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LINEAR AND NON-LINEAR SUBSIDIES

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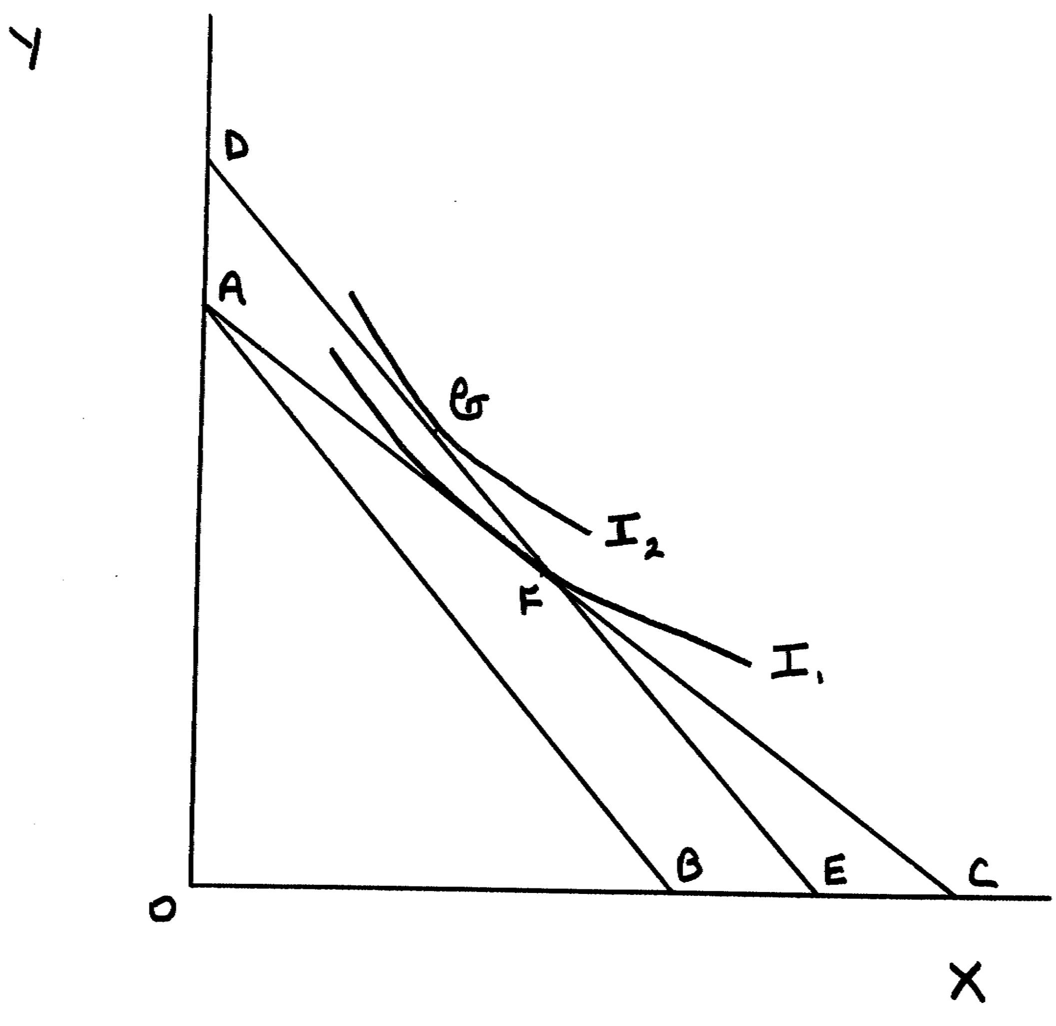
I. Introduction

The analysis of taxation in traditional public finance has been mainly concerned with the problems associated with the "non-neutrality" of the tax system. In this body of analysis the emphasis has been upon the "excess burden", or distortion of economic choices, and the thrust of taxation theory has been to show how a tax system may be devised to minimize distortions. But in spite of this concern with excess burden there also exists a large literature on the use of taxes to correct for external diseconomies and also to protect infant industries. In this latter literature the focus has been upon achieving appropriate adjustments in consumer and producer choices in order to improve social welfare. The objective is to alter the choice parameters of individual decision makers and this is typically done by imposing a tax which is (linearly) related to some economic activity. In the case of the progressive income tax the amount of the tax is (non-linearly) related to the amount of factor services sold and the prices of these.

This paper focuses upon the theory of subsidies and here the emphasis is not symmetric with the analysis of taxation. While there exists a strong justification for attention given to the financing of government expenditure at minimal distortion cost there are few governments confronted with the problem of deciding how to get rid of excess revenue acquired at zero cost. Subsidies may be given to low income people in order to achieve a redistribution of income or they may be given to stimulate some production or consumption activity. Normally a subsidy will produce both redistributive and stimulative effects in its initial impact as well as in its final incidence. But it is possible to design subsidy schemes according to the effects that are to be achieved. While specific subsidies given to improve the housing conditions of the poor may effect a redistribution of income the recipients would be better off with a lump sum payment and no strings attached. Figure I reproduces the conventional diagram of price theory to illustrate this well-known theorem of subsidy theory. If the number of units of housing is measured along the X-axis and expenditures on other goods along the Y axis then a unit subsidy of BC/OC of the cost will place our representative subsidy recipient (whose indifference map is shown in Figure I) on indifference curve I_1. A lump sum subsidy of the same amount (i.e. AD) would place him on a higher indifference curve. The conclusion of this analysis is that the lump sum subsidy is to be preferred unless there are external effects associated with the consumption of housing which do not result from non-housing expenditures.[1] If the consumption of housing by some particular group in society does create external benefits for the other members of

Figure 1



society then this in itself constitutes a distinct reason why the housing consumption of this group should perhaps be subsidized. While a lump sum subsidy would significantly increase housing consumption if its income elasticity were high, this type of subsidy is unlikely to be as effective as a subsidy which reduces the price of housing to the recipient group.

The rest of the paper will examine the effect of various subsidy schemes when the object is to increase the level of consumption of a particular good. The analysis is applied as well to the case of negative income taxes where the objective is to achieve a given minimum level of income without creating a negative effect on work effort. This analysis is in keeping with the rest of the paper since withdrawal of effort by the subsidized group may be viewed as a social cost to the donor group. In the main part of the paper the context will be one in which an agency of social harmony and optimization (e.g. the federal government in a benevolent federation) is attempting to raise the consumption (and production) of a good with which certain external benefits (e.g. benefits to non-members of the provincial jurisdiction providing the good for its inhabitants) are associated.[2] The basic model is one in which one group is attempting to modify the consumption decisions of another group or individual. Other methods than those discussed are available in certain contexts. These include the enactment of legislation designed to modify the behaviour of designated members of society.[3] In most practical situations such action would be considered undue exploitation of one group by another or as unduly authoritarian. Furthermore, in some situations, as in the case of a federation where spheres of constitutional autonomy are clearly demarcated, legislative action would be _ultra vires_. Another method is the use of vouchers and Pauly (1970) has shown that in many realistic situations these are tantamount to income transfers.

In section II below we shall present an analysis of uniform per unit price subsidies. Section III will take up the case of negative income taxes and will discuss non-linear subsidies of both "progressive" and "regressive" varieties. These will be compared with proportional, or linear subsidies, and it will be shown that the intramarginal surplus associated with each of these subsidy schemes will normally differ considerably. Section IV will examine the efficiency of linear and non-linear subsidy schemes in achieving a given target level of consumption. In this section the analysis will deal with the general problem of devising subsidies to cope with external effects. Section V will apply the subsidy analysis to provincial taxation where external effects associated with specific taxes lead to the creation of a "corrective" subsidy system. Section VI concludes the paper.

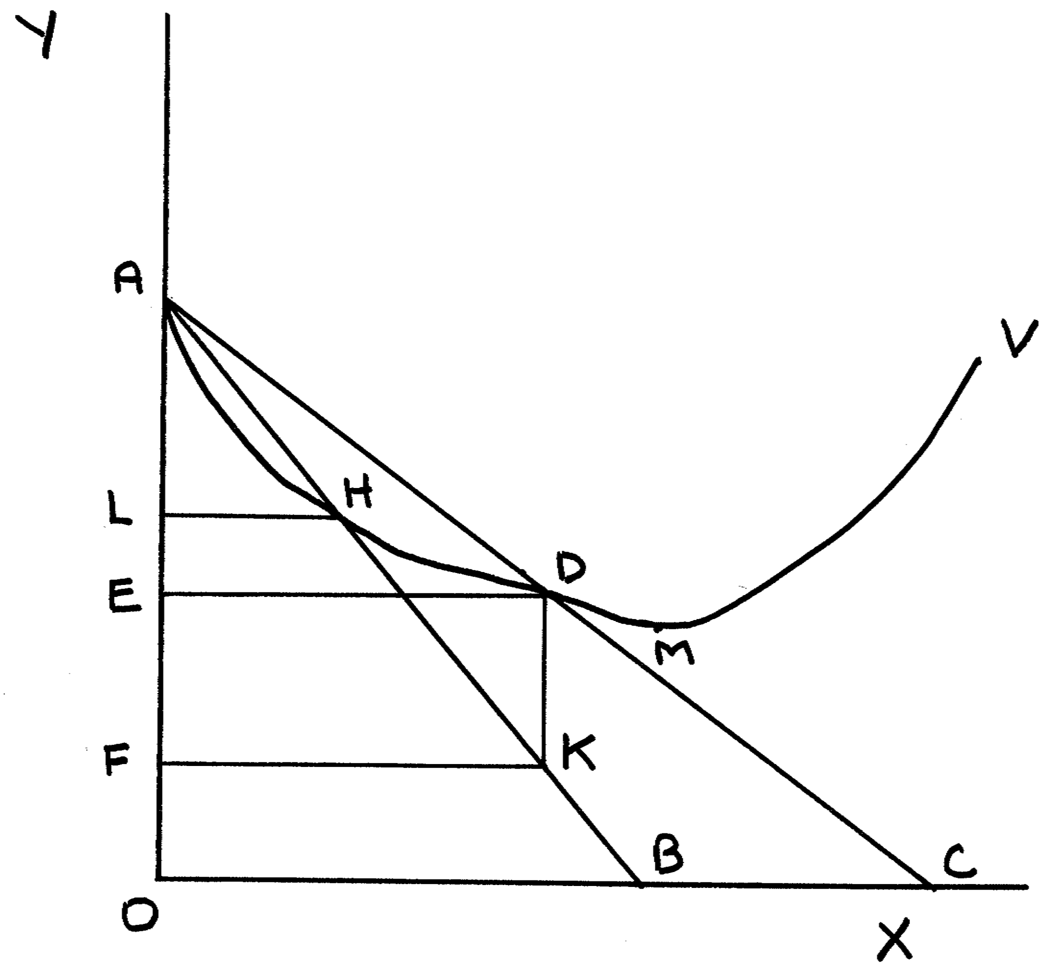
II. Uniform Unit Subsidies

In this section the discussion will center upon price subsidies, where the subsidy per unit is constant. This strategy is an alternative to that of an income subsidy and it has the effect of inducing a greater response. Consider Figure I and note that with true market prices reflected by the slope of AB then a price subsidy whose cost is identical to that of the income subsidy AD evokes a larger increase in consumption of X. The subsidy scheme associated with budget line AC produces both an income and a substitution effect that are normally favourable. The income subsidy scheme which shifts AB out to DE has only an income effect on consumption of X. Hence, price reduction schemes are likely to be more effective in achieving increased consumption.

In the subsidy analysis which follows it will be assumed that the subsidized individual has a "well-behaved" preference function. It is assumed that there are two consumption goods, or that a multitude of these goods can be broken down into two groups. The good (or group of goods) designated by X and also quantified as X units of X will be the externality generating (or merit)[4] good. The good Y will be in essence the expenditure on all other goods, measured in terms of some numeraire good. It is assumed for simplicity that the opportunity cost of X is constant in terms of Y. The preference function of one representative individual is "connected" and the connected ordering is continuous. Continuity of individual indifference curves facilitates the use of simple equalities in defining positions of consumer equilibrium (except for corner optima). The consumer is not satiated with either X or Y and his preference ordering is assumed to be transitive and his indifference curves to be strictly convex. In discussing group behaviour it will be assumed that the group preference function is also "well-behaved" in terms of the assumptions above.

The representative individual has his indifference map portrayed in Figure 2. Again AB represents the real cost of X in terms of Y and AV is the price consumption curve, which enables us to find the appropriate subsidy that will effect some desired level (call it X*) of X. Note that for small price reductions along negatively-sloped segments of AV the demand is elastic and own (i.e. that of subsidy recipient) expenditure increases. Similarly, upward-sloping segments of the price consumption curve are inelastic to price and price reductions elicit corresponding reductions in own expenditures. In Figure 2 a subsidy that reduces the price of X (p_x) from AO/OB to AO/OC also enables the consumer to move from position H to D. Own expenditure on X rises from AL to AE and the

Figure 2



size of the subsidy at D is given by DK. It is clear from the diagram that the size of the subsidy will increase continuously for induced increases in X while own expenditure on X will increase up to point M (where price elasticity is unitary), beyond which it will fall. In what follows AF will be representative of total expenditure on X (i.e. AF=E=rX at point D), AE will be representative of own expenditure (AE=Z=pX at D) while DK represents the external subsidy (DK=S=sX).

If r is used to designate the real unit cost of X (i.e. r=AO/OB) and s is the external subsidy per unit then own unit price is p and p=r-s. The following are the corresponding totals:

(1) $Z(=pX)=rX(=E)-sX(=S).$

The total subsidy cost is equal to real cost minus own expenditure:

(2) $S=E-Z,$

and the marginal subsidy cost of X is given by:

(3) $\frac{dS}{dX} = r - \frac{dZ}{dX}$.

It is well known that

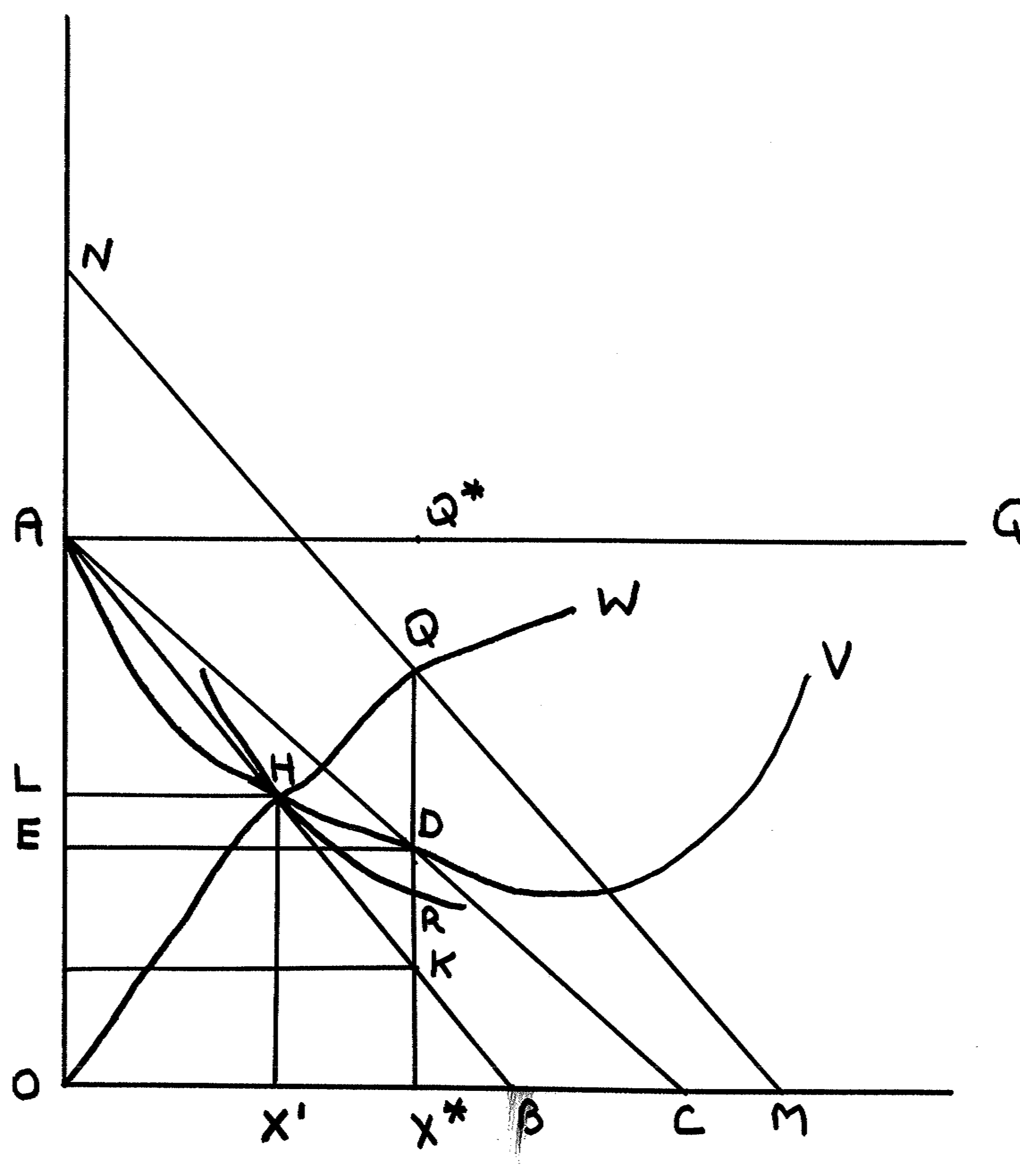
(4) $\frac{dZ}{dX} = p + X\frac{dp}{dX} = p(1-{}^{1}/e),$

where e is own price elasticity of demand. The term $\frac{dZ}{dX}$ in (4) will be negative when e is less than one and positive when e exceeds unity. From (3) this means that the marginal subsidy cost of expanding consumption of X (beyond the level of X associated with a zero subsidy) will be higher, the smaller the elasticity of demand. When demand is elastic the external subsidy cost of increasing X will be relatively low.

Let it now be assumed that the target level of X is given by X*. Figure 3 enables us to compute the uniform per unit subsidy that will achieve consumption of X*. It will be helpful if we erect a vertical line above X*. The intersection of this line with AV enables us to compute the new price line AC and hence the instrumental rate of subsidy BC/OC. The total subsidy is shown by DK. If an income subsidy were used instead of the price subsidy then the subsidy would be QK. This subsidy is found from the intersection of the vertical above X* with the income consumption line OW.

In the case of the price subsidy the uniformity of the per unit subsidy implies the existence of an intramarginal surplus. If we use the Hicksian compensating consumer's surplus then the amount DR is a measure of the intramarginal subsidy associated with the price reduction.[5] The corresponding subsidy in the case of the

Figure 3



income subsidy is RQ. This means that only part of the subsidy (i.e. RK instead of DK) is required as a minimal inducement to the consumer. Our representative consumer would be just as happy at R as he is at the zero-subsidy point H. This suggests that there may be alternative subsidy schemes to those of the uniform type, associated with the price consumption line, or offer curve AV. The next section develops some alternatives in the context of negative income taxes.

III. Negative Income Taxes

In this section we shall discuss the effects of negative income taxes in the context of the general theory of subsidies. The main thrust of most negative income tax proposals has been to reduce poverty by direct redistribution. The aim has been to provide each family with some minimum amount of income without seriously impairing the supply of work effort.[6] In the framework of our analysis above this means that society values the work effort of the subsidized group. It is not the purpose of the present paper to rationalize this concern with work effort. It may be argued by some that disincentive effects of anti-poverty programs upon work effort are not undesirable, even when they are of large magnitude. It will be assumed here that anti-poverty programs should be designed so as to interfere as little as possible with the income-leisure choice. For example, it is assumed that an anti-poverty program ought not to lead to a corner solution where all labour services of subsidy recipients are withdrawn.

Since our attention here is confined to direct redistribution we turn now to some of the possible schemes. First, consider a scheme that will be labelled S_1, where the individual receives a fixed sum s_1, provided that his earned income is equal to zero. Let s represent the amount of the subsidy here. If earned income is positive then the individual receives nothing. If we let y_e denote earned income then it follows that:

(5) $y = y_e + s$,

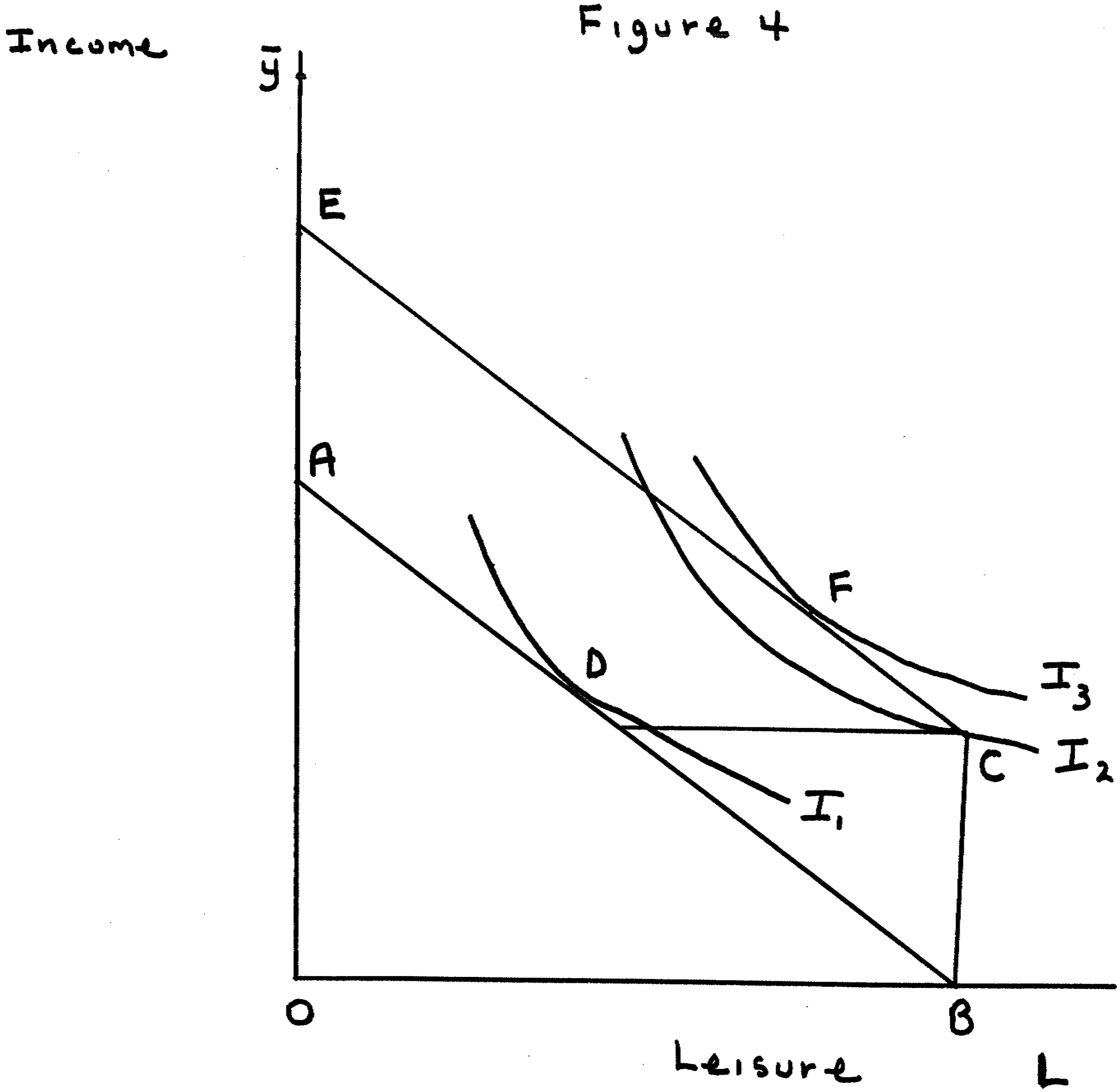
(6) $s = s_1$ if $y_e = o$ and

(7) $s = o$ if $y_e > o$,

where y is total income, earned as well as unearned.

The choice problem confronting our representative individual is illustrated in Figure 4. Total income (y) is measured along the ordinate of Figure 4, while the abscissa measures leisure time. Point B corresponds to the amount of time (OB=K) to be allocated between leisure and work, and its measurement unit (e.g. hours, of which

Figure 4



there are 24 to be allocated per day) defines the decision period. It is assumed that work is available but that feasible earnings at the going wage rate (w=AO/OB) are very low in relation to family expenditure needs. If we let K designate the amount of time to be allocated (e.g. 24 hours) and if L denotes the amount of leisure time chosen then earned income is given by

(8) $y_e = w(K-L)$.

In Figure 4, AB is the earnings opportunity line and its slope is simply the wage rate. Point C corresponds to the subsidy scheme S_1, where $s=s_1$(=BC) for y_e=o. At point C the indifference curve I_2 is higher than I_1, which is tangent to AB at point D. Hence, our representative individual, faced with the alternatives of either working or drawing subsidies under scheme S_1 will opt for the latter and will choose position C.

Now consider subsidy scheme S_2, which differs from S_1 in that S_2 provides that every dollar earned is offset against anti-poverty payments:

(9) $s = s_1 - y_e$ for $s_1 \geq y_e$

The opportunity set is enclosed by OBCDA under S_2 and our representative man (or woman) will continue to opt for the corner solution at C. This scheme is fairly characteristic of most welfare schemes in North America and it is clear that the work disincentive effects are likely to be quite severe.

Scheme S_3 is another simple scheme but here the subsidy is fixed at some level, say s_1(=CB) and the size of the subsidy remains the same as long as income remains below some fixed level, say $\bar{y}$. This scheme is illustrated by CE, where

$$(10) \quad y = s_1 + w(K-L),$$

for $y < \bar{y}$. The point of consumer equilibrium associated with this scheme is at point F.

The remaining subsidy schemes discussed refer to subsidy systems where the subsidy supplement to income is offset by a tax on earned income. In each case a lump sum subsidy is paid but earned income is taxed, and it is the nature of the taxation system that distinguishes the remaining schemes. In the analysis that follows wage subsidies are not discussed, due to shortage of space, but their treatment is fully symmetrical with the following analysis.[7]

For every dollar earned under subsidy scheme S_4 the amount of the subsidy is reduced by some amount less than a dollar and so there is effectively a tax on earned income. Scheme S_4 is characterized by a proportional tax of 100b per cent of earned income $(o < b < 1)$:

(11) $y = (1-b)y_e + s_1$, and

(12) $y = s_1 + (1-b)w(K-L)$

The subsidy and the tax both disappear when the amount of the tax reaches the subsidy level:

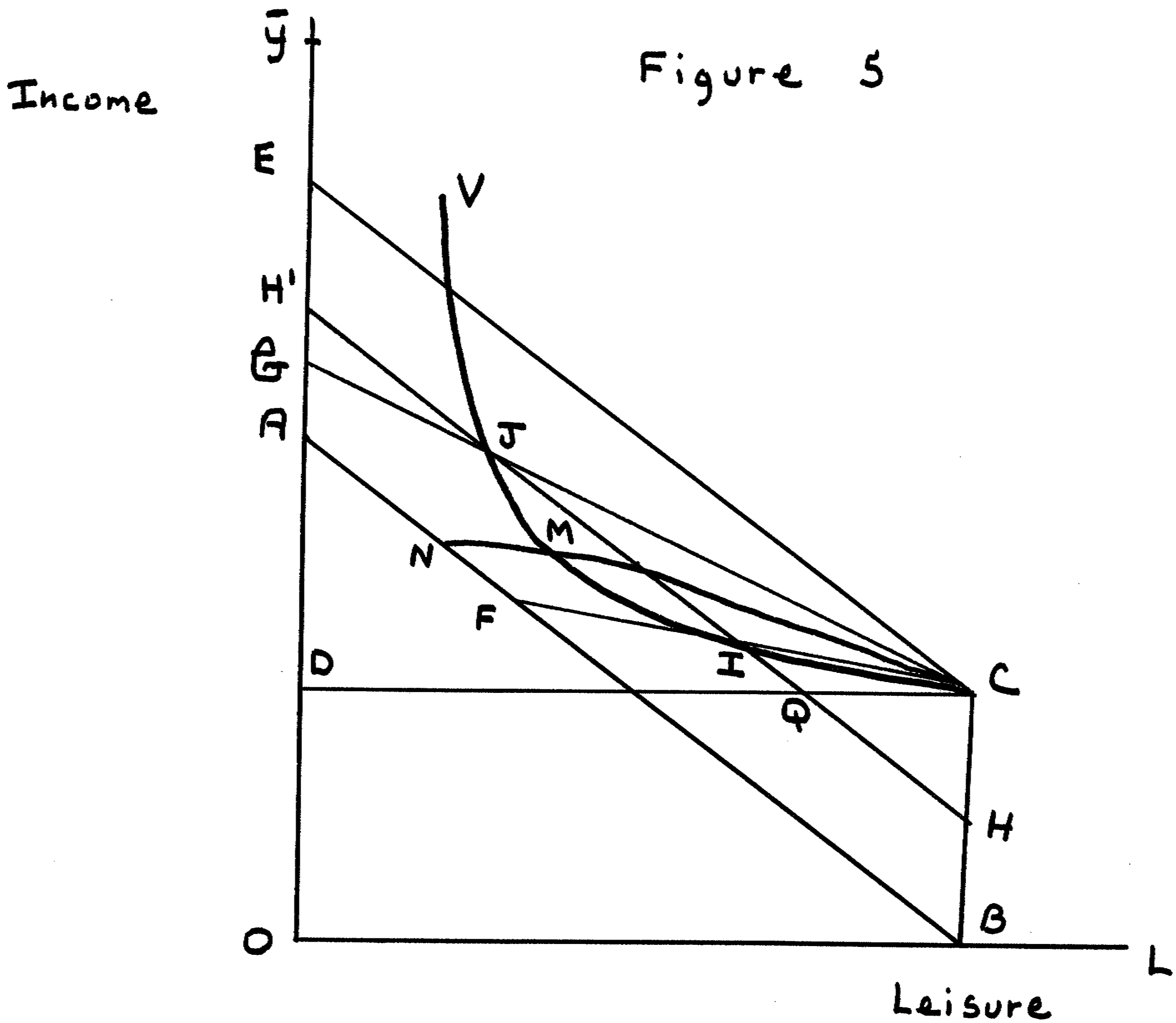
(13) $s = o$ and

(14) $y = y_e$ when $by_e \geq s_1$.

Hence $y_e' = s_1/b$ is the "breakeven" level of y_e since above that level the subsidy vanishes.

Figure 5 illustrates this scheme for a number of specific values of b. Consider the value of b associated with the line CF. As before CB represents the subsidy when earned income is zero. But now earned income is taxed up to point F and the slope of FC is the wage rate multiplied by (1-b). This scheme would have the effect of changing the opportunity set of consumer-worker choice from OAB to OBCFA. At point F the value of the subsidy would be zero and it is assumed for simplicity in what follows that no income taxes (of the conventional sort) are paid unless $y > \bar{y}$.

Let us focus our attention upon this scheme S_4 and examine the behaviour of our representative consumer as the rate of tax is varied from b=1 to b=0. In Figure 5



$\bar{y}$
Figure 5
Income
E
V
H'
G
A
J
M
N
F
I
D
C
Q
H
B
O
L
Leisure

a geometric rendition of this variation is given by rotating the budget line around point C from position CD (corresponding to b=1) to CE (corresponding to b=0). It will be recalled that these positions are representative of schemes S_2 and S_3. Let it be assumed that CV is the wage-work line, or offer curve for our representative worker, and that it is derived by pivoting a line like CF around point C. Offer curve CV is the locus of points of tangency of individual indifference curves with budget lines that come out of point C. These budget lines are associated with different tax rates on the income subsidy. ($s=s_1-by_e$ for $y_e<s_1/b$).[8]

In order to evaluate different subsidy systems, associated with different values of b, we construct an equal subsidy line HH', parallel to AB and passing through point I, the point of worker equilibrium along AFC. This equal subsidy line intersects CV at I and J and enables us to construct CJG, which we shall now proceed to compare with CIF. Corresponding to each of CF and CG there is a tax rate, call these rates b_i and b_j respectively. It is clear from the diagram that b_i exceeds b_j, in spite of the fact that the cost of the anti-poverty program is the same for these two tax rates, since the equilibrium points I and J both lie upon the equal subsidy line HH'. While b_j reflects a more generous anti-poverty program it costs no more to operate than the subsidy associated with b_i, and it results in a higher level of work effort. Furthermore, it makes our representative individual happier since it places him on a higher indifference curve. Clearly, the choice of the optimal value for b is no trivial matter and in designing an anti-poverty program it is important to choose the correct negative tax rate. If a negative income tax is set up with an ungenerously high tax rate then work effort may be unnecessarily reduced. Note that the analysis is inapplicable to a heterogeneous group of individuals and that no attempt is made to suggest policy implications for a non-homogeneous group.

The next scheme to be discussed is S_5 and this scheme is similar to S_4, except that here the tax depends upon the level of income, and is progressive; $b=b(y_e)$ and $\frac{db}{dy_e} > o$ for $y_e < s_l/b$. The purpose of our discussion here is to show that S_5 is likely to be inferior to S_4 in most significant respects and that a proportional negative income tax is to be preferred.

The next stage of our analysis is to find the characteristics of a progressive rate subsidy system and to compare these with those of a proportional system. To do this we look at progressive systems whose subsidy cost, in respect to our representative individual, is equal to

BH(=AH′). This makes possible a comparison of these with the subsidies associated with FC and CG (our two representative S_4 variety subsidies). The analysis will focus upon progressive rates where the rate is a continuously increasing function of earned income. It will be shown that the point of equilibrium for any progressive system with subsidy BH must be between $\underline{I}$ and J. Most significantly it must lie to the right of J and on a lower indifference curve.

The equation for the budget line may be written:

(15) $y = s_1+(1-b)w(K-L), \quad \text{for } w(K-L) < s_1/b,$ and

(16) $y = w(K-L) \quad \text{for } \bar{y} \geq w(K-L) \geq s_1/b$[9]

These equations apply to both progressive and proportional schemes, with the proviso in the former that b is a variable function of income. From (15) we may find the slope of the budget line for the relevant case where $y_e < s_1/b$:

(17) $\dfrac{dy}{dL} = -(1-b)w,$

for S_4, the proportional scheme, where b is a constant, and

(18) $$\frac{dy}{dL} = -(1-b)w - \frac{db}{dy_e}\frac{dy_e}{dL}w(K-L)$$

$$= -(1-b)w+b'w^2(K-L)$$

for the progressive scheme.

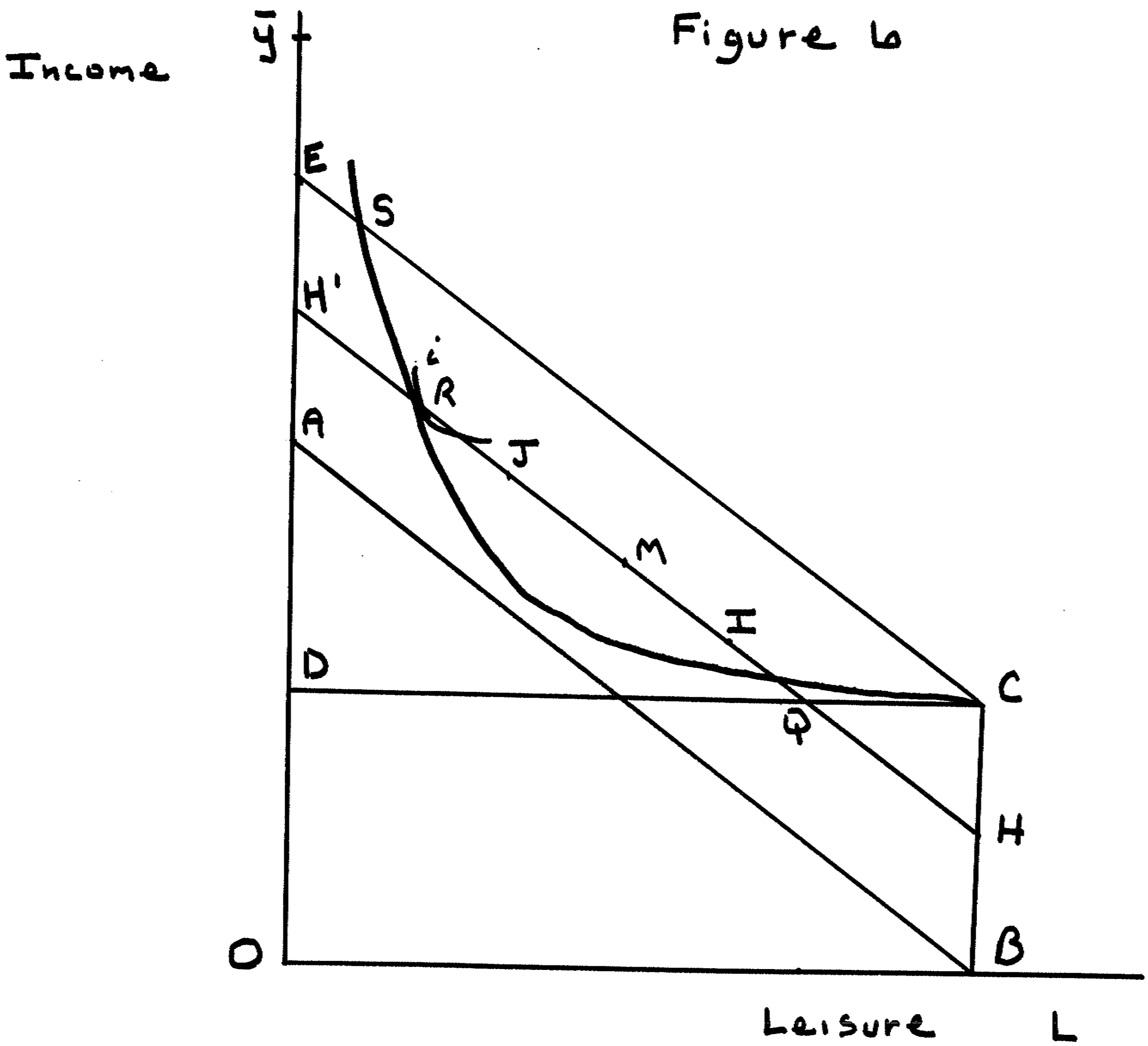
If we now take some point on CV (I, for example) at which some indifference curve is tangent to a proportional tax line then it must be true, from (17) and (18), that any progressive tax line (not shown) passing through that point must be flatter than the indifference curve. Note that at I earned income and work effort will be the same for both the proportional and progressive schemes as will the size of the subsidy. Since $b(y_e)$ is defined as the average tax rate then the b's appearing in (17) and (18) take on the same value at the same point. Furthermore, the second term on the right hand side of the equality sign in (18) is positive. Hence, (17) and (18) prove that the (non-linear) budget line corresponding to any progressive negative tax system will be flatter than the budget line associated with a proportional tax for a given subsidy level. This means that every possible progressive tax which starts at b=o, at point C in Figure 5, will have a budget line that intersects some linear budget line along CV. No point of tangency between an indifference curve and a progressive tax budget line will exist along CV since such budget lines will always be flatter than the indifference curves. Because all indifference curves are assumed to be strictly convex to the origin this means all equilibrium points for potential progressive rate systems must lie to the right of CV. A progressive system that involves a subsidy of HB has therefore to be sought along IJ (which is the only part of HH' that lies to the right of CV).

Let us assume that the progressive system illustrated by budget line CMNA results in an equilibrium position at M. This provides a basis for comparison with the proportional systems shown by CF and CJ. At the equilibrium points I, M, and J the negative income tax payments to our representative individual are given by HB. The point to be made here is that while the progressive tax budget line involves a subsidy of HB it produces a lower level of consumer satisfaction and a smaller supply of work effort than does the proportional tax line CJG.

This section will conclude with a brief discussion of subsidy scheme S_6, in which a subsidy s_1 is combined with a regressive tax on earned income. Richard Perlman (1968) has suggested a negative income tax plan with regressive features. Assume that our regressive scheme is the same as S_4 and S_5, except that $b=b(y_e)$ and $b'(y_e) < 0$. On the first dollar of earned income the tax rate is 100 per cent but this rate falls as work effort and earned income increase.

The budget line associated with a progressive scheme will be concave to the leisure axis of Figure 5. By analogous reasoning budget lines derived from regressive tax systems will be convex. It will be assumed in what follows that the convexity of indifference curves exceeds that of the budget lines that are considered here in order to preserve the uniqueness of equilibrium. Equilibrium in the case of regressive systems will be found only to the left of CV and this result is due to the fact that b' in (18) is negative for subsidy schemes of the S_6 variety. If we concentrate, as above, on subsidy schemes which transfer an amount HB to our representative worker then the segments $\bar{I}Q$ and $H'J$ of HH' are possible candidates. Equilibria along $\bar{I}Q$ will be inferior to those to the left of $\bar{I}$ along $\bar{I}J$ (i.e. S_4 type systems). A regressive tax equilibrium solution along segment $H'J$ at R is illustrated by CS in Figure 6. As it is drawn, position R lies on a higher indifference curve than does point J, although this will not be true in general for regressive type equilibrium positions along $H'J$. It will, however, be true that such S_6-type equilibria will be associated with a higher level of work effort per unit of subsidy than at any equilibrium position along JQ whether the scheme be of the S_4, S_5 or S_6 variety. Note that while a regressive tax scheme achieves a higher level of work effort, or a larger work effort per unit of subsidy, it may do so at the sacrifice of worker welfare in comparison with efficient proportional schemes. There is no reason to believe that a regressive-type budget line which moves the worker to some position of equilibrium

Figure 6



to the left of J, along $H'J$, will move him to a higher indifference curve than the one passing through point J. Since the work effort per unit of subsidy is higher along $H'J$ (for S_6) then this means that the intramarginal surplus per time unit of work will be smaller.

A brief comment will suffice to conclude this section. It is clear that the utility function of the policy maker would have to be known before it is possible to distinguish any one of these six schemes as being most efficient. One may have reason to prefer position J (when the cost of the subsidy scheme is constrained not to exceed HB) and its associated proportional offset (with constant b) to the income subsidy, over positions like R in Figure 6. A possible basis for such a preference is that information about the worker's supply curve of labour would be adequate in order to implement the proportional scheme. The non-linear subsidy schemes (e.g. S_5 and S_6) require much more information about preferences and would be more difficult to implement.

In section IV the analysis of this chapter will be applied to the problem discussed in section II.

IV. Non-linear Subsidies

In section II the problem discussed was that of subsidizing an activity, not to raise the income of the recipient but to increase the activity level. The motivation given for this was that some external effect results from consumption of X. The analysis of this section will apply non-linear subsidies to the problem of increasing consumption of X from the independently chosen level X^{I} to the target level X^{*}.

Let us denote the consumer's income by W, and the real cost of X by r per unit. As before the price of y will be one and p will be the price of X paid by the consumer, while s is the unit subsidy ($p=r-s$). The budget line may now be written:

(19) $W = Y+pX$, or

(20) $Y = W-pX = W-(r-s)X.$

Two non-linear subsidy systems will now be introduced: (1) an increasing subsidy, where $s=s(X)$, and $s^{I} > o$; and (2) a decreasing subsidy, where $s=s(X)$ and $s^{I} < o$. These subsidy systems are continuous for $r > p$ and they may easily be compared with the uniform scheme discussed in section II. Consider the slope of the budget line at D (i.e. AC) in Figure 3:

(21) $\frac{dY}{dX} = -(r-s)$, from (20).

Consider now the slope of an increasing subsidy budget line passing through point D:

(22) $\frac{dY}{dX} = -(r-s)+s^{I}X$, from (20).

From (22) it is clear that the budget line associated with an increasing subsidy will be flatter than the indifference curve passing through point D, and this will be true as well for every point (like D) along the offer curve AV of Figure 3. This means that all equilibrium points on budget lines corresponding to increasing subsidies must lie below AV. For such an equilibrium point above X* the corresponding value for Y must lie between D and R.

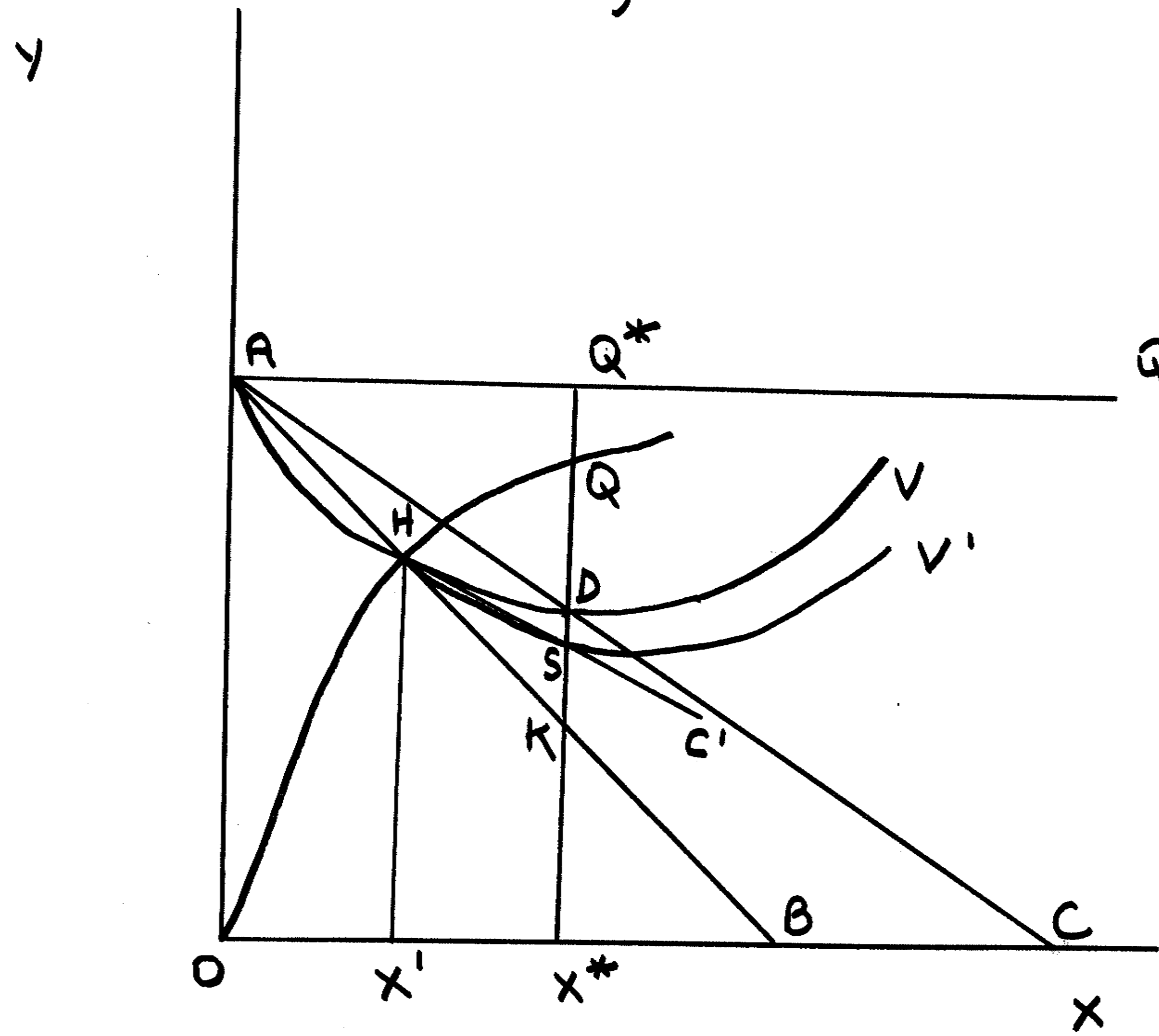
By analogous reasoning it may be shown that decreasing subsidy budget lines originating at A will produce equilibrium points that must lie above AV. Again, for such an equilibrium point on the vertical line erected above X* the corresponding value for Y must lie between D and Q*, where Q* implies that s=r. This subsidy system will cost the subsidizing agency more than a uniform subsidy and so there will be no further discussion of decreasing subsidies in this section.

Increasing subsidies have the potential of stimulating consumption of X at a lower per unit cost than uniform subsidies. This means that they may be used to minimize the redistribution of income associated with a subsidy system. But they will require more information about consumer preferences than will a uniform subsidy scheme.

Note that the subsidy schemes considered up to this point have involved a shifting (and curving) of the budget line within the conic area enclosed by AQ* and AB. One implication of this is that it is likely to involve the payment of intra-marginal surplus upon independent (i.e. no subsidy) purchases of OX^{I} units of X. An alternative to this procedure is to pay the subsidy on extra-marginal units, and this means rotation of the cum-subsidy budget line around H, rather than A. For an extra-marginal uniform subsidy this will generate the offer curve HV^{I}, which must lie below HV. This means that the subsidy cost for an extra-marginal uniform subsidy must be less than that for a uniform subsidy on all units. In Figure 7 this cost is shown as SK for the extra-marginal uniform subsidy scheme, as compared with DK for a subsidy on all units. This scheme, illustrated by the budget line AHC^{I} is essentially an increasing subsidy scheme, with the qualification that it increases discretely, rather than continuously. The scheme has the practical advantage that some benchmark level of consumption (e.g. the value of X for December, 1970) may be used to define "the margin" upon which extra-marginal uniform subsidies are based. In another paper the writer has described an iterative procedure whereby a discretely increasing subsidy system would result in a budget line (non-strictly) convex to the origin, comprised of linear segments.[10]

It is also possible to define two other extra-marginal subsidy schemes, and these are symmetric with

Figure 7



those discussed above. These are (a) increasing and (b) decreasing extramarginal subsidies and the equilibria for (a) will lie below HV^I in Figure 7 while those for (b) will lie above HV^I. Again, the distinguishing feature of these extra-marginal schemes is that there is no intra-marginal surplus on the independently purchased OX^I units.

It is well known that subsidy schemes become very costly when the subsidized good is inferior. While the substitution effect will always be in the right direction the income effect will be perverse, for an inferior good. In the case of the Giffin good the perverse income effect of the subsidy may dominate.[11] The Giffin good case is defined in terms of the dominance of the perverse income effect for a uniform price reduction on all units. A uniform subsidy will therefore be ineffectual in stimulating consumption of X when it is a Giffin good. In general, the dominance of a perverse income effect will depend upon the magnitude of the intra-marginal surplus. In most discussions of the Hicksian income effect the intra-marginal surplus is measured in terms of the compensating income variation.[12] Our analysis suggests that perverse income effects will be suppressed by subsidy schemes that create a minimal transfer of intra-marginal surplus. The extra-marginal subsidy schemes discussed above, the uniform subsidy scheme and particularly the increasing subsidy, are likely to be particularly effective on this count.

This section has drawn attention to some of the characteristics of non-linear subsidy schemes that may be effective in dealing with external effects. Some of the schemes discussed have been especially relevant for situations in which the policy maker is particularly anxious to minimize any redistribution of income associated with the subsidies. There are many reasons, none of which will be discussed in this paper, why a federal government might wish to minimize the redistribution of income connected with its conditional grants. A final point here is that some of the subsidy schemes discussed above are useful in reducing perverse income effects.

V. Provincial Taxes and Federal Subsidies

In this section we consider the tax policy of junior governments (e.g. provincial and municipal) and examine how the central government may attempt to alter their tax mix. This treatment will be symmetric with that of external effects on the expenditure side except that here it will be assumed that a particular tax exerts an external effect. To motivate this it is useful to consider a federal government which is interested in achieving homogeneity of tax rates throughout the country in the interests of fiscal neutrality.[13]

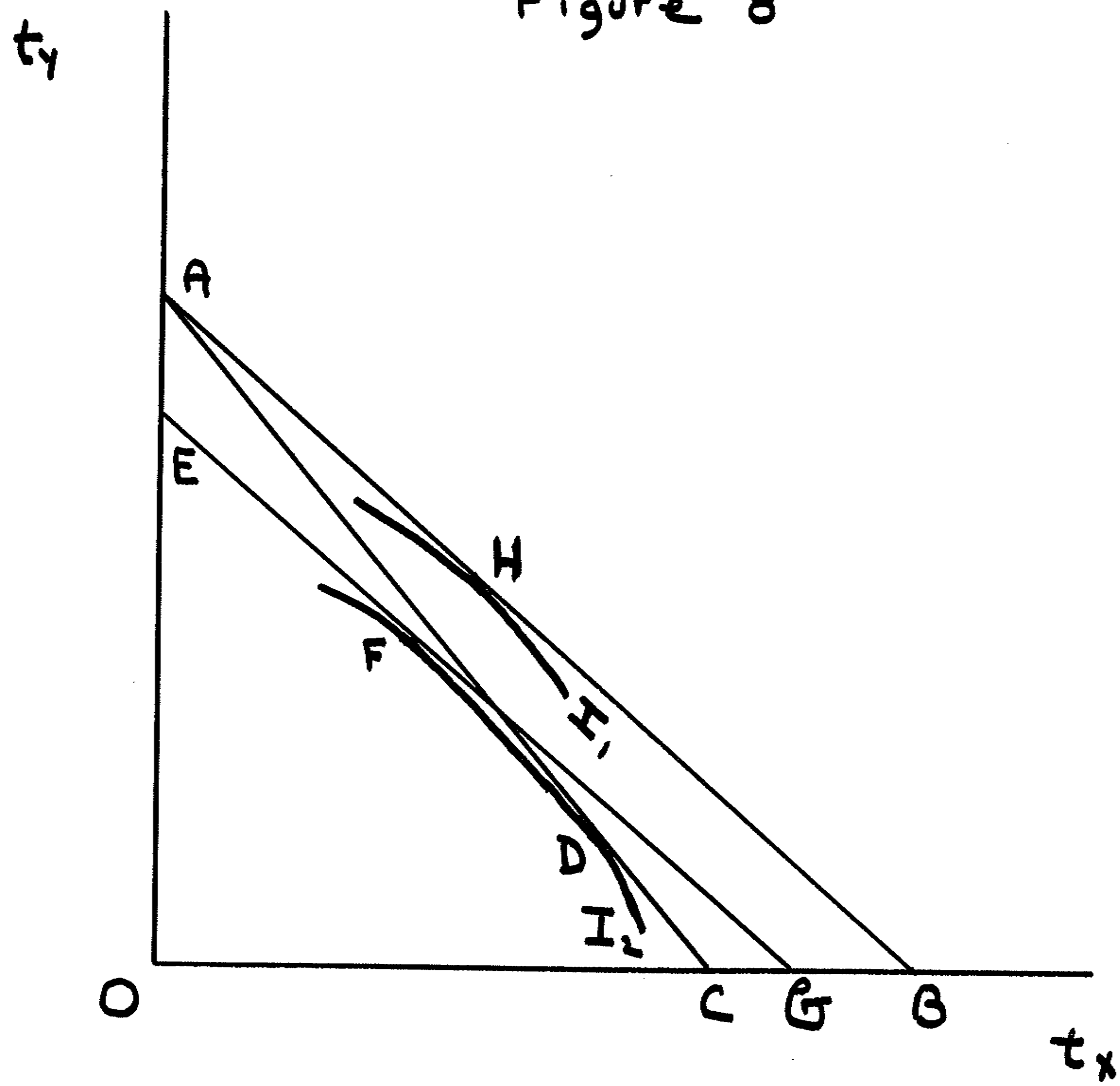
Let it be assumed that we are dealing with a province and that the province uses two tax sources. The size of the two corresponding tax bases are given by X and Y, both measured in dollar terms. The provincial government is a vote-maximizing group and while they try to minimize the distortions associated with taxes there are other characteristics of different taxes that they consider in devising a tax system.[14] Little further will be said about the underlying theory of government and taxation here except to note that taxes are "bads"(rather than "goods"). The tax bases are taken to be fixed, although it is recognized that this is a costly simplification, in view of the federal government's concern with fiscal neutrality.[15] Hence, the variables on the revenue side that have to be chosen are the tax rates and it is assumed that the indifference curves between the two tax rates are downward sloping, but concave to the origin. For a given amount of revenue the budget equation is given by:

(23) $T = t_x X + t_y Y$, or

(24) $t_y = T/Y - t_x X/Y$,

where both taxes are proportional. This equation is graphed as AB in Figure 8 and the position of tax equilibrium is at H.

Figure 8



The slope of the budget line is given by the ratio of the tax bases, from (24). It is assumed that there is a federal subsidy on revenue assessed on tax base X, in the amount of q_x. The budget equation now becomes

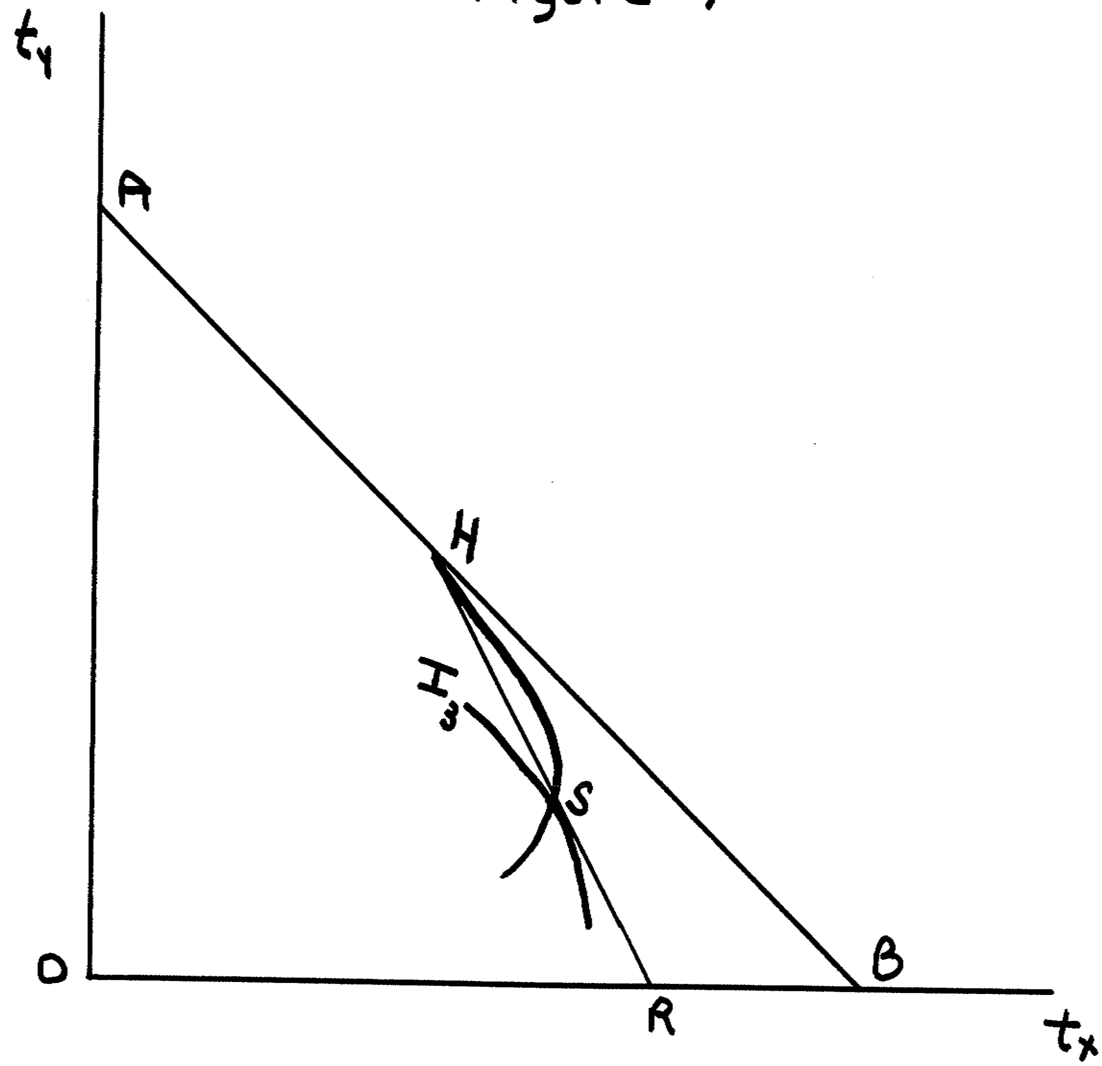
(25) $T = (1+q_x)t_xX+t_yY$, or

(26) $t_y = T/Y-(1+q_x)t_xX/Y.$

The federal subsidy may be viewed as increasing the yield from the tax on X or as effectively raising the tax base.[16] The new budget line in the presence of the subsidy is given by AC and $\frac{CB}{0B} = \frac{q_x}{(1+q_x)}$, $CB/0C=q_x$. The equilibrium set of tax rates now shifts from H to D, which lies on a "higher" indifference curve, I_2. It will be helpful to construct EG, tangent to I_2 at F and parallel to AB. The movement from H to F can now be identified as the "income" effect. In dealing with income effects we shall restrict the analysis to "normal" taxes and rule out the question of inferiority. The income effect of the increase in yield from t_x will be to reduce t_x and t_y. The substitution effect can now be identified as the movement from F to D. The substitution effect of an increase in the yield from t_x will always be to raise t_x. Hence, the income effect and the subsitution effect will operate inopposite directions. This establishes a symmetry with the derivation of the supply curve of work effort for an individual worker and hence a link with the analysis of section III of this paper. In section III there was a dual focus on redistribution and work effort and it was shown that this produces a tradeoff between a regressive tax with a low intra-marginal surplus per unit of work effort and a proportional tax, which involves a larger surplus per work unit. In this section the concern is primarily with tax effort. The effect of a federal tax subsidy is quite likely to reduce tax effort (i.e. to reduce t_x) and non-uniformity of the subsidy is quite likely to be advantageous if it produces a low surplus.

In what follows our analysis will be confined to extra-marginal subsidy schemes, for the sake of brevity. Attention will first be focused upon a linear subsidy scheme which makes the budget line steeper than AB to the right of H. Such a scheme is illustrated by HR, in Figure 9. As the size of q_x is varied a family of lines like HR will be generated and the locus of their tangency with indifference curves will produce the offer curve HV^I. It is easy to show that if q_x is made a function of t_x, for values of t_x to the right of H, so that $q_x = q_x(t_x)$, and $q^I_x > o$, then it is possible to devise an increasing subsidy (i.e. increasing q_x) system that will generate an equilibrium to the right of HV^I. At S, in Figure 9, the slope of the linear budget line HR is given by:

Figure 9



(27) $\frac{dt_y}{dt_x} = -(1+q_x)\ \frac{X}{Y}$.

For an increasing subsidy system, where $q_x = q_x(t_x)$ and $q'_x > o$ the corresponding slope at S is

(28) $\frac{dt_y}{dt_x} = -(1+q_x)\ \frac{X}{Y} - \frac{dq_x}{dt_x}\ \frac{X}{Y}$,

At S the non-linear (concave to the origin) budget line associated with such a system will be steeper than the indifference curve. With concave-to-the-origin indifference curves this means that the equilibrium must lie to the right of HV'.

The implication of the analysis in this section is that it is quite possible that a uniform federal tax-subsidy (on all tax rates) will not create the desired incentive and that resort to an increasing subsidy system may be advantageous. The advantage of such a system is that the increase in tax effort is more likely to be positive and of significant magnitude.

VI. Conclusion

This paper has dealt with the theory of subsidies in the context of external effects. It was assumed that some good X (or alternatively, some tax rate, or some individual's work effort) created an external effect and the discussion was centered upon the subsidy approach to the externality. Our analysis implied that the external effect increased along with the level of X, but without any non-price relationship to the subsidy. If the externality component of X varied with the level of income of the consuming unit then our analysis of a price subsidy, as an instrument of economic policy, would be incomplete. It has been assumed here that the "externality mix" is invariant with respect to the intra-marginal surplus of the subsidy schemes that have been discussed.[17]

The subsidies considered in this paper may be classed as income-lump-sum subsidies or as price subsidies. The latter variety may be sub-classified as constant, increasing and decreasing per unit price subsidies and each of these may be paid on all units or only on extra-marginal units. The paper has evaluated the stimulative effect of each of these subsidies for a number of specific subsidy problems. Those subsidy systems with the smallest intra-marginal surplus have been shown to have the largest stimulative effect.

Most of the analysis in this paper has been based upon the use of indifference curves and upon the assumption that these are convex. One caveat should be noted about the use of indifference curves when the budget line facing the consumer is non-linear. In conventional terms, the marginal rate of substitution of X for Y is the marginal valuation of a unit of X in terms of Y. This valuation is assumed to be independent of theprices of X and Y since the purchase decision is made by comparing the marginal rate of substitution of X for Y with the price of X in terms of Y. The structure of preferences may be assumed to be independent of prices since these prices are parameters when the budget line is linear, and the consumer is a price taker.

The introduction of non-linear budget lines may well vitiate this assumption of independence of prices from the preference function of the consumer. Let us assume that we have a hypothetical consumer whose purchases of good X are being subsidized by an increasing subsidy on the price of X (i.e. $s'(X) > o$). This consumer may now value an extra unit of X not only on the basis of its direct utility but also for its capacity to lower the price on intramarginal units. The magnitude of this effect will depend on the nature of the subsidy scheme and particularly upon the application of the subsidy to intramarginal units. The likely effect, if the increasing subsidy applies to all intramarginal units, is to induce a clockwise shift in the preference map. By putting prices into the utility function in this way the likely effect of an increasing subsidy on purchases of X is to increase the marginal rate of substitution of X for Y. This change in the whole analytic framework that may be wrought by non-linear budget lines will not change the direction of any of the results shown above. In the case of the increasing subsidy mentioned in the present context the stimulative effect will be even greater than it was found to be in the conventional framework in which prices are parameters. Converse reasoning applies in the case of decreasing subsidies.

FOOTNOTES

1. On the question of subsidies in money or in kind see Lucien Foldes (1967, 1968) and James Buchanan (1968).

2. For a general analysis of various subsidy schemes in the context of a federal system of government see A.D. Scott (1952).

3. For a discussion of bargaining and legal approaches to the problems of external effects see S.N.S. Cheung (1970), R.H. Coase (1960) and P. Burrows (1970).

4. Musgrave introduced the concept of a merit good in Musgrave (1959).

5. See Hicks (1956) or Machlup (1957).

6. Traditional microeconomic analysis is applied to the negative income tax problem by M.J. Boskin (1967), P.A. Diamond (1968), C. Green (1968), A.J. Heins (1970), J. Kesselman (1969), and R. Perlman (1968). A skeptical comment on the static work-income theory is provided by J. Conlisk (1968).

7. Kesselman (1969) gives particular attention to the analysis of a wage subsidy and he derives a new labour supply curve for a wage subsidy.

8. It should be noted here that the income and substitution effects on work effort of the change in the net-of-tax wage rate operate in opposite directions. As the net wage line shifts from CF to CJ there is a substitution effect that pushes the offer curve northwest and an income effect that presses northeasterly.

9. Note that $\bar{y}$ is used to separate the group that we are discussing, the disadvantaged class of people whose net income tax payments are non-positive, from those people whose incomes are high enough for them to pay income taxes. The level of income $\bar{y}$ is then the income level below which the individual pays no (net) income taxes.

10. See Vardy (1971).

11. Davis and Whinston (1965) consider the Giffin good obstacle to the use of linear subsidies (on all units). Vardy (1971) treats a similar externality problem with non-linear subsidies.

12. See Machlup (1957) for a discussion of alternative concepts of income variations.

13. It is possible to argue that differences in tax rates between different regions create incentives for inefficient resource movements. For example, a high tax on value added in manufacturing industry imposed by a poarticular state or province may discourage industry from moving in. For a general discussion of fiscal neutrality in a federal context see J.M. Buchanan (1950, and 1952).

14. See Downs (1957).

15. It is fundamentally the induced interjurisdictional movement of tax bases that is at issue in the question of fiscal neutrality.

16. In Canada the Fiscal Arrangements Act (1967) established a new basis for equalization payments to the provinces. The new equalization formula, that first came into use for the 1967-68 fiscal year, is based upon the "tax indicator approach". For a discussion of this approach see J. Lynn (1968). The formula includes all provincial "net general revenues" (a DBS Financial Management concept) except for transfers from the federal government and these revenues are grouped into 16 revenue classes. For each revenue source the national rate of yield is multiplied by the per capita national revenue base. From this the product of the national yield rate and the per capita provincial revenue base is subtracted. The resultant per capita equalization payment is then multiplied by the provincial population to give the equalization payment with respect to that revenue source. The equalization payment to the p^{th} province for the i^{th} revenue source is given as follows:

$$\text{F-1} \quad E_{ip} = P_p \left[\frac{R_i}{B_i} \cdot \frac{B_i}{P} - \frac{R_i}{B_i} \cdot \frac{B_{ip}}{P_p} \right] ,$$

where E_{ip} is the equalisation payment to the p^{th} province with respect to the i^{th} revenue source and the other terms are defined below:

P = national population
P_p = population of p^{th} province
R_i = actual revenue from the i^{th} revenue source

B_i = national revenue base for the i^{th} revenue source
B_{ip} = provincial revenue base for the i^{th} source,

We may rewrite F-1 as:

$$\text{F-2} \quad E_{ip} \equiv R_i\left[\frac{P_p}{P} - \frac{B_{ip}}{B_i}\right] \quad ,$$

where

$$\text{F-3} \quad R_i \equiv \sum_{p=1}^{10} R_{ip} \equiv \sum_{p=1}^{10} t_{ip}B_{ip}$$

and

$$\text{F-4} \quad B_i \equiv \sum_{p=1}^{10} B_{ip}$$

In F-3 it is assumed that all of the taxes of the i-th variety are imposed with proportional tax rates. From F-2 we may investigate the effect on E_{ip} of an increase in t_{ip}:

$$\text{F-5} \quad \frac{\partial E_{ip}}{\partial t_{ip}} = B_{ip}\left[\frac{P_p}{P} - \frac{B_{ip}}{B_i}\right]$$

If E_{ip} is positive then an increase in t_{ip} will raise the equalization payment and the amount of the increase will depend on the size of the population and tax base of the p^{th} province. Total equalization payments to a province are found by adding positive equalization payments and deducting negative payments. The equalization program provides that if the sum of the equalization payments with respect to all revenue sources for a given province is negative then the equalization payment (E_p) for that province is zero. If the province is a large one and E_p is positive then it may have something to gain by raising some of its tax rates. The equalization scheme therefore does produce tax subsidies of the type discussed in the text.

17. For a discussion of the question of "externality mix" see Buchanan (1966).

REFERENCES

1. Boskin, Michael J., "The Negative Income Tax and the Supply of Work Effort," National Tax Journal, December 1967, No. 4, Vol. XX, pp. 353-367.

2. Buchanan, J.M., "Federalism and Fiscal Equity," American Economic Review, September 1950, Vol. XL, pp. 583-599.

3. Buchanan, J.M., "Federal Grants and Resource Allocation," Journal of Political Economy, June 1952, Vol. LX, pp. 208-217.

4. Buchanan, J.M., "Externality", Economica, November 1962, Vol. 29, pp. 371-384.

5. Buchanan, J.M., "Joint Supply, Externality and Optimality," Economica, November 1966, No. 132, Vol. XXXII, pp. 404-415.

6. Buchanan, J.M., "What Kind of Redistribution Do We Want," Economica, May 1968, No. 138, Vol. XXXV, pp. 185-190.

7. Burrows, Paul, "On External Costs and the Visible Arm of the Law," Oxford Economic Papers, N.S., March 1970, No. 1, Vol. XXII, pp. 39-56.

8. Cheung, S.N.S., "The Structure of a Contract and the Theory of a Non-Exclusive Resource," Journal of Law and Economica, April 1970, No. 1, Vol. XIII, pp. 49-70.

9. Coase, R.H., "The Problem of Social Cost," Journal of Law and Economics, October 1960, Vol. III, pp. 1-44.

10. Conlisk, John, "Simple Dynamic Effects on Work Leisure Theory: A Skeptical Comment on the Static Theory," Journal of Human Resources, Summer 1968, No. 3, Vol. III, pp. 324-326.

11. Davis, Otto A. and Andrew B. Whinston, "Some Notes on Equating Private and Social Cost," Southern Economic Journal, October 1965, No. 2, Vol. XXXII, pp. 113-126.

12. Diamond, P.A., "Negative Taxes and the Poverty Problem - A Review Article," National Tax Journal, September 1968, No. 3, Vol. XXI, pp. 288-303.

13. Downs, Anthony, An Economic Theory of Democracy, (New York: Harper, 1957).

14. Foldes, Lucien, "Income Redistribution in Money and in Kind," Economica, February 1967, No. 133, Vol. XXXIV, pp. 30-41.

15. Foldes, Lucien, "A Note on Redistribution," Economica, May 1968, No. 138, Vol. XXXV, pp. 203-205.

16. Green, Christopher, "Negative Taxes and Monetary Incentives to Work: The Static Theory," Journal of Human Resources, Summer 1968, No. 3, Vol. III, pp. 280-288.

17. Heins, A.J., "The Negative Income Tax, Head Grants, and Public Employment Programs: A Welfare Analysis," Journal of Human Resources, Summer 1970, No. 3, Vol. V, pp. 298-303.

18. Hicks, J.R., A Revision of Demand Theory, (Oxford: Clarendon Press, 1956).

19. Kesselman, Jonathon, "Labour-Supply Effects of Income, Income-Work and Wage Subsidies," Journal of Human Resources, Summer 1969, No. 3, Vol. IV, pp. 275-292.

20. Lynn, J.H., Comparing Provincial Revenue Yields: The Tax Indicator Approach, Canadian Tax Foundation, Paper No. 47, March 1968.

21. Machlup, Fritz, "Professor Hicks' Revision of Demand Theory," American Economic Review, March 1957 and reprinted in W. Breit and H. Hockman, Readings in Microeconomics, (Holt, Rinehart and Winston, 1968) pp. 89-103.

22. Musgrave, R.A., The Theory of Public Finance, (New York: McGraw-Hill, 1959).

23. Pauly, Mark V., "Efficiency in the Provision of Consumption Subsidies," Kyklos, Vol. XXIII, Fasc. I, 1970, pp. 33-57.

24. Perlman, Richard A. "A Negative Income Tax Plan for Maintaining Work Incentives," Journal of Human Resources, Summer 1968, No. 3, Vol. III, pp. 289-299.

25. Scott, A.D., "The Evaluation of Federal Grants," Economica, November 1952, No. 76, Vol. XIX, pp. 377-394.

26. Vardy, D.A., "Intergovernmental Transfers and Pareto Optimality," Discussion Paper No. 38, Queen's University Economics Department, February 1971.