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Nominal Exchange Rate Dynamics in the European Monetary System

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Abstract
The EMS is modeled as imposing dual reflecting barriers on the exchange rate process. This policy leads to a state-dependent conditional variance for exchange rate changes. This variance is always less than that under a pure free float regime. A Method of Simulated Moments procedure is employed to estimate the parameters of the model. Simulations with the estimated parameter values show that the model predicts non-normality and non-stationarity in the distribution of exchange rate changes and that these characteristics diminish with aggregation.

Keywords: Brownian motion; Target Zones; leptokurtosis; ARCH; Method of Simulated Moments

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I. Introduction

Stochastic models of exchange rate behaviour under different policy regimes are by now common. In this literature, the exchange rate is modeled as being determined by stochastic (and often unspecified) market fundamentals which are themselves regulated according to some policy rule. Thus for example, Buiter (1989) modeled the shadow exchange rate under a Gold Standard regime as Brownian motion between two absorbing barriers,[1] Flood and Garber (1983) and Smith and Smith (1990) modeled the pound-dollar rate prior to Britain's return to the Gold Standard in 1925 as a process approaching an absorbing barrier, Krugman (1990) modeled a Target Zone regime as two reflecting barriers around a driftless Brownian motion process, Miller and Weller (1989) used a process with drift to explain exchange rate hysteresis in that regime and Flood and Garber (1989) related target zones with discrete intervention to speculative attacks. Bertola and Caballero (1990) and Svensson (1990) have recently used the same methods to model the Exchange Rate Mechanism of the EMS.[2]

While the theoretical literature is now extensive and growing rapidly, much less effort has been devoted to the empirical examination of the relevance of these models. This is the motivation for this paper. The objective is to determine the consistency of a simple stochastic model with the observed characteristics of exchange rate behaviour in the EMS. Section II presents summary statistics on daily and weekly exchange rate levels and changes during a period since the last major realignment of the system. A

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[1] For an introduction to the theory of regulated stochastic processes see Cox and Miller (1965), Harrison (1985) and, for a more advanced treatment, Karatzas and Shreve (1987).

[2] See Giavazzi and Giovannini (1989) for a description and history of the EMS.

model of the EMS must be able to match these statistics.

In keeping with the literature, the EMS is modeled as a regime in which the nominal exchange rate is a stochastic process between two reflecting boundaries. Froot and Obstfeld (1989) provided a simple method for solving for the exchange rate equation in such a regime under the assumption that there is full commitment to defending the announced bands. This model is described and solved in Section III. In Section IV the model is tested by simulation. Here a wide variety of parameter values are used to generate averages across many realizations of the same moments and test statistics presented in Section II. The model is shown to be roughly consistent with the data over wide ranges of parameter values. In Section V a Simulated Method of Moments estimation strategy is used to estimate the free parameters of the model. The estimated parameters are then used to simulate the model. These simulations also are consistent with the behaviour summarized in Section II. The paper concludes with a discussion in Section VI.

II. Summary Statistics on EMS Exchange Rates

Table I summarizes the behaviour of selected EMS exchange rates during the period January 14, 1987 to September 22, 1989[3] based on daily observations. The table gives the first four moments of the levels and changes in the logarithms of Deutschemark exchange rates and test statistics for non-normality and conditional heteroskedasticity in the distribution of

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[3] This period was chosen because the model presented below corresponds to a world in which the authorities are never expected to change the central parity rate. Thus, this period is chosen because it is longer than any interval between previous realignments and ends well before any subsequent realignment (in fact the next realignment did not occur until more than three months later, and that involved only the narrowing of the Dm/Li band). Thus, it is hoped that this period reasonably approximates a regime of full commitment to the announced bands.

exchange rate changes. This was a period of relative strength for the Deutschemark as shown by the mean deviation from the parity rate and the skewed distribution for the exchange rate levels. There are three clear observations to be drawn from these tables. First, the distribution of daily exchange rate changes is not normal. The Bera-Jarque test statistics all reject normality at the 1% level. This non-normality appears to be due almost entirely to excess kurtosis which is significant for each series. Only two of the series exhibit any skewness in changes. Second, this distribution is also nonstationary. The test statistics for the simple ARCH parameterization of conditional heteroskedasticity at various lags all strongly reject the null of constant conditional variance. Finally, both the non-normality and the non-stationarity appear to diminish with aggregation. Table 2 presents the same statistics for mid-week observations. Now for the Dm/Ffr rate for example, the null hypothesis of conditional homoskedasticity cannot be rejected at any lag length even at the 10% level, and for the Dm/Li and Dm/Kr rates this hypothesis is rejected only at shorter lag lengths.

These three observations constitute the kind of behaviour that any model of the EMS must predict. Therefore, the model described below will be tested for its consistency with these facts.

III. A Model with Full Commitment to Intervention

The logarithm of the exchange rate is assumed to satisfy the asset-pricing relationship

$$e(t) = k(t) + \alpha E(de(t)/dt \mid \phi(t)) \qquad (1)$$

where e(t) is the spot exchange rate, k(t) is the market fundamental, α is constant,[4] E is the expectations operator and $\phi(t) \equiv \mathcal{F}_t \cup B$, is the information set in which $\mathcal{F}_t$ is the filtration generated by $\{k(\tau): \tau = 0,1,\ldots,t\}$ and B describes the contingent exchange-rate policy. The solution to equation (1), in the absence of bubbles, is the forward-looking equation,

$$e(t) = \left(\frac{1}{\alpha} \int_t^{\infty} e^{(t-s)/\alpha} \, E\left[k(s) \,|\, \phi(t)\right] ds \right) \tag{2}$$

which simply equates the current exchange rate to the expected discounted value of future fundamentals.

Define a pure free float as a regime in which the authorities never intervene to offset shocks to the fundamentals and therefore always allow these to be reflected in changes in the spot rate. The fundamentals are assumed to evolve as Brownian motion adapted to $\mathcal{F}_t$:

$$dk(t) = \eta dt + \sigma dz(t) \tag{3}$$

where $z(t)$ is a standard Wiener process[5] and η and σ are constant instantaneous drift and variance parameters. Under a pure free float the exchange rate equation reduces to

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[4] For example, in the monetary model $k(t)$ is a linear combination of cross-country differences in money supplies and rates of income and α is the interest semi-elasticity of money demand.

[5] A standard Wiener process has i.i.d. normal increments. By the Central Limit Theorem, if the changes in k are the result of a large number of unobserved, independent random shocks then the increments in k should be normally distributed, which justifies the use of a Wiener process.

$$e(t) = k(t) + \alpha\eta \tag{4}$$

which is linear in k. By differentiating (4) a diffusion equation for the exchange rate under a pure free float is obtained:

$$de(t) = \eta dt + \sigma dz(t) = dk(t). \tag{5}$$

So the exchange rate and the market fundamentals share the same dynamic properties: that is, the exchange rate follows a random walk and its increments are Normally distributed with constant conditional mean η and variance σ^2.[6] Clearly, such a model cannot be reconciled with the observations of non-normality and non-stationarity in this distribution made in Section I. In this framework, the EMS policy cannot easily be specified in terms of bounds on the exchange rate itself since to do so would introduce an important indeterminacy into the problem. Since the exchange rate is a function of future values of the fundamentals variable, any announcement of an intervention policy would have to include an announcement of the extent of intervention when the exchange rate reaches one of its limiting values. At the very least, therefore, the announcement of bounds on the exchange rate would have to include an announcement of the extent of intervention. Since the relationship between k(t) and e(t) is one-to-one, an announcement of exchange-rate boundaries is equivalent to an announcement of boundaries on the fundamental variables. I will therefore assume that the policy announcement includes information on the boundaries on k(t) and on the extent of marginal intervention. The latter would take the form of an announcement

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[6] The constancy of the conditional mean and variance is due to the assumed constancy of the drift and variance parameters in (3). Normality is due to the assumption that z(t) is a Wiener process.

that intervention will be of a sufficient magnitude just to prevent the fundamentals from leaving the band $[\underline{k},\bar{k}]$.[7] I will also assume that the authorities are understood to have fully committed themselves to defending these boundaries.[8]

The general expression for the exchange rate in this case is:

$$e(t) = E\left(\frac{1}{\alpha}\int_t^{\infty} e^{(t-s)/\alpha}\, k(s)\,\middle|\,\mathcal{F}_t,\ k(s) \in [\underline{k},\bar{k}]\right) \tag{6}$$

Solutions to this equation will be of the form

$$e(t) = k(t) + \alpha\eta + A_1\exp(\lambda_1 k) + A_2\exp(\lambda_2 k) \tag{7}$$

$$\lambda_{1,2} = -\eta/\sigma^2 \pm (\eta^2/\sigma^4 + 2/\alpha\sigma^2)^{1/2}$$

where the constants of integration A_1 and A_2 are determined by the boundary conditions. These are the smooth-pasting conditions for infinitesimal intervention (see Krugman (1988) or Dumas (1989)), which are a consequence of continuity and rational expectations (no anticipated discrete jumps in e are allowed):

$$\partial e/\partial\bar{k} = 0 = \partial e/\partial\underline{k}. \tag{8}$$

Imposing these boundary conditions on (7) yields the saddlepath equation for the exchange rate (see Froot and Obstfeld (1989)):

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[7] What is important here is that the authorities do not cause a discrete jump in the fundamentals upon intervening. For the solution in that case see Flood and Garber (1989).

[8] This assumption rules out the effect of anticipated realignments. This possibility will be considered below.

$$e = k + \alpha\eta + A_1\exp(\lambda_1 k) + A_2\exp(\lambda_2 k) \tag{9}$$

$$A_1 \equiv \frac{\alpha\sigma^2}{2\Delta}\left[\exp(\lambda_2\underline{k}) - \exp(\lambda_2\bar{k})\right]\lambda_2 < 0$$

$$A_2 \equiv \frac{\alpha\sigma^2}{2\Delta}\left[\exp(\lambda_1\bar{k}) - \exp(\lambda_1\underline{k})\right]\lambda_1 > 0$$

$$\Delta \equiv [\exp(\lambda_1\bar{k}+\lambda_2\underline{k}) - \exp(\lambda_1\underline{k}+\lambda_2\bar{k})] > 0$$

Note that (9) has two parts, the linear pure free-float solution and the two nonlinear terms that arise due to the anticipation of intervention. If intervention is never expected to occur (as if the boundaries are infinitely-distant) the last two terms disappear and the solution reverts to the pure free float case as expected. Note that the distortion of the exchange-rate equation occurs even between the boundaries (i.e. even if these boundaries are never reached). Equation (9) describes an upward-sloping S-shaped curve in (k,e) space which is horizontal at $\underline{k}$ and $\bar{k}$. It can be shown that $0 \le \partial e/\partial k \le 1$ so that the saddlepath under the EMS regime is everywhere flatter than under the pure free float (where $\partial e/\partial k = 1$). This result is important because differentiating (9) yields the diffusion equation for exchange rate changes under the EMS regime:

$$de(t) = \left[\eta e'(k) + \frac{\sigma^2}{2}e''(k)\right]dt + e'(k)\sigma dz(t). \tag{10}$$

Now the conditional variance of exchange rate changes is $\sigma^2[e'(k)]^2$ which is less than σ^2, the conditional variance of changes in the purely free-floating rate. In fact since the saddlepath is horizontal at the boundaries, this conditional variance approaches zero as the fundamentals approach their boundaries.[9] In this sense, the EMS rules exert a stabilizing influence on the exchange rate. Alternatively, since the the slope of the saddlepath is lower in the EMS regime than in the pure free float regime, the EMS exchange rate always responds less to a change in the fundamentals variable than does a purely free-floating exchange rate. These two interpretations provide theoretical support for the empirical evidence that the EMS has succeeded in reducing the volatility of nominal exchange rates (see Artis and Taylor (1988), Giavazzi and Giovannini (1989))[10]. It is also clear that exchange rate changes are conditionally heteroskedastic in this model even though the conditional distribution for changes in k has constant variance. It is worth emphasizing that these results are not trivially due to limits on the values that the exchange rate may take, but due to the changes in the relationship between the fundamental variables and the exchange rate and so should be observable even if the exchange rate never reaches its limiting values. The instantaneous rate of change of the exchange rate in the EMS is

$$\begin{aligned} \eta_e &= \eta + \eta\lambda_1 A_1 \exp(\lambda_1 k) + \eta\lambda_2 A_2 \exp(\lambda_2 k) + \frac{\sigma^2}{2}\left(\lambda_1^2 A_1 \exp(\lambda_1 k) + \lambda_2^2 A_2 \exp(\lambda_2 k)\right) \\ &= \eta + \lambda_1 A_2 \exp(\lambda_1 k)\left(\eta + \frac{\sigma^2\lambda_1}{2}\right) + \lambda_2 A_2 \exp(\lambda_2 k)\left(\eta + \frac{\sigma^2\lambda_2}{2}\right) \qquad (11) \end{aligned}$$

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[9] This property of the model has been criticized by Bertola and Caballero (1990) who argue that the evidence suggests that exchange rates are actually more volatile near the boundaries than in the middle of the band. They claim that a model with repeated realignments will correct this difficulty. In fact, as I show elsewhere (Spencer (1990)), what is needed is a time-varying probability of realignment. Under general conditions, allowing for realignment with a constant probability will still yield a model which displays the properties described above except that the conditional variance of exchange rate changes will be non-zero at the announced boundaries.

[10] Svensson (1990) demonstrates that this exchange rate stability will likely be at the expense of interest rate instability.

which towards the lower boundary is greater than η and in the upper region is less than η, the instantaneous rate of change of the purely free-floating rate. The intuition behind these results is as follows. As the exchange rate rises towards the upper boundary value, agents are forced to revise downwards their forecasts of future exchange rates (and exchange rate changes). This causes the exchange rate to respond less to changes in current fundamentals, or to "slow down". Conversely, towards the lower boundary, the reverse happens and the speed at which the lower boundary is reached falls as the expectation of intervention increases. This means that even if the drift rate of the fundamentals process is constant, the drift in the exchange rate is time-dependent.[11]

A further observation that can be drawn from equation (10) is that the conditional mean and variance of the EMS exchange-rate changes are both state-dependent, unlike those of the purely free-floating rate. Thus, some kind of conditional heteroskedasticity should be expected. A common specification of conditional heteroskedasticity which has been observed in exchange rate data is autoregressive conditional heteroskedasticity (ARCH) due to Engle (1982) or a more general specification (GARCH) due to Bollerslev (1986). One characteristic of ARCH is that it exhibits the "clustering"of conditional variances that appears to be present in the data. Equation (10), while not necessarily predicting conditional heteroskedasticity of this form, may nonetheless provide an appealing explanation for such behaviour. The conditional variance is much lower near the boundaries than it is in the middle of the band (since $\partial e/\partial k = 0$ at the edges of the band). Thus the

[11]Note that η represents the drift in k relative to the boundaries. Therefore, a regime of sliding parities can easily be incorporated into this framework.

conditional variance of exchange-rate changes will be lower during periods when the exchange rate is close to the edges of the band than when it is in the middle of the band. This may be an explanation of the "clustering" of conditional variances which would result in a positive test for ARCH.

IV. Testing and Estimation

By assumption the fundamental variable follows the Brownian process (3), between reflecting barriers, so its unconditional distribution is exponential for non-zero drift and uniform if $\eta = 0$. Since the exchange rate saddlepath is continuous and monotonic in k the Jacobian of the transformation from k to e exists and a change of variables would allow for the derivation of the unconditional moments of the exchange rate as functions of the parameters. However, this exercise is not carried out because the unconditional moments are of less interest than the conditional ones. Furthermore, the conditional distribution of k is sufficiently complicated and the Jacobian of the transformation is sufficiently non-linear that the derivation of the conditional distribution is intractable. Therefore, these moments will be estimated by simulation. Simulation estimation has the added advantage that it does not require an exact specification of the fundamentals and so permits the level of generality employed above.

The estimation strategy employed is the Method of Simulated Moments. Here, a discrete approximation to the diffusion process (3) is used to generate simulated series of exchange rates from (9). Sample moments are calculated and contrasted to the observed sample moments in a loss function. The parameter estimates are those that minimize this loss. Testing the model then amounts to generating 100 replications with the estimated parameter values and calculating the same sample moments and test statistics. Then, using the actual values as critical values, p-values for the simulated data are reported. Thus, the p-values will typically report the proportion of simulated values that exceed the actual value. A value greater than 0.95 or less than 0.05 would imply a poor fit.

IV.1 Testing the Model with Arbitrary Parameter Values

The first question to be addressed is whether or not the model can match the observed properties of exchange rate levels and changes. This is essentially a question about the statistical properties implied by the DGP, equation (9). This will be pursued by assigning arbitrary values to the parameters of the model and simulating the model a number of times to calculate estimates of these sample moments. This will also provide some insight into the properties of the model. There are five parameters in the model: α, η, σ^2, $\underline{k}$ and $\bar{k}$.[12] For a given set of parameter values 100 replications of 500 observations of the simulated exchange rate series were generated and sample moments calculated. Table 3 reports the mean value and the (bootstrap) standard deviation for each of the first four moments of the simulated exchange rate levels and changes processes for nine different sets of parameters.[13]

The sensitivity of the moments to different parameter values is shown in this table by considering different combinations of parameter values. Thus,

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[12] In fact there is a sixth, k_o, but $\bar{k}$, k_o and $\underline{k}$ only describe ($\bar{k}$ - k_o) and (k_o - $\underline{k}$), so to eliminate the indeterminacy, I have set k_o = 0 in all of the simulations.

[13] The intent of Tables 3 and 4 is partly to examine the general properties of the model. Since equation (10) shows that exchange rate changes are conditionally heteroskedastic ARCH test statistics are not generated for these simulations.

in the second column under 'Case 1' I show a benchmark case against which all other cases will be compared. For each sample moment, simple one-sided p-values are shown in order to compare the performance of the model against the actual daily Dm/Ffr experience of Table 1. These are simply the proportion of replications in which the simulated value exceeded the observed value. In this sense, the first case performs quite well, since all eight moments are consistent with the model's predictions at the 1%. In case 2, the effect of doubling the width of the band is considered. This has little effect on the properties of the exchange rate levels but greatly impairs the model's ability to explain the behaviour of the second and higher moments of exchange rate changes. Note one interesting result from this case: the kurtosis of exchange rate changes is significantly lower when the band is widened. Compared to the benchmark, cases 3 and 4 consider progressively higher values for σ^2. This greatly increases the p-values associated with the two variances and with the kurtosis of exchange rate changes. These cases display much greater variability than the first two in the sense that the variances of exchange rate levels and changes are much larger than observed. However, for moderate values of σ^2 the kurtosis of exchange rate changes is greatly increased so that simulated values are nearly always greater than the observed value. Cases 5 and 6 consider the effect of increasing η from -0.00001 to 0.0 and then to 0.00001. These changes do not have significant effects on the results. However, in case 7 when η is greatly reduced to -0.0001 the p-values for the variance and skewness of exchange rate levels and of the variance, skewness and kurtosis of changes are all greatly increased. Finally, cases 8 and 9 consider the effect of increasing α. Lowering the discount rate lowers the variance of the exchange rate level and also the mean, variance and kurtosis of changes.

While joint tests on these moments have not been performed, the model appears to perform quite well in that it appears that it is possible to find parameter values for which the model will predict small-sample moments similar to those observed. Even with substantial changes in parameter values the simulated moments often remain consistent with the observed data. Note especially that the kurtosis in the distribution of exchange rate changes is significantly different from 3.0 in seven of nine cases even though in most of the replications the boundary values were never hit. This demonstrates that the model can predict this characteristic of exchange rates without a large number of 'hits'. Table 3 also shows a disturbing feature of the model: the large standard errors associated with the simulated moments. Table 4 provides an even more worrying feature of the model — its sensitivity to changes in the simulation sample size. In this table exactly the same tests are performed, but with a sample size of 2000 observations. Increasing the sample size greatly increases the variances of exchange rate levels and changes, and lowers the kurtosis of exchange rate levels and the mean rate of change. This greatly changes the performance of these cases in their ability to explain the properties of exchange rate levels and changes. For example Cases 3, 4 and 7 appear to have lost any explanatory power since most of the p-values indicate that simulated moments differ systematically from observed moments.

The differences between Tables 3 and 4 illustrate an important problem with simulations. There is large sampling variability in the sample moments and there are likely to be significant changes in the observed moments as longer data sets are considered. This means that the choice of sample size in the simulations is not innocuous. A short sample size may well yield mean values for moments and test statistics very close to observed values, but will also likely yield greater standard errors and less efficient, and perhaps biased, parameter estimates. Also, since the simulations are making a discrete-time approximation to a continuous-time process, a small sample size makes this a less acceptable approximation. Unfortunately, I am aware of no convention upon which to base my choice of sample size. However, the role played by sample size is an important one since one of the characteristics of exchange rate behaviour is that the non-normality and non-stationarity in the distribution of exchange rate changes diminish with aggregation. Furthermore, since there is no way to know how many simulated observations correspond to the actual sample size it is important to try to discover the effect of changing the simulation sample size.

IV.2 Simulation Estimators[14]

Estimation by simulation is a natural way to proceed since the model can be completely parametrized, the fundamental variable is unobserved, and the distribution of the exchange rate is unknown, so that analytical derivation of moments is not possible. Estimation by simulation proceeds along the following steps:

1. Generate a series of T random elements, $\{u_t^r\}$, from a Standard Normal distribution.
2. A simulated sample $\{e_t^r\}$ of length T is generated from (9) using the initial condition $k_o = 0$ and arbitrary parameter values. The fundamentals process $\{k_t^r\}$ was generated according to the approximation:

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[14]For more on simulation estimators see McFadden (1990) and Duffie and Singleton (1989).

$$k_t = \begin{cases} \bar{k} & \text{if } \eta + k_{t-1} + \sigma^2 u_t \geq \bar{k} \\ \underline{k} & \text{if } \eta + k_{t-1} + \sigma^2 u_t \leq \underline{k} \\ \eta + k_{t-1} + \sigma^2 u_t & \text{otherwise} \end{cases} \tag{12}$$

3. Simulation sample moments are calculated and the loss function

$$L_{rj} = \sum_i (f_i^{rj} - \bar{f}_i)^2 \tag{13}$$

is evaluated, where the f_i^{rj} are the simulated moments from the j^{th} simulation in the r^{th} replication and $\bar{f}_i$ is the i^{th} observed moment.

4. Repeat steps 2 and 3 for J different values of one parameter, holding all other parameter values fixed.

5. Select θ_r, the parameter value which achieves the lowest value for the loss function.
6. Repeat steps 1-5 for R replications (i.e. R different series $\{u_t^r\}$).

7. The parameter estimate is then given by

$$\hat{\theta}_R = (1/R) \sum_r \theta_r \tag{14}$$

8. Repeat steps 1-7 for each parameter in turn.

In practice this procedure must itself be repeated a number of times in order to search over a finer and finer grid and because substituting in the new parameter estimate will often greatly affect the loss - minimizing values of the other parameters. This procedure was initially conducted over a relatively wide grid with R = J = 10 and T = 2000, and ultimately over a much narrower grid with R = 25 and J = 20. The estimation procedure was halted when it became obvious that further refinements would not significantly affect the parameter value or the loss. Provided the sample size in the simulation is sufficiently large, this procedure will lead to a unique estimate of the true parameter which is consistent, asymptotically normally distributed, but inefficient relative to a procedure which incorporated analytical information about the population moments.[15]

IV.3 Estimation

There are in general five parameters to be estimated. However, only three need to be estimated by this procedure in this case because $\underline{k}$ and $\bar{k}$ can be calculated, for given values of α, η and σ^2, from the smooth-pasting conditions. In this section the results from the simulation estimation procedure are reported. Two different estimation exercises are summarized, corresponding to the use of two slightly different loss functions. In method 1 the parameters α, η, and σ^2 are estimated by imposing the actual exchange rate boundaries and minimizing the loss function:

$$L_1 = (\text{mean}(\Delta e(t)) - \text{mean}(\Delta e(n)))^2 + (\text{var}(\Delta e(t)) - \text{var}(\Delta e(n))^2) + (\text{skew}(\Delta e(t)) - \text{skew}(\Delta e(n)))^2 \qquad (15)$$

In Method 2 the same parameters are estimated using the loss function[16]:

$$L_2 = (\text{mean}(\Delta e(t)) - \text{mean}(\Delta e(n)))^2 + (\text{var}(\Delta e(t)) - \text{var}(\Delta e(n)))^2 + (\text{var}(e(t))-\text{var}(e(n)))^2 \qquad (16)$$

This second loss function is employed for three reasons. First, the sampling variability of the skewness of exchange rate changes is likely to be higher than that of the variance of exchange rate levels, making it less desirable

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[15] See Duffie and Singleton (1989).

[16] Note that while the exchange rate process is clearly not covariance stationary, the presence of boundaries makes it an ergodic process so its variance is not unbounded.

as a means for estimating parameters. Second, the variability, however defined, of exchange rates is of common concern, so including both variances in this function makes that an important characteristic to be included in the estimation strategy. Third, for purely methodological purposes it is of interest to see how the estimates obtained from this procedure depend on the specification of this function.

Table 5 reports the results of the estimation procedure for the two cases. In each case the standard deviation is reported below the parameter estimate. This is the standard deviation of the θ_r around the point estimate $\hat{\theta}_R$. The $\underline{k}$ and $\bar{k}$ values reported are the values given by the smooth-pasting conditions evaluated at the estimated values of the other parameters.

The first observation to be made is that the estimated values of α and η under the two methods are very close, well within two standard deviations of each other. The difference between the two sets of results are in the implied width of the band on the fundamentals and on the variance parameter. This should be noted when the implications of these two sets of estimates are compared below.

IV.4 Testing with Estimated Parameters

Given estimated parameters, attention now turns to evaluating the ability of the estimated model to explain the behaviour of exchange rates: specifically, whether the moments not used to estimate the parameters are captured by the model. This question is again addressed by simulation. Since more moments and test statistics are calculated than are used for estimation the remaining statistics can be used as tests of the model. One hundred replications of the model, with 2000 observations, were simulated using the estimated parameters. Table 6 presents the mean, standard deviation and prob-values for the first four moments of exchange rate levels and changes and also for the LM tests for ARCH effects. The prob-values (in square brackets) indicate the proportion of replications in which the simulated moment or test statistic had a value greater than the value observed. In the case of skewness, kurtosis and ARCH statistics, a second p-value is reported. This indicates the proportion of replications in which the simulated value exceeded the critical value at the 1% level.[17]

The simulations of the Method 1 estimates generally predict much higher means and variances for exchange rate levels and changes than are actually observed, but match the higher moments well. The Method 2 results are more encouraging except that it too predicts a higher mean rate of change. The main difference between the two methods is demonstrated by the great difference in the variance of exchange rate changes, which is much lower in the second case. While 98 of the observations exceed the actual value under Method 1, only 19% exceed it under Method 2. Thus, in terms of matching the first two moments the second method appears more consistent with the data than the first.

The first important result that comes out of this table is that the model predicts highly significant kurtosis in exchange rate changes regardless of which estimation method is considered. In each case the average kurtosis value was significantly different from 3.0 at the 1% level and the majority of values (100% for Method 1 and 83% for Method 2) exceeded the observed value.

More generally, the model predicts significant non-normality in the

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[17] For skewness this is a one-sided test against significant skewness with the sign observed in the data (i.e. positive for levels and negative for changes).

unconditional distribution of exchange rate changes as reflected in the Bera-Jarque test statistics. In Method 1 all of the simulated statistics exceeded the critical value at the 1% level and for Method 2 83% did. Thus, in general the model strongly rejects normality for exchange rate changes, and the results for the individual moments suggest that this rejection is due almost entirely to excess kurtosis.

The second important result concerns the implied test statistics for conditional heteroskedasticity of the ARCH form. For Methods 1 and 2 all of the statistics were significant at the 1% level of test. However, the values for Method 1 are rarely consistent with the observed values of these statistics. Those for Method 2 are more consistent with actual values, with at least 5% of the simulated values exceeding the actual values at all lags greater than 1. Thus, the model is also generally consistent with the observed conditional heteroskedasticity in exchange rate changes.

Attention has already been drawn to the importance of sample size in this model. The results so far are obtained in a moderately-sized sample of 2000 observations. This sample size was chosen because it was hoped to be large enough to yield relatively reliable estimates and was small enough to be computationally manageable. A striking way to see the difference between the small-sample properties of the model and the population properties is to compare the moments and statistics obtained by the model with each set of estimated parameters with a small sample size (T=500) and a very large sample size (T=10000) which is taken as an approximation to the population. Results from one replication are given in Tables 7 and 8. These two tables, combined with Table 6 (T=2000) indicate that as the sample size increases, ARCH effects become more significant and the kurtosis in the distribution of exchange rate changes is greatly increased.

Finally, Tables 1 and 2 demonstrate that if exchange rates are sampled at a lower frequency then their rates of change exhibit less significant non-normality and non-stationarity. This property was investigated by skip-sampling the simulations used to derive Table 6. The results are given in Table 9, where only every tenth observation was sampled. These results show that while the distribution of exchange rate levels is unaffected, the mean rate of change and its variance are both much higher. More importantly, the kurtosis in this distribution is significantly lower as is the Bera-Jarque test statistic for non-normality. Both, however, still reject normality at the 1% significance level. Also, all of the ARCH test statistics are significantly lower than in Table 6 and none are significant at the 1% level. The p-values reported in this table are for comparisons with the data in Table 2 and these show that both methods fit the data very well. Thus, the model does a good job of matching the effects of time aggregation.

The issue of the appropriate simulation sample size has already been raised and deserves final comment here. While it is not possible to know what the number of simulated observations should be in order to simulate a given exchange rate series, it is possible to draw conclusions from the *relative* sample sizes of two data sets. Table 2 shows that the skip-sampled data exhibit less significant excess kurtosis and conditional heteroskedasticity in the distribution of exchange rate changes than the daily data. Table 9 shows that this model has the same property, although with the sample sizes chosen this is less marked in the simulations. Also, a similar result holds when a shorter sample is chosen from a data set sampled at the original frequency (compare tables 7 and 8).

The choice of loss function used in the estimation procedure is also not of crucial importance. While the two loss functions do appear to lead to slightly different sets of parameter estimates, especially the estimate of σ^2, these differences do not appear to be qualitatively important. In the simulations, the two sets of estimates lead to essentially the same predictions of exchange rate behaviour. All of the moments and test statistics implied by each method's results are well within two standard deviations of each other. This is perhaps not too surprising since in Tables 3 and 4 it has been observed that even much greater differences in parameter values often did not lead to great differences in the implied exchange rate behaviour. For this reason, rather than sampling repeatedly from the empirical distribution of the parameter estimates the simulations held the parameters fixed at their point estimates and sampled from the distribution of z(t). Thus, it may appear that this approach will underestimate the true variability in the system. While this possibility is not rejected, it would appear from Tables 3 and 4 that allowing for small differences in parameter values as implied by their empirical distributions would lead to insignificant changes in simulated statistics so this omission does not appear to have been important.

V. Conclusion and Discussion

This paper has investigated the properties of a simple model of exchange rate determination with a stylized EMS intervention policy. The results indicate that this simple model is able to predict behaviour of exchange rate levels and changes that is similar to that which is observed. The analytical results demonstrate that the conditional mean and variance of exchange rate changes are both time-varying, which provides one explanation for the conditional heteroskedasticity in observed series. The conditional variance is shown to be bounded from above by the conditional variance of exchange rate changes under a pure free float regime, thereby providing theoretical support for the observation that the EMS has "stabilized" exchange rates.

Further investigation of the model was conducted by simulation. Four clear results emerged. First, the model clearly rejects normality of the unconditional distribution of exchange rate changes and this is almost entirely due to excess kurtosis. Second, the model also strongly rejects conditional homoskedasticity for exchange rate changes. Third, these results are not due in any sense to the exchange rate having hit the boundaries a large number of times. For example in Table 8, there is strong rejection of normality and conditional homoskedasticity by the simulation of the results of Method 2 even though there are only a total of 807 hits over a sample of 10,000 observations. Finally, when the data are sampled at a lower frequency the non-normality and non-stationarity in the distribution of exchange rate changes are less significant. These four properties mimic those of actual EMS exchange rate data.

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Table 1: Summary Statistics on Selected Daily EMS Exchange Rates *versus* the Deutschemark

	Dm/Ffr	Dm/Li†	Dm/Bfr	Dm/Kr	Dm/Fl
parity(e_c)‡	-1.2101	-6.5802	-3.0265	-1.3388	-0.1193
mean($e-e_c$)	-0.00601	-0.01317	-0.0121	-0.0060	0.0002
variance(e)	0.7287E-4	0.2115E-3	0.2681E-4	0.1343E-3	0.6616E-5
skewness(e)	0.3755*	0.3115*	0.6769*	0.3911*	0.2569*
kurtosis(e)	1.8516	2.0440	2.7081	2.0954	3.9254*
mean(Δe)	-0.2339E-4	0.2658E-4	0.1046E-4	0.3236E-4	0.3849E-5
variance(Δe)	0.2261E-5	0.7887E-5	0.1089E-4	0.1237E-4	0.4980E-5
skewness(Δe)	-0.6131*	0.0417	0.0165	-0.2283	0.0594
kurtosis(Δe)	9.1170*	7.3022*	4.7084*	7.4741*	10.546*
NRSQ(1)	91.948*	116.66*	188.41*	170.66*	167.57*
NRSQ(2)	93.452*	122.35*	231.55*	207.47*	230.66*
NRSQ(3)	94.224*	127.19*	244.94*	220.78*	250.24*
NRSQ(4)	96.368*	133.58*	253.00*	221.39*	261.15*
NRSQ(5)	95.977*	132.72*	258.25*	220.81*	265.28*
NRSQ(8)	97.273*	136.54*	273.12*	223.88*	280.66*
NRSQ(12)	99.067*	137.00*	281.13*	223.47*	290.43*
NRSQ(24)	102.82*	121.09*	276.59*	219.94*	293.57*
BJ(Δe)	1096.29*	521.53*	82.24*	569.70*	1604.27*

Data: Logarithms of daily exchange rate observations over the period Jan.14, 1987 to Sept.20, 1989, taken at 3:00 PM E.T. by the First American Bank, New York, reported by the *Wall Street Journal*.
N = 676 obs
NRSQ(p) is the LM test for conditional heteroskedasticity (ARCH) at p lags
* denotes significance at the 1% level
† the Lira fluctuates within a band ± 6% about the parity rate, while the others have a band of width ± 2%
‡ central parities as of January 12, 1987
BJ(Δe) is the Bera-Jarque test statistic for normality

Table 2: Summary Statistics on Selected Weekly EMS Exchange Rates *versus* the Deutschemark

	Dm/Ffr	Dm/Li	Dm/Bfr	Dm/Kr	Dm/Fl
mean($e-e_c$)	-0.00611	-0.01326	-0.01215	-0.00602	0.00037
variance(e)	0.7401E-4	0.2031E-3	0.2554E-4	0.1346E-3	0.8264E-5
skewness(e)	0.4225*	0.3761*	0.6063*	0.3744*	0.6444*
kurtosis(e)	1.8753	2.0542	2.7613	1.9754	4.7636*
mean(Δe)	-0.8272E-4	-0.7388E-4	-0.6985E-4	-0.1281E-3	-0.8224E-4
variance(Δe)	0.6228E-5	0.1238E-4	0.1269E-4	0.1418E-4	0.8877E-5
skewness(Δe)	-1.8357*	-0.7830*	-0.1678	-0.5603*	-0.0987
kurtosis(Δe)	11.732*	5.9891*	5.0754*	5.4533*	9.0348*
NRSQ(1)	3.7330	12.441*	34.257*	10.014*	29.813*
NRSQ(2)	4.7071	12.407*	44.026*	11.310*	37.688*
NRSQ(3)	5.9839	12.437*	48.517*	11.356*	42.710*
NRSQ(4)	6.6620	12.367	51.188*	12.134	43.636*
NRSQ(5)	6.6271	13.055	52.189*	12.862	45.841*
NRSQ(8)	10.533	13.395	52.588*	14.266	46.248*
NRSQ(12)	13.096	15.713	54.017*	16.454	47.682*
NRSQ(24)	14.832	20.733	59.632*	37.533	46.362*
BJ(Δe)	523.408*	66.425*	25.783*	42.434*	212.670*

Data: Logarithms of Wednesday observations over the period Jan. 21, 1987 to Sept. 22, 1989 taken at 3:00 PM E.T. by the First American Bank and reported by the *Wall Street Journal*.
N = 140 obs.
NRSQ(p) is the LM test for conditional heteroskedasticity (ARCH) at p lags
* denotes significance at the 1% level
BJ(Δe) is the Bera-Jarque test statistic for normality

Table 3: Testing the Model Against the Dm/Ffr Rate (T = 500)

	Actual	Case1	Case2 (wider band)	Case3 (larger σ^2)	Case4 (largest σ^2)
mean(e)	-0.0060	-0.4868E-3	-0.8048E-3	-0.1991E-3	0.2952E-3
		(0.00847)	(0.0129)	(0.0035)	(0.00021)
		[0.89]	[0.80]	[1.00]	[1.00]
variance(e)	0.7287E-4	0.7818E-4	0.7614E-4	0.2372E-3	0.2542E-3
		(0.701E-4)	(0.534E-4)	(0.336E-4)	(0.195E-4)
		[0.28]	[0.39]	[1.00]	[1.00]
skewness(e)	0.3755*	-0.0176	-0.0239	-0.0244	-0.0262
		(0.602)	(0.424)	(0.364)	(0.209)
		[0.22]	[0.19]	[0.18]	[0.02]
		[0.29]	[0.28]	[0.26]	[0.12]
kurtosis(e)	1.8516*	2.4000	2.2759	1.7096	1.5639
		(1.014)	(0.502)	(0.253)	(0.128)
		[0.74]	[0.80]	[0.20]	[0.03]
		[0.08]	[0.03]	[0.00]	[0.00]
mean(Δe)	-0.2339E-4	-0.6117E-5	-0.2912E-5	-0.3946E-5	-0.1496E-5
		(0.273E-4)	(0.431E-4)	(0.296E-4)	(0.319E-4)
		[0.40]	[0.37]	[0.34]	[0.36]
variance(Δe)	0.2261E-5	0.1499E-5	0.1018E-5	0.3358E-4	0.1007E-3
		(0.155E-5)	(0.279E-6)	(0.823E-5)	(0.151E-4)
		[0.15]	[0.01]	[1.00]	[1.00]
skewness(Δe)	-0.6131*	-0.0026	-0.0051	0.0490	-0.0437
		(1.188)	(0.201)	(0.639)	(0.276)
		[0.95]	[0.99]	[0.86]	[0.96]
		[0.12]	[0.03]	[0.31]	[0.20]
kurtosis(Δe)	9.1170*	8.6353*	3.2309	12.617*	7.349*
		(14.29)	(2.264)	(2.948)	(0.779)
		[0.20]	[0.01]	[0.89]	[0.01]
		[0.30]	[0.03]	[1.00]	[1.00]
Hits at $\bar{e}$	0	0.13	0.0	2.99	10.11
		(0.42)	(0.0)	(1.60)	(2.98)
Hits at $\underline{e}$	0	0.15	0.0	3.17	9.09
		(0.41)	(0.0)	(1.56)	(2.54)

( ) denotes a sample standard deviation [ ] denotes a prob-value

Case 1: $\alpha = 0.03$ $\eta = -0.00001$ $\sigma^2 = 0.001$ $\bar{e} = 0.0225$ $\underline{e} = -0.0225$

Case 2: $\alpha = 0.03$ $\eta = -0.00001$ $\sigma^2 = 0.001$ $\bar{e} = 0.05$ $\underline{e} = -0.05$

Case 3: $\alpha = 0.03$ $\eta = -0.00001$ $\sigma^2 = 0.005$ $\bar{e} = 0.0225$ $\underline{e} = -0.0225$

Case 4: $\alpha = 0.03$ $\eta = -0.00001$ $\sigma^2 = 0.01$ $\bar{e} = 0.0225$ $\underline{e} = -0.0225$

Table 3 Cont'd:

	Case 5 (η = 0)	Case6 (η = 0.00001)	Case7 (η = -0.0001)	Case8 (α = 0.1)	Case9 (α = 1)
mean(e)	0.6398E-3	0.001617	-0.003398	-0.7344E-3	-0.5600E-3
	(0.00816)	(0.00814)	(0.00546)	(0.00912)	(0.00823)
	[0.94]	[0.94]	[0.90]	[0.86]	[0.91]
variance(e)	0.7941E-4	0.8045E-4	0.1736E-3	0.6045E-4	0.3064E-4
	(0.739E-4)	(0.801E-4)	(0.811E-4)	(0.574E-4)	(0.182E-4)
	[0.30]	[0.23]	[0.82]	[0.21]	[0.05]
skewness(e)	-0.0376	-0.0209	0.3137*	0.0274	-0.0102
	(0.582)	(0.627)	(0.545)	(0.628)	(0.503)
	[0.21]	[0.23]	[0.38]	[0.24]	[0.22]
	[0.30]	[0.29]	[0.12]	[0.30]	[0.29]
kurtosis(e)	2.3872	2.5927	2.2508	2.4701	2.3112
	(0.844)	(1.093)	(0.954)	(1.136)	(0.619)
	[0.77]	[0.75]	[0.59]	[0.75]	[0.78]
	[0.10]	[0.13]	[0.08]	[0.09]	[0.06]
mean(Δe)	-0.3816E-6	0.2012E-5	0.3789E-5	-0.2582E-4	-0.2215E-5
	(0.293E-4)	(0.286E-4)	(0.299E-4)	(0.293E-4)	(0.259E-4)
	[0.29]	[0.25]	[0.22]	[0.32]	[0.26]
variance(Δe)	0.1667E-5	0.1546E-5	0.2674E-5	0.9799E-6	0.4398E-6
	(0.185E-5)	(0.141E-5)	(0.187E-5)	(0.646E-6)	(0.292E-7)
	[0.17]	[0.18]	[0.47]	[0.06]	[0.00]
skewness(Δe)	-0.2142	-0.3989*	2.2837*	-0.0249	0.0079
	(1.198)	(1.634)	(2.255)	(0.663)	(0.126)
	[0.89]	[0.84]	[0.97]	[0.95]	[1.00]
	[0.17]	[0.17]	[0.05]	[0.08]	[0.04]
kurtosis(Δe)	9.5568*	11.7997*	33.2072*	5.5875*	3.2181
	(15.998)	(20.529)	(27.776)	(7.301)	(0.529)
	[0.21]	[0.27]	[0.77]	[0.12]	[0.00]
	[0.33]	[0.34]	[0.82]	[0.25]	[0.14]
Hits at $\bar{e}$	0.11	0.09	0.85	0.05	0.01
	(0.34)	(0.32)	(0.54)	(0.22)	(0.10)
Hits at $\underline{e}$	0.16	0.18	0.10	0.04	0.0
	(0.42)	(0.41)	(0.33)	(0.19)	(0.0)

* denotes significance at the 1% level of test

Case 5: α = 0.03 η = 0.0 σ^2 = 0.001 $\bar{e}$ = 0.0225 $\underline{e}$ = -0.0225

Case 6: α = 0.03 η = 0.00001 σ^2 = 0.001 $\bar{e}$ = 0.0225 $\underline{e}$ = -0.0225

Case 7: α = 0.03 η = -0.0001 σ^2 = 0.001 $\bar{e}$ = 0.0225 $\underline{e}$ = -0.0225

Case 8: α = 0.10 η = -0.00001 σ^2 = 0.001 $\bar{e}$ = 0.0225 $\underline{e}$ = -0.0225

Case 9: α = 1.0 η = -0.00001 σ^2 = 0.001 $\bar{e}$ = 0.0225 $\underline{e}$ = -0.0225

Table 4: Testing the Model Against the Dm/Ffr Rate (T = 2000)

	Case 1	Case2 (wider band)	Case 3 (larger σ^2)	Case 4 (largest σ^2)	Case 5 (η = 0)
mean(e)	-0.5249E-3	-0.002528	-0.1814E-3	-0.1371E-3	0.6502E-3
	(0.00585)	(0.01661)	(0.00181)	(0.00106)	(0.00574)
	[0.97]	[0.65]	[1.00]	[1.00]	[0.98]
variance(e)	0.1802E-3	0.4073E-3	0.2509E-3	0.2587E-3	0.1706E-3
	(0.653E-4)	(0.349E-3)	(0.153E-4)	(0.975E-5)	(0.706E-4)
	[0.96]	[0.97]	[1.00]	[1.00]	[0.90]
skewness(e)	0.0941	-0.0302	0.0175	0.0158	-0.0653
	(0.596)	(0.658)	(0.182)	(0.107)	(0.601)
	[0.29]	[0.24]	[0.01]	[0.00]	[0.29]
	[0.25]	[0.29]	[0.05]	[0.00]	[0.38]
kurtosis(e)	2.2589	2.6188	1.5516	1.5072	2.2953
	(0.7925)	(1.294)	(0.083)	(0.044)	(0.767)
	[0.67]	[0.79]	[0.00]	[0.00]	[0.66]
	[0.06]	[0.11]	[0.00]	[0.00]	[0.04]
mean(Δe)	-0.1722E-6	-0.2598E-5	0.5634E-6	0.1700E-6	0.1993E-6
	(0.772E-56)	(0.159E-4)	(0.831E-5)	(0.754E-5)	(0.759E-5)
	[0.00]	[0.10]	[0.00]	[0.00]	[0.00]
variance(Δe)	0.2549E-5	0.2532E-5	0.3611E-4	0.9828E-4	0.2182E-5
	(0.134E-5)	(0.304E-5)	(0.443E-5)	(0.609E-5)	(0.117E-5)
	[0.54]	[0.28]	[1.00]	[1.00]	[0.38]
skewness(Δe)	0.2932	0.6211	0.0091	0.0032	-0.4797
	(1.278)	(3.382)	(0.265)	(0.140)	(1.373)
	[0.82]	[0.92]	[0.99]	[1.00]	[0.66]
	[0.25]	[0.11]	[0.14]	[0.02]	[0.49]
kurtosis(Δe)	34.8579*	41.2092*	12.8577*	7.4121*	33.8760*
	(18.552)	(97.396)	(1.237)	(0.381)	(21.162)
	[0.89]	[0.36]	[1.00]	[0.00]	[0.82]
	[0.91]	[0.40]	[1.00]	[1.00]	[0.14]
Hits at $\bar{e}$	1.35	0.28	14.23	34.36	0.88
	(1.17)	(0.51)	(3.12)	(5.04)	(0.89)
Hits at $\underline{e}$	1.05	0.22	13.99	34.23	1.23
	(1.12)	(0.48)	(3.19)	(4.69)	(1.14)

( ) denotes a sample standard deviation
[ ] denotes a prob-value

Table 4 Cont'd

	Case 6 (η = 0.00001)	Case 7 (η = -0.0001)	Case 8 (α = 0.1)	Case 9 (α = 1)
mean(e)	-0.3941E-3	-0.001354	-0.7059E-3	-0.001781
	(0.00592)	(0.00339)	(0.00719)	(0.00975)
	[0.98]	[1.00]	[0.95]	[0.82]
variance(e)	0.1721E-3	0.2156E-3	0.1737E-3	0.1024E-3
	(0.673E-4)	(0.329E-4)	(0.734E-4)	(0.720E-4)
	[0.91]	[1.00]	[0.93]	[0.50]
skewness(e)	0.0379	0.1603	0.0479	0.0542
	(0.649)	(0.339)	(0.677)	(0.756)
	[0.31]	[0.25]	[0.38]	[0.35]
	[0.35]	[0.14]	[0.32]	[0.26]
kurtosis(e)	2.4214	1.8264	2.3052	2.5778
	(1.264)	(0.313)	(0.912)	(1.409)
	[0.66]	[0.72]	[0.36]	[0.73]
	[0.07]	[0.00]	[0.03]	[0.10]
mean(Δe)	-0.6291E-6	-0.2290E-6	-0.6041E-6	-0.1373E-5
	(0.779E-5)	(0.781E-5)	(0.848E-5)	(0.795E-5)
	[0.00]	[0.00]	[0.00]	[0.00]
variance(Δe)	0.2341E-5	0.2823E-5	0.1376E-5	0.4564E-5
	(0.114E-5)	(0.945E-6)	(0.750E-6)	(0.216E-6)
	[0.47]	[0.72]	[0.14]	[0.00]
skewness(Δe)	-0.1288	2.5334	0.1744	0.0387
	(1.735)	(1.333)	(1.034)	(0.368)
	[0.69]	[1.00]	[0.88]	[0.97]
	[0.35]	[0.01]	[0.21]	[0.05]
kurtosis(Δe)	33.8577*	46.5837*	18.7977*	6.5652*
	(26.242)	(18.676)	(14.509)	(5.525)
	[0.85]	[1.00]	[0.70]	[0.21]
	[0.12]	[0.00]	[0.19]	[0.62]
Hits at $\bar{e}$	1.13	3.45	0.81	0.17
	(1.03)	(0.91)	(0.79)	(0.38)
Hits at $\underline{e}$	1.17	0.36	0.54	0.11
	(1.05)	(0.66)	(0.82)	(0.31)

* denotes significance at the 1% level

Table 5: Estimated Parameter Values for Dm/Ffr

	Method 1	Method 2
α	0.03536 (0.00648)	0.026040 (0.00687)
η	-0.000019394 (0.0000004)	-0.000014756 (0.00000043)
σ^2	0.0015264 (0.000119)	0.000732 (0.000143)
$\bar{k}$	0.02389807	0.025338065
$\underline{k}$	-0.02438652	-0.025831081

Table 6: Simulated Sample Moments and ARCH Test Statistics

	Method 1	Method 2
mean(e)	-0.6863E-3	0.001045
	(0.0033)	(0.0065)
	[0.94]	[0.76]
variance(e)	0.1583E-3	0.1545E-3
	(0.268E-4)	(0.730E-4)
	[1.00]	[0.82]
skewness(e)	0.0528	0.0533
	(0.412)	(0.705)
	[0.21]	[0.33]
	[0.30]	[0.39]
kurtosis(e)	1.8271	2.5769
	(0.332)	(1.129)
	[0.41]	[0.76]
	[0.00]	[0.12]
mean(Δe)	0.3212E-6	0.2348E-6
	(0.676E-5)	(0.782E-5)
	[1.00]	[1.00]
variance(Δe)	0.4745E-5	0.1477E-5
	(0.154E-5)	(0.982E-6)
	[0.98]	[0.19]
skewness(Δe)	0.1557	0.8636*
	(0.812)	(1.747)
	[0.84]	[0.92]
	[0.31]	[0.12]
kurtosis(Δe)	26.3383*	43.3427*
	(7.146)	(31.013)
	[1.00]	[0.80]
	[1.00]	[0.83]
BJ(Δe)	50152.87	216229.3
	(33345.9)	(319102.0)
	[1.00]	[0.80]
	[1.00]	[0.83]
Hits at $\bar{e}$	2.87	0.97
	(1.32)	(0.83)
Hits at $\underline{e}$	2.25	128.67
	(1.26)	(126.13)

( ) denotes a sample standard deviation, [ ] denotes a prob-value

Table 6 Cont'd

	Method 1	Method 2
TRSQ(1)	8.4062*	13.729*
	(12.102)	(22.84)
	[0.00]	[0.02]
	[0.38]	[0.42]
TRSQ(2)	15.2535*	26.396*
	(14.514)	(33.47)
	[0.00]	[0.04]
	[0.52]	[0.59]
TRSQ(3)	19.1731*	35.847*
	(16.22)	(36.76)
	[0.00]	[0.60]
	[0.60]	[0.69]
TRSQ(4)	23.7532*	44.580*
	(17.729)	(42.17)
	[0.00]	[0.09]
	[0.68]	[0.74]
TRSQ(5)	27.3588*	49.979*
	(17.715)	(44.46)
	[0.00]	[0.11]
	[0.74]	[0.78]
TRSQ(8)	35.9149*	67.103*
	(19.825)	(48.72)
	[0.01]	[0.23]
	[0.74]	[0.79]
TRSQ(12)	45.607*	83.181*
	(20.812)	(56.96)
	[0.02]	[0.37]
	[0.81]	[0.80]
TRSQ(24)	68.767*	114.672*
	(28.007)	(71.46)
	[0.10]	[0.48]
	[0.85]	[0.81]

* denotes significance at the 1% level

Table 7: Small-Sample Simulated Moments and ARCH Statistics -- 500 obs.

	Method 1	Method 2
mean(e)	0.01126	0.004981
variance(e)	0.4501E-4	0.1321E-4
skewness(e)	-0.4971*	-0.2244*
kurtosis(e)	1.9161	1.9738
mean(Δe)	0.3357E-4	0.1358E-4
variance(Δe)	0.1376E-5	0.5091E-6
skewness(Δe)	-0.0106	0.1537
kurtosis(Δe)	3.7639*	2.9986
BJ(Δe)	12.166*	1.9696
Hits at $\bar{e}$	0	0
Hits at $\underline{e}$	0	0
TRSQ(1)	0.4824	0.2345
TRSQ(2)	1.6919	3.0031
TRSQ(3)	3.5453	3.5236
TRSQ(4)	8.2697**	7.7624
TRSQ(5)	11.522**	7.7141
TRSQ(8)	16.252**	12.791
TRSQ(12)	25.258**	22.378**
TRSQ(24)	40.412**	30.548

Table 8: Estimated Steady-State Moments and ARCH Statistics -- 10000 obs.

	Method 1	Method 2
mean(e)	-0.3459E-3	-0.00251
variance(e)	0.1817E-3	0.1832E-3
skewness(e)	0.0127	-0.08894
kurtosis(e)	1.5342	1.8852
mean(Δe)	0.1047E-5	-0.1329E-5
variance(Δe)	0.3958E-5	0.9405E-6
skewness(Δe)	-0.2211	2.0742*
kurtosis(Δe)	35.289	76.941*
BJ(Δe)	434490.0*	2285200.0*
Hits at $\bar{e}$	15	5
Hits at $\underline{e}$	11	802
TRSQ(1)	6.9075*	71.964*
TRSQ(2)	7.0730*	75.474*
TRSQ(3)	28.306*	77.054*
TRSQ(4)	38.321*	78.283*
TRSQ(5)	38.408*	86.386*
TRSQ(8)	57.452*	90.011*
TRSQ(12)	63.345*	146.56*
TRSQ(24)	88.582*	175.38*

* denotes significance at 1% level, ** denotes significance at 10% level

Table 9: Simulated Sample Moments and ARCH Test Statistics for Aggregated Data

	Method 1	Method 2
mean(e)	-0.6819E-3	-0.1044E-3
	(0.00335)	(0.00651)
	[0.94]	[0.76]
variance(e)	0.1596E-3	0.1559E-3
	(0.274E-4)	(0.737E-4)
	[1.00]	[0.82]
skewness(e)	0.0503	0.0544
	(0.413)	(0.714)
	[0.19]	[0.28]
	[0.28]	[0.41]
kurtosis(e)	1.8222	2.5857
	(0.329)	(1.162)
	[0.40]	[0.74]
	[0.00]	[0.12]
mean(Δe)	0.7132E-5	0.4566E-5
	(0.684E-4)	(0.784E-4)
	[0.87]	[0.81]
variance(Δe)	0.4180E-4	0.1448E-4
	(0.124E-4)	(0.985E-5)
	[1.00]	[0.79]
skewness(Δe)	0.2334	1.3805*
	(0.976)	(2.187)
	[0.98]	[0.95]
	[0.66]	[0.87]
kurtosis(Δe)	12.8224*	24.3778*
	(3.664)	(17.419)
	[0.54]	[0.74]
	[1.00]	[0.83]
BJ(Δe)	947.9746*	6532.892*
	(750.472)	(9750.47)
	[0.66]	[0.75]
	[1.00]	[0.81]

T = 200 obs.
* denotes significance at the 1% level

Table 9 Cont'd.

	Method 1	Method 2
TRSQ(1)	2.1067	3.3911
	(2.944)	(4.806)
	[0.21]	[0.29]
	[0.06]	[0.11]
TRSQ(2)	4.2973	5.3452
	(3.873)	(5.934)
	[0.37]	[0.38]
	[0.13]	[0.18]
TRSQ(3)	6.2664	7.0417
	(4.770)	(6.436)
	[0.42]	[0.43]
	[0.13]	[0.15]
TRSQ(4)	8.0668	9.0682
	(5.501)	(8.029)
	[0.51]	[0.49]
	[0.17]	[0.21]
TRSQ(5)	9.4982	10.3004
	(5.752)	(8.314)
	[0.66]	[0.66]
	[0.14]	[0.21]
TRSQ(8)	13.6674	13.8358
	(8.556)	(6.702)
	[0.59]	[0.64]
	[0.18]	[0.19]
TRSQ(12)	18.1707	17.8159
	(7.327)	(9.157)
	[0.74]	[0.62]
	[0.13]	[0.16]
TRSQ(24)	28.7406	29.0085
	(8.481)	(10.755)
	[0.96]	[0.97]
	[0.07]	[0.11]