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REGIONAL PUBLIC GOODS, SPILLOVERS AND OPTIMIZING FEDERAL SUBSIDIES

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competitive manner. We first consider the case in which there are no spillovers of benefits from one jurisdiction to another and assume that the government produces a Pareto optimal level of output of the public good for its resident population. The public goods are financed by proportional income taxes. It is assumed that these taxes do not affect the income-leisure choice.

There are ℓ_1 workers who are allocated between the private and public sectors in number L_1 and L_2 , respectively, by a competitive hiring process. There are k landlords whose land is allocated competitively between these two industries in the amounts K_1 and K_2 . The tax rate, q_1 , is defined as follows:

$$(101) \quad q_1 = \frac{P_1 R_1}{X_1 + P_1 R_1} .$$

If we write W_1 for the wage rate in province (1), P_{K_1} for the rental on land and Y_1 for the value of provincial output (in terms of the numeraire private good) then we have:

$$(102) \quad Y_1 = P_1 R_1 + X_1 = W_1 \ell_1 + P_{K_1} K_1 .$$

The share of the tax bill paid by workers will be $q_1 W_1 \ell_1$. The tax bill paid by each workers will be $q_1 W_1$ while the tax price per unit of public good will be $\pi_1^e = q_1 \frac{W_1}{R_1}$. By assumption that the production functions are linearly homogeneous we may write:

$$\begin{aligned}
 (103) \quad q_1 W_1 l_1 &= (W_1 l_1) \frac{P_1 R_1}{P_1 R_1 + X_1} \\
 &= \frac{\left(L_1 \frac{\partial X_1}{\partial L_1} + P_1 L_2 \frac{\partial R_1}{\partial L_2} \right) \left(L_2 \frac{\partial R_1}{\partial L_2} + K_2 \frac{\partial R_1}{\partial K_2} \right) P_1}{L_1 \frac{\partial X_1}{\partial L_1} + K_1 \frac{\partial X_1}{\partial K_1} + P_1 L_2 \frac{\partial R_1}{\partial L_2} + P_1 K_2 \frac{\partial R_1}{\partial K_2}} \\
 &= \frac{\left(\frac{1}{R_1} e_{X_1}^{L_1} + \frac{P_1}{X_1} e_{R_1}^{L_2} \right) \left(e_{R_1}^{L_2} + e_{R_1}^{K_2} \right) R_1 P_1}{\frac{1}{R_1} e_{X_1}^{L_1} + \frac{1}{R_1} e_{X_1}^{K_1} + \frac{P_1}{X_1} e_{R_1}^{L_2} + \frac{P_1}{X_1} e_{R_1}^{K_2}} \\
 &= q_1 e_{X_1}^{L_1} X_1 + q_1 e_{R_1}^{L_2} P_1 R_1 \\
 &= q_1 (1 - e_{X_1}^{K_1}) X_1 + q_1 (1 - e_{R_1}^{K_2}) P_1 R_1 .
 \end{aligned}$$

This shows that the tax bill paid by labour depends upon the income shares going to labour and land - where $e_{X_1}^{L_1}$ is the share of income in the private goods industry going to labour, and the other e terms are similarly defined. Let θ_l be the aggregate share going to labour and θ_k be the share going to land, so that we may write:

$$(104) \quad q_1 W_1 l_1 = q_1 \theta_l Y_1 = q_1 (1 - \theta_k) Y_1 ,$$

for taxation of labour. From this the tax price per worker may be written:

$$\begin{aligned}
 (105) \quad \pi_1^l &= q_1 \frac{W_1}{R_1} = q_1 \frac{\theta_l Y_1}{l_1 R_1} = q_1 \frac{(1 - \theta_k) Y_1}{l_1 R_1} . \\
 &= \frac{P_1 R_1}{Y_1} \cdot \frac{\theta_l Y_1}{l_1 R_1} = P_1 \frac{\theta_l}{l_1} ,
 \end{aligned}$$

which is the unit price of the public good multiplied by the income share per worker. By analogous reasoning the tax price per landlord, π_1^k , may be defined as

$$(101) \quad \pi_1^k = P_1 \frac{\theta_k}{k},$$

where k is the number of landlords. The proportional income tax here represents a form of price discrimination between the workers and the landlords.

Since the landlords are immobile their demand for residence will be completely inelastic with respect to the tax price (π_1^k) that they have to pay. Hence, k will be a fixed number in the analysis to follow. Also, it will be convenient to introduce other constants into the analysis. It will be assumed that the elasticity of substitution (in the aggregate production function) between labour and capital is unitary so that the income shares, θ_l and θ_k , are constant. This assumption enables us to write:

$$(107) \quad \pi_1^l l_1 + \pi_1^k k = P_1,$$

$$(108) \quad \pi_1^l l_1 = P_1 - P_1 \frac{\theta_k}{k} k = P_1 (1 - \theta_k) \\ = P_1 \theta_l.$$

Equation (108) now defines a supply of residence function analogous to that described in the preceding section. The difference here is that the relevant mobile population is defined in terms of workers. For a given P_1 the supply

curve of residence is again a rectangular hyperbola, but there is a significant difference in its position. In the present case the vertical asymptote appears at $N=k$ since (108) defines the supply price in terms of l_1 alone. In Figure 3 the vertical asymptote of the rectangular hyperbola was at $N=0$ but the supply price (π) was only defined for $N > k$. If $\frac{\theta_k}{k} > \frac{\theta_l}{l}$ then π_l^l will lie below π_l for a given R_l and N_l so that the supply price of residence will lie below the corresponding supply price for a benefit tax. Ceteris paribus, if $\pi_l^l < \pi_l$ then the non-benefit (proportional income) tax will attract more workers than will a benefit tax. Note that the demand for residence function will include not only the tax price as a variable but the wage rate as well:

$$(109) \quad l_1 = l_1(w_1, \pi_l^l).$$

This analysis suggests that workers will be attracted into a region by a switch from a benefit to a non-benefit system of taxation, where $\pi_l^l < \pi_l$ and that they will accept a lower wage rate, as a consequence. Clearly this represents a loss to society since workers would be responding to a tax price which is lower than the true opportunity cost of the public good.

Assume now that there are two provinces and that provinces (1) and (2) are initially in equilibrium with Pareto-optimal outputs of the public good, financed by benefit taxation. Now province (1) switches to the proportional

income tax system described above in which the supply price of residence is lower than before. The result will be to drive a wedge between the "true" supply price and the demand price of residence. The inefficiency here results from the fact that the two provinces have different tax systems. Note that the new tax price might just as easily have been above the benefit tax ($\pi_1^l > \pi_1$). In the situation described a welfare optimizing federal government might impose a tax on people moving from province (2) to (1) or an income surtax on the population in province (1), if either of these alternatives were feasible. Alternatively, it might want to subsidize the residents of province (2) where the public good is financed by a benefit tax.

Next consider a case where both provinces switch from a benefit system to a proportional income tax system. The supply prices of residence will now become:

$$(110) \quad \pi_1^l = P_1 \frac{\theta^1 l}{l_1}, \text{ and}$$

$$(111) \quad \pi_2^l = P_2 \frac{\theta^2 l}{l_2},$$

in the two provinces, where linear homogeneity and unitary elasticity of substitution are assumed of the aggregate production function in province (2) as well as in (1). If $\pi_1^l > \pi_1$ and $\pi_2^l < \pi_2$ then people will move from province (1), where the tax price has risen, to province (2), where it has fallen. If $\pi_1^l > \pi_1$ and $\pi_2^l > \pi_2$ or if $\pi_1^l < \pi_1$ and

$\pi_2^l < \pi_2$ then one cannot predict a priori what will happen.

If $\frac{\theta^1}{l_1} = \frac{\theta^2}{l_2}$ then induced mobility is not likely to be significant since the ratio $\frac{\pi_1}{\pi_2} = \frac{\pi_1^l}{\pi_2^l}$ for given R_1 and R_2 will be the same.

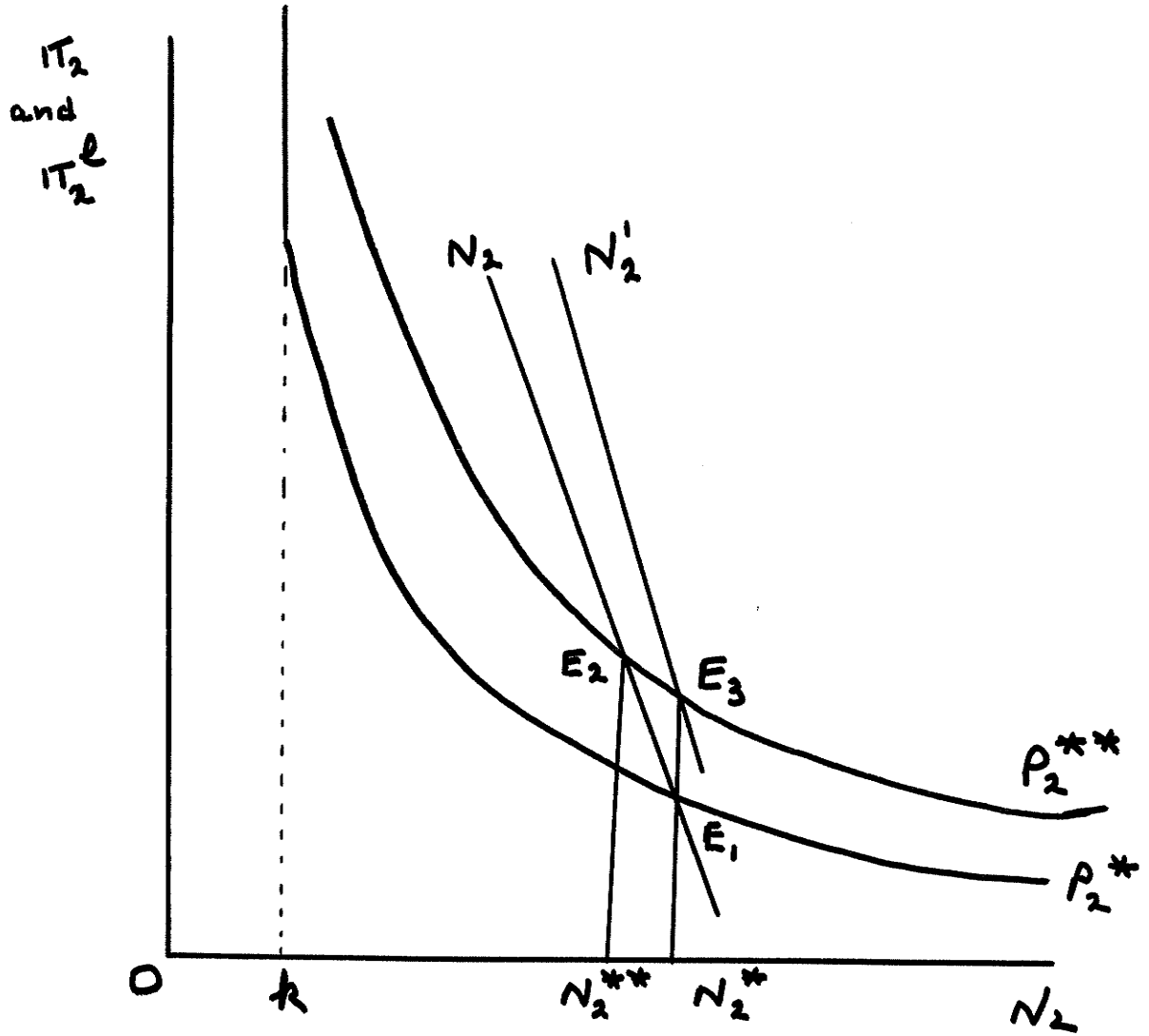
Let us consider a situation where $\pi_1^l > \pi_1$ and $\pi_2^l > \pi_2$ and where $\frac{\theta^1}{l_1} = \lambda \frac{\theta^2}{l_2}$ and $0 < \lambda < 1$. This means that

landowners in province (1) are bearing a larger share of the cost of the public good than those in province (2).

One solution here is for the federal government to pay a unit subsidy to province (2) of λP_2 . However, given our deus ex machina assumption that each province is producing a Pareto optimal level of the public good for the existing resident population then a conditional subsidy might lead to overproduction. The problem is that the government would be faced with the wrong price for the public good. This reasoning suggests that some kind of income subsidies should be paid to the workers of province (2).

Let us examine the problem here in terms of Figure 4, which is based upon quadrant I of Figure 3. The curve P_2^* represents the benefit tax supply price of residence while P_2^{**} represents the non-benefit tax supply price. The curve labelled N_2 is the demand for residence curve. The point E_1 with corresponding population N_2^* reflects the optimal (benefit tax) resident population while N_2^{**} represents the

Figure 4



(smaller) population associated with the non-benefit tax. Our analysis suggests that an income subsidy to workers could be used to shift the N_2 function from N_2 to N_2^I so that the resident population would be restored to N_2^* . This would return the marginal product of labour to its original position.

In the present model the direction of locational distortion depends upon the distributive shares and upon their constancy. A single province switching to a non-benefit system will have excessive in-migration, due to the non-benefit tax system, when the distributive share of each landlord exceeds that of each worker. When the individual distributive shares are equal then the non-benefit tax price will be equal to the benefit tax. If we drop the assumption that the elasticity of substitution in the aggregate production function is unitary then the supply of residence curves will no longer be rectangular hyperbolas. If the elasticity of substitution were greater than one then the P^{**} curve in Figure 4 would be flatter and would move closer to P_2^* as the labour force was reduced. This would be caused by an increase in the share going to land, as shown in (105).¹² It would be possible to present a taxonomy of locational distortion effects, depending upon distributive shares and elasticities of substitution in different provinces, but shortage of space does not permit. Our analysis suggests that where the elasticity of substitution is unity

then the locational effects of a proportional income tax system may be offset by paying subsidies to workers who live in provinces where the distributive share per worker is relatively high.

Given that each provincial government decides correctly on the optimal output of the local public good for domestic consumption then the introduction of benefit spillovers requires the payment of conditional grants from the central government. These grants may cause changes in N_1 and N_2 in (83) and (84) but as long as the grants are adjusted accordingly this presents no problem. However, these grants may lead to a redistribution of income that will change the optimal lump sum subsidies.

The payment of conditional subsidies to adjust for direct spillovers will affect both the benefit tax price as well as the non-benefit tax price. As a result the supply price of residence curves corresponding to P^* and P^{**} in Figure 4 will be shifted. It seems unlikely that the distance between the benefit and non-benefit tax price will remain the same as the equilibrium of the system is displaced by a conditional subsidy. Hence, it is to be expected that the optimizing lump sum subsidies for adjustment of indirect spillovers will depend upon the conditional subsidies and their effects. By the same token the optimal conditional subsidies will depend upon the allocation of population between the provinces and therefore upon the

optimizing lump sum subsidies. This points out the crucial interdependence that must exist in a scheme of optimizing subsidies and adds emphasis to the importance of setting subsidy policy in a general equilibrium framework.

V. Conclusions

The purpose of this paper has been to develop a theoretical framework within which to examine the role of federal subsidies. These subsidies are of increasing importance in most federations, and particularly so in Canada. The literature on federal grants has made it clear that the primary economic rationale for these payments is to be found in the theory of externalities. In the preceding sections two types of external effects have been examined and these have been classified as direct and indirect spillovers. The former of these spillover types has been considered in the context of reciprocal benefit spillovers emanating from the provision of regional public goods. These direct spillovers have been viewed in the context of both benefit and non-benefit regional tax systems and it has been shown that in each case federal (conditional) subsidies may be used to achieve Pareto optimality. These subsidies will depend upon technical spillover ratios, comparative costs, and the closeness of substitution between the public goods.

Indirect spillovers are created as the by-product of a non-benefit taxation system in the form of locational

distortion of worker-consumer choice. In the model of the preceding section the distortion depends upon the existence of differences in the individual income share going to workers and landlords. Federal lump-sum subsidies to workers may be used as a policy instrument to improve resource allocation.

The following equations will be used to summarize the basic two-province model that has been used to convey most of the analysis in the paper:

$$(112) \quad \zeta_{11}N_1s_1 + \zeta_{12}N_2s_2 = T_1 ,$$

$$(113) \quad \zeta_{21}N_1s_1 + \zeta_{22}N_2s_2 = T_2 ,$$

$$(114) \quad N_1 = N_1(\pi_1, \pi_2, w_1, w_2) ,$$

$$(115) \quad N_2 = N_2(\pi_1, \pi_2, w_1, w_2) ,$$

$$(116) \quad \pi_1 = \pi_1(N_1, T_1, \theta_1^1) , \text{ and}$$

$$(117) \quad \pi_2 = \pi_2(N_2, T_2, \theta_2^2) .$$

These equations are meant to summarize some of the analysis above but are not intended as a complete general equilibrium model. Equations (112) and (113) present the optimality conditions for regional public goods with reciprocal externality benefits. They show that the output levels of the public goods R_1 and R_2 will have to be adjusted for the spillover and that the extent of the adjustment will depend upon the

allocation of the national population (N_0) between the two regions. Equations (114) and (115) are the demand functions which show that the demand for residence depends upon the provincial tax prices and wage rates. Equations (116) and (117) are to be interpreted as the tax price and supply price of residence equations. For a non-benefit tax system the tax price will depend upon the distributive shares as well as upon the cost of the public good and the size of the resident population. A significant implication of the analysis presented here is the important role of general equilibrium analysis in examining externality problems. The federal subsidy instruments of "externality policy" are interdependent. Price subsidies and lump sum (wage) subsidies must be determined simultaneously.

List of Symbols Used

- R_j^i Number of units of j th good produced by j th province.
- R_{jk}^i Number of units of j th good produced by i th province and available for consumption by k th region.
- α_{jk}^i Proportion of benefits from public good j produced by province i becoming available in province k .
- X Units of private good.
- U_i Social Welfare function of province i .
- F_i Transformation function for province i .
- S_i Summed marginal rates of substitution of public for private goods in province i .
- T_i Marginal rate of transformation of private into public goods.
- Z_i Subsidy per unit of R_j^i paid by central government to province i .
- β_{jk} Measure of closeness of substitution between R_j and R_k in preferences of province k .
- c_{ij} Conversion factor for transforming units of spillin of R_j into equivalent units of own public good R_j .
- L_0 Fixed number of workers in province (1) and (2) together.
- L_1 Number of workers in X industry in province (1).
- L_2 Number of workers in R industry in province (1).
- L_3 Number of workers in X industry in province (2).
- L_4 Number of workers in R industry in province 2.
- l_1 L_1+L_2
- l_2 L_3+L_4
- K_0 Amount of land in province (1).
- K_1 Amount of land in province (1) used by X industry.
- K_2 Amount of land in province (1) used by R industry.

T_0 Amount of land in province (2).

T_1 Amount of land in province (2) used by X industry.

T_2 Amount of land in province (2) used by R industry.

k Number of landlords in province (1).

t Number of landlords in province (2).

$$N_1 = l_1 + k$$

$$N_2 = l_2 + t$$

$$N_0 = N_1 + N_2$$

$P_{R1} = P_1$ Price of public good in province (1).

$P_{R2} = P_2$ Price of public good in province (2).

$P_X = I$ Price of private goods

π_i Tax price of public good in region i .

π_{ij} Tax price paid by resident of province j for spillin from province i .

$E(N)$ Excess demand (tax) price of residence.

E_{RP} Elasticity of demand for R with respect to P .

$E_{R\pi}$ Elasticity of demand for R with respect to π .

$E_{N\pi}$ Elasticity of demand for residence with respect to π .

W_i Wage rate in province i .

q_i Proportional income tax rate in province i .

Y_i Value of total output in i .

e_{X1}^L Share of X industry output going to labour in province i .

$\theta_l^1 = \theta l_1$ Share of total provincial output in province (1) going to labour.

π^l Income tax price of public goods to mobile population in province (1).

Footnotes

1. See, for example, Breton (1965), Bédard (1967), Bird (1970), Boehne (1969), Break (1967), Harlow (1967), Head (1967), Hirsch and Marcus (1969), Hoffman (1969), Michas (1969), Oates (1968), Pauly (1970), and Williams (1966).
2. See, for example, Gramlich (1968, 1969), Henderson (1968), and Michas (1969).
3. For a more complete treatment of the direct spillover rationale for grants see Break (1967), Breton (1965) and Weisbrod (1964).
4. See Vardy (1971b).
5. A vigorous controversy developed in the early fifties between Anthony Scott (1950, 1952a, 1952b) and James Buchanan (1950, 1952). Scott argued, against Buchanan, that equalizing transfers to provinces would tend to reduce resource mobility and therefore would inhibit the movement of factors of production into areas where they would be more productive.
6. The analysis of reciprocal externalities in Williams (1966) is based on rivalrous benefits.
7. See Vardy (1971a) for a more detailed derivation of OF_{II} .
8. See Vardy (1971b) for a discussion of non-linear subsidies.
9. See Lange (1938) and Kohler (1966).
10. More precisely the function is a rectangular hyperbola for $N \geq k$ and is not defined for $N < k$ since the k landlords do not move. The vertical asymptote for the rectangular hyperbola is, however, at $N=0$.
11. This may be proved as follows for a given P_1 :

$$F_1 \quad l\pi_1^l + k\pi_1^k = P_1 ,$$

$$F_2 \quad \frac{l}{N_1}\pi_1^l + \frac{k}{N_1}\pi_1^k = \frac{P_1}{N_1} = \pi_1 ,$$

$$F_3 \quad \frac{lP_1}{N_1} \frac{\theta l}{l} + \frac{k}{N_1} P_1 \frac{\theta k}{k} = \frac{P_1}{N_1} = \pi ,$$

$$F_4 \quad \pi_l^l = P_l \frac{\theta_l}{l} \stackrel{\geq}{<} P_l \frac{l}{l+k} \frac{\theta_l}{l} + P_l \frac{k}{l+k} \frac{\theta_k}{k} = \pi, \text{ and}$$

$$F_5 \quad \frac{\theta_l}{l} \stackrel{\geq}{<} \frac{\theta_k}{k} .$$

12. A flatter supply curve of residence would make the stability conditions on the demand for residence less stringent.

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