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EFFECTIVE PROTECTION, TRANSPORTATION COSTS, AND THE LOCATION OF FIRMS

John M. Hartwick
Queen's University

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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I. Introduction

A group of well-known models in location theory can be classified according to three salient properties - the presence or absence of homogeneity in the geographic space with respect to first the distribution of consumers and secondly the distribution of inputs for production, and thirdly the presence or absence of boundedness of the geographic space. The model developed in this paper can be readily compared to existing models by noting how it fits into a classification scheme based on the above qualities. It will be shown that this new model, based on the notion of effective protection developed in the literature on tariff protection in international trade, is akin in certain respects to the classic Weber problem. The new model will be referred to as the effective protection model.

At the outset, we shall investigate how this new model relates to existing models in location theory. The basics of the model will be developed and then compared formally to the Weber problem. The location model will be extended to take account of competition among many firms. Notes on the relevance of the model to international trade theory will

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be presented and concluding comments will be made on policy considerations and the place of the model in a spatial general equilibrium context.

II. Models of the Location of Firms

We shall neglect models of the Cournot - Enke - Samuelson genre[1] or of the von Thünen type which focus on spatial pricing as opposed to location of firms in this classification. Throughout, we shall consider space as homogeneous with respect to transportation; that is transportation costs will be assumed to be independent of route or direction and depend only on distance and type of item hauled within the confines of the geographic space. Hills and valleys, lakes and rivers do not figure in these models. We shall assume competition to be atomistic and shortsighted throughout and so collusion among competitors is ruled out as is a location strategy based on multiple time period considerations. Atomistic competition can result in very different models if relocation costs are included in one case and excluded in another. For example, Samuelson's model [10] relies essentially on costs of relocation being sufficiently high to rule relocation out. In this case the sequence of entry determines the pattern. The model in this paper does not consider competitive readjustment and in this sense is similar to Samuelson's. Samuelson sought to emphasize the pathologies inherent in spatial competition with relocation costs.

By homogeneity of consumers, we mean that consuming units dot the geographic space in equal density throughout and that each consuming unit has the same demand function. Heterogeneity can occur here from two sources. First consumers may be concentrated at points in the plain as in Weber's model [13] and secondly consumers' demand functions may be different depending on where they are located as in the Devletoglou model [3]. By homogeneity of in the distribution of inputs, we mean that costs of production are independent of location. In other words, cost functions are the same for any producer wherever he locates in the space. Heterogeneity can arise by having inputs available from fixed points in the space and thus causing costs of production to vary with location. Again Weber's is an example of a model with this latter property.

Geographic spaces without bounds are familiar fictions in location theory. They are useful abstractions for analyzing certain types of phenomena. However it is difficult to conceive of a competitive process taking place in such a space because a new entrant or competitor can simply ignore his rival if the space is homogeneous with respect to costs and demands and locate "far away". Hotelling's [5] beach of given length, the circle, the sphere are all spaces specifically utilized because they are of finite dimension. Lösch when he dealt with hexagonal market areas [7; p.112] and Devletoglou, dealing with duopoly, considered fairly arbitrarily defined geographic spaces of finite area. Mills and Lav [8] dealt with an unbounded homogeneous geographic space.[2]

This brief digression into a comparison of a number of models is intended to be a cautionary exercise rather than as a new synthesis.

TABLE 1

Classification of Firm Location Models

Homogeneity

		Consumers			
		Yes		No	
		Costs		Costs	
		Yes	No	Yes	No
Boundedness	Yes	Hotelling-Smithies Lösch		Devletoglou	
	No	Mills-Lav			Effective-Protection Weber

Note: All models mentioned above except Smithies [11] have been referred to above in the text.

Location models of firms must have incorporated a certain mix of the above qualities in order for the models to be properly anchored in geographic space and economic theory.[3]

The classic Weber Problem treats markets as points fixed in the plain, markets for both transportable inputs and outputs, and sets out to define how to determine where a producer might locate so as to minimize the total transportation bill associated with the production of a unit of output. Constant returns to scale in production and fixed input coefficients are assumed data in the problem. Figure 1 below illustrates the nature of the problem.

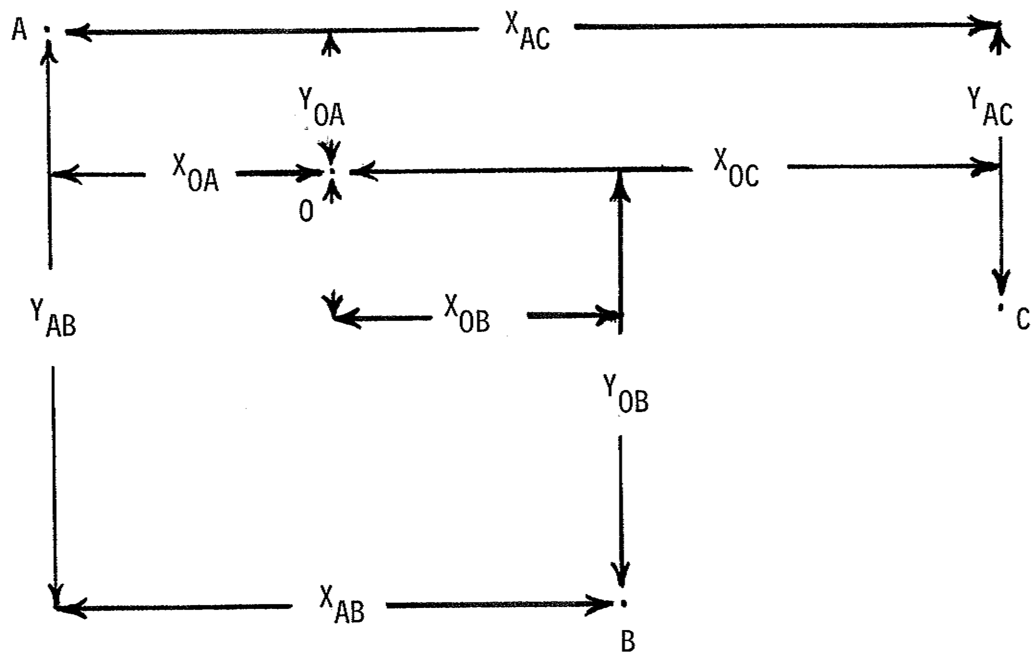

Figure 1.

Weber Problem

A producer will find the point of minimum transportation costs per unit output at O.

The point A in Figure 1 is a market for good j which a firm plans to produce. At points B and C are markets for supplies of inputs required to produce good j. The number of points is arbitrary. Any number greater than 2, one goods market and one input market, will suffice. The producer will locate at point O, somewhere within the area enclosed by three lines joining points A, B, and C. The producer will strive to be close to markets for supplies and output in order to minimize the transportation costs for a unit of output. Close means simultaneously as near each point as will yield a minimization of transportation costs.

In the effective protection problem, we have supply points fixed plus one competitor fixed. Later we shall take up the case of many competitors. Consumers dot the plane homogeneously and each has a demand curve linear in delivered price. Delivered price is price fob plus transport costs. With a linear demand curve, we find that demand drops to zero beyond some distance from the producer.[4] Now at any point, an entrant will have a market given surrounding his plant. What he will seek to do is to keep as far away from his competitor as possible in order to prevent his having his market invaded and as close to his supply points as possible. If we treat initial location point and market area determination separately, which it will be indicated is valid, then we have a straightforward optimization problem the solution of which will indicate where a new entrant should locate so as to maximize his profits. Figure 2 below indicates the nature of this problem.

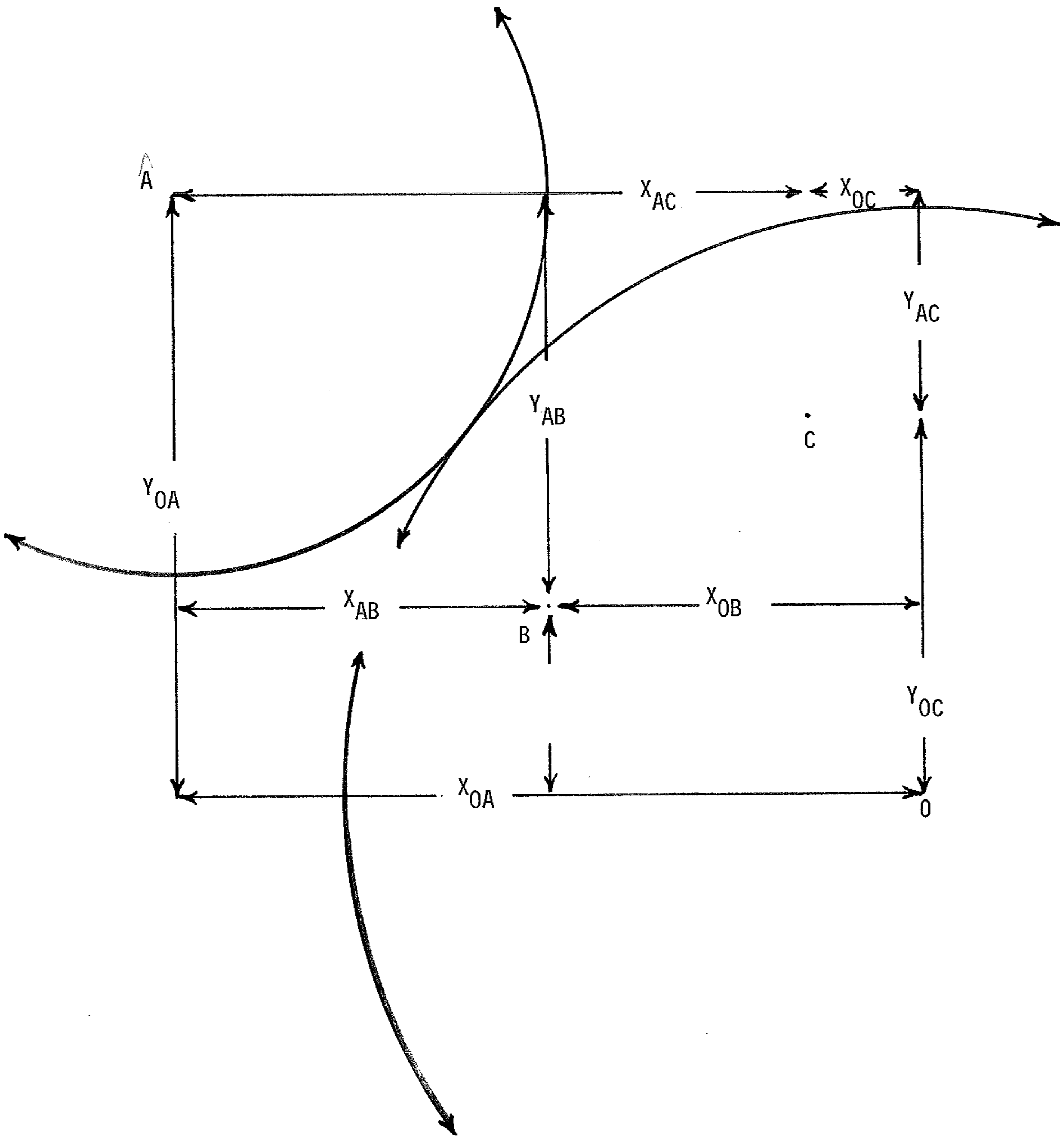

Figure 2.

Effective Protection Problem

A new entrant is maximizing excess profits when located at 0. B and C are supply points for inputs and at point A is located a competitor.

The point A in Figure 2 is the location of a competitor producing good j which our entrant plans to produce. At B and C are markets for supplies required to produce j. Any number of supply points greater than one is required as we shall see below. The producer will locate at 0 in order to maximize his potential excess profits or effective protection. The circles indicate market areas.

The reason that the model is called one of effective protection is because it was inspired by that concept. Johnson [6] defines effective natural protection (ENP) for a production process j in a country as follows. He follows Balassa [1].

$$\mu_j = \frac{d_j t_j - \sum_i^n a_{ij} \, d_i t_i}{1 - \sum_i^n a_{ij}} \qquad (1)$$

where t_j is the cost of transporting a unit of good j a unit distance.

d_j is the distance from one country producing j to another.

t_i is the cost of transporting a unit of the i^{th} input a unit distance.

d_i is the distance from one country producing j to the other.

a_{ij} is the amount of input i required to produce a unit of j.

Definition (1) is the ratio of value-added accruing to process j when it is protected by transportation costs to the value-added accruing to process j if there were no protection afforded by distance. If we add capital and labor requirements plus taxes to the negative term in the numerator of 1, we have a rate of excess profit. Without the denominator, we have excess profit. By letting an entrant vary his location, the distances become variables and excess profits will vary. There will exist a location where excess profits are at a maximum. Such a point was 0 in Figure 2. We will now examine the nature of the problem of determining 0, review the procedure for determining 0 in the Weber Problem and then turn to the case of many competitors in the effective protection model.

III. Maximum Profit Location

Let us consider the problem of locating the point in the plain of maximum effective protection or maximum excess profits. We assume that there is a linear fixed coefficient production function for product j. We assume that there are spatially fixed and separated suppliers of inputs into process j as well as one competitor supplying j from a fixed production point. A producer pays all transportation costs on inputs and outputs. Inputs are available in infinitely elastic supply at fixed prices. The price of an output f.o.b. at any point will be determined by the price at which the given competitor can sell at that point.

Consider the following simplified example. It is diagrammatically presented in Figure 3. At point A is a producer of j. At points B and C, equidistant from A, are two input suppliers and the two inputs are assumed to be required in equal physical amounts to produce a unit of j at a new

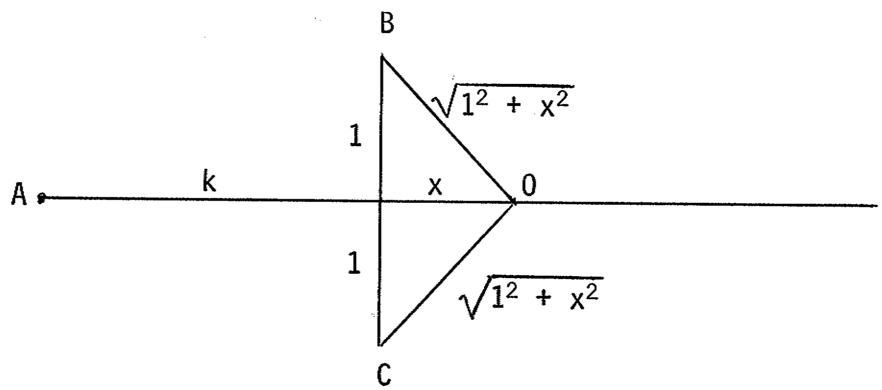

Figure 3

A new entrant will locate at 0, to be close to suppliers at B and C and far from his competitor at A.

production point. Where is the point of maximum excess profits per unit for the new producer? It will be as far from his competitor at A as is compatible with being close to input suppliers at B and C. See Figure 3. Let all transportation costs be \$1 per unit distance and let 1 unit of supplies at B and C be required in the production of a unit of output at 0. Excess profits per unit at the new site will be

$$y = k + x - 2\sqrt{1^2 + x^2}$$

Maximizing y with respect to x,

$$\frac{dy}{dx} = 1 - \frac{2x}{(1^2 + x^2)^{\frac{1}{2}}} = 0$$

or

$$x = \pm\sqrt{\frac{1}{3}}$$

We assume x is positive. Entrant will locate at point 0 where $x = +\sqrt{\frac{1}{3}}$.

<u>Case 1</u>: Transportation costs from the competitor at A rise from \$1 to \$1.5 per distance unit.

We find x rises to $x_1 = +\sqrt{\frac{2.25}{1.75}}$

Observe that if transportation costs from A rise to \$2 per unit or above, then there is no maximum to our problem with a real number for x. Economically, it means that a new entrant should locate infinitely far away. In general if transportation costs are based on physical measure, say pounds per distance unit, and are the same for all similar distances in the plain, then the weight of inputs per unit output always exceed the weight of a unit of output[5].

Case 2: Transportation costs rise from \$1 to \$5 per distance unit between B and 0 and A and 0.

We find x falls relative to its value in the original problem to

$$x_2 = +\sqrt{\frac{1}{99}}$$

The new entrant will desire to be closer to the suppliers at B and C and hence closer to the competitor at A.

It is interesting to note that if all supply points are collinear with the competitor, then x is either 0 or infinite. Thus the case with a competitor and one required input has only the trivial solution.

The generalized problem of locating a producer involves supply points not equidistant from the competitor and the possibility of different rates of transport on all flows between any two points as well as different physical quantities of inputs in the production of a unit of j from different suppliers. The simplifications made in the analysis immediately above must be removed in a general problem. The number of supply points does not affect the problem's qualitative nature provided this number is at least two.

Assume once again that there is a single competitor, located at A, and suppliers of inputs to a potential entrant located at B and C in Figure 2 above.

Point 0 is defined with respect to A, B, and C by three pairs of (x, y) co-ordinates shown in Figure 2. For example

$$y_{OA} + y_{OC} = \bar{y}_{AC}$$

where y_{AC} is fixed and y_{OC} will be a negative number. We have similar relations for y_{BC}, x_{AB}, and x_{AC}.

Our problem is one of maximizing excess profits by varying the prospective location O with respect to A, B, and C. Our objective function is to maximize excess profit per unit output. We can ignore any elements of excess profit not involving distance explicitly since these constants will not affect the optimal location. To simplify the maximand we will assume that all intermediate inputs are imported and that all primary inputs are either available at the final site O or are not required to produce j. The distance d_{OA} between point O and A becomes:

$$(x_{OA}^2 + y_{OA}^2)^{\frac{1}{2}}$$

Similarly for d_{OB} and d_{OC}.

We get the following problem corresponding to the diagrammatic presentation in Figure 2.

maximize

$$t_{OA}\,(x_{OA}^2 + y_{OA}^2)^{\frac{1}{2}} - t_{OB}\,a_{Bj}\,(x_{OB}^2 + y_{OB}^2)^{\frac{1}{2}} - t_{OC}\,a_{Cj}\,(x_{OC}^2 + y_{OC}^2)^{\frac{1}{2}} \tag{1a}$$

subject to:

$$\begin{aligned} y_{OA} - y_{OC} &= \bar{y}_{AC} \\ y_{OA} - y_{OB} &= \bar{y}_{BC} \\ x_{OA} - x_{OC} &= \bar{x}_{AC} \\ x_{OA} - x_{OB} &= \bar{x}_{AB} \end{aligned} \tag{1b}$$

where

t_{OA}, t_{OB}, t_{OC} are transportation costs per unit distance from points A, B, and C respectively to point O for good j, and supplies at B and C respectively.

a_{Bj} and a_{Cj} are coefficients relating the physical amount of input supplied at B and C respectively to a unit output of commodity j.

x_{OA}, x_{OB}, and x_{OC} are the horizontal distances from point O to points A, B and C respectively.

y_{OA}, y_{OB}, and y_{OC} are the vertical distances from point O to points A, B and C respectively.

x_{AC}, x_{AB}, y_{AC}, y_{AB} are fixed co-ordinates of points in Figure 2 defined with respect to point A.

A numerical example is presented in the Appendix.

The problem in (1) is expressed as a constrained maximization problem. Because of its simple form, we could substitute the constraints into the objective function and treat the problem as one of unconstrained maximization. However by treating the problem as one of constrained maximization we can determine the equilibrium conditions of maximization in terms of Lagrangian multipliers which will have the usual economic interpretation as shadow prices. We introduce four Lagrangian multipliers, λ_1, λ_2, λ_3, and λ_4, each corresponding to one of the four respective constraint equations in (1).

The Lagrangian function L is:

$$L = t_{OA}(x_{OA}^2 + y_{OA}^2)^{\frac{1}{2}} - t_{OB}\, a_{Bj}(x_{OB}^2 + y_{OB}^2)^{\frac{1}{2}} - t_{OC}\, a_{Cj}(x_{OC}^2 + y_{OC}^2)^{\frac{1}{2}}$$

$$+ \lambda_1(y_{OA} - y_{OC} - \overline{y}_{AC}) + \lambda_2(y_{OA} - y_{OB} - \overline{y}_{BC}) + \lambda_3(x_{OA} - x_{OC} - \overline{x}_{AC})$$

$$+ \lambda_4(x_{OA} - x_{OB} - \overline{x}_{AB})$$

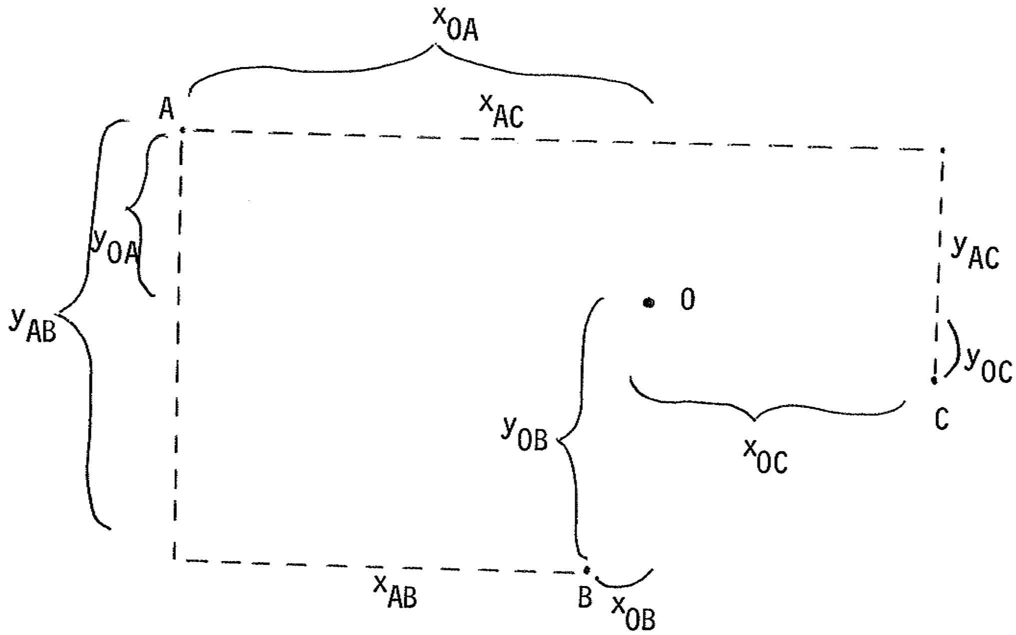

Figure 4.

The Weber Problem: A producer will find the point of minimum transportation costs per unit output at 0.

At point A is a market for good j. At points B and C are markets for supplies of inputs required to produce good j. The producer will locate at point 0, somewhere within the area enclosed by three lines joining points A, B, and C. The producer will strive to be close to markets for supplies and the output in order to minimize the transportation costs for a unit of output.

Mathematically, the problem is another simple optimization problem which, because of its simple form, can be treated either as a constrained minimization problem or an unconstrained one. Let us define it as a constrained one.

Minimize $\quad t_{OA}(x_{OA}^2 + y_{OA}^2)^{\frac{1}{2}} + t_{OB}\ a_{Bj}(x_{OB}^2 + y_{OB}^2)^{\frac{1}{2}} + t_{OC}\ a_{Cj}(x_{OC}^2 + y_{OC}^2)^{\frac{1}{2}}$

subject to:

$$\begin{aligned} x_{OA} + x_{OB} &= \overline{x}_{AB} \\ x_{OA} - x_{OC} &= \overline{x}_{AC} \\ y_{OA} + y_{OC} &= \overline{y}_{AC} \\ y_{OA} + y_{OB} &= \overline{y}_{AB} \end{aligned}$$

where the variables have the same definitions as those given following problem (4).

We could define a Lagrangian function and determine the first order equilibrium conditions for a minimum in a manner similar to that above in Section II.2. Again the Lagrangian multipliers would be shadow prices on the locations of points A, B, and C.

Weber developed iso-valued contours in his exposition. Each contour linked points in the plain corresponding to equally valued total transportation costs per unit of output. In Figure 4, the contours would be concentric with point 0 contained within each contour, point 0 being the location of minimum total transportation costs per unit output. Weber labelled these iso-transportation cost contours "isodapanes".

II.4 Both the Weber Problem and the natural protection location problem neglect the demand side of a more general location model. In the Weber Problem, the optimal location should depend ultimately on where profits are maximized. Profits will depend on gross revenues which in turn will depend on the size of the market for the product. Lösch addressed himself to this deficiency of the Weber model. In fact large sections of Losch's classic

Location Theory are directed to the development of the demand side of a location model.

In the location model dealing with effective natural protection, the neglect of the demand side also detracts from the general usefulness of the model in predicting the location of firms. The model requires that there be a market for the final product either at the point of production or that consumers are willing to pay the transportation costs on the product when the product is supplied by the new producer. Furthermore, maximizing profit per cent of output is an inadequate motivation for a firm. Gross profits are the important consideration for a firm and they depend on the size of the market. There is a final problem and that is defining bounds to the market available to two competitors. If consumers for a final product were homogeneously distributed on a plain then as long as producers were earning positive profits per unit of output it would pay them to move farther and farther apart in order to assume a larger and larger market where the other firm was not competing.

The problem dealing with natural protection is a more general version of Hotelling's well-known duopoly model. It is more general because the competitors have a market extending in two dimensions in the plain rather than simple in one dimension along a line or beach as Hotelling considered it. The above problem presents difficulties for an economic interpretation since the market size is unbounded.

This latter problem will be eliminated when we examine the more realistic case of the decision to locate when there is a plurality of competitors. See Section III.1.

With reference to the problem concerning the proximity of consumers to the point of maximum profit per unit output, we can assume that there are indeed consumers on hand.

II.5. Assume now that we have a geographically fixed point representing a new market for good j. A new entrant is going to locate to supply this market when one competitor is already present. Following Weber, we assume that the supplier pays the transportation costs of getting the product to the market as well as paying the transportation costs for inputs. This problem differs from Weber's in the respect that we have a competitor already present. It is easy to see that the presence of a competitor actually leaves the Weber problem unchanged.

We simply ignore the competitor in the first instance and locate the entrant where unit transportation costs are at a minimum as before. We then calculate the minimum unit price including transportation costs at which the competitor could supply the new market. Our entrant must charge a price marginally lower than that which his competitor would charge and his problem is solved. If the selling price per unit minus costs per unit is negative, then an entrant will never choose to supply the new market.

SECTION III. LOCATION FOR MANY FIRMS

III.1. To this point we have restricted our analysis of location and natural protection to the case where a prospective locator had only to consider the relative position of one competitor in his decision as to where to locate. Let us turn to examining the case where there are many competitors distributed in a plain as well as many suppliers of intermediate inputs.

The optimum location for a new firm entering turns out to be at a point where the rate of natural protection with respect to each of the three nearest competitors is equal for the firm entering. In other words, the new firm, placed in its optimum location could not be moved in any direction without lowering its potential excess profits as a result of competition from one of its previously positioned rivals. Its excess profits defined with respect to one or two competitors may of course be larger because it has moved further away from certain competitors in a shift from its optimum, but this fact is irrelevant given the fact that the competitor to which it has moved closer can always evade apparent gains in potential excess profits to a point where the excess profits would be lower than what the firm could have gleaned when it was optimally located.

Ideally, the firm must compute effective rate of natural protection contours with respect to every competitor in the plain and then determine points of intersection of three equal valued ERNP contours for all combinations of three adjacent competitors. These points of intersection will reveal the maximum excess profit the new firm could enjoy if it were located at that point. All points of intersection could be ranked and the one with the highest excess profit would be selected.

We will only consider how a firm determined one such point given three competitors amongst which it has decided to produce.

It selects the supply points for the required intermediate inputs. It then calculates a map of excess profit contours given the supply points and a fixed competitor. We examine this procedure in Section II. However, given a plurality of competitors, the new firm is not free to locate at the point of maximum excess profit since it will in general be vulnerable to

The Weber Problem is a model of only a small part of the location problem. The level of output is left undetermined unless a saturation bound is placed on consumption at the points of consumption. Profits are undetermined also. Lösch develops these points clearly.

Assume now that we have a geographically fixed point representing a new market for good j rather than consumers dotting the plain homogeneously. A new entrant is going to locate to supply this market when one competitor is already present. Following Weber, we assume that the supplier pays the transportation costs of getting the product to the market as well as paying the transportation costs for inputs. This problem differs from Weber's in the respect that we have a competitor already present. It is easy to see that the presence of a competitor actually leaves the Weber problem unchanged.

We simply ignore the competitor in the first instance and locate the entrant where unit transportation costs are at a minimum as before. We then calculate the minimum unit price including transportation costs at which the competitor could supply the new market. Our entrant must charge a price marginally lower than that which his competitor would charge and his problem is solved. We ignore retaliation. If the selling price per unit minus costs per unit is negative, then an entrant will never choose to supply the new market.

IV. Location with Many Competitors

To this point we have restricted our analysis of location and competition from neighbouring firms other than the one it has just considered. A new map of excess profit contours must be calculated for the new firm with respect to the other competitors (two in number) as well. This is illustrated in Figure 5.

We should note at this point that whether an entrant actually locates at the optimal point, or alternatively whether the entrant earns any profits at the optimal point, depends on whether there is any demand for the product which he can capture. We are required at this point to develop a theory of market encroachment in order to determine in fact whether any excess profits can in fact be earned by an entrant. We will not develop a theory of market encroachment but let it be clear that an "optimal location" may in fact not be a point of feasible production if conditions on demand preclude profitable production. Recall the comments in Section II.4.

In mathematically defining excess profit contours, we found we could associate with each level of excess profits k a group of (x, y) co-ordinates defined with respect to the competitor which the entrant was striving to be protected from. That is, each contour of excess profit k can be defined by a different series of (x, y) with the competitor as the reference point. This is illustrated in Figure 6.

For one excess profit contour illustrated, there are a continuous series of (x, y)'s defining the contour with respect to the competitor C. (x_1, y_1) and (x_2, y_2) are two such defining co-ordinates.

For each of three prospective competitors in a plain, we can define three separate families of contours with three respective families of (x, y)'s.

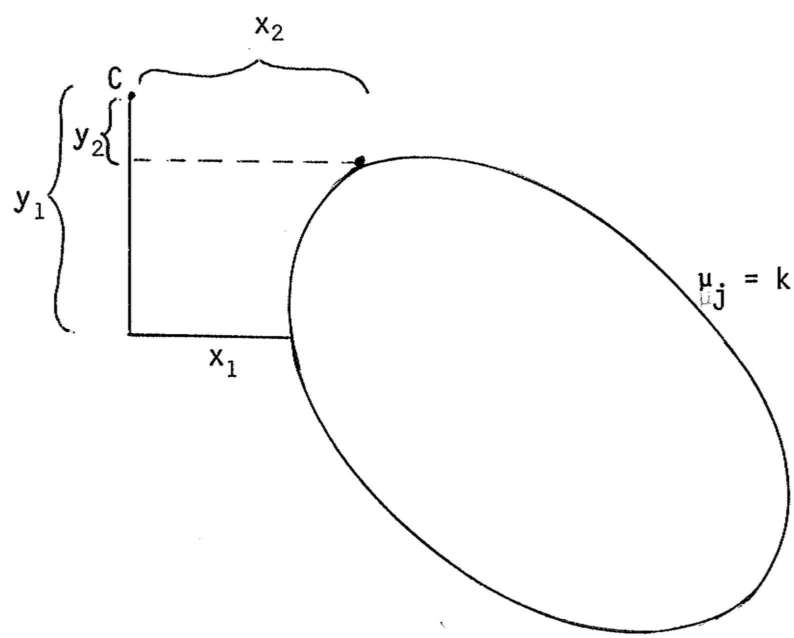

Figure 6.

There exists a schedule of (x, y) defining the locus of constant excess profits per unit in the plain.

For each competitor there will be a series of (x, y)'s for each excess profit contour representing level k and the level k will vary from zero to some maximum value generally different for each competitor. The solution to locating the new firm is to determine the point between the three fixed competitors where the excess profit is the same with respect to all three competitors for the new entrant and also that this excess profit level is a maximum. The common level of excess profit that will occur at a single geographic point will generally be unique provided the excess profit contours have a regular convex shape shown in the above figures.

We can determine the optimum location for the entrant by solving for the (x_A, y_A) , (x_B, y_B) , and (x_C, y_C) in the following non-linear six equation system.

$$\Pi^A(x_{0A}, y_{0A}) - \Pi^B(x_{0B}, y_{0B}) = 0$$

$$\Pi^B(x_{0B}, y_{0B}) - \Pi^C(x_{0C}, y_{0C}) = 0$$

$$x_{0A} + x_{0B} = \bar{x}_{AB}$$

$$x_{0A} - x_{0C} = \bar{x}_{AC}$$

$$y_{0A} + y_{0C} = \bar{y}_{AC}$$

$$y_{0A} + y_{0B} = \bar{y}_{AB}$$

where Π^A, Π^B, and Π^C are excess profit functions for the entrant defined with respect to the competitor at A, B, and C in Figure 6 respectively. The first equation states that the excess profits for the entrant must be the same with respect to the competitors at A and B. The second equation states that the excess profits for the entrant must be the same with respect to competitors at B and C. The last four equations define the new location

0 with respect to the three competitors in the same sense that we previously defined the location of an entrant with respect to a single competitor and two input suppliers.

III.2. We have considered the decision to locate by a single entrant when competitors and supply points are geographically fixed. We focussed on the excess profits per unit output enjoyed by the entrant at alternative locations. It is apparent, however, that the excess profits being enjoyed by an established firm will be altered by the presence of an entrant or new competitor. Moreover, all firms will not be enjoying the same rate of excess profit when entry by a new firm is made in a sequence over some period of time rather than having entry of all firms made simultaneously. This latter phenomenon is manifested in a number of different forms in the economics of location. An early analysis by Hotelling[3] of the location of two producers on a beach-shaped market revealed that sites chosen by entrants independently would be different and socially suboptimal compared with the situation when sites were assigned to entrants by a planner. Samuelson[8] has presented other examples which derive from the Hotelling case but which focus particularly on the fact that the sites chosen by entrants who locate independently and in sequence would be different and socially suboptimal compared with the situation which results when sites are assigned by a central authority and firms are located simultaneously.

Socially suboptimal is defined as follows. The total transportation cost of getting goods to consumers is larger in any suboptimal pattern of production sites than in the optimal pattern.

In view of these established results on firm location and suboptimal patterns under free market conditions, I would like to indicate some parallels arising from the model of optimal location presented in Section II. We remarked that an entrant to an area where established producers are located will alter the excess profit rate enjoyed by its new neighbours and that all firms will in general not be enjoying the same excess profits per unit.

Consider now a pattern of location where all firms are enjoying the same excess profits per unit output and hence none will have an incentive to move. We assume an oval shaped world dotted with numerous points of supply of intermediate inputs. We assume that an outside producer or producers can supply units of commodity j to the perimeter of this oval at a fixed price. To transfer j inland on the oval requires an outlay of transport costs.

We then ask, given the definition of excess profits per unit and the assumptions outlined in Section II, what is the pattern of location of many firms, which would result in the same excess profit per unit for all firms. We can assert without proof that there exist patterns of firm location where the same excess profit per unit is being enjoyed by all firms in the oval shaped world. Furthermore the pattern displayed by the sites of firms when all are enjoying the same excess profit per unit will be different from the pattern displayed by sites when each firm locates independently over a period of time. Since, in reality, the location decision by separate firms producing the same commodity is generally taken at different times, the resulting pattern of sites will result in different levels of excess profit per unit being enjoyed by different firms at the same point in time. We thus have a straightforward explanation for the existence of differences in profit rates for different firms at different locations. It is possible that certain low profit firms will leave the industry and we might expect to find a continuously changing geographic pattern of firm location in an industry[8]. The costs of relocating plants is often such as to preclude the possibility of having firms relocate at points of relatively higher excess profits when their intermeduate market conditions have changed. Thus we might expect firms to remain in business indefinitely and still earn lower rates of profit than their competitors located elsewhere.

FOOTNOTES

1. See Johnson [4].

2. See Balassa [1].

3. See Footnote 7.

4. I have found it useful to consider the product j to be cement. Researchers in industrial organization have observed cement manufacturing plants to be efficiently organized in terms of operating at minimum optimum scale in North America and also in terms of geographic dispersion. With regard to geographic dispersion it has been observed that distinct producers had distinct market areas whose size was determined by the transportation costs for cement and the geographic density of consumers of cement as well as economics of scale. See the discussion in H.C. Eastman and S. Stykolt[2].

5. These considerations have led Dan Usher to label the well-behaved case where X is finite and positive to be the "slag case", i.e. when inputs per unit output exceed in physical terms the physical size of output.

6. See Weber [9] and Kuhn and Kuenne [6].

7. Lösch vigorously argues that the Weber criterion for the optimal location of the firm is deficient since it too ignores the nature of demand, especially the geographical dimension. For example, on p. 28-29 Lösch [7] writes: "Weber's solution for the problem of location proves to be incorrect as soon as not only cost but also sales possibilities are considered. His fundamental error consists in seeking the place of lowest cost. This is as absurd as to consider the point of largest sales as the proper location. Every such one-sided orientation is wrong. Only search for the place of greatest profit is right (sic)."

   Lösch argues further: "There is no scientific and unequivocal solution for the location of the individual firm, but only a practical one: the test of trial and error. Hence, Weber's and all other attempts at a systematic and valid location theory for the individual firm were doomed to failure". p. 29.

8. Koopmans and Beckmann [5, p. 75] observed a similar phenomenon in a spatial general equilibrium model. "It (the conclusion) means that no price system on plants, on locations and on commodities in all locations, that is regarded as given by plant owners, say, will sustain any assignment. There will always be an incentive for someone to seek a location other than the one he holds. In the case of plants on the drawing board, compatible choices cannot be induced or sustained by such a price system. In the case of actual establishments already located, the cost of moving is the only element

of stability, in the technological circumstances we have assumed. Without such a brake on movement there would be a continual game of musical chairs. Whatever the assignment, prices of intermediate commodities and rents on locations cannot be so proportioned as to give no plant an incentive to seek a location other than the one it holds".

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conditions (5) to (10). Thus the Lagrangian multipliers have the dimensions of money units. Moreover it can be shown that the Lagrangian multipliers relate the changes in excess profits resulting from a change in the relative positions of points A, B, and C. Thus λ_1, λ_2, λ_3, and λ_4 are shadow prices on the relative positions of A, B, and C. λ_1 and λ_3 are "prices" on the relative positions of A and C, λ_1 on the y to co-ordinate and λ_3 on the x co-ordinate. λ_2 and λ_4 are "prices" on the relative positions of A and B, λ_2 on the y co-ordinate and λ_4 on the x co-ordinate.

Conditions (11) to (14) indicate that when excess profits are maximized, the relative positions of A, B, and C must be the same as they were when the problem was posed.

II.3 The classic Weber Problem[7] dealing with a producer's point of location for minimum transportation costs per unit output can be expressed in a form similar to that above for the natural protection problem. The Weber Problem treats markets as fixed points in the plain, markets for both transportable inputs and outputs and the problem seeks to determine where a firm might locate so as to minimize the total transportation bill associated with the production of a unit of output. Constant returns to scale in production and fixed input co-efficients are assumed data in the problem.

The following diagram, Figure 4, illustrates the nature of the problem.

The first-order equilibrium conditions are as follows:

$$\frac{\partial L}{\partial t_{OA}} = x_{OA}\ t_{OA}(x_{OA}^2 + y_{OA}^2)^{-\frac{1}{2}} + \lambda_3 + \lambda_4 = 0 \tag{2}$$

$$\frac{\partial L}{\partial x_{OB}} = -\ x_{OB}\ t_{OB}\ a_{Bj}(x_{OB}^2 + y_{OB}^2)^{-\frac{1}{2}} - \lambda_4 = 0 \tag{3}$$

$$\frac{\partial L}{\partial x_{OC}} = -\ x_{OC}\ t_{OC}\ a_{Cj}(x_{OC}^2 + y_{OC}^2)^{-\frac{1}{2}} - \lambda_3 = 0 \tag{4}$$

$$\frac{\partial L}{\partial y_{OA}} = y_{OA}\ t_{OC}(x_{OA}^2 + y_{OA}^2)^{-\frac{1}{2}} + \lambda_1 + \lambda_2 = 0 \tag{5}$$

$$\frac{\partial L}{\partial y_{OB}} = -\ y_{OB}\ t_{OB}\ a_{Bj}\ (x_{OA}^2 + y_{OA}^2)^{-\frac{1}{2}} - \lambda_2 = 0 \tag{6}$$

$$\frac{\partial L}{\partial y_{OC}} = -\ y_{OC}\ t_{OC}\ a_{Cj}\ (x_{OC}^2 + y_{OC}^2)^{-\frac{1}{2}} - \lambda_1 = 0 \tag{7}$$

$$\frac{\partial L}{\partial \lambda_1} = y_{OA} - y_{OC} - \overline{y}_{AC} = 0 \tag{8}$$

$$\frac{\partial L}{\partial \lambda_2} = y_{OA} - y_{OB} - \overline{y}_{BC} = 0 \tag{9}$$

$$\frac{\partial L}{\partial \lambda_3} = x_{OA} - x_{OC} - \overline{x}_{AC} = 0 \tag{10}$$

$$\frac{\partial L}{\partial \lambda_4} = x_{OA} - x_{OB} - \overline{x}_{AB} = 0 \tag{11}$$

Note conditions (2) to (7) express the condition of having a Lagrangian multiplier (or multipliers) equal to an expression with the dimension of money units say dollars. That is, t expresses the cost (in money units) of moving a physical unit of input or output a unit distance and t occurs with no other variable with the dimension of money units in conditions (2) to (7). Thus the Lagrangian multipliers have the dimensions of money units. Moreover it can be shown that the Lagrangian multipliers relate the changes in excess profits resulting from a change in the relative positions of points A, B, and C. Thus λ_1, λ_2, λ_3, and λ_4 are shadow prices on the relative positions of A, B, and C. λ_1 and λ_3 are "prices" on the relative positions of A and C, λ_1 on the y to co-ordinate and λ_3 on the x co-ordinate. λ_2 and λ_4 are "prices" on the relative positions of A and B, λ_2 on the y co-ordinate and λ_4 on the x co-ordinate.

Conditions (8) to (11) indicate that when excess profits are maximized, the relative positions of A, B, and C must be the same as they were when the problem was posed.

Consider the formal statement of the Weber Problem as enunciated at the outset and illustrated in Figure 1. Mathematically, the problem is another simple optimization problem which, because of its simple form, can be treated either as a constrained minimization problem or an unconstrained one. Let us define it as a constrained one. Minimize

$$t_{OA}(x_{OA}^2 + y_{OA}^2)^{\frac{1}{2}} + t_{OB}\, a_{Bj}(x_{OB}^2 + y_{OB}^2)^{\frac{1}{2}} + t_{OC}\, a_{Cj}(x_{OC}^2 + y_{OC}^2)^{\frac{1}{2}}$$

subject to:

$$x_{OA} + x_{OB} = \overline{x}_{AB}$$

$$x_{OA} - x_{OC} = \overline{x}_{AC}$$

$$y_{OA} + y_{OC} = \overline{y}_{AC}$$

$$y_{OA} + y_{OB} = \overline{y}_{AB}$$

where the variables have the same definitions as those given following problem (1). Figure 1 indicates the positions of the various co-ordinates appearing as variables in the above problem.

We could define a Lagrangian function and determine the first order equilibrium conditions for a minimum in a manner similar to that above in the Basic Effective Protection Problem. Again the Lagrangian multipliers would be shadow prices on the locations of points A, B, and C.

Weber developed iso-valued contours in his exposition. Each contour linked points in the plain corresponding to equally valued total transportation costs per unit of output. In Figure 1, the contours would be concentric with point 0 contained within each contour, point 0 being the location of minimum total transportation costs per unit output. Weber labelled these iso-transportation cost contours "isodapanes".

We can turn to the demand side of the model given that a point of maximum excess profits per unit output has been obtained. Each consumer or family in the plain has a linear demand curve.

$$q_F = a - b\ (p_j^o + d_j u)$$

where q_F is quantity demanded per unit (family)

p_j^o is the price of j f.o.b. at the factory at point 0.

d_j is the transport cost per unit distance for commodity j.

u is the distance from the factory at point 0.

So long as profits per unit output are positive at 0, our producer will then select u^*, the frontier of his market by finding the nearest point to 0 of his competitor's market and letting that value be u^*. To determine his f.o.b. price, he will set $q_F = 0$ at u^* and solve for $\overset{*}{p}{}_j^o$. His total excess profits will be total revenue, $p_j^{*o} q^*$, minus total cost Cq^* where q^* is determined by solving for the total market demand by integrating over the area in the usual way.[6] C is cost per unit output which is constant once point 0 is determined. If the demand function were not linear, point 0, the position of maximum excess profits per unit would not necessarily be the point of maximum total excess profits.

We observe that there will be numerous consumer units unserved. This is not an unrealistic state of affairs as Devlefoglou so vigorously argues. In fact Devlefoglou's model is built on the assumption of having consumers quit buying a commodity if they are prohititively distant from the supplier. Prohibitively means simply that the delivered price is sufficiently high to cause consumers not to buy. Recall also the unserved customers in the interstices of the Mills-Lav model. We cannot expect that both competitors will be earning the same level of profits after the entrant is settled. There exists the possibility of price competition and possibly location adjustment. These are important issues to consider but could lead us inevitably far into the realm of competition among the few. I do not believe we should discard this model because this thread is loose especially since unequal rates of profit among producers of similar commodities are a common place in the world around us.

The above development of the demand side is one approach to specifying a demand side to this model. Others could be developed but I believe this one is both simple and plausible.

natural protection to the case where a prospective locator had only to consider the relative position of one competitor in his decision as to where to locate. Let us turn to examining the case where there are many competitors distributed in a plain as well as many suppliers of intermediate inputs. The optimum location for a new firm entering turns out to be at a point where the rate of natural protection with respect to each of the three nearest competitors is equal for the firm entering. In other words, the new firm, placed in its optimum location could not be moved in any direction without lowering its potential excess profits as a result of competition from one of its previously positioned rivals. Its excess profits defined with respect to one or two competitors may of course be larger because it has moved further away from certain competitors in a shift from its optimum, but this fact is irrelevant given the fact that the competitor to which it has moved closer can always evade apparent gains in potential excess profits to a point where the excess profits would be lower than what the firm could have gleaned when it was optimally located.

The firm must compute effective natural protection contours or excess profit contours with respect to every competitor in the plain and then determine points of intersection of three equal valued contours for all combinations of three adjacent competitors. There will in general be one such point and it will be where the firm should locate. These contours are computed simply by varying the potential location of an entrant and determining the level of excess profits earned at every possible point.

If all points of equal excess profit are linked together by a smooth curve, we have one contour indicating all possible locations which would yield a specific and constant level of excess profits. A family of contours must be drawn with respect to each different nearby competitor as is done in Figure 4. A contour shrinks to a point where potential excess profits are at a maximum with respect to one given competitor. It was the determination of that point which we analyzed above in some detail.

In the case of many competitors, an entrant selects a particular area for a possible location and determines the three competitors nearest to his potential location. He then determines possible supply points and evaluates three families of contours of potential excess profits as are illustrated in Figure 4. Three points of maximum excess profit per unit will be found but none will be chosen in general because though it is a best point with respect to one competitor, it will be a vulnerable position with respect to the other two as can be seen in Figure 4. The solution to locating the new firm is to determine the point between the three fixed competitors where excess profit per unit is the same with respect to all three competitors for the new entrant and also that this profit level is positive. The common level of excess profit that will occur at a single geographic point will generally be unique provided excess profit contours have the regular convex shape which they exhibit in Figure 4. They will have this shape provided there are no lakes and streams or hills and valleys which cause transportation costs to vary with direction and route. Recall we assumed no irregularities at the outset. Given the potential

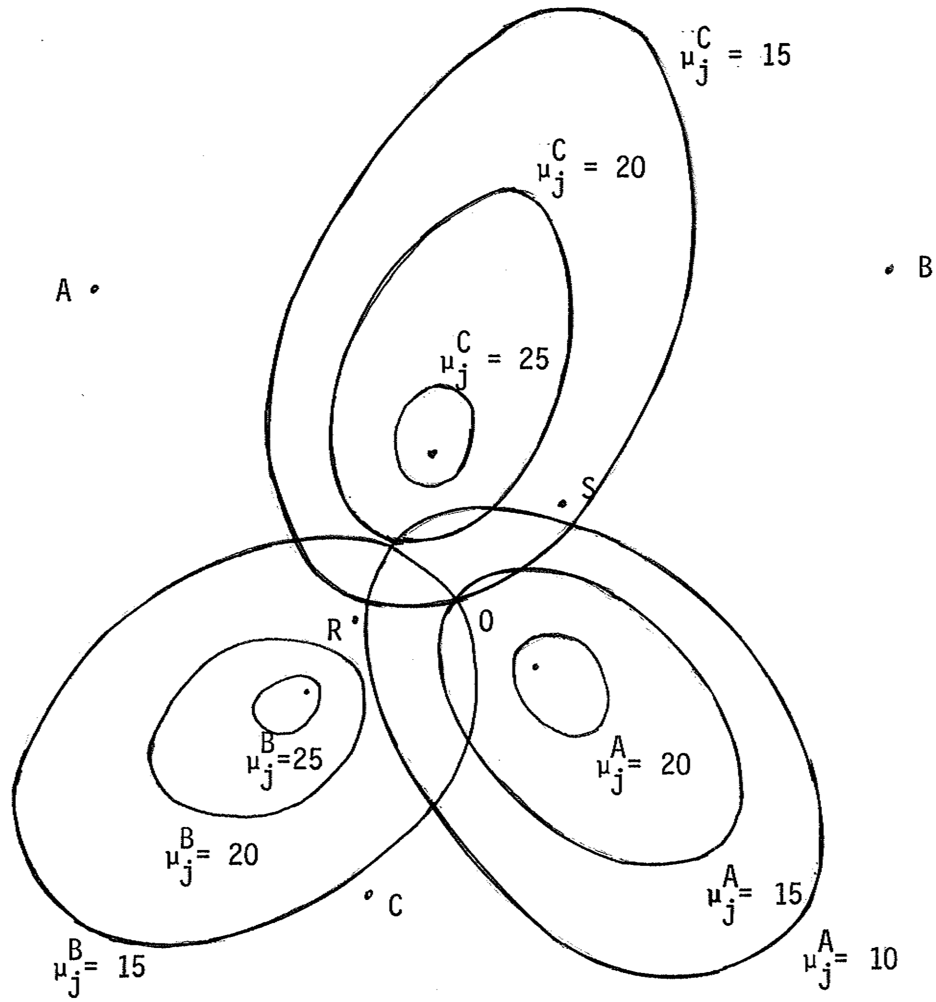

Figure 4.

At points A, B, and C are competitors. At points R and S are suppliers of inputs. At point 0 the maximum excess profits are being earned given the locations of suppliers and competitors.

site, the entrant can determine his market area, level of output and total profit in a manner analogous to that outlined above where we considered the demand side of the simple two competitor model.

V. Concluding Notes

Let us return to effective protection and international trade. Transportation costs vary continuously with distance whereas tariffs do not. Tariffs simply change from 0 to level k at a political frontier. In geographic space we can draw contours of iso-effective natural protection rates or excess profit rates or excess profits which we cannot draw for tariff protection since such iso-valued contours do not exist owing to the property or tariff rates not varying continuously with distance. If we consider countries as points, we can define iso-effective natural protection contours describing the nature of protection at different geographic points. These contours are a descriptive device which could actually be worked out for examining two country trade.

The use of the concept of effective natural protection in empirical regional analysis should be noted. For example, if a government agency is attempting to evaluate the relative merits of various locations for establishing or subsidizing the establishment of a new firm, it can readily compute potential excess profits at various sites in the effective protection framework. Rather then simply examining potential profits at each site in isolation, by using the notion of effective natural protection, it can estimate potential profitability taking account of the presence of existing firms. For example, two firms producing the same product may both estimate the same potential profit but whether the profit is made or not will be contingent on the presence or absence of the adjacent or other nearby competitors. Account has to be taken of the possibility of new firms surviving in the presence of vigorous competition from nearby rivals. The effective protection scheme used to develop an abstract model in this paper has considerable usefulness in this regional policy evaluation problem.

Throughout this presentation of a location model based on the notion of effective natural protection we have considered the actions of one entrant and taken other competitors as fixed. We ruled out competitive relocation by assumption. Relocation costs could easily account for such a phenomenon in reality. We noted that different firms will be generally earning different rates of profit. If we look at the process of location for many firms in this model we will observe the same phenomenon which Samuelson [10] discussed in his very simple framework of location on the circle with no location readjustment. The process is essentially sequential in our model as it was in Samuelson's. Most important also is that there is no sense in which location will be socially optimal. The number of producing units per unit area will not generally be uniform nor will transportation costs to all consumers be minimized subject to production taking place efficiently. Some consumers will pay more for the same product even if they live the same distance from the supplier as another consumer in another region. But this is much the way the real world appears to operate.

In an abstract general equilibrium setting we usually assume all firms are located simultaneously, as in the Mills-Lav model. When they are not, we get a quite different picture of the locational pattern at any point in time. The effective protection location model depicts this phenomena.

APPENDIX

The following example was solved as an illustration.

maximize:

$$4(x_{0A}^2 + y_{0A}^2)^{\frac{1}{2}} - 2.0\ (x_{0B}^2 + y_{0B}^2)^{\frac{1}{2}} - 3.0\ (x_{0C}^2 + y_{0C}^2)^{\frac{1}{2}}$$

subject to:

$$x_{0A} - x_{0B} = 10$$

$$x_{0A} - x_{0C} = 4$$

$$y_{0A} - y_{0B} = 5$$

$$y_{0A} - y_{0C} = 8$$

$$x_{0A},\ x_{0B},\ x_{0C},\ y_{0A},\ y_{0B},\ y_{0C} = 0.$$

The solution was found to be:

$x_{0A} = 10.0$, $y_{0A} = 9.5$, $x_{0B} = 0$, $y_{0B} = 4.5$,

$x_{0C} = 6$, $y_{0C} = 1.5$

and the value at the maximum was 27.617.

The problem is diagrammatically presented in Figure A1.

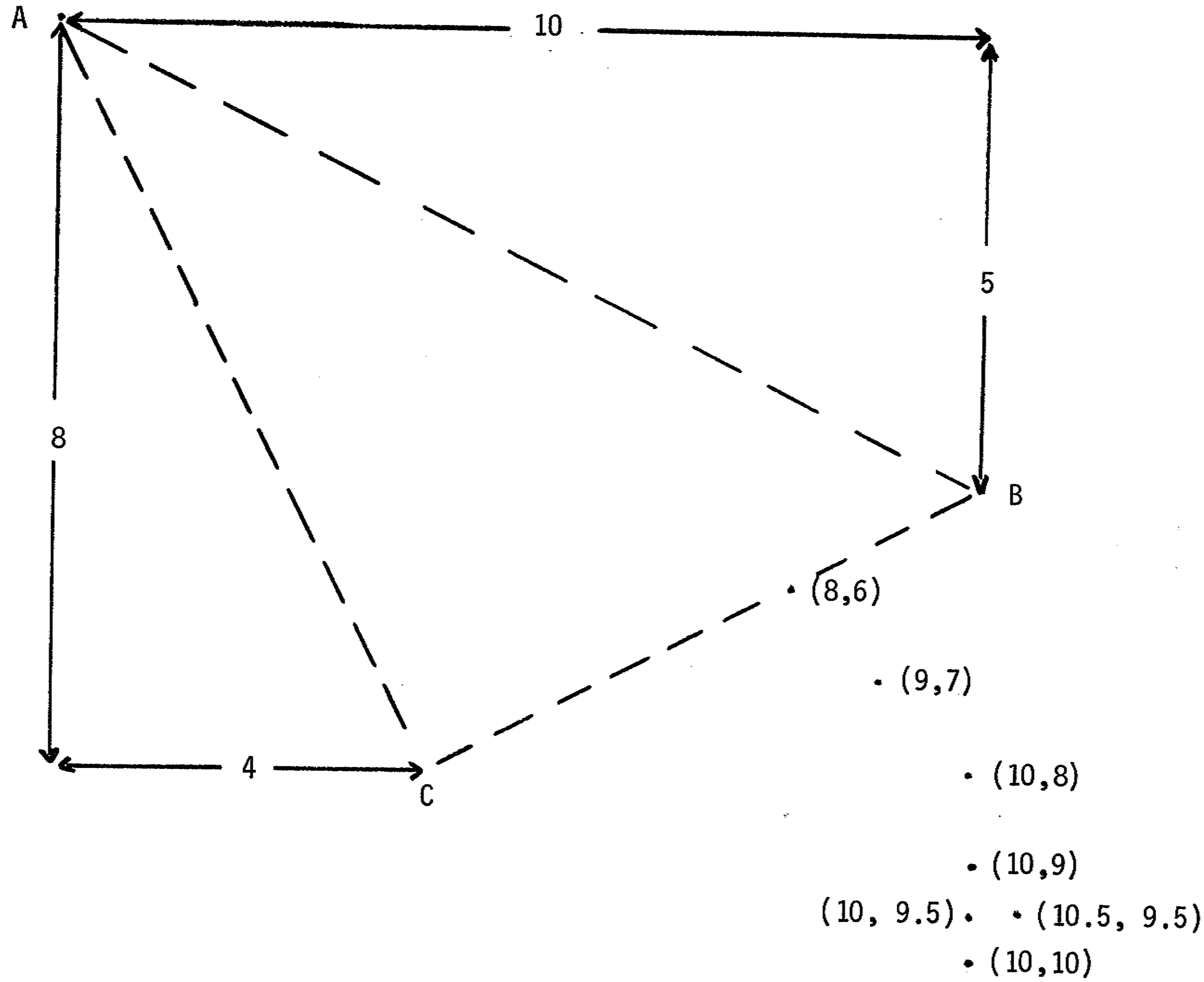

Figure A1

The point of maximum excess unit profits was found to be at 0 for the problem numerically defined above in the Appendix. At A is a competitor. At B and C are suppliers of inputs.

Footnotes

1. See the paper by Samuelson [9] or by Takayama and Judge [12] for the development of this literature.

2. Mills and Lav considered atomistic firms or producing units to be profit maximizers in an infinite homogeneous plain. They then presented their well-known counter examples to Lösch's theorem which purported to demonstrate that space-filling hexagonal market areas would result from a process of simultaneous firm location. However Mills and Lav departed significantly from Lösch's assumptions in order to get their results. Lösch explicitly assumed space to be finite and then considered that firm entry would result in a situation of zero profits in equilibrium. A Chamberlinian equilibrium was assumed to obtain for each firm by Lösch where the demand curve was tangent to the average cost curve. Mills and Lav's case becomes invalid. I have developed this point of view in full analytical detail in [4]. The point is that Mills and Lav required profit maximization for an equilibrium in infinite space whereas Lösch with finite space got a much different,and I believe,more plausible equilibrium condition.

3. See footnote 2 and the reference therein.

4. Linear demand functions have been used by numerous authors concerned with spatial economic theory, e.g., Lösch [7 ; p.111]. The demand curve is

   $$q_F = a - b\ (p + tu)$$

   where q is quantity demanded per unit time, p is price f.o.b., t is transportation cost per unit distance, and u is distance. For $U > \frac{a - bp}{t}$, quantity demanded becomes zero.

   Demand conditions are discussed in detail near the end of section III.

5. These considerations have led Dan Usher to label the well-behaved case where X is finite and positive to be the "slag case", i.e., when inputs per unit output exceed in physical terms the physical size of output.

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The exploration of the similarities between the effects of tariffs and transportation costs in economic models has yielded a variety of interesting analytical results. The theory of effective protection has recently been enlarged by the work of Balassa[1] and Johnson[2] who have substituted transportation costs for tariffs and deduced certain results. It is the purpose of this paper to examine some basic differences between the analytical implications of substituting transportation costs for tariffs in the effective protection theory. By treating countries as points we can develop a graphical technique for describing the effective protection arising from transportation costs. Then, by treating production units as firms rather than countries, a number of interesting problems in location theory arise, in particular, the problem of locating a firm given geographically fixed competitors and suppliers of inputs. The classic Weber problem in location theory is shown to be related to location in the context of effective protection arising from transportation costs.

In Section I, alternative definitions of effective natural protection or effective protection arising from transportation costs are developed and a graphical technique for describing relative rates of effective protection in different areas is presented. In Section II, a problem in the location of a firm is analyzed and compared with the Weber problem. Section III contains a discussion of location in a spatial general equilibrium setting.

SECTION I. EFFECTIVE NATURAL PROTECTION

I.1. Johnson[1] defines effective natural protection for a production process j in a country as follows. He follows Balassa[2].

$$\mu_j = \frac{\hat{t}_j - \sum_i^n a_{ij} \hat{t}_i}{1 - \sum_i^n a_{ij}} \tag{1}$$

where

$\hat{t}_j$ is the cost of transporting a unit of j from a country which produces j to the country which is protected, country A.

$\hat{t}_i$ is the cost of transporting a unit of input i used in the production of j from the nearest producing country to country A. $i = 1, \ldots, n$.

a_{ij} is the amount of input i required to produce a unit of j.

Definition (1) is the ratio of value-added accruing to process j when it is protected by transportation costs to the value-added accruing to process j if there were no protection.

It will be useful later to consider the excess profits accruing to a producer of j arising from natural protection. We shall define the natural rate of profit $\hat{\pi}_j$ at point 0 as follows:

$$\hat{\pi}_j = \frac{d_j t_j - \left\{\sum_{i=1}^n (z_i a_{ij}(p_i + d_i t_i) + (1-z_i) a_{ij} p_i) + k_j(r_k + d_k t_k) + l_j(w_l + d_l t_l)\right\}}{1 - \sum_i a_{ij}} \tag{2}$$

$$\pi_j = \text{numerator of (2) or excess profits} \tag{3}$$

where

d_j is the distance from the point of production of output j to a potential producer at point 0.

d_i is the distance from the point of supply of input i to a potential producer at point 0.

d_k and d_l are also distances from points of supply of inputs capital and labour to point 0.

$\hat{t}_i$, $\hat{t}_j$, $\hat{t}_k$, and $\hat{t}_l$ are transportation costs per unit distance for the respective commodities and factors indicated by the subscripts.

z_i is the fraction of input i required in the production of j which is imported or transported to 0. $(1-z_i)$ is assumed to be supplied at point 0.

a_{ij} is the amount of input i required to produce a unit of j.

p_i is the relative cost of input i f.o.b. at the point of supply. Product j is assumed to have a price of unity.

k_j is the amount of capital required to produce a unit of j.

l_j is the amount of labour required to produce a unit of j.

r_k is the price of a unit of capital f.o.b.

w_l is the price of a unit of labour f.o.b.

We assume that there are no taxes in the economy under consideration.

In moving from the definition in (1) to that in (2), we have made a few changes in form and a number in substance. In matters of form we have defined transport costs in (2) by distance multiplied by cost per unit distance rather than by just unit transport costs per specific non-unit distance as in (1) i.e. $\hat{t}_i = t_i d_i$. In (2), we have included capital and labour costs as costs of production and so we have a term for excess profits in the numerator of (2) rather than the value added in the presence of impediments to trade flows as in (1). We have allowed for the possibility of importing capital and labour from beyond the production point 0.

In (2) we have introduced prices explicitly for inputs and product j at f.o.b. terms. Finally in (2) we allow for the possibility of importing only fractions of supplies from beyond the production point 0 rather than assuming all supplies are imported. For completeness, we should note that we could also allow for the possibility of importing only fractions of capital and labour requirements in accord with the assumption that some supply of factors is available locally. The denominator of (2) is the same as that in (1) and thus defines a profit rate in (2) as a fraction of value-added in the absence of transport costs and tariffs.

Observe that π_j defined in (3) is simply excess profits per unit of output at production point 0.

I.2. Transportation costs vary continuously with distance whereas tariffs do not. Tariffs simply change from 0 to level k at a political frontier. In geographic space we can draw contours of iso-effective natural protection rates or excess profit rates or excess profits which we cannot draw for tariff protection since such iso-valued contours do not exist owing to the property or tariff rates not varying continuously with distance.

If we consider countries as points, we can define iso-effective natural protection contours describing the nature of protection at different geographic points. This is done in Figure 1.

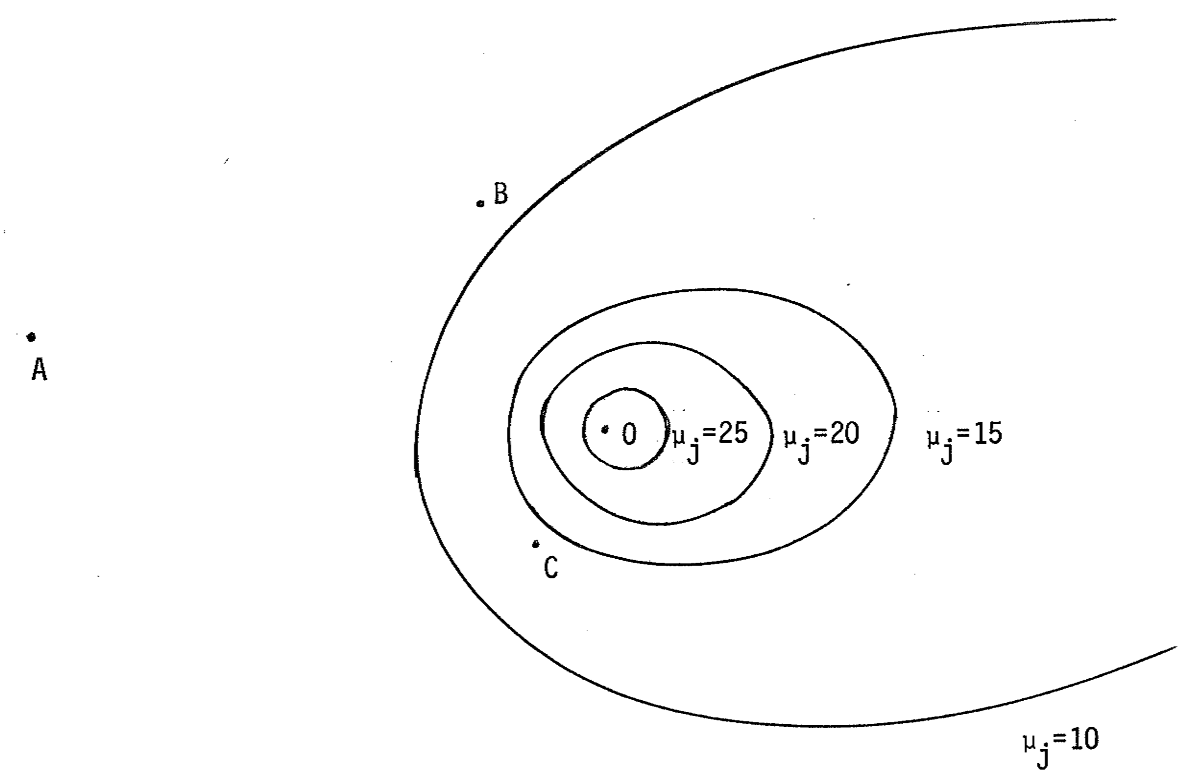

Figure 1.

Iso-Effective Natural Protection Contours.

Country A produces good j. Good j can be produced by importing supplies from points B and C and fabricated at a fixed cost net of transportation costs anywhere in the plain. At point 0, the highest rate of Effective Natural Protection is being enjoyed by another country producing j.

SECTION II. MAXIMUM EFFECTIVE NATURAL PROTECTION

II.1. Let us consider the problem of locating the point in the plain of maximum effective natural protection or excess profits. This problem is similar in form to the Weber Problem, which will be examined in detail below, as well as similar in approach. The Weber Problem was designed to determine the point of production associated with minimum transportation cost per unit of output given geographically fixed points of supply of inputs in production as well as points of demand for a final production. Our problem seeks to determine the point where excess profits per unit output are maximized. Just as in the Weber Problem, the detailed specification of the demand side is ignored in our problem. The inclusion of a realistic demand sector to this model would complicate it considerably as can be seen, for example, in Losch's[3] attempt to attach demand sectors to Weber's model. The problems are discussed in Section II.4.

We assume that there is a linear fixed coefficient production function for product j.[4] We assume that there are spatially fixed and separated suppliers of inputs into process j as well as one competitor supplying j from a fixed production point. A producer pays all transportation costs on inputs and outputs. Inputs are available in infinitely elastic supply at fixed prices. The price of an output f.o.b. at any point will be determined by the price at which the given competitor can sell at that point.

Consider the following simplified example. It is diagrammatically presented in Figure 2. At point A is a producer of j. At points B and C, equidistant from A, are two input suppliers and the two inputs are assumed to be required in equal physical amounts to produce a unit of j at a new

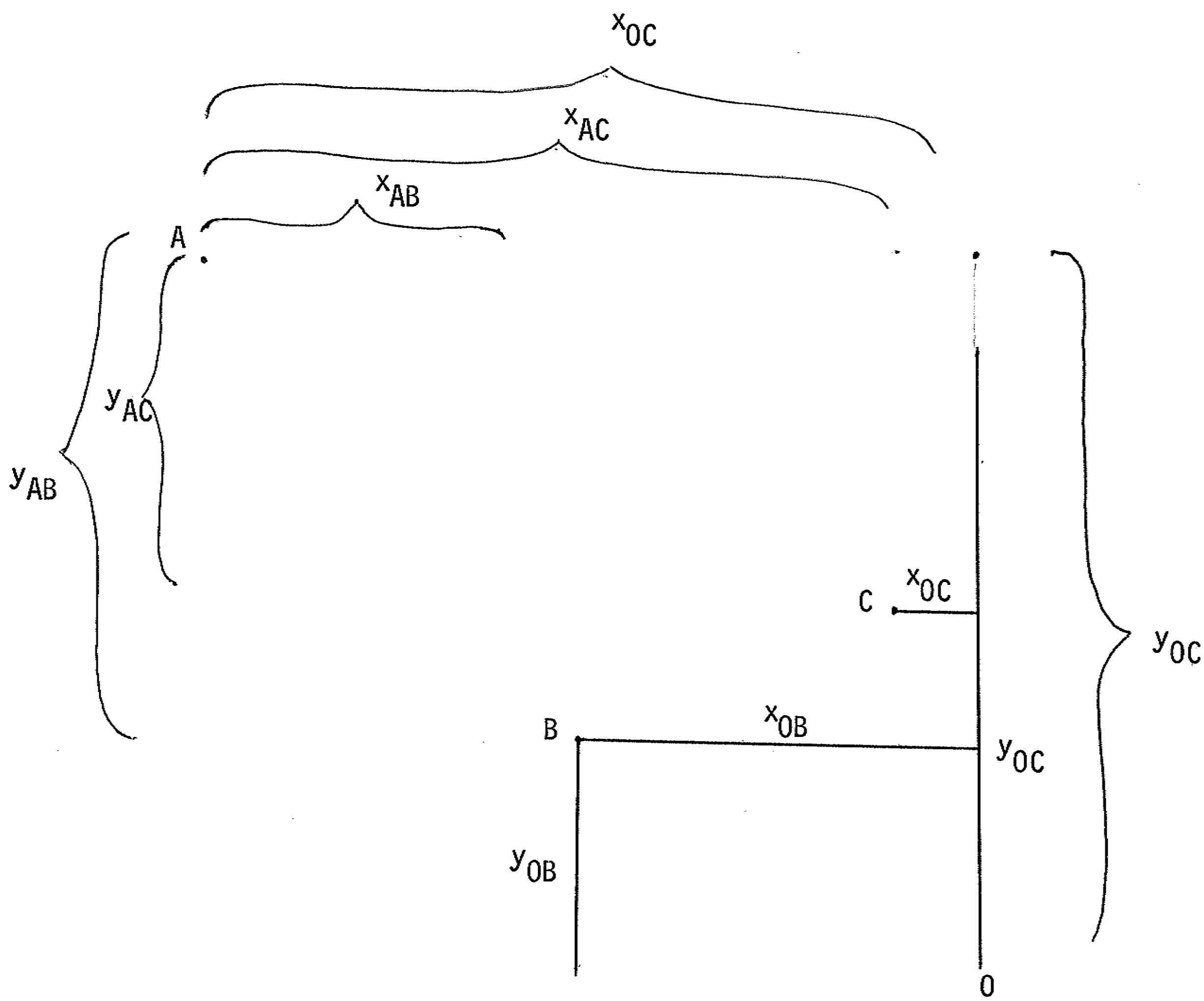

Figure 2.

Effective Protection Problem

A new extrant is maximizing excess profits when located at 0. B and C are supply points for inputs and at point A is located a competitor.