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Ex Post Heterogeneity and the Business Cycle

Jang-Ok Cho

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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by

Jang-Ok Cho
Queen's University
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Abstract

The reason why the assumption of a representative agent is so popular in the equilibrium business cycle literature is mainly that equilibrium allocations are derived by solving a concave programming problem, whereas once heterogeneity is introduced it is necessary to solve for weights on individual utilities and this generally involves solving a complicated fixed point problem. The representative agent framework is extended in a way that permits workers to have different skills. The key feature is that each agent faces a stochastic productivity ex ante but a different realized productivity ex post from others'. This means that there is no ex ante heterogeneity but ex post heterogeneity, and hence an equal weight is applied to the utilities of all agents. The results show how serious the aggregation problem is in aggregate fluctuations. The responsiveness of labor supply in physical units is larger than that of labor supply in efficiency units. On the contrary, productivity in physical units responds to the aggregate productivity shock less vigorously than productivity in efficiency units. So the relative ratio of standard deviation of aggregate hours to that of productivity tends to be exaggerated in the actual data, which measures the aggregate labor supply only in physical unit. In addition, a heterogeneity helps the model economy produce a more plausible correlation structure. Furthermore, the results show that a high elasticity of intertemporal substitution of labor is not required for a real business cycle model to mimic the key features observed in the actual economy.

1. Introduction

In his comments on the Ashenfelter (1984) and Kydland (1984) papers, Heckman (1984) pointed out two very important facts in aggregate fluctuations. First, much of the quarterly and annual variation in aggregate manhours comes through fluctuations in employment rather than hours per man and hence any serious empirical model of business cycle labor market fluctuations must account for manhour variation at the extensive margin, i.e. employment or labor force entry decision, as well as manhour variation at the intensive margin, i.e. hours per worker. Second, there are numerous puzzles in macroeconomics that may simply be aggregation phenomena. He skeptically criticized the representative consumer models and then emphasized the importance of heterogeneity in a model as a step toward accomodating the wealth of microeconomic diversity in preferences and endowments.

However, the assumption of a representative consumer is a common starting point in much theoretical work in macroeconomics. The reason why the assumption is employed so widely is mainly that equilibrium allocations are derived by solving a concave programming problem, whereas once heterogeneity is introduced it is necessary to solve for weights on individual utilities and this generally involves solving a complicated fixed point problem. In this paper I extend the representative agent framework in a way that is precisely in the spirit of Heckman's criticism: workers decide on both participation and hours as well as they face a sort of heterogeneity. In addition, there are fixed costs associated with the decision to participate in employment (see Cho and Cooley (1988)). With this extension, I am able to show how successfully a theoretical model mimics the behavior of hours, employment and productivity in the aggregate data and how serious aggregation phenomena are in the study of the business cycle.

In the real business cycle literature, we have two benchmark models which use the representative agent framework. Kydland and Prescott (1982) constructed a growth model with a representative agent.[1] They introduced time-to-build technology and non-time-separable preference to increase the intertemporal responses of aggregate variables of interest, especially those of aggregate labor supply. However, the adjustment in the labor market in their model takes place only along the intensive margin. On the other hand, Rogerson (1984) constructed a model with representative agent where the labor supply is indivisible, that is, workers either work a given number of hours or not at all. In his model all fluctuations in aggregate hours of work are due to fluctuations in employment. Hansen (1985) extended the model to a growth setting and then calibrated it using the methods of Kydland and Prescott (1982). His results demonstrated that such a model was capable of explaining the high variability in total hours worked and employment. However, it has the unfortunate feature that all fluctuations in aggregate hours are due to fluctuations along the extensive margin. Moreover, it implies a ratio of fluctuations in aggregate hours to productivity nearly twice that found in U.S. data.

Cho and Cooley (1988) constructed a model with labor market adjustment along both margins without disturbing the representative agent framework.

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1 Long and Plosser (1983) also assumed the representative agent. In their model many sectors of production and 100 percent depreciation are assumed. The assumption that capital stock depreciates 100 percent in each period helped them find a closed form solution. But if there are no preference shocks, labor does not fluctuate in their model at all.

Our model assumes infinite number of agents with identical preferences and opportunity sets. The special feature of that model is that agents are assumed to have a fixed cost associated with labor supply that depends on aggregate variables as well as the individual variables, in our example the employment rate and the probability of work of an agent. In particular the cost of participating in the labor force is assumed to be an increasing function of the two variables. This feature might reflect, among other things, the difficulties of replacing home production. Individuals in this model economy must decide on whether to participate in the labor force and on how many hours to work. This feature combined with an employment lottery of the sort introduced by Rogerson (1984) produces an equilibrium that displays fluctuations along both the extensive and intensive margins.

On the other hand, heterogeneities in preferences and/or opportunity set have come to be one of the key issues in real business cycle models. Kydland (1984) extended his model with Prescott (1982) by incorporating a heterogeneity in skills among demographic groups, i.e. skilled and unskilled groups in his model. He fixed the utility weights between the two groups by referring to their relative stock of human capital for the production of market goods. However, his way of weight determination was criticized as ad hoc by King, Plosser and Rebello (1988).[2] To avoid the complicated weight determination problem, Cho and Rogerson (1988)

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[2] Rogerson (1988) reported a very interesting case of heterogeneity where we do not need any ad hoc assumptions. The preferences in his model are linear in leisure. The heterogeneity is in the coefficients in front of the leisures and the initial endowments. However, the coefficient in front of the leisure term pins down the utility weight.

constructed a model with two member family in which the two members have different productivities from each other. Because every family is identical in that model, we did not need to bother about determining the utility weights. One of the common results in Kydland (1984) and Cho and Rogerson (1988) is that aggregation phenomena are very important in explaining the aggregate fluctuations. Especially, the fluctuations in aggregate hours in physical units are much greater than those in efficiency units, which is called aggregation bias in the literature.[3]

My goal in this paper is to extend the representative agent framework in a way that permits workers to have different skills. The key feature is that each agent faces a stochastic productivity ex ante but adifferent realized productivity ex post. In other words, each agent has the same ex ante distribution of productivity, but the realization of an agent's productivity can be different from others'. This means that there is no ex ante heterogeneity in the model but ex post heterogeneity, and hence we do not need to determine the utility weights among agents. To facilitate the characterization of the equilibrium, it is assumed that the ex ante distribution of skills is identical to the ex post distribution of the realized skills. This feature will effectively distinguish the labor supply in efficiency units from that in physical units and show how serious the aggregation phenomena are in aggregate fluctuations as was emphasized

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[3] Bils (1985) and Heckman and Sedlacek (1985) emphasized the aggregation bias in terms of wages. In other words, since employment is much more variable among the agents in less skilled group, real wages in boom times are averaged over a group with lower earning potential than those averaged in depression times. In addition, there is an aggregation bias due to sectoral differences in employment variablities. See the discussions in section 5.

by Heckman(1984).

In the next section of the paper we describe a static version of our economy. In section 3 we characterize the optimal allocations. In the fourth section we present an example and illustrate how the aggregate labor elasticity depends on both margins of adjustment and how serious the aggregation bias problem is. The example illustrates the dramatic differences in aggregate labor supply elasticities that arise in different model economies. In section 5 we will discuss the possible extentions to the model. Section 6 extends the model to a dynamic setting and discusses calibration and simulation. The results presented in section 7 show that this model is able to replicate almost exactly the variability of hours relative to productivity that is found in the U.S. data. More surprisingly, the results show that we don't need such a high elasticity of intertemporal substitution to get the ratio of variability in aggregate hour to that in productivity which is identical to the actual one.

2. The Environment

In this section we describe a model economy with a continuum of agents (or households) uniformly distributed on the unit interval [0,1]. Each agent has identical preferences and the same opportunity set. There are three goods: labor, capital and output. We first describe a static single period model which we later extend to a dynamic setting. Capital and labor are inputs to the production function:

$$f(K,N) : R_+ x R_+ \longrightarrow R_+ . \tag{2.1}$$

It is assumed that the production function is continuous and strictly monotonic in K and N, and concave in K and N separately. In addition, it is assumed that the production function is homogeneous of degree one and $f(0,0)=0$. Anticipating the dynamic version, we introduce a multiplicative productivity shock, λ, and write the production function $\lambda f(K,N)$. For the time being λ will be assumed to be fixed.

Each agent is endowed with one unit of time and one unit of capital. Time is completely divisible and hence there is no indivisibility in supply of labor. The utility function is assumed to be separable between consumption and leisure.

$$U(c,l) = u(c) - v(1-l), \tag{2.2}$$

where c and l are consumption and leisure respectively and hence $n=1-l$ is labor supplied to the market. We further assume that:

(1) u and v are twice continuously differentiable and increasing.

(2) u is strictly concave, v is strictly convex, and $v(0)=0$.

(3) $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$.

(4) $\lim_{n\to 0} v'(n) = 0$, $\lim_{n\to 1} v'(n) = \infty$.

(5) $u'(c+y) + u''(c+y)c \geq 0$ for all $c>0$ and $y\geq 0$.

These are all standard conditions except (5) which is imposed to guarantee that the labor supply curve is not backward bending.

In addition to the above noted standard features we assume that there is a cost in terms of utility associated with labor force participation,[4] following Cho and Cooley (1988). This cost is assumed to capture features of the psychic burden of working which are different in nature from the disutility associated with additional hours of work. The cost of labor supply function will be represented as:

$$\psi : \Omega \longrightarrow R_+, \tag{2.3}$$

where Ω is the space of individual and/or aggregate variables. More specifically, ψ will be assumed to depend on the overall employment rate of the economy and the agent's probability of work in the market. Therefore, $\Omega = [0,1]\times[0,1]$. ψ is specified to be increasing and twice differentiable in both arguments. In addition, we assume that if one of the two argument of ψ is 0, then ψ assumes the value zero.

One way to justify this specification of the fixed cost of labor force participation is to imagine that labor force participation is a substitute for home production[5] that must somehow be replaced. Suppose the utility cost of labor supply is incurred because the agent has children to be cared for. In addition, suppose there are costs associated with other home production that must be replaced. Then, an increase in the rate of employment will make

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4 I developed a model with ex ante heterogeneity in Cho (1987). In my model, every agent faces a different fixed utility cost of labor supply as well as a different productivity from other agents. The economy analyzed in the paper contains the economy studied by Cho and Cooley (1988) as a special case, namely one with the equal weights.

5 See, for example, Becker (1965, 1971) for details.

it more difficult for the agent to find someone who will replace this home production. As a result, an increase in employment rate can raise psychic burden of agents who choose to work. Similarly, we assume that the fixed cost of working are increasing in the worker's own participation; that is, a worker who works six days per week faces a higher fixed cost than one who works one day.

On the other hand, it is assumed that each agent faces stochastic productivity. In other words, if q is the productivity index of an agent, then q has a distribution function G which has a finite support. We assume that the function is twice continuously differentiable on the support, and hence the quantile function of the distribution is well defined. Each agent is assumed to face an identical distribution of q and hence there is no heterogeneity ex ante. However, the realized productivities can be different across agents. It is assumed that the realized productivities across agents fulfill the distribution of q in the sense that the distribution of the realized productivities is identical to the ex ante distribution of q. Note that as for the productivities, there is no uncertainty in the aggregate level but in the individual level.

With this feature of stochastic productivity we extend the basic model described previously to two directions. First of all, we have to redefine the aggregate labor input N because the labor input used in the production process is not in physical units but in efficiency units. That is, we define the labor input in efficiency units as:

$$N^e = \int_L q(t) \cdot n(t) dt, \qquad (2.4)$$

where L is the set of agents who supply labor in the market. Here we define the support of G as $[q,\bar{q}]$. Secondly, it is assumed that the disutility of work defined in (2.2) depends on hours of work in efficiency units rather than those in physical units. This assumption is in the spirit of Heckman (1976) but is different in its detail. He assumed that the utility of an agent depends on the leisure in efficiency units, and this assumption amounts to saying that if an agent is more efficient in work, then the agent enjoys his leisure more efficiently. But the present assumption says that if an agent is more efficient, then the agent's disutility of work is greater per unit of time devoted to work.[6]

With these modifications, the representative agent's utility function defined in (2.2) has to be rewritten as:

$$U(c,l;e,E) = u(c) - v[q\cdot(1-l)] - \psi(e,E)\cdot 1(n>0), \qquad (2.5)$$

where e denotes the probability of work of the agent, E is the employment rate of the economy, and $1(n>0)$ is 1 if $n>0$ and 0 otherwise. Finally, note that the employment rate is not a variable subject to an agent's control. So the agent takes E as given when he decides the participation and hours of work in the market. In this sense the employment rate has an externality effect. On the other hand, the probability of work can take the value of 0 or 1 until we allow the randomization of the participation decision and/or we introduce some employment insurance into the economy. Therefore, given an employment rate $E\in[0,1]$, $\psi(1,E)$ is a fixed cost for each agent.

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[6] The assumption amounts to adding one more feature to Heckman's specification. That is, the endowment of time is stochastic.

But, the aggregate employment rate is a variable which will be determined in equilibrium in the model.

3. Optimal Allocations

If q is stochastic and the ψ function depends on the probability of work of an agent and the overall employment rate of the economy, then each agent maximizes his utility state by state[7]. Here we introduce an employment insurance of the following form. If the realized q for an agent is greater than or equal to some threshhold productivity $q*$, then the agent works $n(q)$ hours in the market. However, if q is smaller than $q*$, then the agent does not work in the market. On the other hand, each agent receives $c(q)$ units of output if the realized productivity is q. So each agent receives an allocation $(c(q), n(q))$ according to the agent's realized productivity q.[8] On the other hand, the ex ante probability of work of an agent is the probability that his realized productivity is greater than $q*$. Since $q*$ depends on the realization of the aggregate uncertainty, i.e. the aggregate productivity shock λ, the probability of work of an agent is a function of λ (not of any individual uncertainty).

Since we are assuming that the distribution of the realized productivity is identical to the ex ante distribution of q, the threshhold

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[7] If q is deterministic and ψ is constant, the environment is the same as those in Rogerson (1984), Grilli and Rogerson (1988), and Cho and Rogerson (1988). On the other hand, if q is deterministic but ψ depends on e and E, the environment is the same as that in Cho and Cooley (1988).

[8] Here is a problem of informational asymmetry if the knowledge of the realized q is private. However, we abstract from this complicated problem in this paper. That is, we assume the information on the realized q of each agent is public.

productivity $q*$ is related to the probability of work of an agent by the following relationship.

$$1 - G(q*) = e. \tag{3.1}$$

Because the quantile function of the distribution function is well defined by assumption, (3.1) can be rewritten as:

$$q* = G^{-1}(1-e). \tag{3.2}$$

That is, the proportion of the agents who have the realized productivity greater than or equal to $q*$ is exactly e. Here we define the labor supply in efficiency unit, N^e, and that in physical unit, N.

$$N^e = \int_{q*}^{1} qn(q) \cdot dG(q) \tag{3.3}$$

$$N = \int_{q*}^{1} n(q) \cdot dG(q). \tag{3.4}$$

Note that the labor input used in the production is not in physical unit but in efficiency unit.

Now the contingent allocation $(c(q),\ n(q))$ and the employment rate e are determined so as to maximize the representative agent's expected utility subject to budget constraint. We can solve for the allocation from the following programming problem.

$$\underset{\{c,n,e\}}{\text{maximize}} \quad EU = \int_{\underline{q}}^{\bar{q}} \{u[c(q)] - [v(qn(q)) + \psi(e,E)] \cdot 1(q \geq q*)\} dG(q) \tag{3.5}$$

$$\text{s.t.} \qquad \int_{\underline{q}}^{\bar{q}} c(q) \cdot dG(q) \leq \lambda f(1, N^e)$$

$$q^* = G^{-1}(1-e)$$

$$c(q) \geq 0, \quad 0 \leq n(q) \leq 1, \quad 0 \leq e \leq 1,$$

where $1(q \geq q^*)$ is an index function which is zero if $q<q^*$ and one if $q \geq q^*$, and $g(q)=G'(q)$. Forming Lagrangian function, we have the following first order conditions.

$$u'[c(q)] = \mu \quad \text{for} \quad q \in [\underline{q}, \bar{q}] \tag{3.6}$$

$$-v'[qn(q)] + \mu\lambda f_2(1, N^e) = 0 \quad \text{for} \quad q \geq q^* \tag{3.7}$$

$$v(q^* n^*) \cdot g(q^*) \cdot \frac{\partial q^*}{\partial e} - [\psi_1(e, E) \cdot e + \psi(e, E)]$$
$$- \mu\lambda f_2(1, N^e) \cdot q^* \cdot n^* \cdot g(q^*) \cdot \frac{\partial q^*}{\partial e} = 0, \tag{3.8}$$

where μ is the Lagrange multiplier attached to the budget constraint and $n^*=n(q^*)$. The first two conditions, (3.6) and (3.7), determine optimal risk sharing among risk averse agents as in Borch(1962), Arrow(1971) and Wilson(1968). The third condition, (3.8), determines the optimal level of the probability of work. If we impose the equilibrium condition or aggregate consistency condition e=E, we can solve for the equilibrium consumptions and hours for each state, and the employment rate.[9]

First of all, because the Lagrange multiplier μ does not depend on the state of an individual, i.e. the realization of q, (3.6) implies that:

$$c(q) = c \quad \text{for all } q \in [\underline{q}, \bar{q}]. \tag{3.9}$$

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[9] See Romer (1987) for the method of obtaining equilibrium and/or optimal allocations in the presence of externalities.

In other words, consumptions are identical across all the realization of q. On the other hand, (3.9) and (3.7) imply that:

$$v'[qn(q)] = u'(c)\lambda f_2(1,N^e) \quad \text{for } q \geq q^*. \tag{3.10}$$

But since c is the same across all q and N^e does not depend on any specific realization of q, the righthand side of (3.10) does not depend on q. This means that:

$$qn(q) = x \quad \text{for all } q \geq q^*. \tag{3.11}$$

That is, the hours worked in efficiency units are the same across all agents who are working in the market. Here we have a direct implications on the supply of labor. The cross sectional elasticity of hours of work in physical units is negative. This is not at all strange result since many empirical work estimating the elasticity have reproted the same result as in (3.11) (for example, see Heckman and Killingsworth (1987), Decker (1987)).

On the other hand, we have the following relationship from (3.1).

$$\frac{\partial q^*}{\partial e} = - \frac{1}{g(q^*)} \tag{3.12}$$

Using (3.9), (3.11) and (3.12), (3.8) can be rewritten as:

$$u'(c)\lambda f_2(1,N^e)x = v(x) + \psi_1(e,E)\cdot e + \psi(e,E). \tag{3.13}$$

We had the same conditions in Cho and Cooley (1988) as (3.10) and (3.13) with one proviso that the labor supply in (3.10) and (3.13) is measured in efficiency units rather than in physical units. But in the Cho-Cooley environment, there is no difference in labor supplies in efficiency units and in physical units. Therefore, the interpretations are the same as those in that paper. That is, (3.10) requires that the marginal disutility of adjustment along intensive margin be equal to the marginal benefit of it. On the other hand, (3.13) requires that the marginal disutility of adjustment along the extensive margin be equal to the marginal benefit of it. If we substitute (3.10) in (3.13), we get the following:

$$v'(x)\cdot x = v(x) + \psi_1(e,E)\cdot e + \psi(e,E). \tag{3.14}$$

This condition says that if the solutions for both x and e are interior, then at the optimum it must be that increasing aggregate hours by an incremental amount along either of these two margins is a matter of indifference. Consider increasing aggregate hours by adding additional working agents at x efficiency hours versus increasing aggregate hours by x efficiency hours through an increase in the hours of individuals already working. The addition to output is the same in both cases. But in the first case the added disutility is $v(x)+\psi_1(e,E)+\psi(e,E)$, whereas in the second case it is $v'(x)x$. Requiring that the economy be indifferent between these two methods produces (3.14).

Now if we use the results in (3.9) and (3.11), we can rewrite the problem in (3.1) as follows:

$$\underset{\{c,x,e\}}{\text{maximize}} \quad u(c) - v(x)\cdot e - \psi(e,E)\cdot e \tag{3.15}$$

$$\text{s.t.} \quad 0 \le c \le \lambda f(1, x\cdot e)$$

$$0 \le e \le 1, \quad 0 \le x \le \bar{q},$$

where x is the labor supply in efficiency unit. If we solve the problem (3.15) with the aggregate consistency condition e=E, we have the same conditions as (3.10) and (3.12). Here we have to note the fact that (3.15) is exactly the same problem as in the Cho-Cooley environment except that the labor supply is measured in efficiency units. Therefore, if we take into account of the efficiency of labor supply, every results in Cho and Cooley (1988) can be applied to the current case. Now note that:

$$\begin{aligned} N &= \int_{q*}^{1} n(q)\cdot dG(q) \\ &= x\cdot\int_{q*}^{1} \left(\frac{1}{q}\right)\cdot dG(q). \end{aligned} \tag{3.16}$$

Therefore, if we get the solution for x, we can transform it into labor supply in physical units by (3.16).

4. An Example

In this section we consider an example. The example is based on the following specification of preferences and technology:

$$u(c) = \frac{1}{\sigma}\, c^{\sigma} \tag{4.1}$$

$$v(n) = \frac{a}{1+\gamma}\, n^{1+\gamma} \tag{4.2}$$

$$\psi(e,E) = \frac{b}{1+\tau}\,[\nu e+(1-\nu)E]^{\tau}, \tag{4.3}$$

where it is assumed that $0<\sigma<1$, $\gamma>0$, $\tau>0$ and $0\leq\nu\leq1$. The technology is given by:

$$\lambda f(N) = \lambda N^{\alpha}, \tag{4.4}$$

where we will abstract from capital stock to make the problem simple.[10] On the other hand, we assume the distribution of the productivities takes the following form:

$$G(q) = (q/\bar{q})^{\kappa} \quad \text{for } q\in[0,\bar{q}]. \tag{4.5}$$

Here we have to note that the lower tail shape of the distribution is very important in the sense that the participation decision is the tail event.

With the specification of the model, the representative agent has to solve the following problem.[11]

$$\begin{aligned} &\text{maximize} \quad \frac{1}{\sigma}\cdot c^{\sigma} - \frac{a}{1+\gamma}\cdot x^{1+\gamma}\cdot e - \frac{b}{1+\tau}\cdot e^{1+\tau} \\ &\quad \text{s.t.} \qquad c \leq wxe \end{aligned} \tag{4.6}$$

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[10] To keep the examples simple we assume that the firm is owned by an agent whose only role is to dispose of the profits associated with this decreasing returns technology.

[11] Since externality plays no significant role in terms of the aggregate fluctuations (see Cho and Cooley (1988)), we will abstract from the externality in the present example.

$$c \geq 0, \quad 0 \leq x \leq q, \quad 0 \leq e \leq 1.$$ [12]

Note in the problem that the hours in efficiency units, i.e. x, are costrained by q.[13] The first order conditions are obtained as:

$$(wxe)^{\sigma-1} \cdot w = a \cdot x^{\gamma} \tag{4.7}$$

$$(wxe)^{\sigma-1} \cdot wx = \frac{a}{1+\gamma} \cdot x^{1+\gamma} + b \cdot e^{\tau}. \tag{4.8}$$

From these, the following supply functions are derived.

$$x^s = J_1 \cdot w^{\frac{\sigma}{R}} \tag{4.9}$$

$$e^s = L_1 \cdot w^{\frac{\sigma(1+\gamma)}{\tau R}} \tag{4.10}$$

$$N^{es} = J_1 \cdot L_1 \cdot w^{\frac{\sigma(1+\gamma+\tau)}{\tau R}}, \tag{4.11}$$

where $J_1 = (H_1^{\alpha-1}/a)^{1/R}$, $H_1 = \left[\frac{a\gamma(1+\tau)}{b(1+\nu\tau)(1+\gamma)}\right]^{1/\tau}$, $L_1 = H_1 \cdot J_1^{(1+\gamma)/\tau}$, $R = \gamma + \frac{(1+\gamma+\tau)(1-\sigma)}{\tau}$, and N^{es} is the aggregate labor supply in efficiency units.

The firm's problem is very simple:

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[12] Note that the wage rate in the example is a payment on a unit of labor supply in efficiency unit. So the wage rate on the labor supply in physical unit is obtained as $w \cdot q$.

[13] In terms of efficiency units of time endowment, here still we have the heterogeneity. If the solution is obtained at the corner, the resulting solutions are quite different from those in Cho and Cooley (1988).

$$\max \quad \lambda N^{\alpha} - wN \tag{4.12}$$

$$\text{s.t.} \quad N \geq 0.$$

The first order condition for (4.12) is:

$$\lambda \alpha N^{\alpha-1} - w = 0, \tag{4.13}$$

and hence the demand for aggregate labor is:

$$N^{d} = G \cdot w^{1/(\alpha-1)}, \tag{4.14}$$

where $G=(\lambda\alpha)^{1/(1-\alpha)}$. Therefore, the equilibrium wage rate for the labor supply in efficiency units by equating (4.11) and (4.14) as:

$$w^{*} = \left(\frac{G}{J_{1} \cdot L_{1}}\right)^{\frac{1}{Q}}, \tag{4.15}$$

where $Q=\frac{\sigma(1+\gamma+\tau)}{\tau(1+\gamma-\sigma)} + \frac{1}{1-\alpha}$. So, assuming an interior solution, there are no differences between the environments in the ex post heterogeneity economy as in Cho and Cooley (1988). But the contents of the equilibria are quite different from each other due to the measurement of labor supplies. In other words, the measured (or physical units) labor supplies are quite different from the ones obtained above.

The aggregate labor supply in physical unit can be obtained using the relationship (3.16) and the assumed distribution of productivity (4.5).

$$N_1^s = x^s \cdot \int_{q^*}^{\bar{q}} (1/q) \cdot dG(q)$$

$$= x^s \cdot (\kappa/\bar{q}^{\kappa}) \cdot \int_{q^*}^{\bar{q}} q^{\kappa-2} \cdot dq$$

$$= x^s \cdot (\frac{\kappa}{\kappa-1}) \cdot (1/\bar{q}) \cdot [1-(q^*/\bar{q})^{\kappa-1}] \quad \text{for } \kappa \neq 1. \qquad (4.16)$$

However, from the relationship (3.1),

$$q^* = G^{-1}(1-e^s)$$

$$= \bar{q} \cdot (1-e^s)^{1/\kappa}, \qquad (4.17)$$

and hence (4.16) can be rewritten in terms of the employment rate as:

$$N_1^s = x^s \cdot (\frac{\kappa}{\kappa-1}) \cdot (1/\bar{q}) \cdot [1-(1-e^s)^{(\kappa-1)/\kappa}] \text{ for } \kappa \neq 1. \qquad (4.18)$$

If $\kappa = 1$, then the distribution of the productivity is uniform and hence we can have the aggregate labor supply as follows:

$$N_2^s = x^s \cdot (1/\bar{q}) \cdot \log[1/(1-e^s)] \quad \text{for } \kappa = 1. \qquad (4.19)$$

Therefore, the aggregate labor supply in physical units is completely different from the labor supply in efficiency units.

Using (4.18) or (4.19), we can obtain the elasticity of the aggregate labor supply in physical units as follows. If $\kappa \neq 1$, then:

$$\varepsilon(N_1^s) = \varepsilon(x^s) + \frac{(\kappa-1) \cdot e^s}{\kappa \cdot [(1-e^s)^{1/\kappa} - (1-e^s)]} \cdot \varepsilon(e^s), \qquad (4.20)$$

and if the distribution of the productivity is uniform, then:

$$\varepsilon(N_2^s) = \varepsilon(x^s) - \frac{e^s}{(1-e^s)\cdot\log(1-e^s)}\cdot\varepsilon(e^s), \tag{4.21}$$

where $\varepsilon(z)$ is the elasticity of a variable z.[14] Note that these elasticities, $\varepsilon(N_1^s)$ and $\varepsilon(N_2^s)$, depend on the alocation of the economy, whereas those of the labor supplies in efficiency unit do not depend on the allocation of the economy.

With the same parameter values specified in Cho and Cooley (1988), we have the following:

$$x^s = .44\cdot w^{0.27} \tag{4.22}$$

$$e^s = .69\cdot w^{1.02} \tag{4.23}$$

$$N^{es} = .30\cdot w^{1.29} \tag{4.24}$$

$$w^* = .99 \quad . \tag{4.25}$$

Therefore, we can see that the elasticity of hours in efficient units is .27, that of employment is 1.02, and that of the aggregate labor supply in efficiency units is the sum of the two elasticities, 1.29. The equilibrium is ($c*$, $x*$, $e*$, N^{e*}, $w*$) = (.29, .46, .64, .28, 1.01).

On the other hand, when the aggregate labor supply is in physical units, the elasticity can be obtained as follows:

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[14] In the calculation of the elasticities, the following fact is used.

$$\frac{\partial x}{\partial \log(y)} = x\cdot\frac{\partial \log(x)}{\partial \log(y)}$$

$$N_1^s = .19, \quad \varepsilon(N_1^s) = 1.63 \quad \text{if } \bar{q} = 2.0 \text{ and } \kappa = 2 \tag{4.26}$$

$$N_1^s = .48, \quad \varepsilon(N_1^s) = 2.07 \quad \text{if } \bar{q} = 2.0 \text{ and } \kappa = .5 \tag{4.27}$$

$$N_2^s = .34, \quad \varepsilon(N_2^s) = 2.04 \quad \text{if } \bar{q} = 1.5 \text{ and } \kappa = 1. \tag{4.28}$$

Here we have to note two facts. First, the distributional assumption on an agent's productivity is very important. According to the assumed distribution of q, the equilibrium value of aggregate labor supply varies a lot. Second, the elasticities of aggregate labor supply in the three cases are uniformly greater than that of the aggregate labor supply when measured in efficiency unit. Once again these elasticities critically depend on the distributional assumption on q.

For reference, Table 1 is taken from Cho and Cooley (1988). Table 1 presents the elasticities in the cases without heterogeneities.

5. Discussion

The ex post heterogeneity environment can approximate many models with heterogeneity already existing in the literature. Most obvious modification is to assume a discrete skill distribution rather than continuous one as in the previous sections. In other words, they assume in many cases finite number of skill groups such as skilled and unskilled, or men, women and youth etc. (see Kydland (1984), Cho and Rogerson (1988)). To replicate this environments using the ex post heterogeneity environment, we have only to assume that the skill distribution is discrete. For example, assume:

$$g(q) = \begin{cases} \frac{1}{m} & \text{if } q=q_i,\ i=1,\ 2,\ \cdots,m \\ 0 & \text{otherwise.} \end{cases} \tag{5.1}$$

Then there are m groups of agents with different skills. Kydland (1984) has two skill groups in his model, i.e. skilled and unskilled.[15] Cho and Rogerson (1988) also have two skill groups, i.e. male and female. Any of these environment can be approximated by the environment studied in the previous sections. However, the employment insurance in the case of a discrete skill distribution takes the following form. An agent with a higher realized skill works in the market first. But among the agents with identical efficiency in skill, market participation is determined by the employment lotteries within the group. For example, suppose m=2 in (5.1) and the aggregate employment rate is 65 percent. Then, all agents belonging to the first group with $q=q_1$ ($>q_2$) work in the market and the remaining 15 percent of the workers in the market are determined by the lotteries within the second group with $q=q_2$ ($<q_1$).

The model can be extended to a multisector setting. Heckman and Sedlacek (1985) constructed an empirical model with two sectors and then analyzed the selection bias as well as the aggregation bias. We can extend the ex post heterogeneity environment to a very similar environment to theirs. Suppose there are two sectors in an economy and each sector produces a distinct product from the other sector. Assume that the utility function is:

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[15] Although the ex post heterogeneity model can capture the essential aspects of his model, there are still a few discrepancies between them. Most of all, the heterogeneity in Kydland (1984) is ex ante and hence the utility weights has to be determined by solving a fixed point problem. But it is very difficult to solve it. In addition, there is no extensive margin adjustment in the labor market in his model.

$$U(c_1, c_2, n; e, E) = u^1(c_1) + u^2(c_2) - v(n) - \psi(e,E)\cdot 1(n>0), \quad (5.2)$$

where c_1 and c_2 are the consumption of goods produced in each sector. Next, assume each sector faces the following production function:

$$y_i = \lambda_i f^i(K_i, N_i^e) \quad \text{for } i = 1,\ 2. \qquad (5.3)$$

Suppose there are five goods: sector specific capital, labor and sector specific output and suppose each agent is endowed with one unit of time and one unit of capital for each sector. Each agent faces a stochastic skill distribution. Let $q=(q_1,q_2)$, where q_i is the skill index specific to sector i, i=1,2. Assume q has the distribution function $G(q)$ for $q\in[q,\bar{q}]\times[q,\bar{q}]$. We assume that each agent works in only one sector of the two and that the following employment insurance is available. If the realized productivity of an agent satisfies:

$$q_2 > h(q_1), \qquad (5.4)$$

then the agent belongs to sector 2; otherwise the agent belongs to the sector 1.[16] In a sector, an agent works in the market if his realized skill specific in that sector is higher than a threshhold productivity

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[16] In fact, this is a sort of selection rule, which is the source of the selection bias. See Heckman (1979) and Heckman and Sedlacek (1985) for details.

level q_i^*, $i=1,2$. With these modifications, we can characterize the equilibrium in the same way as in section 3.

Here we have to note three facts. First, note that the selection rule is specified in a way that each sector contains a fixed proportion of the agents. If we are to allow the variable proportion, we can assume that ϑ depends on some realization of the parameters such as λ_1 and λ_2. In fact, this contract parameter has to be determined in the model in a way that the utility of the representative agent is maximized. Second, the persistence of employment (or unemployment) in a sector can be introduced in the same way as was discussed earlier in this section. Third, Heckman and Sedlacek (1985) discussed the effect of sectoral movement of workers on aggregation bias They showed through a simulation that the aggregation bias can be reversed due to the sectoral movement of workers. That is, employment is much more variable in manufacturing sector, particularly in durables, than in non-manufacturing sectors. Suppose there is an aggregate disturbance such as an oil price increase. Then employment declines in the manufacturing sector, but some of the former manufacturing workers enter nonmanufacturing sectors rather than drop out of the work force altogether. Here we have two effects due to the sectoral shift of these workers. First, the former manufacturing workers turn out to be at the lower tail of skill distribution in manufacturing sector, and hence their exit from the sector raises the average quality of the remaining workers. Thus, this first effect attenuates the decline in measured average wages. This is the aggregation bias effect emphasized in Bils (1985). Second, the former manufacturing workers entering the nonmanufacturing sector turn out to be at the lower tail of the skill distribution in nonmanufacturing sectors, and hence the new entrants lower the average quality of the workers in nonmanufacturing sectors. This reverse aggregation bias effect is emphasized in Heckman and Sedlacek (1985). However, these effects can be easily modeled in the current two sector framework.

6. Dynamics and Calibration[17]

The static model characterized in Section 3 can be extended into a dynamic setting by incorporating capital accumulation and an information structure. Suppose there is a continuum of agents uniformly distributed over the closed interval [0,1] as was assumed in Section 2. Each individual is initially endowed with one unit of time and one unit of capital, and lives forever. There is one firm with technology which can be represented with the production function:

$$Y_t = \lambda_t f(K_t, N_t^e), \tag{6.1}$$

where K_t, N_t^e, and Y_t are aggregate capital, aggregate labor in efficiency units, and aggregate output in period t respectively. We will abstract from population and technological growth. λ_t is a random shock which is assumed to be following an AR(1) process:

$$\lambda_{t+1} = \eta\lambda_t + \varepsilon_{t+1}, \tag{6.2}$$

where ε_t's are assumed to be independently and identically distributed with

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[17] This section is parallel to section 5 of Cho and Cooley (1988).

distribution function F. It is assumed that the distribution has a positive support to guarantee output to be always positive. Since we abstract from growth, λ_t will have an unconditional mean of 1 by assuming the mean of the distribution F to be $1-\eta$. Individuals are assumed to observe λ_t at the beginning of the period t.

The productivity of an agent follows the i.i.d. process with finite support. It is not difficult to incorporate the features of persistence in employment. Since all agents face the same ex ante distribution of their productivity at the beginning of period 0, nothing will be altered even though we assume a distribution which is not i.i.d.. The realized productivity of each agent is assumed to be observed at the begining of each period. For the rest of the paper we will assume the distribution defined in (4.5). If we assume this distribution, the aggregate labor supply in physical units can be obtained from (4.4.8) or (4.4.9).[18]

Output can be either consumed or invested, and hence the following constraint has to be satisfied in the aggregate:

$$C_t + I_t \leq \lambda_t f(K_t, N_t^e), \tag{6.3}$$

where C_t and I_t are aggregate consumption and investment in period t. The law of motion for the aggregate capital stock is given by:

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[18] The estimation of the skill distribution of an economy seems to be a hard task due to the following reasons. First, the shape of the distribution have changed very rapidly over time. Second, there is a growth factor which moves the distribution upwards. Third, the lower tail of the distribution is truncated. Due to these reasons, I cannot report the skill distribution but I think that the estimation of the distribution seems to be a very interesting and valuable project.

$$K_{t+1} = (1-\delta)K_t + I_t, \qquad (6.4)$$

where δ is the rate of capital depreciation and hence $0 \leq \delta \leq 1$. The stock of capital is assumed to be owned by the individuals who sell capital services to the firm. Thus, the aggregate law of motion for the capital stock, (6.4), arises from individual optimizing behavior. In the model, all agents are identical and are treated equally. From now on, the variables in uppercase letters will denote aggregate variables, while those in lowercase letters will denote per capita variables, but anticipating the equilibrium, we use these interchangeably.

The representative agent will maximize the expected value of the discounted sum of temporal utilities. That is, the agent faces the lifetime expected utility given by:[19]

$$E_0[\sum_{t=0}^{\infty}\beta^t\{u(c_t) - v[q_t(1-l_t)]e_t - \psi(e_t,E_t)e_t\}], \qquad (6.5)$$

and the agent maximizes (6.5) subject to the constraints (6.1)-(6.4). As we noted earlier, the nonconvexity in each individual's preference is resolved by the introduction of the employment lotteries or employment insurance. Competitive equilibrium can be shown to exist, but it is suboptimal as far as the cost function ψ depends on the aggregate employment rate.

The programming problem to be solved can be stated as follows:

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[19] We are using in advance the results from section 3.3.

$$\text{maximize } E_0[\sum_{t=0}^{\infty}\beta^t\{u(c_t) - v(x_t)e_t - \psi(e_t,E_t)e_t\}] \quad (6.6)$$

$$\begin{aligned}
\text{s.t.} \quad & c_t + i_t \le \lambda_t f(K_t,N_t^e),\ N_t^e = e_t x_t \\
& K_{t+1} = (1-\delta)K_t + i_t \\
& \lambda_{t+1} = \eta\lambda_t + \varepsilon_{t+1} \\
& c_t \ge 0,\quad 0 \le e_t \le 1 \\
& 0 \le x_t \le q_t,\quad K_t \ge 0.
\end{aligned}$$

Note that the hours of work per worker and the aggregate hours are measured in efficiency units, i.e. $x_t = q_t \cdot (1-l_t)$, rather than in physical units The competitive outcome can be obtained from this programming problem in two steps. First, we need to solve for the first order conditions from the programming problem, taking the aggregate employment rate as given. After obtaining the first order conditions from the programming problem , we use the equilibrium condition, namely $e_t = E_t$, to get the competitive allocation.

We are interested in the quantitative implications of this model since we have constructed it with the express purpose of being able to better understand the observed fluctuations in employment and productivity. We here follow the methods of Kydland and Prescott (1982) and Hansen (1985) because their approach to the quantitative study of these issues provides a convenient standard of comparison. We specify our model as follows:

$$Y_t = \lambda_t K_t^{\alpha}(N_t^e)^{1-\alpha} \quad (6.7)$$

$$u(c_t,x_t;e_t,E_t) = \log(c_t) - \frac{a}{1+\gamma}\cdot x_t^{1+\gamma} - \frac{b}{1+\tau}\cdot e_t^{\tau}\cdot 1(x_t>0) \quad (6.8)$$

Note that we abstract from the externality features in the preferences, i.e. there is no term containing E_t.[20] With these specifications, we can rewrite the problem (6.6) as:

$$\max E_0[\sum_{t=0}^{\infty}\beta^t\{\log(c_t) - \frac{a}{1+\gamma}\cdot x_t^{1+\gamma}\cdot e_t - \frac{b}{1+\tau}\cdot e_t^{1+\tau}\}] \tag{6.9}$$

$$\text{s.t.}\quad c_t + i_t \le \lambda_t K_t^{\alpha}(N_t^e)^{1+\alpha},\ N_t^e = e_t\cdot x_t$$
$$K_{t+1} = (1-\delta)K_t + i_t$$
$$\lambda_{t+1} = \eta\lambda_t + \varepsilon_{t+1}$$
$$c_t \ge 0,\quad 0 \le e_t \le 1$$
$$0 \le x_t \le q_t,\quad K_t \ge 0.$$

Unfortunately, the problem (6.9) cannot be solved analytically for decision rules. But, following Kydland and Prescott (1982) we approximate the model economy with quadratic objective and linear constraints. The details of the approximation method are described in Kydland and Prescott (1982).

The steady state of (6.9) can be solved from the following conditions:

$$I = \delta K \tag{6.10}$$

$$\lambda(1-\alpha)K^{\alpha}(N^e)^{-\alpha} = a\cdot x\cdot c \tag{6.11}$$

$$\lambda(1-\alpha)K^{\alpha}(N^e)^{-\alpha}\cdot x = \{\frac{a}{1+\gamma}\cdot x^{1+\gamma} + b\cdot e^{\tau}\}\cdot c \tag{6.12}$$

$$\lambda\alpha K^{\alpha-1}\cdot(N^e)^{1-\alpha} = \delta+\rho \tag{6.13}$$

$$c + I = \lambda K^{\alpha}(N^e)^{1-\alpha} \tag{6.14}$$

[20] The examples in section 4 show that the externality case is identical to the Pareto optimal one in the sense that they show the same fluctuations in aggregates. If we introduce externality feature, we have to face additional problem in simulation. However, we can solve the problem using the methods in Cooley and Hansen (1988) or in King, Plosser and Rebello (1988). But this is not an issue here.

$$e = E, \tag{6.15}$$

where the steady state of a variable is denoted by the variable's symbol without any script, and $\rho=(1/\beta)-1$. The condition (6.10) is the standard one for a steady state. (6.11) and (6.12) equate the marginal benefits from adjustments along the intensive and extensive margins to the marginal costs of those adjustments respectively. The condition (6.13) requires the rental rate of capital to be equal to the marginal productivity of the capital stock, and (6.14) is the budget constraint. Finally, (6.15) is the equilibrium condition.

We can solve these conditions for x, e, K, I, and c. For notational convenience, we define the following:

$$\Theta = \lambda(1-\alpha)\cdot[\lambda\alpha/(\delta+\rho)]^{\frac{\alpha}{1-\alpha}} \tag{6.16}$$

$$\Lambda = [\lambda\alpha/(\delta+\rho)]^{\frac{1}{1-\alpha}} \tag{6.17}$$

$$\Gamma = \frac{\delta(1-\alpha)+\rho}{\alpha} \tag{6.18}$$

Then the steady state values are obtained as follows:

$$e = \left[\frac{\Theta\gamma}{b(1+\tau)\Gamma\Omega}\right]^{1/(1+\tau)} \tag{6.19}$$

$$x = \left[\frac{\Theta}{a\Gamma\Omega e}\right]^{1/(1+\gamma)} \tag{6.20}$$

$$N^e = x\cdot e \tag{6.21}$$

$$K = \Lambda\cdot N^e \tag{6.22}$$

$$I = \delta\cdot K, \tag{6.23}$$

and the value of consumption can be obtained from (6.14) using these and the labor supply in physical unit can be obtained from the relationship in (3.3.16).

We borrow most of the parameter values from the papers by Kydland and Prescott (1982), Hansen (1985), and Prescott (1986). That is, the following values are used for the calibration: α=.36, β=.99, δ=.025, η=.95. The details of justification for the values can be found in Prescott(1986). The utility parameters, a, b, γ and τ are determined to reflect three facts which can be observed in U.S. economy.

First, the model does not distinguish status of an agent in thelabor force. That is, nobody in the model economy is out of the labor force. Consequently, the appropriate variable in the actual economy corresponds to the employment rate in the model economy is formed by the product of the the employment rate and the participation rate. For the U.S. economy this is about 65%. Second, about one-third of the time endowment is spent in labor market activity. For comparison with the results reported in the papers with models without any heterogeneity, we assume the aggregate hours in efficiency units is one third.[21] In this case, the proportion of time devoted to the market activity seems to be somewhat greater than the actual fraction, but it is not a bad estimate if we take into account the portion of time spent on commuting and preparing for work. Third, one of the features of the business cycle that we stressed earlier is that 75% of the

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[21] The aggregate labor supply in physical unit can be greater or smaller than this number depending on the assumed productivity distribution. In Example 4 in section 4, it is greater if κ = .5, smaller if κ =2, and roughly same if κ = 1.

aggregate labor fluctuation is due to the fluctuation in employment and the remaining 25% is due to the fluctuation in hours of work per person. This ratio of fluctuation in hours per person relative to that in employment has been fixed at one third. We have determined the values of the utility parameters to match these three numbers. There is an additional degree of freedom to the utility parameters since there are four parameters and only three features to pin down. Three cases are simulated: in the first two cases γ is arbitrarily fixed at 1 in the first case and at 1.3 in the second case. The used parameter values are $(a, b, \gamma, \tau) = (6.0, .87, 1, .62)$ and $(7.5, 1.02, 1.3, .72)$. However, the choice of this parameters affects both our estimate values and the implied elasticity of intertemporal substitution. To see the implied elasticity of intertemporal substitution when the ratio of fluctuations in the aggregate hours in physical units relative to productivity is identical to the actual one, the four parameters in the third case are determined to reflect the three facts explained above and an additional fact that the ratio of fluctuations in aggregate hours in physical units relative to those of productivity is 1.47 in U.S. economy. The implied parameter values in the third case are $(a, b, \gamma, \tau) = (29.7, 1.98, 3.7, 1.5)$.

The average deviations of the variables around the steady state have been chosen as .012, .006, .08, .017, .004, and .015 for λ, K, I, N^e, x, and e, respectively. These values are calculated from the actual U.S. economy which are logged and detrended by the same method applied to the model economy.

Using the value function for the economy, we solve for the equilibrium decision rules as functions of the state variables, the technology shock and the capital stock. If we have the equilibrium decision rules, we can generate time series for the model economy. One hundred time series were generated in the first two cases and twenty time series in the third case, and each of the time series was logged and detrended using Prescott-Hodrick filter.[22] Second moments were calculated from each of the time series and means of the repeated simulations were calculated. The results are reported in Table 2 and 3. The statistics for the model economy are computed with the standard deviation of the technology shock equal to .00835 and .00865 in the first two cases and .00952 in the third case. This number, which lies in a range suggested by Prescott (1986), was chosen because it implies the mean of the standard deviation in output from the repeated simulations equal to the standard deviation in actual U.S. output.

7. Results

The results of the Monte Carlo experiments reported in Table 2(a) are those of efficiency units. In fact, since the physical unit is meaningless in terms of output, consumption, investment, capital stock and employment, we need to keep in mind the fact that the hours and productivity are measured in efficiency units in Table 2(a). Basically, the results in Table 2(a) are identical to those reported in Cho and Cooley (1988). The results resemble the statistics from the actual U.S. economy with a few

[22] The choice of filter affects the resulting statistics. For example, King, Plosser and Rebelo (1987) used the linear filter and the resulting statistics seems quite different from Kydland and Prescott (1982) and Hansen (1985). As long as the same filter is applied to the data and the model, this seems relatively unimportant.

notable exceptions. The most important is that the model economy shows less fluctuations than the actual economy. In other words, the standard deviations for the model economy are less than those for the actual U.S. economy overall. Those discrepancies may be due to aggregation bias (see Kydland (1984), Cho and Rogerson (1988)), measurement error(see Hansen and Sargent(1988)), the inclusion of an inventory or a durable component in the data on investment and consumption or other factors not present in the model economy. We will discuss aggregation bias in detail later.

The correlations with output from the model economy are very close to those from the actual economy except that hours, employment and productivity are correlated more highly in the model economy than in the actual economy. This result is due to the fact that the time series in the model economy were created by a single shock. In other words, we need more than one shock to the economy or we need to introduce measurement error to create time series having close correlations to those from the actual economy. This stochastic singularity problem is common to real business cycle models.

In Table 2(a), we can see that the ratio of standard deviation of hours relative to that of employment is one-third in the model economies. This ratio was fixed at that level by adjusting the utility parameters as explained in the previous section. Now look at the ratio of aggregate hour variability relative to productivity variability in the model economy. This ratio is about 1.4 in case of MODEL(1), which is quite close to the ratio implied by the U.S. data. Kydland and Prescott (1982), Hansen (1985), and Prescott (1986) all suggest that this ratio is the most important one to compare with the data from the economy. In the model economy studied by

Kydland and Prescott this ratio turns out to be 1.17, while it is 2.70 for the indivisible labor economy studied by Rogerson (1984) and Hansen (1985). For the U.S. economy, the ratio is about 1.47 in physical units but 1.42 in efficiency unit (see Hansen (1985)).[23]

The implication of this discussion is that the ratio of fluctuations in hours to productivity is too low in a model economy with adjustment only along the intensive margin but too high in a model economy with adjustment only along the extensive margin. It is not surprising that our model economy generates a ratio close to the actual one since it embodies both sources of labor market fluctuations. However, once a model with these features is developed, the issue reduces to the magnitude of the elasticity of intertemporal substitution of aggregate hours of work.

To make the point clearer, let's look at the case of MODEL(2) in Table 2(a). In case of MODEL(2), the assumed parameter values are γ=1.3 and τ=.72. However, if the values of γ and τ increase, then the implied elasticity of intertemporal substitution of aggregate hours decreases. But changes in these parameters do not alter the basic characteristics of the intertemporal substitution (or smoothing) of consumption and hence there are no fundamental changes in the fluctuations in consumption, investment and capital stock. Main changes appear in the labor statistics. First of all, the hours per worker, employment and aggregate hours fluctuate less as the values of γ and τ increase. Productivity fluctuates more since aggregate hours fluctuates less with output variability fixed

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[23] The ratio in efficiency unit is not calculated using an estimated skill distribution but using synthetic cohorts. Thus there is a unresolved aggregation bias yet.

at the actual level. These two effects reduce the ratio of fluctuations in aggregate hours relative to those of productivity from 1.42 in case of MODEL(1) to 1.24 in case of MODEL(2). In fact, the implied elasticity in case of MODEL(1) is about 4.2, while the elasticity in case of MODEL(2) is 3.2.

Many papers have reported that the elasticity of intertemporal substitution is too small for an equilibrium model of aggregate fluctuations to be a good representation of the actual economy (see, for example, Altonji (1982, 1986), Altonji and Ashenfelter (1980), Ashenfelter (1984), Ham (1986) Mankiw, Rotemberg and Summers (1985)). Most of these studies have concluded that it is difficult to identify an elasticity of intertemporal substitution from the data that is large enough to reconcile the much greater fluctuations in aggregate hours than in wage rate which are observed. But no one of them has noted that the labor used in the production process is not in physical units but in efficiency units and that aggregate labor supply in physical units fluctuates much more than that in efficiency units. Table 2(b) shows how important this distinction is in the study of aggregate fluctuations.

Under the assumed distribution of productivity, hours per worker and aggregate hours have much more fluctuation in physical units than in efficiency units in all cases reported in Table 2(b). On the other hand, productivity in physical units shows much less fluctuations than in efficiency units. As a result, the ratios of fluctuations in aggregate hours in physical units relative to productivity in the same units, which are reported in Table 2(b), are much greater than those in efficiency units, which are reported in Table 2(a). Actually, Table 2(b) shows that if we measure the hours and productivity in physical units, then we do not need such a high elasticity of intertemporal substitution to reproduce the key ratio close to the actual one in an equilibrium model of business fluctuations. Table 2(b) also shows that the correlation of productivity in physical units with output critically depends on the magnitude of the fluctuations in aggregate hours. As the fluctuations in aggregate hours increase to those of output, the correlation of productivity with output decreases. Moreover, if the aggregate hours fluctuate more than the output, then the correlation becomes negative.

The last question to be raised in the paper is how large the elasticity of intertemporal substitution has to be for the ratio of fluctuations in aggregate hours in physical units relative to productivity in the same units to be the same as the actual one. The results are reported in Table 3. Once again there are no changes in fluctuations in the variables except those of labor market as in the previous cases. The key ratio is 1.45 in physical units which is very close to the actual one but 0.60 in efficiency units which is much smaller than those in Table 2. Note that the aggregate hours in physical units fluctuate much more than the productivity in physical units, while those in efficiency units show much less fluctuations than the productivity either in physical units or in efficiency units. For the reproduction of the key ratio, we assumed the parameter values as γ=3.7 and τ=1.5. These parameter values imply that the elasticity of intertemporal substitution is 1.12. This elasticity is not at all intractably large. For instance, Alogoskoufis (1987) reported the estimates of the elsticity which are much greater than this value. His smallest elasticity is around 1. This results combined with those in Table 3 imply that an equilibrium business cycle model can be completely consistent with the actual economy if a heterogeneity is taken into account.

TABLE 1

Elasticities in the Examples 1-3

Case	Hours	Employment	Aggregate
Convex Environment(b=0)	$\frac{\sigma}{1+\gamma-\sigma}$ (.40)	0	$\frac{\sigma}{1+\gamma-\sigma}$ (.40)
Pure Fixed Cost(τ=0)	0	$\frac{\sigma}{1-\sigma}$ (4.00)	$\frac{\sigma}{1-\sigma}$ (4.00)
Pareto Optimal(ν=1)	$\frac{\sigma}{R}$ (.27)	$\frac{\sigma(1+\gamma)}{\tau R}$ (1.02)	$\frac{\sigma(1+\gamma+\tau)}{\tau R}$ (1.29)
Middle Case(ν=.5)	$\frac{\sigma}{R}$ (.27)	$\frac{\sigma(1+\gamma)}{\tau R}$ (1.02)	$\frac{\sigma(1+\gamma+\tau)}{\tau R}$ (1.29)
Pure Externality(ν=0)	$\frac{\sigma}{R}$ (.27)	$\frac{\sigma(1+\gamma)}{\tau R}$ (1.02)	$\frac{\sigma(1+\gamma+\tau)}{\tau R}$ (1.29)

NOTE: (1) The assumed utility function is:

$$U(c,l;e,E) = \frac{1}{\sigma}\cdot c^{\sigma} - \frac{a}{1+\gamma}\cdot n^{1+\gamma} - \frac{b}{1+\tau}\cdot[\nu e+(1-\nu)E]^{\tau}\cdot I(n>0).$$

(2) $R = \gamma + \frac{(\tau+\gamma+1)(1-\sigma)}{\tau}$.

(2) The employment lotteries are introduced in the cases of nonconvexities.

(3) The numbers in parenthesis are elasticities when σ=.8, γ=2 and τ=.8.

TABLE 2(a). Calibration Results 1

SERIES	U.S. STDD	U.S. CORR	MODEL(1) STDD	MODEL(1) CORR	MODEL(2) STDD	MODEL(2) CORR
Output	1.76	1.00	1.76(.17)	1.00(.00)	1.76(.18)	1.00(.00)
Consumption	1.29	.85	.53(.06)	.88(2.49)	.55(.08)	.89(3.48)
Investment	8.60	.92	5.63(.57)	.98(.40)	5.53(.58)	.99(.40)
Capital Stock	.63	.04	.47(.08)	.07(6.73)	.48(.01)	.07(5.73)
Agg. Hours	1.74	.77	1.06(.12)	.98(.56)	1.00(.01)	.98(.51)
Hours	.46	.76	.25(.02)	.98(1.24)	.24(.02)	.98(.51)
Employment	1.50	.81	.81(.08)	.98(1.04)	.76(.08)	.98(.51)
Productivity	1.18	.35	.75(.08)	.96(.81)	.80(.01)	.97(.62)
Agg. Hours / Productivity	1.47		1.42		1.24	

Note: The data used are quarterly time series from the third quarter of 1955 to first quarter of 1984. Before the statistics were calculated, the data were logged and detrended by the Prescott-Hodrick filter. STDD = Standard Deviation, CORR = Correlation With Output. Standard Deviations are in percentage terms. The statistics are means of 100 simulations. The numbers in parentheses are standard deviations of the 100 simulations in percentage term. MODEL(1) is the case that $(a, b, \gamma, \tau) = (6.0, .87, 1.0, .62)$ and MODEL(2) is the case that $(a, b, \gamma, \tau) = (7.5, 1.02, 1.3, .72)$.

TABLE 2(b). Hours and Productivity in Physical Units

CASE	SERIES	MODEL(1) STDD	MODEL(1) CORR	MODEL(2) STDD	MODEL(2) CORR
κ=2.0	Hours	.53(.05)	.98(.43)	.50(.05)	.98(.51)
	Agg. Hours	1.34(.13)	.98(.42)	1.26(.13)	.98(.51)
	Productivity	.51(.06)	.87(2.08)	.57(.07)	.91(1.63)
	Ratio	2.63		2.21	
κ=1.0	Hours	.87(.09)	.98(.43)	.83(.08)	.98(.51)
	Agg. Hours	1.68(.16)	.98(.43)	1.59(.16)	.98(.51)
	Productivity	.33(.05)	.34(6.16)	.36(.07)	.56(4.08)
	Ratio	5.09		4.42	
κ=0.5	Hours	1.74(.17)	.98(.44)	1.66(.17)	.98(.51)
	Agg. Hours	2.55(.25)	.98(.43)	2.42(.25)	.98(.51)
	Productivity	.87(.10)	-.84(3.67)	.77(.09)	-.80(5.38)
	Ratio	2.93		3.14	

Note: (1) The assumed distribution of the productivity is:

$$G(q) = (q/\bar{q})^{\kappa}, \text{ where } q \in [0, \bar{q}].$$

(2) STDD=Standard Deviation, CORR=Correlation Coefficient.
(3) Ratio is the ratio of the standard deviation of aggregate hours relative to the standard deviation of productivity.

TABLE 3. Calibration Results 2

SERIES	U.S. STDD	U.S. CORR	EFFICIENCY UNIT STDD	EFFICIENCY UNIT CORR	PHYSICAL UNIT STDD	PHYSICAL UNIT CORR
Output	1.76	1.00	1.76(.21)	1.00(.00)	1.76(.21)	1.00(.00)
Consumption	1.29	.85	.57(.08)	.89(2.97)	.57(.08)	.89(2.97)
Investment	8.60	.92	5.49(.67)	.99(.32)	5.49(.67)	.99(.32)
Capital Stock	.63	.04	.49(.10)	.08(5.38)	.49(.10)	.08(5.38)
Agg. Hours	1.74	.77	.67(.08)	.98(.52)	1.07(.12)	.98(.52)
Hours	.46	.76	.16(.02)	.98(.52)	.56(.06)	.98(.52)
Employment	1.50	.81	.51(.06)	.98(.52)	.51(.06)	.98(.52)
Productivity	1.18	.35	1.11(.06)	.99(.16)	.74(.10)	.96(.79)
Agg. Hours / Productivity	1.47		0.60		1.45	

Note: The data used are quarterly time series from the third quarter of 1955 to first quarter of 1984. Before the statistics were calculated, the data were logged and detrended by the Prescott-Hodrick filter. STDD = Standard Deviation, CORR = Correlation With Output. Standard Deviations are in percentage terms. The statistics are means of 20 simulations. The numbers in parentheses are standard deviations of the 20 simulations in percentage term. The assumed parameter values are $(a, b, \gamma, \tau) = (29.7, 1.98, 3.7, 1.5)$ and the assumed productivity distribution is:

$$G(q) = (q/1.5) \quad \text{for } q \in [0, 1.5],$$

i.e. uniform distribution over the interval $[0, 1.5]$.

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