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Risk Sharing, Indivisible Labor and Aggregate Fluctuations

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Abstract

This paper studies fluctuations in a real business cycle model when there is a risk neutral agent present to offer insurance to workers. This economy is compared with one in which there is no risk neutral agent but labor is indivisible. In static models it is difficult to distinguish the risk sharing and indivisible labor economies, but in dynamic models with capital accumulation the indivisible labor model appears to perform better.

Section One: Introduction

The aggregate labor market has been a persistent problem for economists trying to build equilibrium models capable of reproducing aggregate economic time series. After the seminal work by Lucas and Rapping (1969) there were many papers arguing that elasticities of labor supply were too low for equilibrium models to be consistent with observed magnitudes of fluctuations in total hours and wages (see Altonji and Ashenfelter (1980), Altonji (1986), and Ham (1986) for example). Prescott (1983) argues that competitive theory does not require that payment and delivery of goods be contemporaneous, and suggests that it is more instructive to look at hours and productivity instead of hours and wages. However, this did not change the nature of the problem to any appreciable extent. In another very important paper, Kydland and Prescott (1982) find the total hours to productivity ratio to be the most serious problem in trying to represent post World War II time series with a real business cycle model.

A recent innovation introduced by Rogerson (1988) which greatly enhances the ability of these models to account for labor market fluctuations has been to assume that labor is indivisible. Hansen (1985) showed in a calibration exercise that this assumption does greatly enhance the performance of real business cycle models in accounting for labor market observations. Optimal allocations in this economy involve workers pooling together to offer complete unemployment insurance. This feature is reminiscent of a class of models first introduced by Azariadas (1975) and Bailey (1974) which is characterized by a risk neutral agent offering insurance to workers who face wage uncertainty. Grossman (1981) illustrates how this feature can increase the magnitude of fluctuations in labor relative to productivity or wages.

This paper studies this feature in more detail and contrasts the risk sharing mechanism with the indivisible labor mechanism in causing increased fluctuations. It is shown that the former operates by decreasing curvature in preferences for consumption whereas the latter operates by decreasing curvature in preferences for leisure. In static models these two mechanisms appear to be equally effective in producing an improved fit with actual data. However, when the paper considers dynamic models and calibrations such as Kydland-Prescott (1982) and Hansen (1985), it is seen that the indivisible labor model appears to perform better than a model which is based on risk sharing.

Section Two: Fluctuations in Hours and Productivity: A Parametric Example

This section considers an example which will prove useful in discussing the results obtained later in the paper. Imagine a one period economy where labor is used to produce output according to a linear technology with coefficient θ. There is one representative consumer with preferences over consumption and leisure defined by

$$u(c) - v(h)$$

where it is assumed that

$$u(c) = \frac{c^{\sigma}}{\sigma} \qquad \sigma \leq 1$$

$$v(h) = \frac{h^{\gamma}}{\gamma} \qquad \gamma \geq 1$$

An optimal (and, hence, competitive equilibrium) allocation is found by solving:

$$\max_{h}\ u(\theta h) - v(h)$$

$$\text{s.t. } 0 \leq h \leq \bar{h}$$

where $\bar{h}$ is the time endowment. For the functions u(c) and v(h) defined above, the solution to this problem is given by

$$h = \theta^{\sigma/\gamma-\sigma}$$

As mentioned in the introduction, it is of interest to study how h responds to changes in productivity θ, and, in particular, how this response is affected by the parameters of preferences.

This paper is concerned with the properties of the u(c) function and thus the change in the elasticity (ε) of hours with respect to productivity caused by a change in σ will be of interest. Straightforward substitution gives:

$$\left(\frac{\partial\varepsilon}{\partial\sigma}\right)\frac{\sigma}{\varepsilon} = \frac{\gamma}{\gamma - \sigma}$$

Changes in γ are also of interest. Substitution gives:

$$\left(\frac{\partial\varepsilon}{\partial\gamma}\right)\frac{\gamma}{\varepsilon} = -\frac{\gamma}{\gamma - \sigma}$$

This result says that if $\varepsilon > 0$ (i.e., $\sigma > 0$), then decreasing the curvature in c and h (increases in σ and decreases in γ) are equally effective in increasing fluctuations in hours relative to productivity. Intuition suggests a strong link between curvature and fluctuations: two of the factors which discourage increases in labor supply in response to increases in productivity are that additional units of leisure become more valuable and additional units of consumption become less valuable.

In previous work Rogerson (1988) has shown that the assumption of indivisible labor implies that the economy behaves as if $\gamma = 1$. This paper studies how allowing for risk sharing causes an effective increase in σ.

Section Three: The Economies E and E_a

3.1 The Economy E

The economy E lasts for a single period. Labor (N) and capital (K) are used to produce output according to a concave constant returns to scale production function which is twice continuously differentiable in both arguments and subject to a technological shocks. This function is written as f(K,N,s). The shock s has n realizations, denoted by s_i and occurring with probabilities π_i.

There are two types of agents in the economy. There is a single representative worker endowed with one unit of capital and one unit of time, any fraction of which can be supplied as labor. If a worker receives c units of consumption and supplies h units of labor they receive utility given by:

$$u(c) - v(h)$$

where it is assumed that both functions are twice continuously differentiable, strictly increasing with u concave and v convex. The following boundary conditions are added to insure interior solutions in future sections:

$$\lim_{c \to 0} u'(c) = \lim_{h \to 1} v'(h) = \infty \qquad \lim_{c \to \infty} u'(c) = \lim_{h \to 0} v'(h) = 0$$

The other agent has no endowment of time or capital but is endowed with W units of output. It is assumed that $W > \max_{s_i} f(1,1,s_i)$. The importance of this assumption will become clearer in later sections, but it is to ensure that the second agent has sufficient resources to completely smooth consumption fluctuations of the first agent. The second agent has preferences defined over consumption and is risk neutral so that utility received from consuming c units of consumption is given by c.

Both agents evaluate a state contingent commodity bundle by computing expected utility. The timing of the model is such that the state of nature (the realization of s) is revealed before any production or consumption activity takes place, so it will be possible for agents to enter into contracts contingent upon the realization of s.

3.2 The Economy E_a

The economy E_a is identical to the economy E with two exceptions. The first is that the second agent does not exist, or equivalently, exists but W is set to zero. The second difference is that workers have preferences defined by

$$ac - v(h)$$

where the constant a is the same as the subscript in E_a. The function v(h) is the same as before.

Section Four: Equilibria for E and E_a.

This section characterizes the equilibrium allocations for the two economies E and E_a and proves that an equivalence exists between the two. Prior to doing this, some notation is required. States of nature will be indexed by i. Prices for output, capital and labor respectively in state i will be p_i, q_i, and w_i. Consumption and labor supply of the worker in state i will be denoted c_i and h_i. Supply of capital will always be equal to one. The firm's demand for labor and capital in state i will be denoted by H_i and K_i. The consumption of the risk neutral agent in state i will be denoted by $\hat{c}_i$.

Finding equilibrium allocations for E_a is a straightforward exercise. Because E_a contains only one type of agent the equivalence between competitive allocations and Pareto optima implies the following.

Proposition 1: If (c_i, h_i, H_i, k_i, K_i, p_i, q_i, w_i, $i = 1, \ldots, N$) is an equilibrium for E_a, then (c_i, h_i, $i = 1, \ldots, N$) is the unique solution to:

$$\text{(P-1)} \qquad \max_{(c_i,h_i)} \sum_{i=1}^{N} \Pi_i(ac_i - v(h_i))$$

$$\text{s.t.} \quad 0 \leq c_i \leq f(1,h_i,s_i), \quad i = 1, \ldots, N$$

$$0 \leq h_i \leq 1 \qquad i = 1, \ldots, N$$

Proof: Follows directly from the two welfare theorems (see e.g., Takayama (1974)).

Finding equilibrium allocations for E is at least in principle more difficult. This follows from the fact that there are two different kinds of agents and hence one needs to know the correct weights to attach to their individual utilities in computing an optimal allocation. It will be shown that this does not present a major obstacle for economies like E. In particular, the following holds:

Proposition 2: If (c_i, h_i, $\hat{c}_i$, H_i, K_i, p_i, q_i, w_i, $i = 1, \ldots, N$) is an equilibrium for E then (c_i, h_i, $i = 1, \ldots, N$) is the unique solution to:

$$\text{(P-2)} \qquad \max_{c_i h_i} \sum_{i=1}^{N} \Pi_i(u(c_i) - v(h_i))$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \Pi_i c_i \leq \sum_{i=1}^{N} f(1, h_i, s_i) \qquad i = 1, \ldots, N$$

$$c_i \geq 0 \qquad i = 1, \ldots, N$$

$$0 \leq h_i \leq 1 \qquad i = 1, \ldots, N$$

Proof: See Appendix.

These two propositions can be used to prove the next result.

Proposition 3: If (c_i, h_i, $i = 1, \ldots, N$) is part of an equilibrium allocation for E then there exists an a such that ($\bar{c}_i$, h_i, $i = 1, \ldots, N$) is part of an equilibrium allocation for E_a.

Proof: The proof will follow directly from the first order conditions for problems (P-1) and (P-2). By the assumptions made on the functions involved the solutions will be interior. From proposition two, if (c_i, h_i, $i = 1, \ldots, N$) is part of an equilibrium allocation then the following will hold:

$$\text{(3.1)} \qquad c_1 = c_2 = \ldots = c_N = c$$

$$(3.2) \qquad u'(c)f_2(1,h_i,s_i) = v'(h_i) \qquad i = 1,\dots,N$$

$$(3.3) \qquad c = \sum_{i=1}^{N} \Pi_i f(1,h_i,s_i)$$

Now choose a to satisfy

$$a = u'(c)$$

From proposition one the equilibrium values of h_i for economy E_a must satisfy:

$$(3.4) \qquad af_2(1,\ h_i,\ s_i) = v'(h_i) \qquad i = 1,\dots,N$$

With the chosen value of a, this is simply equation (3.2) and hence the h_i's must be the same in the two equilibria. This proves the proposition.//

The significance of the above result is as follows. In both of the economies E and E_a, capital is supplied inelastically. Hence, the profile of output, and hence, productivity, across states of nature is completely determined by the profile of h across states of nature. The above proposition says that if a is chosen appropriately the economies E and E_a have exactly the same prediction for movements in output, labor supply and productivity. In this sense an economy with complete risk sharing behaves in the aggregate like an economy in which all agents are risk neutral. This is perhaps not so surprising but it is interesting when viewed with the discussion from section two in mind. There it was commented that if one starts with a single agent economy with preferences given by u(c) - v(h) that, loosely speaking, decreasing curvature in both u(c) and v(h) increases the response of hours to changes in productivity. The work by Rogerson on indivisible labor demonstrated that making labor indivisible is equivalent at the aggregate level to making the function v(h) linear, thus increasing the magnitude of fluctuations in hours worked to productivity. Hansen has performed a calibration exercise similar to that of Kydland and Prescott and shown that this effect is potentially important empirically. The results obtained in this paper illustrate that adding a risk neutral agent into this type of an economy is equivalent at the aggregate level to making the function u(c) linear. Again, this will increase the magnitude of fluctuations in hours relative to those in productivity.

This analysis suggests that a risk sharing story could be an alternative to the indivisible labor assumption in improving the labor market behavior in real business cycle models of the Kydland-Prescott variety. Recalling the parametric example of section two:

$$u(c,h) = \frac{c^{\sigma}}{\sigma} - \frac{h^{\gamma}}{\gamma}$$

it was shown that equal per cent changes in σ and γ have the same impact on the elasticity of hours supplied with respect to productivity. Since the profile of h completely determines output, productivity and consumption in a static economy, this one period model suggests that the risk sharing alternative is as persuasive as the indivisible labor alternative.

The next section illustrates that in a dynamic model which allows for capital accumulation the risk sharing alternative appears as less attractive. This is a useful illustration of the importance of dynamic model building in evaluating time series properties.

One final point is worth noting. It is commonly argued that one of the important implications of allowing for risk sharing is that observed wages may be interpreted as consisting of two parts -- one part reflecting marginal productivity, the other part reflecting the net result of the risk sharing. This can be mistakenly interpreted as implying that allowing for risk sharing keeps the equilibrium allocation unchanged but gives a different interpretation of wages. This is incorrect, because as the preceding analysis suggests, adding a risk neutral agent alters the profile of labor supply across states of nature, and hence alters the equilibrium. Without even considering different interpretations of wages this effect predicts increased variability in hours relative to productivity and wages.

Section Five: Dynamic Extension, Calibration and Results

This section considers a dynamic extension to the model of the previous sections and performs some calibrations.

Consider the structure of section three imbedded in the context of a standard growth model. Output can now either be consumed or turned into capital to be used next period, and capital depreciates at a rate of δ. Agents of type one have preferences given by:

$$\sum_{t=1}^{\infty} \beta^t (u(c_t) - v(h_t))$$

where u(c), v(h) are as before and $0 < \beta < 1$ is the discount factor. The second agent has preferences given by

$$\sum_{t=1}^{\infty} \beta^t c_t$$

Assuming that the second agent has a sufficiently large endowment it should be clear that the results of section four will continue to hold in this economy. Whereas in the static economy the risk neutral agent forced prices of contingent consumption to take on certain values, in this model he also forces the interest rate to equal $\beta/1+\beta$. Workers will choose to have consumption constant across time and across states of nature. The final result is that this economy will behave as if populated by a single agent whose preferences are given by

$$\sum_{t=1}^{\infty} \beta^t (ac_t - v(h_t))$$

where a is some positive constant. The remainder of this section is devoted to analyzing the time series properties of this model and comparing them with the results obtained by Hansen (1985) for the indivisible labor and divisible labor cases. The method to be employed here is the same as that used by Hansen, based upon the earlier work of Kydland and Prescott (1982).

This procedure begins by choosing functional forms which have parameters that can be pinned down by sources that don't involve cyclical properties of the economy. Following earlier work we choose:

$$F(k_t, h_t, s_t) = s_t k_t^{\theta} h_t^{1-\theta} \qquad \theta = .36$$

$$s_t = \gamma s_{t-1} + \varepsilon_t, \ \gamma = .95 \qquad \ln\varepsilon_t \sim N(.05, \sigma^2)$$

$$v(h_t) = A\ln(1-h_t)$$

$$\beta = .99 \qquad \delta = .025$$

Note that if the underlying choice of u(c) is

$$u(c) = \ln c$$

then this economy also has the property that steady state hours of work per individual are independent of the wage. With this choice of u(c) there is an implied value for the constant a, however this parameter does not affect the cyclical properties of the economy and hence a is simply set equal to one.

Equilibrium allocations for this economy are generated by the following social planning problem:

(P-3) Maximize $E_0 \sum_{t=1}^{\infty} \beta^t (c_t - v(h_t))$

s.t. $c_t + i_t \leq F(k_t, h_t, s_t)$

$k_{t+1} = (1-\delta)k_t + i_t$

$s_{t+1} = \gamma s_t + \varepsilon_t$

$c_t \geq 0, \; i_t \geq 0 \qquad 0 \leq h_t \leq 1$

k_0 given

Unfortunately this problem does not have a closed form solution. Following the aforementioned authors, we consider a quadratic approximation to this problem in the neighborhood of the steady state when s_t takes on its mean value of one.[1] Following Hansen, the value of A is chosen so that the steady state value of h is equal to .33. This implies A = 1.58. Once this approximation has been carried out the model is used to compute 100 simulations, each with 115 observations. For each simulation the time series are logged and detrended.[2] Standard deviations and correlations with output are computed for each series. Sample means and sample standard deviations are computed by using the 100 simulations. The value of σ^2 is chosen so that series for output displays the same fluctuations as the actual output series for the U.S. during the period (55,3-85,1). This yielded $\sigma^2 = .00326$.

Table One reports the results of this exercise and includes the tables from Hansen (1985) for comparison. As the table shows, the risk sharing case does not perform very well except for the labor market variables. Consumption, investment and the capital stock move far too much relative to output,[3] but more importantly, consumption and investment display virtually no correlation with output. These results are very interesting. As the previous section implied, there is an improvement in the fit of labor market variables relative to the divisible labor case. However, what the previous section did not show was that this comes at the expense of a much worse fit with respect to other variables. This should be contrasted with the indivisible labor model, which improved the fit of the labor market variables without affecting the performance of other variables.

It is possible that the risk sharing alternative considered here was too extreme. Having the second agent risk neutral implied that the entire economy behaved as if it were risk neutral. The economy then has no motive for consumption smoothing and this may explain the poor results. If the second agent were simply less risk averse than the first agent, but not risk neutral, the economy would not behave as if it were risk neutral and the results may improve. Unfortunately, there is not a result which allows aggregation for the case where the second agent is not risk neutral. However, we simply try several specifications of preferences lying between lnc and c in an attempt to obtain information about some intermediate possibilities. Four cases are tried, with u(c) in each case being given by

$$-\frac{c^{\sigma}}{\sigma}$$

where σ takes on the values of .2, .4, .6 and .8. The same procedure as outlined above is used for each case.[4]

Table two reports the result of the calibrations for these four models. A clear pattern emerges from the results. As σ decreases a better fit for consumption results, but at the expense of a worse fit for the labor market variables. For the most part investment behaves well for all four of these cases, but for high values of σ the capital stock is too highly correlated with output.

These results suggest the following conclusions. Starting with an economy with preference of the form

$$\sum_{t=1}^{\infty} \beta^t(\ln c_t + A\ln(1-h_t))$$

improvements in the model's fit to actual time series properties are enhanced more by decreasing curvature in h_t rather than in c_t. Changes in curvature in c_t affect the motive for consumption smoothing, and although this increases the magnitude of labor market fluctuations relative to productivity, it simultaneously weakens the correlation between consumption and output and creates too much correlation between output and the capital stock. On the other hand, changes in the amount of curvature in h_t do not affect the motive for consumption smoothing while at the same time they increase the magnitude of labor market fluctuations.

Conclusion

This paper has compared two methods which can be used to increase fluctuations in hours relative to productivity (or wages) in an equilibrium model. One is to introduce an agent who shares risk with workers and the other is to assume that labor supply is indivisible. In a static setting the two appear to perform comparably, however in a dynamic setting the indivisible labor model appears to perform better. The reason for this is that in a model with risk sharing, any agent who is willing to smooth a worker's consumption across states of nature within a period is also willing to smooth a worker's income across time. In the aggregate this produces a very low correlation between consumption and output.

Appendix

Proof of Proposition 2:

The proof proceeds in series of lemma.

Lemma 1: If $(c_i, h_i, \hat{c}_i, K_i, H_i)$ is an equilibrium allocation for E then it is Pareto optimal.

Proof: Follows directly from first welfare theorem.//

Lemma 2: If (p_i, q_i, w_i) are equilibrium prices for E then there exists $\lambda > 0$ such that $p_i = \lambda \Pi_i$, $i = 1,\ldots,N$.

Proof: This follows from the fact that the second agent has linear indifference curves and that $W > \max f(1, 1, s_i)$. Hence, if this condition is not met this agent's demand is inconsistent with the aggregate resource constraint.//

Lemma 3: If $(c_i, h_i, \hat{c}_i, K_i, H_i)$ is an equilibrium allocation then

$$\sum_{i=1}^{N} \Pi_i \hat{c}_i = W.$$

Proof: The risk neutral agent solves the following problem:

$$\max_{\hat{c}_i} \sum_{i=1}^{N} p_i \hat{c}_i$$

$$\text{s.t. } \lambda \sum_{i=1}^{N} \Pi_i \hat{c}_i \leq \lambda W$$

$$\hat{c}_i \geq 0 \qquad i = 1,\ldots,N$$

By monotonicity, the budget constraint will be binding. Hence $\sum_{i=1}^{N} \Pi_i \hat{c}_i = W$.//

Lemma 3 claims that the risk neutral agent's utility in equilibrium will be equal to W. Hence, by lemma 1 an equilibrium will be the solution to

$$\text{(P-4)} \qquad \max_{c_i, h_i, \hat{c}_i} \sum_{i=1}^{N} \Pi_i (u(c_i) - v(h_i))$$

$$\text{s.t. } c_i + \hat{c}_i \leq f(1, h_i, s_i) + W \qquad i = 1,\ldots,N \qquad \text{(A-1)}$$

$$\sum_{i=1}^{N} \Pi_i \hat{c}_i = W \qquad \text{(A-2)}$$

$$0 \leq h_i \leq 1 \qquad i = 1,\ldots,N$$

$$c_i \geq 0,\ \hat{c}_i \geq 0 \qquad i = 1,\ldots,N$$

The final step is to show that this problem gives the same solution for (c_i, h_i, $i = 1,\ldots,N$) as

$$\text{(P-5)} \qquad \max_{c_i, h_i} \sum_{i=1}^{N} \pi_i (u(c_i) - v(h_i))$$

$$\text{s.t. } \sum_{i=1}^{N} \Pi_i c_i < \sum_{i=1}^{N} \Pi_i f(1, h_i, s_i) \qquad \text{(A-3)}$$

$$0 \leq h_i \leq 1. \quad 0 = 1,\ldots,N$$

$$c_i \geq 0 \qquad i = 1,\ldots,N$$

Note that (P-4) and (P-5) contain only (c_i, h_i, $i = 1,\ldots,N$) in the objective functions. Hence, if the constraint sets imply the same alternatives for these variables, problems (P-4) and (P-5) will imply the same choices for (c_i, h_i, $i = 1,\ldots,N$).

It is straightforward to show that if (c_i, h_i, $\hat{c}_i$, $i = 1,\ldots,N$) satisfy the constraints for problem (P-4) that (c_i, h_i, $i = 1,\ldots,N$)

satisfy the constraints for problem (P-5). Multiply the i^{th} equation in (A-1) by π_i and sum over i to obtain:

$$\sum_{i=1}^{N} \Pi_i c_i + \sum_{i=1}^{N} \Pi_i \hat{c}_i \leq \sum_{i=1}^{N} \Pi_i f(1, h_i, s_i) + W.$$

But by (A-2) $\sum_{i=1}^{N} \Pi_i \hat{c}_i = W$, thus this expression reduces to (A-3).

Now the result needs to be proven in the other direction. Suppose (c_i, h_i, $i = 1,\ldots,N$) satisfies (A-3). Define

$$\hat{c}_i = W + f(1, h_i, s_i) - c_i, \qquad i = 1,\ldots,N.$$

Then by definition:

$$c_i + \hat{c}_i = W + f(1, h_i, s_i), \qquad i = 1,\ldots,N.$$

Also:

$$\sum_{i=1}^{N} \Pi_i \hat{c}_i = \sum_{i=1}^{N} \Pi_i W + \sum_{i=1}^{N} \Pi_i (f(1, h_i, s_i) - c_i) = W \text{ by (A-3).}$$

Hence (A-1) and (A-2) are satisfied. Finally, to show that $\hat{c}_i \geq 0$ observe that the solution to problem (P-2) will involve $c_i = c_j$ for all i,j. Hence it follows that

$$c_j \leq \sum_{i=1}^{N} \Pi_i f(1, h_i, s_i) \qquad j = 1,\ldots,N$$

$$\leq \sum_{i=1}^{N} \Pi_i f(1, 1, s_i)$$

$$\leq W.$$

This completes the proof.

Table One
(a) is standard deviation and (b) is correlation with output.
Sample standard deviations are in parentheses.

	Economy with Risk Sharing		Quarterly U.S. Time Series (55,3-84,1)	
	(a)	(b)	(a)	(b)
Output	1.76 (.23)	1.00 (.00)	1.76	1.00
Consumption	20.51 (2.13)	.18 (.38)	1.29	.85
Investment	70.72 (11.26)	-.06 (.51)	8.60	.92
Capital Stock	1.85 (.22)	.96 (.10)	0.63	.04
Hours	1.17 (.16)	.99 (.00)	1.66	.76
Productivity	.59 (.08)	.99 (.00)	1.18	.42

	Economy with Divisible Labor		Economy with Indivisible Labor	
	(a)	(b)	(a)	(b)
Output	1.76 (.16)	1.00 (.00)	1.76 (.21)	1.00 (.00)
Consumption	.42 (.06)	.89 (.03)	.51 (.08)	.87 (.04)
Investment	4.24 (.51)	.99 (.00)	5.71 (.70)	.99 (.00)
Capital Stock	.36 (.07)	.06 (.07)	.47 (.10)	.05 (.07)
Hours	.70 (.08)	.98 (.01)	1.35 (.16)	.98 (.01)
Productivity	.68 (.08)	.98 (.01)	.50 (.07)	.87 (.03)

Table Two

(a) is standard deviation and (b) is correlation with output.
Sample standard deviations are in parentheses.

	σ = .8		σ = .6	
	(a)	(b)	(a)	(b)
Output	1.76 (.21)	1.00 (.00)	1.76 (.22)	1.00 (.00)
Consumption	.87 (.15)	-.25 (.13)	.52 (.11)	.49 (.08)
Investment	7.97 (.99)	.95 (.01)	6.34 (.81)	.98 (.01)
Capital Stock	.63 (.12)	.23 (.72)	.52 (.11)	.14 (.07)
Hours	1.20 (.14)	.99 (.00)	1.11 (.14)	.99 (.00)
Productivity	.58 (.07)	.98 (.00)	.67 (.09)	.98 (.00)

	σ = .4		σ = .2	
	(a)	(b)	(a)	(b)
Output	1.76 (.24)	1.00 (.00)	1.76 (.20)	1.00 (.00)
Consumption	.53 (.10)	.78 (.04)	.54 (.08)	.86 (.03)
Investment	5.81 (.80)	.98 (.00)	5.57 (.63)	.99 (.00)
Capital Stock	.48 (.10)	.09 (.06)	.46 (.08)	.07 (.06)
Hours	1.02 (.13)	.98 (.00)	.94 (.11)	.99 (.00)
Productivity	.76 (.11)	.98 (.00)	.84 (.10)	.99 (.00)

Footnotes

1. The same approximation method as used by Hansen was used here, including the choice of neighborhood in which the approximation holds.

2. The same procedure as Hansen was used here, based upon the work of Hodrick and Prescott (1980).

3. In formulating this model the endowment consumption has been ignored. Adding it would decrease the magnitude of deviations in consumption (because they are in percentage terms) but would not affect the correlation with output.

4. The implied values of A are 1.69, 1.66, 1.63 and 1.60. The implied values of σ^2 are .00895, .00855, .00809 and .00752.

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