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INTERGOVERNMENTAL TRANSFERS AND PARETO OPTIMALITY

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I. INTRODUCTION

A large part of the literature in the economics of federalism and decentralized government is focused upon the efficiency implications of spillovers or external effects between jurisdictions. These externalities take a number of different forms and the form of the external effect has an important bearing upon the method of analysis and upon the policy implications. Their existence depends upon the nature and extent of the interdependence between jurisdictions.

The external effects with which the present paper is concerned are direct spillovers of benefits from public goods beyond the jurisdiction for which they were produced. The analysis will be limited to the benefit side, although cost spillovers could be integrated into the models treated here without difficulty. The focus of attention is upon public sector efficiency in the context of decentralized government units and any induced effects upon efficiency in the private sector are ignored. These "indirect" or induced effects will be the subject of a later paper.

This paper will examine a variety of models in which the affected governments modify their behaviour in response to spillover benefits. Initially, governments will adjust to the "income effects" of benefit spill-ins but it will be shown that intergovernmental compensation will generally be required in order to achieve Pareto optimality. The tax-and-subsidy solutions shown may be achieved by bargaining between the affected governments or by imposition from a higher level government. The compensation solution presented will be integrated into an independent adjustment reactive movement to equilibrium, where the presence of both "intra-marginal" and marginal spillover benefits (i.e. Pareto relevant externalities) leads to marginal and extra-marginal compensation. Various methods of compensation will be examined, including constant per unit taxes and subsidies as well as compensation which is non-linearly related to output.

The models to be dealt with below are based on a two-region federation, where each region produces one public good and a numeraire private good. The transformation functions will be linear in most of the analysis. The reaction models will be of the Cournot type, following the seminal article by Williams [1966]. In Section II we shall follow Williams in examining the case of "rivalrous" spillover benefits. In Section III the basic model will be extended to deal with "non-rivalrous" spillover benefits. Section IV deals with the social optimum in the rivalrous case and presents a reactive approach to equilibrium where intergovernmental compensation is paid. In Section V a reaction model is introduced for benefits that are non-rivalrous and reciprocal. Section VI presents an analysis of various compensation schemes that will lead to various Pareto-optimal equilibrium solutions. Section VII concludes the paper with a discussion of its leading points.

II. RIVALROUS SPILLOVER BENEFITS

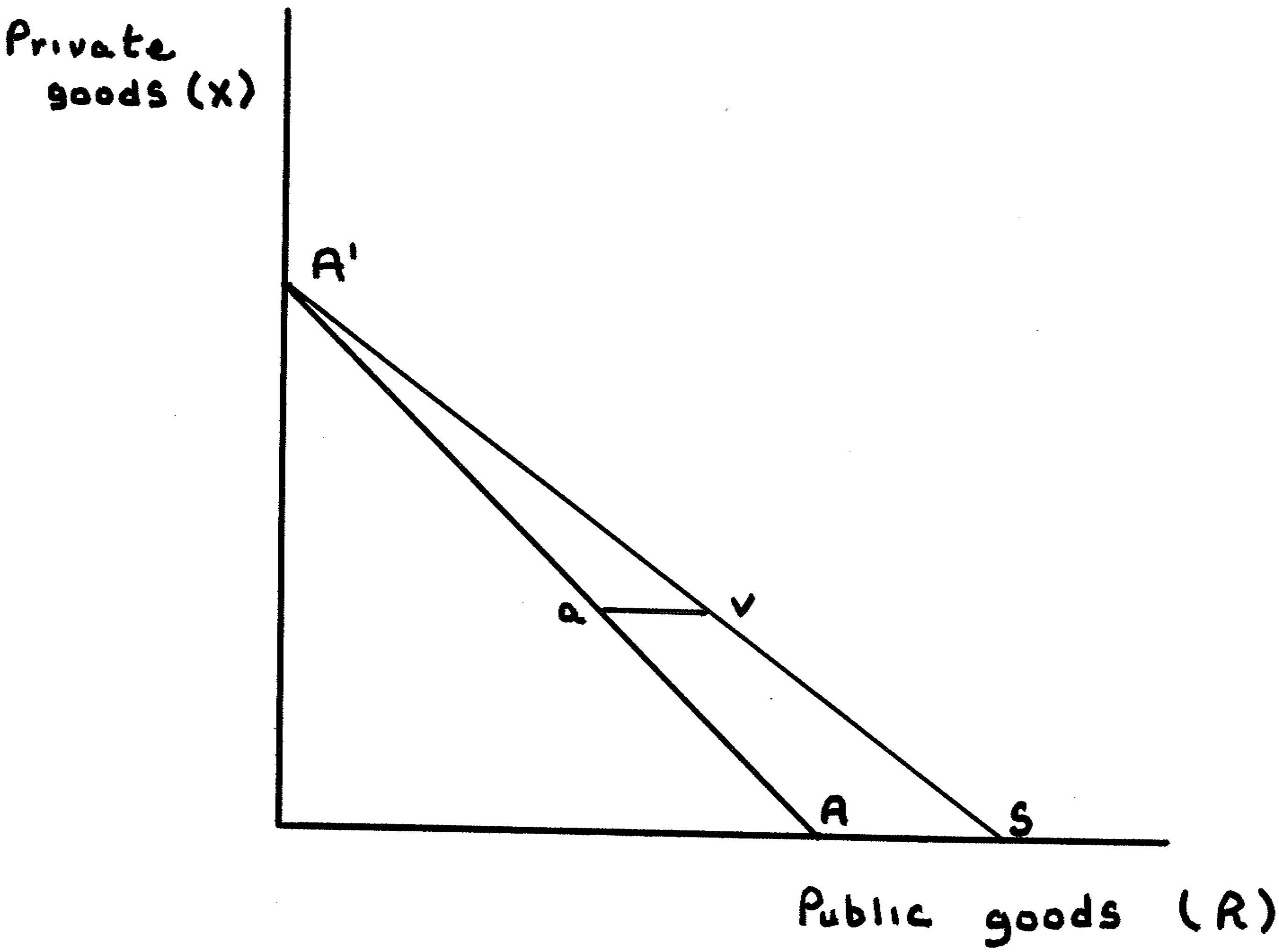
In his paper Williams [1966] attempts to work out the implications of the theory of public goods for a decentralized government.[1] The model posits a set of local governments which produce "local" public goods for which there are spillover effects. The spillover effects radiate out from the producing region and it is the spatial dimension of these goods that makes them differentially accessible. As a simplifying assumption each region produces one public good and this public good is equally available to the residents of any one region but is available in the least adulterated form in the producing region. In spite of the spillover of benefits it is assumed that each local government chooses the output level of its public good on the basis of the benefits and costs to that region alone. In the first Williams model [1966, pp. 19-22], it is assumed that the j-th region makes available certain spillover benefits from its provision of the public good R in amount R_j but that it makes no attempt to recover any of the costs from other regions enjoying the externality benefit.

A major objective of the Williams paper is to refute the Pigouvian assertion that these local governments will tend to undersupply the public goods that they provide. Let R_j^* be the socially optimal level of provision and let R_j' be the level independently provided, then Williams purports to show that $R_j^* \gtreqless R_j'$.

In presenting his graphical model he posits two communities, α and β, and assumes that the reciprocal spillovers are generated in fixed proportions. It is assumed that marginal costs of producing R_α and R_β are constant and the same in all regions. The resident population of each region remains unchanged throughout the analysis.[2]

There are two communities, α and β, and the production possibility curve for α is given by SA′ in Figure 1. Some fixed proportion of the benefit spills over to the other region and this has the effect of reducing the own consumption possibility curve to AA′. The own consumption possibilities refer to the domestic availability of units of the public good produced locally. This means that in the absence of compensation for spill-out benefits the effective opportunity cost line for

FIGURE 1



domestic choice between private and public goods is AA′, rather than SA′. Let us assume that in the absence of spill-in benefits the community α chooses consumption point a, implying that the community is producing at v. The spill-out is av and the <u>spill-out ratio</u> is given by AS/OS.

In the event that community β is producing some positive amount of the public good then α may expect some spill-in from β. It is assumed that production possibilities are identical in the two communities and that their spill-out ratios (AS/OS) are the same.[3] We may expect that different levels of production by β will have the effect of shifting the consumption possibility line for α , and that these shifts will be parallel. Assume, as in Figure 2, that β is consuming at a′ and producing at v′. This amount a′v′ becomes a spill-in for α and it has the effect of shifting α′s consumption possibility curve from AA′ to BB′, and A′B′ = a′v′. The line passing through a and b becomes α′s consumption reaction path for different levels of output by β. As R_β expands, α′s consumption of public goods also increases, but α′s consumption of its own output of these goods falls. This is shown by own consumption points which move leftward along aA′ from a as total consumption increases along ab. Corresponding to total α consumption of a, b and c in Figure 2 are own consumption points a, b′ and c′. It will be useful next to plot the points a, b, c, and a, b′, c′ in R space, in terms of the two public goods.

Units of α and β public goods are measured on the Y and X axes, respectively, of Figure 3, which replicates Figure 2 of the Williams article. Point $\bar{\alpha}$ corresponds to point a of our Figure 2 and is associated with zero production (and spill-out) of R_β. The curve $\bar{\alpha}\ \alpha_c$ is α's consumption reaction curve and point $\bar{c}$ corresponds to point b in Figure 2. The abscissa for $\bar{c}$ is equal to a″v′ of Figure 2. The curve $\bar{\alpha}\ \alpha_Q$, on the other hand, is α's production reaction curve and refers to points on SA′ corresponding to a, b′, c′, etc. On the basis of the positive slope of abc it follows that $\bar{\alpha}\ \alpha_Q$ will lie below $\bar{\alpha}\ \alpha_c$. The implication of these two curves and their respective positions is that α will reduce R_α, its output of public goods, when R_β is increased. The curves $\bar{\beta}\ \beta_c$ and $\bar{\beta}\ \beta_Q$ are based on a similar derivation for region β. On the basis of the construction and underlying assumptions the curves $\bar{\alpha}\ \alpha_Q$ and $\bar{\beta}\ \beta_Q$ are the reaction curves whose consistency defines an equilibrium for the model. At the point Q the expectations of both communities will be satisfied simultaneously and the outputs OQ_α and OQ_β represent the equilibrium solution.

FIGURE 2

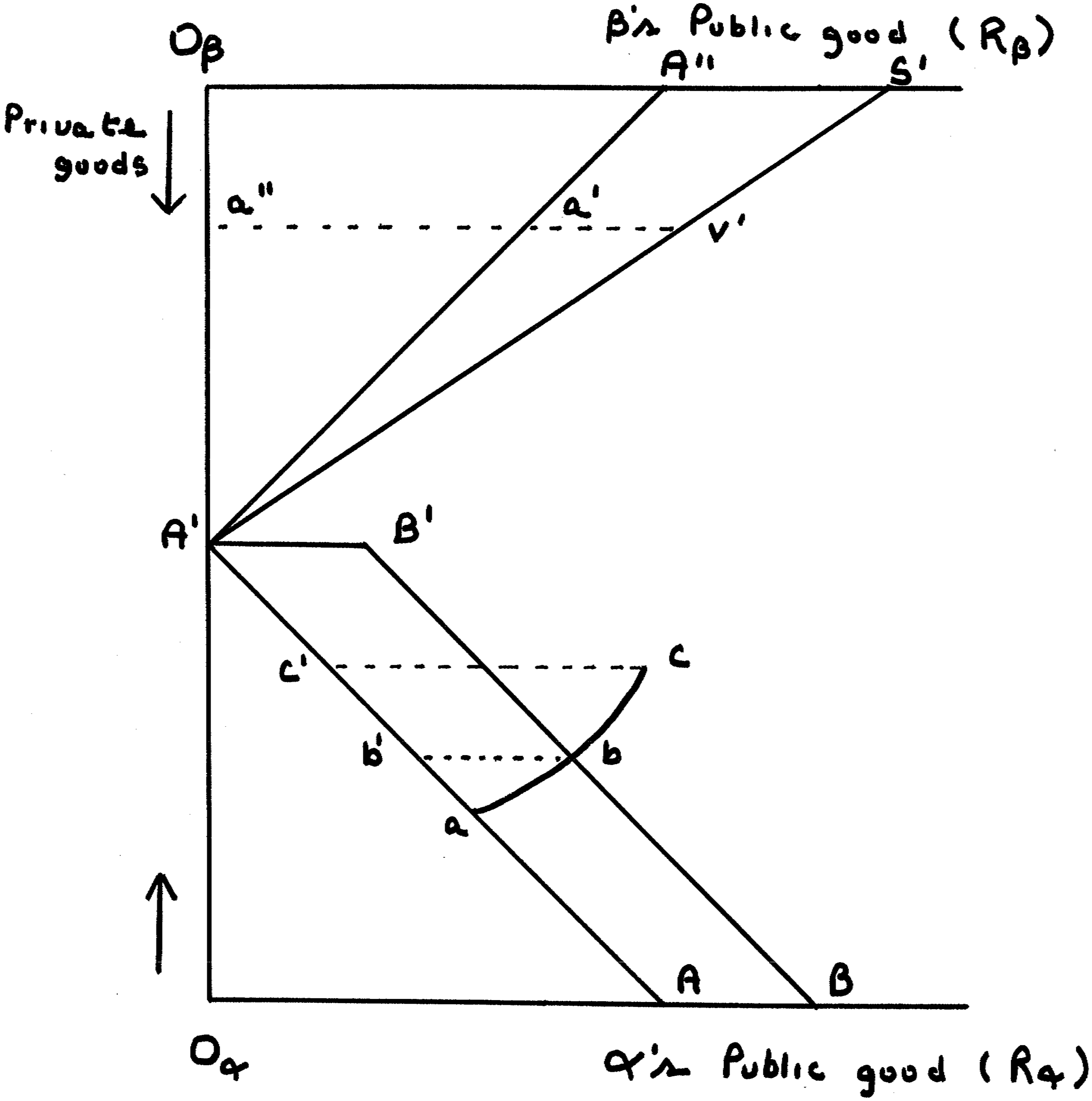
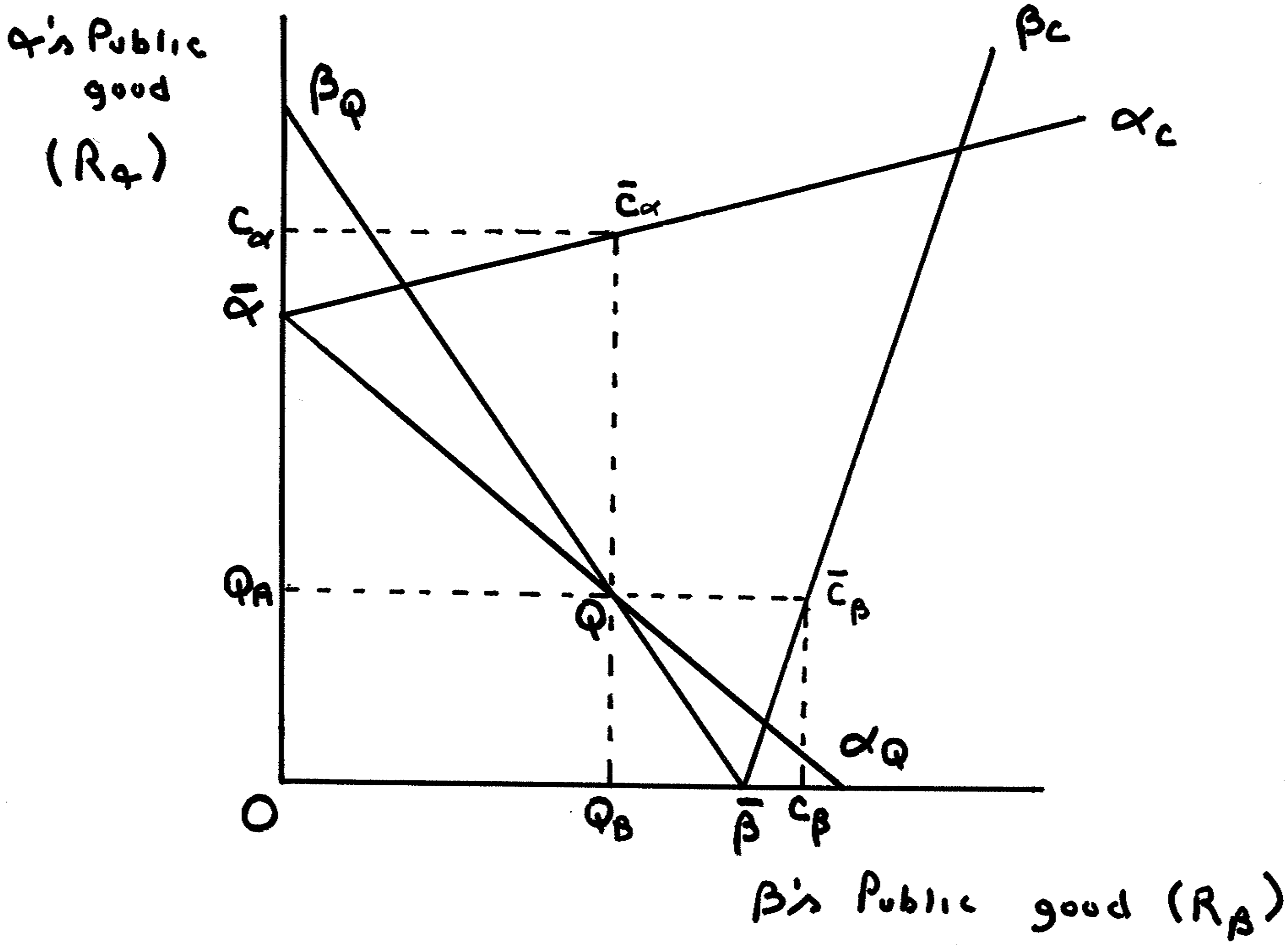

β's Public good (R_β)
O_β
A''
S'
Private goods
a''
a'
v'
A'
B'
c'
c
b'
b
a
A
B
O_α
α's Public good (R_α)

Figure 3


α's Public good (R_α)
β_Q
β_C
α_C
$\bar{C}_\alpha$
C_α
$\bar{\alpha}$
Q_A
Q
$\bar{C}_\beta$
α_Q
O
Q_B
$\bar{\beta}$
C_β
β's Public good (R_β)

Associated with this equilibrium are consumption levels of OC_α and OC_β. As the diagram is constructed α will derive more of its consumption benefits from β's provision than from α's own output.

Note that the equilibrium at Q involves a reduction in output below that provided when the communities ignore spill-ins. This model operates without any compensation between the two regions for spill-out. Each party loses from the point of view of its spill-out, but it gains a spill-in. No mechanism exists in this reaction model to ensure that no party will suffer a net loss. If the recapture of spill-out loss, through compensation or trade, is an inevitable impossibility then fortuitous spill-ins are clearly advantageous to the recipients, and position Q is clearly superior to positions $\bar{\alpha}$ and $\bar{\beta}$.

III. NON-RIVALROUS CONSUMPTION

It was assumed above that the spill-out detracts from domestic consumption possibilities and this assumption of rivalrous consumption is responsible for shifting consumption possibilities from A'S to A'A in Figure 1. If we now drop this assumption it is possible to derive a consumption reaction curve, analogous to abc in Figure 2, by showing how SA' may be shifted out by parallel movement. But in this case the consumption reaction curve corresponding to abc will lie to the right of abc. Consequently the corresponding $\bar{\alpha}\ \alpha_c$ curve in Figure 3 will lie above the one shown. By the same token the production reaction curve will lie above $\bar{\alpha}\ \alpha_Q$. Qualifications of the same kind apply to β's reaction curves. As a result the new Q (call it Q′) at which the production reaction curves meet will lie above and to the right of the old Q. Both parties would produce more than they would if the spillover were rivalrous.

In this non-rivalrous case the extent of the rightward shift in the consumption possibility line, corresponding to the shift of AA′ to BB′ in Figure 2, will depend upon the degree of substitution between R_α and R_β. If they are perfect substitutes, from the point of view of region α, then α's consumption line will shift out by the amount R_β. If they are imperfect substitutes then the rightward shift of the consumption line will be somewhat less than R_β. If, for example, α considers one unit of R_β to be equivalent to AS/OS units (in Figure 1) of R_α then when R_β = a″v′ (in Figure 2) the shift in α's possibility line would be given by a′v′ (= A′B′). This means that the spill-in of a″v′ units of β's public good is equivalent to a′v′ of R_α.

However, in both the rivalrous and the non-rivalrous cases the uncompensated reaction equilibria corresponding to Q (in Figure 3) and Q′ (not shown) are not optimal This is due to the fact that these reaction equilibria lie on the consumption reaction paths of α and β, where the summed marginal rates of substitution within each region are equated to the cost of the public good. Symbolically stated this may be written $\sum_{i \text{ in } \alpha} MRS_{R_\alpha X} = P_R^\alpha$, and $\sum_{i \text{ in } \beta} MRS_{R_\beta X} = P_R^\beta$.

This violates the well known Samuelson optimality condition which requires that

$$\sum_{i \text{ in } \alpha} MRS^{\alpha}_{R_\alpha X} + \sum_{i \text{ in } \beta} MRS^{\beta}_{R_\alpha X} = P_R^\alpha, \text{ and}$$

$$\sum_{i \text{ in } \alpha} MRS^{\alpha}_{R_\alpha X} + \sum_{i \text{ in } \beta} MRS_{R_\beta X} = P_R^\beta.$$

The subscripts and superscripts will be self-explanatory, with X denoting the numeraire private good and $\sum_{i \text{ in } \alpha} MRS^{\alpha}_{R_\alpha X}$ the sum of the marginal rates of substitution in α between α's public good and the numeraire private good.

In the next section we return to the rivalrous case and show how optimality may be achieved through inter-governmental compensation. In section V we deal explicitly with the uncompensated reaction process for spillovers from non-rivalrous public goods. Section VI examines the essential difference between rivalrous and non-rivalrous spillovers and develops compensation systems that will enable optimality to be achieved.

IV. THE SOCIAL OPTIMUM IN THE RIVALROUS CASE

Consider again the reaction equilibrium Q in Figure 3 and note that the 'tâtonnements' involved in achieving Q take place along AA'-type price lines. Spill-ins shift the price line parallel to itself but spill-outs (more correctly, own spill-outs) are disregarded. In order to evaluate this equilibrium from the standpoint of social efficiency we need to establish some optimum Q (call it Q*) which may be used as a benchmark. It should be recognized, however, that Q* will be contingent upon the associated distribution of income.

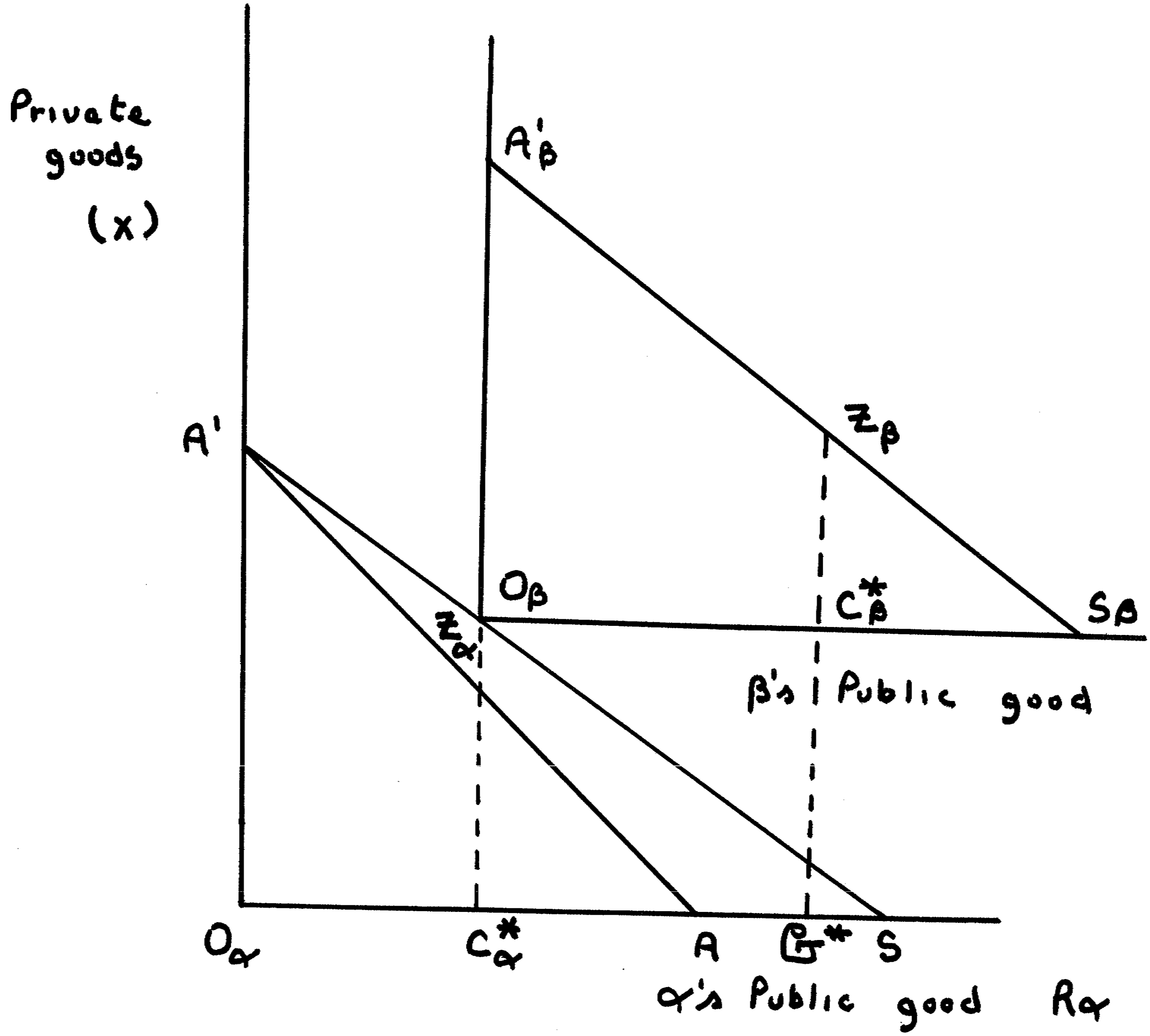
The rivalrous spillover case in question is essentially one in which R_α and R_β are private goods as between communities. There is no sharing in consumption of the same units and hence the problem is not strictly a public good problem. As a result we may define the optimum Q* in a quite orthodox fashion. The institutional analogue to the theoretical procedure to be employed in defining the benchmark is that of creating property rights in the spillover benefits. The problem is then a simple one in welfare maximization. In the present context some procedure is required in order to internalize the spillovers.

Williams [1966, p. 23] invokes some "joint body" or central government whose duty is to ensure that compensation is paid for benefit spill-outs. Interregional compensation will be paid in public and in private goods but the important requirement of the compensation arrangement is that net spill-ins are settled in terms of the numeraire private good. Furthermore, attainment of an optimal solution requires that the transfer of private goods be associated with a mechanism for transforming private into public goods at will. Note that the optimum that will be found is one based upon a given distribution of income. As has been noted above, the opportunity cost of public goods in terms of private goods has been assumed identical in both regions. Figure 3 in the Williams paper will become Figure 4 below.

These assumptions imply that α faces the opportunity curve A'S while β faces $A'_\beta S_\beta$. These curves give the production and consumption possibilities of the two regions and the consumption choices are shown by Z_α and Z_β. The preferred consumption point for α, Z_α, becomes the origin O_β for β. The jointly preferred level of provision of the public good is that given by OG* and this represents the total of OC^*_α and $O_\beta C^*$. These provision levels, represented by Z_α and Z_β, become the Pareto optimal benchmarks which we were seeking. Note that the equilibria are characterized by equalities of the form $MRS = P_R$.

In this section we consider an attempt to achieve optimality using a reaction model similar to that implicit in Figure 3. This reaction model is placed in the context of the implicit federal system with the central government responsible for effecting or enforcing inter-governmental compensation. Each community has a set of contingency plans based upon different levels of expected spill-in. These plans provide some level of own provision, <u>and hence of own spill-out</u>, for each level of spill-in (associated with specific levels of R). These plans revolve around the prior selection of Z_α and Z_β as the preferred consumption positions in the choice calculus of the two regions. We may now turn to Figure 5 to illustrate the derivation of an output reaction curve. If α expects no spill-in of benefits then it produces at q_0

FIGURE 4


Private goods (x)
A'β
Zβ
A'
Oβ
C*β
Sβ
Zα
β's Public good
Oα
C*α
A
G*
S
α's Public good Rα

and this makes available OC^*_α of R and $r_0C^*_\alpha$ of private goods from its own provision. The remainder of its requirements of private goods (i.e. $r_0 Z_\alpha$) is received as compensation for its export of r_0q_0 in the form of public goods. The trade surplus of r_0q_0 in public goods is requited by the payment of r_0Z_α in the numeraire good. If there is a spill-in of S_1Z_α then α will produce at q_1. By so doing the consumption requirement (OC^*_α) of the public good will be satisfied by own production (OG_1) and by spill-in ($G_1C^*_\alpha$). Private goods in the amount r_1S_1 are paid to settle the trade surplus in the public good. It may similarly be shown that α will produce at q_2 when the spill-in is S_2Z_α. In general the level of R_α will vary from that at q_0 to A′ as the spill-in (given by $\frac{AS}{OS} \cdot R_\beta$) increases from zero to S_MZ_α.

The reaction curve for α is shown in Figure 6 (as $R_\alpha R_\alpha'$) along with the output reaction curve for β, derived in a procedure similar to that described for α above. The intersection of these two lines at Q* then becomes an equilibrium in the sense that it is consistent with the objective of each party to achieve the preferred outcomes at Z_α and Z_β. At Q* there will be spillover flows in both directions and an intergovernmental transfer of private goods to the surplus region. The important characteristic of Q* is that the spill-in benefit is equal

FIGURE 5

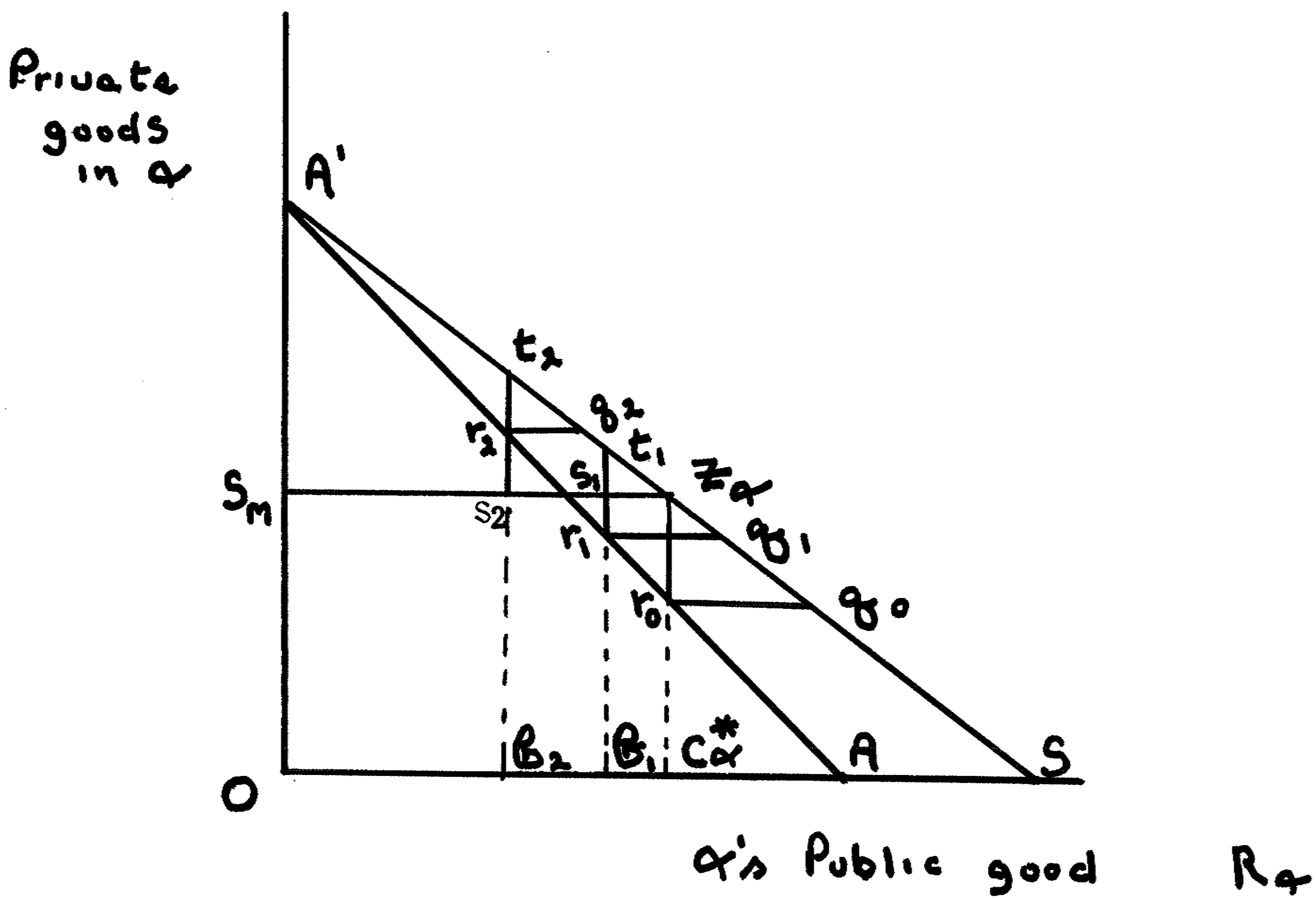
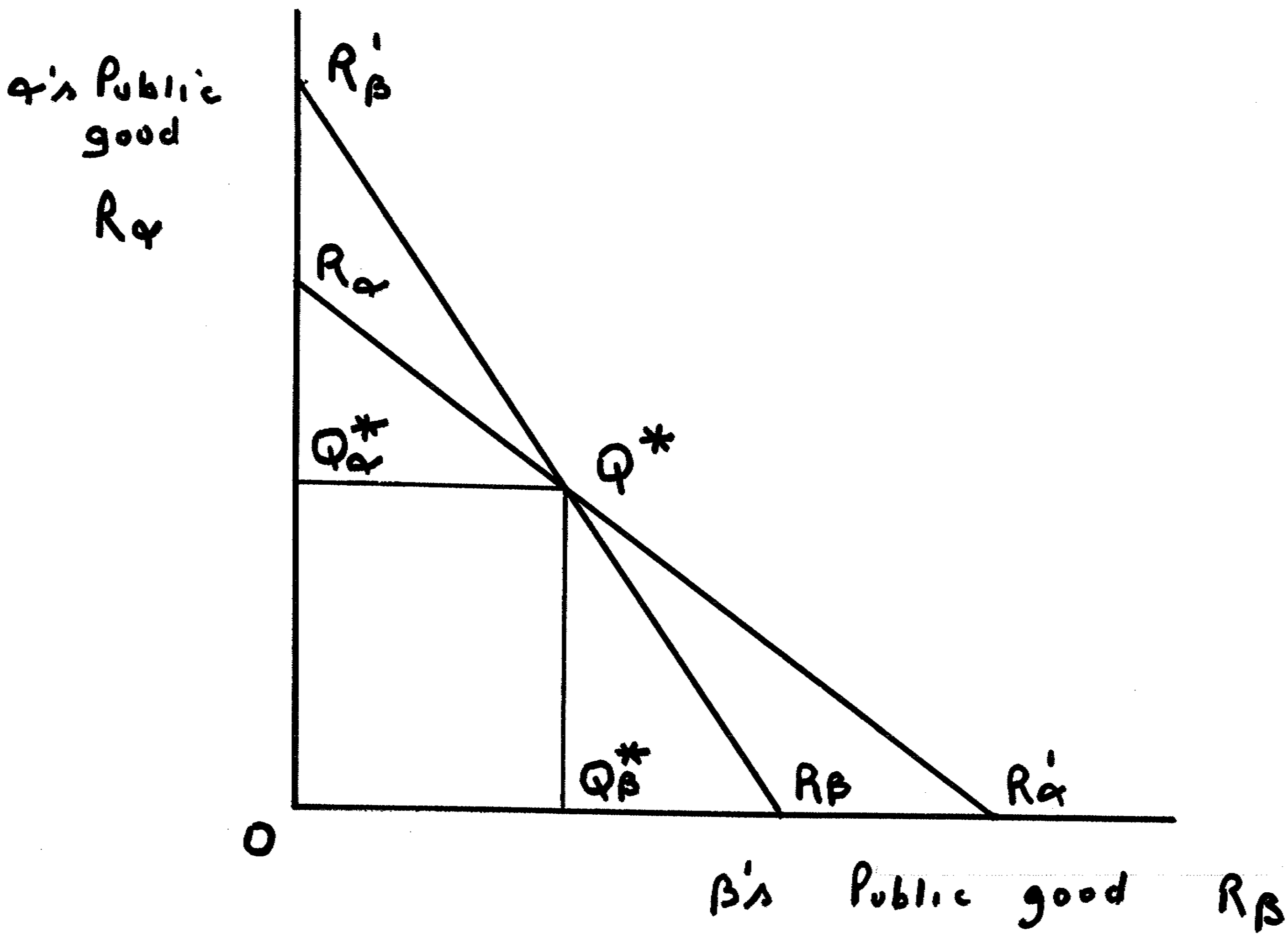

Private goods in α
A'
t2
r2
g2
t1
Sm
s1
Zα
S2
r1
g1
r0
g0
B2
B1
Cα*
A
S
O
α's Public good
Rα

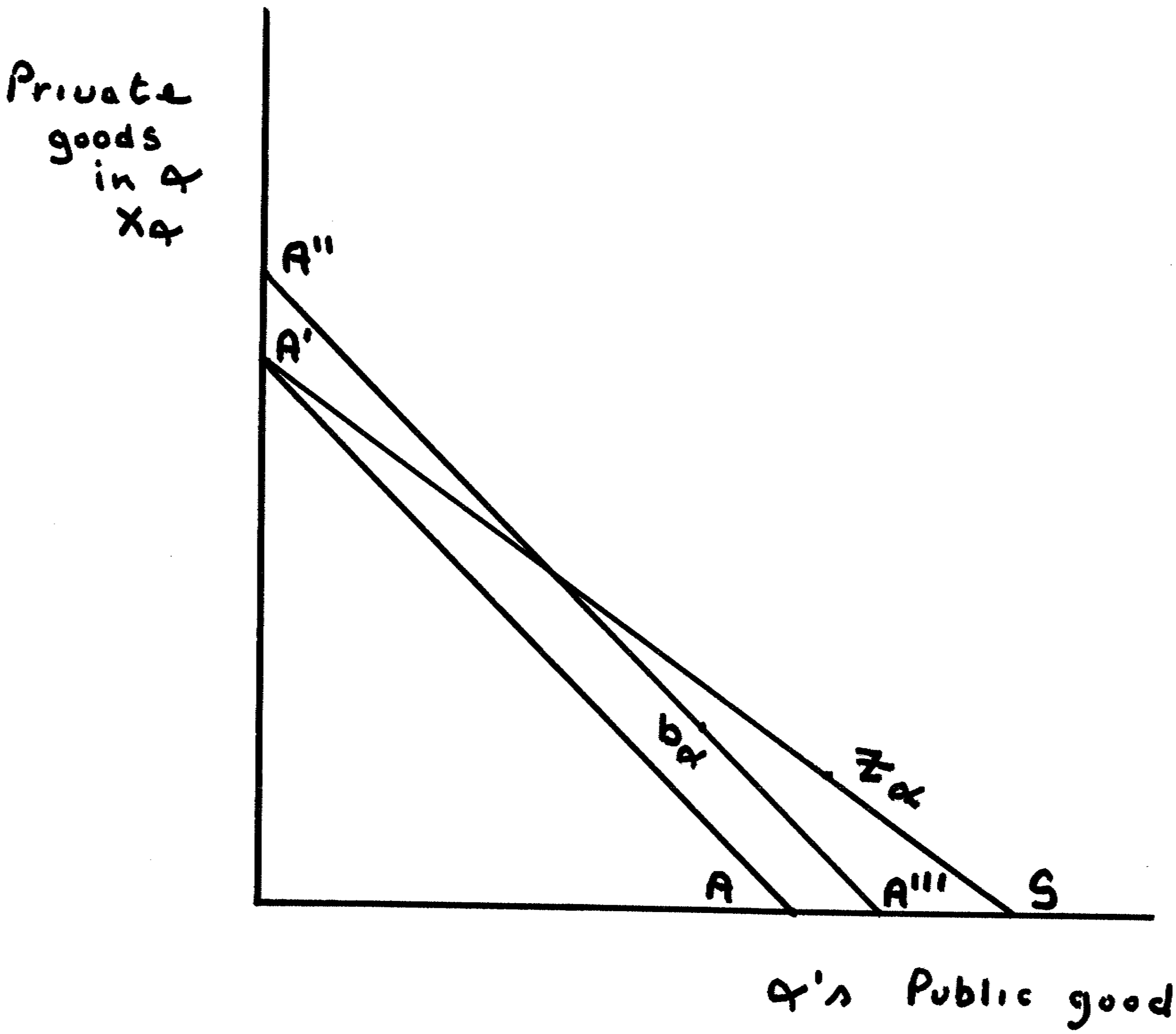
FIGURE 6



to the expected spill in and it is only when these are equal that each party may achieve its welfare optimum. But at no stage of the analysis is it possible for a party to suffer a loss in income. The compensation scheme is crucial here since it ensures that each party will always operate at some point on its opportunity line SA′ or $S_\beta A'_\beta$ (in Figure 4). Only at Q* in Figure 6 will both regions reach their respective welfare optima at Z_α and Z_β but even at non-equilibrium points they will be operating at the same level of real income, albeit at lower levels of welfare. It is the compensation scheme here that plays the crucial role in facilitating the optimizing reactive process.[4]

The process of compensated reactions is to be contrasted with the uncompensated reactions shown in Figures 2 and 3. These reactions are based on the neglect of own spill-out. The income line therefore shifts out parallel to AA′ (for α) and the independent (uncompensated) reaction equilibrium is defined by the mutual consistency of expectations. At this equilibrium the real income levels are likely to be quite different from what they would be at a compensated equilibrium (e.g. at some point along A′S for α). Consider Figure 7 and note that real income at b_α (which is potentially a point of independent adjustment equilibrium) is less than that at

FIGURE 7


Private goods in α X_α
A''
A'
b_α
Z_α
A
A'''
S
α's Public good

Z_α. Hence, if q_α and q_β are the output levels under independent adjustment equilibrium it should not be surprising if $q_\alpha + q_\beta \gtrless q(Z_\alpha) + q(Z_\beta) = q^*$. If α has a lower income elasticity of demand for the public good then β and if the uncompensated adjustment favours β then the outcome $q_\alpha + q_\beta > q^*$ is likely to result.[5] In other words a movement from the non-optimal set of outputs q_α and q_β to the Pareto optimal set associated with Z_α and Z_β may result in a reduction in aggregate provision of the public good. However the move will not be a Pareto optimal move since one party (β in the example given) will gain only at the expense of the other.

V. A REACTION MODEL FOR BENEFITS THAT ARE COMPLETELY NON-RIVALROUS AND RECIPROCAL

In the present section we consider independent and uncompensated adjustments between two regions when each produces a public good which becomes equally available to both regions. The Samuelson efficiency conditions require the following:

$$(1) \qquad \sum_{i \text{ in } \alpha} MRS^i_{R_\alpha X} + \sum_{i \text{ in } \beta} MRS^i_{R_\alpha X} = MRT_{R_\alpha X},$$

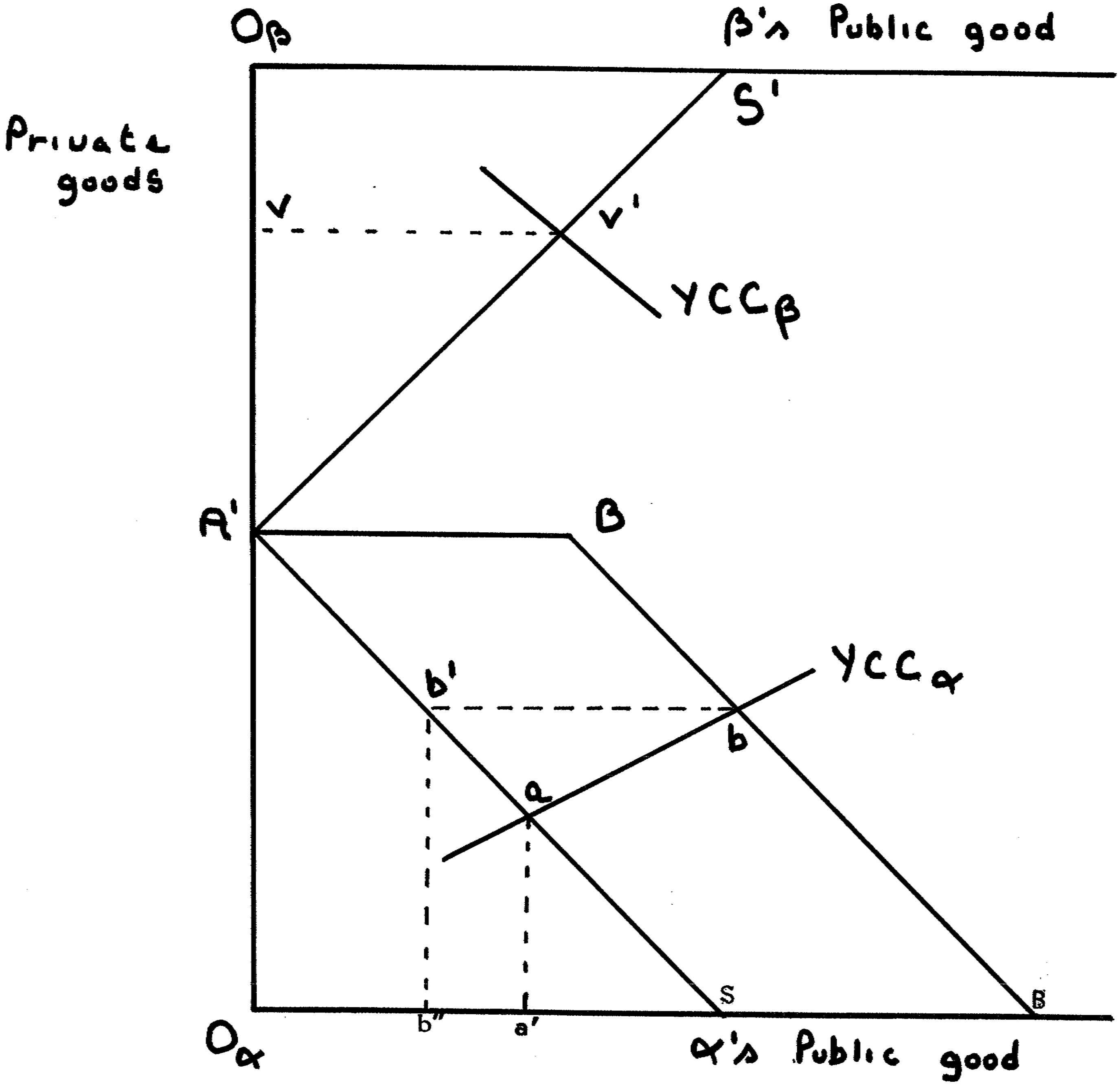
and

$$(2) \qquad \sum_{i \text{ in } \alpha} MRS^i_{R_\beta X} + \sum_{i \text{ in } \beta} MRS^i_{R_\beta X} = MRT_{R_\beta X}.$$

in what follows it will be shown that an independent reaction model typically leads to an equilibrium where $R_\alpha + R_\beta$ falls below the optimal output level. In the next section it will be shown that only by a compensated adjustment mechanism is it possible to achieve a Pareto optimal solution.

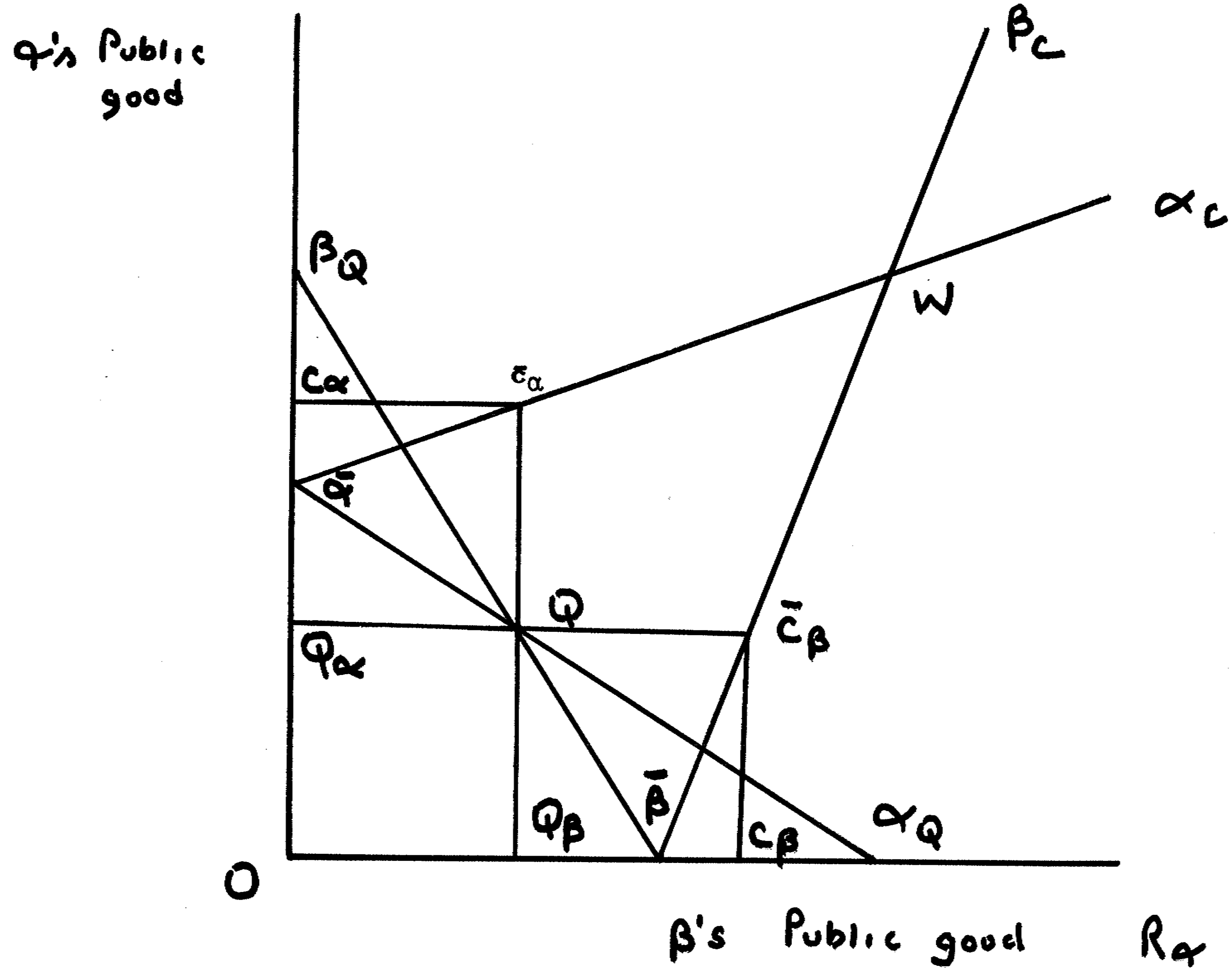
In Figure 8 we show the opportunity loci for both α and β as A'S and A'S', respectively. It is assumed that the cost of the public good, R, is identical in both regions, in terms of the numeraire private good. If β chooses to produce at v' then this will shift A'S out to BB', where A'B' is equal to vv'. This will shift α's equilibrium consumption position from a to b but will reduce α's own provision of R from $Q_\alpha a'$ to $Q_\alpha b''$. By finding α's desired consumption of R as its spill-in increases we get YCCα which enables us to derive consumption and production reaction curves for both regions, which are shown in Figure 9. The curves in this diagram are labelled identically as in Figure 3. Note that $\bar{\alpha}\alpha_c$ and $\bar{\beta}\beta_c$, the consumption reaction curves for α and β, represent consumption of $R_\alpha + R_\beta$. For example, consider point $\bar{c}_\alpha$ on $\bar{\alpha}\alpha_c$ and note that OC_α is equal to OQ_α, provided by α, and OQ_β, provided by β. As shown in the diagram Q is the independent adjustment equilibrium and it is easily seen that Q is stable.

FIGURE 8


Oβ
β's Public good
S'
Private goods
v
v'
YCCβ
A'
B
YCCα
b'
b
a
S
B
b''
a'
Oα
α's Public good

The question of stability in this model will be dealt with very briefly. The reaction curves of Figure 9 have been linearized to simplify this discussion. The assumption that R_α and R_β are perfect substitutes has been shown to imply that $\bar{c}_\alpha Q = Q_\alpha Q$. Let us write the equation for the output reaction curve $\bar{\alpha}\alpha_Q$ as $R_\alpha^Q = a - bR_\beta$ and the equation for the corresponding consumption reaction curve as $R_\alpha^c = R_\alpha^Q + R_\beta = a + R_\beta(1-b)$. The slope of the consumption reaction curve depends upon the income consumption line and the positive sign on this slope reflects the non-inferiority of the public good. If we assume that both private and public goods are non-inferior then we obtain a restriction on the slope of $\bar{\alpha}\alpha_c$. From α's point of view $R_\beta > 0$ implies an outward shift in its opportunity curve and hence an externally generated increase in real income. The curve $\bar{\alpha}\alpha_c$ (with slope $\frac{\Delta R_\alpha^c}{\Delta R_\beta}$) is tantamount to an Engel curve and our non-inferiority restrictions imply the following slope restriction; $0 \leq (1-b) \leq 1$.[6] If we require the strict inequality restriction on the slope of $\bar{\alpha}\alpha_c$ then we get the following associated restriction upon the slope of $\bar{\alpha}\alpha_Q$: $0 < b < 1$.[7] By analogous reasoning non-inferiority restricts the slope of $\beta_Q\bar{\beta}$ to be greater than one. Instability in the present model can be ruled out as long as $\bar{\alpha}\alpha_Q$ cuts $\beta_Q\bar{\beta}$ from below. Hence our non-inferiority

FIGURE 9


α's Public good
β_C
α_C
β_Q
W
C_α
ε_α
ᾱ
Q
C̄_β
Q_α
β̄
Q_β
C_β
α_Q
O
β's Public good
R_α

assumptions are sufficient to restrict equilibrium positions such as Q to those which are stable.

Before leaving the independent adjustment model here it is important to point out that Q, whether it be stable or unstable, is not a Pareto optimal position. At Q each of α and β is in equilibrium in the sense that it is equating its MRS with the price of the public good. But as long as $MRS^{\alpha} + MRS^{\beta} > P_R$ then the level of provision of the public good will be sub-optimal.

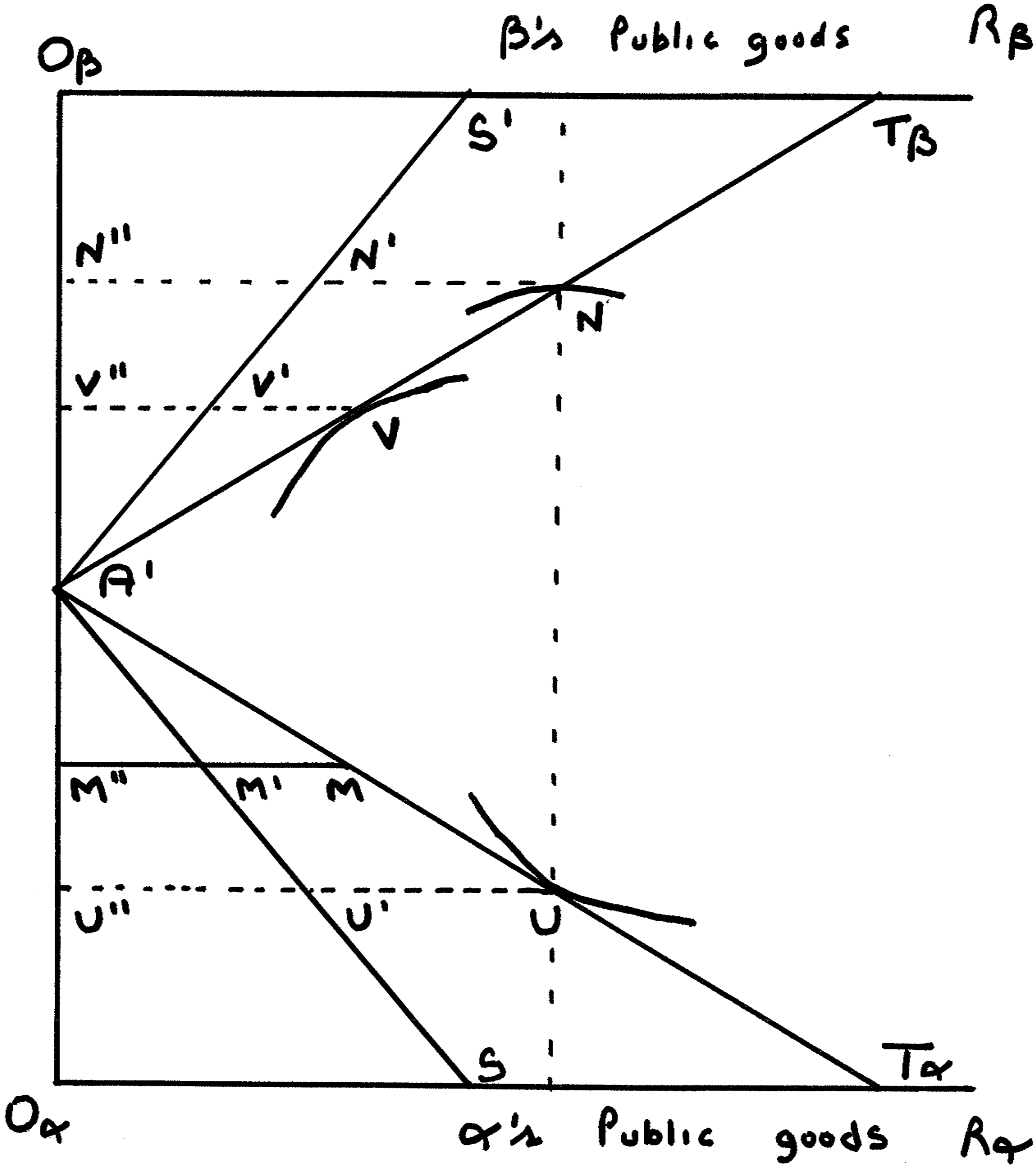
VI. COMPENSATED ADJUSTMENT WITH PUBLIC GOODS

As in the case of benefit spillovers for rivalrous consumption goods it is impossible to achieve optimality without compensation or trade. Participants in an adjustment process involving externalities must revise their choice calculus to incorporate the value of benefits and costs passed on by others and those created by their own activities. In the Williams case described above it was shown that a set of subsidies and taxes could be employed for this purpose.

In what follows various compensation schemes will be presented for the case of non-rivalrous spillovers for public goods which are perfect substitutes. The basic two region model will be retained, subject to the appropriate modification. In the Williams model the region creating spillover benefits was vested with property rights in the amount of spill-out benefit. The assumption of an identical production transformation rate between private and public goods in the two regions vested the model with an unambiguous numeraire.

In the case of non-rivalrous spillover benefits vestment of property rights is more difficult to achieve. Even more difficult is it to value the benefits (to an external beneficiary) since the number of beneficiaries may vary without detracting from own consumption. Even when it is assumed that the price of the public good is the same in both regions the problem remains. Assume that there exists some joint welfare optimizing body with the same duties as those possessed by the central government invoked by Williams. Then this body would have to know the MRS (of the public good available by spill-in, in terms of the private good) before assessing compensation. One solution would be to require that β pay α one-half of the cost of its output and vice versa. The adjustment process could then be defined in terms of Figure 10. The line A'S' reflects the cost of producing R_β and the introduction of α's 50 per cent subsidy shifts the price line from A'S' to $A'T_\beta$. We are assuming that subsidy receipts are in principle spendable on private or public goods. If β chooses to produce at v then both

FIGURE 10


Oβ
β's Public goods
Rβ
S'
Tβ
N''
N'
N
V''
V'
V
A'
M''
M'
M
U''
U'
U
S
Tα
Oα
α's Public goods
Rα

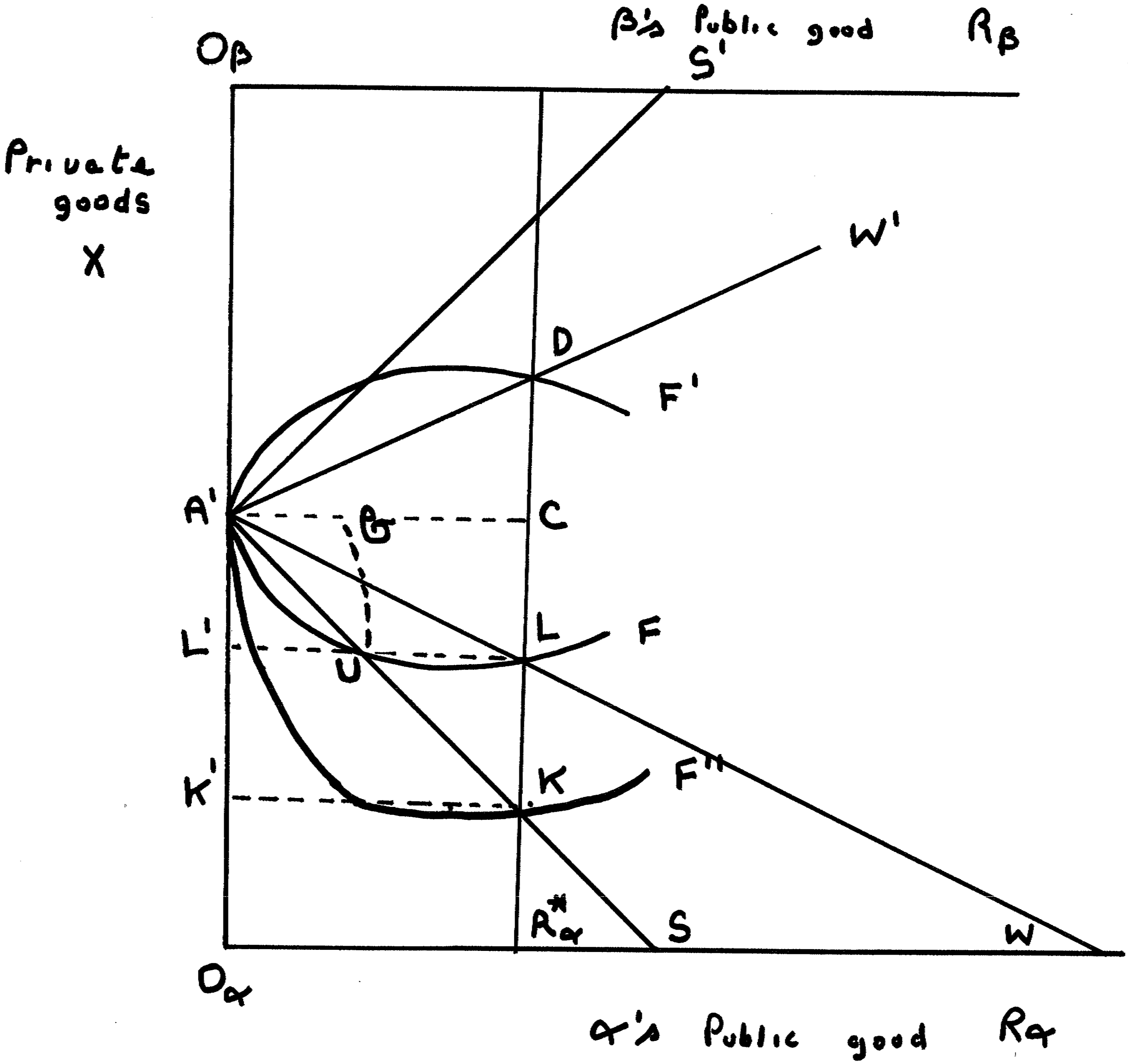
α and β will consume benefits in the amount VV″ and will contribute an equal amount to the cost. Since the true cost in terms of private goods is given by the slope of A′S′ (or A′S) then the cost of VV″ is V″M″ and the cost shares for α and β will be A′M″ and A′V″, respectively. The opportunity set for α now becomes $0\alpha T\alpha$MM″ since β has committed α to public good consumption of MM″ (=VV″) and to foregone private good consumption of A′M″. If M also represents α's preferred position along A′Tα then we have a Pareto optimal solution which turns out as well to be a Lindahl solution. A Lindahl solution is one in which the marginal rate of substitution of the public good for private goods is equal for each participant to his tax price.

In our example there is no reason to expect a Lindahl solution. Assume that α's preferred position is at point U. By its choice of U region α will have displaced β from its preferred position at V to a less preferred position at N. In fact at U and N the summed marginal rates of substitution will fall short of the price of the public good. Pareto optimality will not in general obtain and pressure will be forthcoming to alter the tax financing arrangements. If these tax arrangements are inflexible then some solution will be found at V or U, or somewhere in between. At any rate it should be clear that the compensation arrangement shown here will not in general be satisfactory to both parties. Enough has been said about this compensation scheme to point out the problems. The main problem stems from the difficulty in assigning property rights to public goods and the consequent ambiguous optimization to which arbitrary compensation schemes give rise.

Let us turn now to a more conventional tax and subsidy solution in the context of a bargaining process. It will be convenient first to consider a process based upon offers by one region to pay a uniform per unit subsidy to the other. In Figure 11 we reproduce the opportunity lines A′S and A′S′ of Figure 10. The curves AF and AF′ are the price consumption lines, or offer curves, of α and β, respectively. Let us assume that region β offers to pay α a subsidy so that α may provide the public good for both regions. Consider A′F′, the offer curve for β, and note that if we measure upward from A′ (i.e. we designate A′ as origin) then the ordinate represents the amount of private goods that β is willing to forego for the corresponding quantity of public goods. For example, at point D the amount DC measures the amount of private goods that β will sacrifice for A′C in public goods at an opportunity cost given by DC/A′C.

If the offer curve A′F′ is added vertically to

FIGURE II


β's Public good
R_β
O_β
S'
Private goods
X
W'
D
F'
A'
B
C
L'
L
F
U
K'
K
F''
R^*_α
S
W
O_α
α's Public good
R_α

A′F then the combined offer curve A′F″ represents the amount of private goods that the two regions together will sacrifice for a given provision of public goods. The point K represents a position of joint welfare maximization since K is the point at which the combined marginal rates of substitution are equal to the opportunity cost of the public good, in terms of the numeraire private good. If β pays α a subsidy of (100SW/0_αW per cent of unit cost then α will provide $0_\alpha R_\alpha^*$ units of public goods. This solution is not only Pareto-optimal. It is also a Lindahl solution since at L and D the tax cost per unit is equal to the sum of the marginal rates of substitution in α and β, respectively. The cost of producing $0_\alpha R_\alpha^*$ is A′K′ and β carries L′K′ of the cost.

Note that this is one possible solution that has the property that it is Pareto optimal. Each region is better off than it would have been in independent equilibrium. However, the joint solution at K is not the only Pareto-optimal solution associated with the given initial distribution of income. Its practical feasibility is suspect because of the complete specialization in the public good that it produces. In a realistic situation there are likely to be political reasons for existing jurisdictions to be loathe to give up their powers. We shall return to this line of reasoning.

Another practical problem that may be associated with a uniform subsidy is that demand for the public good in the subsidized region may be highly inelastic. The subsidy cost may be very high. Perhaps the most intractable case is that of the Giffen good and this is illustrated in Figure II by the offer curve A'UG (with UG broken line). If the public good were a Giffen good for α then the effect of β's subsidy would be to reduce the level of public good provision by α.

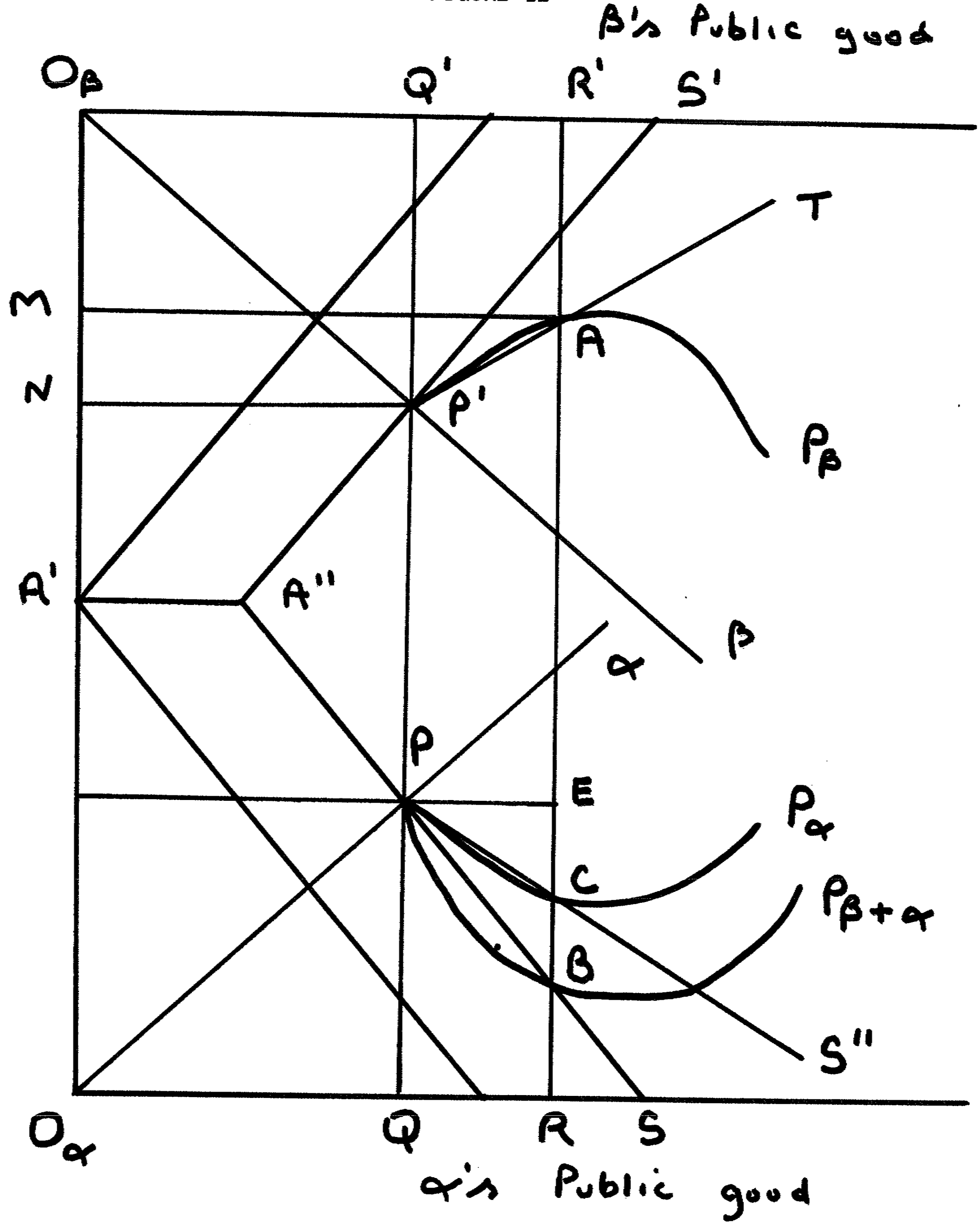
An alternative to the use of a linear (i.e. uniform on all units of production) subsidy system is a system using an increasing marginal subsidy rate. Such an increasing subsidy could be constructed so as to minimize the intra-marginal subsidy surplus and its purpose would be to maximize the stimulative effect per unit of subsidy. A continuously increasing (and efficient) marginal subsidy system would not only reduce the cost of the subsidy scheme to the donor but it would minimize the possibly adverse income effects (in the case of an inferior good). The above considerations will be relevant to the argument below in spite of the fact that the subsidy system treated is not a continuously increasing non-linear subsidy.

Let us turn now to an independent adjustment situation. Consider some equilibrium position that has been achieved by a non-compensated adjustment process between α and β. In Figure 12 the positions P and P′ have been reached by a process of action and reaction between α and β. It has already been shown that positions such as these are non-optimal since the sum of the marginal rates of substitution for each region is equal to the cost of the public good. Hence the summed marginal rates of substitution over both regions are equal to only one half of the rate of product transformation and both regions would benefit from an increase in the level of public good provision.

Positions P and P′ are points from which a trading process will enable a preferred position to be achieved by each region. In general, an efficient allocation of resources will be achieved if the region which produces the public good most efficiently is compensated for expanding its output. The result will be an exchange of private for public goods.

The process of trading and exchange may take place in a number of different ways. The bargaining strength of the participants in trade will be of crucial importance in determining the outcome. Bargaining in terms of each extra unit of output may enable the stronger participant to extract all the consumer surplus from the weaker. But it will be assumed here that bargaining

FIGURE 12


β's Public good
O_β
Q'
R'
S'
T
M
A
N
P'
P_β
A'
A''
β
α
P
E
P_α
C
$P_{\beta+\alpha}$
B
S''
O_α
Q
R
S
α's Public good

takes place in terms of a uniform subsidy for each unit produced beyond P. To illustrate this type of bargaining consider the offer curves PP_{α} and $P'P_{\beta}$, for α and β respectively, shown in Figure 12. Point A reflects the offer of an amount MN of private goods in exchange for an expansion in public goods provision from $O_{\beta}Q'$ to $Q_{\beta}R'$, at the relative prices reflected by the slope of P'AT. If we now add β's marginal offer curve $P'AP_{\beta}$ to that of α then we get the joint marginal offer curve $PP_{\beta+\alpha}$. For example, at point C, which corresponds to point A (on $P'P_{\beta}$) the joint offer curve is constructed by adding MN (in the direction of the R_{α} axis) to the ordinate of C so that MN = CB. Clearly, the new joint equilibrium appears at B, where the joint offer curve $PP_{\beta+\alpha}$ intersects the opportunity line PS. At B region α is paying a price indicated by PCS" while β is operating at the price given by P'AT.

Note that B is now a Pareto-optimal equilibrium position since at A the $\sum_{i \text{ in } \beta} MRS^{i}_{RX}$ is equal to the slope of P'AT while at C the $\sum_{i \text{ in } \alpha} MRS^{i}_{RX}$ is equal to the slope of PCS" and hence the slope of PBS is equal to the sum of the MRS_{RX} for both regions. By subsidizing region α region β attains a higher level of welfare while at the same time improving the welfare position of α.

This kind of marginal compensation process leads to incomplete specialization in the public good even when the transformation functions are linear. Incomplete specialization may also be invoked by specifying a neo-classical transformation function with decreasing returns. The intergovernmental compensation system that has been superimposed upon an independent adjustment equilibrium here is a constant per unit subsidy on "extra-marginal" output. It has been shown above that, in the context of the present model, a constant subsidy on all units produced may be used to achieve Pareto-optimality through specialization in the public good.

The Pareto optimal solution achieved in Figure 12 will be a different one from that achieved by the linear subsidy scheme shown in Figure 11. The final distribution of real income will differ between the two situations because of the difference in intra-marginal surplus associated with each. It can be shown that the marginal subsidy scheme achieves Pareto optimality with a smaller redistribution of real income, since the intra-marginal surplus is smaller than it is for the constant per unit subsidy scheme.

Another scheme which would achieve Pareto optimality at an even lower transfer cost than that associated with the marginal subsidy described above will be briefly discussed. This is an iterative scheme by which the subsidizing region offers to pay $\lambda_k(100)$ per cent of the cost of the public good (P_R) to the other region for an expansion of the other region's output beyond independent adjustment equilibrium output by some amount Q_k. These offers of λ_k (associated with Q_k) will be raised until they elicit a positive response and until $\lambda_k P_R$ is equal to the $\sum$ MRS of the paying government.

If R′ is the independent adjustment equilibrium output and R'_α is the corresponding output of α then this scheme involves an offer of $\lambda_k P_R Q_k^\alpha$ (where Q_k^α is some small increment beyond R'_α and P_R is the constant real cost per unit of R). Region α will accept the offer if:

$$\sum_{i \text{ in } \alpha} MRS_{RX}^{\alpha}\Big|_{R=R'} > (1-\lambda_k) P_R ,$$

and will continue to accept such offers for every additional λ_k and Q_k^α until

$$\sum_{i \text{ in } \alpha} MRS_{RX}^{\alpha} = (1-\lambda_k) P_R .$$

This kind of iterative scheme, as well as the constant marginal subsidy scheme described above, has the advantage that it lowers the income effect on intramarginal units. This lowers the transfer cost and prevents the subsidy plan from being thwarted by the inferiority of the subsidized good (to the producing region).

The introduction of decreasing returns and of imperfect substitution between the two public goods would alter the nature of the solution and enhance the economic case for incomplete specialization. This is likely to be true as well when our model is modified to encompass variable substitutability between the two public goods in each of the two regions. At low levels of own provision the substitutability between own and external provision may be quite low. The question of complete spcialization is likely to be associated with the question of local autonomy. In constructing a model to deal with external effects in the context of a federal system there are important positive and normative reasons why Pareto-optimal movements should focus around marginal adjustments to a private equilibrium. It makes more sense to talk in terms of mutually satisfactory marginal changes, rather than in terms of structural changes. Recognition of interdependence in terms of the possibility of joint consumption leads to expansion of public good provision.

The duopoly approach to federalism is not really necessary, although it was helpful in our graphical analysis. It is possible to use the models described in this paper to deal with the relations between a central, federal government and the member jurisdictions of the federation. The federal government may be viewed as the responsible agency for joint welfare maximization. Its role in internalizing external effects between jurisdictions would then be performed by a system of subsidies.[8] When these subsidies are financed by an "imperfect" tax system, which produces "distortions" in economic choices, then the onus will be upon the federal government to use its subsidy funds in the most efficient manner possible. This means that it will have strong motivation to minimize intra-marginal surpluses and to employ the marginal subsidy schemes outlined above. In a forthcoming paper this writer will provide further cogency for such marginal subsidy schemes, which have minimal redistributive effects.

When the federal government performs the role of Paretian guardian angel it is acting as a cooperative agency for the citizens of its member jurisdictions. It will typically assume this role when the direct bargaining costs of the optimizing adjustment process are relatively high, and this situation will normally obtain when there are more than two jurisdictions. It is the absence of direct recontracting between parties that facilitates a subsidy (without corresponding taxation of beneficiary regions) or unilateral solution.[9]

VII. CONCLUSION

The purpose of this paper has been to show how intergovernmental transfers may be used to achieve Pareto optimality. In the case of rivalrous spillover benefits it is a simple matter to establish property rights in the spillover. Real income is capable of unambiguous evaluation in terms of a numeraire private good. A bargaining model following that of Williams can be constructed to illustrate the reactive process that leads to optimality.

The case of non-rivalrous spill-outs is more difficult. A model has been developed to deal with the case of non-rivalrous spillover where benefit spill-out is complete and where all public goods are perfect substitutes for one another. It has been shown that non-compensated adjustments will lead to a sub-optimal provision of public goods, as was found to be the case for rivalrous spillovers. Starting from some independent equilibrium, with sub-optimal provision achieved by an uncompensated adjustment process, it was shown how the introduction of explicit trade may lead to Pareto optimality. Whatever may be the mechanism for intergovernmental compensation, whether it be a central government vested with the authority and the information to tax and subsidize, or simply a process of direct bargaining, the compensating flows are essential in achieving optimality.

This is true when the externality is a partial spillover of a rivalrous benefit and also when the spillover is non-rivalrous and complete. The introduction of explicit trade and compensation enables an optimal solution to be achieved by voluntary mutual agreement.

A large part of the paper has been devoted to the exploration of spillover effects in the context of a two region public goods model. A theoretical treatment of various marginal subsidy schemes was presented and these schemes were compared with optimizing uniform subsidies. It was shown that the latter type of subsidy was likely to be less effective in dealing with external effects than marginal subsidies. The analysis was also generalized to deal with conditional grants made by federal governments to regional jurisdictions.

FOOTNOTES

1. The problems to which Williams addresses his paper arise in the context of what Musgrave (1969) refers to as "rivalrous" spillover benefits.

2. In another paper the present author presents a general model where the size of the resident population in each region and the output of local public goods are simultaneously determined. See Vardy (1971).

3. Note that this is not a necessary assumption of the model but is adopted here so that the analysis of Williams may be followed closely.

4. The assumption of identical transformation curves in this model enables the compensation to take place in terms of private goods whose prices are identical in both regions. If neo-classical production conditions were assumed along with trade in private goods then again the model would not lack a numeraire.

5. This is in fact the result that Williams reaches (1966, p. 31).

6. This states that an increase in the availability of R_β will not lead to a reduction in α's consumption of private goods and will lead to an increased consumption of public goods no greater than the amount of the spill-in.

7. The strict inequality rules out the possibility that the income elasticity of demand may be zero, for public or private goods.

8. If λ_k^{fj} is the subsidy share paid to the j-th jurisdiction for each Q_k^j expansion in its public good then in equilibrium the marginal cost to the j-th jurisdiction will be $(1-\lambda_k^{fj})\ P_R^j$. This analysis reflects a slightly less nihilistic approach to the economics of federalism than does the tone of the following comment by Anthony Scott (1952):

   > If the satisfactions of the central community are greater when the province spends its grants on (say) higher education than when it endows separate schools, and the order of the satisfactions of the province runs in the opposite direction, we have a deadlock welfare situation analogous to two points on a contract curve, about which very little of interest can be said by economists. (Scott, 1952, p. 392).

9. Buchanan and Stubblebine (1962) and Ralph Turvey (1963) deal with small number (two participants) externality problems where a bilateral (tax and subsidy) scheme is required. A unilateral (tax or subsidy) approach would be thwarted by renegotiation to a non-optimal solution.

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