F(\cdot)$ and $G(\cdot)$ are strictly concave, there will be positive profits in this model. Those profits are assumed to be fully taxed and thus will enter the government budget constraint. Letting Π denote profits, we have:

$$\Pi(\tau_1, T_1, \tau_2, T_2, U) = F\left((N_1 - U)L_1\right) + G\left(N_2 L_2\right) - (N_1 - U)L_1 w_1 - N_2 L_2 w_2. \quad (9)$$

Differentiating equation (9) and using equations (2)–(5), we obtain (where Π_{τ_i} denotes the total derivative of Π with respect to τ_i, etc.):

$$\begin{aligned}
\Pi_{\tau_1} &= -(N_1 - U)L_1 \frac{\partial w_1}{\partial \tau_1} > 0 \\
\Pi_{T_1} &= -(N_1 - U)L_1 \frac{\partial w_1}{\partial T_1} > 0 \\
\Pi_{\tau_2} &= -N_2 L_2 \frac{\partial w_2}{\partial \tau_2} > 0 \\
\Pi_{T_2} &= -N_2 L_2 \frac{\partial w_2}{\partial T_2} > 0 \\
\Pi_U &= -(N_1 - U)L_1 \frac{\partial w_1}{\partial U} < 0.
\end{aligned} \quad (10)$$

We now turn to the behaviour of households. Following the optimal income literature, we assume that all households have identical preferences given by the utility function $u(C_i, L_i)$ where C_i is the consumption of household i. The utility function is assumed to be concave with:

$$u_1^i > 0; \quad u_{11}^i < 0; \quad u_2^i < 0; u_{22}^i < 0.$$

In our later analysis we also assume $u_{12}^i \leq 0$, that is, that leisure and consumption are either complementary or separable. Households face a budget constraint stating that their consumption equals their after-tax income. The government is assumed to be able to observe income, but not individual wage rates or labour supplies. It can therefore choose a non-linear income income tax schedule of the form $\Lambda(w_i L_i)$ so that a worker of type i supplying L_i units of labour obtains after-tax income $Y_i = w_i L_i - \Lambda(w_i L_i)$. Given the non-linearity of the tax schedule, in equilibrium households will face different marginal and average tax rates. It is useful for analytical purposes to rewrite the budget constraint in the virtual form by linearizing it at the equilibrium. To do so, we introduce the virtual tax parameters τ_i and T_i, which represent the marginal tax rate and the implicit lump-sum tax rate at the equilibrium. After-tax income can then be written:

$$Y_i = \tau_i w_i L_i - T_i.$$

Note that the fact that $\Lambda(\cdot)$ is non-linear implies that $\tau_1(T_1)$ and $\tau_2(T_2)$ do not have to be equal.

Since $C_i = Y_i$, we may rewrite the utility function of a type i household as:

$$u_i = u(\tau_i w_i L_i - T_i, L_i). \tag{11}$$

The problem of household i is simply to maximize utility by choice of labour supply taking τ_i, T_i, and w_i as given:

$$\max_{L_i} u(\tau_i w_i L_i - T_i, L_i).$$

The first order condition of this maximization problem is:

$$u_1^i \tau_i w_i + u_2^i = 0. \tag{12}$$

The solution to this problem yields the labour supply function which we used above:

$$L_i = L^i(\tau_i, T_i, w_i). \tag{13}$$

Recall that we have assumed it to be non-decreasing in all its arguments. Substituting the labour supply in the utility function, we obtain the indirect utility function:

$$v^i(\tau_i, T_i, w_i) = u(\tau_i w_i L^i(\tau_i, T_i, w_i), L^i(\tau_i, T_i, w_i)). \tag{14}$$

Using the envelope theorem, it is then straightforward to obtain:

$$v_\tau^i = u_1^i w_i L_i \tag{15}$$

$$v_T^i = -u_1^i \tag{16}$$

$$v_w^i = u_1^i \tau_i L_i \tag{17}$$

where $v_\tau^i = \frac{\partial v^i}{\partial \tau_i}$, etc.

The above comparative static effects of taxes on the household's indirect utility represent only the direct effects. As shown above, the tax rates as well as the unemployment rate will affect the equilibrium wage rates and therefore will indirectly affect household utilities. Denoting by δv^i_τ the total derivative of v_i with respect to τ_i, and similarly for T_i and the unemployment rate U, we obtain:

$$\delta v^1_\tau = v^1_\tau + v^1_w \frac{\partial w_1}{\partial \tau_1}; \qquad \delta v^1_T = v^1_T + v^1_w \frac{\partial w_1}{\partial T_1} \tag{18}$$

$$\delta v^1_U = v^1_w \frac{\partial w_1}{\partial U} \tag{19}$$

$$\delta v^2_\tau = v^2_\tau + v^2_w \frac{\partial w_2}{\partial \tau_2}; \qquad \delta v^2_T = v^2_T + v^2_w \frac{\partial w_2}{\partial T_2}. \tag{20}$$

For future reference, we also denote by δL^i_τ and δL^i_T the total derivative of L_i in (13) with respect to the tax rates taking account of the effect on w_i (i.e., $\delta L^1_\tau = L^1_\tau + L^1_w \frac{\partial w_1}{\partial \tau_1}$). Similarly, δL^1_U is the total derivative of L_1 with respect to the unemployment rate.

The above discussion of household behaviour applied only to those households who were working and behaving as utility maximizers. In our model, two other outcomes are possible for households. The first arises because households may find themselves unemployed. The second arises because of the possibility that high-skilled households may wish to mimic the income of low-skilled households to exploit the income tax system. Consider each of these in turn.

Suppose there are U low-skilled workers unemployed. We assume the unemployment is involuntary and is induced by the government setting MW above the market clearing wage for the low-skilled workers. For simplicity and since all low-skilled workers are identical, we assume the unemployed to be drawn randomly from the N_1 low-skilled workers so that, *ex ante*, the low-skilled workers will face a probability (U/N_1) of being made unemployed. Suppose that b is the UI benefit paid to the unemployed. Then, the expected utility of the low-skilled workers is:

$$Ev_1 = \left(\frac{N_1 - U}{N_1}\right) v^1(\tau_1, T_1, U) + \frac{U}{N_1} u(b, 0). \tag{21}$$

The wage rate of the high-skilled workers will be above MW so there will be no unemployment of those high skilled workers who do not attempt to mimic the low-skilled. For them, their expected utility will be equivalently $v^2(\tau_2, T_2)$.

However, in formulating the planner's redistributive policy, we must take account of the fact that high-skilled workers may have an incentive to mimic the low-skilled. This is because we presume that the redistributive policy is to transfer income from the high-skilled to the low-skilled workers. If the tax rates on the income of high-skilled workers are too high, those workers will have an incentive to mimic the income of the low-skilled workers by providing less units of labour. By doing so, they will then face the low-skilled workers' income tax parameters and this could be advantageous. As is well-known, this possibility of mimicking restricts the amount of redistribution that can take place. Technically, the government must take account of a self-selection constraint on the high-skilled workers. To formulate this constraint, we need to characterize the mimicking behaviour of the high-skilled persons. In this context where the possibility of unemployment of the low-skilled workers exists, this is not as straightforward a task as it is in the ordinary optimal income tax problem of Stiglitz (1982).

Denote with a "hat" those variables applying to the mimicking high-skilled workers. To mimic the income of a low-skilled worker, high-skilled workers will have to supply $\hat{L}_2 = w_1 L_1 / w_2$ units of labour. Since $w_1 < w_2$, this implies that $\hat{L}_2 < L_1$. The mimicking worker has no decision variables in this model. We may write the utility of the mimicking households when working by:

$$u\left(\tau_1 w_1(\cdot) L_1(\cdot) - T_1, \frac{w_1(\cdot) L_1(\cdot)}{w_2(\cdot)}\right) = \hat{v}^2(\tau_1, T_1, \tau_2, T_2, U). \tag{22}$$

Note that $\hat{v}^2 > v^1$ since the consumption levels are the same for the two persons but the mimicker supplies less labour than the low-skilled worker. Total differentiation of (22) yields (using the same notation as before):

$$\delta \hat{v}^2_{\tau_1} = \hat{u}^2_1 \left[w_1 L_1 + \tau_1 L_1 \frac{\partial w_1}{\partial \tau_1} + \tau_1 w_1 \delta L^1_\tau \right] + \frac{\hat{u}^2_2}{w_2} \left[L_1 \frac{\partial w_1}{\partial \tau_1} + w_1 \delta L^1_\tau \right] \tag{23}$$

$$\delta \hat{v}^2_{T_1} = \hat{u}^2_1 \left[\tau_1 L_1 \frac{\partial w_1}{\partial T_1} + \tau_1 w_1 \delta L^1_T - 1 \right] + \frac{\hat{u}^2_2}{w_2} \left[L_1 \frac{\partial w_1}{\partial T_1} + w_1 \delta L^1_T \right] \tag{24}$$

$$\delta \hat{v}^2_{\tau_2} = -\hat{u}^2_2 \left(\frac{w_1 L_1}{w_2^2} \right) \frac{\partial w_2}{\partial \tau_2} \tag{25}$$

$$\delta \hat{v}^2_{T_2} = -\hat{u}^2_2 \left(\frac{w_1 L_1}{w_2^2} \right) \frac{\partial w_2}{\partial T_2} \tag{26}$$

$$\delta \hat{v}^2_U = \left[\hat{u}^2_1 \tau_1 + \frac{\hat{u}^2_2}{w_2} \right] \left[L_1 \frac{\partial w_1}{\partial U} + w_1 \delta L^1_U \right]. \tag{27}$$

These utility effects apply only when the mimicking person is employed. Depending on the institutional and informational assumptions we make, the mimicking person may also face the prospect of being unemployed. Essentially that depends upon whether the mimicking person is required to mimic both the income and the employment of the low-skilled person or only the income. We will treat both types of mimicking behaviour in what follows. The first one will be called "full mimicking" behaviour and the second one "partial mimicking" behaviour. Much of our analysis will be done for the case of full mimicking. As we show later, partial mimicking can be treated as a simplification of full mimicking and similar qualitative results will apply. Guesnerie and Roberts (1987), in their informal discussion of mimicking behaviour of the high skilled under unemployment, assumed the full mimicking framework.

In the case of full mimicking, the high-skilled household who mimics the low-skilled household will face, as do the low-skilled workers, a probability (U/N_1) of being made unemployed. This probability presumes that in the equilibrium derived below when the planner is optimizing, the self-selection constraint will be binding, and no high-skilled worker will, in fact, choose to mimic. Nonetheless, for the purposes of formulating the self-selection constraint, it is necessary to specify the expected utility that would be achieved in the event of mimicking behaviour. It will be given by:

$$E\hat{v}^2 = \left(\frac{N_1 - U}{N_1}\right) \hat{v}^2(\tau_1, T_1, \tau_2, T_2, U) + \frac{U}{N_1} u(b, 0). \tag{28}$$

In the case of partial mimicking, where only the income level has to be mimicked, the mimicking household can escape the possibility of being unemployed. In this case, a type $\hat{2}$ worker will simply have utility given by $\hat{v}^2(\tau_1, T_1, \tau_2, T_2, U)$.

3. The Planner's Optimal Income Tax Problem

The planner is assumed to want to redistribute income from the high to the low income persons. As in Stiglitz (1982), virtually all the interesting qualitative results on optimal income taxation can be treated as efficiency results; that is, as finding the optimal income tax system which makes, say, the high income persons as well off as possible consistent with a given level of utility of the low income person given the constraints faced by the planner. Our objective is to determine whether further Pareto improvement can be achieved by adding the additional instruments of MW and UI for redistributive purposes. Thus, our analysis should be treated as one of efficiency rather than equity. However, for heuristic reasons we shall conduct the analysis as a social welfare-maximizing problem using a utilitarian social welfare function. It should be clear from our analysis, however, that any quasi-concave social welfare function will yield the same results. We proceed in two stages. In the first stage, the planner chooses the optimal income tax structure, given a level of unemployment U and UI benefits b. We will be particularly concerned with the case in which there is no unemployment. In the second stage, we consider whether social welfare can be improved by inducing some unemployment using MW as an instrument. The main analysis will be conducted for the case of full mimicking. As we proceed, we point out the difference that partial mimicking will make to the results.

The planner's problem then is to maximize the sum of expected utilities of the households by choice of the non-linear income tax parameters (τ_i, T_i) subject to a resource constraint and to a self-selection constraint. The resource constraint takes the form of a government budget constraint and must account for income tax revenues as well as unemployment benefits and all pure profits, which are assumed to accrue to the government. The self-selection constraint requires that the high-wage household can do no better by mimicking the low-income household than by not mimicking.

The planner's problem may be written:

$$\max_{\tau_1, T_1, \tau_2, T_2} \quad N_1 \left[\left(\frac{N_1 - U}{N_1} \right) v^1(\tau_1, T_1, U) + \left(\frac{U}{N_1} \right) u(b, 0) \right] + N_2 v^2(\tau_2, T_2)$$

subject to:

$$v^2(\tau_2, T_2) \geq \left(\frac{N_1 - U}{N_1} \right) \hat{v}^2(\tau_1, T_1, \tau_2, T_2, U) + \left(\frac{U}{N_1} \right) u(b, 0) \qquad (\gamma)$$

$$(N_1 - U)[(1 - \tau_1) w_1 L_1 + T_1] - Ub + N_2[(1 - \tau_2) w_2 L_2 + T_2] + \Pi(\cdot) = 0. \qquad (\lambda)$$

Equation (γ) is the self-selection constraint for the case of full mimicking behaviour, and (λ) is the resource constraint, where γ and λ are the Lagrangian multipliers that will be associated with these constraints. We write the Lagrangian for this problem:

$$\begin{aligned} &\Omega(\tau_1, T_1, \tau_2, T_2, \gamma, \lambda; U, b) = \\ &\quad N_1 \left[\left(\frac{N_1 - U}{N_1} \right) v^1(\tau_1, T_1, U) + \left(\frac{U}{N_1} \right) u(b, 0) \right] + N_2 v^2(\tau_2, T_2) \\ &\quad + \gamma \left[v^2(\tau_2, T_2) - \left(\frac{N_1 - U}{N_1} \right) \hat{v}^2(\tau_1, T_1, \tau_2, T_2, U) - \left(\frac{U}{N_1} \right) u(b, 0) \right] \\ &\quad + \lambda \{ (N_1 - U)[(1 - \tau_1) w_1 L_1 + T_1] - Ub \\ &\quad + N_2[(1 - \tau_2) w_2 L_2 + T_2] + \Pi(\cdot) \}. \end{aligned} \qquad (29)$$

The first order conditions for this maximization problem are:

$$N_1 \left(\frac{N_1 - U}{N_1} \right) \delta v^1_\tau - \gamma \left(\frac{N_1 - U}{N_1} \right) \delta \hat{v}^2_{\tau_1}$$

$$+\lambda \left\{ (N_1 - U) \left[-w_1 L_1 + (1 - \tau_1) L_1 \frac{\partial w_1}{\partial \tau_1} + (1 - \tau_1) w_1 \delta L^1_\tau \right] + \Pi_{\tau_1} \right\} = 0 \qquad (\tau_1)$$

$$N_1\left(\frac{N_1-U}{N_1}\right)\delta v_T^1-\gamma\left(\frac{N_1-U}{N_1}\right)\delta\hat{v}_{T_1}^2$$

$$+\lambda\left\{(N_1-U)\left[(1-\tau_1)L_1\frac{\partial w_1}{\partial T_1}+(1-\tau_1)w_1\delta L_T^1+1\right]+\Pi_{T_1}\right\}=0 \qquad (T_1)$$

$$N_2\delta v_\tau^2+\gamma\left[\delta v_\tau^2-\left(\frac{N_1-U}{N_1}\right)\delta\hat{v}_{\tau_2}^2\right]$$

$$+\lambda\left\{N_2\left[-w_2L_2+(1-\tau_2)L_2\frac{\partial w_2}{\partial\tau_2}+(1-\tau_2)w_2\delta L_\tau^2\right]+\Pi_{\tau_2}\right\}=0 \qquad (\tau_2)$$

$$N_2\delta v_T^2+\gamma\left[\delta v_T^2-\left(\frac{N_1-U}{N_1}\right)\delta\hat{v}_{T_2}^2\right]$$

$$+\lambda\left\{N_2\left[(1-\tau_2)L_2\frac{\partial w_2}{\partial T_2}+(1-\tau_2)w_2\delta L_T^2+1\right]+\Pi_{T_2}\right\}=0 \qquad (T_2)$$

where the variables involving δ are defined as above. Note that the two multipliers will have a positive sign: $\gamma > 0$ and $\lambda > 0$.

These first order conditions along with the constraints could be used to solve for the parameters of the optimal tax system. It is useful for our purposes to rewrite them more explicitly for the case in which $U = 0$ since we will be evaluating the desirability of MW at that point. Note that, in this case, the value of the Lagrangian is obviously independent of U but also of b. Also note that here the first order conditions will be the same with both full and partial mimicking behaviour. After some manipulation of the first order conditions using equations (6), (8), (10), (18), (20) and (22)—(27), we obtain the following for $U = 0$:

$$N_1\left(u_1^1w_1L_1+u_1^1\tau_1L_1\phi L_\tau^1\right)+\lambda N_1\left[-w_1L_1-\tau_1L_1\phi L_\tau^1+(1-\tau_1)w_1(1+\phi L_w^1)L_\tau^1\right]$$

$$-\gamma\left\{\hat{u}_1^2\left[w_1L_1+\tau_1L_1\phi L_\tau^1+\tau_1w_1(1+\phi L_w^1)L_\tau^1\right]\right.$$

$$\left.+\left(\frac{\hat{u}_2^2}{w_2}\right)\left[L_1\phi L_\tau^1+w_1(1+\phi L_w^1)L_\tau^1\right]\right\}=0 \qquad (\tau_1')$$

$$N_1\left(-u_1^1+u_1^1\tau_1 L_1\phi L_T^1\right)+\lambda N_1\left[-\tau_1 L_1\phi L_T^1+(1-\tau_1)w_1(1+\phi L_w^1)L_T^1+1\right]$$

$$-\gamma\Big\{\hat{u}_1^2\left[\tau_1 L_1\phi L_T^1+\tau_1 w_1(1+\phi L_w^1)L_T^1-1\right]$$

$$+\left(\frac{\hat{u}_2^2}{w_2}\right)\left[L_1\phi L_T^1+w_1(1+\phi L_w^1)L_T^1\right]\Big\}=0 \qquad (T_1')$$

$$(N_2+\gamma)\left(u_1^2 w_2 L_2+u_1^2\tau_2 L_2\Theta L_\tau^2\right)+\gamma\left(\frac{\hat{u}_2^2}{w_2}\right)\left(\frac{w_1 L_1}{w_2}\right)\Theta L_\tau^2$$

$$+\lambda N_2\left[-w_2 L_2-\tau_2 L_2\Theta L_\tau^2+(1-\tau_2)w_2(1+\Theta L_w^2)L_\tau^2\right]=0 \qquad (\tau_2')$$

$$(N_2+\gamma)\left(-u_1^2+u_1^2\tau_2 L_2\Theta L_T^2\right)+\gamma\left(\frac{\hat{u}_2^2}{w_2}\right)\left(\frac{w_1 L_1}{w_2}\right)\Theta L_T^2$$

$$+\lambda N_2\left[-\tau_2 L_2\Theta L_T^2+(1-\tau_2)w_2(1+\Theta L_w^2)L_T^2+1\right]=0 \qquad (T_2')$$

where the following new notation has been introduced:

$$\phi=\Phi\Big|_{U=0}<0.$$

This, in turn, implies:

$$\frac{\partial w_1}{\partial\tau_1}\Big|_{U=0}=\phi L_\tau^1\leq 0 \qquad \text{and} \qquad \frac{\partial w_1}{\partial T_1}\Big|_{U=0}=\phi L_T^1\leq 0$$

and

$$\delta L_\tau^1\Big|_{U=0}=(1+\phi L_w^1)L_\tau^1\geq 0 \qquad \text{and} \qquad \delta L_T^1\Big|_{U=0}=(1+\phi L_w^1)L_T^1\geq 0.$$

Also note that $(1+\Theta L_w^2)>0$.

To interpret these optimal tax conditions, we first combine (τ_2') and (T_2') to obtain:

$$\left\{(N_2+\gamma)u_1^2\tau_2 L_2\Theta+\gamma\left(\frac{\hat{u}_2^2}{w_2}\right)\left(\frac{w_1L_1}{w_2}\right)\Theta\right.$$
$$\left.+\lambda N_2\left[-\tau_2L_2\Theta+(1-\tau_2)w_2(1+\Theta L_w^2)\right]\right\}\cdot\left[L_\tau^2+w_2L_2L_T^2\right]=0. \qquad (30)$$

The term $(L_\tau^2 + w_2L_2L_T^2)$ is the compensated change in the labour supply for a change in τ_2.[6] In general, this term will not be equal to 0 so that (30) becomes:

$$(N_2+\gamma)u_1^2\tau_2L_2\Theta+\gamma\left(\frac{\hat{u}_2^2}{w_2}\right)\left(\frac{w_1L_1}{w_2}\right)\Theta+\lambda N_2\left[-\tau_2L_2\Theta+(1-\tau_2)w_2(1+\Theta L_w^2)\right]=0 \qquad (31)$$

Note that for the case where the wage w_2 is exogenous i.e. $\Theta = 0$, we obtain $(1-\tau_2) = 0$ which is the standard result according to which the marginal tax rate on the high-skilled workers should equal 0. Here, however, the wage rates are endogenous. According to Stiglitz (1982, 1985) and Stern (1982), the marginal tax rate on the high-skilled workers should then be negative. This result can be verified in our model. Substituting equation (31) in (T_2'), we obtain:

$$-(N_2+\gamma)u_1^2+\lambda N_2=0. \qquad (32)$$

Substituting (32) back into (31) yields:

[6] To show that, one simply has to derive the Slutsky equation for the workers' problems using:

$$\max_{C,Z} \quad u(C,K-Z) \quad \text{s.t.} \quad C=\tau wL-T \quad \text{and} \quad L+Z=K$$

and

$$\min_{C,Z} \quad C+\tau wZ \quad \text{s.t.} \quad u(C,K-Z)=\overline{u}$$

where Z is leisure and K is the time endowment. Denoting the compensated labour supply function by L^c, it is then possible to obtain:

$$\frac{\partial L}{\partial \tau}=\frac{\partial L^c}{\partial \tau}-wL\frac{\partial L}{\partial T}$$

which is the desired result.

$$\gamma\left(\frac{\hat{u}_2^2}{w_2}\right)\left(\frac{w_1 L_1}{w_2}\right)\Theta + \lambda N_2(1-\tau_2)w_2(1+\Theta L_w^2) = 0. \tag{33}$$

For this equality to be satisfied, it is necessary that $(1-\tau_2) < 0$.

Consider now equations (τ_1') and (T_1'). Combining these equations yields:

$$\left\{N_1 u_1^1 \tau_1 L_1 \phi - \gamma\left[\hat{u}_1^2 \tau_1 + \frac{\hat{u}_2^2}{w_2}\right]\left[L_1\phi + w_1(1+\phi L_w^1)\right]\right.$$

$$\left. +\lambda N_1\left[(1-\tau_1)w_1(1+\phi L_w^1) - \tau_1 L_1 \phi\right]\right\} \cdot \left[L_\tau^1 + w_1 L_1 L_T^1\right] = 0. \tag{34}$$

Again, normally, $(L_\tau^1 + w_1 L_1 L_T^1)$ is not equal to 0. Equation (34) then becomes:

$$N_1 u_1^1 \tau_1 L_1 \phi - \gamma\left[\hat{u}_1^2 \tau_1 + \frac{\hat{u}_2^2}{w_2}\right]\left[L_1\phi + w_1(1+\phi L_w^1)\right]$$

$$+\lambda N_1\left[(1-\tau_1)w_1(1+\phi L_w^1) - \tau_1 L_1 \phi\right] = 0. \tag{35}$$

From (35), it is straightforward to show that for both endogenous $(\phi < 0)$ and exogenous $(\phi = 0)$ wage w_1, the marginal tax rate on the low-skilled workers should be positive i.e. $(1-\tau_1) > 0$, both of which are standard results [see Stiglitz (1982, 1985)].

For future reference, it is useful to derive two further equations. Substituting equation (35) in (T_1'), we obtain:

$$N_1(\lambda - u_1^1) + \gamma \hat{u}_1^2 = 0. \tag{36}$$

Substituting (36) back into (35) gives:

$$\left\{N_1 u_1^1(1-\tau_1) - \gamma\left[\hat{u}_1^2 + \frac{\hat{u}_2^2}{w_2}\right]\right\}\left[w_1(1+\phi L_w^1)\right] - \gamma\left(\frac{\hat{u}_2^2}{w_2}\right) L_1 \phi = 0. \tag{37}$$

It is worth stressing that equations (τ_1'), (T_1'), (τ_2'), (T_2'), and (30)—(37) hold for both full mimicking and partial mimicking, but only for the case with no unemployment $(U = 0)$. Let us denote the tax parameters derived for the no-unemployment case $(U = 0)$ by $\tau_1^*, T_1^*, \tau_2^*, T_2^*$. For these tax parameters, there is a corresponding pair of wages:

$$w_1^* = w_1(\tau_1^*, T_1^*, 0) \tag{38}$$

$$w_2^* = w_2(\tau_2^*, T_2^*). \tag{39}$$

Note that as long as $U = 0$, the value of b is irrelevant. It can be set at any arbitrary value we chose and the same optimal tax rules will apply. We are now ready to assess the usefulness of a MW combined with UI.

4. The Usefulness of a Minimum Wage with Unemployment Insurance

Beginning at the no-unemployment solution described above, suppose that the government can set a minimum wage for the low-skilled workers w_1.[7] To be effective, the minimum wage must exceed the market clearing wage for given tax parameters, $w_1 > w_1^*$. Here we consider a marginal increase in w_1 evaluated at $w_1 = w_1^*$, or, equivalently, at $U = 0$. Since U is the only variable that can adjust on the labour market, increasing w_1 will cause U to rise from 0 to some positive amount. Since we are considering only a marginal increase, we can use all the equations obtained under $U = 0$: (τ_1'), (T_1'), (τ_2'), (T_2'), and (30)—(37).

Before turning to the explicit examination of this problem, we have to modify the model to take into account the fact that U is now endogenous and w_1 exogenous. The following now hold, all evaluated at $\tau_1^*, T_1^*, \tau_2^*, T_2^*$:

i) $U = U(w_1)$ where $U(w_1^*) = 0$. From (2),

$$\frac{\partial U(w_1^*)}{\partial w_1} = \frac{N_1 F'' L_w^1 - 1}{F'' L_1} > 0$$

ii) $v^1(\tau_1^*, T_1^*, U)$ can now be written $v^1(\tau_1^*, T_1^*, w_1)$ so that:

$$Ev_1 = \left[\frac{(N_1 - U(w_1))}{N_1}\right] v^1(\tau_1^*, T_1^*, w_1) + \left[\frac{U(w_1)}{N_1}\right] u(b, 0).$$

Differentiating this with respect to the MW yields:

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[7] The government is assumed to be able to enforce such a regulation even though it cannot use data on individual wages in the optimal tax formulation. This assumption is also adopted by Guesnerie and Roberts (1987) and Allen (1987). Guesnerie and Roberts do, however, recognize that their is some inconsistency between assuming that the hourly wage is not observable for income tax purposes, although a minimum wage is enforceable. They refer to this as a "somewhat mixed observability assumption."

$$\left.\frac{dEv^1}{dw_1}\right|_{w_1=w_1^*} = u_1^1\tau_1^* L_1 + \left(\frac{\partial U(w_1^*)}{\partial w_1}\right)\frac{u(b,0) - v^1(\tau_1^*, T_1^*, w_1^*)}{N_1}.$$

iii) $\hat{v}^2(\tau_1^*, T_1^*, \tau_2^*, T_2^*, U)$ is now written $\hat{v}^2(\tau_1^*, T_1^*, \tau_2^*, T_2^*, w_1)$. For the case of full mimicking behaviour,

$$E\hat{v}_2 = \left[\frac{(N_1 - U(w_1))}{N_1}\right]\hat{v}^2(\tau_1^*, T_1^*, \tau_2^*, T_2^*, w_1) + \left[\frac{U(w_1)}{N_1}\right] u(b,0).$$

Taking the derivative with respect to MW, we obtain:

$$\left.\frac{dE\hat{v}^2}{dw_1}\right|_{w_1=w_1^*} = \left[\hat{u}_1^2\tau_1^* + \frac{\hat{u}_2^2}{w_2^*}\right][L_1 + w_1^* L_w^1]$$
$$+ \left(\frac{\partial U(w_1^*)}{\partial w_1}\right)\frac{u(b,0) - \hat{v}^2(\tau_1^*, T_1^*, \tau_2^*, T_2^*, w_1^*)}{N_1}.$$

For the case of partial mimicking behaviour, these simplify to:

$$\hat{v}_2 = \hat{v}^2(\tau_1^*, T_1^*, \tau_2^*, T_2^*, w_1)$$

and

$$\left.\frac{d\hat{v}^2}{dw_1}\right|_{w_1=w_1^*} = \left[\hat{u}_1^2\tau_1^* + \frac{\hat{u}_2^2}{w_2^*}\right][L_1 + w_1^* L_w^1].$$

iv) $\Pi(\tau_1^*, T_1^*, \tau_2^*, T_2^*, U)$ becomes $\Pi(\tau_1^*, T_1^*, \tau_2^*, T_2^*, w_1)$ so that:

$$\left.\frac{d\Pi}{dw_1}\right|_{w_1=w_1^*} = -N_1 L_1.$$

We are now in a position to formulate two propositions concerning the possibility of MW being welfare-improving starting at the no-unemployment optimal tax equilibrium. Recall that these can be interpreted as pure efficiency gains. The two propositions apply for the cases of full mimicking and partial mimicking behaviour, respectively. Consider them in turn.

Proposition 1: Given full mimicking behaviour, a set of sufficient conditions for MW and UI to be welfare-improving are:

i) The total tax liability of the low-income workers is negative, and
ii) The elasticity of the utility function with respect to consumption is decreasing in the labour supply (given consumption) and the marginal utility of consumption is decreasing in the labour supply ($U_{12} < 0$).

These sufficient conditions are in addition to those which we have already assumed to apply throughout the analysis (i.e., $U_{ii} < 0$ and $L_w \geq 0$).

Proof of Proposition 1:

Using the Lagrangian in (29), modified to take into account the dependence of U, v_1, $\hat{v}_2$, and Π on w_1, and the envelope theorem, we obtain, for a given level of b:

$$\begin{aligned}\xi = \frac{d\Omega}{dw_1}\bigg|_{w_1=w_1^*}^{\mathrm{FM}} =& N_1 \left\{ u_1^1 \tau_1^* L_1 + \left(\frac{1}{N_1} \frac{\partial U(w_1^*)}{\partial w_1} \right) \left[u(b,0) - v^1(\tau_1^*, T_1^*, w_1^*) \right] \right\} \\ & - \gamma \left\{ \left(\hat{u}_1^2 \tau_1^* + \frac{\hat{u}_2^2}{w_2^*} \right) (L_1 + w_1^* L_w^1) \right. \\ & \left. + \left(\frac{1}{N_1} \frac{\partial U(w_1^*)}{\partial w_1} \right) \left[u(b,0) - \hat{v}^2(\tau_1^*, T_1^*, \tau_2^*, T_2^*, w_1^*) \right] \right\} \\ & + \lambda \left\{ N_1 \left[(1-\tau_1^*) L_1 + (1-\tau_1^*) w_1^* L_w^1 \right] \right. \\ & \left. - \left(\frac{\partial U(w_1^*)}{\partial w_1} \right) \left[(1-\tau_1^*) w_1^* L_1 + T_1^* + b \right] - N_1 L_1 \right\}\end{aligned}$$

Substituting equation (36) in ξ and making use of the fact that $(1-\tau_1)w_1L_1 + T_1 = w_1L_1 - C_1$, we obtain:

$$\begin{aligned}\xi =& \left\{ N_1 u_1^1 (1-\tau_1) - \gamma \left[\hat{u}_1^2 + \frac{\hat{u}_2^2}{w_2} \right] \right\} w_1 L_w^1 \\ & - \gamma \left(\frac{\hat{u}_2^2}{w_2} \right) L_1 \\ & + \left(\frac{1}{N_1} \frac{\partial U}{\partial w_1} \right) \left(N_1 u_1^1 - \gamma \hat{u}_1^2 \right) (C_1 - w_1 L_1) \\ & + \left(\frac{1}{N_1} \frac{\partial U}{\partial w_1} \right) \left\{ N_1 \left[u(b,0) - v^1(\cdot) \right] - \gamma \left[u(b,0) - \hat{v}^2(\cdot) \right] - \left[N_1 u_1^1 - \gamma \hat{u}_1^2 \right] b \right\}\end{aligned}$$

where we have dropped the *'s for simplicity. It should be understood that all variables are evaluated at $w_1 = w_1^*$.

We now show that under the conditions previously stated, $\xi > 0$ and thus, a welfare improvement is possible with the implementation of a MW. From equation (37), we can infer that:

$$\left\{ N_1 u_1^1 (1-\tau_1) - \gamma \left[\hat{u}_1^2 + \frac{\hat{u}_2^2}{w_2} \right] \right\} > 0.$$

Thus, the first term of ξ is positive. Since $\hat{u}_2^2 < 0$, the second term is also positive. Now consider the third term. From equation (36), it is clear that:

$$\left(N_1 u_1^1 - \gamma \hat{u}_1^2 \right) > 0.$$

Thus, this third term will be positive if $C_1 > w_1 L_1$, that is, if low income persons have negative tax liabilities. Note that none of the three first terms depends on the UI benefit b. Finally, consider the fourth term. Note that:

$$\left(\frac{1}{N_1} \frac{\partial U}{\partial w_1} \right) > 0.$$

Thus, the fourth term will be positive if:

$$N_1 \left[u(b,0) - v^1(\cdot) \right] - \gamma \left[u(b,0) - \hat{v}^2(\cdot) \right] - \left[N_1 u_1^1 - \gamma \hat{u}_1^2 \right] b > 0$$

or, if

$$N_1 \left[v^1(\cdot) + u_1^1 b - u(b,0) \right] - \gamma \left[\hat{v}^2(\cdot) + \hat{u}_1^2 b - u(b,0) \right] < 0.$$

From (36), $u_{12} < 0$ implies that $(N_1 - \gamma) > 0$. Given this, a sufficient condition for for the fourth term to be positive is:

$$N_1 \left[v^1(\cdot) + u_1^1 b \right] - \gamma \left[\hat{v}^2(\cdot) + \hat{u}_1^2 b \right] < 0$$

where, with $u_{12} < 0$: $N_1 > \gamma$; $v^1(\cdot) < \hat{v}^2(\cdot)$; $u_1^1 < \hat{u}_1^2$. Equivalently, a sufficient condition is:

$$\frac{N_1}{\gamma} < \frac{\hat{v}^2(\cdot) + \hat{u}_1^2 b}{v^1(\cdot) + u_1^1 b} = \eta(b). \tag{40}$$

It should be here clear that none of N_1, γ, $v^1(\cdot)$, $\hat{v}^2(\cdot)$, u_1^1, or $\hat{u}_1^2$ depends on b since ξ is evaluated at a no-unemployment situation. There will be a value for b such that inequality (40) is satisfied if its right-hand side is increasing in b. Differentiating $\eta(b)$, we obtain:

$$\eta'(b) = \frac{\hat{u}_1^2 v^1(\cdot) - u_1^1 \hat{v}^2(\cdot)}{\left(v^1(\cdot) + u_1^1 b \right)^2}.$$

Obviously, $\eta'(b) > 0$ if the numerator is positive, or:

$$\frac{\hat{u}_1^2}{\hat{v}^2(\cdot)} > \frac{u_1^1}{v^1(\cdot)}.$$

Multiplying both sides by C_1, the consumption of workers of both type 1 and type $\hat{2}$, the condition for $\eta'(b)$ positive becomes:

$$\varepsilon_{uc}(\hat{L}_2) = \frac{\hat{u}_1^2 C_1}{\hat{v}^2(\cdot)} > \frac{u_1^1 C_1}{v^1(\cdot)} = \varepsilon_{uc}(L_1)$$

where $\varepsilon_{uc}(L)$ is the elasticity of the utility function with respect to consumption. This condition requires this elasticity to be decreasing in the labour supply L.[8]

To summarize, ε_{uc} decreasing in L insures that $\eta'(b)$ is positive. The fact that $\eta'(b)$ is positive will then allow us to set b sufficiently high for condition (40) to be satisfied.[9] And if (40) is satisfied, we know that the fourth term in ξ is positive, which in turn implies that ξ will be positive for sure given the first three terms are positive. Since $\xi > 0$ for an appropriate b, implementing a MW and UI can be welfare improving. ■

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8 An example of a utility function satisfying this property is:

$$u = AC^{\alpha}(K - L)^{\beta} + \nu$$

where K is the time endowment and where the parameters satisfy the following: $0 < \alpha < 1$; $0 < \beta < 1$; $\nu > 0$. To show that $\varepsilon_{uc}(L)$ is decreasing in L, write:

$$\varepsilon_{uc}(L) = \frac{u_1 C}{u} = \frac{\alpha(u - \nu)}{u}$$

Taking the derivative, we obtain:

$$\left.\frac{\partial \varepsilon_{uc}(L)}{\partial L}\right|_C = \frac{\alpha \nu u_2}{u^2} < 0$$

This utility function also satisfies the other properties required throughout the paper: $u_1 > 0$; $u_{11} < 0$; $u_2 < 0$; $u_{22} < 0$; $u_{12} < 0$; $L_w \geq 0$.

9 To ensure that unemployment is involuntary, we would like it to be the case that $v^1(\cdot) > u(b, 0)$. However, we have not at this stage imposed that restriction. Since we are deriving only sufficient conditions, it is not clear that much can be gained analytically by doing so. Clearly it would seem reasonable that the sufficient conditions could be satisfied with this restriction imposed. If the restriction were not satisfied, the welfare-improving MW outcome could still be achieved if the government could restrict b to persons involuntarily rather than voluntarily unemployed.

The intuition behind this result can be explained in the following way. Following Stiglitz (1982), we can characterize the optimal income tax problem with full employment [equation (29)] as the solution to a Pareto-optimizing problem in the following equivalent way:

$$\max_{\tau_1, T_1, \tau_2, T_2} \quad Ev^1(\tau_1, T_1)$$

subject to

$$v^2(\tau_2, T_2) \geq K$$

$$v^2(\tau_2, T_2) \geq E\hat{v}^2(\tau_1, T_1, \tau_2, T_2)$$

$$(N_1 - U)[(1-\tau_1)w_1 L_1 + T_1] - Ub + N_2[(1-\tau_2)w_2 L_2 + T_2] + \Pi(\cdot) = 0$$

where $Ev^1(\cdot) = v^1(\cdot)$, $E\hat{v}^2(\cdot) = \hat{v}^2(\cdot)$, and $U = 0$ because of full employment. Also, K has been chosen to correspond with the utility level achieved by the high-wage person in problem (29). The first constraint sets the utility level of a high-wage person; the second is a self-selection constraint; and the third is the government budget constraint.

Now, we consider whether an increase in w_1 by an MW policy can make person 1 better off without making person 2 worse off. Consider the effect of w_1 on each of the terms in this problem. It will not have an effect on $v^2(\cdot)$ so it will not affect the first constraint. The effect of an w_1 increase on $Ev^1(\cdot)$ can go either way. However, it is clear that if b is large enough, it will be positive. It will be unambiguously positive if unemployment is voluntary i.e. $u(b, 0) \geq v^1(\cdot)$. Furthermore, there will be some ϵ such that $u(b, 0) + \epsilon = v^1(\cdot)$, that is, unemployment is involuntary, and yet $Ev^1(\cdot)$ will rise. Similarly, the increase in w_1 can cause $E\hat{v}^2(\cdot)$ to fall, and thus, the self-selection constraint to be relaxed. In fact, it is quite possible to have $Ev^1(\cdot)$ rising and $E\hat{v}^2(\cdot)$ falling at the same time. For example, if $u_{12} < 0$ and $L^1_w = 0$, that will unambiguously be the case. Thus, it is clearly possible that an increase in w_1 causes persons of type 1 to be better off at the same time as relaxing the self-selection constraint. The only remaining issue is the revenue cost of imposing the MW. Here again, the effect of an w_1 increase on government revenue is ambiguous. Employed workers of type 1 incur more tax liabilities since their income are higher. At the same time, those made unemployed pay less taxes as well as receiving UI benefits b. There is also a fall in pure profits. Thus, total revenues may rise or fall. Obviously, there is a great deal going on here and the net effect can go either way. Our sufficient conditions listed in Proposition 1 (and 2 below) indicate when the net effect of these three terms is positive.

Now turning to the partial mimicking behaviour, a proposition can be demonstrated that is similar to Proposition 1.

Proposition 2: Given the partial mimicking behaviour, a sufficient condition for MW and UI to be welfare-improving is that the tax liability of the low-skilled workers is negative ($C_1 > w_1 L_1$).

Proof of Proposition 2:

The proof follows the same procedure as for Proposition 1. Differentiating the Lagrangian in (29) and using the envelope theorem, we obtain, for a given level of b:

$$\begin{aligned}\zeta = \left.\frac{d\Omega}{dw_1}\right|^{PM}_{w_1=w_1^*} =& N_1\left\{u_1^1\tau_1^* L_1 + \left(\frac{1}{N_1}\frac{\partial U(w_1^*)}{\partial w_1}\right)\left[u(b,0) - v^1(\tau_1^*, T_1^*, w_1^*)\right]\right\} \\ & -\gamma\left(\hat{u}_1^2\tau_1^* + \frac{\hat{u}_2^2}{w_2^*}\right)(L_1 + w_1^* L_w^1) \\ & +\lambda\left\{N_1\left[(1-\tau_1^*)L_1 + (1-\tau_1^*)w_1^* L_w^1\right]\right. \\ & \left. -\left(\frac{\partial U(w_1^*)}{\partial w_1}\right)\left[(1-\tau_1^*)w_1^* L_1 + T_1^* + b\right] - N_1 L_1\right\}.\end{aligned}$$

Substituting equation (36) in ζ and making use of the fact that $(1-\tau_1)w_1 L_1 + T_1 = w_1 L_1 - C_1$, we obtain:

$$\begin{aligned}\zeta =& \left\{N_1 u_1^1(1-\tau_1) - \gamma\left[\hat{u}_1^2 + \frac{\hat{u}_2^2}{w_2}\right]\right\} w_1 L_w^1 \\ & -\gamma\left(\frac{\hat{u}_2^2}{w_2}\right) L_1 \\ & +\left(\frac{1}{N_1}\frac{\partial U}{\partial w_1}\right)(N_1 u_1^1 - \gamma\hat{u}_1^2)(C_1 - w_1 L_1) \\ & +\left(\frac{1}{N_1}\frac{\partial U}{\partial w_1}\right)\left\{N_1\left[u(b,0) - v^1(\cdot)\right] - \left[N_1 u_1^1 - \gamma\hat{u}_1^2\right] b\right\}\end{aligned}$$

where again we have dropped the *'s for simplicity. All terms except the fourth are the same as in the previous proof. We then know that the first two terms are necessarily positive and that the third term will be positive if the tax liability on low-skilled workers is negative, i.e., $C_1 > w_1 L_1$. Now consider the fourth term. Given that:

$$\left(\frac{1}{N_1}\frac{\partial U}{\partial w_1}\right) > 0$$

the fourth term will be positive if:

$$N_1\left[v^1(\cdot)+u_1^1 b-u(b,0)\right]-\gamma\hat{u}_1^2 b<0.$$

Since $-N_1 u(b,0)<0$, a sufficient condition for this is:

$$\frac{N_1}{\gamma}<\frac{\hat{u}_1^2 b}{v^1(\cdot)+u_1^1 b}=\rho(b) \tag{41}$$

where, as in the previous proof, N_1, γ, $v^1(\cdot)$, u_1^1, and $\hat{u}_1^2$ are all independent of b. Since $\rho'(b)$ is positive, it is possible to set b sufficiently high so that inequality (41) is satisfied.Thus, ζ can be made positive which in turn implies that it is possible to increase the social welfare by implementing a MW and UI. ■

It should be emphasized here that the conditions that are making ξ and ζ positive in the above two propositions are only sufficient conditions. In particular, it is quite possible to have ξ and ζ positive even if the fourth terms are negative. Indeed, it may also be the case that MW can be welfare-improving in the absence of UI ($b=0$), since the magnitude of b only affects the fourth term in these expressions. Everything depends upon the specific parameters of the case being considered.

5. Conclusion

In this paper, we have considered the conventional problem of a government that can use an optimal non-linear income tax schedule but that is not able to observe the skill level of the workers in the economy. The government, in its attempt to redistribute, was thus facing a self-selection constraint. In a decentralized framework, we have established the standard results with regard to the tax parameters when the wages are endogenously determined.

In this framework we have investigated whether minimum wage and unemployment insurance policies could be welfare-improving when the minimum wage caused involuntary unemployment. We have derived a set of sufficient conditions that appear to be not too restrictive for this to be the case. Clearly letting the MW cause involuntary unemployment is critical to the analysis. Previous analyses by Guesnerie and Roberts (1987) and Allen (1987) assumed that MW induced underemployment (reduced hours of work), and had found little support for MW as a redistributive device. Guesnerie and Roberts had made the conjecture that it would make a difference if the unemployment were involuntary. The results of this paper confirm their intuition. The argument of this paper is also in the spirit of Blackorby (1990) who argued that economic policies that are harmful in a first-best world might well be optimal in a second-best world as the one described in our paper.

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