Queen's Economics Department Working Paper No. 801

Strategic Innovation and Economic Growth

Allen C. Head Beverly J. Lapham

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

12-1990

DISCUSSION PAPER # 801

Strategic Innovation and Economic Growth

by

Allen C. Head* and Beverly J. Lapham**

*Department of Economics
University of Minnesota
Minneapolis, MN 55455

**Department of Economics
Queen's University
Kingston, Ontario K7L 3N6

December 1990

We thank Dan Bernhardt and the participants of the Queen's Macroeconomics Workshop for helpful comments.

Abstract

Strategic interaction among oligopolistic innovators and its implications for economic growth are examined in two dynamic computable general equilibrium models. In each environment, technologies for producing a final good are such that the profits of any intermediate good producer depend on the quality of <u>all</u> intermediate goods. This leads to strategic innovation choices which affect equilibrium growth rates. In the first economy, these spillovers lead to long-run output growth rates which fluctuate around a constant level. This level is affected by the degree of spillovers, the length of firms' planning horizons, the number of firms, and average innovation costs. In this economy a firm unequivocally benefits from innovation by other firms. A second economy is also considered in which innovation by other firms may harm intermediate goods producers. This results as innovation leads to rising factor costs. This second economy also exhibits continual output growth but at a decreasing rate. The economy can easily be modified to exhibit long-run growth rates which fluctuate about a constant. We consider two examples: population growth at a constant rate and intermediate goods' technologies affected by average quality.

I. INTRODUCTION

I.1 Motivation

Recent papers by Aghion and Howitt (1989), Grossman and Helpman (1989), and others examine economic growth driven by quality improvements in dynamic, general equilibrium frameworks. In each of these papers, entrepreneurs engage in costly innovation to develop higher quality intermediate goods which produce a final good at lower cost. This feature of these environments leads to sustained growth. These studies have typically examined theoretical structures in which the innovating behavior of entrepreneurs is not explicitly affected by the behavior of others.

In this paper, we examine dynamic general equilibrium environments in which quality improvements are the engine of growth and in which the rates of innovation and output growth are determined in a dynamic game among heterogeneous oligopolistic innovators. Growth results as oligopolists continually innovate to capture profits generated by intermediate goods production.

Strategic interaction arises as the marginal product of any intermediate good in the production of the final good depends on qualities of all intermediate goods. This specification is motivated by the observation that quality improvements in one intermediate good are likely to affect the productivity of others. For example, improvements in software have greatly increased the usefulness of personal computers. Firms, recognizing that these spillovers directly affect the return to their own innovation decisions, react strategically to innovative behavior by their competitors. Equilibrium time series of the artificial economies are generated. Factors which contribute to growth in these economies are analyzed in a series of computational experiments.

I.2 General Framework

In this section, a general class of games with asymmetric agents is described and the time series properties of agents' choice variables are discussed. This discussion illustrates that it is difficult to make general statements about properties of growth rates in dynamic games of this type. This paper presents two example economies which illustrate possible outcomes of dynamic games in this class.

Consider the following dynamic game with asymmetric agents. There are N players, each with strategy choice $x_j \in \mathbb{R}_+$ and objective function at time t given by

$$R_j(x_1,x_2,\ldots,x_N,s_{jt},\Theta) = \Pi_j(x_1,x_2,\ldots,x_N,\Theta) - .5\theta_j(x_j-s_{jt})^2,$$

where $\Theta \equiv \{\theta_1,\theta_2,\ldots,\theta_N\} \in \mathbb{R}^N_+$ is a vector of firm specific parameters. For ease of exposition, Θ is assumed to be constant over time; however, in the example economies presented below these parameters are time varying. Θ and s_{jt} describe the state for agent j at time t. $\Pi_j(.)$ represents future returns to agent j net of current adjustment costs, $.5\theta_j(x_j-s_{jt})^2$. This return function incorporates the effects of current decisions on all future Nash equilibrium returns. $\Pi_j(.)$ is increasing and convex in its j^{th} argument and $R_j(.)$ is concave in its j^{th} argument.

Let $\mathbf{x}_{/j} \equiv \{x_i\}_{i \neq j}$. Then, $\forall$j, best response functions $\hat{x}_j(\mathbf{x}_{/j},s_{jt},\Theta)$ are implicitly defined by the following equations:

$$\partial\Pi_j(x_1,x_2,\ldots,x_{j-1},\hat{x}_j(\mathbf{x}_{/j},s_{jt},\Theta),x_{j+1},\ldots,x_N,\Theta)/\partial x_j = \theta_j(\hat{x}_j(\mathbf{x}_{/j},s_{jt},\Theta)-s_{jt}) \tag{1}$$

When a solution to the above system of N equations exists, the solution is a Nash equilibrium at state $(\mathbf{s}_t,\Theta)$. Let $x^*_j(\mathbf{s}_t,\Theta)$ denote the set of Nash equilibria associated with the state vector $(\mathbf{s}_t,\Theta)$. The elements of this set are implicitly defined by the following equations $\forall j$:

$$\partial\Pi_j(\{x^*_i(\mathbf{s}_t,\Theta)\}^N_{i=1},\Theta)/\partial x_j = \theta_j(x^*_j(\mathbf{s}_t,\Theta)-s_{jt}) \tag{2}$$

Given $\mathbf{s}_0$, a subgame perfect equilibrium for the game is a sequence $\{\mathbf{s}_t\}^\infty_{t=0}$ that satisfies:

$$s_{jt} = x^*_j(\mathbf{s}_{t-1},\Theta).$$

We seek to examine the time series properties of subgame perfect equilibria. In particular, we focus on the behavior of first differences of the series, $\Delta s_{jt} \equiv s_{jt}-s_{jt-1}$, and the growth rate of the series, $g_{jt} \equiv \Delta s_{jt}/s_{jt-1}$.

From Equation (1), it is clear that both in and out of equilibrium, maximization of the agents' objective functions requires that the first differences of the agents choice variable must satisfy the following:

$$\forall j,\forall t: \quad \hat{x}_j(\mathbf{x}_{/j},s_{jt},\Theta)-s_{jt} = (1/\theta_j)\partial\Pi_j(x_1,x_2,\ldots,x_{j-1},\hat{x}_j(\mathbf{x}_{/j},s_{jt},\Theta),x_{j+1},\ldots,x_N,\Theta)/\partial x_j \tag{3}$$

Now, since $\theta_j>0$ and Π_j is increasing, the right hand side of this equation is positive. Therefore, $\forall j$, $\forall t$ $\hat{x}_j(.,.,.)-s_{jt}>0$ and $\forall j$ the s_{jt}

are increasing sequences. Hence, the first differences, Δs_{jt}, are positive sequences.

Lemma

If the choice variables are strategic complements, the sequences of first differences, Δs_{jt}, are increasing over time. If the choice variables are strategic substitutes, the behavior of these sequences of first differences over time depends on economy parameters.

Proof:

By definition, if the choice variables are strategic complements, $\partial\Pi_j(.)/\partial x_j$ is increasing in x_i $\forall i\neq j$, (See Bulow, Geanakopolos, and Klemperer (1985)). Furthermore, since Π_j is a convex function of x_j, $\partial\Pi_j(.)/\partial x_j$ is increasing in x_j. Therefore, since the s_{jt} are increasing sequences, the right hand side of (3) must be increasing over time. Hence, in the case of strategic complements, the sequences of first differences, Δs_{jt}, are increasing over time.

By definition, if the choice variables are strategic substitutes, the $\partial\Pi_j(.)/\partial x_j$ are decreasing in x_i $\forall i\neq j$. Since the x_i are increasing over time, this will tend to cause $\partial\Pi_j(.)/\partial x_j$ to decrease over time. However, increases in x_j over time will tend to cause $\partial\Pi_j(.)/\partial x_j$ to increase over time. Hence, in the case of strategic substitutes, whether or not the series of first differences are increasing, decreasing, or constant over time will depend on parameters.■

The above indicates that, except for the case of strategic complements, it is not possible to provide a general characterization of the first differences of the equilibrium choices. Even if the choice variables are strategic complements, it is difficult to make any general statements about the growth rates of these series. Although first differences increase over time in this case, their rate of increase must be greater than or equal to the rate of increase of the series itself to guarantee that growth rates do not asymptotically converge to zero. These rates of increase will in general depend on economy parameters. In the case of strategic substitutes, we cannot say anything about growth rates in general as first differences may be decreasing, increasing, or constant depending on economy parameters.

In the following sections two economies are examined which are consistent with the general framework described here. In the first economy, the choice variables are strategic complements and resulting equilibrium growth rates are constant. In the second economy, the choice variables are strategic substitutes and equilibrium growth rates, while always positive, asymptotically converge to zero.

The remainder of the paper is organized as follows. Sections two and three describe the environments of the two model economies and examine properties of their equilibria. Relationships among economy parameters and growth rates are examined in a series of computational experiments. Section four concludes and discusses possible extensions.

II. ECONOMY I - An Economy in Which Qualities are Strategic Complements

In this economy, the technology for producing final goods is characterized by spillovers among qualities of intermediate goods. In particular, the marginal product of any intermediate good is positively affected by the quality of other intermediate goods employed. This specification leads to demands for intermediate goods which depend on qualities of all goods and, therefore, to a strategic innovation game among oligopolistic producers of intermediate goods. Section II.1 describes the economy, section II.2 discusses the equilibrium concept, section II.3 presents results of computational experiments, and section II.4 concludes.

II.1: The Economy

II.1.1: Environment

Technologies

Final Goods:

Our solution technique for solving the dynamic, game among innovators limits our attention to environments which give rise to linear best response functions. This places restrictions on the functional form for the technology for production of the final good. This technology is given by:

$$y_t = \sum_{j=1}^{N} [a + q_{jt}(b\sum_{i \neq j} q_{it})]k_{jt} \qquad \forall t$$

where q_{jt} is a quality index for the j^{th} intermediate good at time t and k_{jt} is the quantity of the j^{th} intermediate good employed in production of the final good at time t. The marginal product of intermediate good j in the production of the final good is:

$$a + q_{jt}(b_i \sum_{i \neq j} q_{it})$$

Note that this depends on qualities of all intermediate goods employed.

Intermediate Goods:

The technology for producing intermediate goods is given by

$$k_{jt} = \ell_{jt}$$

where ℓ_{jt} is labor input into intermediate good j production and k_{jt} is restricted to the set {0,1}. Constraints on production levels in the intermediate goods sector are required as the marginal product of intermediate goods in production of the final good are independent of the level of intermediate goods employed. Furthermore, the constant returns to scale technology for producing intermediate goods leaves the supply of intermediate goods indeterminate. Therefore, output levels of intermediate goods are restricted to one unit of each good produced each time period.

Innovation:

The technology for innovations leading to quality changes is summarized by a total cost function. The total amount of the final good at date t required to alter the quality of the j^{th} good by the amount Δq_{jt} is given by

$$C(\Delta q_{jt}) = .5\theta_{jt}(\Delta q_{jt})^2,$$

where θ_{jt} is a time varying, firm-specific technology parameter on the quality innovation process. For all j, the sequences, $\{\theta_{jt}\}_{t=0}^{\infty}$, are known at time 0. This assumption of perfect foresight is one of convenience rather than necessity; issues related to the introduction of uncertainty into this environment are discussed in the extensions in section IV.

Endowments

It is convenient to normalize the time endowment of consumers to the number of firms operating in the intermediate goods sector, N. Since there is no disutility of labor, time is supplied inelastically to the intermediate goods sector.

Preferences

A representative consumer has preferences over consumption of the final good ordered by the following utility function:

$$\sum_{t=0}^{\infty} \beta^t U(c_t)$$

where $U(c_t)$ is increasing and concave. The measure of consumers is normalized to one.

II.1.2: Market Arrangements

Producers

The final good sector is perfectly competitive. Producers in this sector take prices of the final good and intermediate goods as given. Firms seek to maximize discounted profits. As there are no dynamic factors affecting firms' decisions in this sector, firms face the following static maximization problem:

$$\forall t \quad \max_{\{k_{jt}\}_{j=1}^{N}} \quad p_t\{y_t - \sum_{j=1}^{N} r_{jt}k_{jt}\}$$

$$\text{subject to} \quad y_t \le \sum_{j=1}^{N} [a + q_{jt}(b\sum_{j\neq i} q_{it})]k_{jt},$$

where r_{jt} is the price of intermediate good j at time t in units of the date t final good.

The solution to the representative firm's problem is:

$$(1) \qquad k_{jt} = \begin{cases} 0 & \text{if} \quad a + q_{jt}(b\sum_{j\neq i} q_{it}) < r_{jt} \\ \geq 0 & \text{if} \quad a + q_{jt}(b\sum_{j\neq i} q_{it}) = r_{jt} \\ \infty & \text{if} \quad a + q_{jt}(b\sum_{j\neq i} q_{it}) > r_{jt} \end{cases}$$

The intermediate goods sector is a differentiated oligopoly. Firms in this sector are restricted to produce either one or zero units of output of their intermediate good. Given this restriction and the normalization of the time endowment, any $w_t>0$ will clear the labor market at time t. Attention is restricted to constant wages measured in units of the date t final good, $w_t = w \ \forall t$.

Firms in this sector must expend resources to alter the quality of the good they produce according to the innovation technology. Firms face a finite planning horizon of integer length δ, $\delta\geq 0$. That is, although the economy continues forever and firms operate indefinitely, firms' current choices maximize discounted profits over the next δ periods only. We restrict our attention to finite planning horizons for two reasons. Firstly, our solution algorithm requires a terminal condition. Secondly, finite planning horizons guarantee concavity of firms' objective functions and, therefore, existence of a solution. Casual empiricism suggests that this is not an unreasonable assumption. The effect of the length of the planning horizon on growth is analyzed using computational experiments in section II.3.

Producers of intermediate goods play a dynamic, quality setting game. Firms seek to maximize discounted Nash profits over the length of the planning horizon in units of the date t final good. If a firm produces, its profits at time t in units of the date t final good are given by

$$r_{jt} - w - .5\theta_{jt}(q_{jt} - q_{jt-1})^2$$

Letting $\mathbf{q}_t = \{q_{jt}\}_{j=1}^{N}$ and substituting demand from (1) for an interior solution implies that one-period profits to intermediate producer j at time t are given by

$$\Pi_j(\theta_{jt}, q_{jt-1}, \mathbf{q}_t) = a + q_{jt}(b_j \sum_{i \neq j} q_{it}) - w - .5\theta_{jt}(q_{jt} - q_{jt-1})^2$$

Let $\theta_t = \{\theta_{jt}\}_{j=1}^{N}$ and $\theta^t = \{\theta_s\}_{t \leq s \leq t+\delta-1}$. Letting R_{jt} denote the objective function of firm j at time t, then

$$R_{jt}(\theta^t, q_{jt-1}, \mathbf{q}_t) = \Pi_j(\theta_{jt}, q_{jt-1}, \mathbf{q}_t) + \beta V_{jt}(\mathbf{q}_t, \theta^{t+1})$$

where $\beta V_{jt}(\mathbf{q}_t, \theta^{t+1})$ is the discounted value of future profits at time t over the length of the planning horizon given that all other firms play their Nash equilibrium strategies. That is, defining a Nash Equilibrium at time t to be a set of strategy functions, $q_j^*(\mathbf{q}_{t-1}, \theta^t)$, that satisfy $\forall j = 1 \ldots N$:

$$q_j^*(\mathbf{q}_{t-1}, \theta^t) = \underset{q_{jt}}{\text{argmax}}\, R_{jt}(\theta^t, q_{jt-1}, q_{jt}, q_{i \neq j}^*(\mathbf{q}_{t-1}, \theta^t))$$

then

$$V_{jt}(\mathbf{q}_{t-1}, \theta^{t+1}) = R_{jt+1}(\theta^{t+1}, q_{jt}, q^*(\mathbf{q}_t, \theta^{t+1})).$$

Therefore, intermediate producer j faces the following maximization problem at time t:

$$\max_{\{q_{js}\}_{t \leq s \leq t+\delta}} R_{jt}(\theta^t, q_{jt-1}, \mathbf{q}_t)$$

Subgame perfect equilibrium qualities at time t in this dynamic game are determined by solving the game backward from the end of the planning horizon, $t+\delta-1$. This method is discussed in further detail in section II.3.

Consumers

Consumers own an equal share of each firm in the economy. Since goods are not storable and consumers are identical, the representative consumer has no means by which to borrow or lend. Therefore, the consumer seeks to maximize period by period utility subject to a sequence of budget constraints:

$$\forall t \quad \max_{c_t} U(c_t)$$

$$\text{subject to:} \quad p_t c_t \le p_t\{Nw + (\sum_{j=1}^{N} \Pi_{jt})\}$$

where p_t is the price of date t final good in units of date zero final good, and Π_{jt} are profits from intermediate producer j in units of date t final goods.

Since U(.) is increasing, a first order condition for a solution to the consumer's problem is:

$$\forall t \quad c_t = Nw + \sum_{j=1}^{N} \Pi_{jt}$$

II.2 Equilibrium

II.2.1: Definition of Equilibrium

A *subgame perfect equilibrium* for this economy is a collection of sequences for prices $\{p_t\}$, $\{r_{jt}\}$, allocations $\{c_t,y_t\}$, $\{k_{jt}\}$, and quality functions $\{q_{jt}\}$ such that

(i) $\{c_t\}$ maximizes the representative consumer's utility subject to a sequence of budget constraints.

(ii) $\{q_t\}$ is a Nash equilibrium quality function for each subgame of the dynamic game.

(iii) Goods markets clear: $\forall t$

Final Goods:

$$c_t = y_t - .5\sum_{j=1}^{N} \theta_{jt}(q_{jt}-q_{jt-1})^2$$

Intermediate goods:

$$k_{jt} = 1 \quad \forall j=1...N$$

(iv) Labor market clears: $\forall t$

$$\sum_{j=1}^{N} k_{jt} = N$$

II.2.2: **A Special Case**

It is useful to examine a case in which analytic Nash equilibrium quality functions and growth rates of qualities and output can be determined. Consider the case where firms have a one period planning horizon (δ=1), and the industry is a duopoly, (N=2). Examining this case provides insight into the factors which affect the evolution of qualities and growth in the economy.

Since firms have a single period planning horizon, firm j faces the following maximization problem at time t:

$$\max_{q_{jt}} \; a + q_{jt}(b\sum_{i\neq j} q_{it}) - w - .5\theta_{jt}(q_{jt}-q_{jt-1})^2$$

subject to q_{it} given for $i\neq j$

q_{jt-1} given

A first order necessary condition for a solution to this maximization problem gives the best response function for firm j=1,2 at time t:

$$\hat{q}_j(q_{jt-1},\theta_{jt},q_{i\neq jt}) = (1/\theta_{jt})(bq_{i\neq jt}) + q_{jt-1}$$

Note that $\partial\hat{q}_j(.)/\partial q_{i\neq jt}>0$. This illustrates that qualities are strategic complements in this economy and best response functions are upward sloping. This is also true in more general cases with longer planning horizons and more firms.

Combining these best response functions gives Nash equilibrium qualities at time t as functions of last period qualities and current innovation technology parameters:

$$q_1^*(q_{t-1},\theta_t) = (\theta_{2t}/\gamma_t)(\theta_{1t}q_{1t-1} + bq_{2t-1})$$

$$q_2^*(q_{t-1},\theta_t) = (\theta_{1t}/\gamma_t)(\theta_{2t}q_{2t-1} + bq_{1t-1})$$

where $\gamma_t = \theta_{1t}\theta_{2t} - b^2$. These Nash equilibria exist if $\gamma_t>0$.

Imposing the initial condition, $q_{jt} = q_o\ \forall j$, implies that Nash qualities at time t can be written as a function of q_o and $\{\theta_s\}_{1\leq s\leq t}$. These functions are determined as follows. Define the sequences $\{\psi_{1t}\}_{t=1}^{\infty}$, $\{\psi_{2t}\}_{t=1}^{\infty}$:

$$\psi_{11} = \theta_{21}(\theta_{11} + b)$$

$$\psi_{21} = \theta_{11}(\theta_{21} + b)$$

and $\forall$ t>1:

$$\psi_{1t} = \theta_{2t}(\theta_{1t}\psi_{1t-1} + b\psi_{2t-1})$$

$$\psi_{2t} = \theta_{1t}(\theta_{2t}\psi_{2t-1} + b\psi_{1t-1})$$

Let $\Gamma_t = \prod_{s=1}^{t} \gamma_s$.

Then, Nash equilibrium qualities at time t are given by:

$$\forall t \quad q^*_{1t}(q_o, \{\theta_s\}_{1\le s\le t}) = (q_o\psi_{1t}/\Gamma_t)$$

$$\forall t \quad q^*_{2t}(q_o, \{\theta_s\}_{1\le s\le t}) = (q_o\psi_{2t}/\Gamma_t)$$

Equilibrium growth rates of qualities at time t are given by

$$\forall t \quad g_{q1t}(b, \{\theta_s\}_{1\le s\le t}) = (b/\gamma_t)(\theta_{2t}(\psi_{2t-1}/\psi_{1t-1}) + b)$$

$$\forall t \quad g_{q2t}(b, \{\theta_s\}_{1< s\le t}) = (b/\gamma_t)(\theta_{1t}(\psi_{1t-1}/\psi_{2t-1}) + b)$$

If $\forall t\ \theta_{1t}\theta_{2t} > b^2$, then $\forall t\ \gamma_t > 0$, (the same condition for existence) and growth rates of qualities are positive since the ψ_{jt} sequences are positive, $\forall j=1,2$. Since output of the final good is an increasing function of qualities, growth rates of output are positive as well.

These growth rate functions demonstrate that growth of the quality of good j at time t is affected by the sequence of innovation parameters of both firms up to time t. Furthermore, given the relationship between qualities of intermediate goods and output of the final good, growth rates of output also depend on the entire history of innovation parameters.

Consider an economy with constant, symmetric, adjustment costs, i.e. $\theta_{jt} = \bar{\theta}\ \forall j, \forall t$. In this economy $\psi_{1t} = \psi_{2t}\ \forall t$. Therefore, growth rates of qualities in this economy are constant and equal:

$$\bar{g}_q(b, \bar{\theta}) = b/(\bar{\theta}-b)$$

Note that $\partial\bar{g}_q/\partial b > 0$ and $\partial\bar{g}_q/\partial\theta < 0$. That is, economies described by this example will exhibit higher growth rates of quality in equilibrium the higher the spillover of qualities and the lower the costs of innovating.

Equilibrium output of the final good at time t is given by

$$y_t = 2\{a + bq_0^2[\bar{\theta}/(\bar{\theta}-b)]^{2t}\},$$

and the growth rate of output at time t is

$$g_{yt}(b,\bar{\theta},q_o) = \frac{(bq_o)^2\left[\frac{2\bar{\theta}-b}{\theta^2}\right]}{a\left[\frac{\bar{\theta}-b}{\bar{\theta}}\right]^{2t} + bq_o^2\left[\frac{\bar{\theta}-b}{\bar{\theta}}\right]^2}.$$

Since $[(\bar{\theta}-b)/\bar{\theta}] < 1$, the long-run growth rate of output converges to

$$\bar{g}_y(b,\bar{\theta}) = b(2\bar{\theta}-b)/[(\bar{\theta}-b)^2]$$

As with growth rates of quality, higher b and lower $\bar{\theta}$ are associated with higher equilibrium growth rates of output.

This example demonstrates that a version of this economy characterized by constant innovation costs and symmetric duopolists with static maximization problems exhibits constant long-run growth rates of output. This rate is affected by spillovers and the cost of innovating. Constant long-run growth results because of the way in which spillovers are modeled in this economy. The behavior of other intermediate good producers affects the slope of a particular firm's profit function net of innovation costs. The slope of the profit function increases over time as other firms' qualities increase. This leads firms to choose larger increases in quality over time and results in constant growth rates of qualities and constant long-run growth rates of output.

To clarify this point, consider an example in which the technology for producing final goods is not characterized by spillovers in qualities:

$$y_t = \sum_{j=1}^{N} [a + bq_{jt}]k_{jt}$$

Then, equilibrium growth rates of quality in this economy with constant, symmetric innovation costs at time t are

$$b/[b(t-1) + \bar{\theta}q_0]$$

Therefore, although qualities continually increase in this example, growth rates will asymptotically converge to zero. It is spillovers in Economy I which lead to constant long-run growth rates.

In the next section, computational experiments examine the economy in equilibrium when firms may face longer planning horizons and when there may be more than two intermediate goods producers. Because of the dynamic, strategic interaction of innovators, these environments are too complicated to be analyzed analytically. The effects of the spillover parameter, the length of the planning horizon, the number of firms, and the mean of the innovation technology parameters on growth rates of qualities and output are examined in those experiments.

II.3 Computational Experiments

In what follows, the economy described above is simulated under various values of economy parameters. The dynamic game is solved backwards at each time t from the end of the planning horizon. The procedure involves determining Nash equilibrium qualities from date t+1 to date $t+\delta-1$ as linear functions of the vector of qualities at time t. Given these Nash quality functions, future discounted Nash profits as a function of qualities at t are determined. Linear best response functions at t are calculated which maximize current and future Nash profits taking into account the effect of current quality choices on future returns. Finally, combining these best response functions determines Nash qualities at time t.

For all j, the sequences of innovation technology parameters, $\{\theta_{jt}\}_{t=0}^{T}$, are generated using the following stochastic process:

$$\theta_{jt+1} = \rho\theta_{jt} + \varepsilon_{jt+1}.$$

For all j, θ_{jo} is set equal to $\bar{\theta}$, and T realizations of ε_{jt} are generated using a random number generator where $\varepsilon_{jt} \sim LnN(\bar{\theta}(1-\rho), \sigma_{\varepsilon}^{2})$. This specification guarantees positive innovation parameters which fluctuate about $\bar{\theta}$.

The standard set of parameters used in the computational experiments are given in the following table.

Table II.1: Standard Parameter Values

$a = 5$	$\rho = .8$	$\sigma_{\varepsilon}^{2} = .04$	$q_o = 1$	$w = 1$	$\beta = .95$
$\delta = 2$	$N = 2$	$b = .2$	$\bar{\theta} = 30$		

The computational experiments examine the effects of different δ, N, b, and $\bar{\theta}$ on equilibrium growth rates of qualities and output of the final good. In each experiment, the remaining parameters are set at the levels given above. Long-run average growth rates reported below are averages of 100 time periods (periods 700-800) over 10 realizations of the θ process.

II.3.1. Equilibrium Time Paths

Figures 1.A - 1.D depict the behavior of average quality and output in equilibrium when the economy parameters are set at the values in Table II.1. Quality growth fluctuates around a constant level while output growth initially increases and then fluctuates around a constant. These figures demonstrate that the economy exhibits continual growth in equilibrium and constant growth in the long run. Growth results in this economy because of the strategic element of firm's quality choices (the spillovers). Without this effect, the economy would grow, but growth rates would asymptotically converge to zero as firms will choose to increase qualities each period by an absolute amount which fluctuates around a constant.

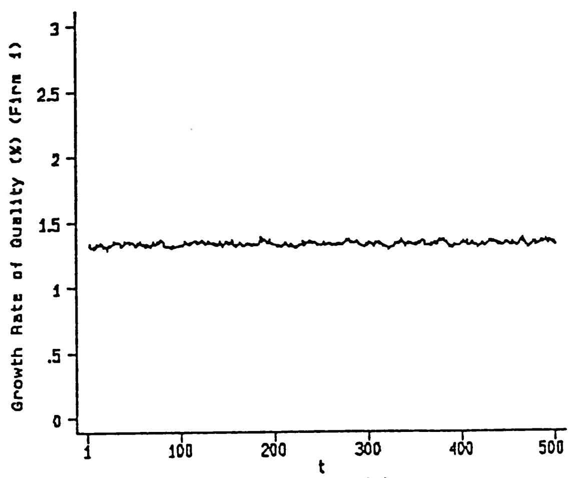

Figure 1.A

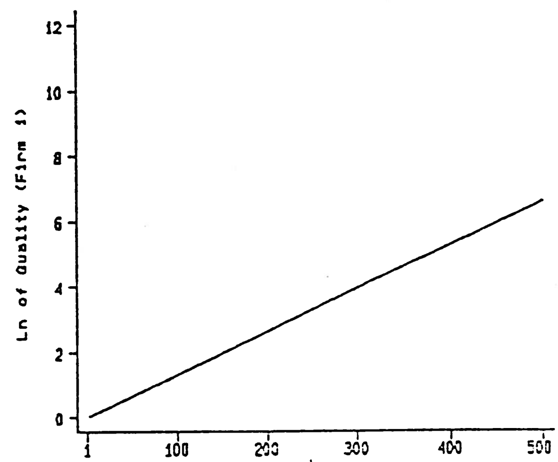

Figure 1.B

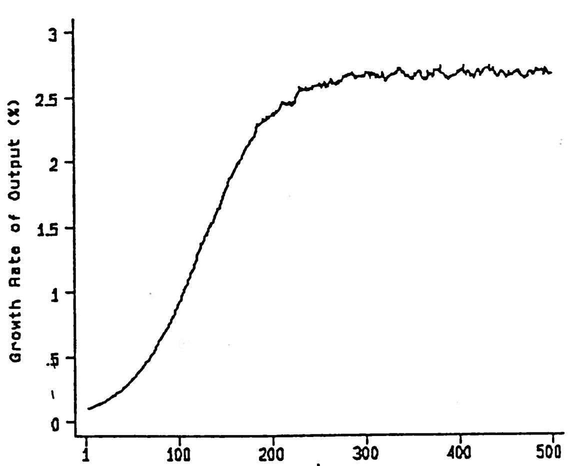

Figure 1.C

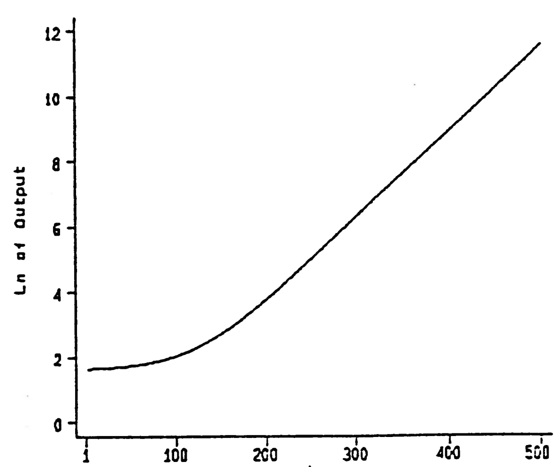

Figure 1.D

II.3.2. Effect of the Length of the Planning Horizon

Figures 2.A and 2.B depict the long-run growth rates of qualities and output as a function of the length of the planning horizon, δ. These figures illustrate that economies characterized by firms with longer planning horizons will grow faster. This result is expected as firms, taking into account future profits when making current innovation decisions, recognize that costs associated with higher levels of innovation in the current period will be offset by higher future profits. This leads to higher growth rates of qualities and, therefore, of output.

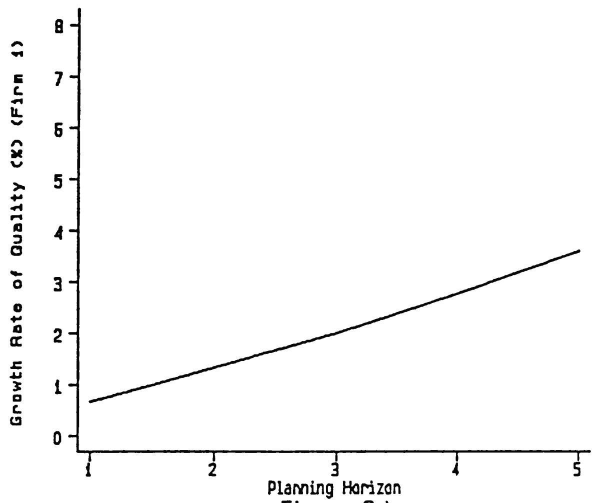

Figure 2.A

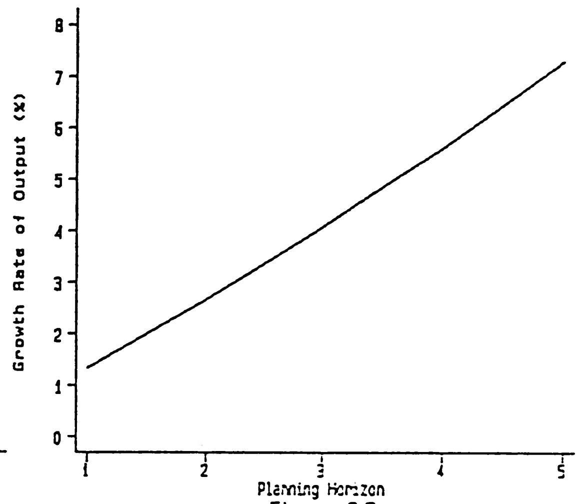

Figure 2.B

II.3.3. Effect of the Number of Firms

Figures 3.A and 3.B depict the growth rates of qualities and output as a function of the number of firms operating in the intermediate goods sector, N. These figures illustrate that a larger number of intermediate goods is associated with higher growth rates. This results from two effects. By assumption, labor endowment increases with the number of firms providing a larger resource base for production. In addition, the specification of the final good production technology implies that more firms in the intermediate sector is associated with higher profits for an individual firm for a given quality choice. Therefore, firms are willing to undertake larger innovation expenditures if there are a large number of firms in the industry and higher growth rates result.

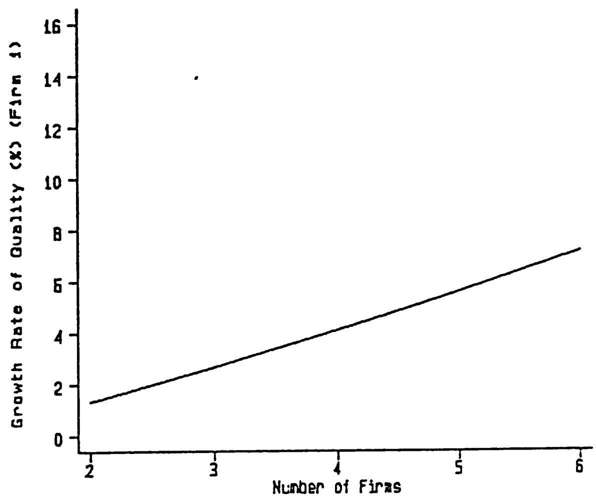

Figure 3.A

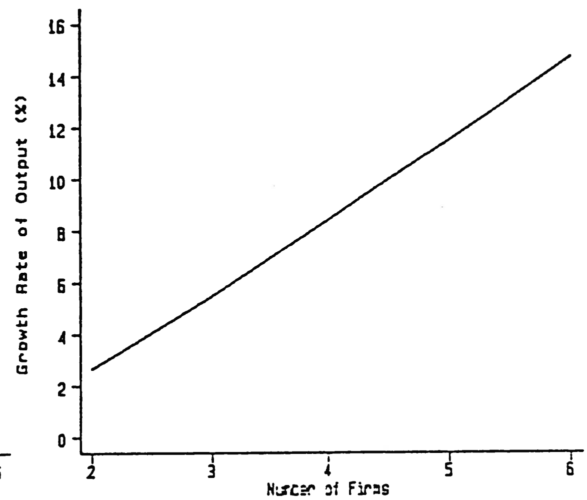

Figure 3.B

II.3.3 Effect of Spillovers

Figures 4.A and 4.B depict equilibrium growth rates of qualities and output as a function of the level of spillovers, b. These figures illustrate that higher spillovers in quality are associated with higher growth rates.

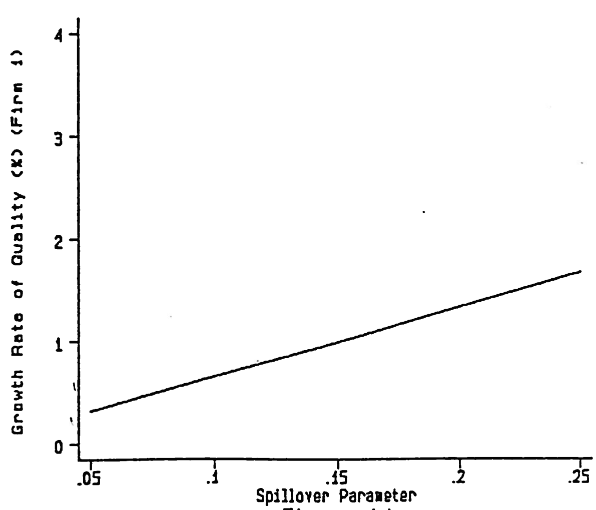

Figure 4.A

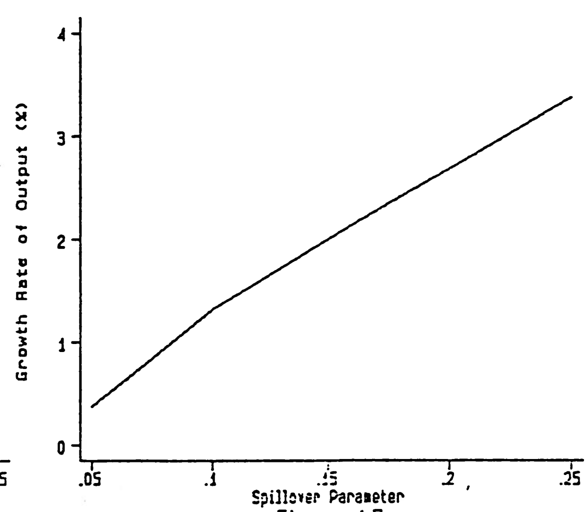

Figure 4.B

II.3.4 Effect of the Mean of Innovation Technology Shocks

Figures 5.A and 5.B depict equilibrium growth rates of qualities and output as a function of the mean of the innovation technology parameters, $\bar{\theta}$. These figures demonstrate that higher average costs of innovating are associated with lower growth rates.

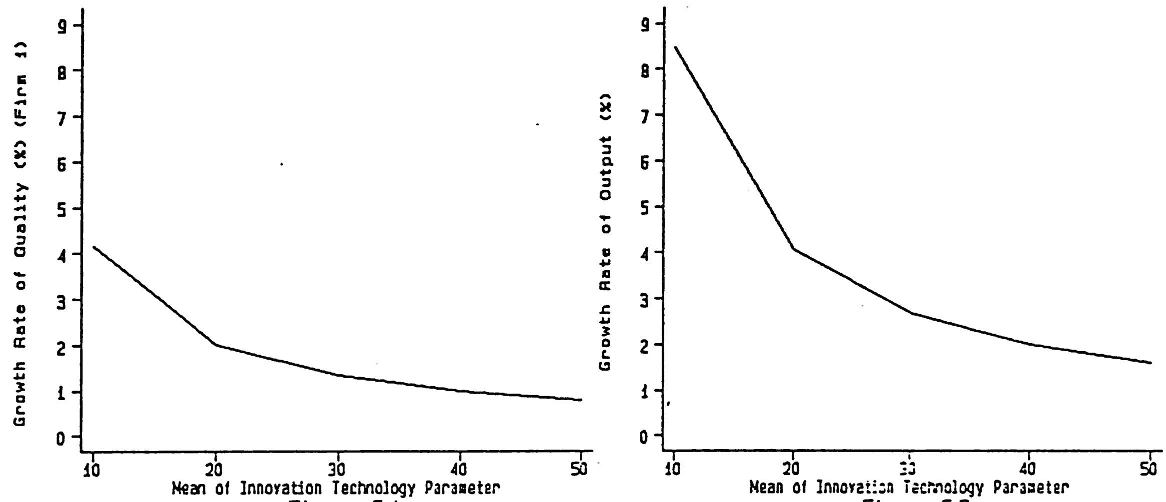

Figure 5.A Figure 5.B

II.4: Conclusions

The economy studied in this section exhibits continual output growth and long-run growth rates which fluctuate around a constant level. Growth rates increase toward that constant level along the dynamic equilibrium path. Long-run growth rates which fluctuate around a constant result in this economy because of spillovers in quality among intermediate goods. These spillovers lead to strategic quality choices by intermediate goods producers in a dynamic game. The length of firms planning horizons, the number of intermediate goods, the size of spillovers, and the mean of innovation technology parameters significantly affect long-run growth rates.

In this economy, firms unequivocally benefit from the quality improvements of other firms, i.e. qualities are strategic complements.

In part, this arises because of the way equilibrium wages are determined. In the next section, we examine an economy in which the marginal product of an intermediate good depends on the quantity of the good employed. This modification implies that innovation by other firms can harm intermediate producers by pushing up wages, i.e. qualities are strategic substitutes.

III. ECONOMY II - An Economy in Which Qualities are Strategic Substitutes

In Economy I, the supply of intermediate goods and, therefore, the demand for labor were indeterminate. This required an arbitrary specification of the quantity of each intermediate good produced and the wage. That specification allowed us to focus on the effects of strategic behavior in innovation on growth while abstracting from other issues. In this section, we examine an economy in which the quantity of intermediate goods and the wage are determined in equilibrium. This modification leads to qualities being strategic substitutes. Section III.1 describes the economy, section III.2 discusses the equilibrium concept and considers an example, section III.3 presents results of computational experiments, section III.4 four considers two modifications, and section IV.5 concludes.

III.1 The Economy

III.1.1 Environment

This economy differs from the previous economy primarily in its specification of the technology for producing the final good:

$$y_t = \sum_{j=1}^{N} (a + bq_{jt} + d\sum_{i\neq j} q_{it})k_{jt} - [.5(k_{jt})^2]/c,$$

where b>d. The marginal product of intermediate good j is given by:

$$\partial y_t/\partial k_{jt} = a + bq_{jt} + d\sum_{i\neq j} q_{it} - k_{jt}/c.$$

Note that with this specification, the marginal product of intermediate goods depends on the qualities of all intermediate goods and, unlike Economy I, on the quantity of the good employed. This specification is chosen because it makes wages and quantities endogenous while maintaining the linearity of best response functions.

This technology for the final good generates linear demand functions for intermediate goods while maintaining concavity of the production function. We restrict our attention to the portion of the production function which exhibits non-negative marginal product for each intermediate good. That is, we restrict k_{jt}, $\forall j, \forall t$, as follows:

$$0 \leq k_{jt} \leq c[a + bq_{jt} + d\sum_{i\neq j} q_{it}]$$

It can be shown that this constraint never binds as long as the prices of intermediate goods are positive.

The technologies for producing intermediate goods and innovation are as specified in the previous economy. The time endowment is set equal to L>0. Preferences are as specified in Economy I.

III.1.2 Market Arrangements

Producers

Final good producers face the following static maximization problem:

$$\max_{\{k_{jt}\}_{j=1}^{N}} \quad p_t\{y_t - \sum_{j=1}^{N} r_{jt}k_{jt}\}$$

$$\text{subject to} \quad y_t = \sum_{j=1}^{N} (a + bq_{jt} + d\sum_{i\neq j} q_{it})k_{jt} - [.5(k_{jt})^2]/c$$

The solution to this maximization problem gives rise to the following demand function for intermediate good j at time t:

$$k_j(r_{jt},q_t) = \begin{cases} c[a + bq_{jt} + d\sum_{i\neq j} q_{it} - r_{jt}] & \text{if } >0 \\ 0 & \text{otherwise} \end{cases}$$

In this economy, intermediate goods producers solve a two-stage maximization problem. In the first stage, firms choose qualities in a dynamic game taking into account how prices will be determined in the second stage. We first examine the second stage in which prices are chosen. In this stage at time t, firm j chooses r_{jt} to maximize current period profits net of innovation costs in units of the date t final good:

$$\max_{r_{jt}} \quad (r_{jt} - w_t)k_j(r_{jt},q_t)$$

$$\text{subject to} \quad k_j(r_{jt},q_t) = c[a + bq_{jt} + d\sum_{i\neq j} q_{it} - r_{jt}]$$

$$w_t, q_t \quad \text{given}$$

The solution to the second stage problem gives equilibrium prices, quantities, and profits net of innovation costs as functions of current qualities and wages:

$$r_{jt} = r_j(w_t,q_t) = .5(a + bq_{jt} + d\sum_{i\neq j} q_{it} + w_t)$$

$$k_{jt} = k_j(w_t,q_t) = .5c(a + bq_{jt} + d\sum_{i\neq j} q_{it} - w_t)$$

$$\text{Profits}_{jt} = .25c(a + bq_{jt} + d\sum_{i\neq j} q_{it} - w_t)^2$$

In the first stage of the firms' maximization problems, firms play a dynamic, quality setting game taking into account the relationship between qualities and prices as determined in the second stage. Firms seek to maximize discounted Nash profits over the length of the planning horizon in units of the date t final good. If a firm produces, its profits at time t in units of the date t final good are given by

$$\Pi_j(w_t, \theta_{jt}, q_{jt-1}, q_t) = .25c(a + bq_{jt} + d\sum_{i \neq j} q_{it} - w_t)^2 - .5\theta_{jt}(q_{jt} - q_{jt-1})^2$$

The objective function of firm j at time t is

$$R_{jt}(w^t, \theta^t, q_{jt-1}, q_t) = \Pi_j(w_t, \theta_{jt}, q_{jt-1}, q_t) + \beta V_{jt}(w^{t+1}, \theta^{t+1}, q_t)$$

where $w^t = \{w_s\}_{t \le s \le t+\delta-1}$ and $\beta V_{jt}(w^{t+1}, \theta^{t+1}, q_t)$ is the discounted value of future profits at time t over the length of the planning horizon given that all other firms play their Nash equilibrium strategies.

Therefore, intermediate producer j faces the following maximization problem:

$$\max_{\{q_{js}\}_{t \le s \le t+\delta}} R_{jt}(w^t, \theta^t, q_{jt-1}, q_t)$$

As in the previous economy, subgame perfect equilibrium qualities at time t in this dynamic game are determined by solving the game backward from the end of the planning horizon, t+δ-1. At time t, w^t is determined by simultaneously imposing market clearing in all labor markets based on future labor demand in the Nash equilibrium.

III.2 Equilibrium

III.2.1: Definition of Equilibrium

A *subgame perfect equilibrium* for this economy is a set of sequences for prices $\{w_t, p_t\}$, $\{r_{jt}\}$, allocations $\{c_t, y_t\}$, $\{k_{jt}\}$, and quality functions $\{q_{jt}\}$ such that

(i) $\{c_t\}$ maximizes the representative consumer's discounted utility subject to a date zero budget constraint.

(ii) $\{q_t\}$ is a Nash equilibrium quality function for each subgame of the dynamic game

(iii) Goods markets clear: $\forall t$

Final Goods:

$$c_t = y_t - .5\sum_{j=1}^{N} \theta_{jt}(q_{jt}-q_{jt-1})^2$$

Intermediate goods:

$$k_{jt} = .5c[a + bq_{jt} + d\sum_{i\neq j} q_{it} - w_t] \quad \forall j=1...N$$

(iv) Labor market clears: $\forall t$

$$\sum_{j=1}^{N} k_{jt} = L$$

III.2.2: A Special Case

Again, we consider a simple case which can be examined analytically. Consider the case where firms have a one period planning horizon (δ=1), and the industry is a duopoly, (N=2). Labor market clearing in this example implies

$$\forall t \quad L = k_{1t} + k_{2t} = c(a - w_t) + .5c(b+d)(q_{1t} + q_{2t})$$

or

(3) $$\forall t \quad a - w_t = L/c - .5(b+d)(q_{1t} + q_{2t})$$

Since firms have a single period planning horizon, firm j faces the following maximization problem at time t:

$$\max_{q_{jt}} \ .25c[a + bq_{jt} + dq_{i\neq jt} - w_t]^2 - .5\theta_{jt}(q_{jt}-q_{jt-1})^2$$

$$\text{subject to } q_{it} \text{ given for } i\neq j$$

$$q_{jt-1} \text{ given}$$

A first order necessary condition for an interior solution to this maximization problem for firm j=1,2 at time t is:

$$.5cb(a + bq_{jt} + dq_{i\neq jt} - w_t) - \theta_{jt}(q_{jt}-q_{jt-1}) = 0$$

Substituting for $(a-w_t)$ from the labor market clearing condition in equation (3) above gives the following best response function for firm j at time t:

$$\hat{q}_j(q_{i\neq jt},\theta_{jt},q_{jt-1}) = (1/Z_{jt})[\alpha L/c + .5\alpha(d-b)q_{i\neq jt} + \theta_{jt}q_{jt-1}]$$

where $\alpha = .5cb$ and $Z_{jt} = \theta_{jt} + .5\alpha(d-b)\ \forall j$. Note that if $Z_{jt}>0$, then since $b>d$, $\partial q_j(.)/\partial q_{i\neq jt}<0$. This illustrates that qualitites are strategic substitutes in this economy.

Combining these best response functions gives Nash equilibrium qualities at time t as a function of last period qualities and current innovation technology parameters:

$$q_1^*(q_{t-1},\theta_t) = (1/\gamma_t)(.5bL[\theta_{2t}+ \alpha(d-b)] + Z_{2t}\theta_{1t}q_{1t-1} + .5\alpha\theta_{2t}(d-b)q_{2t-1})$$

$$q_2^*(q_{t-1},\theta_t) = (1/\gamma_t)(.5bL[\theta_{1t}+ \alpha(d-b)] + Z_{1t}\theta_{2t}q_{2t-1} + .5\alpha\theta_{1t}(d-b)q_{1t-1})$$

where $\gamma_t= \theta_{1t}\theta_{2t} +.5\alpha(d-b)(\theta_{1t}+ \theta_{2t})$. These Nash equilibria will exist if $\gamma_t>0\ \forall t$.

Imposing the initial condition, $q_{jt} = q_o\ \forall j$, implies that Nash qualities at time t can be written as a function of q_o and $\{\theta_s\}_{1\le s\le t}$ as follows. Define the sequences $\{\eta_{1t}\}_{t=1}^{\infty}$, $\{\eta_{2t}\}_{t=1}^{\infty}$, $\{\psi_{1t}\}_{t=1}^{\infty}$, $\{\psi_{2t}\}_{t=1}^{\infty}$:

$\forall t$:

$$\eta_{1t} = .5bL(\theta_{2t} + \alpha(d-b))$$

$$\eta_{2t} = .5bL(\theta_{1t} + \alpha(d-b))$$

and

$$\psi_{11} = \eta_{11}$$

$$\psi_{21} = \eta_{21}$$

$\forall t>1$:

$$\psi_{1t} = \Gamma_{t-1}\eta_{1t} + Z_{2t}\theta_{1t}\psi_{1t-1} + .5\alpha(d-b)\theta_{2t}\psi_{2t-1}$$

$$\psi_{2t} = \Gamma_{t-1}\eta_{2t} + Z_{1t}\theta_{2t}\psi_{2t-1} + .5\alpha(d-b)\theta_{1t}\psi_{1t-1}$$

Let $\Gamma_t = \prod_{s=1}^{t} \gamma_s$.

Then, Nash equilibrium qualities are given by: $\forall t$

$$q^*_{jt}(q_o, L, \{\theta_s\}_{1\le s\le t}) = \psi_{jt}/\Gamma_t + q_o \qquad \forall\ j=1,2.$$

Also, Nash equilibrium growth rates of qualities are given by: $\forall t$

$$g_{1t}(q_o, L, \{\theta_s\}_{1\le s\le t}) = \frac{\Gamma_{t-1}\eta_{1t} + .5\alpha(d-b)\theta_{2t}(\psi_{2t-1}-\psi_{1t-1})}{\gamma_t\psi_{1t-1} + \Gamma_t q_o}$$

$$g_{2t}(q_o, L, \{\theta_s\}_{1\le s\le t}) = \frac{\Gamma_{t-1}\eta_{2t} + .5\alpha(d-b)\theta_{1t}(\psi_{1t-1}-\psi_{2t-1})}{\gamma_t\psi_{2t-1} + \Gamma_t q_o}$$

Now, $\psi_{1t} - \psi_{2t} = \sum_{s=2}^{t} .5bL\Gamma_s(\theta_{1s}-\theta_{2s})\left(\prod_{\tau=s+1}^{t}\theta_{1\tau}\theta_{2\tau}\right) + \prod_{\tau=2}^{t} .5bL\theta_{1\tau}\theta_{2\tau}(\theta_{11}-\theta_{21})$

These growth rates imply that if a firm has high innovation costs relative to its competitor for enough periods, the firm may choose to decrease quality. This results because increases in quality lead to increases in demand for labor and increases in equilibrium wages. Therefore, profit maximizing behavior for a firm which faces consistently high innovation costs may induce decreases in quality for that firm, losses in market share, and perhaps exit. In the computational experiments below we focus on equilibria which are characterized by positive growth rates of qualities. However, this feature of the economy opens interesting questions regarding exit of firms due to obsolescence of products. These issues are discussed briefly in the extensions but are generally beyond the scope of this paper.

Consider an economy with constant, symmetric innovation costs; i.e. $\theta_{jt} = \bar{\theta}\ \forall j,\ \forall t$. In this economy, $\psi_{1t} = \psi_{2t}\ \forall t$. Therefore equilibrium growth rates of qualities at time t equal:

$$g_{qt}(b, q_o, L, \bar{\theta}) = .5bL/[.5bL(t-1) + \bar{\theta}q_o]$$

Now as $t \to \infty$, $g_{qt} \to 0$. However, it should also be noted that

$$\forall t \quad q_t - q_{t-1} = .5bL/\bar{\theta}.$$

That is, qualities continually increase in this economy but by a constant increment. Therefore growth rates of quality approach zero in the long run.

Further note that $\partial g_{qt}/\partial b > 0$, $\partial g_{qt}/\partial L > 0$, and $\partial g_{qt}/\partial \theta < 0$. That is, economies described by this example will exhibit higher equilibrium growth rates of quality in the short-run the higher is b, the larger the labor endowment, and the lower the cost of innovating. Also, in this example, with constant, symmetric innovation costs, the spillover parameter, d, has no effect on short-run growth rates of qualities. This result is particular to the symmetric equilibrium as will be demonstrated in the computational experiments in section III.3.

The equilibrium growth rate of output at time t in this example is:

$$g_{yt}(b,q_o,L,\bar{\theta}) = [.5bcL(b+d)]/[ac\bar{\theta} - .25L\bar{\theta} + \bar{\theta}q_o(b+d) + .5bL(b+d)(t-1)]$$

Therefore, the growth rate of output approaches zero in the long-run. Short-run growth rates are affected positively by b, d, and L, and negatively by $\bar{\theta}$. In addition,

$$y_t - y_{t-1} = [.5bL^2(b+d)]/\bar{\theta}$$

Therefore, output is unbounded in this economy but grows at a decreasing rate.

This example demonstrates that a version of this economy characterized by symmetric duopolists with constant innovation costs and static maximization problems will exhibit growth rates of output which decline over time. Short-run growth rates of output are affected by spillovers, the cost of innovating, and the resource base. The next section examines the effects of these parameters on the equilibria of more general versions of this economy.

III.3 Computational Experiments

In what follows, the economy described above is simulated for 300 periods under various values of economy parameters. The evolution of innovation technology parameters is governed by the same process as described in Economy I. The standard set of parameters used in the computational experiments are given in the following table.

Table III.1: Standard Parameter Values

$a = 5$	$\rho = .8$	$\sigma_{\varepsilon}^{2} = .04$	$q_{o} = 1$	$\beta = .98$	$L = 2$	$c = 1$
$\delta = 2$	$N = 2$	$b = .8$	$d = .6$	$\bar{\theta} = 30$		

The computational experiments examine the effects of different δ, N, b, d, and $\bar{\theta}$ on equilibrium growth rates of qualities and output of the final good. In each experiment, except for those which vary d, the remaining parameters are set at the levels given above.

III.3.1. Effect of the Length of the Planning Horizon

Figure 6 depicts equilibrium levels of average quality and output and their growth rates as the length of the planning horizon varies, (δ=1,2,3,4,5). These figures illustrate that growth rates decline over time in this economy. They further demonstrate that economies characterized by firms with longer planning horizons will exhibit higher short-run growth rates.

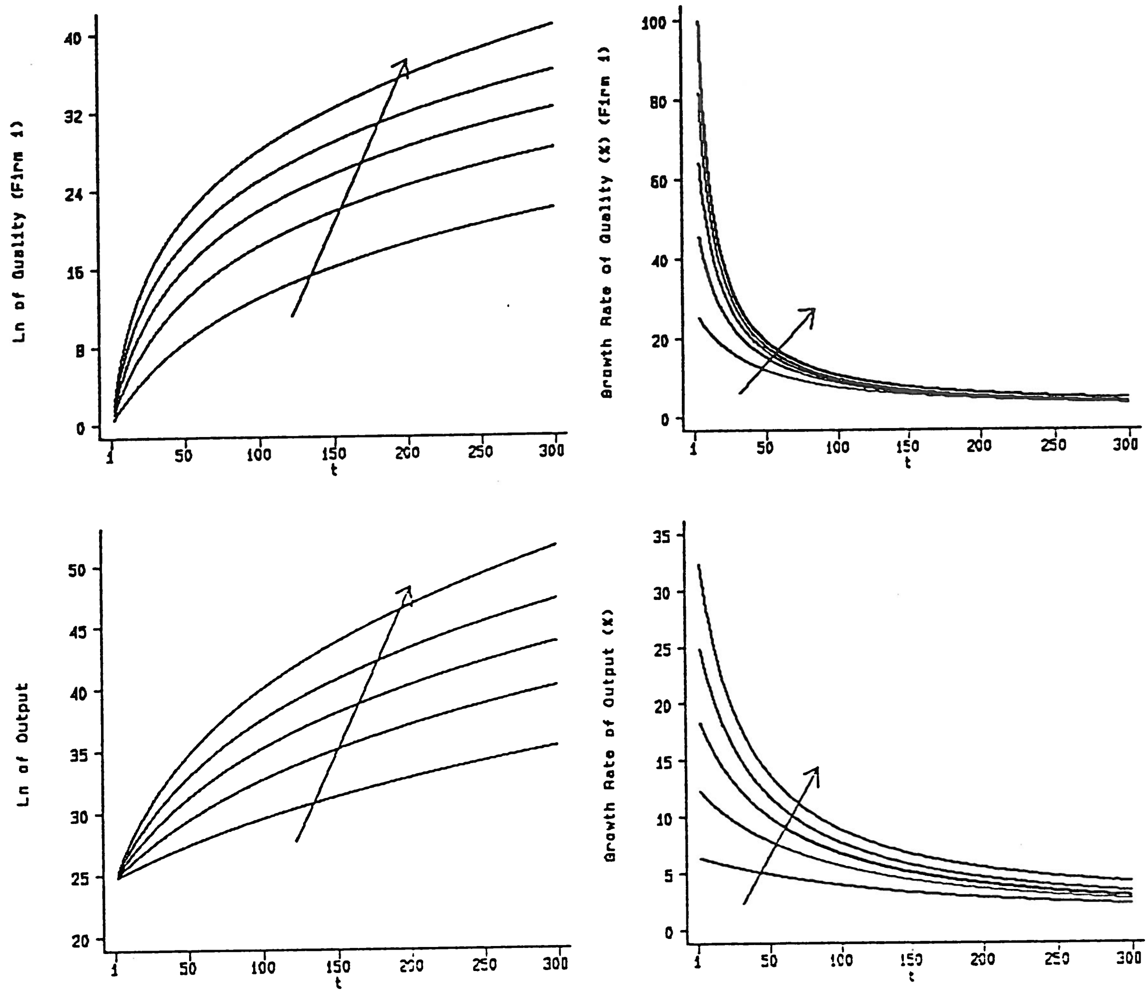

Figure 6: Length of Planning Horizon

II.3.2. Effect of the Number of Firms

Figure 7 depicts equilibrium levels and growth rates as the number of firms operating in the intermediate goods sector increases, (n=2,3,4,5,6). Increases in the number of firms without an associated increase in the labor endowment leads to lower growth in this economy. This results as total demand for labor is higher in economies with more firms and therefore, equilibrium wages are higher. Since the costs of producing intermediate goods is higher, lower quantities and therefore qualities result. Hence, qualities and output grow more slowly the larger the number of intermediate goods producers.

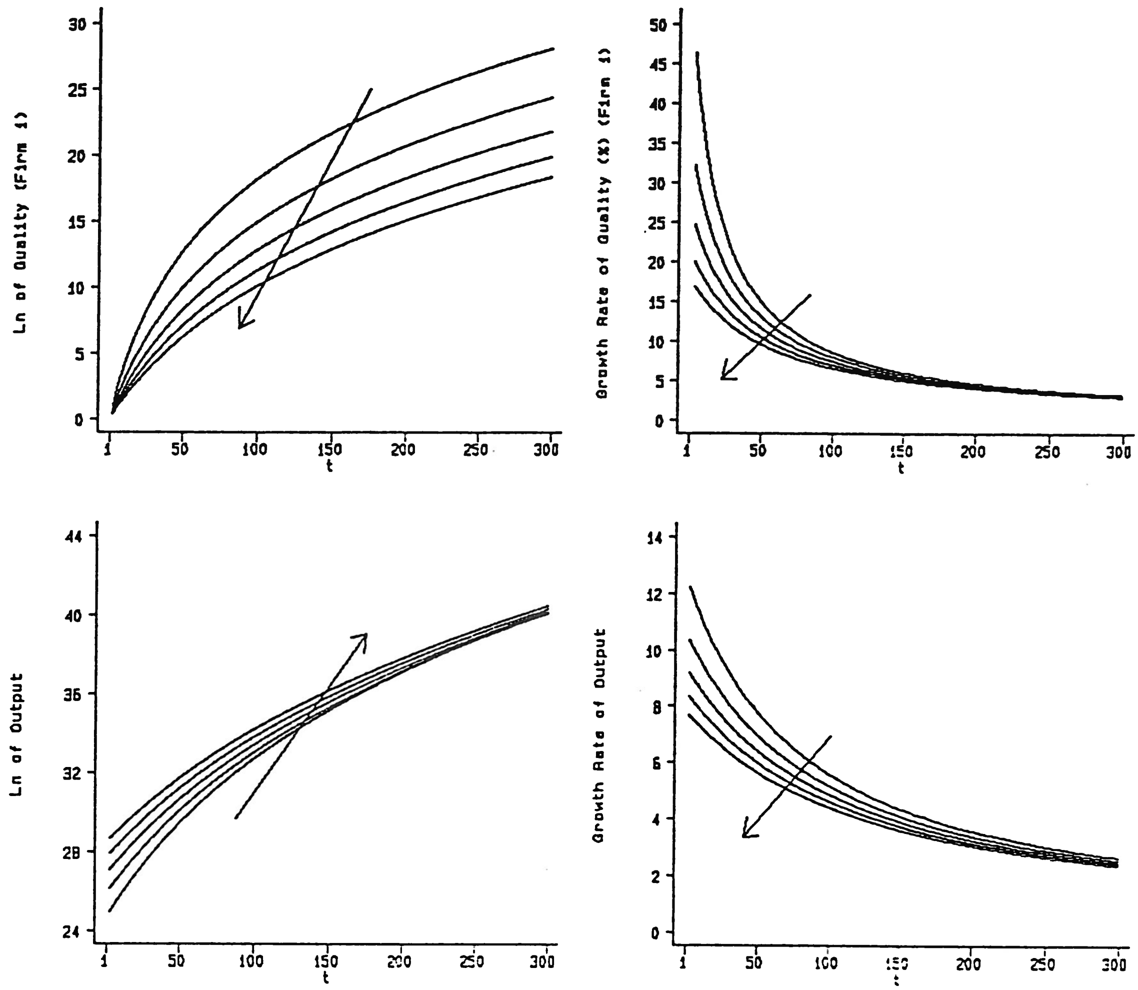

Figure 7: Number of Firms

II.3.3 Effect of Parameter b

Figure 8 depicts equilibrium levels and growth rates as the parameter determining the impact of own quality on profits varies, (b=.7,.8,.9,1.,1.1). These figures illustrate that higher values of this parameter are associated with higher short-run growth rates.

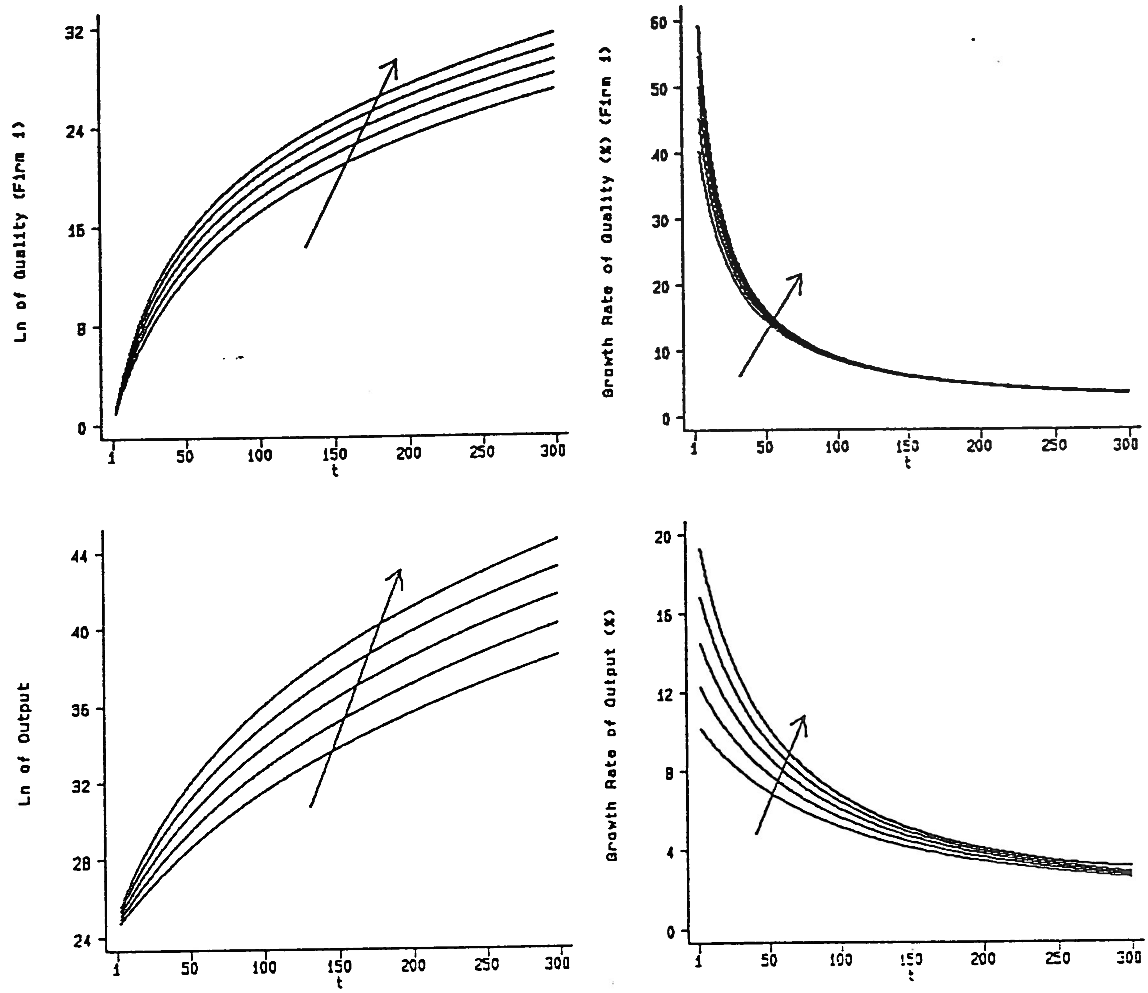

Figure 8: Size of b

III.3.3 Effect of Spillovers

As the example in section II demonstrated, in a symmetric equilibrium, the level of spillovers will not affect quality growth but will affect output growth. This results only because of the symmetric nature of intermediate goods producers. If firms are asymmetric, different levels of d will result in different equilibrium growth rates of qualities as well as of output. To highlight these asymmetries but avoid equilibria in which one firm decreases qualities, the parameters associated with the process used to generate innovation technology parameters are altered as follows:

$$\rho = .6 \qquad \sigma_\varepsilon^2 = 1.$$

This specification lowers the probability that a single firm will have a series of bad realizations relative to the other firm over a reasonable time horizon (300 periods). The higher variance leads to more prominent asymmetries among firms and highlights effects of different spillovers.

Even in this case, variations in d have only small effects on the growth rates of qualities in equilibrium. This results as averaging over different realizations of the innovation parameters averages out asymmetries across firms. Effects of different levels of spillovers are more apparent if we examine a single realization of the innovation technology parameters. In the experiment here, the differences are apparent graphically only after a number of periods from the starting period. The simulations indicate that there does not exist a monotonic relationship between the level of d and the equilibrium growth rate of qualities. However, changes in this parameter does affect the equilibrium path of qualities in this economy.

Figure 9 shows resulting equilibrium qualities and output when d varies from .3 to .75. For the particular realizations of θ in these simulations, higher d are associated with higher growth rates of qualities for one of the firms. However, closer examination of simulations indicate that this does not hold for all realizations of the innovation technology parameters. Output growth rates are consistently higher for economies with larger spillovers among qualities.

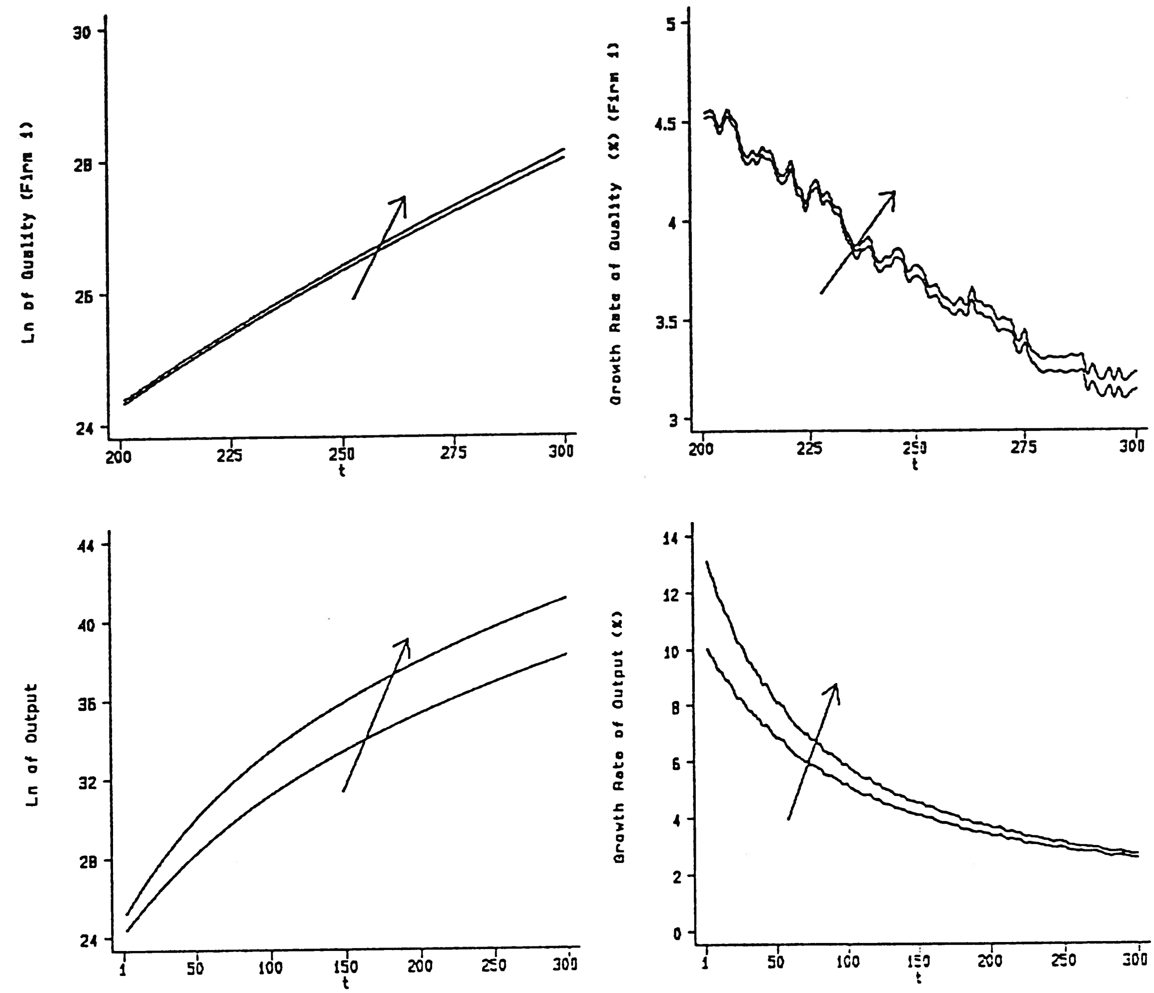

Figure 9: Size of Spillovers

III.3.5 Effect of the Mean of Innovation Technology Shocks

Figure 10 depicts equilibrium levels and growth rates as the mean of the innovation costs varies, (θ=10,20,30,40,50). These figures demonstrate that higher average innovation costs are associated with lower short-run growth rates.

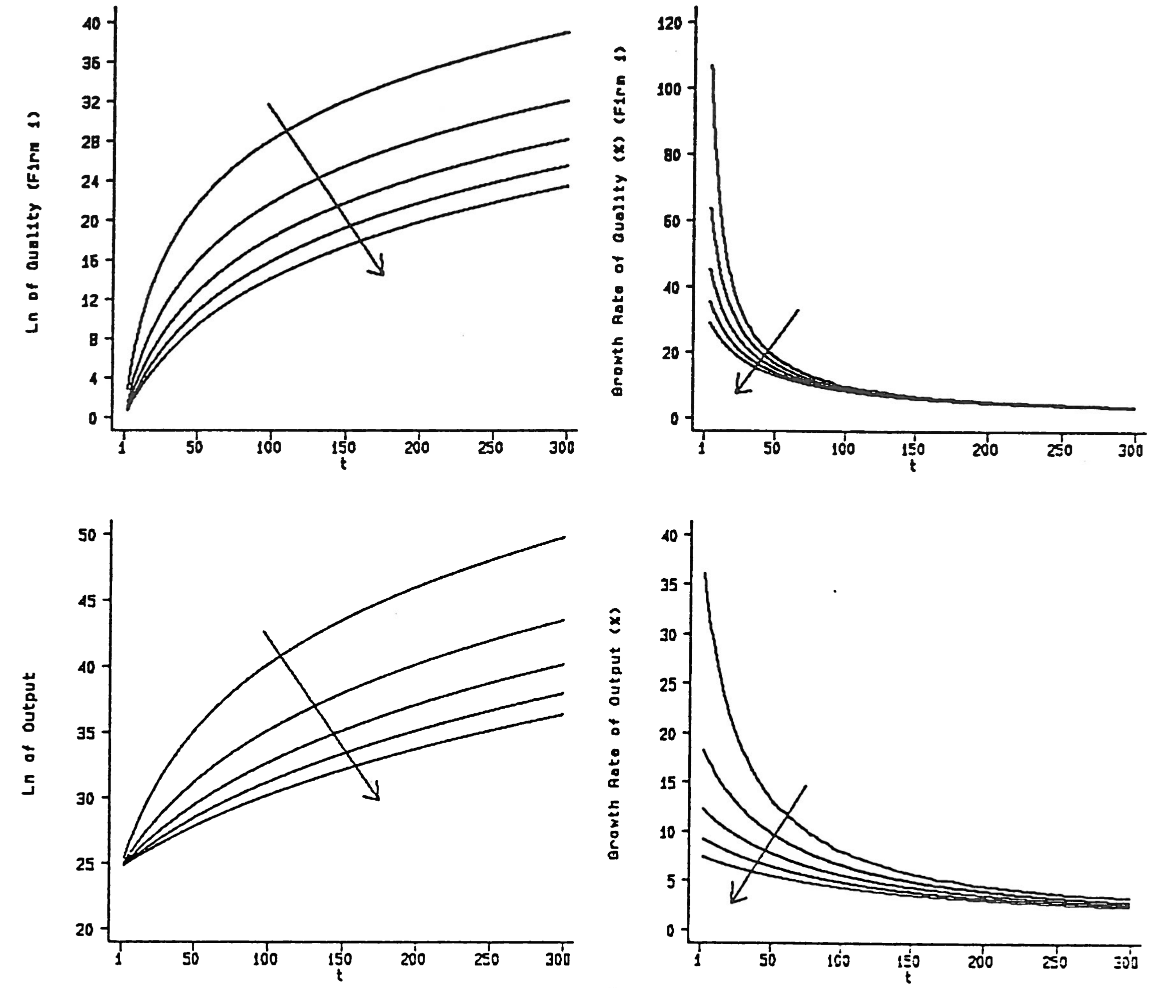

Figure 10: Mean of Technology Shocks

III.3.6: Summary

The computational experiments above indicate that the economy exhibits continual growth of output but with growth rates of output which asymptotically converge to zero. It should be noted that the reasons for convergence of growth rates in the long-run differs from those in the standard neo-classical growth model without exogenous productivity growth. In that model, output converges to a constant in the steady stage and growth ceases. In the economy examined in this section, output is unbounded but growth rates asymptotically converge to zero as output increases in absolute increments which fluctuate about a constant.

Furthermore, short-run growth rates of output are higher the longer the planning horizon of intermediate goods producers, the lower the number of intermediate goods (because of a fixed labor endowment), the higher the level of spillovers, and the lower the average cost of innovating. For certain parameters and series of innovation technology parameters, a firm may lag behind other firms and exit as production becomes unprofitable due to increasing costs of production. These equilibria were not considered in the above experiments but open interesting issues of exit due to obsolescence.

The economy studied in this section exhibits continual increases in output but long-run growth rates which asymptotically converge to zero. The question arises as to what plausible modifications can be made to this economy to generate equilibria with sustained growth. In the following section, two modifications are considered. In the first experiment, population growth is introduced leading to a constant long-run growth rate depending only on the rate of population growth. Short-run growth rates, however, are affected by economy parameters. In a second experiment, average quality affects productivity of labor in the intermediate goods sector. This economy exhibits constant long-run growth, the level of which depends on the parameters b and d, the resource base, and the mean of innovation costs.

III.4: Modifications

The economy described above exhibits growth rates which asymptotically converge to zero because the supply and productivity of labor is fixed. Increasing demand for labor as firms innovate leads to increasing wages and changes in quality which fluctuate around a constant. Increasing the supply of labor or its productivity will alleviate this effect and generate constant long run growth. Population growth is considered in section III.4.1 and endogenous increases in labor productivity are considered in section III.4.2.

III.4.1 Population Growth

Consider an economy characterized by the technology and preferences of Economy II but with constant population growth:

$$\forall t: \quad L_t = (1+\phi)^t L_0$$

In the computational experiment considered here economy parameters are set at the values in Table III.1 and ϕ=.02. As Figure 11 demonstrates, this economy exhibits quality growth rates which decrease to the rate of population growth. Output growth rates increase to a constant level which is determined by the number of intermediate goods producers and the rate of population growth.

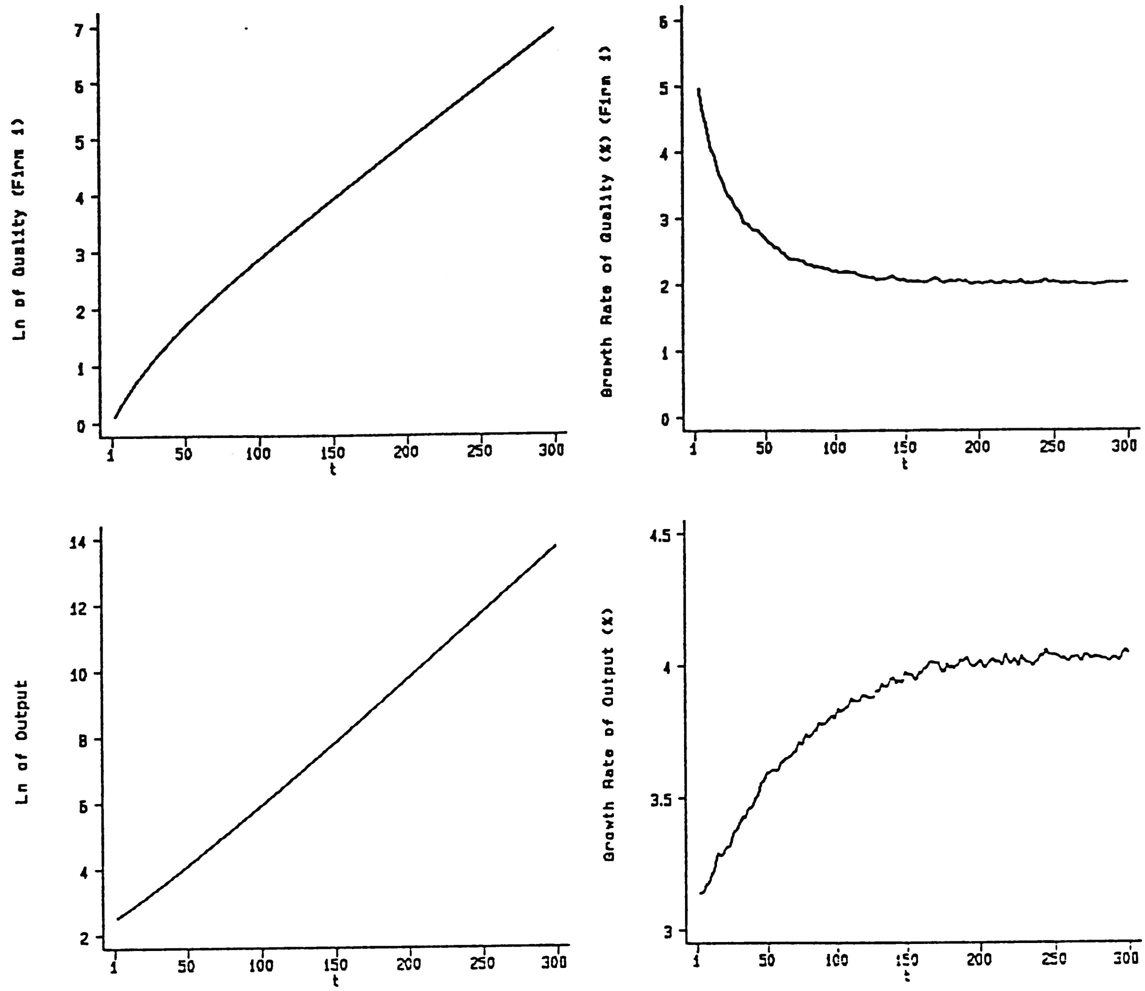

Figure 11: Population Growth

Other computational experiments were conducted which varied δ, N, b, d, and $\bar{\theta}$. As in the earlier version of the economy, short-run growth rates are affected positively by δ, N, b, and d and negatively by $\bar{\theta}$. Long-run growth rates are unaffected by these parameters.

III.4.2 Endogenous Labor Productivity

Consider an economy characterized by the preferences, endowments, and technologies for final goods production and innovation of Economy II. We modify the technology for producing intermediate goods as follows:

$$k_{jt} = \bar{q}_{t-1}\ell_{jt}$$

where $\bar{q}_{t-1}$ is average quality of intermediate goods at time t-1. For simplification, we assume that firms do not take into account their effect on average quality when making innovation decisions. With this modification, the productivity of labor evolves endogenously with innovation.

In the computational experiment considered here, economy parameters are set at their values in Table III.1 except d=.2 and b=.4. Even with lower values for these parameters, the economy exhibits exceptionally high growth rates. As Figure 12 illustrates, growth rate of qualities fluctuate about a constant while the growth rate of output increases to a level which fluctuates about a constant.

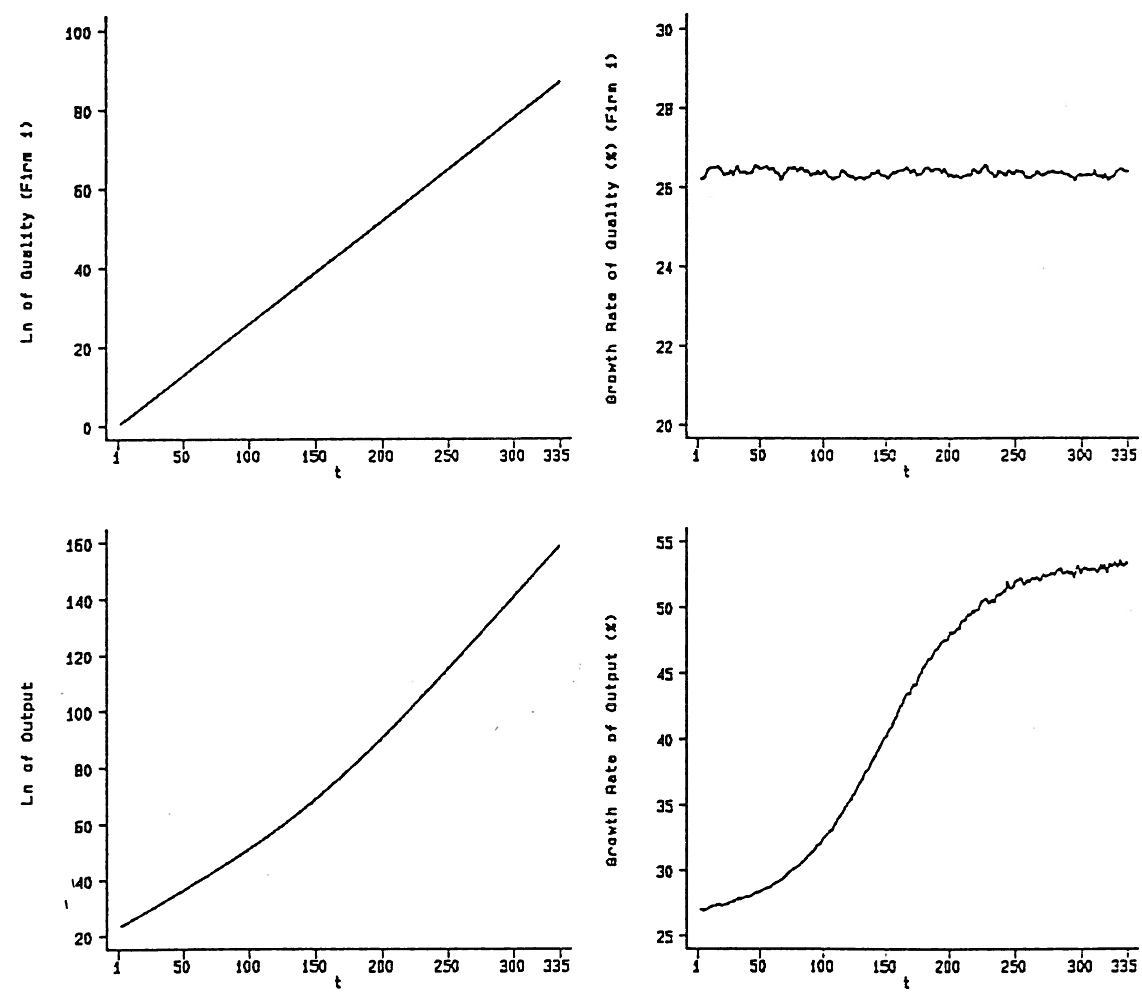

Figure 12: Effects of Average Quality

Other computational experiments, not presented here, indicate that δ, b, d, N, and $\bar{\theta}$ affect short- and long-run growth rates.

These examples help to illustrate why growth rates asymptotically converge to zero in Economy II. Without increases in the supply of labor or increases in its productivity, rising wages lead to declining growth rates.

III.5 Conclusions

The economy studied in this section exhibits continual output growth and long-run growth rates which, without modification, asymptotically converge to zero. Growth rates asymptotically converge to zero, not because growth ceases, but because output increases in increments which fluctuate around a constant. This results as firms innovation activities leads to increasing demands for labor and increasing wages. The length of firms planning horizons, the number of intermediate goods, the size of parameters on own and others qualities (spillovers), and the mean of innovation technology parameters affect short-run growth rates.

Two modifications were considered which alleviate the effects of increasing wages in this economy. Constant population growth and labor productivity which depends on average quality lead to positive long-run growth rates.

IV. CONCLUSIONS AND EXTENSIONS

Strategic innovation and its implications for economic growth were analyzed in two dynamic model economies. Both economies exhibit growth of output as profit maximizing firms continually improve the productivity of intermediate goods. In both economies there are spillovers from the quality of one intermediate good to the productivity of all others. Analysis of this particular type of spillover from innovative activity necessitates examining environments in which firms interact strategically.

In the first economy, where qualities are strategic complements, these spillovers lead to long-run growth rates that fluctuate around a constant. In this economy each firm unequivocally benefits from innovation by other firms as the only effect of the spillover is to make all intermediate goods more productive. In the second economy, innovation leads to rising factor costs. This effect implies that an intermediate good producer is harmed by the innovation activities of other firms and gives rise to an environment in which qualities which are strategic substitutes. This economy exhibits continual growth of output but growth rates asymptotically converge to zero.

An interesting feature of the second economy is that firms may choose to exit. Exit results in this economy as high cost firms innovate less, lag behind their competitors, and lose market share. A series of persistently high innovation costs may lead to firm exit. A planned extension of this paper includes analysis of equilibria in which exit and entry results from product innovation and obsolescence.

It is possible in these environments to examine the effects of heterogeneities in the length of planning horizons across firms. It has been conjectured in the popular press that differences in the length of firms' planning horizons across countries (e.g. U.S. vs. Japan) contribute to differences in growth rates across countries. It would be interesting to analyze these differences in open economy versions of the environments explored in this paper.

Although perfect foresight was assumed in the analysis in this paper, it is not necessary. Uncertainty can be incorporated by allowing innovation technology parameters to follow a finite state Markov process. It is conjectured that the major results of this paper would not be altered by the incorporation of uncertainty.

REFERENCES

Aghion, Philippe, and Peter Howitt, "A Model of Growth Through Creative Destruction," MIT Working Paper No. 527, 1989.

Bulow, J., J. Geanakopolos, and P. Klemperer, "Multimarket Oligopoly: Strategic Substitutes and Complements," Journal of Political Economy, 488-511, 1985.

Grossman, Gene M., "Explaining Japan's Innovation and Growth: A Model of Quality Competition and Dynamic Comparative Advantage," NBER Working Paper No. 3194, 1989.

_____, and Elhanan Helpman, "Comparative Advantage and Long-Run Growth," American Economic Review, September 1990, 796-815.

_____, Innovation and Growth, Forthcoming, the MIT Press, 1990.

_____, "Trade, Knowledge Spillovers, and Growth," Foerder Institute of Economic Research Working Paper No. 26-90, 1990.

_____, "Trade, Innovation, and Growth," Foerder Institute for Economic Research Working Paper No. 6-90, 1990.

_____, "Quality Ladders in the Theory of Growth," Foerder Institute of Economic Research Working Paper No. 32-89, 1989. Forthcoming in the Review of Economic Studies.

Helpman, Elhanan, "International Links of Innovation Rates," Foerder Institute of Economic Research Working Paper No. 25-90, 1990.

Krugman, Paul R., "Endogenous Innovation, International Trade and Growth," presented at the SUNY-BUffalo Conference on 'The Problem of Development,' 1988.

Lucas, Robert, "On the Mechanics of Economic Development," Journal of Monetary Economics, 1988, 3-42.

Romer, Paul M., "Increasing Returns and Long-Run Growth," Journal of Political Economy, 1986, 1002-1037.

_____, "Endogenous Technological Change," Forthcoming in the Journal of Political Economy, 1990.

Segerstrom, Paul S., T.C.A. Anant, and Elias Dinopoulos, "A Schumpeterian Model of the Product Life Cycle," American Economic Review, (80) December 1990, 1077-91.

Stokey, Nancy L., "Human Capital, Product Quality, and Growth," Center for Mathematical Studies in Economics and Management Science, Northwestern University Discussion Paper No. 883, 1990.

____, "Learning by Doing and the Introduction of New Goods," Journal of Political Economy, 1988, 701-717.