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The Use of Public Expenditure for Distributive Purpose

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Abstract

Governments typically use expenditures extensively as redistributive devices. Examples include the public provision of health, education, welfare, and public pensions. The purpose of this paper is to investigate the normative rationale for such policies. In particular, we study the role of government expenditures as purely redistributive devices given that the government also has available to it an optimal non-linear income tax. We do so in the context of specific types of quasi-private expenditures meant to represent education and pensions, both of which could have been provided privately. We assume that public provision to an individual cannot be related to individual characteristics or income, so it is uniform across individuals. We derive a set of sufficient conditions for the use of public expenditures in the presence of optimal taxes. The conditions are similar to those which would make subsidies to private provision welfare-improving. Subsidization and public provision appear to be substitute policies. Which one would be preferable depends upon the global characteristics of the economy.

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by

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I. Introduction

Governments pursue redistributive goals through a wide variety of instruments including taxes, transfers and public expenditures. Much of the economic analysis of redistribution has stressed the role of taxes and transfers in achieving equity objectives, neglecting that of public expenditures. Yet, one of the lessons to be learned from the optimal income tax literature is that there are limits to the amount of redistribution that can be achieved by progressive taxation.[1] This is true even in the absence of tax evasion which presumably limits the possibilities of redistribution even further. It could be argued that governments recognize this and accomplish much of their redistribution through the expenditure side of the budget. For example, many countries provide public education, health services, unemployment insurance, welfare services and pensions on a universal basis to their citizens. It would be difficult to justify such massive public sector intervention solely on efficiency grounds. If anything, the private sector may have an advantage over the public sector at providing such services since many of them are essentially private in nature, except to the extent that they generate externalities. But these public services also have a sizable redistributive component to them, and it could be argued that public provision is ultimately driven heavily by this redistribution. The purpose of this paper is to investigate whether the use of government expenditures for redistributive purposes alone can be justified when the government has also available to it a general non-linear income tax, given that it cannot observe individual abilities and so cannot impose lump-sum taxes and transfers. We do so in the context of specific types of expenditures meant to be representative of education and pensions, though other types of expenditures could be captured by our analysis as well.

The redistributive component of government expenditures has been recognized in the literature. Two different strands exist — one in which the public expenditure is a quasi-private good provided in equal amounts to all persons, the other in which the expenditure can be earmarked to different persons. The seminal analysis of the effects of uniform public provision was by Usher (1977). He considered a population of persons who had identical tastes but differed by incomes. The government, guided by a median voter voting rule, had to determine how much of a quasi-private good to provide through the public sector on a uniform basis to all, financed by a proportional tax. Given that the good in question was normal, the median voter was the median income person, who, in turn had less than the mean income given

[1] See Mirrlees (1971) and Roberts (1984).

the assumption that the exogenously-given income distribution was skewed toward the lower end. The median voter outcome generated redistribution towards the lower income persons, as well as generating a non-preferred level of output of the quasi-private good for all but the median voter, and thus a deadweight loss.[2]

A similar model was used recently by Besley and Coate (1989) with some important differences. One was that the quasi-private good being provided was defined to be fixed in quantity, but not in quality. The public sector provided a uniform quality of the good to all. This was less than the quality that the higher income persons would have chosen. Another difference was that persons had the option to acquire the good on their own at their preferred quality level. However, by the nature of the good (e.g. health, education) they could only consume one type. Thus, there would be some income level beyond which all persons would choose to substitute their own quality for that provided by the government and forgo the freely available public provision entirely. Besley and Coate investigate whether uniform provision by the public sector would improve social welfare. They find that this is possible, but that optimal public provision entails welfare costs. The quality provided exceeds that which the low income persons prefer, but is less than that preferred by the high. The high income persons would prefer to forgo the free public service and purchase a higher quality on their own. In this way, redistribution is achieved, but at a cost.

In both Usher and Besley-Coate, the use of the tax system for redistribution is assumed away. Given that individual incomes are exogenous and the redistribution via expenditures generates some inefficiency, redistributive taxation would seem to dominate expenditures as devices for redistribution. Besley and Coate recognize this, but are essentially analyzing a situation that exists in less developed countries where redistributive taxation systems are not so well established because of administrative reasons. They suggest that a case for public expenditures might be made if labour supplies were variable, and if leisure and the public service were not separable. This follows directly from an argument by Hylland and Zeckhauser (1979) intended to provide a normative rationale for using government expenditures for redistributive purposes.

The second strand of literature was initiated by Arrow (1971) who investigated fully the normative characteristics of redistribution using targeted government expenditures. Households are assumed to be distributed by a characteristic x and to obtain utility $U(x,y)$ where y is government expenditure and $U_x, U_y > 0$ and $U_{yy} < 0$. Utility of income is abstracted from entirely (e.g., by separability). The government can observe x and can provide a differing amount of expenditure y to each person. It adopts a utilitarian social welfare function so that optimal policy is characterized by equalizing U_y across all persons. Arrow studies whether optimal

[2] The Usher analysis was applied in a fiscal federalism context by Wilson and Katz (1983) who used it to justify why certain types of public expenditures were initiated first at the lower level of government. The argument was based on differing patterns of income distribution at the state level compared with the national level.

policy is *input-progressive* ($dy/dx < 0$), or not.[3] He also defines *output-progressive* policy as one which results in $dU/dx < 0$, and vice versa, and studies conditions for output progressivity and regressivity. He suggests that education is characterized by a correlation of ability x with the securing of benefits from expenditures at the margin and in total.[4] This yields output regressivity. However, health expenditures are the opposite. Here x is the state of health, and expenditures will be less productive of increased utility for a healthier individual[5] so output progressivity is optimal.

As mentioned, Arrow assumes implicitly that ability or health status is observable so that the optimal expenditure policy can be implemented. Given this assumption, tax policy would dominate expenditure policy. That is, if the government could use the information at its disposal to make income transfers among households, it should do so and allow households to purchase whatever quantities of goods y they desired. In that sense, Arrow does not establish the normative basis for using expenditures for redistributive purposes. Instead, he assumes it.[6]

Our purpose is to investigate whether the use of expenditure policies can be justified on redistributive grounds. We do so in the context of a model in which public expenditures have similar properties to those in Arrow, but, in the spirit of the optimal income tax literature, the government does not have full information about individual characteristics. We allow the government to implement the optimal income tax in the sense of Mirrlees (1971), and ask whether social welfare could be improved by instituting universal provision of some service by the government. Given our assumption of non-observability of individual characteristics, government expenditures must be provided in the same amount for all. Two prototypical types of government expenditures are considered, both of which are quasi-private goods.[7] The first, which might correspond with education, allows the good to interact with ability and affect the wage rate received by the household. The second is simply the public provision of a quasi-private good which enters the utility function directly and cannot be resold. Examples of this might include public pensions and health care.

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3 It will be input-progressive iff $U_{xy} < 0$, and vice versa.

4 That is, $V_{ux} < 0$, where $V(u,x)$ is the inverse of $U(x,y)$.

5 i.e., one for which $V_{ux} > 0$.

6 Bruno (1976) has extended Arrow's analysis by allowing public expenditures to be financed by a progressive income tax. However, the source of variability in income is government expenditure itself rather than variable factor supplies. Furthermore, though the government can observe x for the purposes of selecting government expenditures on each household, the option of allowing the tax to depend directly on x is not considered. If so, progressive taxation combined with private purchase of y would dominate public provision. Besley (1989) has also used the Arrow model to investigate whether user charges should be implemented for the publically-provided public good. As with Bruno, he assumes that, although the government has enough information to implement the optimal provision of y to each household, it cannot charge taxes differentially by household. Instead it implements optimal linear commodity taxes and a common user charge on y. However, since households cannot vary y, the user charge plays no allocative role.

7 These were also the two examples used by Arrow.

In the former case, the good provided by the government is future consumption. The latter case it is health care. In each case, households are allowed to supplement public expenditures with their own private provision.

In the optimal income tax problem underlying our analysis, we consider a finite number of household types, following Stiglitz (1982), Guesnerie and Seade (1982) and Stern (1982). In these models, the self-selection constraint restricts the amount of redistribution that can be achieved. Government expenditures will be social-welfare-improving to the extent that they serve to relax the self-selection constraint. Our analysis will be aimed at determining when that is possible. We also consider whether an alternative instrument, subsidies for private provision, should be used in addition to, or instead of, public provision. The spirit of the analysis is similar to that of Blackorby and Donaldson (1988) who investigated public provision in a slightly different context. They considered the case for in-kind provision of public services in an example of a two-person economy in which the persons had differing tastes (needs) for the good provided. Public provision screened those who needed the good most and thereby provided a form of redistribution which could not be achieved by cash transfers. More generally, Guesnerie and Roberts (1984) have argued the case for quantitative restrictions (rationing) as policy devices in a second-best world of imperfect information with optimal taxes. They applied the principle to the case of minimum wages in Guesnerie and Roberts (1987). Our analysis, as well as that of Blackorby and Donaldson, could also be taken to be applications of that principle.

The interaction of optimal income taxation and education decisions has been analyzed in models similar to ours by Hare and Ulph (1979) and Tuomala (1990). As Arrow did, Hare and Ulph assume that ability to benefit from education can be observed by the government for the purposes of allocating education expenditures among households. However, this same information cannot be used for determining taxes. Their purpose is to examine whether introducing optimal income taxation in Arrow's model makes public provision of education less input- and output-regressive. In the present paper, we depart from Hare and Ulph's asymmetric treatment of information available to the government by assuming that public provision of education cannot be related to an individual's ability or income. In a sense this stacks the deck against the use of public expenditures as redistributive devices and thus strengthens our results. Tuomala focusses on the other side of the coin, that is, how education choices affect the progressivity of the optimal income tax. As in our analysis, one of his models considers the optimal choice of a uniform provision of public education, but he does not look for the circumstances which makes public provision desirable in the first place. Nor does he allow for the possibility of private expenditures as a supplement to public provision. This prevents him from being able to consider simultaneously a subsidy on those expenditures.

The analysis of optimal taxation in our context requires a modified version of the self-selection approach as presented by Stiglitz. The following section sets up the basic self-selection model used for the education case. Then, public expenditure and subsidy policies are analyzed given that taxes are set optimally. The analysis is then applied to the other type of expenditures. Throughout, a two-person economy is used since the principles can be analyzed most clearly in that context.

II. The Self-Selection Model of Public Expenditures and Optimal Income Taxation: The Case of Education

The analysis will be conducted using the standard optimal income tax assumptions as devised by Mirrlees. Persons will have identical utility functions $u(x_i, l_i)$ where x_i is consumption and l_i is labour supplied by household i for $i = 1, 2$. Households differ only in their exogenously-given ability with 2 being the high-ability person. In the standard optimal income tax problem, ability is normalized to equal the wage rate. Here, we make the wage rate for a person of given ability endogenous and dependent on education expenditures, with the high-ability person getting a higher wage from given education expenditures than a low-ability person.. Let z_i be education expenditures made privately by household i and let g be education expenditures provided uniformly by the public sector, where $g, z_i \geq 0$. Note that we allow households to supplement public with private provision. For example, public provision could be associated with mandatory schooling and private provision with further education. As mentioned, the government is unable to observe ability, so public education expenditures cannot be conditional on ability. Also, we assume that they cannot be conditional on incomes either, which the public sector can observe. For example, the planner may have to commit to such expenditures before knowing the incomes that will be produced. We assume there is a wage function for each person given by $w_i(g + z_i)$ where $w_i' > 0$. A different wage function applies for persons of different ability such that $w_2(g + z_i) > w_1(g + z_i)$. Other relationships between the wage functions of high and low ability persons are critical to the case for using public expenditures for redistributive purposes, so we leave the possibilities until discussing our results. The production technology is linear, and units of labour are normalized such that w_i equals the wage rate.

If the government could observe abilities, it would be possible to reach any point on the utility frontier through appropriate lump-sum transfers. Private education expenditures would be first best and there would be no reason for the government to provide public education. However, we adopt the standard assumption that the government cannot observe abilities, wage rates or labour supplies directly, but can observe total labour incomes. Consequently, they choose an income tax function $\Gamma(w_i l_i)$, which can take any general shape. It turns out to be useful for our purposes to linearize the income tax schedule for each household at the equilibrium point by defining a *virtual* budget constraint.[8] Thus, we define after-tax virtual income for the household to be $\tau_i w_i l_i - T_i$ where τ_i is one minus the marginal tax rate and T_i is the lump-sum component. The virtual tax parameters τ_i and T_i are defined implicitly with respect to the preferences of the household as in Stiglitz (1982). As he points out, optimal marginal tax rates are likely to be discontinuous so these tax parameters are virtual only. Because the income tax schedule is non-

[8] The procedure is analogous to that used in the empirical labour supply literature. See, for example, Hausman (1985).

linear, they can vary across households. Those parameters together with public education expenditures are chosen by the government. As in Stiglitz, it is inefficient to have a pooling equilibrium in which the two types of individuals have the same labour incomes. Instead, the government uses its policy instruments to implement a separating equilibrium. The equilibrium must satisfy an appropriate self-selection constraint. Before introducing it, we derive some properties of household indirect utility functions that are used elsewhere.

The household budget constraint is:

$$x_i + z_i = \tau_i w_i l_i - T_i.$$

The problem of the household can be written as:

$$\max_{l_i, z_i} \; u\left[\tau_i w_i(g + z_i) l_i - T_i - z_i, l_i\right] \tag{1}$$

where the budget constraint has been substituted directly into the utility function. The first-order conditions may be written:

$$\begin{gathered} \alpha_i \tau_i w_i + u_l^i = 0 \\ w_i' \tau_i l_i \leq 1; \quad z_i(w_i' \tau_i l_i - 1) = 0 \end{gathered} \tag{2}$$

where $\alpha_i = u_x^i$ is the marginal utility of income for household i. The solution to the household utility maximization problem yields supply-of-labour and demand-for-education functions $l_i(\tau_i, T_i, g)$ and $z_i(\tau_i, T_i, g)$ which, when substituted into the utility function, give the indirect utility function $v^i(\tau_i, T_i, g)$. Using the envelope theorem, we obtain:

$$\begin{aligned} v_\tau^i &= \alpha_i w_i l_i, \\ v_T^i &= -\alpha_i. \\ v_g^i &= \alpha_i w_i' \tau_i l_i. \end{aligned} \tag{3}$$

For the purposes of analysis, we proceed in two steps. In the first step, the planner chooses an income tax schedule optimally, given the level of government expenditures per person g. The second step involves evaluating changes in g with taxes set optimally. For the optimal income tax problem, the planner chooses τ_i and T_i for $i = 1, 2$ to maximize a social welfare function subject to a self-selection constraint. In our analysis we use a utilitarian social welfare function, though any quasi-concave social welfare function would do. The self-selection constraint effectively rules out the high income person mimicking the income of the low income person.[9] It is important to note that while household 2 mimics household 1's income, labour supply and educational expenditures will differ for the two households. The mimicking problem for household 2 is:

$$\underset{\bar{z}_2}{\text{Max}} \quad u\left[\tau_1 w_1 l_1 - T_1 - \bar{z}_2, \frac{w_1 l_1}{w_2(g+\bar{z}_2)}\right]$$

where $\bar{z}_2$ is household 2's private educational expenditures and $w_1 l_1$ is taken as given by (2). A "bar" will always refer to variables when 2 is mimicking 1's income. Note that $\bar{l}_2 = w_1 l_1/\bar{w}_2$. The first order condition for this problem is:

$$\bar{\alpha}_2 + \frac{\bar{u}_l^2}{\bar{w}_2}\bar{w}_2'\bar{l}_2 \geq 0; \quad \bar{z}_2\left(\bar{\alpha}_2 + \frac{\bar{u}_l^2}{\bar{w}_2}\bar{w}_2'\bar{l}_2\right) = 0. \tag{4}$$

The solution to this problem gives the indirect utility function $\bar{v}(\tau_1, T_1, g, w_1 l_1)$, where $w_1 l_1$ itself is a function of government policy variables. From the envelope theorem,

$$\begin{aligned}
\bar{v}_\tau &= \bar{\alpha}_2\left(w_1 l_1 + \tau_1 \frac{\partial(w_1 l_1)}{\partial \tau_1}\right) + \frac{\bar{u}_l^2}{\bar{w}_2}\frac{\partial(w_1 l_1)}{\partial \tau_1} \\
\bar{v}_T &= \bar{\alpha}_2\left(\tau_1 \frac{\partial(w_1 l_1)}{\partial T_1} - 1\right) + \frac{\bar{u}_l^2}{\bar{w}_2}\frac{\partial(w_1 l_1)}{\partial T_1} \\
\bar{v}_g &= \bar{\alpha}_2\left(\tau_1 \frac{\partial(w_1 l_1)}{\partial g}\right) + \frac{\bar{u}_l^2}{\bar{w}_2}\left(\frac{\partial(w_1 l_1)}{\partial g} - \bar{l}_2\bar{w}_2'\right).
\end{aligned} \tag{5}$$

Suppose there are n_i persons of type i. The planner's problem is to maximize the sum of utilities subject to the self-selection constraint $v^2(\tau_2, T_2, g) \geq \bar{v}(\tau_1, T_1, g)$ and a government revenue constraint. Assuming that the only revenue requirement is to finance g, the Lagrangian expression may be written:

$$\begin{aligned}
\Omega(\tau_i, T_i, \mu, \gamma; g) = {} & n_1 v^1(\tau_1, T_1, g) + n_2 v^2(\tau_2, T_2, g) + \mu\left[v^2(\tau_2, T_2, g) - \bar{v}(\tau_1, T_1, g)\right] \\
& + \gamma\left[n_1\left((1-\tau_1)w_1 l_1 + T_1 - g\right) + n_2\left((1-\tau_2)w_2 l_2 + T_2 - g\right)\right]
\end{aligned} \tag{6}$$

where $w_i l_i$ is a function of τ_i, T_i and g. The first order conditions are:

$$n_1 \alpha_1 w_1 l_1 - \mu\left[\bar{\alpha}_2\left(w_1 l_1 + \tau_1 \frac{\partial(w_1 l_1)}{\partial \tau_1}\right) + \frac{\bar{u}_l^2}{\bar{w}_2}\frac{\partial(w_1 l_1)}{\partial \tau_1}\right]$$

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[9] Stiglitz (1982) analyzes the efficiency of optimal income taxes and allows the self-selection constraint to apply to either individual depending on the circumstances. Since we are considering the case in which the planner wants to redistribute from the high to the low income person, applying the self-selection constraint to the high income person is appropriate. The low income person would never choose to mimic the high. This is what Stiglitz refers to as the "normal" case.

$$+\gamma n_1 \left[-w_1 l_1 + (1-\tau_1)\frac{\partial(w_1 l_1)}{\partial \tau_1}\right] = 0 \qquad (\tau_1)$$

$$-n_1\alpha_1 - \mu\left[\bar{\alpha}_2\left(\tau_1\frac{\partial(w_1 l_1)}{\partial T_1} - 1\right) + \frac{\bar{u}_l^2}{\bar{w}_2}\frac{\partial(w_1 l_1)}{\partial T_1}\right]$$
$$+\gamma n_1 \left[(1-\tau_1)\frac{\partial(w_1 l_1)}{\partial T_1} + 1\right] = 0 \qquad (T_1)$$

$$(n_2+\mu)\alpha_2 w_2 l_2 + \gamma n_2 \left[-w_2 l_2 + (1-\tau_2)\frac{\partial(w_2 l_2)}{\partial \tau_2}\right] = 0 \qquad (\tau_2)$$

$$-(n_2+\mu)\alpha_2 + \gamma n_2 \left[(1-\tau_2)\frac{\partial(w_2 l_2)}{\partial T_2} + 1\right] = 0. \qquad (T_2)$$

These conditions can be simplified considerably and rewritten in a way which will prove useful below. Combining the equations for τ_2 and T_2, we obtain,[10]

$$1 - \tau_2 = 0. \qquad (7)$$

This is the usual optimal income tax result that the marginal tax rate on the highest wage person is zero. Using (7), the condition on T_2 reduces to:

$$\gamma n_2 - (n_2+\mu)\alpha_2 = 0. \qquad (8)$$

From the conditions on τ_1 and T_1 we obtain:[11]

$$\gamma n_1(1-\tau_1) = \mu\left(\bar{\alpha}_2\tau_1 + \frac{\bar{u}_l^2}{\bar{w}_2}\right). \qquad (9)$$

Since the multipliers μ and γ are both positive, this implies that the marginal tax rate $(1-\tau_1)$ has the same sign as the derivative of $\bar{u}^2$ with respect to $w_1 l_1$ (that is, the expression in parentheses on the right-hand side of (9)). This is quite intuitive. Take the case where the derivative is positive. An income-compensated increase in the tax rate $1-\tau_1$ then weakens the incentive-compatibility constraint because it

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[10] Multiplying (T_2) by $w_2 l_2$ and adding to τ_2 yields:

$$(1-\tau_2)\left[\frac{\partial(w_2 l_2)}{\partial \tau_2} + w_2 l_2\frac{\partial(w_2 l_2)}{\partial T_2}\right] = 0$$

where the expression in brackets is the income-compensated effect on labour income of a change in τ_2. It normally differs from zero.

[11] Multiplying (T_1) by $w_1 l_1$ and adding to (τ_1) gives:

$$\left[\gamma n_1(1-\tau_1) - \mu\left(\bar{\alpha}_2\tau_1 + \frac{\bar{u}_l^2}{\bar{w}_2}\right)\right]\left[\frac{\partial(w_1 l_1)}{\partial \tau_1} + w_1 l_1\frac{\partial(w_1 l_1)}{\partial T_1}\right] = 0.$$

reduces $w_1 l_1$ and so makes worse off household 2 mimicking household 1. Therefore, it is optimal in this case to have a positive tax rate. It can be shown[12] that the derivative of $\bar{u}^2$ is positive if x is a normal good and if $\bar{z}_2 \geq z_1$. The latter will be satisfied, for example, if $\bar{z}_2$ and z_1 are both crowded out by g ($\bar{z}_2 = z_1 = 0$). Those are only sufficient conditions.[13] Using (9), the condition on T_1 reduces to:

$$n_1(\gamma - \alpha_1) + \mu\bar{\alpha}_2 = 0. \tag{10}$$

Consider now the welfare effects of changing g while maintaining taxes at their optimal values (and therefore satisfying (7)–(10)). Differentiating the Lagrangian expression (6) (partially, because of the envelope theorem) with respect to g and using (7)–(10) gives:

$$\frac{d\Omega}{dg} = n_1\alpha_1\left(\tau_1 w_1' l_1 - 1\right) + (n_2 + \mu)\alpha_2\left(w_2' l_2 - 1\right) + \mu\left(\bar{\alpha}_2 + \frac{\bar{u}_l^2}{\bar{w}_2}\bar{w}_2'\bar{l}_2\right). \tag{11}$$

From this we can readily infer the circumstances in which increases in g will be welfare-improving. Note first that, from the first order conditions for the households, if $z_1, z_2, \bar{z}_2 > 0$, $d\Omega/dg = 0$. In this case, the change in g is inframarginal to everyone and is equivalent to an equal lump sum transfer. Thus, the effect of g can be replicated by the tax system so g is redundant. However, as g increases, private provision z_i falls and eventually z_i will fall to zero. Different persons will become crowded out at different levels of g and, depending on the order of crowding out, the welfare consequences will differ.

To see this, note that when persons 1 and 2 become crowded out, the first two terms in (11) change from zero to becoming negative by (2). When 2 gets crowded out when mimicking 1 ($\bar{z}_2 = 0$), the last term in (11) becomes positive rather than zero by (4). Denote by $g_1, g_2, \bar{g}_2$ the levels of g at which persons 1 and 2 and mimicking person 2 just become fully crowded out by public provision. Then the following result is apparent:

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12 Proof: Suppose first that $\bar{z}_2 = z_1$. Since household 2 when mimicking household 1 supplies less labour but consumes the same amount as household 1,

$$-\frac{\bar{u}_l^2}{\bar{\alpha}_2} < -\frac{u_l^1}{\alpha_1} = \tau_1 w_1$$

assuming x is a normal good. Therefore, $\bar{\alpha}_2\tau_1 + \frac{\bar{u}_l^2}{w_1} > 0$. Since $\bar{w}_2 > w_1$, this implies that $\bar{\alpha}_2\tau_1 + \frac{\bar{u}_l^2}{\bar{w}_2} > 0$. In other words, the derivative of $\bar{u}^2$ with respect to $w_1 l_1$ is positive. Now take the case where $\bar{z}_2 > z_1$. This will decrease the value of $-\bar{u}_l^2/\bar{\alpha}_2$ (because of the quasi-concavity of the utility function) and increase that of $\bar{w}_2$. Both these changes will keep the inequality satisfied.

13 In a model similar to ours, Tuomala (1990) also obtains the result that the sign of the marginal tax rate is ambiguous.

Proposition 1. If $\bar{g}_2 < g_1, g_2$, then public spending will be welfare-improving up to some amount strictly above $g = \min(g_1, g_2)$.

The intuition behind this result is as follows. Increasing g beyond the point at which person 1 or person 2 gets crowded out simply makes those persons worse off by constraining their choice, and thus reduces social welfare. However, pushing g beyond the point at which $\bar{z}_2$ is crowded out makes the mimicking person worse off and therefore relaxes the self-selection constraint. If the latter occurs before persons 1 and 2 are crowded out, a gain in social welfare can be achieved.

We can derive the conditions under which the mimicking person gets crowded out before either person 1 or person 2. First, $\bar{g}_2$ will be less than g_2 if x is a normal good.[14] The real issue, however, is whether $\bar{g}_2$ is greater or less than g_1. The following result can be proven: if x is a normal good, $\bar{g}_2 < g_1$ if $\epsilon_1(\bar{g}_2) \geq \epsilon_2(\bar{g}_2)$ where ϵ_i is the elasticity of the wage function.[15] Therefore, we obtain:

Proposition 2. If $\epsilon_1 \geq \epsilon_2$, and if x is a normal good, then public spending will be welfare-improving up to some amount strictly above $g = \min(g_1, g_2)$.

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14 Proof: Suppose the mimicking person is just crowded out so $g = \bar{g}_2$. Accordingly, $\bar{z}_2 = 0$ and (4) is satisfied with equality, i.e., $\frac{-\bar{u}_l^2}{\bar{\alpha}_2}\frac{\bar{w}_2'\bar{l}_2}{\bar{w}_2} = 1$. We shall prove by contradiction that if x is a normal good, $z_2 > 0$. Let us suppose that household 2's optimum is such that $z_2 = 0$. Then, $w_2'l_2 \leq 1$, or

$$w_2'l_2 \leq -\frac{\bar{u}_l^2}{\bar{\alpha}_2}\frac{\bar{w}_2'\bar{l}_2}{\bar{w}_2}.$$

Since $\bar{z}_2 = z_2 = 0$, we have $\bar{w}_2 = w_2$ and $\bar{w}_2' = w_2'$. Also for the incentive-compatibility constraint we must have $w_2 l_2 > w_1 l_1$, which implies $l_2 > \bar{l}_2$ since $w_1 l_1 = \bar{w}_2 \bar{l}_2$. Therefore, the above inequality implies $w_2 \leq -\bar{u}_l^2/\bar{\alpha}_2$, or

$$-\frac{u_l^2}{\alpha_2} \leq -\frac{\bar{u}_l^2}{\bar{\alpha}_2}$$

which contradicts the hypothesis that x is a normal good.

15 Proof: As in the previous footnote, suppose the mimicking person is just crowded out ($g = \bar{g}_2$). We show by contradiction that under the stated hypotheses, $z_1 > 0$. Suppose that $z_1 = 0$, which implies that $\tau_1 w_1' l_1 \leq 1$. Using the first order conditions on l_1, we obtain:

$$\frac{-u_l^1}{\alpha_1 w_1} w_1' l_1 \leq 1 = \frac{-\bar{u}_l^2}{\bar{\alpha}_2}\frac{\bar{w}_2'\bar{l}_2}{\bar{w}_2}.$$

We know that $\bar{l}_2 < l_1$, and we have by hypothesis that $\epsilon_1 \geq \epsilon_2$. Therefore, the above inequality implies

$$\frac{-u_l^1}{\alpha_1} \leq \frac{-\bar{u}_l^2}{\bar{\alpha}_1}$$

which cannot be satisfied if x is a normal good.

Once either 1 or 2 gets crowded out, further increases in g will have conflicting effects. Increases in g will continue to weaken the self-selection constraint thereby improving welfare (the third term in (11)), but it will worsen welfare as 1 or 2 is forced to consume more education than is desired. The optimal level of public education is that which equates these two effects at the margin.

III. Subsidizing Education

The optimal income tax allows only for taxing households. It is well-known that in a multi-commodity world it may be desirable to tax commodities differentially as well. In this context, a subsidy (tax) on education is like differential commodity taxation, although education enters only indirectly into the utility function through the wage function. In this section we investigate the case for subsidizing the purchase of private education as an alternative policy instrument, again assuming that optimal income taxation is in place. The analysis is done first for a given level of government expenditures g, and then the effect of changing g is considered given that the size of the subsidy has been chosen optimally.

Let the subsidy rate be $1-\sigma$ per unit of education purchased. Then, the consumer price of education is σ. The problem of household i becomes:

$$\max_{l_i,\, z_i} \; u\left[\tau_i w_i(g+z_i)l_i - T_i - \sigma z_i, l_i\right]. \tag{1.s}$$

The first-order conditions may be written:

$$\alpha_i \tau_i w_i + u_l^i = 0$$
$$w_i' \tau_i l_i \leq \sigma; \quad z_i(w_i' \tau_i l_i - \sigma) = 0. \tag{2.s}$$

This gives the indirect utility function $v^i(\tau_i, T_i, \sigma, g)$. Using the envelope theorem, we obtain equations (3) as before plus:

$$v_\sigma^i = -\alpha_i z_i. \tag{3.s}$$

The mimicking problem for household 2 is:

$$\max_{\bar{z}_2} \; u\left[\tau_1 w_1 l_1 - T_1 - \sigma \bar{z}_2, \frac{w_1 l_1}{w_2(g+\bar{z}_2)}\right]$$

where $w_1 l_1$ is taken as given by 2. The first order condition for this problem is:

$$\bar{\alpha}_2 \sigma + \frac{\bar{u}_l^2}{\bar{w}_2} \bar{w}_2' \bar{l}_2 \geq 0; \quad \bar{z}_2\left(\bar{\alpha}_2 \sigma + \frac{\bar{u}_l^2}{\bar{w}_2} \bar{w}_2' \bar{l}_2\right) = 0. \tag{4.s}$$

The solution to this problem gives the indirect utility function $\bar{v}(\tau_1, T_1, \sigma, g, w_1 l_1)$, where $w_1 l_1$ itself is a function of government policy variables. From the envelope theorem, we obtain the same expressions for $\bar{v}_\tau$, $\bar{v}_T$ and $\bar{v}_g$ as in (5) as well as:

$$\bar{v}_\sigma = \bar{\alpha}_2 \left(\tau_1 \frac{\partial (w_1 l_1)}{\partial \sigma} - \bar{z}_2 \right) + \frac{\bar{u}_l^2}{\bar{w}_2} \frac{\partial (w_1 l_1)}{\partial \sigma}. \tag{5.s}$$

Taking account of the revenue needed to finance the subsidy, the Lagrangian expression for the planner's' problem may now be written:

$$\begin{aligned} \Omega(\tau_i, T_i, \mu, \gamma; \sigma, g) = n_1 v^1(\tau_1, T_1, \sigma, g) + n_2 v^2(\tau_2, T_2, \sigma, g) + \mu \left[v^2(\tau_2, T_2, \sigma, g) \right. \\ \left. - \bar{v}(\tau_1, T_1, \sigma, g) \right] + \gamma \left[n_1 \left((1 - \tau_1) w_1 l_1 + T_1 - (1 - \sigma) z_1 - g \right) \right. \\ \left. + n_2 \left((1 - \tau_2) w_2 l_2 + T_2 - (1 - \sigma) z_2 - g \right) \right] \end{aligned} \tag{6.s}$$

where $w_i l_i$ is now a function of τ_i, T_i, σ and g.

The first order conditions on the tax parameters are:

$$\begin{aligned} n_1 \alpha_1 w_1 l_1 - \mu \left[\bar{\alpha}_2 \left(w_1 l_1 + \tau_1 \frac{\partial (w_1 l_1)}{\partial \tau_1} \right) + \frac{\bar{u}_l^2}{\bar{w}_2} \frac{\partial (w_1 l_1)}{\partial \tau_1} \right] \\ + \gamma n_1 \left[-w_1 l_1 + (1 - \tau_1) \frac{\partial (w_1 l_1)}{\partial \tau_1} - (1 - \sigma) \frac{\partial z_1}{\partial \tau_1} \right] = 0 \end{aligned} \tag{τ_1.s}$$

$$\begin{aligned} -n_1 \alpha_1 - \mu \left[\bar{\alpha}_2 \left(\tau_1 \frac{\partial (w_1 l_1)}{\partial T_1} - 1 \right) + \frac{\bar{u}_l^2}{\bar{w}_2} \frac{\partial (w_1 l_1)}{\partial T_1} \right] \\ + \gamma n_1 \left[(1 - \tau_1) \frac{\partial (w_1 l_1)}{\partial T_1} + 1 - (1 - \sigma) \frac{\partial z_1}{\partial T_1} \right] = 0 \end{aligned} \tag{T_1.s}$$

$$(n_2 + \mu) \alpha_2 w_2 l_2 + \gamma n_2 \left[-w_2 l_2 + (1 - \tau_2) \frac{\partial (w_2 l_2)}{\partial \tau_2} - (1 - \sigma) \frac{\partial z_2}{\partial \tau_2} \right] = 0 \tag{τ_2.s}$$

$$-(n_2 + \mu) \alpha_2 + \gamma n_2 \left[(1 - \tau_2) \frac{\partial (w_2 l_2)}{\partial T_2} + 1 - (1 - \sigma) \frac{\partial z_2}{\partial T_2} \right] = 0. \tag{T_2.s}$$

Combining (τ_2.s) and (T_2.s) we obtain:

$$(1 - \tau_2) \left[\frac{\partial (w_2 l_2)}{\partial \tau_2} + w_2 l_2 \frac{\partial (w_2 l_2)}{\partial T_2} \right] - (1 - \sigma) \left[\frac{\partial z_2}{\partial \tau_2} + w_2 l_2 \frac{\partial z_2}{\partial T_2} \right] = 0. \tag{12}$$

This yields (7) and (8) as before when $\sigma = 1$. When $\sigma \neq 1$, the marginal tax rate on the highest income person is no longer zero. However, for our purposes we are not interested in the particular properties of the optimal tax system. Similarly, combining (τ_1.s) and (T_1.s), we obtain:

$$\left[\gamma n_1(1-\tau_1)-\mu\left(\bar{\alpha}_2\tau_1+\frac{\bar{u}_l^2}{\bar{w}_2}\right)\right]\left[\frac{\partial(w_1l_1)}{\partial\tau_1}+w_1l_1\frac{\partial(w_1l_1)}{\partial T_1}\right]$$
$$-\gamma n_1(1-\sigma)\left[\frac{\partial z_1}{\partial\tau_1}+w_1l_1\frac{\partial z_1}{\partial T_1}\right]=0. \tag{13}$$

This gives (9) and (10) when $\sigma=1$ as before.

Consider now the optimal size of the subsidy on private education, given g. Differentiating (6.s) with respect to σ yields:

$$\frac{d\Omega}{d\sigma}=-n_1\alpha_1z_1-(n_2+\mu)\alpha_2z_2-\mu\left[\bar{\alpha}_2\left(\tau_1\frac{\partial(w_1l_1)}{\partial\sigma}-\bar{z}_2\right)+\frac{\bar{u}_l^2}{\bar{w}_2}\frac{\partial(w_1l_1)}{\partial\sigma}\right]$$
$$+\gamma\left[n_1\left((1-\tau_1)\frac{\partial(w_1l_1)}{\partial\sigma}+z_1-(1-\sigma)\frac{\partial z_1}{\partial\sigma}\right)\right. \tag{14}$$
$$\left.+n_2\left((1-\tau_2)\frac{\partial(w_2l_2)}{\partial\sigma}+z_2-(1-\sigma)\frac{\partial z_2}{\partial\sigma}\right)\right].$$

The optimal level of σ, given g, is obtained by setting (14) to zero. Eliminating α_1 and α_2 by ($T_1.s$) and ($T_2.s$) and then using (12) and (13), this expression can be simplified to yield:

$$\frac{d\Omega}{d\sigma}=\mu\bar{\alpha}_2(\bar{z}_2-z_1)+\gamma(1-\sigma)\left\{n_1\left[\frac{d\tilde{z}_1}{d\tau_1}\frac{d\tilde{w_1l_1}}{d\sigma}\left(\frac{d\tilde{w_1l_1}}{d\tau_1}\right)^{-1}-\frac{d\tilde{z}_1}{d\sigma}\right]\right.$$
$$\left.+n_2\left[\frac{d\tilde{z}_2}{d\tau_2}\frac{d\tilde{w_2l_2}}{d\sigma}\left(\frac{d\tilde{w_2l_2}}{d\tau_2}\right)^{-1}-\frac{d\tilde{z}_2}{d\sigma}\right]\right\} \tag{15}$$

where we use the tilde to identify income-compensated effects. Since the cross effects $d\tilde{z}_1/d\tau_i$ and $d\tilde{w_il_i}/d\sigma$ are equal and so of the same sign, the expression in braces is positive. Setting (15) equal to zero, we conclude that at the optimum $1-\sigma$ is of opposite sign to $\bar{z}_2-z_1$. From this we immediately obtain the following result:

Proposition 3. For given g, $\sigma >,=,< 1$ as $\bar{z}_2 >,=,< z_1$.

It can be shown that $\bar{z}_2 < z_1$ if $\epsilon_1(g+\bar{z}_2)\geq\epsilon_2(g+\bar{z}_2)$ and x is a normal good.[16] Thus, the same circumstances which make g welfare-improving also make a subsidy welfare-improving.

[16] This can be proved along the same lines as the previous footnote. With minor modifications, (σ now appears in the formulae and τ_2 is different from 1), the proofs in the previous two footnotes carry over to the present case with the same sufficient conditions.

Proposition 3 is actually quite intuitive. Focussing on (15) first at $\sigma = 1$, the substitution of an increment in σ for T_1 and T_2 can be done so as to leave persons 1 and 2 with the same level of utility (as long as $z_1, z_2 > 0$). Thus, T_i would have to decrease by $z_i d\sigma$ to ensure that there was no net income effect on person i.[17] However, for the mimicking person, a change in σ accompanied by $dT_1 = z_1 d\sigma$ would have an income effect of $(-z_1 + \bar{z}_2)d\sigma$, the latter term being the revenue paid on $\bar{z}_2$ due to the increment in σ. This income effect on the mimicking person is evaluated at $\bar{\alpha}_2$ by the mimicking person, and increments of the mimicking person's utility are evaluated at μ by the planner thus leading to (15). As soon as we move σ away from 1, we need to account for the distortion effect on private education decisions. This appears in the second term of (15). Notice that if g is set so as to crowd out both z_1 and $\bar{z}_2$, a subsidy or tax on education should not be used.

Next we want to know how the optimal values of g and σ interact so as to know what combination of the two should be used. Consider the welfare effect of changes in g given that σ is being set optimally. Suppose first that g is set such that $z_1, z_2 > 0$. Then differentiating eq. (6.s) with respect to g and using the optimal tax conditions $(T_1.s)$, $(\tau_1.s)$, $(T_2.s)$ and $(\tau_2.s)$, we obtain:[18]

$$\frac{d\Omega}{dg} = \mu\left[\bar{\alpha}_2\sigma + \frac{\bar{u}_l^2}{\bar{w}_2}\bar{w}_2'\bar{l}_2\right] - (1-\sigma)\gamma(n_1 + n_2). \tag{16}$$

For $\bar{z}_2 > 0$, the first term is zero and, assuming $\bar{z}_2 < z_1$, the second term is negative. Again, this is quite intuitive. The increase in g will have only an inframarginal effect. An increase in g of one "dollar" crowds out one dollar of z_i and the lump-sum tax T_i can be reduced by $\sigma(< 1)$ dollars and still keep utility constant for person i. Thus an increase in g of one dollar to each person would cause government revenue to change by $-(1-\sigma)(n_1 + n_2)$ which has a social value of γ per dollar. Once g increases by enough to crowd out $\bar{z}_2$ entirely, the first term becomes positive.

To fix ideas, we shall concentrate on the case in which $\bar{g}_2 < g_1 \leq g_2$. A sufficient condition for $g_1 \leq g_2$ can be shown to be:[19]

17 At $\sigma = 1$, the distortion effect is of the second order so can be neglected.

18 We have also used the fact that as long as $z_i > 0$, changes in g are inframarginal and have in all respects the same effect on household i as a lump-sum subsidy. In particular, dg has the same effect on utility as $-\sigma dT_i$. Therefore,

$$\frac{\partial(w_i l_i)}{\partial g} = \frac{\partial(w_i l_i)}{\partial T_i}\frac{\partial T_i}{\partial g} = -\sigma\frac{\partial(w_i l_i)}{\partial T_i}.$$

19 Suppose that g just crowds out z_2 ($g = g_2$), which means that $\tau_2 w_2'(g_2)l_2 = \sigma$. We show by contradiction that under the stated hypothesis, $z_1 > 0$. Thus, suppose that $z_1 = 0$. Therefore, $\tau_1 w_1'(g_2)l_1 = \sigma$, and we have $\tau_1 w_1'(g_2)l_1 > \tau_2 w_2'(g_2)l_2$, or $\epsilon_1(g_2)\tau_1 w_1 l_1 > \epsilon_2(g_2)\tau_2 w_2 l_2$, which contradicts the hypothesis.

$$\frac{\epsilon_2(g_2)}{\epsilon_1(g_2)} > \frac{\tau_1 w_1 l_1}{\tau_2 w_2 l_2}.$$

This is not too restrictive given that for incentive compatibility it is required that $\tau_2 w_2 l_2 > \tau_1 w_1 l_1$.[20] Figure 1 depicts for this case a typical pattern of social welfare as g is changed, assuming that the taxes and education subsidy are set optimally. As long as g is kept below g_1, the slope of the curve is given by (16). Note that it is negative at $g = \bar{g}_2$ (since the first term is still zero while $\sigma < 1$ according to Proposition 3), and positive at $g = g_1$ (where σ becomes equal to 1.) Beyond $g = g_1$, the slope of the curve is provided by expression (11), and a local maximum of σ is reached where its first two (negative) terms just equal the third (positive) one. The figure is drawn showing two local optima, one in which education is subsidized ($\sigma < 1$) and there is no public provision ($g = 0$), and the other in which there is public provision ($g > 0$) and no subsidy ($\sigma = 1$). Thus, here public provision and subsidization of education are alternative policy instruments. Which one is optimal requires a global comparison. This is summarized in the following proposition.[21]

Proposition 4. If $\bar{g}_2 < g_1 \leq g_2$, and if income taxes are set optimally, there are at least two locally optimal education policies which can be globally optimal:

i. $g \geq g_1$ and $\sigma = 1$
ii. $\sigma < 1$ and $g = 0$.

IV. The Case of Pensions

In the case of education, public spending entered indirectly into the utility function through the wage function. In this section we consider the simpler case of the public spending being on a private good which enters directly into the utility function. Given that the public sector will be providing what is essentially a private good to the households in the economy, it is critical that the good in question cannot be resold. Otherwise, it would be equivalent to providing a lump-sum transfer to all households.[22] This case, which is similar to that analyzed by Usher (1977) and Besley and Coate (1989), is somewhat simpler and also more familiar since it is related to the problem of optimal taxation in a multi-commodity world. Persons

20 A sufficient condition for $g_1 > \bar{g}_2$ was shown above to be $\epsilon_2(\bar{g}_2)/\epsilon_1(\bar{g}_1) < 1$. Assuming, e.g., constant elasticities, it means that there is a range of elasticities which satisfy both sufficient conditions:

$$\frac{\tau_1 w_1 l_1}{\tau_2 w_2 l_2} < \frac{\epsilon_2}{\epsilon_1} < 1.$$

Furthermore, those are sufficient but not necessary conditions.

21 Technically, we cannot rule out the possibility of there being another local optimum between $\bar{g}_2$ and g_1 with $\sigma < 1$ and $g > 0$. We have been unable to determine a set of conditions which would characterize this possibility so we have left it out of Proposition 4. Furthermore, there may also be multiple local optima beyond g_1 where $\sigma = 1$ and $g > g_1$.

22 Blackorby and Donaldson (1988) have avoided this problem be assuming that the good is only demanded by one of the two types of persons in the economy.

differ only by an ability parameter normalized to equal the wage rate. We use the example of public pensions for this case, where the good provided is future consumption. The pension is assumed to be fully funded so as to concentrate on intra-generational redistribution and thereby avoid the dynamic complications that arise from intergenerational redistribution. Other interpretations of the publicly-provided good are possible however. For example, the case of health care could be analyzed as a private consumption good. In this case we might want to modify the analysis to allow persons to vary not only by an ability parameter but also by a second characteristic, health status. While ability is not observable, health status is, so differing amounts of health care can be provided to persons of different health status, independent of ability (wage rate). The analysis for this case is a straightforward extension of the pension case, except that a different amount of public spending is provided to each health class. Similar conditions for public provision to each class to be welfare-improving will apply in this case. Also, the optimal income tax rates will vary by health status.[23]

The household is assumed to consume two goods, x and z, and to supply labour, l. Good x can be thought of as present consumption and z as future consumption. The government may also supply an amount g of future consumption uniformly to all persons and finance it by an optimal income tax. The budget constraint of a household of type i is:

$$x_i + z_i = \tau_i w_i l_i - T_i,$$

where the wage rate is exogenous and $w_2 > w_1$. Again, we assume fixed producer prices and normalize quantities so that prices of consumer goods are unity. Given the value of g the problem of the household can be written:

$$\max_{l_i, z_i} \; u\left[\tau_i w_i l_i - T_i - z_i, g + z_i, l_i\right].$$

The first order conditions are:

$$\begin{aligned} &\alpha_i \tau_i w_i + u_l^i = 0 \\ &-\alpha_i + u_z^i \leq 0; \quad z_i\left[-\alpha_i + u_z^i\right] = 0. \end{aligned} \tag{17}$$

This problem yields the indirect utility function $v(\tau_i, T_i, g)$. Applying the envelope theorem to the indirect utility function we obtain:

$$\begin{aligned} v_\tau^i &= \alpha_i w_i l_i, \\ v_T^i &= -\alpha_i. \\ v_g^i &= u_z^i. \end{aligned} \tag{18}$$

[23] The case for an optimal linear income tax whose parameters vary with health status was made by Blomqvist and Horn (1984).

The problem for household 2 when mimicking the income of household 1 is:

$$\underset{\bar{z}_2}{\text{Max}} \quad u\left[\tau_1 w_1 l_1 - T_1 - \bar{z}_2, g + \bar{z}_2, \frac{w_1 l_1}{w_2}\right].$$

The first-order conditions for this problem are:

$$-\bar{\alpha}_2 + \bar{u}_z^2 \leq 0; \quad \bar{z}_2\left(-\bar{\alpha}_2 + \bar{u}_z^2\right) = 0. \tag{19}$$

This yields an indirect utility function $\bar{v}(\tau_1, T_1, g)$. From the envelope theorem,

$$\begin{aligned}
\bar{v}_\tau &= \bar{\alpha}_2\left(w_1 l_1 + \tau_1 w_1 \frac{\partial l_1}{\partial \tau_1}\right) + \bar{u}_l^2 \frac{w_1}{w_2}\frac{\partial l_1}{\partial \tau_1} \\
\bar{v}_T &= \bar{\alpha}_2\left(\tau_1 w_1 \frac{\partial l_1}{\partial T_1} - 1\right) + \bar{u}_l^2 \frac{w_1}{w_2}\frac{\partial l_1}{\partial T_1} \\
\bar{v}_g &= \bar{\alpha}_2 \tau_1 w_1 \frac{\partial l_1}{\partial g} + \bar{u}_z^2 + \bar{u}_l^2 \frac{w_1}{w_2}\frac{\partial l_1}{\partial g}.
\end{aligned} \tag{20}$$

The planner maximizes total utility subject to a self-selection constraint and a revenue constraint. The Lagrangian expression is identical to (6) for the education case. The first order conditions are the same as (τ_1), (T_1), (τ_2) and (T_2) with the exception that w_1 and w_2 are parametric here. Equations (7)–(10) characterizing the optimal tax structure apply here as well. Differentiating (6) with respect to g using (20) gives:

$$\begin{aligned}
\frac{d\Omega}{dg} &= n_1\left(u_z^1 - \gamma\right) + n_2\left(u_z^2 - \gamma\right) + \mu\left(u_z^2 - \bar{u}_z^2\right) \\
&+ \left(\gamma n_1(1-\tau_1)w_1 + \gamma n_2(1-\tau_2)w_2 - \mu(\bar{\alpha}_2\tau_1 w_1 + \bar{u}_l^2 w_1/w_2)\right)\frac{\partial l_1}{\partial g}.
\end{aligned}$$

Using (7)–(10), this reduces to:

$$\frac{d\Omega}{dg} = n_1\left(u_z^1 - \alpha_1\right) + (n_2 + \mu)\left(u_z^2 - \alpha_2\right) + \mu\left(\bar{\alpha}_2 - \bar{u}_z^2\right). \tag{21}$$

The first two terms are ≤ 0 by (17), being equal to zero if private expenditures on z are positive. On the other hand, the last term is ≥ 0 by (19), being zero for $\bar{z}_2 > 0$. Again defining g_i as the level of g which just crowds out person i's spending on z_i, the following result is apparent from (21):

Proposition 5. If $\bar{g}_2 < g_1, g_2$, then public spending will be welfare-improving at least until $g = \min(g_1, g_2)$.

The question then become when will the mimicking person become crowded out before either persons 1 or 2. The following result can be demonstrated:[24]

> A necessary and sufficient condition for $\bar{g}_2 < g_1$ is that leisure and z be substitute goods. This is also a sufficient condition for $\bar{g}_2 < g_2$.

Thus we conclude with the following Proposition.

> *Proposition 6.* A sufficient condition for public provision up to at least $g = \min(g_1, g_2)$ to be welfare-improving is that leisure and z be substitutes.

Next, consider the use of a subsidy on private purchases of z as an alternative policy instrument. The analysis is almost the same as for the case of subsidizing education. If the subsidy rate is $1-\sigma$ as before, the problem of household i becomes:

$$\max_{l_i, z_i} \; u\left[\tau_i w_i l_i - T_i - \sigma z_i, g + z_i, l_i\right].$$

The first-order conditions may be written:

$$\alpha_i \tau_i w_i + u_l^i = 0$$
$$-\alpha_i \sigma + u_z^i \leq 0; \quad z_i\left(-\alpha_i \sigma + u_z^i\right) = 0.$$

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24 The proof of this is as follows. Set g equal to $\bar{g}_2$. At this point, $\bar{z}_2 = 0$ and from (19), $-\bar{\alpha}_2 + \bar{u}_z^2 = 0$, or $\frac{\bar{u}_z^2}{\bar{\alpha}_2} = 1$. Now, suppose that person 1 is constrained to have $z_1 = 0$. From the envelope theorem, $\frac{\partial v^1}{\partial z_1} = -\alpha_1 + u_z^1$. This will be positive at $z_1 = 0$ if

$$\frac{u_z^1(x_1, \bar{g}_2, l_1)}{\alpha_1(x_1, \bar{g}_2, l_1)} > 1 = \frac{\bar{u}_z^2(x_1, \bar{g}_2, \bar{l}_2)}{\bar{\alpha}_2(x_1, \bar{g}_2, \bar{l}_2)}.$$

Since $l_1 > \bar{l}_2$, this will be satisfied if and only if:

$$\frac{\partial\left[\frac{u_z^i}{\alpha_i}\right]}{\partial l_i} > 0,$$

that is, if l and z are complements, i.e., if leisure and z are substitutes. Similarly, for person 2 set and $z_2 = 0$. The effect on utility from increasing z_2 will be $\frac{\partial v^2}{\partial z_2} = -\alpha_2 + u_z^1$. This will be positive at $z_2 = 0$ if

$$\frac{u_z^2(x_2, \bar{g}_2, l_2)}{\alpha_2(x_2, \bar{g}_2, l_2)} > 1 = \frac{\bar{u}_z^2(x_1, \bar{g}_2, \bar{l}_2)}{\bar{\alpha}_2(x_1, \bar{g}_2, \bar{l}_2)}.$$

Since $l_2 > \bar{l}_2$ and $x_2 > x_1$, this will also be satisfied if l is complementary with z since both the higher value of l_2 and the higher value of x_2 (for a given value of g) will tend to increase the value of the left-hand side. (Note that substitutability is not necessary here.)

This gives the indirect utility function $v^i(\tau_i, T_i, \sigma, g)$. Using the envelope theorem, we obtain equations (18) as before plus equation (3.s).

The mimicking problem for household 2 is:

$$\underset{\bar{z}_2}{\text{Max}} \quad u\left[\tau_1 w_1 l_1 - T_1 - \sigma \bar{z}_2, g + \bar{z}_2, \frac{w_1 l_1}{w_2}\right]$$

where $w_1 l_1$ is taken as given. The first order condition for this problem is:

$$-\bar{\alpha}_i \sigma + \bar{u}^i_z \leq 0; \quad z_i\left(-\bar{\alpha}_i \sigma + \bar{u}^i_z\right) = 0.$$

The solution to this problem gives the indirect utility function $\bar{v}(\tau_1, T_1, \sigma, g)$. From the envelope theorem, we obtain the same expressions for $\bar{v}_\tau$, $\bar{v}_T$ and $\bar{v}_g$ as in (20) as well as equation (5.s) for $\bar{v}_\sigma$. Since w_i is exogenous here, it may be written:

$$\bar{v}_\sigma = \bar{\alpha}_2 \left(\tau_1 w_1 \frac{\partial l_1}{\partial \sigma} - \bar{z}_2\right) + \bar{u}^2_l \frac{w_1}{w_2} \frac{\partial l_1}{\partial \sigma}.$$

The Lagrangian expression for the planner's problem is identical to (6.s) and the first order conditions on the tax rates are the same as (τ_1), (T_1), (τ_2) and (T_2) for the education case, where again the w_i are exogenous and can be taken out of the partial derivative expressions. These first order conditions reduce to (12) and (13), which in turn reduce to (7)–(10) when $\sigma = 1$. The expression determining $1 - \sigma$ at the optimum is given by an equation equivalent to (15). Thus Proposition 3 applies here as well. A subsidy on z will be welfare-improving in the presence of an optimal income tax if z and leisure are substitutes, and, conversely, a tax will be welfare-improving if z and leisure are complements. This result should not be surprising in light of the finding of Atkinson and Stiglitz (1976) that the absence of separability between goods and leisure is enough to justify differential commodity taxation alongside an optimal income tax. The sign of the relative commodity tax distortion goes back to the famous Corlett and Hague (1953–4) result, which was recast in the context of optimal taxation by Harberger (1964).

As in the education case, the subsidy is zero once g crowds out $\bar{z}_2$ and z_1. As before, we can derive the welfare effect of increasing g in the presence of the subsidy when $z_1, z_2 > 0$. Differentiating (6.s) with respect to g and using the first order conditions for the households as well as the optimal tax conditions we obtain:

$$\frac{d\Omega}{dg} = \mu\left(\sigma \bar{\alpha}_2 - \bar{u}^2_z\right) - (1 - \sigma)\gamma(n_1 + n_2). \tag{21}$$

Consider the case in which z and leisure are substitutes. As long as $g < \bar{g}_2$, the first term is zero and social welfare is decreasing in g. This is the same result as was obtained for the education case. Proposition 4 applies here as does the diagrammatic illustration of Figure 1.

V. Conclusion

Governments apparently accomplish a good deal of their redistributive objectives through the expenditure side of the budget using such instruments as public education, public health provision and public pensions. We have investigated whether a theoretical case can be made for using expenditures for redistributive purposes. This has been done in a model in which the government is able to pursue redistribution fully through an optimal non-linear income tax, and in which the only role for expenditures is redistributive. That is, public expenditures are on items which are otherwise purely private and could be allocated by markets. Doing so abstracts from the fact that there may be significant externalities associated with their use in practice which would justify public provision on market failure grounds. By adopting these assumptions, we are forced to make the strongest case for using expenditures for redistributive purposes. We have considered two types of expenditures — one which affects the wage rate and we have identified with education, and one which is like an ordinary private good, which we have called future consumption (public pensions). The latter case could represent health care with some minor amendments. In each case we assumed that the public provision was of a good (or service) which could not be re-traded among households.

For both these instances of potential public provision, a similar set of results is obtained. When an optimal income tax is in place, uniform public provision which crowds out the private provision of at least one household is welfare-improving if the following condition holds: the level of public provision which crowds out the private provision of the high income person when mimicking the income of the low income person is less that the level of public provision which crowds out either person individually. In the case of education, this would be the case if the elasticity of the wage function with respect to education expenditures is higher for the low wage person. For public pensions and health, leisure and the good in question must be substitutes.

We also found that the same circumstances which make public provision welfare-improving also make a uniform subsidy on private expenditures welfare-improving. However, the two instruments are substitutes in the sense that the use of them gives rise to two local optima — one in which the subsidy alone is used, and one in which government expenditures is used and in which the subsidy is likely not used. Which of these two local optima is globally optimal cannot be inferred from the marginal techniques we have used here. It depends upon the particular forms of taste and technology of the economy.

Our analysis has been restricted to a two-person economy and to analyzing individual types of expenditure individually. It would be useful to extend the analysis to a more complicated setting. For example, the methodology of Guesnerie and Seade (1982) could be used to consider a multi-person economy. A priori, it seems apparent that similar arguments could be extended to this case.

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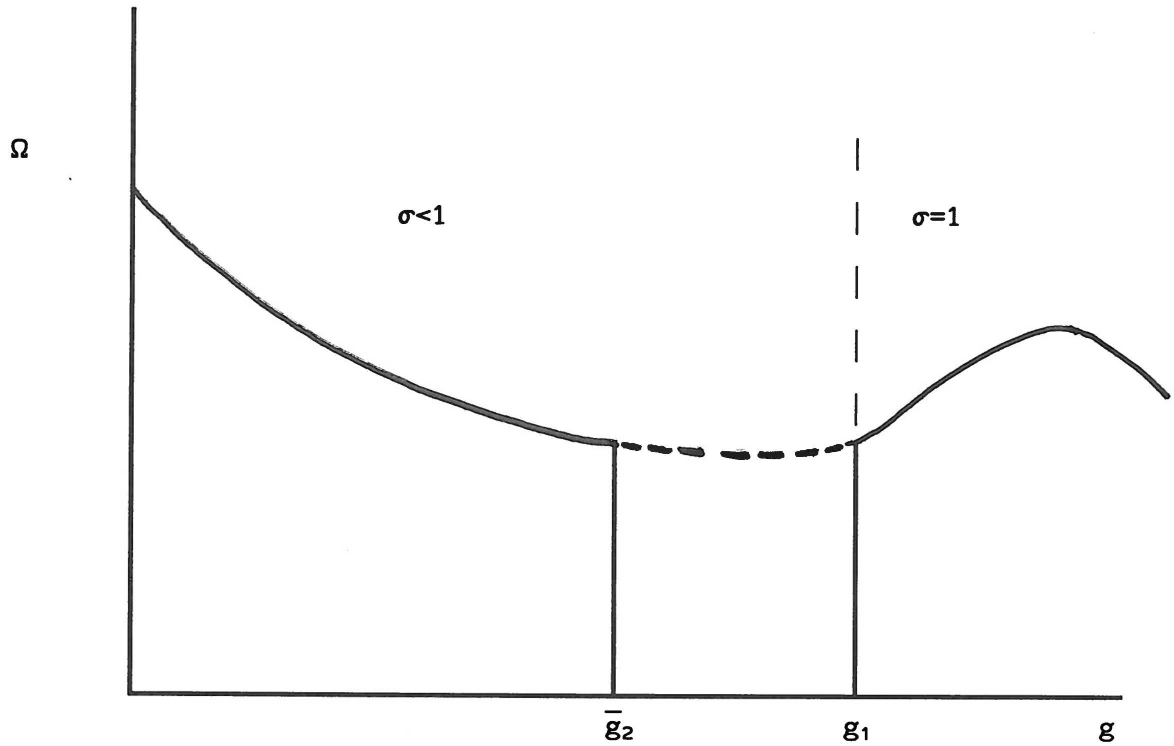

Figure 1