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Money, Nominal Contracts, and the Business Cycle: I. One-Period Contract Case

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by

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Abstract

A modified version of the nominal contract developed by Gray (1976) and Fischer (1977) is introduced in a general equilibrium model with money which has been used in the real business cycle literature. Money is introduced in the model through cash-in-advance constraint. The contract studied is more efficient than that studied by Gray-Fischer in the sense that the processes involved in the calculation of the nominal contract are not from any other model but from the contract model itself. Two kinds of contract are examined, namely a nominal wage contract and a nominal price contract. A nominal wage contract improves the fit of the model in every respect. In other words, nominal wage contract resolves almost all controversies related to the real business cycle approach. The output volatility increases significantly and the correlation structure becomes much closer to the actual one. However, although a nominal price contract increases the output volatility enormously, it has some unrealistic features. The real and nominal wage rate have very high correlations with output, and a surprise in the technology shock has a negative effect on total hours, output, and real wage rate.

1. Introduction

After the seminal works by Finn Kydland and Edward Prescott (1982) and John Long and Charles Plosser (1983), small general equilibrium models have been widely used in research on the business cycle. These models have been successful in mimicking various aspects of the business cycle implied by the actual data. Given the simplicity of these models, their success is astounding. However, almost all of them are real models in the sense that they abstract from monetary side of the economy and so they are called models of "the real business cycle", following the terminology moulded by Long and Plosser (1983). Each of these uses small dynamic general equilibrium setup as a common starting point but introduces very distinct features, such as time-to-build technology and nonseparable preferences (Kydland and Prescott (1982)), many sectors of production (Long and Plosser (1983)), indivisibility of labor (Rogerson (1984), Hansen (1985)), work weeks of capital (Kydland and Prescott (1988)), deterministic, stochastic and/or endogenous growth (Hansen (1986), King, Plosser and Rebelo (1988)), heterogeneity (Cho and Rogerson (1988), Rebelo (1988)), inventory (Christiano (1988)), private information (Townsend (1988)), pubilc sector (Baxter and King (1988), King, Plosser and Rebelo(1988), Wynne (1989)), and international trade (Backus, Kehoe and Kydland (1989), Devereux, Gregory and Smith (1990)).

To the best of my knowledge there are two real business cycle models with money. Although they explicitly have a monetary sector, they are classified as real business cycle models since the dominant shock in the economy is a technology shock, while the monetary shock has a passive role. The role of inside money was emphasized by King and Plosser (1984). In their model, output and money are correlated due to the procyclical movement in inside money which is determined by the demand for financial services by agents. Cooley and Hansen (1989) introduced money in Rogerson-Hansen indivisible labor model through cash-in-advance constraint and examined the role of the inflation tax, which had been suggested to have some role in the business cycle by Lucas (1987). However, what they found is that inflation tax matters little along the business cycle, although changes in the average growth rate of money have significant effects in the long run.

These results may be considered to be unsatisfactory according to one's view on the propagation of monetary shocks. As Lucas (1987) pointed out, the inflation tax cannot be considered as the main transmission mechanism through which money affects the real side of an economy. Although it is still an open question how to model nominal rigidities in a dynamic general equilibrium setup, these are firmly believed by Keynesians to be the main machines magnifying the responsiveness of real variables to a nominal shock. However, the emphasized rigidities have been changing over time. In late seventies and early eighties, nominal wage rigidity was popular in Keynesian macroeconomics (for example, see Fischer (1977), and Taylor (1979, 1980)), but after that rigidity in nominal output prices has gained popularity (for example, see the extensive survey by Rotemberg (1986)). Mankiw (1986) effectively explained why they prefer nominal price rigidity to nominal wage rigidity. First, if the nominal wage contracts are responsible for large and inefficient fluctuations in employment, rational workers should not agree to them. Second, rigid nominal wages can be installment payments according to a long-term employment relationship and hence wages may not be important for determining employment. Third, it is predicted by the rigid nominal wage story that real wages are countercyclical along the business cycle, but this is not supported by the actual data. Contrary to these drawbacks of the nominal wage rigidity (or contract) story, nominal price rigidity story has the following advantages. First, since the benefit of changing prices is of second order, a small menu cost of changing prices can make rational firms not change their prices (see Mankiw (1985), Akerlof and Yellen (1985), and Parkin (1986)). Second, rigid prices like newspaper or vending machine prices represent transactions prices, not installment payments. Third, real wage moves procyclically in a rigid output price model. So Mankiw called nominal wage contract theories the "old" Keynesian view and rigid price theories the "new" Keynesian view.

Although the new Keynesian microfoundations are precious in the development of macroeconomics, it seems that the old Keynesian view has been considered "old" too easily. This paper introduces "old" and "new" nominal rigidities in a small dynamic general equilibrium model of the sort used in the real business cycle literature and examines the role of nominal rigidities over the business cycle. But it is very important how the rigidities are introduced in a model. Here a modified version of the idea developed by Gray (1976) and Fischer (1977) is followed. As McCallum (1989) emphasizes in his textbook in monetary economics, Gray-Fischer's setup has a couple of good features. In the Gray-Fischer nominal wage contract the nominal wage rate in period t is determined in period t-j as an expected market clearing wage rate and so the unexpected innovations in shocks between periods t-j and t (inclusive of period t) matter. However, as time period moves on far in the future, the effects of the innovations disappear and all variables return to their normal level. This means that the natural rate hypothesis holds in the long run but that monetary shocks have short run effects on real variables.

The contract nominal wage rate is exogenous in Gray-Fischer's setup in the sense that it is borrowed from the market clearing model without any contracts which is associated with a contract model. In other words, the processes involved in the calculation of the expected market-clearing wage rate are not from the contract model itself but from the market clearing version of the model. This may cause an extra inefficiency in the allocation in the model. The nominal contracts considered in this paper are endogenous in the sense that the processes involved in the calculation of the nominal contracts are not from any other model but from the contract model itself. However, this does not mean that the contracts considered in the paper do not involve any welfare costs. Since the contracts considered in this paper imply smaller volatility of real variables than Gray-Fischer's nominal contracts, the former is more efficient than the latter. However, the nominal contracts studied in this paper share the same steady state with the associated models without contracts and with Gray-Fischer contracts, and hence the natural rate hypothesis holds in the model studied in the paper.

The paper introduces nominal wage and price contracts in a cash in advance economy with capital accumulation of the sort studied by Lucas (1987) and Cooley and Hansen (1989). Implications of the rigidities are examined and the effects of nominal rigidities are studied using a numerical solution method. The method developed in this paper is a variation of the method developed by Kydland (1987) and applied by Cooley and Hansen (1989). The method simply adds a couple of steps for evaluating conditional expectations involved in the nominal wage and price contracts to Cooley and Hansen's method. The simulation results show that nominal rigidities have huge effects on the real side of the economy and that the effects of the rigidities are very different. A nominal wage contract improves the fit of the model virtually in all aspects but a nominal price contract has very counterintuitive effects. However, since multi-period contracts with capital accumulation imply a long memory (even though the underlying forcing processes are Markovian and the preferences are time-separable) in the sense that far past states matter in the current decision-making, one period contracts are looked into in this paper(see the Appendix for this point).

The next section describes the environment. Equilibria with nominal contracts are defined and a couple of important facts are discussed in section 3. Section 4 describes the solution method, specifies the model which will be simulated, and pins down parameter values. Results are reported in section 5, and Section 6 concludes.

2. The Economy

The economy consists of a continuum of identical agents (or households) distributed over the closed interval [0, 1]. Each agent is endowed with one unit of time in each period, initial capital stock k_0, and initial nominal money holdings m_0. The variables in lowercase letters will denote individual variables and those in capital letters denote the aggregate counterparts. Each agent maximizes his expected lifetime utility which is assumed to be time separable.

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \cdot u(c_t, \ell_t, e_t), \tag{2.1}$$

where c_t is consumption, ℓ_t is leisure in a week, e_t is proportion of weeks worked in a period, and β is a utility discount factor. That is, each period, say a quarter, is divided up into weeks and each individual in the economy faces two kinds of choices; he must choose the number of weeks in each period in which to work and also he must choose the number of hours to work in each of the week he does work. Since all agents are identical, e_t can be interpreted as the employment rate of the economy (see Cho and Cooley (1988) for a different justification). Even though we can allow complicated nonseparability between the leisure in a week and proportion of weeks worked, we assume the following intuitive form of temporal preferences.

$$u(c_t, \ell_t, e_t) = u(c_t) - v(1-\ell_t)e_t - \psi(e_t) \tag{2.2}$$

We assume the following regularity and Inada conditions.

(i) $u'(c_t) \geq 0$, $u''(c_t) \leq 0$, $\lim_{c_t \to 0} u'(c_t) = \infty$, and $\lim_{c_t \to \infty} u'(c_t) = 0$

(ii) $v'(n_t) \geq 0$, $v''(n_t) \geq 0$, $\lim_{n_t \to 0} v'(n_t) = 0$, and $\lim_{n_t \to 1} v'(n_t) = \infty$

(iii) $\psi'(e_t) \geq 0$, $\psi''(e_t) \geq 0$, $\lim_{e_t \to 0} \psi'(e_t) = 0$, and $\lim_{e_t \to 1} \psi'(e_t) = \infty$,

where $n_t = 1-\ell_t$ is hours worked in a week.

There are many identical firms producing homogeneous output. Each firm produces output with an identical production function homogeneous of degree one. Each firm is assumed not to have any market power, so firms do not care about the level of the output production and hence we can treat all firms as one production entity. The production function is subject to an aggregate productivity shock.

$$Y_t = \lambda_t F(K_t, Q_t), \tag{2.3}$$

where λ_t is the productivity shock common to all firms, and K_t and Q_t are aggregate capital stock and aggregate labor input in period t. $Q_t = N_t E_t$, where N_t is aggregate hours worked per week and the E_t is employment rate of the economy. We assume (2.3) is twice continuously differentiable and concave. If we define z_t as $z_t = log(\lambda_t)$, then z_t is assumed to follow an AR(1) process.

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \quad 0 \le \rho \le 1, \tag{2.4}$$

where ε_t is an i.i.d. random variable. We assume the following.

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{2.5}$$

It is assumed that the technology shock is known at the beginning of each period before any decisions are made.

Money is injected into the economy through lump sum transfer. If we let g_t denote the growth rate of money in period t, money follows the following process.

$$M_{t+1} = g_{t+1} M_t \tag{2.6}$$

Newly created money is distributed to each household in a lump-sum way and the growth rate of money is assumed to follow AR(1) process.

$$log(g_{t+1}) = \eta \cdot log(g_t) + \omega_{t+1}, \quad 0 \le \eta \le 1, \tag{2.7}$$

where ω_t is white noise. We assume the following.

$$\omega_t \sim N((1-\eta)log(g),\ \sigma_\omega^2) \tag{2.8}$$

So the unconditional mean of the money growth rate is g, which we will assume to be the steady state money growth rate. Individual agents hold money for the purpose of consumption, i.e. they face a cash-in-advance constraint.

$$P_t c_t \le m_{t-1} + (g_t-1)M_t, \tag{2.9}$$

where m_{t-1} is money holdings carried over from previous period, $(g_t-1)M_t$ is lump sum money transfer, and P_t is the price level in period t. We assume the growth rate of money is known together with the technology shock at the beginning of each period.

The last feature of the model is nominal contracts. We assume that the nominal wage rate or output price in period t is determined in a contract before the period arrives. There can be a couple of ways to write a nominal wage or price contract. We will postulate the following story. After the technology and monetary shocks have revealed themselves in each period, each agent chooses their consumption, investment and working hours and weeks etc., and then at the end of the period the future nominal price or wage rate is determined. We assume this process repeats over time. We will consider two types of contracts which last only one period.

$$log(W_t^c) = E[log(W_t)|\Omega_{t-1}] \tag{2.10}$$

$$log(P_t^c) = E[log(P_t)|\Omega_{t-1}] \tag{2.11}$$

where W_t^c and P_t^c are the nominal contract wage and contract price, W_t and P_t are the equilibrium processes of the nominal wage rate and price level implied by the contract model itself, and Ω_{t-1} is the information set available in period t. Here we have to note that Ω_{t-1} include the period t capital stock K_t. All agents involved in the economy know the structure of the model and the implied processes of the variables of interest. The nominal wage contract (2.10) has been emphasized by "old" Keynesians (for example, see Fischer (1977), Gray (1976), and Taylor (1979, 1980)) but the nominal price contract (2.11) has been emphasized by "new" Keynsians (for example, see Blinder and Mankiw (1984)). Under the contract (2.10), workers are assumed to cede the firm the right to determine the employment volume in period t, and under the contract (2.11) firm is assumed to commit itself to supply the quantity of output demanded in the market place at the pre-set price.

The differences between Gray-Fischer contracts and those used in the paper can be explained more clearly as follows. In the case of nominal wage contract, the nominal wage rate used in the expectation in (2.10) is the value of marginal product of labor and so, assuming a Cobb-Douglas production function, we have the following:

$$log(W^c_t) = E[log(P_t)+log(\lambda_t)+\theta log(K_t)-\theta log(Q_t)+log(1-\theta)|\Omega_{t-1}]. \tag{2.10'}$$

To evaluate the conditional expectations in (2.10'), we need the processes governing the technology shock, the price level, and the aggregate total hours (note that K_t is contained in Ω_{t-1}). However, the processes for the price level and the aggregate total hours are from the market clearing version of the model in the case of the Gray-Fischer contracts, and those are from the contract model itself in the contracts in this paper. This means that the contracts studied in the paper is invoking a fixed point concept in the sense that they are implied by the model and the allocations in the model are affected by the contracts.

3. Equilibrium and Discussions

To get an equilibrium, we need to look at the problems facing households and producers separately. Since money creates an efficiency wedge and there exists nominal contract(s), it is not possible to obtain an equilibrium by solving a programming problem. To solve the problem we need a definition of an equilibrium which will be operational in later sections.

A. Nominal Wage Contract

Under the nominal wage contract the firm solves the following profit maximizing problem.

$$\Pi_t = \max \ P_t \lambda_t F(K_t, Q_t) - W_t^c Q_t - R_t K_t \tag{3.1}$$
$$\text{s.t. } log(W_t^c) = E[log(W_t)|\Omega_{t-1}] \text{ is given.}$$
$$K_t \geq 0, \quad Q_t \geq 0,$$

where R_t is the rental rate of capital and W_t is the equilibrium nominal wage rate in period t. Note that the firm does not care about the composition of aggregate hours[1]. The first order conditions for the problem can be obtained

[1] If there are some fixed costs associated with hiring and firing of a worker, a firm does care about the composition of aggregate hours between aggregate hours worked per week and the number of workers in a period. In other words, a spectrum of fixity or Quasi-fixity of labor appears in the presence of fixed costs of hiring. A classical reference is Oi (1962).

in a straightforward way.

$$P_t \lambda_t F_2(K_t, Q_t) = W_t^c \tag{3.2}$$

$$P_t \lambda_t F_1(K_t, Q_t) = R_t \tag{3.3}$$

Taking expectations of both sides of (3.2) conditional on the information set at the end of period t-1, the following is obtained.

$$log(W_t^c) = E[log\{P_t \lambda_t F_2(K_t, Q_t)\}|\Omega_{t-1}], \tag{3.4}$$

where the fact that $E[log(W_t^c)|\Omega_{t-1}] = log(W_t^c)$ is used. That is, the contract wage rate is the expected equilibrium wage rate, which is from the model in the sense that the processes governing P_t, K_t, and Q_t are the equilibrium processes[2]. From (3.2) and (3.4), we have the following.

$$\{log(P_t) - E[log(P_t)|\Omega_{t-1}]\} + \{log(\lambda_t) - E[log(\lambda_t)|\Omega_{t-1}]\} + \{log(F_2(K_t, Q_t)) - E[log(F_2(K_t, Q_t))|\Omega_{t-1}]\} = 0 \tag{3.5}$$

So the surprises in price and productivity should be offset by the unexpected changes in marginal product of labor and so (3.5) states the well-known proposition in rational expectations that only unexpected parts of the shocks matter. If we use the Cobb-Douglas production function,

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[2] Note that the contract wage rate cannot be obtained until the equilibrium processes are known. In fact, setting the contract wage rate cannot be separated from the problem of solving the equilibrium. This is the feature that makes the problem more complicated than that studied by Cooley and Hansen (1989).

$$Y_t = \lambda_t K_t^{\theta} Q_t^{1-\theta} \tag{3.6}$$

we can write (3.5) as follows.

$$\begin{aligned} log(Q_t) = E[log(Q_t)|\Omega_{t-1}] &+ \frac{1}{\theta}\cdot\{log(P_t) - E[log(P_t)|\Omega_{t-1}]\} \\ &+ \frac{1}{\theta}\cdot\{log(\lambda_t) - E[log(\lambda_t)|\Omega_{t-1}]\} \end{aligned} \tag{3.7}$$

So we can see that the labor input in period t consists of three components. The first component is the expected labor input as of contract period t-1, which can be interpreted as the natural rate of employment. The second component is the price surprise and the third component is the productivity surprise. Using (3.7), we can perform very simple multiplier analysis on the supply side of the economy. Price and productivity surprises increase the use of labor input by more than one-to-one and the multipliers associated with those shocks are the same. Suppose the share of the capital stock in the production is $\theta=0.36$, which is the value used by Kydland and Prescott (1982) and others. Then the employment multipliers are 2.8 on the supply side. This means that under the assumed nominal wage contract unexpected disturbances in money and productivity are equally powerful sources of aggregate fluctuations. However, we have to emphasize the fact that these multipliers are obtained only from the production side responses to those shocks. If we take into account of the household side responses to those shocks, we can distinguish the effects between the shocks. Given nominal wage rate determined by the nominal contract, a price increase lowers the real wage rate and this discourages the supply of labor. On the contrary, a productivity increase lowers the price level (given the money supply) and so raises the real wage, which encourages the supply of labor further. So we can conjecture that the productivity shock will be a more powerful source of the business fluctuations than the price shock. If in addition we take into account the fact that both shocks have some effects on capital accumulation, we can see that the above argument is more plausible.

To define an equilibrium, we need to identify the state variables involved in the processes. Under the assumption that the contract is a one period nominal wage contract and money follows the process (2.6), state variables can be identified as follows. Since the contract wage in period t is set at the end of the period t-1, the money supply process between the two periods matters. That is, state variables from the money supply process are $log(g_{t-1})$, and ω_t, where $log(g_{t-1})$ is the log of the growth rate of money in the contract period and ω_t is a monetary innovation after the contract wage has been set. State variables from the productivity shock process are $log(\lambda_{t-1})$ and ε_t. Once again $log(\lambda_{t-1})$ is the size of the technology shock in the contract period and ε_t is the innovation in it after the contract. The third state variable is the current capital stock. Since the contract wage paid in period t is set at the end of period t-1, the capital stock in period t is known at the time the contract wage is set. These are the state variables facing the economy. We denote the aggregate state as:

$$S_t = (log(\lambda_{t-1}),\ \varepsilon_t,\ log(g_{t-1}),\ \omega_t,\ K_t)^T,$$

where superscript T denote the transpose of a matrix. In addition, we have to note that individuals take their own capital stock as one of the state variables and that money holdings from the previous period is also an individual state variable. However, to make the problem stationary we introduce a change in variables as $\hat{m}_t=m_t/M_t$ and $\hat{P}_t=P_t/M_t$ and take $\hat{m}_{t-1}$

as a state variable. So the state facing a household can be defined as:

$$s_t = (S_t, k_t, \hat{m}_{t-1})^T,$$

where k_t is the individual capital stock. Of course, $k_t=K_t$, $\hat{m}_{t-1}=1$ should hold in equilibrium.[3]

As was mentioned in the previous section, the right to determine the quantity of labor employed in period t is ceded to the firm by workers. Basically, (3.7) can be considered as the equation determining the quantity of labor. That is, after the technology and monetary shocks are revealed at the beginning of period t, the firm determines the quantity of labor employed in that period by adding the adjustments due to the price and technology surprise to the expected market clearing (or time-varying natural rate) quantity of labor. We denote the quantity of labor determined by the firm as Q_t^F. Once the quantity of aggregate labor input is determined by the firm, the representative agent's choice of hours per week and fraction of weeks worked in the market sector should be restricted by the the quantity chosen by the firm.

$$e_t \cdot n_t = Q_t^F \qquad (3.8)$$

So one of the two labor decisions made by the household is not a variable which can be chosen independently of the other labor decision.

Each household maximizes its lifetime expected utility subject to the

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[3] There can be other ways of identifying state variables, but the way in the text is used due to the fact that we are interested in the effects of surprises in money and productivity processes.

budget constraint, the cash in advance constraint, and the equations of motion involved. The equations of motion involved are those of the technology shock, the money growth, and the capital stocks. The processes governing the technology shock and money have been described in the previous section. Individual capital stock follows the usual process.

$$k_{t+1} = (1-\delta)k_t + x_t, \tag{3.9}$$

where x_t is the investment made in period t and δ is that rate of capital depreciation. Simply aggregating (3.9), we have the process governing the aggregate capital stock.

$$K_{t+1} = (1-\delta)K_t + X_t, \tag{3.10}$$

where X_t is the aggregate investment in period t. Finally, the budget constraint for the household can be written as follows.

$$c_t+x_t+\hat{m}_t/\hat{P}_t = (\hat{W}^c_t/\hat{P}_t)n_t e_t+(\hat{R}_t/\hat{P}_t)k_t+(\hat{m}_{t-1}+g_t-1)/(\hat{P}_t g_t) \tag{3.11}$$

Now the problem facing the household is to maximize the lifetime expected utility (2.1) subject to the constraints specified so far. For later references, we state the problem.

$$\text{maximize} \quad E_0 \sum_{t=0}^{\infty} \beta^t \cdot \{u(c_t) - v(1-\ell_t)e_t - \psi(e_t)\} \tag{3.12}$$

$$\text{s.t.} \quad c_t+x_t+\hat{m}_t/\hat{P}_t = (\hat{W}^c_t/\hat{P}_t)n_t e_t+(\hat{R}_t/\hat{P}_t)k_t+(\hat{m}_{t-1}+g_t-1)/(\hat{P}_t g_t) \tag{3.11}$$

$$c_t \le (\hat{m}_{t-1}+g_t-1)/(\hat{P}_t g_t) \tag{3.13}$$

$$log(\lambda_{t+1}) = \rho log(\lambda_t) + \varepsilon_{t+1} \tag{2.4}$$

$$log(g_{t+1}) = \eta log(g_t) + \omega_{t+1} \tag{2.6}$$

$$k_{t+1} = (1-\delta)k_t + x_t \tag{3.9}$$

$$K_{t+1} = (1-\delta)K_t + X_t \tag{3.10}$$

$$e_t \cdot n_t = Q_t^F \tag{3.8}$$

$$c_t \geq 0, \quad x_t \geq 0, \quad \hat{m}_t \geq 0, \quad 0 \leq n_t \leq 1, \quad 0 \leq e_t \leq 1,$$

where $\hat{W}_t^c = W_t^c/M_t$ and $\hat{R}_t = R_t/M_t$. Note here that the real contract wage and rental price of capital are the marginal product of those inputs respectively. We can represent the problem (3.12) using a recursive structure. Define $V(s_t)$ to be the equilibrium maximized present value of the utility stream of the representative household as of period t. Then, we can represent the problem (3.12) as follows.

$$V(s_t) = \max \ \{[u(c_t) - v(1-\ell_t)e_t - \psi(e_t)] + \beta E_t[V(s_t)|\Omega_t]\} \tag{3.14}$$
$$\text{s.t. the constraints in (3.12)}$$

However, if there are some periods when the cash in advance constraint is not binding, the problem is much too complicated to be solved, even numerically. So we will assume that cash in advance constraint is always binding. In fact, if the growth rate of money is sufficiently high, we can easily show that it is binding. The equilibrium for this economy can be defined as follows.

<u>Definition 1</u>: A stationary competitive equilibrium for the economy consists of a set of decision rules, $c(s_t)$, $x(s_t)$, $\hat{m}(s_t)$, $n(s_t)$, and $e(s_t)$, a set of aggregate decision rules, $C(S_t)$, $X(S_t)$, $N(S_t)$, and $E(S_t)$, price functions,

$\hat{P}(S_t)$ and $\hat{R}(S_t)$, and a value function $V(s_t)$ such that:

(i) the functions $V(s_t)$, $X(S_t)$, $N(S_t)$, $E(S_t)$, and $\hat{P}(S_t)$ satisfy (3.14) and $c(s_t)$, $x(s_t)$, $\hat{m}(s_t)$, $n(s_t)$, and $e(s_t)$ are the associated decision rules;

(ii) $x(s_t) = X(S_t)$, $n(s_t)=N(S_t)$, $e(s_t) = E(S_t)$, and $\hat{m}(s_t) = 1$ when $k_t=K_t$, and $\hat{m}_{t-1}=1$;

(iii) decision rules and pricing functions satisfy (3.7), and

(iv) the functions $C(S_t)$ and $X(S_t)$ satisfy $C(S_t) + X(S_t) = Y(S_t)$.

The key feature of the contract equilibrium is in (iii), which simply says that the quantity of labor is determined by the firm using the aggregate processes implied by the problem itself in the equation given by the wage contract. In fact, in this process the firm equates the marginal product of labor to expected marginal product of labor, which is the contract wage. However, the contents of the definition are not so self-revealing since equilibrium pricing functions and decision rules are used for the calculation of an equilibrium itself.

B. Nominal Price Contract

Suppose the period t price is determined at the end of t-1 but the wage rate is flexible. The condition we have to use in the case of price contract is the equilibrium condition in the output market. To get the condition, we need to assume that all firms are owned by households but that ownership and management are separated. Under this assumption the budget constraint becomes.

$$c_t + x_t + \hat{m}_t/\hat{P}^c_t = (\hat{W}_t/\hat{P}^c_t)n_t e_t + (\hat{R}_t/\hat{P}^c_t)k_t + \Pi_t/P^c_t$$

$$+ (\hat{m}_{t-1}+g_t-1)/(\hat{P}^c_t g_t), \tag{3.15}$$

where P^c_t is contract price and Π_t is profit of the firm distributed as dividends. Here we have to note two facts. First, since the firm commits itself to supply the quantity of output demanded in the market place at the pre-set contract price, the real wage rate is not necessarily the same as the marginal product of labor. For example, if the demand for output increases in the market place, the firm should increase its production regardless of the technical conditions of the firm. This means that the firm should pay higher nominal wages to attract more worker and/or increase the hours of those already working. Second, since labor is not paid its marginal product, profit is not necessarily zero even though the technology has constant returns to scale. This is why we include profit in the budget constraint.

By aggregating individual budget constraints we obtain the following market equilibrium condition in the final goods market under the assumption that the cash in advance constraint is binding.

$$X_t + 1/\hat{P}^c_t = \lambda_t F(K_t,\ Q_t), \tag{3.16}$$

where we used the fact that $\hat{m}_t=1$ in aggregate. Now to make the problem simpler, we linearize the condition (3.16) around the steady state in log-linear form. Assuming a Cobb-Douglas production function (3.6), the following can be obtained.

$$log(\hat{P}^c_t) = \gamma_{10} + \gamma_{11} log(X_t) + \gamma_{12} log(\lambda_t) + \gamma_{13} log(K_t) + \gamma_{14} log(Q_t), \tag{3.17}$$

where $\gamma_{10} = \{X[1-log(X)]+[1+log(\hat{P})]-\lambda K^{\theta}Q^{1-\theta}[1-log(\lambda)-\theta log(K)-(1-\theta)log(Q)]\}\hat{P}$,

$\gamma_{11} = X\hat{P}$, $\gamma_{12} = -\hat{P}\lambda K^{\theta}Q^{1-\theta}$, $\gamma_{13} = -\theta\hat{P}\lambda K^{\theta}Q^{1-\theta}$, $\gamma_{14} = -(1-\theta)\hat{P}\lambda K^{\theta}Q^{1-\theta}$, and the variables without subscript denote the steady state values of their subscripted counterparts. Here we know that $\gamma_{11}>0$, $\gamma_{12}<0$, $\gamma_{13}<0$, and $\gamma_{14}<0$. By taking expectations of both sides of (3.17) conditional on the information in period t-1, the contract price is obtained as follows.

$$log(\hat{P}^c_t) = \gamma_{10} + \gamma_{11}E[log(X_t)|\Omega_{t-1}] + \gamma_{12}E[log(\lambda_t)|\Omega_{t-1}] \\ + \gamma_{13}E[log(K_t)|\Omega_{t-1}] + \gamma_{14}E[log(Q_t)|\Omega_{t-1}], \quad (3.18)$$

where we used the fact that $E[log(P^c_t)|\Omega_{t-1}] = log(P^c_t)$.

Substituting the definition of contract price (3.18) in (3.17) and rearranging, we have the following relationship.

$$log(Q_t) = E[log(Q_t)|\Omega_{t-1}] + \gamma_{22}\{log(\lambda_t) - E[log(\lambda_t)|\Omega_{t-1}]\} \\ + \gamma_{23}\{log(X_t) - E[log(X_t)|\Omega_{t-1}]\}, \quad (3.19)$$

where $\gamma_{22}=-\gamma_{12}/\gamma_{14}<0$, $\gamma_{23}=-\gamma_{11}/\gamma_{14}>0$. (3.19) tells us a quite different story from (3.7). First, an unexpected increase in investment demand increases use of the labor input. This propagation channel has been emphasized in Keynesian macroeconomic models. Even though we abstract from the government sector, we can imagine the role for government expenditure in a nominal price contract model. Second, since γ_{22} is negative in (3.19), an unexpected increase in the technology shock reduces use of the labor input. However, an unexpected technology shock affects investment, so it has an indirect effect on the use of labor input by increasing demand for output. If the indirect effect through the demand increase is smaller than the direct effect through supply in the use of labor input, it is possible for the firm to reduce the use of labor input. If this is true, a nominal price contract has a couple of implications. Most of all, an unexpected productivity increase can affect the labor adversely, which is very counter-intuitive. In addition, if the firm is to reduce the use of labor input given the contract price, it should lower the wage rate even though the productivity has increased. This is also very counter-intuitive. This will be discussed again in the simulation section. Third, an unexpected supply of money increases aggregate demand without affecting aggregate supply initially, so a nominal shock has an undoubtedly positive effect on employment and output.

Before we formally define the equilibrium in this case, we have to postulate the firm's behavior more specifically. First, capital is assumed to be paid its value of marginal product[4]. That is, the following holds

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[4] Some may argue that the firm is different from the one assumed in the text. That is, the firm can be assumed to minimize costs of producing output determined in the market.

$$\begin{aligned} \min \quad & W_t \cdot Q_t + R_t \cdot K_t \\ \text{s.t.} \quad & \lambda_t K_t^{\theta} Q_t^{1-\theta} = X_t + 1/\hat{P}_t^c \\ & Q_t \geq 0, \quad 0 \leq K_t \leq \bar{K}_t \end{aligned}$$

But since the capital stock is predetermined in each period and the firm behaves competitively in the factor market, the first order condition under the assumption that the capital stock is fully utilized can be obtained as follows.

$$W_t/R_t \geq [(1-\theta)/\theta] \cdot (\bar{K}_t/Q_t)$$

However, there are a couple of problems here. First, multiple solutions cannot be ruled out. Second, the case that equality holds in the first order condition was used for a simulation of the model but it was found that the system does not converge and so it seems not so practical to have the cost-minimizing behavior. The assumption made can be justified in a few ways. If there is a potential foreign competitor, the firm should pay capital the value of its marginal product. In addition, it is not believed that the results obtained in the paper depend on this specific firm behavior.

in the capital market.

$$R_t = P^c_t \lambda_t F_1(K_t, Q_t) \tag{3.20}$$

Second, since the firm commits itself to supply the output demanded in the market, the market-clearing condition in the goods market (3.16) determines the quantity of labor demanded by the firm. The nominal wage should be determined in the system by equating the labor supply and its demand determined by the demand for goods. This means that the wage rate prevailing in the market place is not the same as the value of the marginal product of labor.

Now the problem facing the household can be written recursively as follows.

$$V(s_t) = \max \ \{[u(c_t) - v(1-\ell_t)e_t - \psi(e_t)] + \beta E_t[V(s_t)|\Omega_t]\} \tag{3.21}$$

$$\text{s.t.} \quad c_t+x_t+\hat{m}_t/\hat{P}^c_t = (\hat{W}_t/\hat{P}^c_t)n_t e_t+(\hat{R}_t/\hat{P}^c_t)k_t+\Pi_t/P^c_t + (\hat{m}_{t-1}+g_t-1)/(\hat{P}^c_t g_t) \qquad (3.15)$$

$$c_t = (\hat{m}_{t-1}+g_t-1)/(\hat{P}^c_t g_t) \tag{3.22}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \tag{2.4}$$

$$log(g_{t+1}) = \eta log(g_t) + \omega_{t+1} \tag{2.6}$$

$$k_{t+1} = (1-\delta)k_t + x_t \tag{3.9}$$

$$K_{t+1} = (1-\delta)K_t + X_t \tag{3.10}$$

$$c_t \geq 0, \quad x_t \geq 0, \quad \hat{m}_t \geq 0, \quad 0 \leq n_t \leq 1, \quad 0 \leq e_t \leq 1,$$

where $\hat{W}_t=W_t/M_t$, $\hat{R}_t=R_t/M_t$, $\hat{P}^c_t=P^c_t/M_t$ and the contract price is in (3.18). An equilibrium under the one-period nominal price contract can be defined as follows.

<u>Definition 2</u>: A stationary competitive equilibrium for the economy consists of a set of decision rules, $c(s_t)$, $x(s_t)$, $\hat{m}(s_t)$, $n(s_t)$, and $e(s_t)$, a set of aggregate decision rules, $C(S_t)$, $X(S_t)$, $N(S_t)$, and $E(S_t)$, price functions, $\hat{W}(S_t)$ and $R(\hat{S}_t)$, and a value function $V(s_t)$ such that:

(i) the functions $V(s_t)$, $X(S_t)$, $N(S_t)$, $E(S_t)$, and $\hat{W}(S_t)$ satisfy (3.21) and $c(s_t)$, $x(s_t)$, $m(s_t)$, $n(s_t)$, and $e(s_t)$ are the associated decision rules;

(ii) $x(s_t) = X(S_t)$, $n(s_t)=N(S_t)$, $e(s_t) = E(S_t)$, and $\hat{m}(s_t) = 1$ when $k_t=K_t$ and $\hat{m}_{t-1}=1$;

(iii) decision rules and pricing functions satisfy (3.18) and (3.20);

(iv) the functions $C(S_t)$ and $X(S_t)$ satisfy $C(S_t) + X(S_t) = Y(S_t)$.

4. Simulation

A. *Specification*

The model used in the simulation in this section is specified as follows.

$$u(c_t, n_t, e_t) = log(c_t) - \frac{\alpha_1}{1+\gamma}\cdot n_t^{1+\gamma}\cdot e_t - \frac{\alpha_2}{1+\tau}\cdot e_t^{1+\tau} \tag{4.1}$$

$$Y_t = \lambda_t K_t^{\theta} Q_t^{1-\theta} \tag{4.2}$$

The preference specification (4.1) allows us to identify the elasticity of intertemporal substitution of labor easily. It can be obtained as:

$$\mathcal{E}(Q) = \frac{1+\gamma+\tau}{\gamma\tau}. \tag{4.3}$$

The Cobb-Douglas production function is common in most equilibrium business cycle models.

B. Calibration

Among the preference parameters those determining the elasticity of intertemporal sustitution are estimated using PSID data and the first order conditions implied by the household's optimization problem. The values obtained are $\gamma=1.2$ and $\tau=2$[5]. The other preference parameters are obtained considering the following two facts. First, the steady state total hours is about one third of the time endowment. Second, since the population is the labor force in the model, the steady state employment population ratio is assumed to be 65 percent, which is the ratio from the U.S. economy. These two facts require that the parameter values be $\alpha_1=6$ and $\alpha_2=1.5$. The other technology parameters and discounting factors are from Prescott (1986) as follows: $\beta=.99$, $\delta=.025$, $\theta=.36$. The remaining parameters are those in technology and monetary shock processes. For the technology shock the persistence and size of the shock are assumed as $\rho=.95$ and $\sigma_\varepsilon=.009$. The assumed size of the shock is on the upper bound of the range estimated by Prescott (1986). The value of parameters involved in the growth rate of money process are taken from the estimates in Cooley and Hansen (1989) as $\eta=.48$, $\sigma_\omega=.009$ and the mean growth rate is assumed as $\bar{g}=1.15$. Actually the mean growth rate of money does not play a significant role in the model economy.

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5 The implied elasticity of intertemporal substitution of labor is 1.75, which is in the range of estimates by Alogoskoufis (1987).

C. Steady State

Three cases are simulated, namely the model without a nominal contract, the model with a nominal wage contract, and the model with a nominal price contract. All three cases share the same steady state, which can be obtained as follows.

$$\Omega_1 = \left[\frac{1-\beta(1-\delta)}{\lambda\beta\theta}\right]^{1/(\theta-1)}$$

$$\Omega_2 = \frac{\beta\lambda(1-\theta)\cdot\Omega_1^{\theta}}{\alpha_1(\lambda\cdot\Omega_1^{\theta} - \delta\cdot\Omega_1)\cdot\bar{g}}$$

$$E = \left[\frac{\Omega_2\cdot\alpha_1\cdot\gamma}{\alpha_2\cdot(1+\gamma)}\right]^{1/(1+\tau)}$$

$$N = (\Omega_2/E)^{1/(1+\gamma)}$$

$$Q = E\cdot N$$

$$K = \Omega_1\cdot Q$$

$$X = \delta\cdot K,$$

where the variables without a subscript are the steady state values of the counterparts with a subscript. Output and consumption can be obtained from the production function and resource constraint respectively. Ω_1 is the capital-labor ratio and Ω_2 is defined for convenience.

D. Solution Method: Nominal Wage Contract

The solution method used in the paper is a variation of the method suggested by Kydland (1987) and applied by Cooley and Hansen (1989). If there is no nominal contract, the problems in section 3 are the same in its dynamic nature as the model studied by Cooley and Hansen (1989). As a reference case, we will use the economy without contracts. Simulation of the model without contracts involves computing a linear-quadratic approximation to the household's problem and then solving the problem by iterating on Bellman's equation. The special feature in the solution method in the case without any contract is that it is required at each step of the iteration that the second condition in the Definition 1 or Definition 2 holds. The method used to solve the cases with a nominal contract contains these steps and in addition steps calculating conditional expectation involved in the nominal contract.

Step 1: Since we have to evaluate conditional expectations for the purpose of obtaining a contract nominal wage rate, we need a guess on the system of equilibrium decision rules. However, the equation determining the quantity of labor demanded by the firm (3.7) involves only the equilibrium processes for $log(Q_t)$ and $log(P_t)$. Suppose the guesses are as follows.

$$log[Q(S_t)] = \pi_{10} + \pi_{11} log(\lambda_{t-1}) + \pi_{12}\varepsilon_t + \pi_{13} log(g_{t-1}) + \pi_{14}\omega_t + \pi_{15} log(K_t) \quad (4.4)$$

$$log[\hat{P}(S_t)] = \pi_{20} + \pi_{21} log(\lambda_{t-1}) + \pi_{22}\varepsilon_t + \pi_{23} log(g_{t-1}) + \pi_{24}\omega_t + \pi_{25} log(K_t), \quad (4.5)$$

where $\hat{P}(S_t) = P(S_t)/M_t$. One important fact to note is that the guess should be from a model sharing the same steady state. Since the coefficient in front of the term $E[log(Q_t)|\Omega_{t-1}]$ in (3.7) is one, a guess from a model without sharing the same steady state with the wage contract model can have a serious bias which cannot be corrected through iteration. In fact, we have only one possible guess known to us, namely the rules from the model without nominal contracts, so we use the decision rules for total hours and price as our initial guess.

Using the guess, the total hours demanded by the firm can be obtained as follows.

$$log(Q_t^F) = \gamma_{30} + \gamma_{31}log(\lambda_{t-1}) + \gamma_{32}\varepsilon_t + \gamma_{33}log(g_{t-1}) + \gamma_{34}\omega_t + \gamma_{35}log(K_t), \quad (4.6)$$

where $\gamma_{30} = \pi_{10}+[\pi_{14}-1/\theta(1+\pi_{24})](1-\eta)log(\bar{g})$, $\gamma_{31} = \pi_{11}$, $\gamma_{32} = (1/\theta)(1+\pi_{22})$, $\gamma_{33} = \pi_{13}$, $\gamma_{34} = (1/\theta)(1+\pi_{24})$, and $\gamma_{35} = \pi_{15}$.

Step 2: The next step is to approximate the problem with quadratic objective function and linear constraints. Since each household's labor choice is constrained by the firm's demand for labor (4.6), we have the following.

$$n_t = Q_t^F/e_t \quad (4.7)$$

Using the pricing functions of labor and capital in the budget constraint, the following can be obtained.

$$\hat{P}_t = \hat{m}_t/[(1-\theta)(\exp(\rho z_{t-1}+\varepsilon_t)\cdot K_t^{\theta}\cdot(Q_t^F)^{1-\theta} + \theta(\exp(\rho z_{t-1}+\varepsilon_t)\cdot K_t^{\theta-1}\cdot(Q_t^F)^{1-\theta}\cdot k_t - x_t] \quad (4.8)$$

Substituting (4.7), (4.8) and the binding cash-in-advance constraint (2.8) into the temporal utility function (4.1) and then approximating the resulting utility function around the steady state by the method developed by Kydland and Prescott (1982), the quadratic temporal preference can be obtained as follows.

$$u_t = (\bar{s}_t^T \; X_t \; v_t^T)\Gamma \begin{bmatrix} \bar{s}_t \\ X_t \\ \bar{v}_t \end{bmatrix}, \tag{4.9}$$

where $\bar{s}_t^T=(1,\ z_{t-1},\ \varepsilon_t,\ log(g_{t-1}),\ \omega_t,\ K_t,\ k_t,\ \hat{m}_{t-1})$, $v_t^T=(x_t,\ \hat{m}_t,\ e_t)$ and Γ is the symmetric matrix obtained from the quadratic approximation.

Step 3: The Bellman's equation associated with the quadratic version of the utility function can be written as:

$$\bar{s}_t^T V \bar{s}_t = \max \left\{ (\bar{s}_t^T \; X_t \; v_t^T)\Gamma \begin{bmatrix} \bar{s}_t \\ X_t \\ v_t \end{bmatrix} + \beta\cdot\bar{s}_{t+1}^T V \bar{s}_{t+1} \right\} \tag{4.10}$$

s.t. (2.4), (2.6), (3.8), (3.9),

and a perceived process governing X_t as a function of aggregate state $\bar{S}_t=(1\ z_{t-1}\ \ \varepsilon_t\ \ log(g_{t-1})\ \ \omega_t\ \ K_t)$. We iterate on the approximated quadratic Bellman's equation with a starting guess for the matrix V, say V_0, and solve the maximization problem on the righthand side of (4.10). Using the law of motions in the objective function, we have the individual decision rules.

$$v_t = D_1\bar{s}_t + D_2X_t \tag{4.11}$$

By imposing the equilibrium condition (ii) of Definition 1, i.e. $\hat{m}_t=\hat{m}_{t-1}=1$, $k_t=K_t$, and $x_t=X_t$, we have the aggregate decision rule for investment and the number of workers as functions of state of the economy $\bar{S}_t$.

follows.

$$u_t = (\bar{s}_t^T \; X_t \; v_t^T)\Gamma \begin{bmatrix} \bar{s}_t \\ X_t \\ \bar{v}_t \end{bmatrix}, \tag{4.9}$$

where $\bar{s}_t^T=(1,\ z_{t-1},\ \varepsilon_t,\ log(g_{t-1}),\ \omega_t,\ K_t,\ k_t,\ \hat{m}_{t-1}),\ v_t^T=(x_t,\ \hat{m}_t,\ e_t)$ and Γ is the symmetric matrix obtained from the quadratic approximation.

Step 3: The Bellman's equation associated with the quadratic version of the utility function can be written as:

$$\bar{s}_t^T V \bar{s}_t = \max \left\{ (\bar{s}_t^T \; X_t \; v_t^T)\Gamma \begin{bmatrix} \bar{s}_t \\ X_t \\ v_t \end{bmatrix} + \beta \cdot \bar{s}_{t+1}^T V \bar{s}_{t+1} \right\} \tag{4.10}$$

s.t. (2.4), (2.6), (3.8), (3.9),

and a perceived process governing X_t as a function of aggregate state $\bar{S}_t=(1\ z_{t-1}\ \ \varepsilon_t\ \ log(g_{t-1})\ \ \omega_t\ \ K_t)$. We iterate on the approximated quadratic Bellman's equation with a starting guess for the matrix V, say V_0, and solve the maximization problem on the righthand side of (4.10). Using the law of motions in the objective function, we have the individual decision rules.

$$v_t = D_1\bar{s}_t + D_2X_t \tag{4.11}$$

By imposing the equilibrium condition (ii) of Definition 1, i.e. $\hat{m}_t=\hat{m}_{t-1}=1$, $k_t=K_t$, and $x_t=X_t$, we have the aggregate decision rule for investment and the number of workers as functions of state of the economy $\bar{S}_t$.

$$X_t = \gamma_{40} + \gamma_{41} log(\lambda_{t-1}) + \gamma_{42}\varepsilon_t + \gamma_{43} log(g_{t-1}) + \gamma_{44}\omega_t + \gamma_{45}K_t, \tag{4.12}$$

$$E_t = \gamma_{50} + \gamma_{51} log(\lambda_{t-1}) + \gamma_{52}\varepsilon_t + \gamma_{53} log(g_{t-1}) + \gamma_{54}\omega_t + \gamma_{55}K_t, \tag{4.13}$$

Using the aggregate decision rules, individual decision rule, the laws of motion involved, and the initial guess V_0 on the righthand side of (4.12), we have the value function for the next iteration, say V_1, and using V_1, we repeat the maximization process again. We repeat iteration until value function from the (j+1)-th iteration V_{j+1} is sufficiently close to the j-th iteration V_j.

Step 4: The next step is to derive decision rules of interest using decision rules obtained from the final iteration in *Step 5*. First, we need to obtain the process governing aggregate hours. For this we rewrite the decision rules obtained in *Step 4* in the following way.

$$log(X_t) = \hat{\gamma}_{40} + \hat{\gamma}_{41} log(\lambda_{t-1}) + \hat{\gamma}_{42}\varepsilon_t + \hat{\gamma}_{43} log(g_{t-1}) + \hat{\gamma}_{44}\omega_t + \hat{\gamma}_{45}K_t \tag{4.14}$$

$$log(E_t) = \hat{\gamma}_{50} + \hat{\gamma}_{51} log(\lambda_{t-1}) + \hat{\gamma}_{52}\varepsilon_t + \hat{\gamma}_{53} log(g_{t-1}) + \hat{\gamma}_{54}\omega_t + \hat{\gamma}_{55}K_t, \tag{4.15}$$

where $\hat{\gamma}_{40}=(\gamma_{40}/X)-(1-log(X))$, $\hat{\gamma}_{50}=(\gamma_{50}/E)-(1-log(E))$, $\hat{\gamma}_{4j} = \gamma_{4j}/X$, and $\hat{\gamma}_{5j} = \gamma_{5j}/E$ for j=1, 2,$\cdots$, 5. From the first order conditions for the household's problem, we can have the following.

$$log(N_t) = \frac{1}{1+\gamma}\cdot log\left[\frac{\alpha_1(1+\gamma)}{\alpha_1\gamma}\right] + \frac{\tau}{1+\gamma}\cdot log(E_t) \tag{4.16}$$

The equilibrium process governing aggregate hours then becomes:

$$log(N_t) = \gamma_{60} + \gamma_{61}log(\lambda_{t-1}) + \gamma_{62}\varepsilon_t + \gamma_{63}log(g_{t-1}) + \gamma_{64}\omega_t + \gamma_{65}K_t, \tag{4.17}$$

where $\gamma_{60}=\frac{1}{1+\gamma}\cdot log\left[\frac{\alpha_1(1+\gamma)}{\alpha_1\gamma}\right] + \frac{\tau}{1+\gamma}\cdot\hat{\gamma}_{50}$ and $\gamma_{6j}=\frac{\tau}{1+\gamma}\cdot\hat{\gamma}_{6j}$ for $j=1,\cdots,5$. From (4.15) and (4.16), we have the aggregate hour process.

$$log(Q_t) = \gamma_{70} + \gamma_{71}log(\lambda_{t-1}) + \gamma_{72}\varepsilon_t + \gamma_{73}log(g_{t-1}) + \gamma_{74}\omega_t + \gamma_{75}K_t, \tag{4.18}$$

where $\gamma_{7j}=\hat{\gamma}_{5j}+\gamma_{6j}$ for $j = 0, 1,\cdots, 5$.

On the other hand, linearizing the aggregate resource constraint yields:

$$log(\hat{P}_t) = \gamma_{10} + \gamma_{11}log(X_t) + \gamma_{12}z_t + \gamma_{13}log(K_t) + \gamma_{14}log(Q_t), \tag{4.19}$$

where γ_{1j}'s are defined in (3.17). Using the processes of aggregate investment and hours, (4.19) can be rewritten as:

$$log(\hat{P}_t) = \gamma_{80} + \gamma_{81}log(\lambda_{t-1}) + \gamma_{82}\varepsilon_t + \gamma_{83}log(g_{t-1}) + \gamma_{84}\omega_t + \gamma_{85}K_t, \tag{4.20}$$

where $\gamma_{80}= \gamma_{10} + \gamma_{11}\hat{\gamma}_{41} + \gamma_{41}\gamma_{71} + \gamma_{13}(log(K)-1)$, $\gamma_{81}=\gamma_{11}\hat{\gamma}_{42} + \gamma_{12}\rho + \gamma_{14}\gamma_{72}$,

$\gamma_{82} = \gamma_{11}\hat{\gamma}_{42} + \gamma_{12} + \gamma_{14}\gamma_{72}$, $\gamma_{83} = \gamma_{11}\hat{\gamma}_{43} + \gamma_{14}\gamma_{73}$, $\gamma_{84}=\gamma_{11}\hat{\gamma}_{44} + \gamma_{14}\gamma_{74}$, and $\gamma_{85} = \gamma_{11}\hat{\gamma}_{45} + \gamma_{14}\gamma_{75} + \gamma_{13}/K$.

Step 5: The decision rule for aggregate hours (4.18) and the process governing price (4.20) is used as a new initial guess and then steps from *Step 1* to *Step 4* are repeated[6].

Step 6: Steps 1 - 5 are repeated until the initial guess in *i*-th repetition is sufficiently close to the rules obtained in Step 4 in that repetition. If the differences between those decision rules are close enough, then we claim that iterations have converged. We use the converged decision rules to generate data sets. The generated data sets are detrended by Hodrick-Prescott filter. Finally using the detrended data set, we calculate statistics of interest from each data set and then calculate the averages and variances of the statistics obtained from each data set.

E. *Solution Method: Nominal Price Contract*

Step 1: We need an initial guess on the decision rules of investment and aggregate labor. Let the conjecture be the following.

$$\log[X(S_t)] = \pi_{30} + \pi_{31}\log(\lambda_{t-1}) + \pi_{32}\varepsilon_t + \pi_{33}\log(g_{t-1}) + \pi_{34}\omega_t + \pi_{35}K_t \tag{4.21}$$

$$\log[Q(S_t)] = \pi_{40} + \pi_{41}\log(\lambda_{t-1}) + \pi_{42}\varepsilon_t + \pi_{43}\log(g_{t-1}) + \pi_{44}\omega_t + \pi_{45}K_t \tag{4.22}$$

We will use the decision rules from the case of no contracts as the initial

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[6] In fact, the decision rules obtained from the first round of the iteration are those in the case of Gray-Fischer nominal wage contract.

guess once again. Using the guess, we calculate the contract price from (3.17) as follows.

$$log[\hat{P}^c(S_t)] = \gamma_{90} + \gamma_{91}log(\lambda_{t-1}) + \gamma_{93}log(g_{t-1}) + \gamma_{95}K_t, \qquad (4.23)$$

where $\gamma_{90}=\gamma_{10}+\pi_{30}+\pi_{40}+(\gamma_{11}\pi_{34}+\gamma_{14}\pi_{44})(1-\eta)log(g)+(log(K)-1)$, $\gamma_{91}=\gamma_{11}\pi_{31}+\gamma_{12}\rho+\gamma_{14}\pi_{41}$, $\gamma_{93}=\gamma_{11}\pi_{31}+\gamma_{14}\pi_{43}$, $\gamma_{95}=\gamma_{11}\pi_{35}+\gamma_{13}/K+\gamma_{14}\pi_{45}$. We use (4.30) in the binding cash in advance constraint and the budget constraint.

Step 2: This step is the same as *Step 2* in nominal wage contract case with a couple of provisos. First, since the labor demand is determined by the demand for output, it is determined by the market clearing condition (3.16). Using the Cobb-Douglas production function in (3.16), labor demand can be obtained as follows.

$$Q_t^d = \left[\frac{X_t+1/\hat{P}_t^c}{\lambda_t K_t^{\theta}}\right]^{1/(1-\theta)} \qquad (4.24)$$

Using (4.24) and the value of marginal product of capital, the budget constraint can be solved for n_t, and then the resulting n_t and the binding cash-in-advance constraint are substituted in the temporal utility function. One fact we have to note is that the nominal wage rate should be determined in the system and so it is one of the aggregate decisions. That is, the aggregate decisions contained in the approximated preferences are aggregate investment, X_t, and the nominal wage rate, W_t.

The remaining steps are the same as in the case of nominal wage contract.

5. Results

The summary statistics from the U.S. economy are in Table 1. The main differences in the statistics presented here from those found in other papers are in the labor and price variables. Total hours and total number of employed persons were taken from Citibase database and the per capita hours are obtained by dividing total hours by total number of employed persons. Most of the real business cycle studies have looked at either the number of employed persons or per capita hours. Since about one quarter of the volatility in aggregate hours is explained by the fluctuations in per capita hours and the remaining three quarters is explained by the fluctuations in the number of workers, we need to model both margins of fluctuations in the aggregate labor market as in the paper. One notable fact is that aggregate hours fluctuate more than output. Another fact which is worthwhile to note is that nominal price and nominal wage rate are significantly countercyclical and that the correlations of productivity and real wage rate with other variables are low. These are the findings in Christiano and Eichenbaum (1988) and Cooley and Hansen (1989). Although many real business cycle models are successful in a qualitative sense, some of the correlations seem to be very difficult to mimic with a model which has only a technology shock.

The statistics from a model without a nominal contract are reported in Table 2. Here we have to note that the size of the technology shock is around the upper bound among the estimates reported by Prescott (1986) and that the elasticity of intertemporal substitution of labor is 1.75 in the model. With the elasticity implied by the PSID data, the model does not seem to explain the volatility of output successfully. Even though the size of the fluctuations of output in the model is 5 percent below that in the U.S. economy, the technology shock variance should be increased far more than 5 percent to hit the actual number from the data. However, the most unsuccessful features of the model are in the implied correlation structure. Overall all variables except consumption are too highly correlated with each other. The reason why consumption series shows low correlation is that it is constrained by the cash-in-advance constraint. Especially, labor variables such as total hours, per capita hours, number of employed persons and real wage rate have extremely high correlation with output and this is a very stubborn feature of a model with one technology shock. The other unsuccessful feature in the labor variables is the ratio of standard deviation of total hours relative to the productivity. The ratio is 1.65 in the U.S. economy, but it is .85 in the model economy. In fact, this ratio has been emphasized as a measure of goodness of fit of a model economy by many authors (for example, see Kydland and Prescott (1982), Hansen (1985), and Cho and Cooley (1988)).

Table 3 contains the summary statistics from a model with nominal wage contract. As soon as one period nominal wage contract is introduced in the model, the fit of the model improves amazingly. First of all, with the same size of the shocks and the elasticity of intertemporal sustitution, the volatility of output increases about 58 percent and the magnitudes of fluctuations of all variables increase. This means that to have the same size of the fluctuations in the model as in the U.S. economy we need far smaller shocks than those assumed in the simulation. The ratio of standard deviation of the total hours to that of productivity is also increased to 2.17. Correlations of every real variable with other variables closely mimic those from the actual U.S. economy. Especially the improvement in the fit of labor variables is surprising. The most notable fact from the table is in the correlation of real wage rate with output. In fact, the real wage rate in the model is slightly procyclical.

Here we need to recall Mankiw's evaluation of "old" Keynesian view which was summarized in the introduction. He argued that the most serious one among the problems with the view is that real wage is countercyclical. However, his conclusion is based on the assumption that the dominant shock in the economy is from the nominal side. If there is a real shock whose size is comparable to the monetary shock, his argument is not neccessarily true. In other words, since a real shock tends to reduce the price pressure, it is highly likely that the real wage rate is procyclical in a model with a real shock as well as a nominal shock. In addition, the result answers the question raised by Christiano and Eichenbaum (1989). Looking at the fact that the real wage rate from the actual data has a pretty low correlation with output (it is .2 according to their data set), they cast doubt on the fit of real business cycle models. It is .09 in the model with a nominal wage contract. We can call the improvement in the fit of the model a "contract effect". However, the contract effect is not solely from nominal side of the model but also from the real side of the model. In fact, the real shock seems to be more important in a nominal wage contract model than the nominal shock. This will be discussed below.

We can find the fit of the model with a nominal price contract in Table 4. This is a very different economy. Although the nominal price contract is of one period, its effects are significant in every aspect. Output fluctuations increase more than five times that found in the model without contract, and every variable except consumption also has incredible volatility. A good feature of the model is that the size of the shocks do not have to be large. To have the same volatility of output in the model as in the U.S. economy we can reduce the size of the shocks to one tenth of the size assumed in the simulation. However, there are problems in correlation coefficients. Consumption is countercyclical, but since this feature is from the binding cash in advance constraint, we cannot blame the nominal price contract. Labor variables including real wages still have very high correlations with output. In addition, even nominal variables are procyclical. Especially, nominal wage has a correlation with output that is too high. Although the nominal price contract has a really large effect on volatility of all variables, it does not improve the correlation structure of the model at all. On the contrary, the correlation structure has very uncomfortable features.

Differences among the models can be found in the decision rules. Table 5 reports these for each case. The decision rules in the case of no nominal contract are very similar to those in Cooley and Hansen (1989) except that the magnitutes of the coefficients are slightly smaller because of the difference in the values of the elsticity of intertemporal substitution assumed in the models. The technology shock has a positive effect on total hours and output but the monetary shock has a negative effect on them. The decision rules in case of the nominal wage contract can be found in Table 5 B. At a glance we can find a couple of huge differences from the case without any contracts. First of all, the responsiveness of total hours to a surprise in technology shock increases by about 140 percent and that of output increases by 40 percent without changes in those of the known part of the shock. More dramatic changes in effects of a contract can be found in the thechanges in the responsiveness in the variables to the surprise in monetary shock. Output and total hours respond negatively in the model without nominal contract, but respond to the surprise positively and strongly. However, overall contract effects cannot be evaluated by simply looking at these facts. Since the contract is for one period, the surprise in the monetary shock does not have such a propagation over time as the surprise in the technology shock. In other words, even though the surprise in the money growth has a positive effects on the output and total hours, it will have a negative effect on them in the next period. On the other hand, this period's surprise in the technology shock has a positive effect on the next period's output and total hours as well as on the current period's. In this sense technology shock has a more powerful role in a nominal contract model.

The decision rules for the nominal price contract model can be found in Table 5 C. Here we have a most striking features of the model. The surprise in the technology shock has a very strong negative effect on output, total hours etc.. This confirms the guess in the previous section, namely that if the surprise in the technology shock has a larger supply effect than demand effect, innovations in technology shock can have anegative effect on output and the factors of production. In addition, we note that the price contract increases the responsiveness of variables to innovations in monetary shock far more significantly than in nominal wage contract model. In fact, responsiveness of output and total hours to the money growth innovation is about 4.9 times larger in the nominal price contract case than in the nominal wage contract model.

One final note is on the decision rules implied by the Gray-Fischer contracts. The decision rules in case of the nominal wage contract are defferent from those obtained from the endogenous contract by less than 5 percent, but the decision rules in case of the nominal price contract are virtually the same as those rules from the endogenous contract. This means that the Gray-Fischer contracts are good enough to be used in the studies of nominal rigidities.

6. Conclusion

One-period nominal contracts are introduced in a small general equilibrium model. The simulation results show that nominal contracts are a powerful means of magnifying the responsiveness of endogenous variables to innovations in real and nominal shocks. Even a one-period contract can have huge effects on the business cycle. However, there are notable differences between different contracts in their propagation of shocks. The nominal wage contract improves the fit of the model in virtually all respects. On the other hand, the nominal price contract magnifies the volatility of all variables in the model dramatically and has a few uncomfortable features. In particular all labor variables, including the real wage rate, have correlations with output that are too high compared with the actual data, and innovations in the technology shock have strong negative effects on output, total hours etc. These features of the nominal price contract can be considered to be a serious drawback.

If an economy consists of sectors where nominal contracts and spot transactions are dominant, and if the latter is substantial, the results could be somewhat modified. The other features which may affect the results in the paper are multi-period contracts and inventory held by the firm. As shown in the appendix, a multi-period contract with capital accumulation in the model similar to that studied in the paper implies a long memory in the sense that a state variable in the far past affects the current decision. So a new feature is required to remove this long memory from the model. In a companion paper the effect of multi-period contracts are examined by introducing a time-to-build technology in the model economy. On the other hand, if the firms are allowed to hold inventory in the case of nominal price contract, there can be substantial changes in the results obtained in the paper. However, if there are some periods with binding market equilibrium condition (3.16), most of the results, I expect, will hold. One interesting point worthwhile to note is that if inventory is introduced in a nominal price contract, one puzzling fact in the actual economy can be reconciled. That is, it is widely known in the inventory literature that production is more volatile than sales (for example see Kahn (1988)). Since the fact is contrary to the production smoothing role of inventory, many economists consider that the fact is puzzling. However, in an environment with nominal price contract the motive for holding inventory is to make use of the favorable production opportunities and so it seems to be natural to have more volatile production than sales.

One final note is on the compatibility of real business cycle view with nominal rigidity views. A nominal wage contract is compatible with the real business cycle view in the sense that it makes the propagation of the technology shock richer. However, a nominal price contract is not compatible with the real business cycle paradigm in the sense that it prevents a surprise in the technology shock from having positive effect on output and total hours, etc.. If the technology shock is one of the dominant shocks in an economy, the nominal price contract theory needs some modification.

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Appendix: Capital Accumulation and Multi-period Contract

A multi-period contract combined with capital accumulation imply a long memory. To see this, assume that the nominal wage rate prevailing in period t is determined at the end of period t-2 as an expected market clearing wage rate. Then the problem facing the representative household is the same as the problem (3.13) with a modification in the labor input determined by the firm. With a Cobb-Douglas production function, the following can be obtained.

$$\begin{aligned} log(Q_t) = E[log(Q_t)|\Omega_{t-2}] &+ \frac{1}{\theta}\cdot\{log(\hat{P}_t) - E[log(\hat{P}_t)|\Omega_{t-2}]\} \\ &+ \frac{1}{\theta}\cdot\{log(\lambda_t) - E[log(\lambda_t)|\Omega_{t-2}]\} \\ &+ \frac{1}{\theta}\cdot\{log(M_t) - E[log(M_t)|\Omega_{t-2}]\} \end{aligned} \quad (A.1)$$

Since the labor demanded by the firm depends on a two period expectation error, we guess that the state of the economy is the following.

$$S^1_t=(1, log(\lambda_{t-2}), \varepsilon_{t-1}, \varepsilon_t, log(g_{t-2}), \omega_{t-1}, \omega_t, K_{t-1}, K_t)^T \quad (A.2)$$

Now by applying the method described in the main text, we can obtain decision rules as functions of the state of the economy. Suppose the decision rules obtained are as follows.

$$X_t = \Phi^1_1 S^1_t \quad (A.3)$$
$$P_t = \Phi^1_2 S^1_t \quad (A.4)$$
$$Q_t = \Phi^1_3 S^1_t \quad (A.5)$$

Here $\Phi_j=(\phi^1_{j1}, \phi^1_{j2}, \phi^1_{j3}, \phi^1_{j4}, \phi^1_{j5}, \phi^1_{j6}, \phi^1_{j7}, \phi^1_{j8}, \phi^1_{j9})$, j = 1, 2, 3, is a coefficient vector. However if the expectations in (A.1) are evaluated using the decision rules (A.3) - (A.4) and the equations of motion for the capital stock, it can be easily shown that $log[Q_t]$ depends on $log(\lambda_{t-3})$, ε_{t-2}, $log(g_{t-3})$, ω_{t-2}, and K_{t-2} as well as S_t^1 and so it is clear our guess on the state of the economy is wrong. Now then we can guess the state of the economy as follows.

$$S_t^2=(1, log(\lambda_{t-3}), \varepsilon_{t-2}, \varepsilon_{t-1}, \varepsilon_t, log(g_{t-3}), \omega_{t-2}, \omega_{t-1}, \omega_t, K_{t-2}, K_{t-1}, K_t) \quad \text{(A.5)}$$

In addition, suppose the decision rules are obtained as follows.

$$X_t = \Phi_1^2 S_t^2 \quad \text{(A.6)}$$

$$P_t = \Phi_2^2 S_t^2 \quad \text{(A.7)}$$

$$Q_t = \Phi_3^2 S_t^2 \quad \text{(A.8)}$$

Here $\Phi_j=(\phi_{j1}^2, \phi_{j2}^2, \phi_{j3}^2, \phi_{j4}^2, \phi_{j5}^2, \phi_{j6}^2, \phi_{j7}^2, \phi_{j8}^2, \phi_{j9}^2, \phi_{j9}^2, \phi_{j10}^2, \phi_{j11}^2)$, j = 1, 2, 3, is coefficient vector. However, if the conditional expectations (A.1) are evaluated using (A.6) - (A.8) and the law of motion for capital stock, it can be easily shown that $log[Q_t]$ depends on $log(\lambda_{t-4})$, ε_{t-3}, $log(g_{t-1})$, ω_{t-2}, and K_{t-3} as well as S_t^2 and so once again it is clear that our guess is wrong. If we repeat this procedure, we can find that the only state compatible with a model with a muti-period contract and capital accumulation is one that consists of all of the histories of real and nominal shocks and capital stock.

$$S_t = (log(\lambda_0),\ \{\varepsilon_j\}_{j=1}^{t},\ log(g_0),\ \{\omega_t\}_{j=1}^{t},\ \{K_j\}_{j=0}^{t}) \quad \text{(A.9)}$$

This phenomenon appears due to the fact that the contract nominal wage rate depends on the lagged aggregate investment through the law of motion for capital stock. In the case of any multi-period nominal price contract, we have the same result. Note here that the number of state variables increases as time period moves on.

Table 1. Summary Statistics from U.S. Data

A. Standard Deviations

Y	C	X	K	Q	H	E	Pdty	W	Rw	P
1.74	.81	8.49	.38	1.80	.46	1.50	1.09	1.00	.99	.98

B. Cross Correlations

	Y	C	X	K	Q	H	E	Pdty	W	Rw	P
Y	1.00										
C	.65	1.00									
X	.91	.42	1.00								
K	.28	.20	.10	1.00							
Q	.87	.47	.81	.51	1.00						
H	.76	.32	.78	-.02	.72	1.00					
E	.81	.46	.74	.61	.98	.55	1.00				
Pdty	.56	.46	.55	-.38	.14	.42	.04	1.00			
W	-.45	-.17	-.51	.32	-.33	-.59	-.22	-.43	1.00		
RW	.40	.52	.36	-.30	.20	.23	.17	.60	-.24	1.00	
P	-.53	-.38	-.53	.27	-.36	-.45	-.29	-.56	.84	-.57	1.00

Note: Y = GNP, C = consumption of nondurables and services plus the flow of services from durables constructed by Lawrence Christiano, X = gross private domestic investment, K = nonresidential equipment and structures, Q = total hours of all person, H = hours per person, E = total number of employed person, Pdty = output divided by total hours, W = nominal compensation per hour, Rw = real compensation per hour, P = GNP deflator. All series except consumption were taken from Citibase database. All series are seasonally adjusted, logged and detrended using Hodrick-Prescott filter. The standard deviations are in percentage term. Sample periods: 1955.3 - 1984.1.

Table 2. Summary Statistics: No Nominal Contract

A. Standard Deviations

Y	C	X	K	Q	H	E	Pdty	W	Rw	P
1.66	.66	5.25	.43	.77	.37	.41	.91	3.02	.91	3.02
(.15)	(.06)	(.55)	(.09)	(.07)	(.03)	(.04)	(.09)	(.41)	(.09)	(.40)

B. Cross Correlations

	Y	C	X	K	Q	H	E	Pdty	W	Rw	P
Y	1.00										
C	.73	1.00									
	(.05)										
X	.97	.53	1.00								
	(.01)	(.06)									
K	.07	.40	-.06	1.00							
	(.08)	(.11)	(.06)								
Q	.98	.67	.97	-.11	1.00						
	(.00)	(.05)	(.01)	(.07)							
H	.98	.67	.97	-.11	1.00	1.00					
	(.00)	(.05)	(.01)	(.07)	(.00)						
E	.98	.67	.97	-.11	1.00	1.00	1.00				
	(.00)	(.05)	(.01)	(.07)	(.00)	(.00)					
Pdty	.99	.76	.94	.22	.94	.94	.94	1.00			
	(.00)	(.05)	(.01)	(.08)	(.02)	(.02)	(.02)				
W	.13	-.19	.24	-.04	-.17	.11	.11	.15	1.00		
	(.15)	(.12)	(.16)	(.21)	(.16)	(.16)	(.16)	(.15)			
RW	.99	.76	.36	.22	.94	.94	.94	1.00	.15	1.00	
	(.00)	(.05)	(.01)	(.08)	(.02)	(.02)	(.02)	(.00)	(.15)		
P	-.16	-.42	-.05	-.40	-.67	-.17	-.17	-.15	.95	-.15	1.00
	(.15)	(.11)	(.17)	(.11)	(.16)	(.16)	(.16)	(.15)	(.02)	(.15)	

Note: The statistics reported are the sample means of statistics from each of twenty five simulations of 115 observations. The numbers in parentheses are sample standard deviations. Each simulated series was logged and detrended by Hodrick-Prescott filter. Standard deviations are in percentage term.

Table 3. Summary Statistics: Nominal Wage Contract

A. Standard Deviations

Y	C	X	K	Q	H	E	Pdty	W	Rw	P
2.32	.51	8.03	.50	2.49	1.18	1.30	1.15	2.84	1.15	2.82
(.51)	(.12)	(1.76)	(.14)	(.53)	(.25)	(.28)	(.25)	(.70)	(.25)	(.71)

B. Cross Correlations

	Y	C	X	K	Q	H	E	Pdty	W	Rw	P
Y	1.00										
C	.71	1.00									
	(.15)										
X	.95	.63	1.00								
	(.19)	(.13)									
K	-.04	.51	-.13	1.00							
	(.05)	(.13)	(.05)								
Q	.85	.39	.89	-.22	1.00						
	(.18)	(.09)	(.18)	(.07)							
H	.85	.39	.89	-.22	.96	1.00					
	(.18)	(.09)	(.18)	(.07)	(.20)						
E	.85	.39	.89	-.22	.96	.96	1.00				
	(.18)	(.09)	(.18)	(.07)	(.20)	(.20)					
Pdty	.09	.59	-.01	.40	-.35	-.35	-.35	1.00			
	(.12)	(.14)	(.12)	(.12)	(.12)	(.12)	(.12)				
W	.01	.22	-.03	.31	-.09	-.09	-.09	.21	1.00		
	(.17)	(.21)	(.15)	(.17)	(.08)	(.09)	(.09)	(.17)			
RW	.09	.59	-.01	.40	-.35	-.35	-.35	.96	.21	1.00	
	(.12)	(.14)	(.12)	(.12)	(.12)	(.12)	(.12)	(.20)	(.17)		
P	-.02	-.02	-.02	.15	.06	.06	.06	-.18	.88	-.18	1.00
	(.16)	(.20)	(.15)	(.17)	(.09)	(.09)	(.09)	(.16)	(.18)	(.16)	

Note: See note for Table 2.

Table 4. Summary Statistics: Nominal Price Contract

A. Standard Deviations

Y	C	X	K	Q	H	E	Pdty	W	Rw	P
8.39	.43	32.83	1.12	13.60	6.47	7.12	5.34	8.84	7.99	2.38
(1.78)	(.11)	(6.97)	(.27)	(2.89)	(1.38)	(1.51)	(1.14)	(1.88)	(1.70)	(.60)

B. Cross Correlations

	Y	C	X	K	Q	H	E	Pdty	W	Rw	P
Y	1.00										
C	-.08	1.00									
	(.08)										
X	.96	-.12	1.00								
	(.20)	(.07)									
K	-.31	.67	-.33	1.00							
	(.09)	(.16)	(.09)								
Q	.96	-.10	.96	-.28	1.00						
	(.19)	(.08)	(.19)	(.09)							
H	.96	-.10	.96	-.28	.96	1.00					
	(.19)	(.08)	(.19)	(.09)	(.20)						
E	.96	-.10	.96	-.28	.96	.96	1.00				
	(.19)	(.08)	(.19)	(.09)	(.20)	(.20)					
Pdty	-.92	.13	-.93	.23	-.95	-.95	-.95	1.00			
	(.19)	(.09)	(.19)	(.10)	(.20)	(.20)	(.19)				
W	.91	.10	.91	-.06	.91	.91	.91	-.88	1.00		
	(.19)	(.09)	(.19)	(.10)	(.19)	(.19)	(.19)	(.18)			
RW	.96	-.05	.95	-.23	.96	.96	.96	-.94	.93	1.00	
	(.19)	(.08)	(.19)	(.09)	(.20)	(.20)	(.20)	(.19)	(.19)		
P	.19	.54	.17	.55	.17	.17	.17	-.13	.46	.22	1.00
	(.07)	(.17)	(.07)	(.14)	(.07)	(.07)	(.07)	(.08)	(.10)	(.07)	

Note: See note for Table 2.

Table 5. Decision Rules

A. No Nominal Contract

	Constant	$ln(\lambda_{t-1})$	ε_t	$ln(g_{t-1})$	ω_t	K_t	$ln(M_{t-1})$
$ln(Y_t)$	-0.40112	1.35083	1.42193	-0.01162	-0.02421	0.21438	0
$ln(C_t)$	-1.51342	0.36726	0.38658	-0.21345	-0.44468	0.57174	0
$ln(X_t)$	0.60446	4.20335	4.42457	0.57369	1.19519	-0.82197	0
$ln(K_{t+1})$	1.48282	0.10906	0.16531	-0.00107	0.15140	0.08252	0
$ln(Q_t)$	-0.62676	0.62630	0.65927	-0.01816	-0.03783	-0.22752	0
$ln(N_t)$	-0.48421	0.29824	0.31394	-0.00865	-0.01801	-0.10834	0
$ln(E_t)$	-0.14255	0.32806	0.34533	-0.00951	-0.01982	-0.11918	0
$ln(Pdty_t)$	1.51321	0.72453	0.76266	0.00654	0.01362	0.44191	0
$ln(W_t)$	1.29256	0.35720	0.37600	0.69994	1.45821	-0.12971	1
$ln(Rw_t)$	1.06692	0.72453	0.76266	0.00654	0.01362	0.44191	0
$ln(P_t)$	1.51321	-0.36734	-0.38667	0.21340	0.44460	-0.57161	1

B. Nominal Wage Contract

	Constant	$ln(\lambda_{t-1})$	ε_t	$ln(g_{t-1})$	ω_t	K_t	$ln(M_{t-1})$
$ln(Y_t)$	-0.21010	1.35083	2.03302	-0.01162	1.61910	0.01853	0
$ln(C_t)$	-0.75388	0.36606	0.41893	-0.00085	0.08926	0.04943	0
$ln(X_t)$	-0.85327	4.20680	6.71405	-0.04285	6.05580	-0.07107	0
$ln(K_{t+1})$	1.48282	0.10517	0.16785	-0.00107	0.15140	0.08252	0
$ln(Q_t)$	-1.14286	0.62630	1.61409	-0.01816	2.52985	-0.01967	0
$ln(N_t)$	-0.73002	0.29814	0.76825	-0.00864	1.20435	-0.00936	0
$ln(E_t)$	-0.41284	0.32816	0.84583	-0.00951	1.32550	-0.01031	0
$ln(Pdty_t)$	0.93276	0.72453	0.42893	0.00654	-0.91075	0.03821	0
$ln(W_t)$	1.24035	0.35847	0.00000	0.48739	0.00000	-0.01123	1
$ln(Rw_t)$	0.48647	0.72453	0.41893	0.00654	-0.91074	0.03821	0
$ln(P_t)$	0.75388	-0.36606	-0.41893	0.48085	0.91074	-0.04943	1

C. Nominal Price Contract

	Constant	$ln(\lambda_{t-1})$	ε_t	$ln(g_{t-1})$	ω_t	K_t	$ln(M_{t-1})$
$ln(Y_t)$	-0.66115	1.35339	-5.48250	-0.00537	7.96513	0.01761	0
$ln(C_t)$	-0.71505	0.36494	-0.00292	-0.21308	0.00293	0.04919	0
$ln(X_t)$	-2.72501	4.22002	-21.37419	0.59698	31.05639	-0.07395	0
$ln(K_{t+1})$	1.43602	0.10550	-0.53435	0.01492	0.77641	0.08244	0
$ln(Q_t)$	-1.84762	0.63030	-10.12890	-0.00840	12.44551	-0.02111	0
$ln(N_t)$	-1.06557	0.30014	-4.82329	-0.00340	5.92643	-0.01005	0
$ln(E_t)$	-0.78205	0.33016	-5.30562	-0.00440	6.51908	-0.01106	0
$ln(Pdty_t)$	1.18647	0.72309	4.64641	0.00302	-4.48038	0.03872	0
$ln(W_t)$	0.59660	0.35939	-5.77472	0.70532	8.57500	-0.00757	1
$ln(Rw_t)$	-0.11846	0.72432	-5.77754	0.01224	7.57793	0.04162	0
$ln(P_t)$	0.71505	-0.36494	0.00282	0.69308	0.99707	-0.04919	1