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Realistic Cross-Country Consumption Correlations in a Two-Country, Equilibrium, Business Cycle Model

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Abstract

A well-known feature of one-good, multi-agent, Arrow-Debreu economies with identical, additively-separable, homothetic preferences is that the consumptions of all agents are perfectly correlated. Such economies are widely used in interpreting business cycles but seem to be inconsistent with observed cross-country correlations of aggregate consumption. This paper provides an example of a two-country real business cycle model in which preferences are not separable between consumption and labor supply. The model has a simple closed-form solution, and allows for fluctuations in labor supply in equilibrium. Moreover, it generates correlations between national consumption rates which are close to some of those observed in historical data.

1. Introduction.

A well-known property of Arrow-Debreu economies with one good and stationary, additively-separable preferences is that the consumption of each agent is a deterministic, increasing function of aggregate consumption. Moreover, if preferences are identical and homothetic then the consumptions of two agents are virtually perfectly correlated (see for example Townsend (1987), Brennan and Solnik (1989), and Stulz (1981)).[1] While preferences with these features are adopted widely, this implication of optimal risk-sharing is not evident empirically. For example, the correlation between U.S. and Canadian quarterly private consumption from 1971:1-1988:4 (Source: OECD Department of Economics and Statistics *Quarterly National Accounts*) in deviations from trends is 0.564. Alternative transformations such as first differences produce even lower consumption correlations. Moreover, the correlation for the U.S. and Canada appears to be higher than that for other pairs of countries (see Backus, Kehoe, and Kydland, 1989; Table 2 and Tesar, 1989; Table 1).

So far, these empirical correlations constitute a challenge even for models with non-separability between consumption and labor supply. An example is the model of Backus, Kehoe, and Kydland (1989) who extend the equilibrium business cycle model of Kydland and Prescott (1982) to a

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[1] Strictly speaking one consumption is a monotone increasing and deterministic function of the other. Since this function need not be linear the correlation coefficient need not be one. For the preferences typically adopted in economics it is very close to one and for simplicity we refer to it as taking that value.

two-country setting. Their model captures many features of the international business cycle but does not reproduce realistic cross-country consumption correlations.

This paper alters the preferences commonly used in real business cycle studies in a way which may resolve the discrepancy between data and theory with respect to the cross-country consumption correlations. We do not address other shortcomings of business cycle models (see McCallum (1989) for a survey). We construct a simple, two-country model economy in which preferences exhibit a particular non-separability between consumption and labor supply. To make the argument as transparent as possible we study an example which has a closed-form, analytical solution as in Long and Plosser (1983) and Cantor and Mark (1988). In contrast to those papers the model allows for fluctuations in labor supply in equilibrium. In contrast to Backus, Kehoe, and Kydland (1989) the form of the non-separability between consumption and labor supply generates correlations between national consumption rates which are close to those observed in data.

Section 2 of the paper describes the model economy. We focus on a technology with one hundred percent depreciation; this allows analytical solutions but (as Appendix B shows) is not necessary for the results on consumption comovements across countries. Section 3 discusses the empirical implications of the model and considers a simple method for gauging the closeness of the model's consumption correlation to that in data. While the model has some obvious deficiencies, its consumption correlation is not significantly different from that observed in the U.S. and Canada during the post-war period. Section 4 concludes the paper.

2. A Two-Country Economy.

We develop a two-country model of a world economy in which there is a common good produced in each country. The countries follow identical production techniques but each national technique is subjected to independent, country-specific, productivity shocks. Let the countries be called 'home' and 'foreign' for concreteness, and denote all foreign variables with an asterisk. The model is characterized through the following series of assumptions:

Assumption 1. Preferences of the home and foreign countries are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t); \qquad u = \log(c_t - \gamma n_t^{\mu}) \tag{1}$$

$$\sum_{t=0}^{\infty} \beta^t u^*(c_t^*, n_t^*); \qquad u^* = \log(c_t^* - \gamma n_t^{*\mu}) \tag{2}$$

where $\beta \in (0,1)$, $\mu > 1$, c_t is home consumption, and n_t is home employment. The special feature of these preferences is that the income elasticity of leisure is zero. Note in particular that the marginal utility of consumption is <u>not</u> independent of labor supply. A similar representation of preferences is adopted by Greenwood, Hercowitz, and Huffman (1988).

Assumption 2. Technologies are Cobb-Douglas;

$$y_t = \theta_t K_t^{\alpha} n_t^{(1-\alpha)} \tag{3}$$

$$y_t^* = \theta_t^* K_t^{*\alpha} n_t^{*(1-\alpha)} \tag{4}$$

where $\alpha \in (0,1)$, y_t and y_t^* are output levels for each country, K_t and K_t^* are capital stocks, and θ_t and θ_t^* are productivity coefficients, identically and independently distributed across both time <u>and</u> countries. Each shock has a mean of unity and a constant variance σ^2.

Assumption 3. The rate of physical depreciation in the capital stock of each country is unity. Thus $I_{t+1} = K_{t+1}$ and $I_{t+1}{}^* = K_{t+1}{}^*$. This assumption is required for exact closed-form analytical solutions for policy functions in the program outlined below. A similar assumption is made, for identical reasons, by Long and Plosser (1983) and Cantor and Mark (1988). While this assumption is useful for illustrative purposes, it is in no way required for the main point of the paper, as is shown Appendix B.

It is assumed that there is a unified world capital market whereby households from either country can trade in goods and assets, but that labor is immobile across national boundaries. One way to approach the problem would be to define and derive a recursive competitive equilibrium for the world economy. This would give consumption, capital stocks, employment rates, and prices for each country as a function of the state vector $\{K_t, K_t{}^*, \theta_t, \theta_t{}^*\}$. However, since a world competitive equilibrium with complete markets is obviously Pareto efficient, we can exploit the equivalence between a competitive equilibrium and a social planning optimum by solving instead a social planning problem in which a weighted sum of national utilities is maximized subject to technologies and aggregate resource constraints. By the results of Negishi (1960) and Mantel (1971), there exist weights such that the allocations that solve the social planning problem for these weights are identical to those of a competitive equilibrium for a given set of initial endowments. The attractiveness of this approach is that it avoids the detailed specification of trading institutions involved in solving directly for a competitive equilibrium. It is the exact analogue for the two-country model of the social planning problem solved by Long and Plosser (1983) for a closed, linear logarithmic economy.

The social planner then faces the following problem:

(P1) Choose $\{c_t, c_t^*, n_t, n_t^*, K_{t+1}, K_{t+1}^*\}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t [u(c_t, n_t) + u^*(c_t^*, n_t^*)]$$

subject to

$$c_t + c_t^* + K_{t+1} + K_{t+1}^* = \theta_t K_t^{\alpha} n_t^{(1-\alpha)} + \theta_t^* K_t^{*\alpha} n_t^{*(1-\alpha)}$$

$$\theta_t \sim \text{i.i.d.}(1, \sigma^2) \qquad \theta_t^* \sim \text{i.i.d.}(1, \sigma^2)$$

$$K_0 = K_0^* \text{ given.}$$

Since the model is entirely symmetric and the initial capital stocks of each country are assumed equal, the weights in the social planner's objective function are equal.

To solve (P1) define the value function $V(K_t, K_t^*, \theta_t, \theta_t^*)$ as

$$V(K_t, K_t^*, \theta_t, \theta_t^*) = \text{Max}\{u(c_t, n_t) + u^*(c_t^*, n_t^*) + \beta E_t V(K_{t+1}, K_{t+1}^*, \theta_{t+1}, \theta_{t+1}^*)\} \qquad (5)$$

subject to

$$c_t + c_t^* + K_{t+1} + K_{t+1}^* = \theta_t K_t^{\alpha} n_t^{(1-\alpha)} + \theta_t^* K_t^{*\alpha} n_t^{*(1-\alpha)}$$

where E_t is the conditional expectation operator. Appendix A derives the solution for optimal consumption, hours worked and capital stocks. Given Assumption 1 the marginal rate of substitution between consumption and labor supply must be independent of consumption. Equality between (the negative of) this marginal rate of substitution and the marginal product of labor in each country must then imply that hours are determined by the condition[2]:

$$\mu\gamma n_t^{\mu-1} = (1-\alpha)\theta_t K_t^{\alpha} n_t^{-\alpha}$$

$$\mu\gamma n_t^{*\mu-1} = (1-\alpha)\theta_t^* K_t^{*\alpha} n_t^{*-\alpha}$$

These give

$$n_t = \left[\frac{(1-\alpha)\theta_t K_t^{\alpha}}{\mu\gamma}\right]^{\frac{1}{\omega}} \tag{6}$$

$$n_t^* = \left[\frac{(1-\alpha)\theta_t^* K_t^{*\alpha}}{\mu\gamma}\right]^{\frac{1}{\omega}} \tag{7}$$

where $\omega \equiv \mu-(1-\alpha) > 0$. Conditional on a given capital stock, hours worked for each country respond positively to current domestic productivity shocks, with an elasticity of $1/\omega$, but do not depend on productivity shocks in the other country. Thus Assumption 1 allows for *intra*-temporal response of labor supply to productivity shocks, but no *inter*-temporal response, in the sense that real interest rate movements will have no affect on hours worked.

It is clear given equation (5) that an optimal program will entail marginal utilities of consumption being equalized across countries. This implies that that $(c_t-\gamma n_t^{\mu}) = (c_t^*-\gamma n_t^{*\mu})$ must hold in equilibrium. However, given $\gamma \neq 0$, consumption is not equalized across countries.

Substituting equations (6) and (7) into the home and foreign production

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[2] See also Greenwood, Hercowitz, and Huffman (1988).

functions, we derive the output levels for each country,

$$y_t = \nu(\theta_t K_t^{\alpha})^{\mu/\omega} \tag{8}$$

$$y_t^* = \nu(\theta_t^* K_t^{*\alpha})^{\mu/\omega} \tag{9}$$

where $\nu = \left[\frac{(1-\alpha)}{\mu\gamma}\right]^{\frac{(1-\alpha)}{\omega}}$. Given $\mu/\omega > 1$, the elasticity of output with respect to an innovation in the current productivity coefficient is greater than one. The reason is that hours worked can respond immediately to a productivity shock. Because labor supply is determined by the atemporal conditions (6) and (7), however, a productivity shock in one country in a given period has no effect within that period on production in the other country. This effect can occur only through time as investment responds to the productivity disturbance.

Given these solutions for output levels, it is easy to manipulate the first order conditions (see Appendix A) to arrive at the following 'portfolio-type' characterization of the optimal choice of capital stocks across countries:

$$E_t \left\{\nu(\theta_{t+1}K_{t+1}^{\alpha})^{\mu/\omega}/\{K_{t+1}[\nu(\theta_t K_{t+1}^{\alpha})^{\mu/\omega} + \nu(\theta_t^* K_{t+1}^{*\alpha})^{\mu/\omega} - K_{t+2} - K_{t+2}^*]\}\right\}$$

$$= E_t \left\{\nu(\theta_{t+1}^* K_{t+1}^{*\alpha})^{\mu/\omega}/\{K_{t+1}^*[\nu(\theta_t K_{t+1}^{\alpha})^{\mu/\omega} + \nu(\theta_t^* K_{t+1}^{*\alpha})^{\mu/\omega} - K_{t+2} - K_{t+2}^*]\}\right\} \tag{10}$$

Since θ_t and θ_t^* are i.i.d., it is clear that $K_{t+1} = K_{t+1}^*$ solves this expression[3]. Manipulating further gives the optimal policy rule for the capital stock

$$K_{t+1} = \tfrac{1}{2}\alpha\beta\nu[(\theta_t)^{\mu/\omega} + (\theta_t^*)^{\mu/\omega}]K_t^{\alpha\mu/\omega} \tag{11}$$

Thus the capital stock in each country depends on the sum of a convex function of the two productivity coefficients. Productivity shocks in either country contribute equally to investment in each country for the next period. Since $\mu\alpha/\omega < 1$, it is clear that the capital stock will converge to a stationary distribution.

Now using the solution for the capital stock, (11), the fact that the effective consumption index $c_t - \gamma n_t^{\mu}$ is equated across countries, and the solution for hours worked in each country, (6) and (7), we may show directly that consumption in the home and foreign countries is

$$c_t = \tfrac{1}{2}(\nu\omega/\mu)(1-(\mu\alpha\beta/\omega))[(\theta_t)^{\mu/\omega}+(\theta_t^*)^{\mu/\omega}]K_t^{\alpha\mu/\omega} + \gamma\nu(\theta_t K_t^{\alpha})^{\mu/\omega} \tag{12}$$

$$c_t^* = \tfrac{1}{2}(\nu\omega/\mu)(1-(\mu\alpha\beta/\omega))[(\theta_t)^{\mu/\omega}+(\theta_t^*)^{\mu/\omega}]K_t^{\alpha\mu/\omega} + \gamma\nu(\theta_t^* K_t^{\alpha})^{\mu/\omega} \tag{13}$$

The first term is common to each of these equations. It is the value of

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[3] We could have assumed more generally, as in Cantor and Mark (1988) that the θ_t and θ_t^* processes are only i.i.d. across time, and not across the two countries. Then it is easy to show that condition (10) gives an implicit equation in the share of world investment carried out by the home and foreign countries, where the special case we have chosen gives a share of one half. However, in the more general case there is no exact solution for the share, and we require the exact solution below.

effective consumption that optimal risk-sharing equates across countries. It depends upon the sum of a function of each of the productivity outcomes θ_t and θ_t^*. The second term however, is country-specific, and captures the fact that domestic hours worked and so domestic consumption will respond directly to the domestic productivity shock. Clearly the model predicts that consumption rates are not perfectly correlated across countries for $\gamma \neq 0$. The conditional (given K_t) correlation coefficient between period t consumption rates, based on period t-1 information, is

$$\rho_{t-1}(c_t, c_t^*) = 1/(1+\eta^2/(2(\eta+\zeta)\zeta)) \tag{14}$$

where $\zeta = \frac{1}{2}(\nu\omega/\mu)(1-(\mu\alpha\beta/\omega))$ and $\eta = \gamma\nu^{(\sigma/(1-\alpha))}$.

Thus only when $\gamma = 0$ would we expect a correlation of unity between home and foreign consumption. In the numerical analysis below, we compute the unconditional correlation and its sampling variability by simulating the model with sequences of shocks.

3. Empirical Implications.

Given parameter values and sequences of shocks, the model generates time paths for domestic and foreign output, consumption, labor supply, savings, investment, and the trade balance. Thus the predictions of the model can be compared with historical evidence on such facts as the cyclicality of the trade balance or the correlation between domestic savings and investment. Since our interest is in the consumption correlation implied by this model, it seems natural to focus on this moment by first parameterizing the model and then determining both the population consumption correlation and the sampling distribution for the corresponding sample moment. The latter provides useful information in measuring the match between data and model in this dimension. We set parameter values as follows: the discount factor, β = 0.95; γ = 1.00; μ = 2.00; the share of capital, α = 0.30; so that ω = 1.30.[4] For these parameter values the population conditional correlation coefficient is $\rho_{t-1}(c_t, c_t^*) = 0.6134$, from equation (14).

The shocks are drawn independently from a common density such that $\theta = \exp(x)$ with $x \sim \text{n.i.d.}(-0.002, 0.2^2)$; thus θ is lognormally distributed with mean unity. The shocks thus have the same distribution and moments as that of the unconditional shock density in Kydland and Prescott (1982) and Backus, Kehoe, and Kydland (1989), based on Solow residuals.[5] Initial conditions for the two capital stocks are set arbitrarily and then fifty initial observations are discarded.

We generate 1000 replications of 71 observations of the model. For each replication we calculate the sample correlation between consumptions. We estimate nonparametrically the probability density function of this sample moment. The density is estimated by kernel methods, with a quartic kernel and a variable window width given by Silverman (1986). Table 1 gives the population moment (obtained by letting the sample size become arbitrarily

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[4] These numbers are similar to those of other studies, e.g. Kydland and Prescott (1983) and Greenwood Hercowitz, and Huffman (1988). We could estimate some of the parameters using a simulation estimator such as that adopted by Kydland and Prescott (1982) and studied by Ingram and Lee (1987), Duffie and Singleton (1988), and Gregory and Smith (1989).

[5] One could also adopt the moments of the country-specific shocks identified by Costello (1989).

large until convergence is achieved) and the approximate 95 percent confidence interval based upon the estimated density function. Figure 1 illustrates the estimated density function for this sample size.

Table 1 also presents the observed sample correlation between U.S. and Canadian private consumption expenditures, quarterly from 1971 to 1988 and thus also based on 71 observations. We quote sample correlations found after using several different detrending methods. In each case, a p-value gives the estimated probability of finding a correlation less than or equal to the corresponding sample correlation. The p-value is simply the proportion of replications in which the simulated correlation coefficient is less than the historical value. Thus we treat these sample moments as critical values in testing the business-cycle model, using as a metric the variability in the model itself.[6] Table 1 and Figure 1 show that the population correlation coefficient between consumptions in the model is well below one. They also show that the sampling variability (based on that of Solow residuals, as in other business cycle models) of this moment is sufficient to reconcile the model with some observed, historical correlations.

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[6] This method is outlined in greater detail in Gregory and Smith (1988).

Table 1: Consumption Correlations†

	Sample Moment [p-value] (U.S.A. - Canada)	Population Moment (95%)
		0.68 (0.53, 0.79)
Detrending		
difference, levels	0.348 [0.001]	
difference, logs	0.358 [0.001]	
linear trend, levels	0.564 [0.06]	
linear trend, logs	0.436 [0.008]	

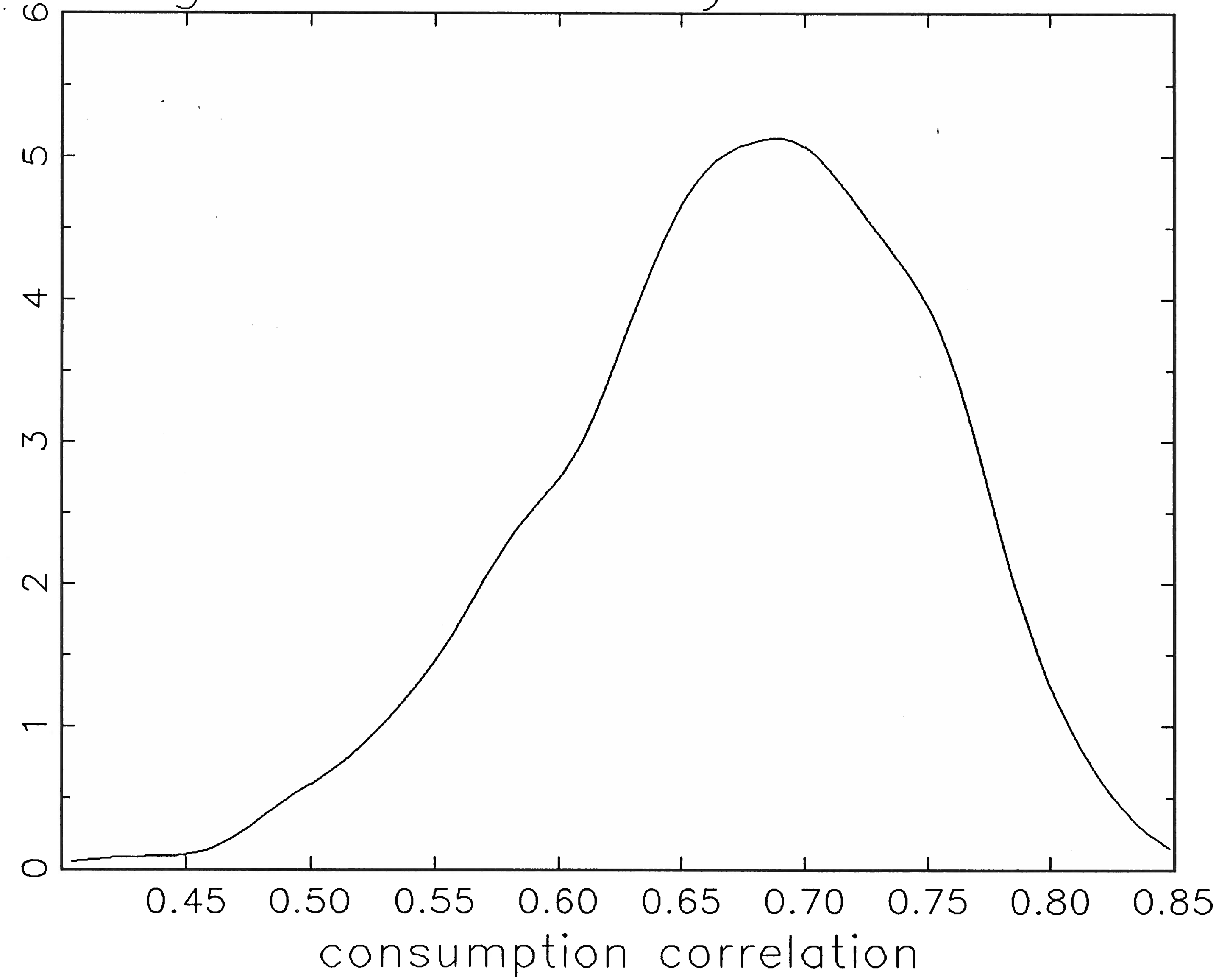
† Correlations use quarterly private consumption: 1985=100, s.a., 1971:1-1988:4. OECD Department of Economics and Statistics *Quarterly National Accounts*; comparative tables; number 1 1989 pp 167, 173, and number 3 1985 pp 151 and 157. We lose the first observation in differencing, leaving calculations for 1971:2-1988:4, *i.e.* 71 observations.

4. Extensions.

Although the parameter settings used in Section 3 are not unusual, one also could examine other settings (for γ and μ, for example) and shock densities. We have varied both the capital share, α, and the variance of the productivity shocks with little effect on the consumption correlations. One could also allow for persistence in the productivity shocks, although with this extension numerical methods would be needed to solve the model.

Our aim has been to demonstrate the point concerning consumption correlations rather than to construct a complete numerical business-cycle model. For this reason we have not studied the correspondence with data for the other moments of the model (for instance, the autocorrelation coefficient of consumption or of labor supply). However, as a check on the accuracy of our model under more general specifications, in Appendix B we solve the model under the assumption of less than one hundred percent depreciation rates. For a depreciation rate of ten percent the unconditional correlation coefficient for consumption across countries is calculated to be 0.552, which again is comparable to the historical values in Table 1.

Figure 1: Density Estimates



Appendix A: Solution of the Social Planning Problem

Given the solutions (6) and (7) of the text, the first-order conditions for the dynamic programming problem (5) can be obtained by substituting the aggregate resource constraint into the expression $u(c_t, n_t)$.[7] This leaves three control variables to be chosen at any date: c_t^*, K_t, and K_t^*. First order conditions are given by

$$[Y_t+Y_t^*-K_{t+1}-K_{t+1}^*-c_t^*-\gamma n_t^{\mu}]^{-1}=(c_t^*-\gamma n_t^{*\mu})^{-1} \quad \text{(A1)}$$

$$[Y_t+Y_t^*-K_{t+1}-K_{t+1}^*-c_t^*-\gamma n_t^{\mu}]^{-1}=\beta E_t V_1(K_{t+1},K_{t+1}^*,\theta_{t+1},\theta_{t+1}^*) \quad \text{(A2)}$$

$$[Y_t+Y_t^*-K_{t+1}-K_{t+1}^*-c_t^*-\gamma n_t^{\mu}]^{-1}=\beta E_t V_2(K_{t+1},K_{t+1}^*,\theta_{t+1},\theta_{t+1}^*) \quad \text{(A3)}$$

By the derivative condition on the value function,

$$V_1=[Y_t+Y_t^*-K_{t+1}-K_{t+1}^*-c_t^*-\gamma n_t^{\mu}]^{-1}\alpha\theta_t K_t^{\alpha-1}n_t^{(1-\alpha)} \quad \text{(A4)}$$

$$V_2=[Y_t+Y_t^*-K_{t+1}-K_{t+1}^*-c_t^*-\gamma n_t^{\mu}]^{-1}\alpha\theta_t^* K_t^{*\alpha-1}n_t^{*(1-\alpha)} \quad \text{(A5)}$$

The full solutions are then obtained by substituting for n_t and n_t^*. (A1) simply gives the condition of equal marginal utilities of consumption in an optimal program. (A2) and (A3), when combined with (A4) and (A5) updated by one period, give condition (10) of the text.

Appendix B: Realistic Depreciation

In this Appendix we construct a version of the model which does not rely upon the artificial construct of one hundred percent depreciation rates per period. We show that the main argument of the paper - that the predicted

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[7] Alternatively, (6) and (7) come out of the first order conditions of the full problem P1.

consumption correlation coefficient can be realistically low, remains unchanged with this alteration.

The model with less than one hundred percent depreciation cannot be solved analytically. To obtain solutions we linearize the first order conditions of the social planning problem around the deterministic steady state as in King, Plosser and Rebelo (1988). Population moments can easily be computed from the resulting linear stochastic difference equations.

Maintain assumptions *1* and *2*, but instead of assumption *3* let investment be given by

$$I_{t+1} = K_{t+1}-(1-\delta)K_t \text{ and } I_{t+1}^* = K_{t+1}^*-(1-\delta)K_t^* \tag{B1}$$

The appropriate resource constraint is then

$$c_t + c_t^* + K_{t+1}-(1-\delta)K_t + K_{t+1}^*-(1-\delta)K_t^* = \theta_t K_t^{\alpha} n_t^{(1-\alpha)} + \theta_t^* K_t^{*\alpha} n_t^{*(1-\alpha)} \tag{B2}$$

The Euler conditions relevant to the optimal rate of investment are derived in a straightforward fashion, and are written as

$$E_t\left\{\nu((\theta_{t+1}K_{t+1}^{\alpha})^{\mu/\omega}/K_{t+1})-(1-\delta)/[\nu(\theta_t K_{t+1}^{\alpha})^{\mu/\omega}+\nu(\theta_t^* K_{t+1}^{*\alpha})^{\mu/\omega}-I_{t+2}-I_{t+2}^*]\}\right\}$$

$$= E_t\left\{\nu((\theta_{t+1}^* K_{t+1}^{*\alpha})^{\mu/\omega}/K_{t+1}^*-(1-\delta)/\{[\nu(\theta_t K_{t+1}^{\alpha})^{\mu/\omega} + \nu(\theta_t^* K_{t+1}^{*\alpha})^{\mu/\omega} - I_{t+2} - I_{t+2}^*]\}\right\} \tag{B3}$$

For the same reasons as in the specification in the text of the paper, the solution implies identical capital stocks across countries. Then taking the Euler equation for optimal consumption in either country and linearizing around the deterministic steady state, we may derive the following solution difference equation, where $\hat{X}_t$ denotes the deviation of the time t variable X from its steady state level:

$$\hat{K}_{t+1} - \Omega\hat{K}_t + \beta\hat{K}_{t-1} = -(\beta\nu\varphi/2)(\theta_t+\theta_t^*) \tag{B4}$$

where $\phi = (1/\beta) - 1$ and

$$\Omega = (2+\phi)/(1+\phi) + (\phi+\delta)^{\omega/(1-\mu)(1-\alpha)}((\omega+\delta)/\mu\alpha-\delta)(\nu\alpha(\mu-1)(1-\alpha)/(1+\phi)^2\omega)^{(1-\omega/(1-\mu)(1-\alpha))}$$

$$\varphi = ((\phi+\delta)/\nu\alpha)^{\omega/(1-\mu)(1-\alpha)}$$

This has the solution

$$\hat{K}_{t+1} = \lambda\hat{K}_t + (\lambda\beta\nu\varphi/2)(\theta_t+\theta_t^*) \quad \text{(B5)}$$

where λ is the stable root of the difference equation (B4). Using the same approach as in the text, it is then easy to demonstrate that consumption in each country may be written (in terms of deviations from the deterministic steady state) as

$$\hat{c}_t = a\hat{K}_t + b\hat{K}_{t+1} + c\theta_t + d\theta_t^* \quad \text{(B6)}$$

$$\hat{c}_t^* = a\hat{K}_t + b\hat{K}_{t+1} + c\theta_t^* + d\theta_t \quad \text{(B7)}$$

where a, b, c, and d are functions of the parameters. Using the solution (B5) the population correlation coefficient for consumption across countries can be computed readily. Given the assumption that the technology shocks are drawn from the same distribution across countries this does not depend upon the moments of the θ process. With the parameter settings in the text and the additional assumption that $\delta = 0.1$, (ten percent depreciation) the unconditional population correlation coefficient, $\rho(c_t, c_t^*)$, is computed to be 0.552.

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