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THE PRICE OF CAPITAL AND THE REAL REAL OF INTEREST

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Few subjects in economics have been more exhaustively discussed than relation between productivity and thrift in the determination of the rate of interest, yet leading authors continue to hold markedly divergent views. The issues may be brought into focus by considering the following questions.

1) Is the time-structure of real interest rates endogenous in a competitive economy in the way that commodity prices are endogenous? To answer this question in the affirmative is almost to deny that the real rate of interest may serve as an instrument in monetary or fiscal policy. To answer it in the negative is almost to assert that a competitive economy, left to itself, will generate sub-optimal rates of investment and economic growth.

2) Can a change in technology influence real rates of interest a) instantaneously, and b) in the course of time?

3) Can a change in taste influence real rates of interest a) instantaneously, and b) in the course of time?

4) What is the relation between the rate of interest and marginal product of capital?

A spot check of some of the principle source books and expositions of capital theory reveals a diversity of points of view. Differences of opinion among authors are not always as marked as table 1 suggests because in many cases authors have knowingly chosen simple cases to illustrate particular aspects of the theory. The student comparing expositions of capital theory faces a formidable task of discovering and comparing assumptions to determine why authors differ in their conclusions.

TABLE 1: VIEWS OF SOME LEADING AUTHORS ON CAPITAL THEORY

Question	Fisher (1)	Knight (2)	Hayek (3)	Dewey (4)	Clower (5)	Friedman (6)	Solow (7)	Kaldor (8)	Achley (9)
1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	?	No
2a	Yes	Yes	Yes	Yes	No	Yes	Yes	No	No
2b	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No
3a	Yes	No	No	No	No	Yes	No	Yes	No
3b	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	No

Notes to table 1:

(1) Irving Fisher, The Theory of Interest (1930)

(2) Frank Knight "Capital, Time and the Rate of Interest", Economica, 1934. In his early writings Knight seems to take the view that neither in the long run nor in the short run can taste affect the rate of interest. This view seems to have been modified in "Diminishing Returns to Investment", Journal of Political Economy, 1944.

(3) Friedrich Hayek, The Pure Theory of Capital (1941) especially appendix I.

(4) Donald Dewey, Modern Capital Theory, (1965). Dewey's views of capital theory are based to a great extent on Knight's crusonia analogy.

(5) R.W. Clower "Productivity, Thrift and the Rate of Interest" Economic Journal, (1954). See especially paragraphs 7 and 8.

(6) Milton Friedman, Price Theory: A Provisional Text, (1962), Chapter 13.

(7) Robert Solow, Capital Theory and the Rate of Return, (1962). Most of Solow's results follow if one grants the assumption on page 29 that there is "an economy producing one all-purpose commodity which can be consumed or else used as a capital good".

(8) Nicholas Kaldor "Capital Accumulation and Economic Growth" a chapter in The Theory of Capital, edited by D.C. Hague. See page 218: we shall follow the orthodox Keynesian lines in assuming that the rate of interest is determined by the liquidity preference function and/or monetary policy". If the interest rate is determined by the demand for money exclusively, then it is affected by a change in taste for money-holding, but not by a productivity change unless productivity influences the demand for money.

(9) Gardner Ackley, Macroeconomic Theory, (1961) pp. 477 to 485

In this article I propose to outline a theory of the determination of the term structure of interest rates - a theory which draws heavily on the ideas of the authors cited above - and to use the theory in evaluating and comparing the different points of view.

A competitive economy may be thought of as maximizing the function

$$W = \int_0^\infty e^{-\rho t} U(C(t))\, dt \ldots U' > 0, \quad U'' < 0 \qquad \ldots\ldots (1)$$

subject to constraints consisting of a production function

$$Q = E \cdot F(K,L) \ldots \quad F_k > 0 \quad F_{kk} < 0, \quad F_L > 0 \quad F_{LL} < 0 \qquad \ldots\ldots (2)$$

a disposition of output equation

$$Q = C + R\dot{K} \qquad \ldots\ldots (3)$$

and cost of capital equation

$$R = R(\dot{K}) \qquad \ldots\ldots (4)$$

where the variables Q, K, W and $\dot{K}$ are functions of time, where ρ, L and E are constant, where F and R are functions (though the symbol R is also representative of a variable) and where

W is the index of welfare of the economy derived from the time-stream of consumption

U is a function indicating the utility in any given year of consumption in that year

ρ is an indicator of time-preference; it is the rate at which utils are discounted

C is consumption <u>measured in apples per year</u>

Q is the maximum amount of consumption (measured in apples per year) that could be produced with the given capital stock and labour force

E is the measure of efficiency in the production function

K is the capital at time t <u>measured as a stock of axes</u>

L is the labour force measured in available man-hours per year and

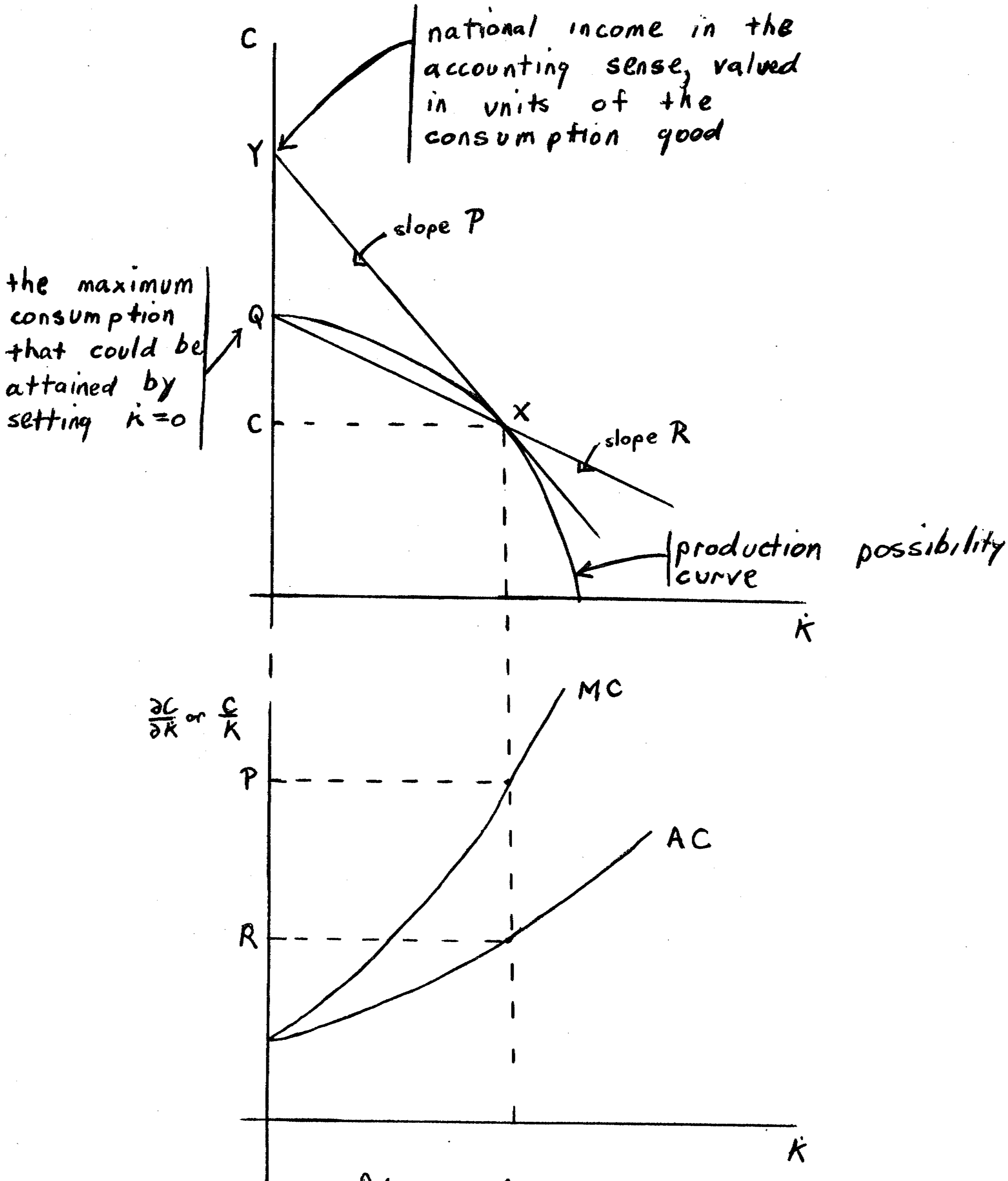
C
national income in the accounting sense, valued in units of the consumption good
Y
slope P
the maximum consumption that could be attained by setting $\dot{K}=0$
Q
C
X
slope R
production possibility curve
$\dot{K}$
$\frac{\partial C}{\partial \dot{K}}$ or $\frac{C}{\dot{K}}$
MC
P
AC
R
$\dot{K}$

figure 1

is assumed to remain constant over time.

$\dot{K}$ is dk/dt

and R is the average cost of capital in terms of consumption goods.

Some aspects of the constraints are illustrated in figure 1. The resources of the economy, the labour and the capital, may be used to produce consumption goods and new capital goods in varying proportions, and the options open to the economy are illustrated in the production possibility curve in the top half of figure 1. The relation between factor supplies and outputs could be described by two production functions, one for consumption and one for new capital goods. Instead we suppose that the height of the production possibility curve is determined by a production function (2) and that the shape of the production possibility curve is determined by a separate equation (4). It is as though labour and capital were combined to produce the consumption good, Q, some of which was subsequently transformed into new capital goods, $\dot{K}$. This way of looking at the economy simplifies things to some extent for it saves us from having to account for separate and distinct capital-labour ratios in the consumption industry and in the capital formation industry.

The value R in equation (4) is not the price of capital. It is the average cost, $Q-C/\dot{K}$, when consumption is the numeraire. The corresponding total cost is $R\dot{K}$ and the marginal cost, designated as P, is

$$P = \frac{\partial}{\partial \dot{K}} (R\dot{K}) = R\left(1 + \frac{1}{\eta}\right) \text{..............................(5)}$$

where η, equal to $\left[\partial \dot{K} / \partial R\right] \left[R/\dot{K}\right]$ may be thought of as the elasticity of the average cost curve of new capital goods. The price of capital is P, the rate of trade-off in production between consumption goods and new capital goods.

Similarly, Q must be distinguished from income. variable Q represents the maximum consumption that could be obtain the economy chose not to invest, while income in the accounting sense the sum of the values of consumption and investment. When the consump good is the numeraire, income Y, becomes

$$Y = C + P\dot{K} \quad \text{.......................................} \quad (6)$$

where $P\dot{K}$ may be thought of as investment, the value in consumption units of new capital produced. The value of Q is at any time a technical magnitude, completely independent of taste. The value of Y on the other hand is determined in an interaction between technology and taste. Technology furnishes the production curve but taste influences the choice of a mix of C and $\dot{K}$ on that curve and thereby influences the value of Y. The national income is sometimes defined "the maximum consumption that could be attained in a year without depleting the capital stock". It is obvious from Figure 1 that this definition is inappropriate except in the special case where the production possibility curve is a straight line.

<u>The Derivation of the Term-Structure of Interest Rates</u>

The real rate of interest is defined as the rate of trade-off between consumption at different periods of time. If r(t) is the term structure of interest rates on loans contracted at time 0, then

$$e^{r(t)t} = \frac{\Delta C(t)}{\Delta C(0)}$$

where C(t) is the amount of consumption at time t that may be exchanged for an amount C(0) at time 0.[1] The short-term (instantaneous) rate of interest at time t is defined as

$$\hat{r}(t) = \frac{\frac{d}{dt}\left(e^{r(t)t}\right)}{e^{r(t)t}} = r\left(1+\frac{r't}{r}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

In the ordinary demand supply diagram, there are two prices corresponding to any quantity, a demand price indicated by the height of the demand curve and a supply price indicated by the height of the supply curve. The demand price pertains to taste and the supply price pertains to technology. Interest rates can also be specified on the demand side and on the supply side of an economy; rates of transformation between consumption now and consumption later may be evaluated with respect to technology or with respect to taste; one may calculate the rate of exchange between present and future consumption attainable with existing technology or one may calculate the rate at which welfare derived from the stream of consumption is held constant.

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[1] We have no occasion to discuss finite-term rates of interest on loans commencing at time t.

From the definition of ρ it follows that:

$$e^{\rho t} = \frac{\Delta\ U\ (C(t))}{\Delta\ U\ (C(0))} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

where $\Delta\ U(C(t))$ and $\Delta\ U(C(0))$ are changes in utility at times 0 and t that leave total welfare W invariant. Define $r(t)_D$ as the rate of exchange between $\Delta\ C(0)$ and $\Delta\ C(t)$ that leaves W invariant. It follows from (9) that

$$e^{\rho t} = \frac{\Delta\ U(C(t))}{\Delta\ U\ (C(0))} = \frac{\dfrac{\partial\ U\ (C(t))}{\partial\ C}}{\dfrac{\partial\ U(C(0)}{\partial\ C}} \quad \frac{\Delta C(t)}{\Delta C(0)}$$

$$= \frac{U'(C(t))}{U'(C(0))} \quad e^{r(t)_D\ t}$$

OR

$$e^{-r(t)_D t} = \frac{U'(C(t))}{U'(C(0))}\ e^{-\rho t} \quad \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

The short-term rate of interest on the demand side is

$$\hat{r}(t)_D = \frac{\dfrac{d}{dt}\left[\dfrac{U'(C(0))}{U'(C(t))}\ e^{\rho t}\right]}{e^{r(t)_D t}} = \rho - \frac{U''(C(t))}{U'(C(t))}\ \frac{dC}{dt} \quad \ldots\ldots(11)$$

It follows from equation 11 that the rate of interest must fall to ρ whenever the economy is in a stationary state with consumption constant, that $r(t)_D$ exceeds ρ whenever consumption is growing and that $\hat{r}(t)_D$ falls over time when consumption grows at a steadily decreasing rate.

The short-term rate of interest on the supply side of the economy is

$$\hat{r}(t)_s = \frac{\Delta\ C(t+\ \Delta\ t)\ -\ \Delta\ \ C(t)}{\Delta\ C(t)\quad \Delta\ t}$$

where $\Delta\ C(t)$ may be equated with the price of capital and $\Delta\ C(t\ +\ \Delta\ t)$ - $\Delta C(t)$ may be equated to the sum of the yield of capital and appreciation over the period $\Delta\ t$.

and when Δt is small

$$\hat{r}(t)_s = \frac{MPC(t) + \frac{d}{dt} P(t)}{P(t)} \qquad \ldots\ldots(13)$$

The short rate equals the marginal product of capital plus the appreciation of the price of capital divided by the price of capital.

In the ordinary demand and supply diagram, the equilibrium quantity is that for which the demand price equals the supply price and for which the rates of trade-off between the commodity and the numeraire are the same in production and in use. Similarly, the term structures of short-term rates of interest in production and in use are the same in the equilibrium where the intertemporal utility (1) is maximized subject to the constraints (2), (3) and (4). Substituting equations (2), (3) and (4) into equation (1), we see that

$$W = \int_0^\infty e^{-\rho t} U(Q - R\dot{K})\, dt \qquad \ldots\ldots(14)$$

The equilibrium time paths of C and $\dot{K}$ are indicated by the Euler equations:

$$e^{-\rho t} U'(C(t))\, E\, F_K + \frac{d}{dt} \left[e^{-\rho t} U'(C(t)) \left(R + \dot{K} \frac{\partial R}{\partial \dot{K}}\right) \right] = 0 \ldots(15)$$

The integral form of the Euler equation is

$$\int_t^\infty e^{-\rho \tau} U'(C(\tau))\, EF_K\, d\tau = e^{-\rho t} U'(C(t))\, P \qquad \ldots\ldots(16)$$

when $t = 0$, equation (16) becomes

$$\int_0^\infty e^{-\rho t} \frac{U'(C(t))}{U'(C(0))} EF_K\, dt = P \qquad \ldots\ldots(17)$$

Both the differential form (15) and the integral form (17) of the Euler equations have significant economic interpretations.

The differential form indicates that the term-structure of interest rates on the supply side of the economy is the same as the term structure on the demand side when utility is maximized i.e. equation (15) implies that

$$r(t)_s = r(t)_D \qquad (18)$$

for all values of t.[2] On substituting equation (10) into equation (17), and letting r(t) represent the equilibrium term structure, equation (17) becomes

$$\int_0^\infty e^{-r(t)} EF_K \, dt = P \qquad (19)$$

which has the simple economic interpretation that the discounted marginal product of capital is equal to the price of capital . In principal, the Euler equations can be solved to yield optimal time paths of C and K, in which case r(t) would be determined as well.[3]

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[2] Equation (15) is $e^{-\rho t} U'(C(t)) \, E \, F_K + \frac{d}{dt}\left[e^{-\rho t} U'(C(t))P\right] = 0$

The second term of the left hand side of this equation is

$$\frac{d}{dt}\left[e^{-\rho t}U'(C(t))P\right] = e^{-\rho t} U'(C(t)) \left[-P\left(\rho - \frac{U''(C(t))}{U'(C(t))} \frac{dc}{dt}\right) + \frac{dP}{dt}\right]$$

Dividing equation (15) by $e^{-\rho t} U'$ and substituting for the value of $\frac{d}{dt}\left[e^{-\rho t}U'(C(t))P\right]$ we have

$$EF_K = MPC = P\left(\rho - \frac{U''(C(t))}{U'(C(t))} \frac{dC}{dt}\right) - \frac{dP}{dt}$$

OR

$$\frac{MPC + \frac{dP}{dt}}{P} = \rho - \frac{U''(C(t))}{U'(C(t))} \frac{dC}{dt}$$

which is equation (18) above.

---

[3] From the equivalence between equations (15) and (17) as variants of the Euler equations, it follows that the statements "investment is carried up to the point at which the price of capital equals its discounted marginal product" and "the short term rate of interest equals the marginal product of capital plus the rate of appreciation of the price of capital", are fully compatible, and describe two aspects of one and the same maximization procedure.

Stocks, Flows and the Term-Structure of Interest Rates

It is possible to speak of the demand and supply of permanent consumption streams in a comparison of stationary states. An economy in a stationary state could, if it chose to do so, exchange some consumption now for a stream of consumption in perpetuity, and the price of permanent consumption in terms of present consumption is the inverse of the interest rate $\bar{r}$. From the definition of the interest rate, it follows that the rate of interest in a stationary state is

$$\bar{r} = \frac{\Delta\, C(\text{permanent})}{\Delta\, C(\text{present})}$$

where Δ C (present) is the amount that consumption this year that might be exchanged for a stream of consumption, Δ C (permanent), in perpetuity. The demand curve for permanent consumption is defined as the locus of all combinations of $\bar{r}$ and C (permanent) at which there would be no desire to invest or to disinvest. The supply curve for permanent income streams is defined as the locus of all combinations of $\bar{r}$ and C (permanent) that could be attained eventually with the technology available to the economy.

These curves are illustrated in figure 2b. The demand curve for permanent consumption streams is derived from figure 2a which represents the economy's indifference curves between present consumption and future permanent consumption. In interpreting this diagram, think of C (present) as the rate of consumption during the current year and C(permanent) as the rate thereafter. The rate of interest corresponding to any combination of C(present) and C(permanent) is indicated as the slope of the indifference curve at that combination. All rates of interest in stationary states, indicated by slopes of indifference curves where they cross the 45^{o} line, are the same if the utility function is homogeneous. In general, the function need not be homogeneous and interest rates need not be the same

FIGURE 2

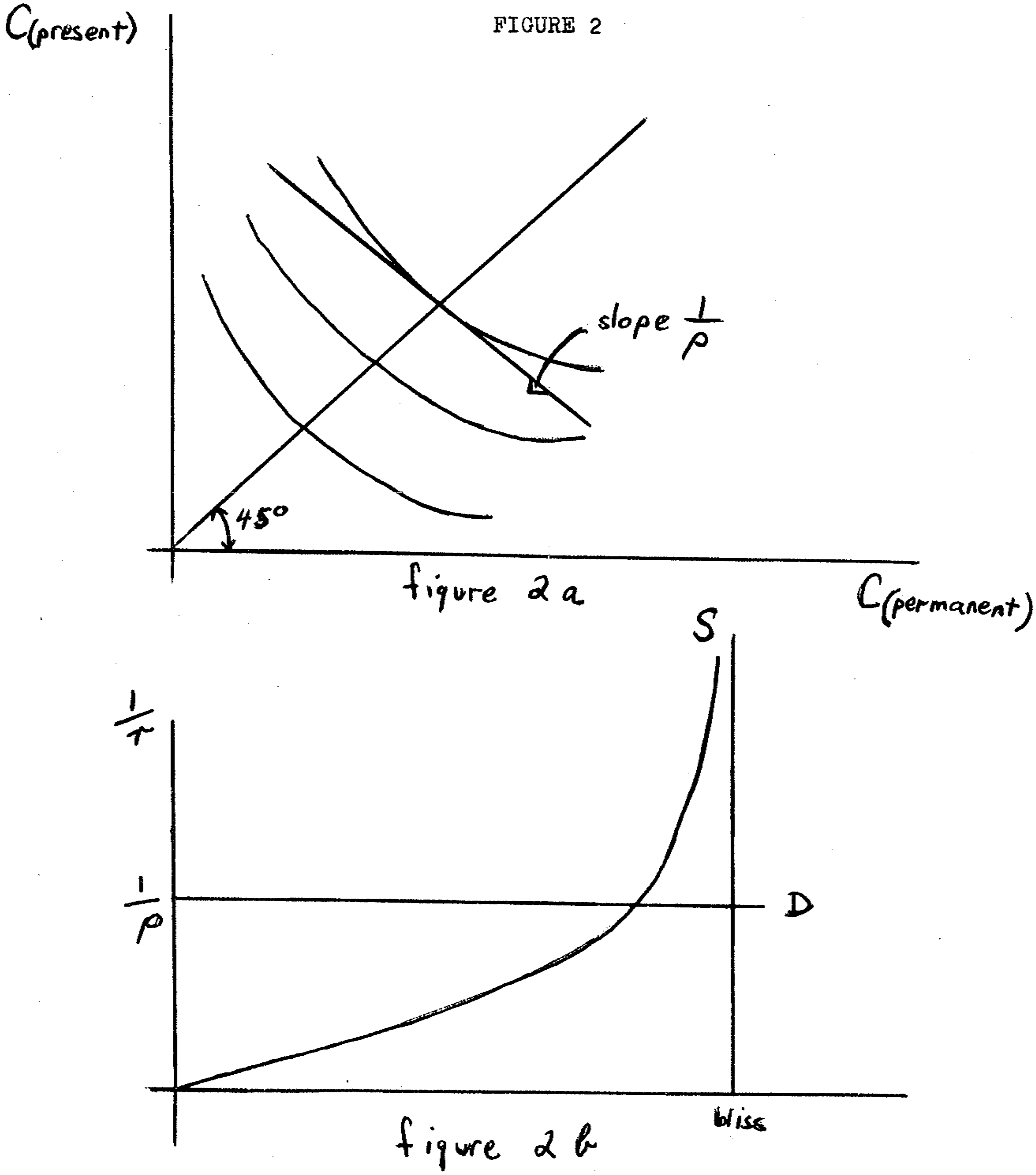


figure 2a

figure 2b

in all stationary states, but I can think of no economic basis for deciding whether interest rates rise or fall as consumption increases among stationary states. It is a consequence of the form of the welfare function, equation (1), and of equation (11) that the demand curve in figure 2b is flat, for $\bar{r}_D = \rho$ whenever the economy is in a stationary state.

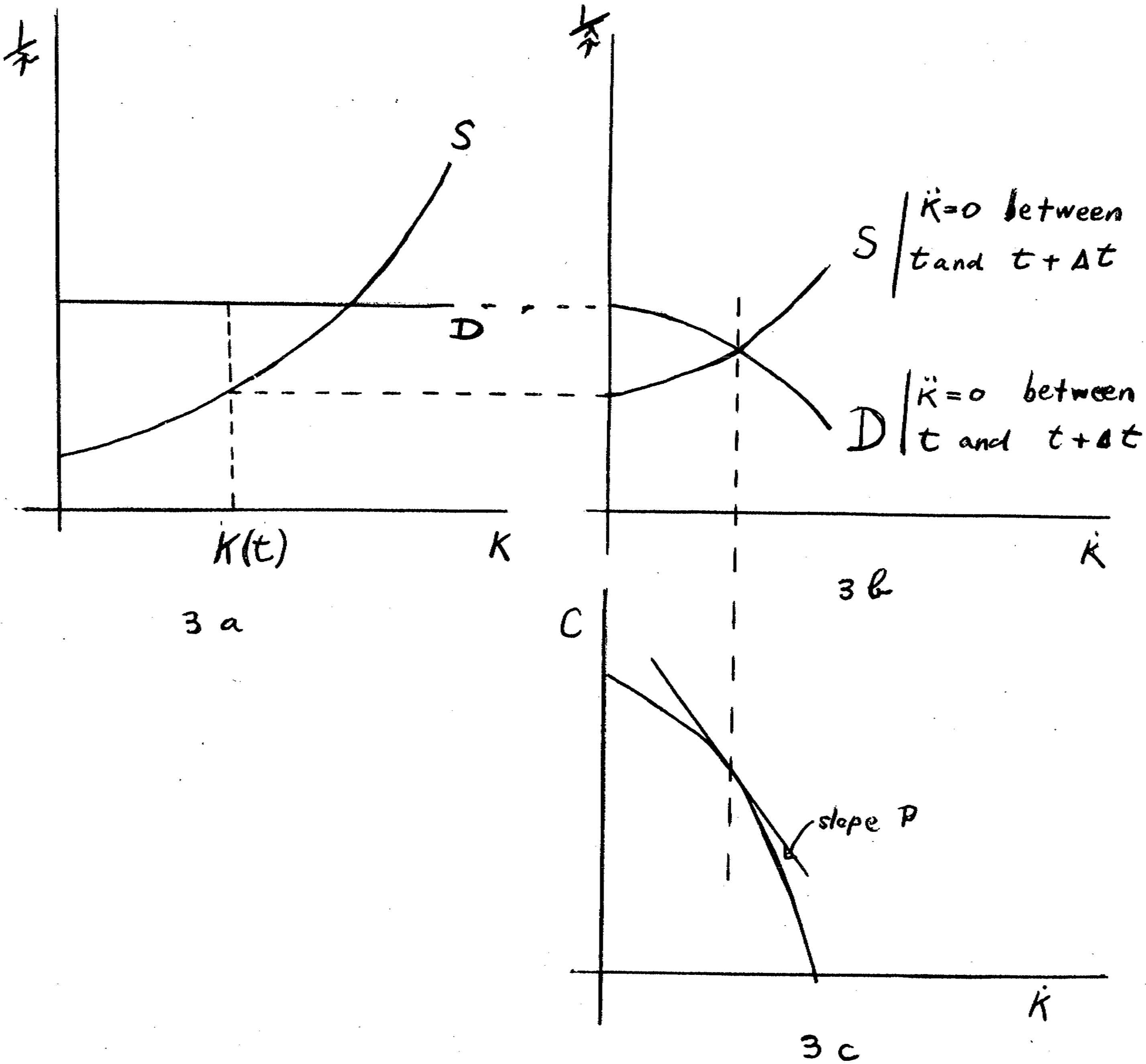
To construct a supply curve corresponding to the demand curve of figure 2b, observe that the rate of interest on the supply side of the economy is

$$\bar{r}_s = \frac{EF_K}{P}\bigg|_{\dot{K}=0} \qquad \text{.....(21)}$$

where P is constant because $\dot{K} = 0$ and F_K is a diminishing function of K. The meaning of equation (21) is that the technically feasible rate of interest in a stationary state is a decreasing function of the capital stock. Furthermore, since the attainable C(permanent) is an increasing function of the capital stock, the variable $1/\bar{r}_s$ is an increasing function of C(permanent) among stationary states; this is the relation labeled S in Figure 2b. Though this was not part of our assumptions, the figure is drawn to show S becoming vertical eventually, implying that there is a rate of consumption, called bliss, which is the maximum that can be sustained permanently with the technology available to the economy.

The stationary state toward which the economy is moving is indicated by the intersection of S and D. One cannot infer from figure 2 what the rate of interest would be at any moment of time before the economy has settled into a stationary state, but the heights of S and D are upper and lower limits upon the short-term rates of interest at all times, and therefore upon the long-term rates as well.

FIGURE 3


S
D
K(t)
K
3 a
S
K̈=0 between
t and t+Δt
D
K̈=0 between
t and t+Δt
K̇
3 b
C
slope P
K̇
3 c

The time paths of capital, consumption and the interest rate can be calculated by solving the Euler's equations, but I cannot find a way of representing the solutions exactly on a two-dimensional diagram. Figure 3b illustrates an approximation to the equilibrium in the market determining r(t) and $\dot{K}(t)$ at any time t. From this diagram and from its relation to figure 3a, we can visualize the main forces at work. Part a of figure 3 is the same as figure 2b except that the horizontal axis measures the quantity of capital required to sustain C (permanent) rather than C(permanent) itself; the transformation is according to the formula C(permanent) = F (K,L). If at time t the capital stock were equal to K(t), the stationary state that could be sustained with the existing technology would have an interest rate $\bar{r}_s$ indicated by the height of the S curve; and the interest rate $\bar{r}_D$, at which the economy would not want to invest would be indicated by the height of the D curve. As the figure is drawn $\bar{r}_s > \bar{r}_D$, which signifies that investment is taking place. Capital formation will not stop altogether until the capital stock has grown to the point where the S and D curves cross.

Imagine that the rate of capital formation $\dot{K}(t)$ is held constant between time periods t and t + Δ t where Δ t is a small, finite increment of time; we shall consider the implications of choosing alternative values of $\dot{K}$ (t) on the assumption that $\ddot{K}(t) = 0$. Figure 3c illustrates the relation between P and $\dot{K}(t)$ on the supply side of the economy. Since P is an increasing function of $\dot{K}(t)$, and since $\ddot{K} = 0$ over the interval t to t + Δ t, $\frac{dP}{dt} = 0$ as well and the short term rate of interest is:

$$\hat{r}_s(t) = \frac{MPC}{P(K)} \quad \text{.......................................(22)}$$

Since Δt is small, MPC, dependent only on K(t), may be thought of as constant, and the rate of interest is highest when $\dot{K} = 0$ and declines steadily as $\dot{K}$ increases. This relation between $\hat{r}_s$ and $\dot{K}$ is illustrated on the flow supply curve of figure 3b. The S curve of figure 3b begins at the vertical axis at the hieght of the S curve of figure 3a corresponding to the existing capital stock and rises steadily as $\dot{K}$ increases, thus ensuring that the short termrate of interest is less than the rate that would obtain in the stationary state that could be achieved with the existing capital stock.

From equation (11) it follows that $\hat{r}_D(t)$, the short term rate of interest on the demand side of the economy is greater than ρ whenever consumption is increasing. To introduce $\dot{K}$ into equation (11) note that

$$\frac{dC}{dt} = \frac{dC}{dK} \cdot \frac{dK}{dt} = MPC \cdot \dot{K}$$

where MPC is virtually constant because K is virtually constant in the interval t to t+ Δ t. Equation (11) becomes

$$\hat{r}_D(t) = \rho - \frac{U''(C(t))}{U'(C(t))} \; MPC. \; \dot{K} \quad \text{...................(23)}$$

It follows from this equation that the flow demand curve of figure 3b is at its maximum height, $1/\rho$, when $\dot{K} = 0$ and that it declines steadily as $\dot{K}$ increases because $\dot{K}$ is a term in equation (23) and because an increase in $\dot{K}$ reduces C and thereby increases the marginal utility of income. The negative slope of the flow demand curve ensures that whenever there is capital formation, the short term rate of interest exceeds the rate at which the economy would be content with a stationary state.

Unfortunately, the "equilibrium" rate of interest indicated by the intersection of the S and D curves of figure 3b is not a true representation of the short-term rate of interest whenever $\ddot{K}$ differs from zero, for investors are making capital gains and the rate of interest is greater than that indicated in figure 3b.

With this qualification in mind, the history of the rate of interest as K increases may be characterized by shifts of the flow demand and supply curves for capital goods. As the capital stock increases, the marginal productivity of capital declines, the rate $\bar{r}_s$ declines as indicated by the supply curve of figure 3a, and the flow supply curve of figure 3b shifts upwards until, in the end, the S and D curves intersect at the vertical axis, $\dot{K} = 0$, and $\hat{r}(t) = \rho$. At the same time the flow demand curve, the position of which on the vertical axis is fixed, rotates anti-clockwise until it becomes horizontal when $\frac{dc}{dt}$ has fallen to zero. The shift of the flow supply curve and the rotation of flow demand curve cause $\dot{K}$ to fall to 0 and r to fall to ρ as the stationary state indicated by the crossing of the S and D curves of figure 3a is approached.[4]

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[4] The designation of the S and D curves of figure 3 as supply and demand curves is not strictly correct for prices and quantities do not correspond to one another in the proper way. Since the rate of interest is essentially a relative price of consumption goods now and consumption goods in the future, the appropriate quantity to be combined with interest rate should be consumption goods (as in figure 2) rather than capital goods. Nevertheless, I have chosen to label the curves S and D because they are related to the true supply and demand curves of figure 2 and because, even in figure 3, the S curves represent technology and the D curves represent taste.

Productivity and Thrift

We are now in a position to attempt to answer the questions posed at the outset of this paper. The answer to question 1 is obvious: in an economy like the one described here, the time-structure of interest rates is endogenous and originates, like all prices, as a property of equilibrium when welfare is maximized.

Questions 3a and b may also be answered in the affirmative. Taste in the choice between present and future consumption may be represented by the parameter ρ ; the smaller ρ , the more future-oriented is taste. The effect of a sudden change in taste, causing future goods to become more valuable relative to present goods, is to decrease ρ raising the height of the D curve of figure 3a, and raising in turn the height of the flow demand curve of figure 3b. The new equilibrium, to the north-east of the original equilibrium along the flow supply curve, contains more capital formation and a lower rate of interest. Thus a change in taste affects the time-structure of interest rates in three respects a) there is an instantaneous change in the short-term rate of interest b) short-term rates remain higher or lower (depending on the direction of the change of ρ) than they would have been for a sustained period of time and c) the rate that is to be attained in the stationary state toward which the economy is moving (being ρ itself) is affected by the change in taste.

The influence of technical change on interest rates is less readily assessed than the influence of taste because it is less clear with technology than with taste what a future-oriented change might be. One might examine the impact on the time-structure of interest rates of a once-and-for-all-change in the marginal product of capital occuring now (an increase or decrease in E in equation (2)), a change in the marginal product of capital occuring in the future, a rate of change of the marginal product of capital, a once-and-for-all change in the function $\dot{R}(K)$ occuring now or in the future, or a rate of change of $\dot{R}(K)$. All of these changes in technology affect interest rates. We shall consider only one sort of technical change, an unexpected and once-and-for-all change in the marginal product of capital.

Suppose there is a 10% increase in E. It follows from equation (21) that the stationary state supply curve of figure 3a shifts down by 10%. The corresponding flow supply curve also shifts down by 10%, causing an increase in $\dot{K}(t)$ and a rise in the short-term rate of interest (since the flow demand curve is downward sloping) by something less than 10% of its original value. The rate of interest remains higher than it would have been without the increase in the marginal product of capital for a long time. However, the rate of interest in the eventual stationary state remains at ρ and is not affected by the change in E.

The answer to question 2a is in the affirmative. A change in technology, represented by an increase in E, raises the stationary state supply curve of figure 3a thereby raising the flow supply curve of figure 3b and increasing the short-term rate of interest. The answer to question 2b is in the affirmative if the phrase "in the course of time" refers to future short-term rates of interest in the economy before it enters the stationary state. However, it follows from assumptions in equations (1), (2), (3) and (4) that the stationary state demand curve is flat and that ρ is the only possible rate of interest in the stationary state itself. A productivity change may raise the rate of interest permanently because an infinite period of time may have to elapse before the economy reaches the stationary state toward which it is always moving, but a productivity change cannot influence the limiting value of the term-structure of interest rates. If the phrase "in the course of time" in question 2b refers to stationary state toward which the economy is moving, the assumptions we have made oblige us to answer the question in the negative.

Had a different form of community welfare function been assumed, the stationary state demand curve need not have been flat, and productivity could influence the interest rate in the eventual stationary state. The general form of a community welfare function is:

$$W = W(C(1),\ C(2),\dots\dots\dots C(n))\dots\dots\dots\dots\dots\dots\dots\dots(24)$$

where time periods may be as short as we please. To get equation (1) from equation (24) it was assumed that the function W is separable in its arguments and that utils at different time periods are compared by means of a fixed rate of discount. Both assumptions are needed to ensure that D in figure 3a is flat. For instance it could have been assumed that

$$\rho \;=\; \rho\,(C(\text{permanent}))\dots\dots\dots\dots \qquad \rho' > 0 \;\dots\dots\dots(25)$$

The demand curve of figures 3a would be downward sloping and there would be complete symmetry between the demand and supply sides of the market for interest rates.

Types of Interest Theory

With one reservation about question 2b, our answers to question 1, 2 and 3 posed at the outset of this paper are all in the affirmative. In the remainder of this paper I shall examine the assumptions that have led other authors to different conclusions.

a) The assumption that P is constant

This assumption accounts for the most important divergence of opinion between Friedman, Ackley, and Fisher on one side and Knight, Clower, Solow and Dewey on the other.[5] It can take several forms.

i) It may be assumed explicitly that the production possibility curve in figure 3c is a straight line.

ii) The economy may be thought of as a perpetual plant that grows either at a fixed rate or at a rate dependent on its size. (Knight's crusonia model)

iii) It may be supposed that the output of the economy, designated as Y, is the sum of consumption C and investment I, and investment is the rate of capital formation $\dot{K}$.

The implication of this assumption is that the interest rate is independent of taste and unaffected by changes in taste in the short run. If P is independent of $\dot{K}$, the production possibility curve of figure 3c is a straight line, the short-run supply curve of figure 3b is flat, and the interest rate is indicated by the height of S over K(t) in figure 3a.

[5] Though Ackley answers all questions in the negative and Fisher answers all questions in the affirmative, the differences between them are more apparant than real.

Changes in taste shift the stationary-state demand curve of figure 3a and change the position of the flow demand curve of figure 3b. There is no immediate effect on the short-term rate of interest because the flow supply curve of figure 3b is flat; the fall impact of the shift in the flow demand curve is on the rate of capital formation $\dot{K}(t)$. Only in the course of time are interest rates affected. Since $\dot{K}$ is larger from now on, the capital stock in the future will be larger than otherwise, the flow supply curve will be higher than it would be otherwise, and the short-term rate of interest will be lower.

Hayek cited approvingly by Dewey, has described the situation as follows: "Time preference is a subordinate factor compared with the productivity of investment in determining the rate of interest, since it operates only by way of determining the rate of savings and the rate of capital accumulation, and hence the productivity of investment. In the short run it merely adapts itself to the given marginal productivity of investment"[6]. This statement is false unless P is assumed to be independent of changes in $\dot{K}$.

Variants ii and iii of this assumption - the crusonia model and what might be called the national accounting variant of the assumption- share the property that they set $P = 1$ by assuming that capital goods and consumption goods are made of the same, homogenous stuff. By oversight or by explicit assumption, many authors pass from the accounting identity that

[6] See D.Dewey, Modern Capital Thoery (Columbia University Press, 1965), p.70 and F. Hayek, The Pure Theory of Capital (Routledge Kegan Paul, 1941) p.413. Curiously, on p.234 Hayek admits to being persuaded on this point by an article by Frank Knight.

the dollar value of income equals the sum of the dollar values of consumption and the dollar value of investment, to models in which real income is the sum of real consumption and real investment. For many purposes it is convenient and relatively harmless to work with such a one-sector model of an economy. The model is dangerous in analyzing interest rates because it proscribes important relations among taste, thrift and the interest rate that emerge as soon as the relative price of capital goods and consumption goods is allowed to vary.

The assumption that P is independent of $\dot{K}$ is, I think, exceedingly implausible. A better case could be made for the opposite assumption that, in the short-run at least, the economy yields consumption goods and capital goods in fixed proportions. Farming and services, for instance, produce consumption goods in the most part, and their output cannot be converted to capital of any kind except at a very marked rise in the relative price of capital goods. Even manufacturing firms specialize to a great extent, and the composition of output of a factory cannot be altered without loss of profit unless relative prices change too. In short, the flow supply curve of figure 3b slopes upward because it is a supply curve, and because constant returns to scale, though they may exist in small sectors of an economy, are most unlikely to exist between two sectors that together comprise the whole economy.

We have not so far distinguished between short-run and long-run supply curves of capital goods. We have even ruled out the distinction by supposing that

$$R = R(\dot{K}) \quad \ldots\ldots (4)$$

which indicates that the average cost of capital goods depends only on the rate at which they are being produced and not on the length of time at which they have been produced at that rate. To introduce the distinction between long and short run assume

$$R = R(\dot{K}, \ddot{K}) \;.........\; R_{\dot{K}} > 0,\; R_{\ddot{K}} > 0 \;...............(25)$$

Relations between curves of different runs are illustrated in figure 4 which is similar to parts b and c of figure 3. The supply curves (1,1), (2,2) and (3,3) are derived from the corresponding production possibility curves. The longest-run curve is the one for which $\ddot{K} = 0$ and is illustrated as (3,3). The shortest-run curve is the one for which $\ddot{K} = 0$ and is illustrated as (1,1). The meaning of (1,1) is that $\dot{K}$ cannot be increased instantly by any finite amount.

b) "The Rate of Interest Equals the Marginal Product of Capital". This statement is true of two conditions: that the price of capital is independent of $\dot{K}$, and that capital is measured not in its own units (not as a stock of axes) but in units of the consumption good (apples). Recall that the short-term rate of interest is

$$\hat{r}(t)_s = \frac{MPC + \frac{d}{dt} P}{P}$$

Only if $\frac{dP}{dt} = 0$ can one ignore obsolence or appreciation in the determination of interest rates and it is necessary to suppose that P=1 to ensure that the rate of interest and the marginal product of capital have the same dimension. Both of these conditions hold in a model where output Y is defined to be the sum of $C + \dot{K}$. As discussed above these assumptions hide important aspects of the determination of interest rates.

FIGURE 4

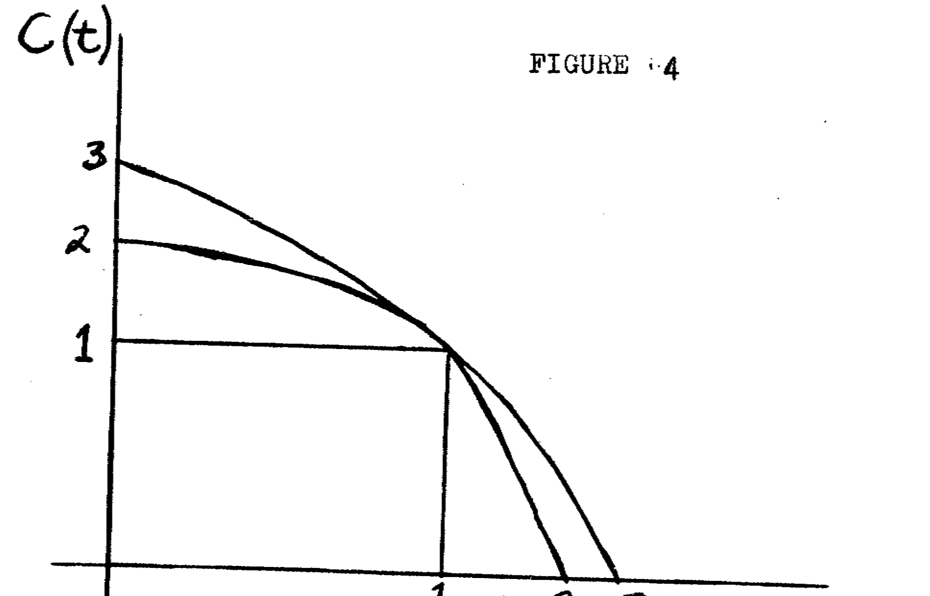

figure 4c

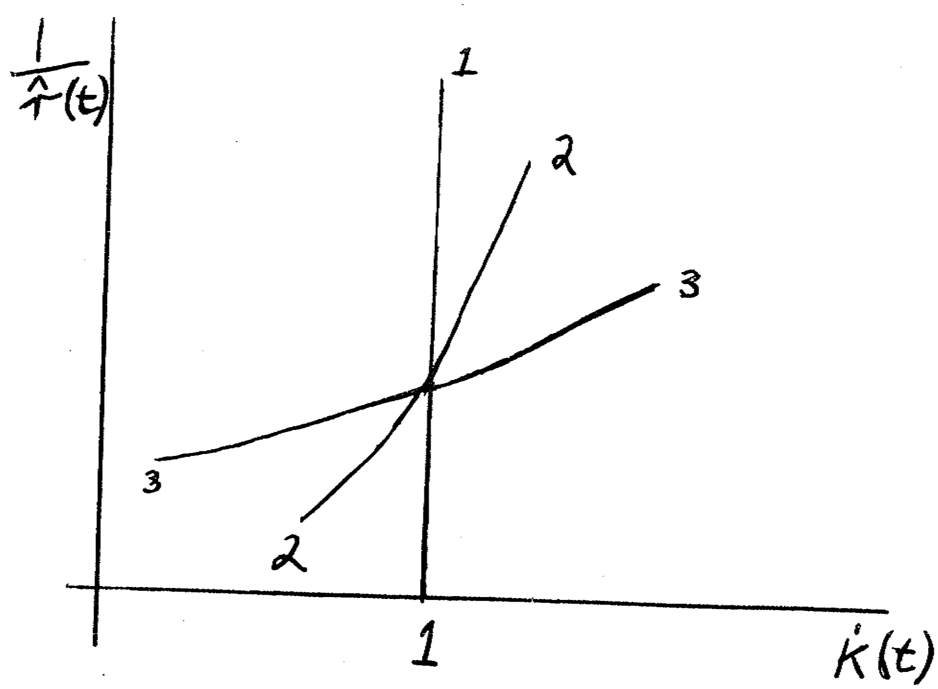

figure 4b

Some authors have emphasized the equivalence between interest rates and the marginal product of capital in the hope of deriving the real rate of interest as a property of technology exclusively and as a function of the capital-labour ratio in the economy. Such derivations are necessary fallacious whenever P is a function of $\dot{K}$ because a given stock of capital $K(t)$ is consistent with a wide range of interest rates $\hat{r}(t)$ and the economy's choice of one rate from the spectrum depends on the value of ρ and other taste parameters[7].

c) The Keynesian Assumption that the Interest rate is Exogenous to the Competitive Economy.

The supply side of figure 3 is virtually identical to Ackley's diagram for explaining the rate of capital formation as a function of the rate of interest. Yet, Ackley excludes the demand side altogether and treats the rate of interest as exogenous to the competitive economy.

Like any price, the rate of interest might be exogenous to a small country in international trade. If the world market establishes a rate of interest 5% and the rate that would emerge if the country were left to itself were 10%, the country would produce capital to sell abroad and would add to its own capital stock only up to the point where $r_B(t)$ equals the world rate of interest. As shown in figure 6, production of new capital

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[7] See R. Solow, Capital Theory and the Rate of Return, Chapter III.

goods is $\dot{K}_s$ but domestic capital formation is only $\dot{K}_D$ and capital is exported in the amount $\dot{K}_s - \dot{K}_D$ per unit of time.

Ackley following Keynes appears to reason as though the government of a country can lend to or borrow from the private sector in much the same way that a country might lend or borrow from the world market. On this view, the government can, if it chooses, make the real interest rate exogenous to the private sector by offering to lend or borrow any amount at a fixed real rate of interest on the strength of its powers of taxation now and in the future. By and large, governments do not behave in this way, and the more typical effect on interest rates of monetary and fiscal policy is to shift curves in figure 3 without affecting substantially the mechanism in which interest rates are determined by the interaction of technology and taste.

d) Capital as Title to a Flow of Consumption goods.

Friedman and Clower, following Knight, use the word capital to denote what I have called C(permanent). A unit of capital in Friedman's system is not an axe but is instead title to one apple per year in perpetuity. This system of capital, though it seems to yield the correct results (and I have drawn heavily on it above), is in some respects less satisfactory for analysing the rate of interest than is a system of capital that makes use of the concept of real capital measured in its own units. It is not possible to draw consistent flow curves like those of 3b when capital is defined as title to a permanent income stream because the size of the permanent income stream that can be bought with a given sacrifice of consumption today depends on the future marginal product of capital which in turn depends on the extent to which the economy invests tomorrow. To some extent, Friedman circumvents this problem by

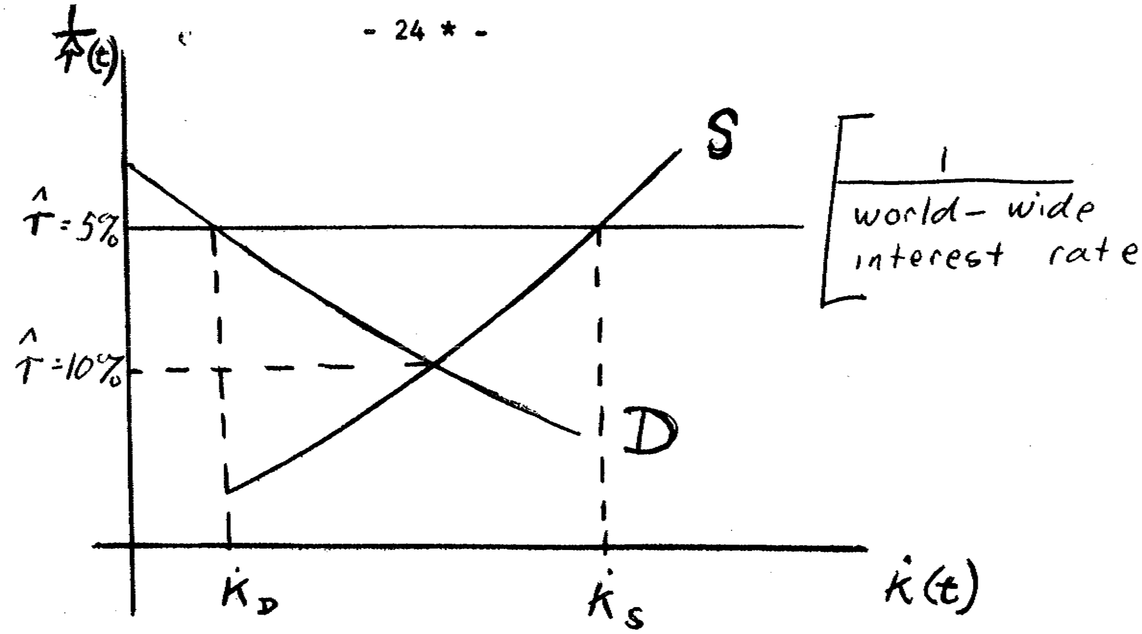

figure 6

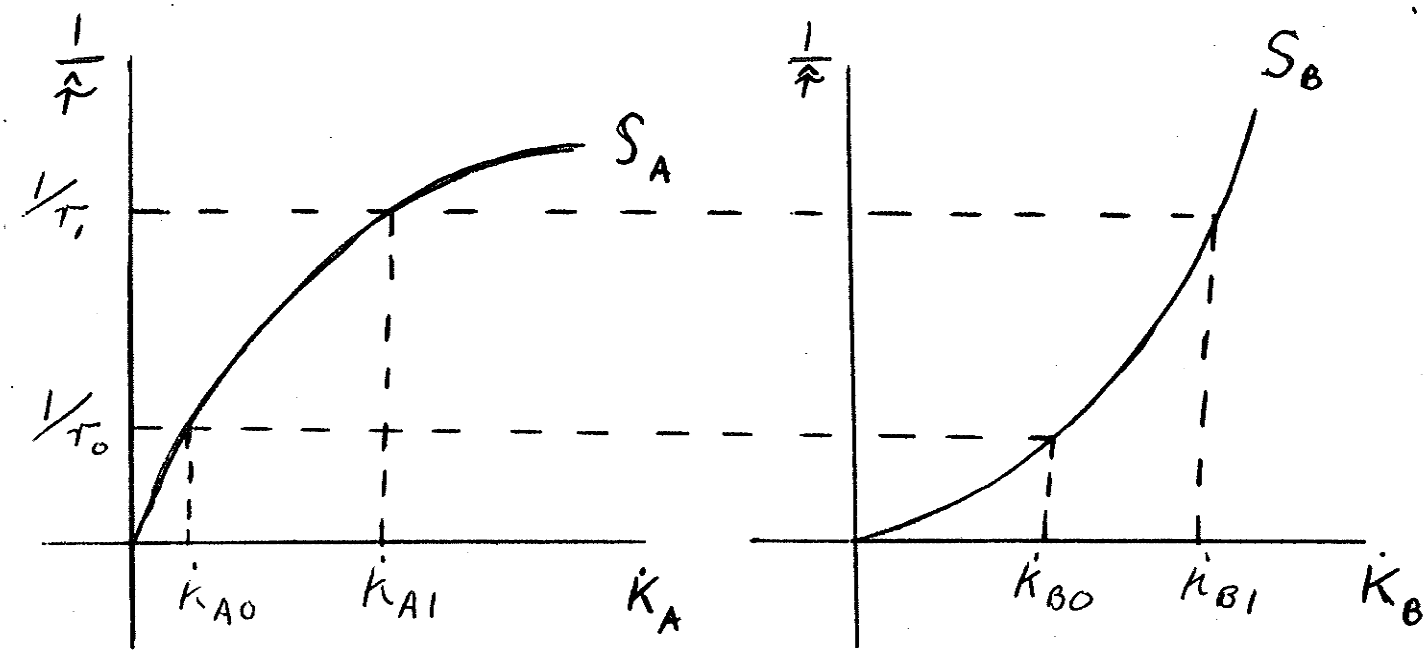

figure 7

drawing families of curves like S and D of figure 3a, a separate curve for each rate of saving.

e) Turnpikes and thrift.

In the Von Neumann model there is a priveleged path called the turnpike to which an economy tends. A property of the Von Neumann model that contributes to this result is that there are fixed rates of trade-off, rates independent of time and of rates of investment, between mixtures of consumption goods foregone now and mixtures of consumption goods acquired next year. Though actual paths diverge from the turnpike depending on the mix of output at which the economy begins and the mix that is eventually attained, the turnpike itself is a property of technology alone. I conjecture that there is no corresponding technically-given privileged path in an economy in which the rate of trade-off between present and future consumption in any technique of investment depends on the rate at which the technique is utilized. The optimal commodity composition of investment would depend on the rate of investment. The best route from one mix of output to another may well demand on how fast the route is travelled and on how large is consumption in between. Figure 7 is a modification of the supply side of figure 3b. There are two industries, A and B, and capital formation in each industry increases as the rate of interest falls. However, investment in industry A is more easily expanded than investment in industry B so that as r falls, the ratio of $\dot{K}_A$ to $\dot{K}_B$ increases. Since the ratio of outputs of industries depends ultimately on rates of capital formation, and since these rates are not independent of taste, it is hard to see how any technically-given proportion of outputs of A and B could be approached in all possible efficient time paths of consumption.

Conclusion

The main division among theorists in their views about the rate of interest is between those who believe that the short-term rate of interest is determined by technology alone, and those who believe that a given stock of capital is consistent with a wide spectrum of interest rates and that the economy's choice of a rate from the spectrum is in part a consequence of thrift. The former view is based on the assumption that capital goods and consumption goods are the same stuff, that $Y = C + \dot{K}$ in real terms, that the economy may be thought of as a perpetual plant growing at a rate dependent on its size, that the price of capital is independent of the rate of investment, or that the rate of interest equals the marginal productof capital (measured in axes). Members of the latter group, who I believe are correct, emphasize that the rising supply curve of capital goods causes the short-term rate of interest tofall whenever investment is increased, and that productivity and thrift are equally important determinants of the rate of interest in the long run and in the short run as well.