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The Term Structure of Interest Rates: Departures from Time-Separable Expected Utility

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THE TERM STRUCTURE OF INTEREST RATES: DEPARTURES FROM TIME-SEPARABLE EXPECTED UTILITY

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Abstract

This paper assesses the ability of general equilibrium models of asset pricing using two recently developed sets of preferences to quantitatively account for the observed variability in the Canadian term structure of interest rates. The preference structures are non-expected utility and habit persistence associated with Epstein and Zin (1989a) and Constantinides (1990) respectively. The framework adopted follows Backus, Gregory and Zin (1989) where a numerical version of the theory is specified and empirical features of the artificial economy are compared against actual data. Neither preference structure is able to satisfactorily mimic the magnitude or the variability of the risk premiums.

1. Introduction

Empirical research on the term structure of interest rates suggests that the expectations hypothesis of the term structure is rejected. Most empirical research has identified the rejection of the expectations hypothesis as evidence of time varying risk premiums (an excellent survey is Melino, 1987). Unfortunately models of risk premiums based upon the general equilibrium theory of asset pricing (Merton, 1973; Lucas, 1978; Breeden, 1979; Brock, 1982; and Cox, Ingersoll and Ross, 1985) have been unable to account for the observed fluctuations in multiperiod bond returns. For example, Backus, Gregory and Zin (1989) find that the representative agent model with additively time-separable expected utility fails to account for the magnitude of risk premiums on forward contracts of United States treasury bills. Within this class of preferences, the variability of intertemporal prices (identified with marginal rates of substitution) is too small to explain the large variability observed in data.

Recently, Epstein and Zin (1989a, 1989b) and Constantinides (1990) have attempted to reconcile the representative agent asset pricing model with market data by relaxing the assumption of additively time-separable, von Neumann-Morgenstern preferences. Epstein and Zin develop a non-expected utility theory for intertemporal optimization problems. The advantage of this class of preferences is that it separates the agent's measure of risk aversion from the measure of intertemporal substitution. In the constant elasticity of substitution utility function, the two measures are constrained to be equal. Epstein and Zin (1989b) find some empirical support for modelling preferences in this manner.

Constantinides (1990) presents an alternative approach. He relaxes the assumption of additive time-separability while still maintaining the von Neumann-Morgenstern axioms of expected utility. The preferences are specified to capture intertemporal habit forming behaviour similar to that developed by Ryder and Heal (1973). Constantinides has shown that this kind of nonseparability leads to large average excess returns (risk premiums) in equity. Recently Backus, Gregory and Telmer (1990) have employed these preferences to study the forward foreign exchange market.

The purpose of this paper is to assess whether these kinds of important preference modifications can quantitatively account for the observed variability in the Canadian term structure of interest rates. As in Backus, Gregory and Zin (1989) a numerical version of the theory is specified (commonly called an artificial economy) and empirical features of the artificial economy are compared with those of actual data.

The results of this investigation are at best mixed. Both the non-expected utility theory and the habit persistence theory are unable to provide a risk premium which can fully explain the rejections of the expectations hypothesis. Although there are parameter settings that can generate sufficient variability in the risk premium, such models generate important counterfactual features which qualify any conclusions in favour of these models.

The organization of the paper is as follows. Section 2 briefly reviews the expectations hypothesis of the term structure of interest rates and provides some empirical analysis with Canadian treasury bill data. Section 3 develops the artificial economy and the asset pricing model under the two different preference structures. Section 4 provides Monte Carlo evidence on the model's ability to generate risk premiums and explain the rejections of the expectations hypothesis. Section 5 concludes.

2. The Term Structure of Interest Rates

The discussion is in terms of prices rather than rates since the former link immediately with the theoretical model. Define the price of a k-period treasury bill to be q_{kt}. Forward prices are defined as:

$$f_{kt} \equiv q_{k+1,t} \, / \, q_{kt}. \tag{2.1}$$

The expectations hypothesis states that the k-period forward price is an unbiased predictor of the one-period spot price k periods ahead:

$$f_{kt} = E_t(q_{1t+k}), \tag{2.2}$$

where E_t is the expectation conditional on date t information. Alternatively we may consider the forward prices as consisting of a combination of risk premiums and forecast errors in which case we define the forward risk premiums as:

$$fp_{kt} \equiv E_t(q_{1t+k}) - f_{kt}. \tag{2.3}$$

The risk premium fp_{kt} may be time-varying and is defined to be positive when the expected future spot price exceeds the current forward price.

Table 2.1 presents summary statistics of the Canadian three month and six month treasury bill market for the time period 1961:2 to 1988:3. The time interval is a quarter so that q_{1t} is the price of a three-month treasury bill. In addition to the spot and forward prices, Table 2.1 also reports statistics for $q_{1t+1} - f_{1t}$; if the forecasts of the one-period future spot rates are unbiased then the sample mean of this variable is an estimate of the average risk premium. For the Canadian data, the risk premium is small but significantly different from zero. A comparison with the same measure for the United States treasury bill market (Backus, Gregory and Zin, 1989, Table 1) shows that the average Canadian risk premium is about one third the size of the average United States risk premium (0.00142).

A common test of the expectations hypothesis applied in the literature is based on the regression:

$$q_{1t+1} - f_{1t} = a + bx_t + \varepsilon_{t+1} \ . \tag{2.4}$$

The variable x_t is any element or set of elements of the information set. Under the expectations hypothesis: (i) ε_{t+1} is serially uncorrelated (possibly heteroskedastic) and orthogonal to x_t and (ii) $a = b = 0$. That is, $q_{1t+1} - f_{1t}$ cannot be predicted using current information.

Various choices for x_t have been used in the literature. In Table 2.2 we consider three examples, all of which have been used in previous empirical studies:

$$q_{1t+1} - f_{1t} = a + \varepsilon_{t+1} \tag{2.5}$$

$$q_{1t+1} - f_{1t} = a + b(f_{1t}-q_{1t}) + \varepsilon_{t+1} \tag{2.6}$$

$$q_{1t+1} - f_{1t} = a + b(q_{1t}-f_{1t-1}) + \varepsilon_{t+1} \ . \tag{2.7}$$

Table 2.2 reports Wald tests of the hypothesis $a = b = 0$, $b = 0$, and $b = -1$. The Wald test for $b = -1$ is based on the linear transformation:

$$q_{1t+1} - q_{1t} = a + (b+1)(f_{1t}-q_{1t}) + \varepsilon_{t+1} \tag{2.8}$$

and is interesting because its rejection implies that the forward premium is a useful predictor of the change in spot prices.

Table 2.2 also contains some specifications tests based upon residual analysis. Godfrey's (1978) tests for first and fourth order serial correlation are calculated, as well as Engle's (1982) tests for first and fourth order autoregressive conditional heteroskedasticity (ARCH). In all three regressions there is some evidence of fourth order serial correlation and ARCH. Recall that the expectations hypothesis implies serially uncorrelated errors.

In light of possible non-constant variances, hypothesis tests are based upon heteroskedastic-consistent covariances (White, 1980). For both the forward premium regression (2.6) and the lagged forecast error regression (2.7), the joint hypothesis $a = b = 0$ is rejected at the five per cent significance level. The test of $b = 0$ is rejected in the forward premium regression (2.6) but retained in the forecast error equation (2.7). Finally, the hypothesis $b = -1$ is not rejected indicating a lack of predictive power for the forward premium; a result which differs with Shiller, Campbell, and Schoenholtz (1983).

The regressions of Table 2.2 also provide lower bound estimates of the variance of the risk premium. Equation 2.3 and the assumption of rational expectations imply that

$$q_{1t+1} - f_{1t} = fp_{1t} + \varepsilon_{t+1}$$

where ε_{t+1} is a mean zero forecast error which is orthogonal to the risk premium. This decomposition then implies

$$\mathrm{Var}(q_{1t+1} - f_{1t}) = \mathrm{Var}(fp_{1t}) + \mathrm{Var}(\varepsilon_{t+1}).$$

Since the risk premiums can be predicted with date t information, the variability of the fitted values in any regression explaining $q_{1t+1} - f_{1t}$ is a lower bound estimate of the variability of the risk premium fp_{1t}. From the estimated regression of (2.6) in Table 2.2, the implied lower bound for the standard deviation of the risk premium is 0.0008. This value is one half of a similar calculation for the United States treasury bill data reported in Backus, Gregory, and Zin (1989) (the estimate there is 0.00157).

3. The Economy

We next consider whether or not some general equilibrium asset pricing models can provide a sufficiently variable risk premium to account for the observed rejections of the expectations hypothesis of the term structure of interest rates. We specify a monetary version of the economy similar to that of Backus, Gregory and Zin (1989) which is due to Mehra and Prescott (1985). We investigate the term structure of interest rates, with special attention to risk premiums, under two different preference structures: non-expected utility associated with Epstein and Zin (1989a, 1989b), and habit persistence associated with Constantinides (1990). Although we briefly outline the preferences for each, the interested reader is advised to consult the original sources.

We first describe those components of the economy which are common to both the non-expected utility and the habit persistence problems. The representative agent receives an endowment each period, y_t, of the perishable consumption good. The endowment evolves according to

$$y_{t+1} = x_{t+1} y_t, \tag{3.1}$$

where x_t is the growth rate which follows a stationary Markov processes defined on the discrete state space $\Lambda = \{\lambda_1, \lambda_2, \ldots \lambda_l\}$.

Money is introduced by cash-in-advance constraints in a manner following Lucas (1982). Without going into the details (see Sargent, 1987, Chapter 5), we may identify inflation, p_{t+1}/p_t, with money growth. Inflation is also assumed to be a stationary Markov process with discrete state space $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_g\}$.

We define an equilibrium for this economy to be a set of state contingent prices for which consumption equals the endowment at all time periods in all states. With fully specified preferences, the relevant state vector, and the underlying stochastic process, we may then price various assets.

(i) **Non-Expected Utility**

At any time t, the representative agent chooses a consumption plan $\{c_{t+i}\}_{i=0}^{\infty}$ to maximize the recursive preference ordering,

$$U_t = \left(c_t^{1-\rho} + \beta (E_t U_{t+1}^{1-\alpha})^{(1-\rho)/(1-\alpha)} \right)^{1/(1-\rho)} \tag{3.2}$$

where U_t is utility defined at time t, c_t is consumption, $\beta \in [0,1]$ is the discount factor, $\alpha > 0$ is the constant measure of relative risk aversion, and $\rho > 0$ is the inverse of the elasticity of intertemporal substitution. Epstein and Zin (1989b) provide a thorough discussion of the utility structure given in equation (3.2) and Epstein (1988) and Weil (1989) investigate its application for asset pricing. If $\alpha = \rho$, then equation (3.2) collapses to the standard expected utility specification with constant elasticity of substitution (CES) preferences.

The optimal consumption plan is characterized by the following Euler equation:

$$E_t\left(\beta^{\frac{(1-\alpha)}{(1-\rho)}} \left(c_{t+1} / c_t \right)^{-\rho \frac{(1-\alpha)}{(1-\rho)}} \left(M_{t+1} \right)^{\frac{(1-\alpha)}{(1-\rho)} - 1} R_{t+1} \right) = 1 \tag{3.3}$$

where M_{t+1} is the real return on the one equity share which lays claim to the future endowment process of the output $\{y_{t+1}\}_{i=1}^{\infty}$ and R_{t+1} is the gross return of any one-period inside asset with zero net supply.

The real return M_{t+1}, which in this economy is the real return on the market, may be written $M_{t+1} = [p^e_{t+1}(x_t,y_t) + y_{t+1}]/p^e_t(x_t,y_t)$ where $p^e_t(x_t,y_t)$ is the price of the equity share.[1] $p^e_t(x_t,y_t)$ is homogeneous of degree one in y_t (see Mehra and Prescott, 1985), and we may write $p^e_t(x_t,y_t) = w(x_t)y_t$, where $w(x_t)$ is the equity pricing function. Since (3.3) holds for all inside assets, we may replace R_{t+1} with M_{t+1} and with the above decomposition of M_{t+1} we get the following non-linear difference equation for $w(x_t)$:

$$w(x_t)^{\frac{(1-\alpha)}{(1-\rho)}} = E_t\left(\beta^{\frac{(1-\alpha)}{(1-\rho)}} x_{t+1}^{(1-\alpha)} \left(1 + w(x_{t+1})\right)^{\frac{(1-\alpha)}{(1-\rho)}} \right). \tag{3.4}$$

A non-linear difference equation solution technique is required to solve for $w(x_t)$; once a solution has been obtained it is possible to price all other assets on the basis of equation (3.3). In particular, we are interested in

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[1] We assume that the dividends are paid in real terms. See Sargent, 1987 for a discussion of the distortion to equity prices from cash dividends.

nominal, one-period riskfree bonds with gross return R_t^*; the associated real return is then $R_t^* p_t / p_{t+1}$. Substitution into (3.3) gives the following pricing relationship for the nominal riskfree asset:

$$q_{1t} = E_t \left(\beta^{\frac{(1-\alpha)}{(1-\rho)}} x_{t+1}^{-\alpha} \left[\frac{(1 + w(x_{t+1}))}{w(x_t)} \right]^{\frac{(1-\alpha)}{(1-\rho)} - 1} \frac{p_t}{p_{t+1}} \right) \tag{3.5}$$

$$= E_t n_{t+1},$$

which defines the nominal marginal rate of substitution. Multiperiod nominal bonds may be priced by iterating on the one-period definition:

$$q_{kt} = E_t \prod_{i=1}^{k} n_{t+i}. \tag{3.6}$$

For this particular problem, the state variables $(x_t, p_t/p_{t-1}, y_t)$ with the known joint Markov chain of the two stochastic variables, x_t and p_t/p_{t-1}, fully characterize the economy. Let $z_{t+1} = (x_{t+1}, p_{t+1}/p_t)$ be described by a stationary Markov process on discrete state space $\Omega = \Lambda \otimes \Gamma$. The stochastic behaviour of z_t is characterized by the matrix π of transition probabilities with typical element

$$\pi_{ij} = \Pr\left[z_{t+1} = s_j \mid z_t = s_i \right] \qquad s_i, s_j \in \Omega. \tag{3.7}$$

The equilibrium or unconditional probabilities are $\pi_i^* = \Pr[z_t = s_i]$ for all t. The special advantage of the finite state Markov assumption is that analytical solutions for evaluating expectations of nonlinear functions may be found (see Tauchen, 1987).

This framework allows us to calculate expected future spot prices, forward prices, and the forward risk premiums for any state s_i. Implicit in these pricing functions (3.5) and (3.6) are forward rates and risk premiums. The forward pricing function corresponding to (2.1) is:

$$f_k(s_i) = q_{k+1}(s_i)/q_k(s_i). \tag{3.8}$$

The risk premium, fp, defined in (2.3) may be written:

$$fp_{kt} = \frac{-\mathrm{cov}_t\left[\prod_{i=1}^{k} n_{t+i},\ q_{1t+k}\right]}{q_{kt}}, \tag{3.9}$$

where cov_t is the conditional covariance operator. Since this covariance depends upon the current state, the risk premium is a function of the state and is time-varying.

(ii) **Habit Persistence**

This discussion is similar to section 5.2 of Constantinides (1988). The infinitely-lived, representative agent has preferences which satisfy the von Neumann-Morgenstern axioms but which are not additively time-separable. At any time t, the agent chooses a consumption plan $\{c_{t+i}\}_{i=0}^{\infty}$ to maximize the lifetime utility function U_t:[2]

$$U_t = E_t\left(\sum_{i=0}^{\infty} \beta^i\, u(c_{t+i} - d_1 c_{t+i-1} - d_2 c_{t+i-2})\right) \tag{3.10}$$

and,

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[2] While we could have more than two lags of consumption appearing in the utility function we have found additional lags to be unhelpful quantitatively.

$$u(z) = (z^{1-\alpha}-1)/(1-\alpha).$$

This preference specification, like the one used in the non-expected utility problem, is directly related to CES preferences. If $d_1 = d_2 = 0$, then (3.10) reduces to the standard additively time-separable expected utility problem.

For these preferences, a one period, nominal riskfree bond may be priced

$$q_{1t} = E_t n_{t+1} = E_t\left[\frac{\partial U_t/\partial c_{t+1}}{\partial U_t/\partial c_t} \; \frac{p_t}{p_{t+1}} \right].$$

Prices are characterized at any point in time by the state vector $(x_t, p_t/p_{t-1}, x_{t-1}, y_t)$. The inclusion of x_{t-1} is necessary since $c_t = y_t$, $c_{t-1} = y_{t-1} = y_t/x_t$, and $c_{t-2} = y_{t-2} = y_t/(x_t x_{t-1})$ all appear in the expressions for marginal utility at time t and hence are relevant in the pricing equation of nominal bonds. It is also straightforward to adapt the original state space of the joint Markov process $(x_t, p_t/p_{t-1})$ to the expanded state space of $(x_t, p_t/p_{t-1}, x_{t-1})$. Finally, prices of multiperiod bonds, the forward prices, and risk premiums are again derived as (3.6), (3.8) and (3.9) respectively.

4. Artificial Economy Results

We now select values for the parameters of the economy (for both utility functions) and calculate state contingent prices of nominal bonds, expected spot prices, forward prices and forward risk premiums. We compare the properties (moments) of these variables and some regression results against Canadian treasury bill data from Table 2.1 and Table 2.2.

The experimental design is as follows. We first choose a discrete state space Markov process for consumption growth and inflation. Then, for each of the two economies, we choose the structural parameters and calculate population moments for the artificial economy. We then generate data using random number generator routines (NAG subroutines G05CBF and G05CAF) and an estimated transition probability matrix to create a time series of state realizations. This also yields a time series for all prices and risk premiums. The length of each of the artificial series is chosen to be the same as the actual Canadian data, 110 quarterly observations.

One advantage of the artificial economy is that we know what the risk premiums are at all times and can gauge their influence by comparing regressions (similar in form to equations 2.5-2.7) with and without them. While we cannot say that the rejections in actual data are caused by the risk premiums, we can determine whether the model is capable of generating results similar to those observed in data. To account for sampling error we repeat each simulation experiment 1000 times and follow techniques in Gregory and Smith (1990b) to investigate more formally the correspondence of model and data.

We impose no initial restrictions on possible choices for structural preference parameters; this allows us to investigate the performances of the models under a wide variety of parameterizations. However, we do calibrate our model to certain features of the Canadian economy by estimating a Markov process from Canadian consumption growth and inflation data. An additional criterion for these economies is that treasury bill prices are required to have a population mean less than one (positive interest rates) which does not rule out the possibility of state contingent prices which exceed one.

Table 4.1 presents summary statistics for Canadian consumption growth and inflation. The quarterly growth rates are calculated from seasonally unadjusted data from the CANSIM database (database numbers are given in the table); the growth rates are then seasonally adjusted using ordinary least squares and seasonal dummies (results for the officially adjusted data series are comparable). These summary statistics in Table 4.1 are similar to those of the American data in Backus, Gregory and Zin (1989, Table 4) except for higher negative serial correlation in consumption growth. We calibrate the economies using consumption of non-durables which conforms to the theoretical model.

We estimate the Markov transition probabilities and equilibrium distribution for consumption growth and inflation by maximum likelihood and use this to simulate the model. We classify each observation for consumption and inflation as either being in a high or low state at time t determined by whether the observation is above or below the sample mean. This leads to a joint classification for the two variables with four states. The maximum likelihood estimates of the transition probabilities are $\hat{\pi}_{ij} = n_{ij}/n_i$, where n_{ij} is the number of transitions from state i to j over the sample and n_i is the number of times in state i with $i, j = 1, \ldots, 4$. These results are reported in Table 4.2. The actual values for the high (low) consumption growth and inflation necessary for simulating the model are obtained as the the sample average of the observations above (below) the entire sample average. These state values are also reported in Table 4.2.

The choice of the preference parameters is less straightforward since there are few existing empirical studies based on the two classes of preferences considered here. While we have done some sensitivity analysis over the parameter space we have attempted to limit the number of experiments as much as possible. The discount rate, β, is chosen to be 0.975 for all experiments which is consistent with much of the literature. Backus, Gregory, and Zin (1989) find that it is necessary to have an implausibly large coefficient of relative risk aversion to generate risk premiums with similar properties to data. However since our focus is on preferences which are less dependent upon the single risk aversion parameter, we choose modest values for α of 1.500 or 4.500 which are consistent with previous empirical studies.

All that remains to characterize the economy is to choose values for the intertemporal substitution parameter in the non-expected utility economy and the persistence parameters in the habit persistence economy. Epstein and Zin (1989b) estimate (using U.S. data) values for $\alpha \in [1.0,2.0]$, which is consistent with our first choice above, and $\rho \in [2.0,9.0]$. We consider both their full sample estimate of $\rho \cong 4.5$ and an alternative, $\rho \cong 15.500$. As we shall see, the higher value of ρ is necessary to obtain sufficient variation in the risk premium. We can view this setting of parameter values on the basis of moment-matching as estimation (see Gregory and Smith, 1990a).

For habit persistence there again is little direct empirical evidence to guide us in our choice of parameters. Recently, Ferson and Constantinides (1989) estimate a one-period distributed lag on consumption ($d_2 = 0$) with $d_1 \in [0.79,0.95]$. Unfortunately these values imply certain counterfactual properties for bond prices. Instead we choose two experiments $\{\alpha = 1.500,\ d_1 = 0.200,\ d_2 = 0.200\}$ and $\{\alpha = 4.500,\ d_1 = 0.100,\ d_2 = 0.100\}$ which are similar to those of Constantinides (1988) in his investigation of the equity premium puzzle. The persistence parameters are chosen to ensure a sufficient degree of variability in the risk premium without increasing the mean risk premium to excessively high levels.

Population moments for five experiments are given in Table 4.3 and a number of features emerge. Both non-expected utility and habit persistence formulations (especially parameterizations for Experiment 3 and 5) are able to produce a standard deviation of the risk premium which is "close" to 0.0008, the estimated lower bound. As expected, Experiment 1 (expected utility) is the worst in this regard; however, notice that the non-expected utility preferences require rather high values of ρ to match the lower bound. Also we note that all experiments imply a one period bond price which is too variable relative to the data. Other parameterizations of these preferences produce even less satisfactory results.[3]

The value 0.0008 represents an estimate of the lower bound of the standard deviation of the risk premium. Since comparisons of population moments (as in Table 4.3) with sample moments are informal, we try to gauge the probability that this model would yield a sample standard deviation greater than 0.0008. The methodology follows Gregory and Smith (1990b). We simulate the model 1000 times, calculate the sample standard deviation for each replication and then estimate the density of this sample moment by nonparametric methods. The density is estimated using a quartic kernel and a variable window width as described in Silverman (1986). Figure 1 shows the densities of the sample standard deviations for Experiment 3 with non-expected utility preferences (denoted NEU) and Experiment 5 for habit persistence (denoted HP). While Experiment 5 gives a population mean of 0.0007 and would

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[3] A sensitivity analysis was performed for a wide range of parameter choices. For both types of preferences, the experiments chosen adequately summarize the relationship between the nature of the risk premium and the structural parameters.

appear to be "close" to 0.0008, the density indicates that there is zero probability that a standard deviation greater than 0.0008 would occur. On the other hand, the non-expected utility preferences with parameter settings for Experiment 3 generate estimates greater than the lower bound with probability one.

Regression results from Monte Carlo simulation for these 5 experiments are reported in Tables 4.4-4.8. Each Monte Carlo experiment is based on the same realization of the state process so that differences are solely due to preference differences. For each replication, we perform the same tests presented in Table 2.2 but now on the artificial data arising from simulating the model. We then perform the same tests after removing the risk premium from the dependent variable in each regression; this gives some indication of the power of the tests of the expectations hypothesis. For each test, the number of rejections at five per cent tests are recorded; chance alone (test size) should account for approximately 50 rejections in each experiment.

For the time-additive expected utility case presented in Table 4.3, tests of the expectations hypothesis only reject the hypothesis about five per cent of the time. A similar conclusion is reached for tests of first and fourth order serial correlation and ARCH. Similar evidence leads Backus, Gregory, and Zin (1989) to conclude that the intertemporal asset pricing model with time additive expected utility preference structure is not capable of generating a sufficiently variable risk premium to cause rejections of the expectations hypothesis.

The remaining experiments (Tables 4.5-4.8), based upon the non-expected utility preference structure and upon the habit persistence preference structure, provide conclusions only slightly more favourable than those of the expected utility case. Consider first the two Wald tests of the expectations hypothesis: $a = b = 0$ and $b = 0$. For the latter test, the hypothesis is rejected only slightly more than five per cent of the time in both regressions, whether the risk premium has been extracted or not. For experiments 2 and 4, the test $a = b = 0$ is rejected roughly ten percent of the time in both the forecast error regression and the lagged forward premium regression, regardless of the presence of the risk premium. For experiments 3 and 5, the regressions which do not extract the risk premium reject this hypothesis approximately 15 per cent of the time, while those that do extract the risk premium only do so roughly ten per cent of the time. This provides some weak evidence that the model, under both non-expected utility preferences and habit persistence preferences, can generate a time-varying risk premium sufficiently large and variable enough to cause rejections of the expectations hypothesis. However both experiments have the large (counterfactual) bond price variability discussed earlier.

In addition, all experiments overwhelmingly reject the Wald test $b = -1$, a fact that is consistent with the U.S data but not with the Canadian evidence in Table 2.2. The other residual-based diagnostic tests presented, however, are less encouraging. Although fourth-order serial correlation and ARCH are present in Table 2.2, all experiments yielded only a five per cent rejection frequency for these tests.

5. Conclusion

Finding general equilibrium models of asset pricing that can quantitatively capture important features of bill prices is not going to be easy. While certain preference structures like habit persistence and non-expected utility are better able to meet this challenge than time-additive expected utility, there nevertheless remain aspects that are simply counterfactual. In the present study there appear to be two principal tensions between model and data: (i) variability in risk premiums and variability in bond prices and (ii) predictability in the risk premium. The first point simply reflects the fact that to generate the observed large risk premiums in the model we require more volatile bond prices than observed in data. The second is due to regression results using actual Canadian data which suggest that the risk premium can be predicted using current information like the forward premium. However the artificial economy is, for the most part, unable to deliver this correlation.

What seems clear from this and from other exercises is that more substantial changes are required to the general equilibrium model of asset pricing. Perhaps we need to drop the representative agent assumption and address the heterogeneity of agents in the market. While there might be a preference structure within the representative agent framework which yields predictions consistent with term structure data, we are not optimistic.

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Table 2.1

Treasury Bill Market
Selected Statistics, 1961:2-1988:3

Bond Prices

Statistic	Spot Price 3-Month q_{1t}	Spot Price 6-Month q_{2t}	Forward Price f_{1t}	Risk Premium $q_{1t+1}-f_{1t}$
Mean	0.98196	0.96364	0.98128	0.00050
	(0.00166)	(0.00320)	(0.00162)	(0.00025)
Standard Deviation	0.00817	0.01759	0.00801	0.00292
	(0.00121)	(0.00223)	(0.00111)	(0.00059)
Autocorrelation	0.93667	0.93479	0.92451	0.15074
	(0.03499)	(0.02756)	(0.02382)	(0.14341)

Notes to table: Data are from the seasonally unadjusted series of the 1989 CANSIM tape. The price series are constructed from three month and six month treasury bill prices (database numbers B0014007 and B0014008); the last price of each quarter is used. Numbers in parentheses are Newey-West (1987) standard errors, computed by GMM using four lags of the autocorrelation function.

Table 2.2

Regressions of $q_{1t+1} - f_{1t} = a + bx$ with Canadian treasury bill data

Variable or test	Estimated coefficient or statistic		
x	none	$f_{1t} - q_{1t}$	$q_{1t} - f_{1t-1}$
a	0.0005 (0.0003)	0.0000 (0.0003)	0.0004 (0.0003)
b	—— ——	-0.7071 (0.2980)	0.1495 (0.1260)
s	0.0029	0.0028	0.0029
DW	1.7003	1.8417	1.9139
AR1	0.1170	0.4104	0.6536
AR4	0.0015	0.0002	0.0028
ARCH1	0.2262	0.9028	0.3926
ARCH4	0.0000	0.0000	0.0000
Wald(a=b=0)	——	0.0253	0.0140
Wald(b=0)	——	0.0177	0.2355
Wald(b=-1)	——	0.3257	——

Notes to table: Data are from the 1989 CANSIM tape; the sample is from 1961:2 to 1988:3 (110 observations). Numbers in parentheses are heteroskedasticity-consistent standard errors, s is the standard error of the estimate, DW is the Durbin-Watson statistic, and ARn and ARCHn are marginal significance levels from LM tests for serial correlation and autoregressive heteroskedasticity of order n. The marginal significance levels for the Wald tests are based on the heteroskedasticity-consistent covariance matrix estimator.

Table 4.1

Canadian Consumption Growth and Inflation
Selected Statistics, 1961:2-1988:3

Per capita consumption growth (c_t/c_{t-1})

Statistic	Total	Durables	Non-durables	Services
Mean	1.01180	1.03197	1.00763	1.00780
	(0.00156)	(0.00299)	(0.00149)	(0.00113)
Standard Deviation	0.01798	0.05323	0.02095	0.01442
	(0.00073)	(0.00330)	(0.00188)	(0.00124)
Autocorrelation	-0.55901	-0.16350	-0.54700	-0.57019
	(0.07736)	(0.06366)	(0.07147)	(0.09580)

Price level growth (p_t/p_{t-1})

	Total	Durables	Non-durables	Services
Mean	1.01399	1.00866	1.01550	1.01497
	(0.00166)	(0.00174)	(0.00231)	(0.00150)
Standard Deviation	0.00884	0.01081	0.01411	0.00802
	(0.00077)	(0.00087)	(0.00107)	(0.00094)
Autocorrelation	0.75251	0.49778	0.50787	0.76326
	(0.05789)	(0.09796)	(0.08438)	(0.06388)

Notes to table: Data are from the seasonally unadjusted series of the 1989 CANSIM tape. The per capita consumption series, created using the series D0000001, D0010131, D0010132, D0010141, and D0010147, are quarterly data. The price series are constructed from the monthly CPI series, database numbers D0484486, D0484488, D0484490, and D0484491; the last price of each quarter is used. Each growth series is seasonally adjusted using OLS on seasonal dummies. Numbers in parentheses are Newey-West (1987) standard errors, computed by GMM using four lags of the autocorrelation function.

TABLE 4.2

Markov Estimation

Markov Process

$x_t \in \{ 0.9921,\ 1.0250\}$ $\qquad\qquad$ $p_t/p_{t-1} \in \{ 1.0061,\ 1.0291\}$

$$\hat{\pi} = \begin{bmatrix} 0.20000 & 0.48571 & 0.11429 & 0.20000 \\ 0.65517 & 0.17241 & 0.13793 & 0.03448 \\ 0.13043 & 0.26087 & 0.17391 & 0.43478 \\ 0.27273 & 0.04545 & 0.50000 & 0.18182 \end{bmatrix}$$

$$\hat{\pi}^* = [\ 0.32110 \quad 0.26606 \quad 0.21101 \quad 0.20183\ ]$$

Population moments

	μ	σ	ρ
x_t	1.0075	0.0164	-0.4740
p_t/p_{t-1}	1.0156	0.0113	0.3944

Correlation matrix:

x_t	1.0000	
p_t/p_{t-1}	0.0353	1.0000

Where $\hat{\pi}^*$ the stationary equilibrium distribution calculated by successive multiplication of the state contingent transition matrix.

Table 4.3

Population Moments

Prices

	$\bar{q}_1$	σ_{q1}	$\bar{fp}_1$	σ_{fp}
Observed:	0.98196	0.00166	0.00050	0.00080*
Experiment 1: Expected Utility $\alpha = 1.5$	0.94994	0.01233	0.00017	0.00002
Experiment 2: $\alpha = 1.5$ $\rho = 4.5$	0.93019	0.03314	0.00096	0.00012
Experiment 3: $\alpha = 1.5$ $\rho = 15.5$	0.86627	0.10417	0.00731	0.00114
Experiment 4: $\alpha = 1.5$ $d_1 = 0.2$ $d_2 = 0.2$	0.95156	0.03889	0.00180	0.00023
Experiment 5: $\alpha = 4.5$ $d_1 = 0.1$ $d_2 = 0.1$	0.93442	0.05706	0.00473	0.00072

Notes: $\bar{q}_1$, $\bar{fp}_1$ are the mean one period bond price and the mean one period forward risk premium respectively. σ_{q1}, σ_{fp} are the associated standard deviations. The actual value for σ_{fp} (0.00080*) is the estimated lower bound for the standard deviation of the risk premium discussed in the text.

Table 4.4

Experiment 1: Expected Utility
Number of rejections by 5 per cent tests in 1000 replications

$\alpha = 1.5$

equation	(1)	(2)	(3)	(4)	(5)	(6)
independent variable	none		$f_{1t} - q_{1t}$		$q_{1t} - f_{1t-1}$	
AR(1)	49	47	46	45	61	60
AR(4)	49	49	48	48	46	48
ARCH(1)	63	63	61	61	58	58
ARCH(4)	46	46	48	47	47	47
Wald (a=b=0)	-	-	85	86	85	81
Wald (b=0)	-	-	61	60	63	65
Wald (b=-1)	-	-	1000	1000	-	-

Notes: The odd numbered equations have $q_{1t+1}-f_{1t}$ as the dependent variable while the even numbered equations have $q_{1t+1}-f_{1t}-fp_{1t}$ as the dependent variable. The tests are the same as those presented in table 2.1. The number of observations for each replication is 110.

Table 4.5

Experiment 2: Non-Expected Utility
Number of rejections by 5 per cent tests in 1000 replications

$\alpha = 1.5,\ \rho = 4.5$

equation	(1)	(2)	(3)	(4)	(5)	(6)
independent variable	none		$f_{1t} - q_{1t}$		$q_{1t} - f_{1t-1}$	
AR(1)	57	56	50	50	47	44
AR(4)	51	50	49	49	45	45
ARCH(1)	51	50	59	56	62	60
ARCH(4)	54	55	53	53	57	57
Wald (a=b=0)	-	-	97	99	98	95
Wald (b=0)	-	-	64	58	73	70
Wald (b=-1)	-	-	1000	1000	-	-

Notes: See notes to table 4.4.

Table 4.6

Experiment 3: Non-Expected Utility
Number of rejections by 5 per cent tests in 1000 replications

$\alpha = 1.5,\ \rho = 15.5$

equation	(1)	(2)	(3)	(4)	(5)	(6)
independent variable	none		$f_{1t} - q_{1t}$		$q_{1t} - f_{1t-1}$	
AR(1)	57	52	55	55	46	48
AR(4)	49	47	47	47	47	48
ARCH(1)	57	55	63	62	61	59
ARCH(4)	53	51	55	55	57	57
Wald (a=b=0)	-	-	142	95	148	94
Wald (b=0)	-	-	74	60	75	68
Wald (b=-1)	-	-	1000	1000	-	-

Notes: See notes to table 4.4.

Table 4.7

Experiment 4: Habit Persistence
Number of rejections by 5 per cent tests in 1000 replications

$\alpha = 1.5,\ d_1 = 0.2,\ d_2 = 0.2$

equation	(1)	(2)	(3)	(4)	(5)	(6)
independent variable	none		$f_{1t} - q_{1t}$		$q_{1t} - f_{1t-1}$	
AR(1)	53	56	48	48	49	48
AR(4)	49	50	50	51	47	47
ARCH(1)	55	56	61	61	67	67
ARCH(4)	56	56	56	56	54	53
Wald (a=b=0)	-	-	101	95	104	95
Wald (b=0)	-	-	70	69	71	70
Wald (b=-1)	-	-	1000	1000	-	-

Notes: See notes to table 4.4.

Table 4.8

Experiment 5: Habit Persistence
Number of rejections by 5 per cent tests in 1000 replications

$\alpha = 4.5,\ d_1 = 0.1,\ d_2 = 0.1$

equation	(1)	(2)	(3)	(4)	(5)	(6)
independent variable	none		$f_{1t} - q_{1t}$		$q_{1t} - f_{1t-1}$	
AR(1)	57	52	51	52	48	48
AR(4)	48	50	46	48	45	46
ARCH(1)	59	59	61	61	65	63
ARCH(4)	56	53	54	54	57	55
Wald (a=b=0)	-	-	150	91	160	96
Wald (b=0)	-	-	71	65	76	68
Wald (b=-1)	-	-	1000	1000	-	-

Notes: See notes to table 4.4.

Fig.1 Volatility of Risk Premiums