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SHORT RUN ADJUSTMENT IN MODELS OF MONEY AND GROWTH

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In a recent article, "Monetary Growth Theory in Perspective," Jerome Stein has elucidated the differences between the neoclassical and the Keynes-Wicksell (K-W) approaches to monetary growth theory. In particular, he has elaborated on the different implications the two models have for the dynamics of price change, and he proposes a synthesis, or what may be termed a reconciliation, of the two models.

The essential features of the KW models are independent investment-savings decisions, and the explicit representation of a price equation in which prices rise only in response to excess demand in the goods markets. Then, from Walras' law, corresponding to the excess demand in the goods market there must be excess flow supply in the money market. Stein then treats the special case where the flow excess supply of money corresponds to a stock excess supply of money, and hence prices move in response to stock disequilibrium in the asset markets.[1]

It is argued here that this possibility of stock disequilibrium is in fact the crucial point of departure from the neoclassical scenario, and by considering the implications of stock disequilibrium on the demand behavior of the economic agents, the KW and neoclassical approaches are more easily reconcilable.[2] Specifically, adjustment costs are explicitly introduced to explain the stock disequilibrium, and wealth holders act to adjust their asset holdings along an adjustment path which eliminates the stock disequilibrium <u>over time</u>. This analysis leads to consideration of the short run behavior of the economy, but it is inappropriate to call it disequilibrium dynamics. It is equilibrium analysis in the flow sense: given the stock disequilibrium and the instantaneous cost of adjustment, wealth holders adjust their asset holdings in an optimum manner along an equilibrium path. That is, the flows dominate in the short run, and flow equilibrium is sustained. Long run equilibrium is characterized by stock equilibrium in addition to flow equilibrium.

It is the purpose of this note to explicitly analyze the flow aspects of a simple model which reflects the essential details of that used by Stein, and to derive simple dynamics of price change consistent with possible stock disequilibrium. Finally, we briefly consider the special case analyzed by Stein.

For simplicity, we explore only the case where money is held as a consumers good.[3] Per capita output is given by (1), and disposable income includes the value of the government transfer payment, $m(u - \pi)$, and is given by (2):

(1) $$y = y(k)$$

(2) $$y_d = y(k) + m(u-\pi)$$

where m is actual real balances per capita, k is the existing capital-labor-ratio, u is the rate of increase in the nominal money stock, and π is the actual rate of inflation. Assume that the labor force is growing at the exogenous rate n and that, for now, per capita consumption, c, is a constant ratio, (1-s), of disposable income.[4] Then, denoting the operator d/dt by a dot over the variable, and physical savings per capita by S_p, we have:

(3) $$\dot{k} = (y - c) - nk = S_p - nk.$$

Desired per capita real balances are positively related to per capita income, the income coefficient b varying inversely with the nominal interest rate, i, defined as $y'(k) + \pi^e$, where π^e is the expected rate of inflation (assumed equal to u - n).

(4) $$m^d = b(i) \cdot y(k).$$

Equilibrium or steady state, occurs when $\dot{k} = 0$, or alternatively, when the level of per capita physical savings, S_p, given by (5), equals nk.

(5) $$S_p = [s-(1-s)\cdot b\cdot(u-\pi)]y(k) = \sigma(k, \pi)\cdot y(k).$$

Following Sidrauski [2], we set S_p = nk and solve for equilibrium real balances:

(6) $$m^* = \frac{sy\ (k^*) - nk^*}{(1-s)n} = m(k^*).$$

The neoclassical model is characterized by perpetual stock equilibrium, so all adjustment is made along $OR(\pi_o)$ in Figure 1, where OR plots equation (4) for a given value of π^e and m^d identically equals actual m. The price dynamics of the neoclassical model can be seen by differentiating equation (4) with respect to time, and solving for π:

(7) $$\pi = u - n - \frac{B(k, \pi^e)}{m} \cdot \dot{k}$$

where $B(k, \pi^e)$ is equal to $\partial m^d/\partial k$ equals $[y(k)\cdot b'(i)\cdot y''(k) + b(i)\cdot y'(k)]$ which is positive under normal assumptions on y(k). Hence for π to exceed

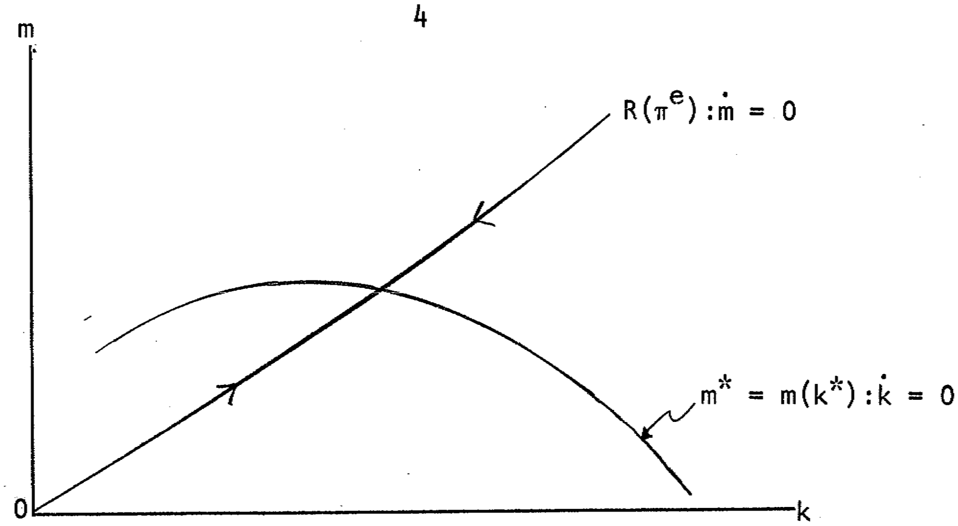

FIGURE 1

its steady state value, u - n, in the neoclassical model, capital must be decumulating. That is, in Figure 1, the section of 0R northeast of E corresponds to a rate of inflation in excess of the steady-state rate u - n, and conversely for the section southwest of E.

We wish now to analyze the case where the assumption of perpetual stock equilibrium is relaxed, in the sense that we no longer wish to treat m^d given by equation (4) as being identically equal to actual per capita real balances, m. This must ultimately derive from the existence of transactions costs in asset markets--in particular, we assume that there exists rising marginal costs to the rate of increase in the holdings of capital at any point in time. This is just like the well known Penrose effect facing the individual investor. (For a detailed discussion, see Uzawa [5] and the references cited there.) Then the flow demand for real balances, f^d, will include a stock adjustment component $t\hat{m}$, where $\hat{m}$ is the stock excess supply of real balances, $m - m^d$. As a first approximation, we will treat t as a constant. The flow demand for real balances is given by (8):

(8) $$f^d = nm^d + B(k, \pi^e)\cdot\dot{k} - t\hat{m}$$

where the first term is the steady state flow demand and the second term is the adjustment in desired real balances for changes in the capital intensity as the economy moves to the steady state.[5]

The concept of the stock demand for money now deserves some further comment. The quantity m^d given by equation (4) defines, for given k, the stock of real balances that would be held in the absence of transactions costs, and can be regarded as the long run desired real balances. Also, at any point in time there exists the distinct concept of the desired stock of real balances given the existing stock and the instantaneous cost of adjustment. This latter concept can be regarded as the short run desired real balances, m^s, and in the case of perpetual flow equilibrium which we will be analyzing in some detail, is always equal to actual real balances. That is, flow equilibrium implies short run stock equilibrium in the sense that, if given the stock they actually are holding they are changing that stock at the desired rate; then, given the rate of change of the stock they are happy with the stock actually being held!

Flow supply, f^s, is given by (9), and the excess flow supply of real balances, $\hat{f}$, by the difference $f^s - f^d$, in (10):

(9) $$f^s = m(u - \pi)$$

(10) $$\hat{f} = m(u - \pi) - nm^d - B(k, \pi^e)\cdot\dot{k} + t\hat{m}.$$

Now, although there may be stock disequilibrium at any point in time ($m^d \neq m$), agents are able to add to their existing real balances at the desired rate--this is the nature of the term $t\hat{m}$ in (8). Then $\hat{f}$ identically equals zero, and we can solve (10) for π:

$$\pi = u - n + \frac{\hat{m}}{m} \cdot (n+t) - \frac{B(k, \pi^e)}{m} \cdot \dot{k}. \tag{11}$$

In the steady state, $\hat{m}=\dot{k}=0$, so (11) has the desired steady state solution, $\pi = u - n$. For π to exceed its steady state value, $\hat{m}\cdot(n+t)$ must exceed $B(k, \pi^e)\cdot\dot{k}$. Now, contrary to the neoclassical model, it is possible for capital to be accumulated while the rate of inflation exceeds its steady state value, if there is a large enough excess stock supply of money!

If we define a as the excess of π over its steady state value, from (11) we see that $\dot{k}$ is related to $\hat{m}$ according to:

$$\dot{k} = - \frac{1}{B(k, \pi^e)} \cdot [am - \hat{m}(n+t)]. \tag{12}$$

Equation (12) is the flow equilibrium condition in the money market. For any given value of a, $\dot{k}$ is positively related to $\hat{m}$--this seems eminently reasonable since $\hat{m}$ positive would imply dishoarding of real balances, and some of this dishoarding could be expected to be directed towards accumulation of physical capital. This relationship is plotted in Figure 2 where we get a family of positively sloped curves mm each corresponding to different values of π (or of a). For any point in $(\dot{k}, \hat{m})$ space there exists a value of π consistent with flow equilibrium in the money market.

However, $\dot{k}$ must in fact move in accordance with (3)--the equilibrium condition in the goods market--except that we are no longer free to interpret the savings rate as a constant (cf. footnote 4). To show this we need to introduce explicitly the long run desired capital stock, k^d. Of course, k^d cannot be specified independently of m^d since in the steady state when long run desired stocks are being held, they must satisfy the wealth constraint, $w = k + m = k^d + m^d$. However, when we allow for stock disequilibrium it is possible at any point in time to have an excess demand (or supply) for wealth in the sense that k^d plus m^d exceeds (falls short of) k plus m.[6] Then we wish to postulate the following behavior of the savings ratio:

$$s = s(\hat{m}, \hat{k}); \; s_1 < 0, \; s_2 < 0 \tag{13}$$

where $\hat{k}$ is the excess of actual k over k^d. The signs of the partial derivatives arise from the belief that an excess stock supply of either asset will induce people to consume a larger proportion of any given income, and hence the savings rate falls. Then (3) can be parameterized as:

$$\dot{k} = \phi(\hat{m}, \hat{k}, \pi); \quad \begin{aligned} \phi_1 &= s_1[y(k) + m(u - \pi)] < 0 \\ \phi_2 &= s_2[y(k) + m(u-\pi)] + (sy' - n) \gtrless 0 \\ \phi_3 &= (1-s) \cdot m > 0. \end{aligned} \tag{3'}$$

Although the second sign is theoretically ambiguous, it will be negative for all but very large negative values of $\hat{k}$, and we shall treat it as negative. Equation (3'), for $k^*=k$ and $\pi=\pi^e$ is plotted in Figure 2 as cc, and when we recognize that goods market equilibrium implies flow equilibrium in the money market, we can draw an equilibrium curve, for $k=k^*$, by allowing for the required change in π. This is seen as c'c' in Figure 2, which is flatter than cc due to the sign of ϕ_3. An increase in the capital stock would shift c'c' down.

Steady state in the economy occurs at the origin in Figure 2. At any point in time, m, p, k, μ, π^e and y are all given, and flow equilibrium determines π--and hence i--and $\dot{k}$.

Now consider a once-for-all increase in the nominal stock of money at t_0 followed by steady growth in the money supply at the previous steady

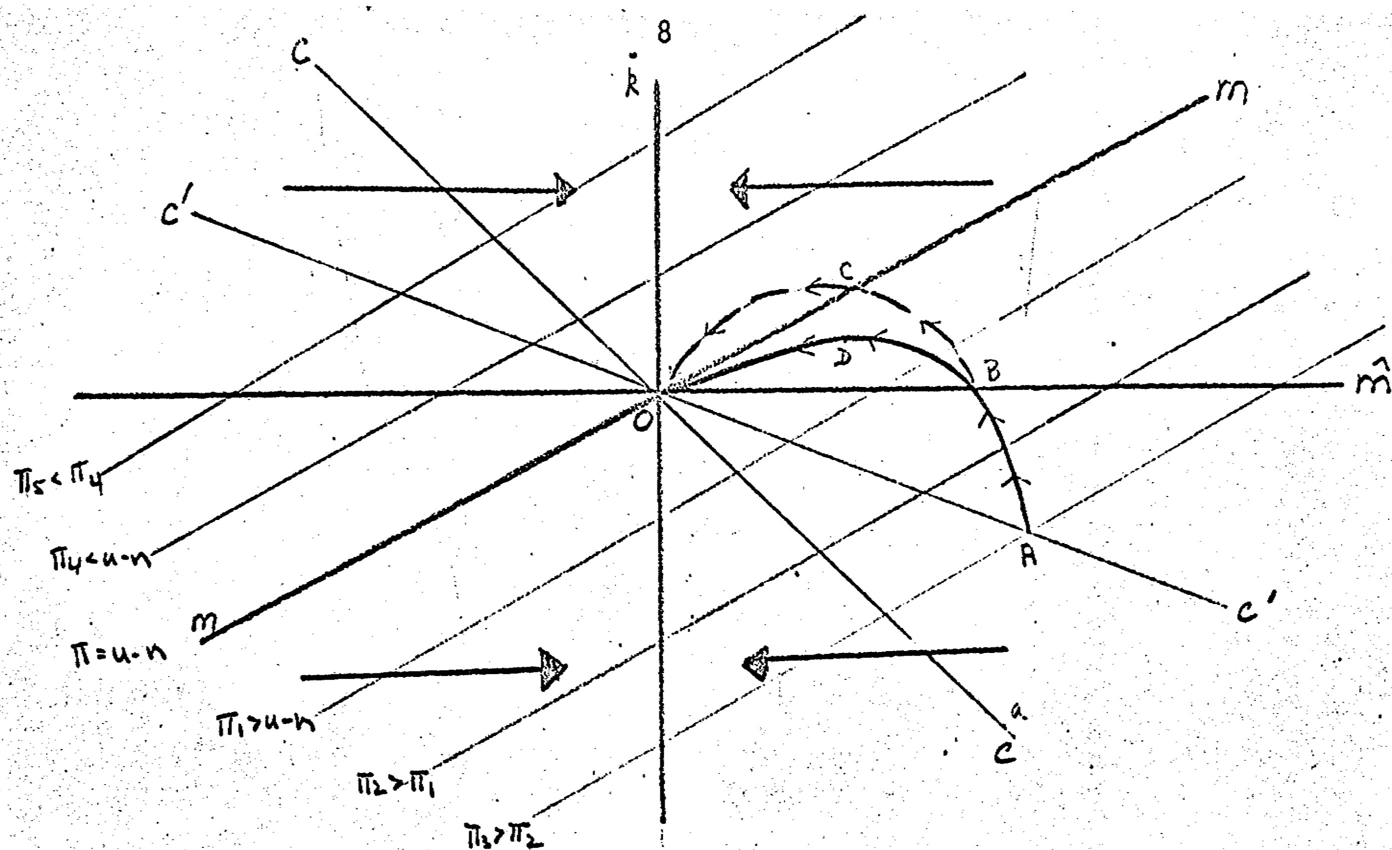

FIGURE 2

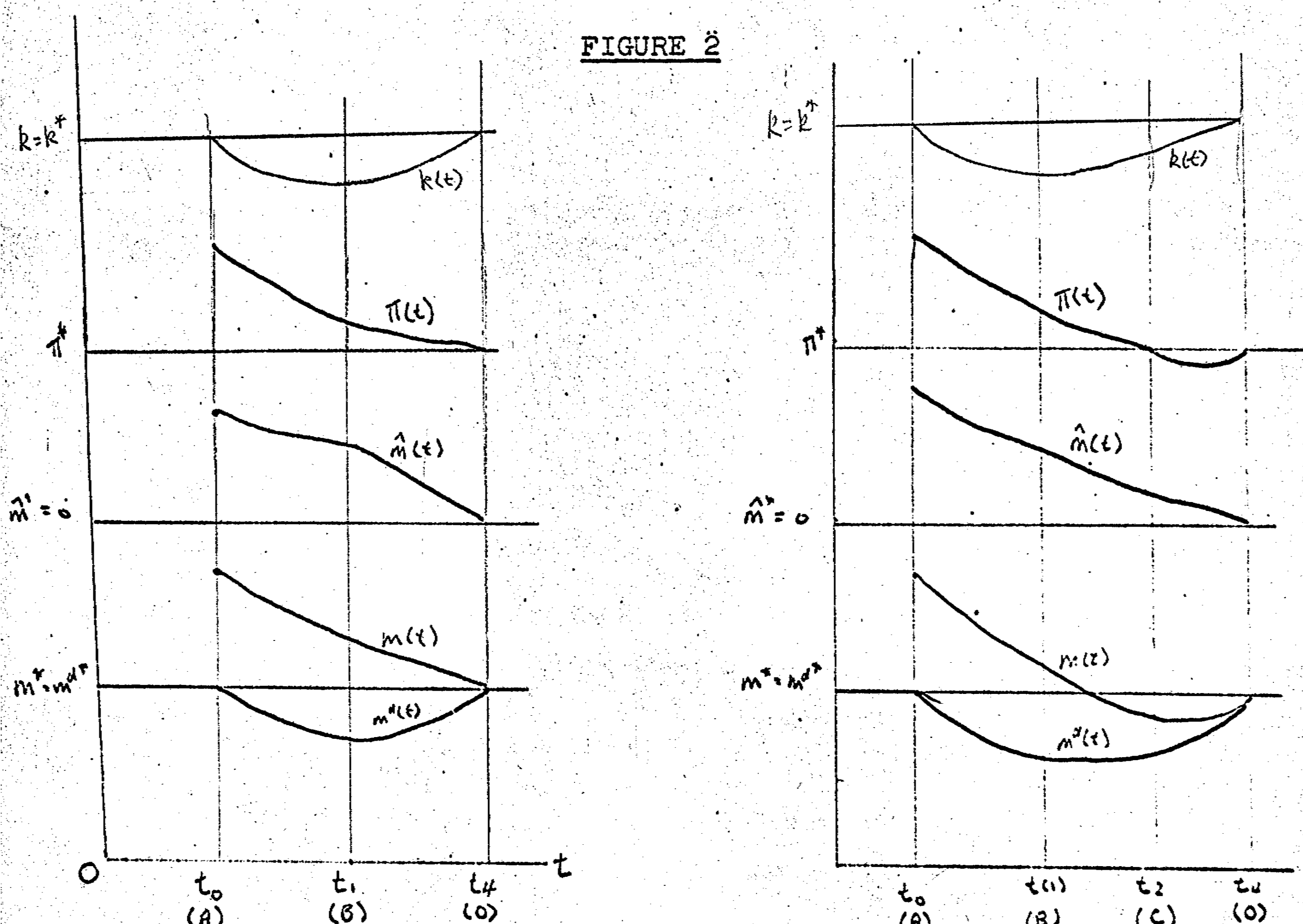

FIGURE 3

state rate u.[7] Instantaneously, at t_0, $\hat{m}$ will be positive and wealth holders immediately try to dishoard by entering the goods market. The increased consumption reduces output available for capital accumulation and $\dot{k}$ falls (below zero). The increased demand in the goods market produces a 'blip' in the price level and we move to point A in Figure 2--π is also bid up instantaneously.

At A, $\hat{m}$ is positive, and due to our expectations hypothesis, unambiguously falling.[8] On the other hand, $\hat{k}$ is negative and falling ($\dot{k} < 0$), causing c'c' to shift upwards. Then the economy is moving northwesterly from A, and π is falling. Our expectations hypothesis is sufficient to require that $\hat{m}$ fall asymptotically to zero, while the fact that u is returned to its previous steady state rate constrains the adjustment path to satisfy $\int_{\tau_A}^{\tau_O} \dot{k}\, dt = 0$ since our comparative dynamics tells us that k^* is unchanged. Two possible adjustment paths ABCO and ABDO are plotted; m^d attains its minimum value at B, then rises directly to its long run equilibrium value. Along ABDO, m and π fall asymptotically to their long run equilibrium values, whereas along ABCO both m and π overshoot and then approach their long run equilibrium values from below. Along the segment AB the dishoarding is reducing $\hat{m}$, but increasing the negative value of $\hat{k}$--both effects serving to increase the savings rate. Northwest of B, $\hat{m}$ falls and $\hat{k}$ (k) rises, both effects tending to restore portfolio equilibrium.

It is interesting to compare the behavior of the capital stock in this model to that obtained in Sidrauski's analysis [2] in which long run desired balances are always held, but expectations are adaptive. In that model, an increase in the rate of inflation causes the capital stock to initially fall, and then rise to its steady state. In that model, the increased real balances are treated initially like an income transfer, and wealth holders tend to consume out of their "excess stock supply of real balances." Only after a period of time sufficient for the increased rate of inflation to become fully anticipated do they adjust their portfolios in favor of physical capital. If that model had our static expectations, the capital stock would rise initially as the new steady state inflation rate would be anticipated fully. In the present model, the monetary authority gives the lump sum to the wealth holders who then proceed to inject it into the economy over time --the analogy to an economy with no adjustment costs but in which the rate of expansion in the money supply is varied over time is immediate.

If, instead of the money creation experiment, we consider an increase in the rate of increase in the money supply, we again get initially an excess stock supply of money. This can be seen by observing that the stock demand for money instantaneously falls by our static price expectations assumption, while the stock supply of money rises initially. The portfolio disequilibrium thus caused is very similar to that analyzed in the money creation case, and the adjustment mechanism described above is just as applicable here. However, the adjustment path is not constrained by the condition that k^* is unchanged, but instead by $\int_{\tau_A}^{\tau_o} \dot{k}\, dt = \lambda$, where λ is the difference between the new and old steady state values of the capital stock.

Now consider briefly the case analyzed by Stein, which implies the following price equation:[9]

$$(14) \qquad \pi = u - n + h \cdot \hat{f}$$

$$= u - n + \frac{h}{1+hm} [\hat{m}(n+t) - B(k, \pi^e) \cdot \dot{k}].$$

Again, in the steady state, $\hat{f} = \hat{m} = \dot{k} = 0$, so (14) has the desired steady state solution, $\pi = u - n$; and $\dot{k}$ can be related to $\hat{m}$ and a by (15):

$$(15) \qquad \dot{k} = - \frac{1}{B(k, \pi^e)} [a(\frac{1}{h} + m) - \hat{m}(n + t)]$$

which is similar to (12). We have the same conditions in the goods market, except we are not constrained to flow equilibrium and hence would conduct the analysis in terms of the cc curves, and not the c'c' curves.

Thus, either equations (11) and (12), or equations (14) and (15) may be used to analyze non-steady state behavior. The first set represents a neoclassical view wherein the price level is essentially driven by the money market, but in which costs of adjustment in asset markets which give rise to stock disequilibria phenomena are allowed. The latter set is more in the spirit of the Keynes-Wicksell paradigm.

It is interesting to compare these versions with their predecessors which had virtually no analysis of the flow market for real balances. As Fischer [1] has emphasized, the neoclassical models have suffered from their inability to explain short run price behavior, while the Keynes-Wicksell suffered from undesirable steady state properties. The present version of the neoclassical model still requires 'blips' in the price level in response to 'blips' in the money stock, but we are able to handle non-steady state behavior in a richer manner, allowing for stock disequilibrium over time. Our version of the Keynes-Wicksell model possesses more acceptable steady state properties than previous ones, inasmuch as a non-zero steady state rate of inflation no longer requires the invocation of a <u>deus ex machina</u> to distribute frustrations between investors and savers. Further, we are able to handle non steady state behavior in a more consistent manner.

Whichever version is used will depend on one's view of the world and upon the particular question being asked of the model--the important thing is that either is characterized by consistent stock-flow relationships.

The diagrammatic framework used here is not well suited for dynamic analysis--it would be more useful to work in the state variables space. However, Figure 2 clearly sets out what is going on in the model, and how the portfolio adjustment and goods market equilibrium interact to determine π and $\dot{k}$.

FOOTNOTES

[1] See Stein [4], pp. 98-99. Fischer [1] also constructs a model in which prices move in direct accordance to stock disequilibrium. This, however, is due to the stock adjustment term in the investment function.

[2] As Fischer [1] has made clear, it is the neoclassical assumption of perpetual stock equilibrium which leads to undesirable short run properties.

[3] Money is produced costlessly, and is distributed independently of present real balances as a transfer payment.

[4] The limitations of this assumption are well known--however it will suffice for our purposes except for the following qualification. In a model with stock disequilibrium, any semblance of maximizing behavior would indicate a savings rate varying with the stock disequilibrium. Of course the most satisfactory treatment would be a model of full dynamic optimization.

[5] It seems unreasonable that wealth holders would demonstrate the kind of price expectations specified and still express demand functions in terms of current variables. It would be more appropriate to have them demonstrate similar degrees of sophistication in the various markets. For example, the expectations function specified would be more consistent if demands were in terms of permanent incomes, and the present demands would be more consistent if combined with expectations like $\pi^e = \pi(t)$. However, for present purposes we follow the case set out by Stein.

[6]Note that when we are not in the steady state, it is $k^s + m^s$ that is constrained by $w = k + m$; where the interpretation of k^s is analogous to m^s. However, by the flow equilibrium postulate, $k^s - k = 0$ as does $m^s - m$, so that not only do the "short run excess demands" sum to zero, but each individual short run excess demand equals zero.

[7]Assume that all nominal balances are increased overnight, but no one expects it to happen again, and initially, no one knows that anyone else's balances have increased. Hence there are no changes in expectations over the level or rate of change of prices.

[8]To see this, consider $\dot{\hat{m}} = m(u - n - \pi) - (B/m)\cdot\dot{k}$, so for $\dot{\hat{m}} < 0$, we require $\pi > u - n - (B/m)\cdot\dot{k}$, which from (11) occurs whenever $\hat{m} > 0$; and similarly $\hat{m} < 0$ implies that $\dot{\hat{m}} > 0$. Hence we can draw the heavy black horizontal arrows in Figure 2.

[9]Stein's analysis implies $\hat{f} = q\cdot\hat{m}$, where we can solve for q from equation (8), which yields: $q = n - a\frac{m}{\hat{m}} - \frac{B(k, \pi^e)}{\hat{m}}\cdot\dot{k} + t$ so it is not correct to treat q as a constant, but in fact $q = q(\pi^e, u, m, k, \dot{k})$. Note that the limits of (14) and (15) as $h \to \infty$ are (11) and (12). This, in fact, gives us the most restrictive case of all with only steady state behavior possible.

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