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Margaux MacDonald
Queen's University

Michal Popiel
Queen's University

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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Margaux MacDonald[†] Michał Ksawery Popiel[‡]
Queen's University Queen's University

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Abstract

This paper investigates the effects of unconventional monetary policy in Canada. We use recently proposed methods to construct a shadow interest rate that captures monetary policy at the zero lower bound (ZLB) and estimate a small open economy Bayesian structural vector autoregressive (B-SVAR) model. Controlling for the US macroeconomic and monetary policy variables, we find that Canadian unconventional monetary policy increased Canadian output by 0.23% per month on average between April 2009 and June 2010. Our empirical framework also allows us to quantify the effects of US unconventional monetary policy, which raised US and Canadian output by 1.21% and 1.94% per month, respectively, on average over the 2008–2015 period.

JEL Codes: E52, E58, F42.

Keywords: Small open economy; unconventional monetary policy; Bayesian structural VAR; zero lower bound; international monetary policy transmission.

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[†]Email: macdonald@econ.queensu.ca (Corresponding author)
[‡]Email: popielm@econ.queensu.ca

1 Introduction

Following the 2008 global financial crisis, many central banks quickly exhausted their ability to stimulate economic activity as policy rates reached the zero lower bound (ZLB).[1] To continue to encourage lending, they turned to unconventional measures including direct market interventions through large scale asset purchases and forward guidance to influence expectations of the future short rate. A focus of much of the recent literature has been to quantify the effects of these policies on long term rates, asset prices, credit costs and, to a lesser extent, the real economy.[2] Moreover, a majority of these studies have focused on the domestic effects of such policies in large open economies, most notably the US. In contrast, this paper analyzes the effects of unconventional monetary policy measures in a small open economy, a topic which has hitherto received relatively little attention.[3] Specifically, we measure the effects of the Bank of Canada's actions while at the ZLB on the real Canadian economy.

There are two main challenges that we face in conducting our analysis. First, small open economies often respond strongly to foreign shocks. Therefore, to gain meaningful insight into the role of domestic monetary policy we must control for international variables. This motivates our selection of Canada, among several candidate small open economies that engaged in unconventional monetary policy following the global financial crisis, since international effects can be mostly captured by US variables. This greatly simplifies our analysis. Moreover, since the Canadian economy is relatively small, we can assume that its domestic shocks have little impact on the US, which also reduces the complexity of our study. In fact there is evidence that the direction of monetary spillovers globally is generally from the US to non-US markets, with limited evidence that

[1]The ZLB, also called the effective lower bound, refers to the rate at which households and firms began to prefer to hold physical currency over bank deposits. Although this bound could potentially be negative (*e.g.* due to convenience costs) we define it as 25 basis points (bps) to be consistent with central bank definitions at the time. At a target rate of 25 bps, banks earn zero interest on overnight loans but still pay a positive rate when they borrow. Note that if depository institutions are required to hold a certain level of balances at the central bank above the required reserves level, or if they simply desire to do so, then the policy rate can be set below zero. The Swiss National Bank, the Danish National Bank, the Swedish Riskbank, and the European Central Bank (ECB) were able to lower their policy rates below zero in 2014 and 2015 because of such requirements and desires for excess reserves. The Bank of Canada has recently indicated it would be willing to implement negative rates as well.

[2]See, for example, Hamilton and Wu (2012), Swanson and Williams (2014), D'Amico and King (2013), Krishnamurthy and Vissing-Jorgensen (2011), and Gagnon, Raskin, Remache, and Sack (2010), among others, for studies that look at the impact on interest rates, term spreads, asset prices, and credit costs, and Dahlhaus, Hess, and Reza (2014) and Gambacorta, Hofmann, and Peersman (2014) for studies that look at the impact on the real economy.

[3]Note that there is a large literature on Japanese unconventional monetary policy from the early 2000s, which is separate from the literature we refer to here on US or small open economy unconventional monetary policies. See Bernanke and Reinhart (2004) for an overview of the state of the literature at that time.

there are spillovers in the reverse direction, even from other large open economies.[4] We impose these assumptions via block-exogeneity restrictions à la Cushman and Zha (1997) in a two-country Bayesian structural vector autoregressive (B-SVAR) model. More recently SVAR models with block-exogeneity restrictions have been used to study the impact of U.S. monetary policy shocks on the Euro Area (Neri and Nobili, 2010), on Latin American countries (Canova, 2005), on a range of emerging markets (Maćkowiak, 2007), and on Canada (in an alternative model characterization) (Bhuiyan, 2012). Block-exogeneity assumptions can also been useful in non-monetary policy spillover models, such as models of global commodity shocks on small open economies (Charnavoki and Dolado, 2014).

Second, capturing the stance of monetary policy is complicated by the fact that there is no variation in the target interest rate — the standard choice for a policy variable — at the ZLB. To get around this problem, the literature provides two alternatives: event study methods or standard SVAR models with a different policy variable such as the central bank balance sheet (Gambacorta *et al.*, 2014; Dahlhaus *et al.*, 2014; MacDonald, 2016), the term spread (Baumeister and Benati, 2013) or a shadow rate (Krippner, 2013; Wu and Xia, 2016). Event studies measure the response of financial variables, such as long term yields and asset prices, to monetary policy announcements. By focusing on the response around a very short time interval (typically 30 minutes), they are able to isolate monetary policy shocks that are not contaminated by any other important events. While insightful on the immediate response of financial variables, event studies are unable to capture the effects of monetary policy shocks on real variables, such as prices and output, because their response is much slower. Additionally, the event study captures only changes in market expectations that occur within the specified time interval, which itself may be misspecified. Using the central bank balance sheet as a policy variable in an otherwise standard SVAR directly tackles this issue. However, the balance sheet only captures the effects of large scale asset purchases and does not reflect other policies such as forward guidance.[5] The term spread captures the effect of both

[4] Rogers, Scotti, and Wright (2014) find this result for unconventional monetary policy, Ehrmann and Fratzscher (2005) find a similar result for conventional monetary policy.

[5] Importantly, the balance sheet only reflects the actual implementation of LSAPs and not their announcements, which have been shown to have a significant stimulating effect on the economy. Krishnamurthy and Vissing-Jorgensen (2011) and Gagnon *et al.* (2010), among others, show unconventional monetary policy announcements had significant impacts on US financial markets. Neely (2015), Chen, Filardo, He, and Zhu (2015), and Bowman, Londono, and Sapriza (2015), among others, show that unconventional monetary policy announcements had significant impacts on foreign country financial markets. Eichengreen and Gupta (2014), and Dahlhaus and Vasishtha (2014) show the same is true for unconventional monetary policy tapering announcements.

announcements and large scale asset purchases but by measuring these effects through the compression of the yield curve it cannot be constructed as an uninterrupted variable through conventional and unconventional episodes, as is also the case with the balance sheet. Moreover, counterfactuals using the term spread rely on outside estimates of the effects of large scale asset purchases on the spread. The alternative that we adopt in this study is to define the policy variable using an estimated shadow interest rate, which uses the term structure of interest rates to predict the level of the short-term rate as if it were not bounded below by zero.

Although the use of shadow rates comes at a cost of not being able to disentangle the effects of specific policies, it has several distinct advantages. The shadow rate provides a consistent measure of monetary policy stance for *both* central banks in our empirical model. Since the Bank of Canada's unconventional monetary policy measures did not include large scale asset purchases, the shadow rate is more appropriate than the balance sheet as a policy variable. Furthermore, its flexibility also makes it appropriate as a policy variable for the Federal Reserve (the Fed) because it captures the effects of both forward guidance and large scale asset purchases. Moreover, since the shadow rate can be viewed as an extension of the traditional policy variable uninterrupted by the ZLB, we are able to begin our sample long before the ZLB episode and use the additional observations to improve the precision of our results.

We estimate our model for the period 1994–2015 and quantify the impact of unconventional monetary policies through counterfactual experiments that restrict the shadow rate to the ZLB. Our main finding is that Canadian unconventional monetary policy had expansionary effects on the Canadian economy, boosting output by approximately 0.23 percent per month on average during the ZLB episode. This result is robust to several different specifications. Out setup also allows us to investigate the effects of US unconventional monetary policies on the Canadian economy. We find that Canadian output would have been approximately 1.21 percent lower per month on average without US unconventional monetary policies.

This paper is most closely related to other studies seeking to quantify the effects of unconventional monetary policy measures in Canada. An early effort to assess the impact of forward guidance on Canadian interest rates by He (2010) considers the relationship between interest rates, inflation and unemployment, and shows that the Bank of Canada's conditional commitment on interest rates effectively reduced yields, albeit not statistically significantly. Chang and Feunou (2013) study changes in implied and realized volatility around important Bank of Canada announcement days, and find that forward guidance, the expansion of liquidity, and policy rate cuts successfully reduce market volatility. Gambacorta, Hofmann, and Peersman (2014) use central bank balance sheets to assess the effectiveness of monetary policy at the ZLB domestically. For Canada, as well as 7 other countries, the authors find that a positive shock to the balance sheet increased economic activity and consumer prices but, compared to conventional times, the effect on inflation was weaker and less persistent. There are two important distinctions between these studies and ours. First, they do not control for potential spillovers from the US and second, they do not evaluate the total effect of all unconventional monetary policies on the economy.

Including the US in our analysis ties our work to a strand of literature that measures the effects of US unconventional monetary policy spillovers. Fratzscher, Lo Duca, and Straub (2013) analyze the impact of 12 key quantitative easing (QE) announcements by the Fed and their impact on net inflows of bonds and equities, equity price returns, long-term bond yields, and exchange rate returns of 65 advanced- and emerging-market-economy countries (including Canada) in a panel regression.[6] They find that QE 1 and 2 lowered sovereign yields and raised equity prices but that there was substantial heterogeneity in spillover effects between emerging market economies and advanced economies. Bauer and Neely (2014) use an event study within a dynamic term structure framework to identify the transmission channels for unconventional US monetary policy. According to their results, the LSAP programs had a large effect on Canadian long-term yields, bringing them down significantly through a signalling and portfolio balance channels. Neely (2015) also uses an event-study and finds large effects on Canadian yields and the exchange rate. Dahlhaus *et al.* (2014) evaluate the spillovers from QE on the Canadian economy with quarterly data from 2008Q4 to 2013Q3.[7] Using a factor-augmented VAR (FAVAR) model and the Fed's balance sheet as the policy instrument, they compare the impact of QE on Canadian variables to a counterfactual scenario in which the balance sheet continues to grow at the pre-crisis rate. They find that QE had a positive impact on Canadian output and that Canadian asset prices and interest rates move in tandem with their US counterparts. Our study is distinct from these because we carefully model the

[6] Fratzscher, Lo Duca, and Straub (2014) perform a similar analysis but for European Central Bank QE announcements.

[7] Their approach closely follows Charnavoki and Dolado (2014) who analyze the effects of commodity price shocks on Canada using a structural dynamic factor model.

domestic monetary policy in the spillover country, and, by using the shadow rate, we incorporate both announcement and balance sheet spillovers.

Recent concerns about the large adverse cross-border spillovers of unconventional monetary policies have rekindled the discussion of international coordination of monetary policy (Taylor, 2013; Engel, 2015). It has been argued that countries who are responsible for substantial international monetary policy spillovers need to acknowledge their role in influencing foreign economies, and to consider feasible remedies to limit such spillovers (Blanchard, Ostry, and Ghosh, 2013; Mishra and Rajan, 2016). Our work contributes to an alternative perspective on unconventional monetary policy spillovers, and shows that under certain circumstances they can be favourable.

This paper is structured as follows. The next section describes our measure of unconventional monetary policy. Section 3 presents the data and method. Our main analysis, which studies the effect of Canadian and US monetary policy on the Canadian economy from 1994–2015, is discussed in Section 4. We conduct several robustness exercises, by considering alternative model specifications. Results for these exercises are reported in Section 5. Section 6 concludes.

2 Measuring unconventional monetary policy

This section briefly discusses the scope of unconventional monetary policies enacted by the Bank of Canada and the Fed and presents the shadow rate as an appropriate proxy for capturing their effects.

The Bank of Canada targets inflation by influencing its target for the overnight rate (the "target rate"). There are eight fixed announcement dates annually, upon which it reports whether or not it will adjust the target rate. It also has an explicit policy *not* to intervene in the Canadian currency market. On April 21 2009, in coordination with other central banks, the Bank of Canada lowered the target rate from 50 bps to 25 bps, considered at the time its effective lower bound. Along with the rate cut, the Bank of Canada reduced the operating band from 50 bps to 25 bps and set the target rate to the lower bound of this range (25 bps). Aiming to reduce uncertainty in Canadian financial markets, the Bank of Canada committed to keep the target rate unchanged until the end of the second quarter of 2010, conditional on inflation expectations. The Bank of Canada further reinforced the upper bound on the operating band (at 50 bps) through its standing liquidity facility (SLF), which provided access to overdraft loans at the bank rate for the Bank's Large Value

Transfer System (LVTS) participants, and by prorating access to new standing purchase and resale agreements (PRAs) at the bank rate for Canadian primary dealers.[8] The Bank of Canada reinforced the lower bound on the operating band by providing LVTS participants with access to a standing deposit facility on which they could earn the deposit rate and by conducting sale and repurchase agreements (SRAs) intraday at 25 bps with primary dealers, if required. Finally, the Bank of Canada targeted daily settlement balance levels at $3 billion, a dramatic increase from the small positive balance target during conventional times (Bank of Canada, 2010). With market conditions improving, the statement of this commitment was removed on April 20 2010. On June 1 2010 the target rate was raised to 50 bps, and the standard operating framework was reestablished.

The federal funds rate reached the ZLB in December 2008 at which point the Fed began engaging in unconventional monetary policies, including LSAP purchases and announcing commitments to remaining at the ZLB and continue the LSAP program. LSAPs involved purchases of asset with medium- and long-term maturities, including US Treasuries, mortgage-backed securities, Federal agency debt, and currency swaps. These purchases were meant to lower the cost of long-term private borrowing, or flatten yield curves. The Fed conducted several rounds of LSAPs: QE1 (December 2008–March 2010), QE2 (November 2010–June 2011), and QE3 (September 2012–October 2014).[9] The Fed's commitment to LSAPs and the 0 to 0.25 bps rate range was meant to reduce uncertainty in US financial markets and stimulate the economy. Importantly, the central bank target rates do not reflect any of these actions because of the ZLB.

2.1 Shadow rates

The shadow rate term structure model (SRTSM), first proposed by Black (1995), confronts the issue of modelling interest rates at the ZLB. Since economic agents have the option to hold physical currency for a rate of return of zero, deposit rates are bounded below by this constraint. However, standard term-structure (Gaussian affine term-structure or GATSM) models are linear in factors and thus allow for the possibility of estimating negative yields.[10] Black (1995) proposed thinking

[8] The Bank of Canada targets the overnight rate in the LVTS, which allows transfers of large payments with a guarantee of settlement.

[9] The Fed also conducted Operation Twist in September 2011, which involved purchases of bonds with long-term maturities and sales of bonds with short-term maturities. This operation left the overall size of the Fed's balance sheet unchanged.

[10] Some models specify the short-rate diffusion process as quadratic or square-root to avoid negative rates. However, these specifications do not have theoretically consistent assumptions as they treat the ZLB as a reflecting barrier rather than an absorbing one. See Christensen and Rudebusch (2014) for a detailed discussion.

of the observed nominal rate, r_t, as the sum of an unobserved and unrestricted shadow rate, sr_t, which can go negative and the option value of holding physical currency that is exercised at the effective lower bound, $\underline{r}$. Specifically,

$$r_t = \max\{sr_t, \underline{r}\} = sr_t + \underbrace{\max\{0, \underline{r} - sr_t\}}_{\text{option value of currency}} . \tag{1}$$

Note that when the shadow rate is above the ZLB, the option value of holding currency is zero. However, when the shadow rate begins to dip below the ZLB, the currency option begins to have an effect. Fitting this model to the data allows us to back out an estimate of the shadow rate process.

There are several benefits to using the shadow rate as a measure of unconventional monetary policy. First, the shadow rate captures both the signalling and portfolio balance channels of central banks actions. While the Fed's LSAPs were meant to flatten yield curves, Fed announcements were meant to reduce both future rate expectations and uncertainty in financial markets. Measuring monetary policy by either the balance sheet or Fed announcements alone would then only capture one of the two channels. By taking information from the entire yield curve, the shadow rate captures both channels as well as forward guidance and other actions. Second, the shadow rate provides us with an uninterrupted measure of monetary policy through conventional and unconventional-ZLB episodes. This allows for more precise estimation as we can extend the time series beyond the ZLB period. The nature of the variable also allows us to study whether there are significant differences in the effects of monetary policy at or away from the ZLB.

Despite its many advantages, estimating the SRTSM is significantly more involved than estimating a short rate in a standard term structure model. The main difference between the shadow rate defined by a SRTSM and a traditional short rate defined by a GATSM model is the non-linearity that the max operator introduces. This non-linearity implies that there is no analytical solution to the model. While Krippner (2012, 2013) and Bauer and Rudebusch (2013) use numerical simulation methods to solve for the rate, Wu and Xia (2016) take an alternative approach and derive an approximate solution to the shadow rate as a function of the yield curve and the probability that the shadow rate will fall below the effective lower bound.[11] This method provides a tractable

[11]Specifically, they solve the shadow rate by defining it in a nonlinear state space model, where the shadow forward

analytical approximation of the shadow rate, and does not require numerical simulation.

We take the estimate of the US shadow rate directly from Wu and Xia (2016), and we adopt their method to estimate a Canadian shadow rate which is briefly outline here.[12] The Canadian SRTSM is constructed as a nonlinear state space model estimated with the extended Kalman filter. The Canadian shadow rate is assumed to be an affine function of state variables, X_t:

$$sr_t = \delta_0 + \delta_1' X_t. \tag{2}$$

Our state variables are the forward rates derived from the Bank of Canada's zero-coupon bond yield data for 3 month, 6 month, 1, 2, 5, 7, and 10 year maturities (Bolder, Johnson, and Metzler, 2004). We assume the state variables follow a first order vector autoregressive (VAR(1)) process under the physical measure $\mathbb{P}$,

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I), \tag{3}$$

which is the transition equation. The log stochastic discount factor is essentially affine

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right), \tag{4}$$

where the price of risk λ_t is linear in the factors: $\lambda_t = \lambda_0 + \lambda_1 X_t$. The dynamics under the risk neutral measure $\mathbb{Q}$ also follow a VAR(1)

$$X_{t+1} = \mu^{\mathbb{Q}} + \rho^{\mathbb{Q}} X_t + \Sigma \varepsilon_{t+1}^{\mathbb{Q}}, \quad \varepsilon_{t+1}^{\mathbb{Q}} \sim^{\mathbb{Q}} N(0, I). \tag{5}$$

The parameters under the $\mathbb{P}$ and $\mathbb{Q}$ measures are related as follows: $\mu - \mu^{\mathbb{Q}} = \Sigma\lambda_0$ and $\rho - \rho^{\mathbb{Q}} = \Sigma\lambda_1$.

An analytical approximation of the forward rate in the SRTSM is generated by defining a forward rate, $f_{n,n+t,t}$, as the rate at time t for a loan starting at $t+n$ and maturing at $t+n+1$. The forward rate is a linear function of the yields on risk-free n and $n+1$ period pure discount

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rate depends on the probability of the short rate being at its ZLB and a vector of state variables. The model is solved using the extended Kalman filter. The observed state variables are the forward rates associated with the 3 month, 6 month, 1, 2, 5, and 10 year yields on zero coupon bonds from the Gürkaynak, Sack, and Wright (2007) dataset.

[12] See Wu and Xia (2016) for the complete derivation of the state space model for the SRTSM.

bonds q

$$f_{n,n+t,t} = (n+1)q_{n+t,t} - nq_{nt}. \tag{6}$$

The forward rate in the SRTSM described in equations (2)–(5) is approximated by

$$f^{SRTSM}_{n,n+t,t} = \underline{r} + \sigma^{\mathbb{Q}}_n g\left(\frac{a_n + b'_n X_t - \underline{r}}{\sigma^{\mathbb{Q}}_n}\right) \tag{7}$$

where

$$\begin{aligned}
a_n &= \delta_0 + \delta'_1 + \left(\sum_{j=0}^{n-1}(\rho^{\mathbb{Q}})^j\right)\mu^{\mathbb{Q}} - \frac{1}{2}\delta'_1\left(\sum_{j=0}^{n-1}(\rho^{\mathbb{Q}})^j\right)\Sigma\Sigma'\left(\sum_{j=0}^{n-1}(\rho^{\mathbb{Q}})^j\right)'\delta_1,\\
b'_n &= \delta'_1(\rho^{\mathbb{Q}})^n, \text{ and}\\
\mathbb{V}ar^{\mathbb{Q}}_t(sr_{t+n}) \equiv (\sigma^{\mathbb{Q}}_n)^2 &= \sum_{j=0}^{n-1}\delta'_1(\rho^{\mathbb{Q}})^j\Sigma\Sigma'(\rho^{\mathbb{Q}})^j\delta_1.
\end{aligned}$$

The function $g(z) \equiv z\Phi(z) + \phi(z)$ consists of a normal cumulative distribtuion function $\Phi(\cdot)$ and a normal probability density function $\phi(\cdot)$. Its non-linearity comes from the moments of the truncated normal distribution.[13]

Finally, we define a measurement equation that relates the observed forward rate, $f^o_{n,n+t,t}$, to the factors, based on equation (7), as follows

$$f^o_{n,n+t,t} = \underline{r} + \sigma^{\mathbb{Q}}_n g\left(\frac{a_n + b'_n X_t - \underline{r}}{\sigma^{\mathbb{Q}}_n}\right) + \eta_{nt}, \tag{8}$$

where the measurement error η_{nt} is i.i.d. normal, $\eta_{nt} \sim N(0,\omega)$. Finally, using the same identification assumptions as Wu and Xia (2016), we assume a three factor model and impose normalizing restrictions on the $\mathbb{Q}$ parameters: $\delta = [1,1,0]'$; $\mu^{\mathbb{Q}} = 0$; and Σ is lower triangular. We also assume $\rho^{\mathbb{Q}}$ is real Jordan form with eigenvalues in descending order: $\rho^{\mathbb{Q}} = [\rho^{\mathbb{Q}}_1\ 0\ 0;\ 0\ \rho^{\mathbb{Q}}_2\ 0;\ 0\ 0\ \rho^{\mathbb{Q}}_3]$. The model is estimated using the extended Kalman filter with equation (8).

Figure 1 plots our estimated Canadian shadow rate and the US shadow rate, spliced with the Bank of Canada target rate and federal funds target rates when these policy rates were at the ZLB.

[13] For details of the derivation of (7), see the appendix in Wu and Xia (2016).

The Canadian shadow rate reached its lowest point of approximately -0.34 percent in November 2009, at which point the Bank of Canada was midway through its conditional commitment mandate and was actively engaging in liquidity provision activities. The US shadow rate was decreasing from 2008 through 2014, reaching a low of -2.99 percent. Throughout this period the Fed engaged in three rounds of QE and made numerous statements regarding continued policy easing.

2.2 Event study

Since we are the first to estimate a shadow rate for Canada, we begin by assessing whether its movements are consistent with the actions taken by the Bank of Canada during the ZLB.[14] Specifically, we conduct an event study around Bank of Canada policy announcements. We note, however, that an event study is not ideal. Our shadow rate can be estimated at a daily frequency at most, which is often considered too low to identify any effects from central bank announcements.[15] Furthermore, the ZLB episode was relatively short for Canada and thus the number of monetary policy events is limited. As a result, we are cautious when interpreting our results and treat the event study more as a consistency check than a careful empirical exercise.

The Bank of Canada made several announcements during the ZLB period, three of which were particularly important: the announcement that the short rate was being lowered to 25 bps and would remain there until the second quarter of 2010 (April 21, 2009), the announcement that the Bank of Canada's conditional commitment to the ZLB was being removed (April 20, 2010), and the announcement that the short rate was being raised to 50 bps (June 1, 2010). We include several other announcements in our event study that relate to the Bank of Canada's extraordinary liquidity operations during the ZLB, but we suspect, *a priori*, that these events may have a lesser impact on the shadow rate than the others as they were less widely reported or known outside of the traditional banking sector.

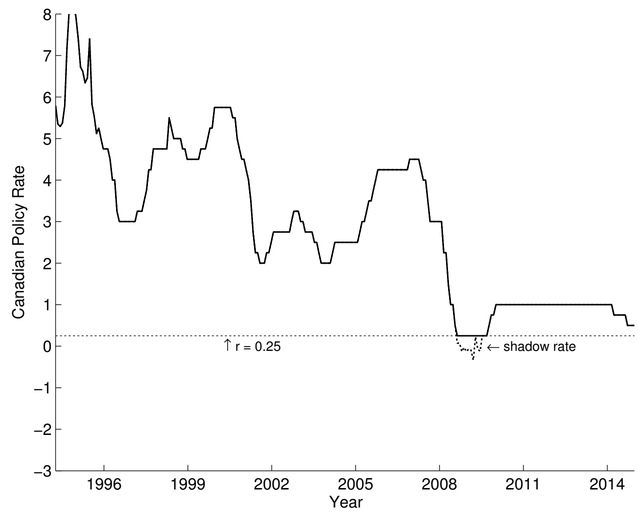
First, we look at the one-day changes in the shadow rate on announcement dates. We define the change in the shadow rate on announcement days as tail events if there was a statistically significant change in the shadow rate, based on standard errors calculated assuming a normal distribution (following Bowman *et al.*, 2015). On several occasions the Bank of Canada made their

[14]In their paper, Wu and Xia (2016) show that there was no substantive difference in the response of US macroeconomic variables to the US shadow rate at and away from the ZLB. We do no repeat that exercise here.

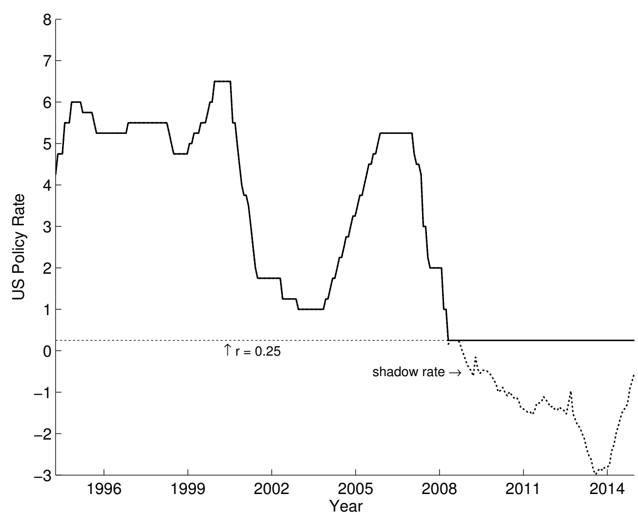
[15]This is because the Bank of Canada's zero coupon bond yield curve data, which is the observed data used in the state space model to estimate the shadow rate, is available only at the daily frequency.

Figure 1: Canadian and U.S. Policy Rates (1994–2015)

(a) Canadian Policy Rate



(b) US Policy Rate



Source: FRED, Statistics Canada CANSIM, Bank of Canada, Wu and Xia (2016), and authors' own calculations. Note: The federal funds rate is the target federal funds rate before November 2008 and the upper bound of the federal funds rate from November 2008 onwards.

announcements late in the day, either near to or after financial market closing. In these cases we look at the next-day change in the shadow rate for our analysis. The dates, event descriptions and one-day changes in the shadow short rate are reported in Table 1. We see that two of the three main ZLB announcements by the Bank of Canada were associated with tail events in changes in

Table 1: Bank of Canada announcements and shadow short rate changes

Date	Δsr_t	**Event Description**
April 21, 2009	-0.0964***	BoC announces operating framework for the implementation of monetary policy at the effective lower bound for the target rate, lowers target rate target to 1/4 percent, introduces conditional commitment to hold current policy rate until the end of the second quarter of 2010 and announces term PRA transaction schedule.
April 24, 2009	-0.0408	BoC announces term PRA, term PRA for private sector instruments, and TLF transaction schedules.
June 25, 2009	-0.0097	BoC announces extension of TLF and expanded swap facility with the Fed as well as temporary reciprocal currency arrangements (swap lines) between the Fed and other central banks extended to 1 February 2010.
July 21, 2009	-0.0014	BoC announces term PRA, term PRA for private sector instruments, and TLF transaction schedules.
September 23 2009†	-0.0124	Reflecting the improved conditions in funding markets the BoC announced term PRA Facility for private sector instruments (after a final operation on October 27, 2009) and TLF (after a final operation on October 28, 2009) will expire at the end of October 2009.
October 20, 2009	-0.0914***	BoC announces another term PRA transaction schedule.
November 6, 2009†	-0.0056	Given improved conditions in funding markets the BoC announces that temporary measure of allowing LVTS participants to assign their non-mortgage loan portfolios as eligible collateral for LVTS and SLF purposes will be gradually reduced from 100 per cent to 20 per cent of each participant's total pledged collateral starting starting February 2, 2010.
December 16, 2009†	0.0282	Given improved conditions in funding markets the BoC announces term PRA operations will be held on a monthly basis, rather than bi-weekly, and that only Canadian dollar securities eligible as collateral under the BoC's SLF will be eligible for term PRAs effective January 19, 2010. Affiliated-dealer bank-sponsored ABCP, and BBB corporate bonds will no longer be eligible.
April 20, 2010	0.1110***	BoC maintains target rate at 1/4 per cent and removes conditional commitment.
May 10, 2010†	0.0873**	BoC and the Fed reestablish US$30 billion swap facility (reciprocal currency arrangement) that had expired on February 1, 2010.
June 1, 2010	-0.0478	BoC increases target rate to 1/2 per cent and reestablishes normal functioning of the overnight market as well as the standard operating framework for the implementation of monetary policy. The target for the target rate is set to the midpoint of the operating band and the width of the operating band to 50 bps.

Notes: This table contains a list of major policy events and the corresponding changes in the shadow rate. *, **, and *** represent tail events with respect to a normal distribution at the standard 10, 5, and 1 percent confidence levels. † marks announcement dates that were made the day before but after markets closed.

the shadow short rate — the 21 April 2009 announcement of the shift to the ZLB and the 20 April 2010 removal of conditional commitment. The announcement of exit from the ZLB on 1 June 2010 was not associated with a significant shift in the ZLB, we suspect because after the removal of conditional commitment the increase in the interest rate was widely expected.

We also estimate an event study regression to verify the robustness of the results. We regress the daily change in the estimated shadow rate on a set of dummy variables for expansionary and contractionary announcements. Results are reported in Table 2, column (1). As expected, we see

Table 2: Effect of Bank of Canada Announcements on Shadow Short Rate

Dependent Variable: Δsr_t		
	(1)	(2)
Expansionary Announcement	-0.025*	-0.019
	(0.01)	(0.02)
Contractionary Announcement	0.015	0.020
	(0.02)	(0.02)
FAD		-0.014
		(0.01)
MPR		0.002
		(0.02)
N	280	280
R^2	0.02	0.02

Notes: Standard errors in parentheses. Data frequency is daily. Period is April 20, 2009 – June 1, 2010, corresponding to the Canadian ZLB period when the Bank of Canada Bank Rate was at 25 basis points. Expansionary announcements include all PRA announcements, TLF announcements, and SLF announcements. Contractionary announcements include announcements regarding the end or reduction of the PRA, TLF, or SLF operations. FAD and MPR dummy variables are equal to one on dates that the Bank of Canada released FAD document or the monetary policy report.

that expansionary announcements were associated with a significant fall in the shadow rate and contractionary announcements were associated with a rise (albeit not statistically significant). The signs on the coefficient estimates are robust to including dummy variables for the Bank of Canada's fixed announcement dates (FADs) and monetary policy report releases (MPR), reported in column (2), which we include to control for market reaction on anticipated announcement days.

Both the magnitude and direction of the change in the shadow rate in response to policy announcements are consistent with the behaviour of the Bank rate during conventional times. Therefore, we proceed with our empirical analysis and use the shadow rate as our policy indicator during the ZLB.

3 Small open economy B-SVAR model

We model the dynamic interaction of the variables in the two countries using a structural vector autoregression,

$$AY_t = C + \sum_{l=1}^{p} B_l Y_{t-l} + \varepsilon_t,$$

where Y_t is an $n \times 1$ vector of endogenous variables, A and B_l are $n \times n$ parameter matrices, ε_t is an $n \times 1$ vector of structural shocks with $E(\varepsilon_t | Y_1, ..., Y_{t-1}) = 0$ and $E(\varepsilon_t \varepsilon_t' | Y_1, ..., Y_{t-1}) = I_n$.

Introducing the small open economy assumption that Canada cannot influence US variables involves imposing block exogeneity restrictions on the parameters A and B_l. This greatly reduces the number of parameters to be estimated, despite the number of overall parameters increasing with the inclusion of US variables, which are meant to capture international influence on Canadian variables. We follow Cushman and Zha (1997) and assume that the endogenous variable vector Y_t comprises two blocks: a Canadian block Y_t^{CAN} and a US block Y_t^{US}. We allow the Canadian block to respond to the US variables both contemporaneously and with a lag, but restrict the US block to be self contained and not influenced by the dynamics in the Canadian variables. Specifically, we impose the restrictions in the following way

$$\begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} Y_t^{\text{CAN}} \\ Y_t^{\text{US}} \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ 0 & F_{22} \end{bmatrix} \begin{bmatrix} Z_t^{\text{CAN}} \\ Z_t^{\text{US}} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\text{CAN}} \\ \varepsilon_t^{\text{US}} \end{bmatrix},$$

where $F = [C, B_1, \ldots, B_p]$ and $Z_t^i = [I, Y_{t-1}^i, \ldots, Y_{t-p}^i]'$ for $i = \{\text{CAN, US}\}$.

For the Canadian block, our main specification includes the Canadian policy rate, r; seasonally-adjusted real Canadian industrial production, y; the Canadian consumer price index, p; the Canadian/US dollar exchange rate, s; and the Canadian current account balance, CA. For the US block, we include seasonally-adjusted real US industrial production, y^*; the US consumer price index, p^*; commodity export prices, wpx; the implied volatility of the S&P 500 index, VIX; and the US policy rate, r^*. Detailed data definition and sources are provided in Table 4 of Appendix A.

All variables are in logs except for the two policy rates, which we construct using (1), *i.e.* splicing the Bank of Canada target rate and the federal funds rate with their respective shadow rates whenever the shadow rates are below the 25 bps ZLB. The Canadian shadow rate is calculated as described in Section 2.1, using Canadian zero coupon bond yield curve data derived by Bolder, Johnson, and Metzler (2004).[16] We use the US shadow rate provided by Wu and Xia (2016). Using the policy rate, as defined in (1), allows us to extend the data much farther and treat both central banks as if they have a constant reaction function but a varying set of policy instruments at the ZLB. Including multiple business cycles gives our estimates more precision. The data is monthly and covers July 1994 to October 2015. The beginning of our sample coincides with a notable

[16]Data is available at http://www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/.

shift in the Bank of Canada's operating procedure as it adopted a corridor system and shifted to targeting the overnight rate as its key monetary policy instrument.[17] This change was part of a broader transition in the 1990s to targeting two percent inflation and improving both the clarity and efficiency of monetary policy.[18]

Our identification is based on exclusion restrictions in the contemporaneous coefficient matrix A. We assume that the Bank of Canada reacts contemporaneously to the exchange rate, the US policy rate, the VIX and commodity prices, but not to any other variables because information on output, prices and current account balances arrives with a delay. Including commodity prices is particularly important for decision making by the Bank of Canada for two reasons. First, commodity prices adjust very quickly to market conditions and hence they control for future price expectations, *i.e.* they help mitigate the *price puzzle* often found in similar SVAR analyses, and second, since Canada is a commodity exporter, commodity prices have a large impact on Canadian output as well as the value of the Canadian dollar. Following Gambacorta *et al.* (2014), we include the VIX as a proxy for financial turmoil, economic risk, and uncertainty, which played a critical role in the latter part of our sample. We control for current account balances to capture both the trade balance as well as international receipts and payments of income. From 1994–2015 receipts of income from abroad accounted for 9 percent of all current account receipts, and payments of income from abroad account for 15 percent of current account payments, on average, in Canada. By simply looking at the trade balance we miss a substantial component of Canadian international borrowing and lending.[19] We let the exchange rate react contemporaneously to all variables, domestic and foreign, in the model and assume that the production sector of the Canadian economy takes an upper triangular form with the variables ordered (CA, p, y).[20] The US block takes an upper triangular form, ordered $(r^*, VIX, wxp, p^*, y^*)$, so that output and prices cannot respond to a monetary policy shock within the same month.

The total number of restrictions on the contemporaneous coefficient matrix is 64. However, since we need only $n(n-1)/2 = 45$ restrictions for exact identification, the model is overidentified.

[17] The corridor system establishes a 50bp operating band target around the target rate.

[18] The Bank of Canada officially began targeting inflation in 1991, introduced the LVTS in 1999 and adopted eight annual fixed announcement days in 2001.

[19] Secondary income (transfers) accounts for only one percent of current account credits and 2 percent of current account debits, on average.

[20] Results are robust to ordering CA last, and are available upon request.

Although overidentification permits a more sensible set of restrictions than a recursive ordering, it imposes restrictions on the reduced-form covariance matrix, $A^{-1}A^{-1\prime} = \Omega$, which complicates the estimation. The posterior distribution of Ω does not have a convenient form and regular Monte Carlo integration methods cannot be used. Cushman and Zha (1997) use the importance sampler, but Waggoner and Zha (2003) show that it is inefficient in the presence of overidentifying restrictions. They propose a Gibbs sampler which yields more accurate results. We follow this method, and use 2000 draws, discarding the first 1000 to ensure that the initial values do not affect the posterior distribution. We use the Sims and Zha (1998) prior and set loose values for all hyper-parameters except for the lag decay.[21] We estimate the model with 12 lags and report 68% error bands as well as the medians of the posterior distribution.

Prior to estimation, we check the validity of the overidentifying restrictions. Since the contemporaneous coefficient matrix is overidentified, we cannot directly test block exogeneity in the reduced form VAR. Similarly, we cannot directly test overidentifying restrictions because of block exogeneity imposed in the contemporaneous coefficient matrix. As a result, we follow Cushman and Zha (1995, 1997) and perform a joint likelihood ratio test for overidentification and block exogeneity in the contemporaneous coefficient matrix and block exogeneity in the lagged coefficient matrices (a total of 319 restrictions). This test rejects the null hypothesis, but we nevertheless choose to keep the model as specified. We have explored a variety of different models based on alternative variables (including deterministic trends), lag selections, and time periods and did not find any specification that resulted in a sufficiently large increase in the p-value of the likelihood ratio test, while also satisfying the underlying small open economy theory. We suspect this result could be driven by an omitted variable or poor small sample properties of this non-standard test. We plan to explore these possibilities in future research.

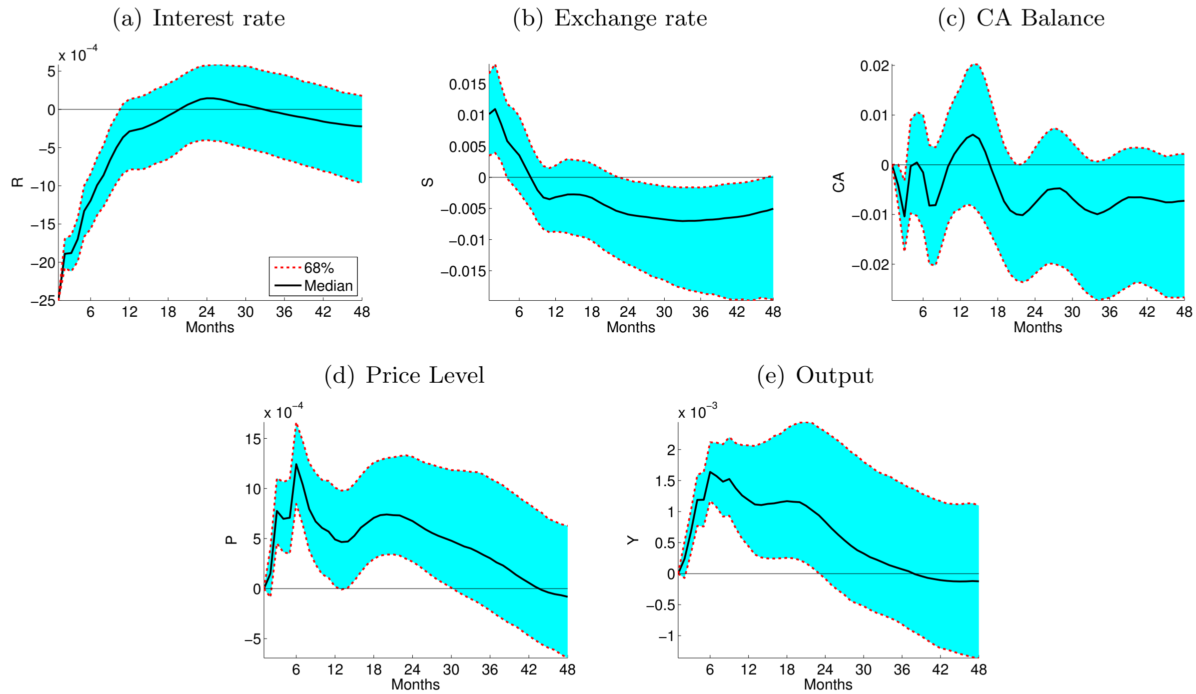
4 Results

We first consider the dynamic response of the variables in our system to both domestic and foreign expansionary monetary policy shocks and then use the estimated system to perform counterfactual experiments based on the historical decomposition.

The impulse response functions from a 25 bps expansionary shock to Canadian monetary policy

[21] We set $\mu_i = 10$ for $i = 1, \ldots, 6$.

Figure 2: Response to domestic (Canadian) expansionary monetary policy shock (Jul 1994 – Jul 2015)



Notes: This figure contains the impulse response functions for the Canadian variables in response to a 25bp expansionary Canadian monetary policy shock. Error bands are constructed based on 2000 Gibbs sampling draws from the posterior distribution.

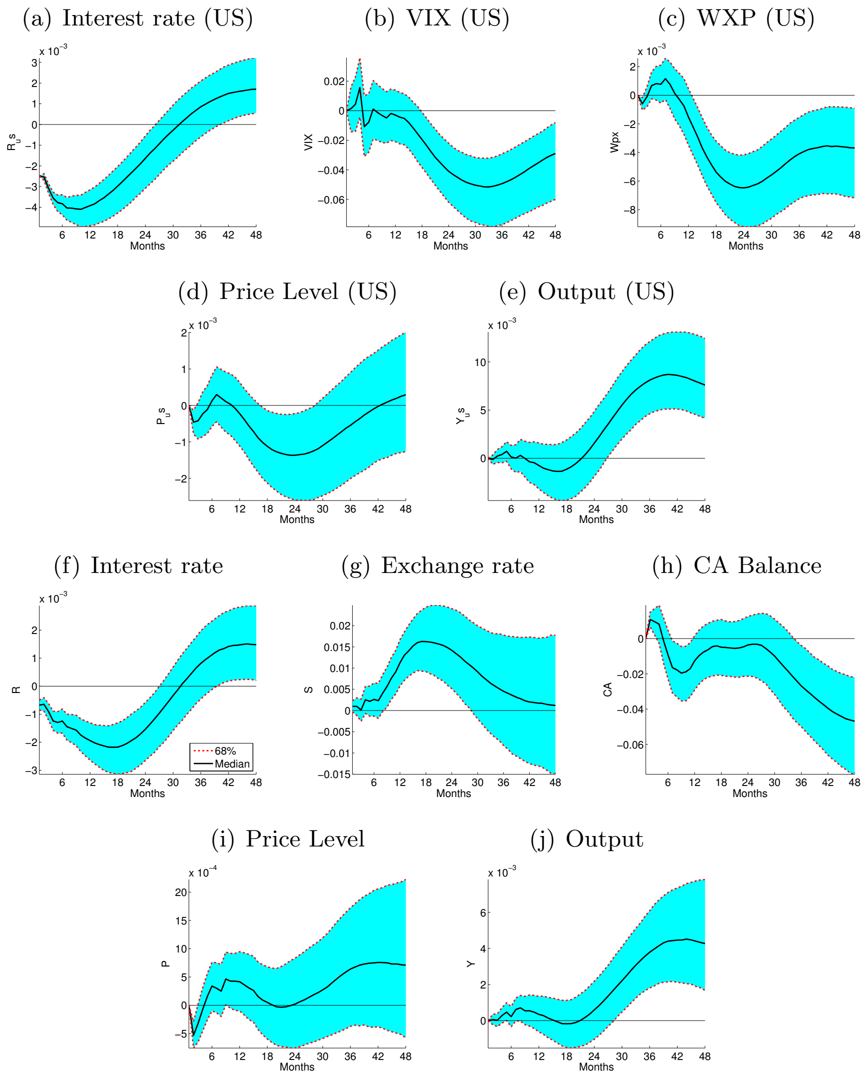
are shown in Figure 2. Prices, output, and the exchange rate respond to this shock in accordance with theoretical models and existing empirical evidence. That is, an expansionary monetary policy shock raises output and prices significantly on impact, and for approximately 24 months after the initial shock. The currency depreciates on impact, and appreciates as the interest rate rises. While the Canadian dollar is weaker, a fall in the current account is consistent with a fall in the trade balance driven by rising Canadian income increasing demand for imports. This suggests an income effect is outweighing any exchange rate effect on the current account balance.[22]

The impulse response functions from a 25 bps expansionary shock to US monetary policy for both Canadian and US variables are shown in Figure 3. Though our primary focus is on the impact to Canadian variables, we find the response of US variables in figures 3(a)–3(e) is generally consistent with theoretical and empirical literature.[23] Following the US monetary policy shock, US

[22]One would also expect that as Canadian interest rates fall, net investment income payments to foreigners would also fall, which would imply a rise in the current account balance. However, as net exports make up approximately 90 percent of the Canadian current account balance, it is likely that the income effect on net exports outweighs any change in investment income payments due to lower interest rates.

[23]The impact on the US economy has been studied extensively, as discussed in section 1. We have not attempted

Figure 3: Response to foreign (US) expansionary monetary policy shock (Jul 1994 – Jul 2015)



Notes: This figure contains the impulse response functions for the all variables in response to a 25bp expansionary US monetary policy shock. Error bands are constructed based on 2000 Gibbs sampling draws from the posterior distribution.

equity market volatility falls significantly, output rises significantly, and prices fall significantly (this price puzzle is commonly found in the recursive SVAR literature), all with a delay of approximately 1–2 years.

Figure 3(f) shows a strong response from the Canadian policy rate both on impact and for over two years following the initial shock. The Canadian dollar depreciates, as shown in Figure

to model the US economy explicitly here, but rather include those US variables which are important to Canadian spillovers.

3(g), along with a fall in the policy rate. Both variables reach their minimum after approximately 18 months. Canadian output, shown in Figure 3(j), rises in response to the US monetary policy shock and exhibits the same delay as US output, demonstrating the strong spillover effects from US monetary policy. Canadian prices fall in the first month and then rise albeit not significantly. The current account balance rises on impact, briefly falls after approximately eight months, and then falls again after three years. This pattern is consistent with an initial fall in investment income payments as the Canadian interest rate falls, followed by a fall in net exports as Canadian output rises and the income effect becomes more important.

In summary, we find that a Canadian expansionary monetary policy shock boosts domestic prices and output and depreciates the Canadian dollar. A US expansionary monetary policy shock increases US output, and has strong spillover effects on Canada, operating through both the exchange rate and the endogenous response of the Bank of Canada.

4.1 Effects of unconventional monetary policy

In order to quantify the effects of unconventional monetary policy we conduct a counterfactual experiment proposed by Wu and Xia (2016), where we simulate a scenario in which the shadow rate remains at 25 bps (the ZLB) while the other variables remain unrestricted. We assume that, since the nominal rate is bounded by 25 bps, any movement in the policy rate below this bound is driven by unconventional monetary policies. As a result, our estimates can be considered an upper bound on the effect of unconventional monetary policy.

We begin with the historical decomposition, which decomposes each variable in Y_t into the contribution from the initial value, the constant term and the structural shocks,

$$Y_t = \underbrace{G^t Y_0}_{\text{initial condition}} + \underbrace{\sum_{i=1}^{t} G^i C}_{\text{constant}} + \underbrace{\sum_{i=1}^{t} \Psi_{i-1}\varepsilon_{t-i+1}}_{\text{structural shocks}}, \tag{9}$$

where $G = A^{-1}F$ and Ψ_i is the set of coefficients for the impulse response function in period i. We use this equation to calculate the paths of the variables under different scenarios by manipulating the set of contributing structural shocks. The scenarios we are interested in are those that restrict the shadow rate to 25 bps for Canada and the US.

For Canada, the counterfactual is implemented by replacing the realized monetary policy shock, $\varepsilon_t^{MP_{CAN}}$ in (9), with a counterfactual shock, $\tilde{\varepsilon}_t^{MP_{CAN}}$, such that the shadow rate respects the lower bound, $\tilde{s}_t = 0.25$ when the actual target rate was at 25 bps. Similarly for the US, the counterfactual is implemented by replacing replace $\varepsilon_t^{MP_{US}}$ in (9) with $\tilde{\varepsilon}_t^{MP_{US}}$, such that $\tilde{s}_t^* = 0.25$ when the federal funds rate was at the ZLB.[24] The path of each variable is then simulated under these two scenarios.

4.1.1 Restricted monetary policy in Canada

Figure 4 reports the results for our first experiment which restricts Canadian unconventional monetary policy. We show each original series with its counterfactual path, which we interpret as what the state of the economy would have been had the Bank of Canada been unable to provide any additional stimulus beyond lowering the target rate to 25 bps.

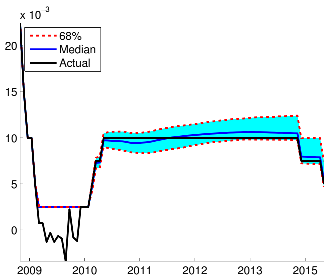
Figure 4(a) visually demonstrates how the counterfactual scenario is constructed: while the shadow rate is below 25 bps during the period 2009–2010, we restrict it to be bound by this lower limit for the counterfactual path. Figure 4(e) (which is zoomed-in to the ZLB period only) shows that under the counterfactual scenario Canadian output would have been significantly lower. Table 3 contains the average percentage difference between each series and their counterfactual paths. Output and prices would have been 0.23% and 0.14% lower on average during the Canadian ZLB period, respectively, had the Bank of Canada been unable to provide any additional stimulus.

Importantly, although output and prices are significantly lower under the counterfactual scenario, they would have returned to their observed path within two years. This means the unconventional monetary policy operations conducted by the Bank of Canada while the target rate was at 25 bps sped up the recovery of the Canadian economy significantly, but had relatively short term transitory effects.

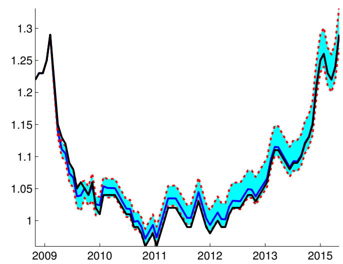
[24] We have also conducted a third counterfactual experiment that replaces both the Canadian and US monetary policy shocks with their counterfactual shock so as to make both the Canadian and US shadow rates constrained at $\tilde{s}_t = \tilde{s}_t^* = 0.25$. Because the impact of US unconventional monetary policy is so large relative to Canadian unconventional policy (as reported in this section), the results from this experiment are essentially unchanged from the US counterfactual experiment. We thus do not report these results, but note that they are available upon request.

Figure 4: Counterfactual paths: Canadian ZLB imposed

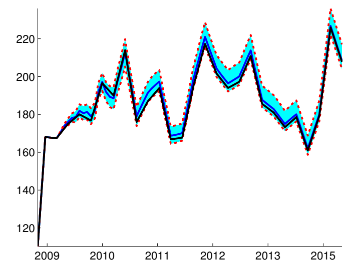
(a) Policy Interest rate



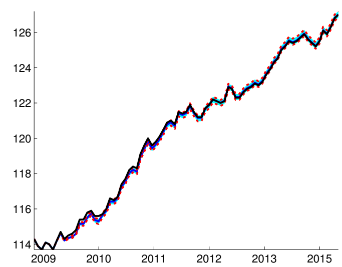
(b) Exchange rate



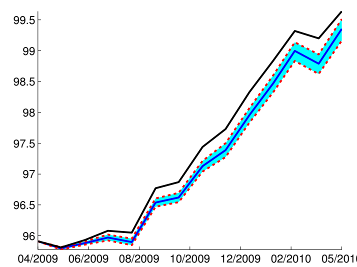
(c) CA Balance



(d) Price Level



(e) Output



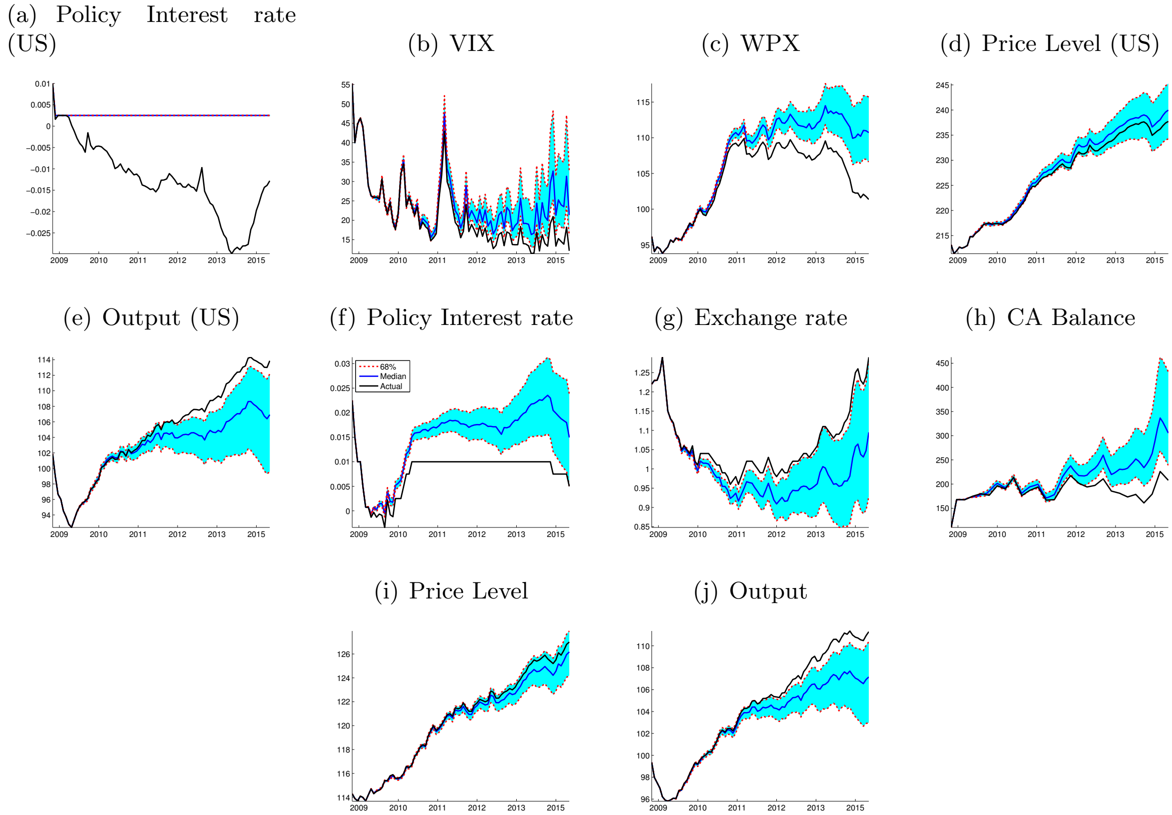
Notes: This figure contains the plot of each series along with its counterfactual path constructed by generating a set of structural shocks for the policy rate such that it is forced to respect the ZLB. Error bands are constructed based on 2000 Gibbs sampling draws from the posterior distribution.

Table 3: Average percent difference between data series and counterfactual path (full sample)

	Canadian ZLB Imposed			US ZLB Imposed		
Variable	Median	Max	Min	Median	Max	Min
Canadian Interest rate	0.00	0.00	0.00	0.01	0.01	0.00
Canadian Interest rate†	0.00	0.00	0.00			
Canadian CPI	−0.05	0.07	−0.17	−0.26	0.19	−0.73
Canadian CPI†	−0.14	−0.07	−0.21			
Canadian output	−0.07	0.13	−0.29	−1.21	−0.30	−2.32
Canadian output†	−0.23	−0.15	−0.33			
US interest rate				0.02	0.02	0.02
US CPI				0.38	1.15	−0.43
US Output				−1.94	−0.29	−4.09

Note: This table contains the average percentage difference between the actual series and the counterfactual median as well as 68% bounds. The calculation uses the formula $\frac{1}{\tilde{T}}\sum_{t\in ZLB}\frac{y_t - y_t^{CF}}{y_t}$, where y_t is the series y at time t and y_t^{CF} is the same series under the counterfactual scenario, ZLB denotes the time period for the experiment and $\tilde{T}$ denotes the number of observations for the experiment. † indicates the estimates are for the Canadian ZLB period only. Also note that although the max and min bounds may include 0 this does not exclude the possibility that the paths were significantly different from zero for individual months during the experiment period.

Figure 5: Counterfactual paths: US ZLB imposed



Notes: This figure contains the plot of each series along with its counterfactual path constructed by coming up with a set of structural shocks for the policy rate such that it is forced to respect the ZLB. Error bands are constructed based on 2000 Gibbs sampling draws from the posterior distribution.

4.1.2 Restricted monetary policy in the US

Figure 5 reports the results from our second counterfactual experiment in which we restrict the US shadow rate to 25 bps after December 2008, while the federal funds rate was at the ZLB. As shown in Figure 5(a), US unconventional monetary was significantly more expansionary than Canadian unconventional monetary policy and the ZLB episode lasted much longer. As a result, the measured magnitude of the effects of US unconventional monetary policy is much larger as well. US output, shown in Figure 5(e), is substantially and persistently lower under the counterfactual experiment. The trajectory of output reveals that without unconventional interventions from the Fed, US output would have significantly diverted from its growth path. The summary statistics from the counterfactual experiment shown in Table 3 indicate that US output would have been 1.94 percent lower on average without the Fed's unconventional monetary policy. This estimate is similar in magnitude to those found in other studies. Dahlhaus *et al.* (2014) find that US GDP would have been 2.3% lower on average over the period from 2008Q4 to 2013Q3 if the Fed's balance sheet continued to grow at pre-crisis levels; Chung, Laforte, Reifschneider, and Williams (2012), find that US GDP would have been 3% lower in 2012 without the first instalment of quantitative easing (QE I); Baumeister and Benati (2013) find that QE I boosted US GDP growth by 2% in 2009; and Wu and Xia (2016) find that the US industrial production index would have been 101.0 rather than 101.8 in December 2013, a 0.78% difference. In contrast to these studies (with the exception of Wu and Xia (2016)), we consider the expansionary effects of both large scale asset purchases and forward guidance.

Importantly, we find that US unconventional monetary policy had a significant impact on the Canadian economy. Figure 5(f) shows that the Canadian policy rate would have been much higher under this counterfactual scenario, remaining around 2 percent for much of the experiment. The higher Canadian policy rate highlights the sensitivity of the Bank of Canada's reaction function to movements in the US policy rate. Canadian output, shown in Figure 5(j), clearly benefited from the US expansionary policies. Not only is it significantly lower — about 1.21 percent lower on average as shown in Table 3 — but it would be on a different trajectory without US policy intervention. Our estimate is smaller in magnitude compared to the only study considering a similar question; Dahlhaus *et al.* (2014) find that US QE alone increased Canadian GDP by 2.2 percent on average over the period from 2008Q4 to 2013Q3.

A preeminent result from this US counterfactual experiment, which can be seen clearly in Figure 5, is that not only would Canadian output have been lower on average during the US ZLB, but its trend would be altered for a considerable length of time. In the last period of our sample, July 2015, we estimate that Canadian output would have still been 3.56 percent lower.[25] Similarly, US output would have been on a very different path for substantially longer without the Fed's actions. The much more persistent effect of the Fed's unconventional monetary policy shocks stands in stark contrast to the Bank of Canada's.

[25] This is the estimated median value, and is significantly different from zero with the 68 percent confidence band (-7.3025, -0.7755).

5 Robustness

In this section we assess the robustness of our main result. In Section 5.1 we use an alternative definition of the current account and in Section 5.2 we control for government expenditure. In both cases our results are robust to these different specifications.

5.1 Alternative current account measure

It is possible that the monthly current account balance measure we use in Section 4 misses some important variation because it is interpolated from quarterly data. Here we use an alternative estimation method to construct a monthly current account balance variable, and inspect the robustness of our main results. We follow Miao and Pant (2012) and construct the alternative variable as monthly Canadian net exports minus monthly changes in Canadian international reserves.[26] This proxy is based on the accounting identity:

$$\begin{aligned}\text{change in reserves} = {} & \text{net exports} + \text{net income from abroad} + \text{net transfers} \\ & + \text{capital and financial flows} + \text{errors and omissions} + \text{valuation effects},\end{aligned}$$

and the assumption that valuation effects, transfers, and income from abroad are all small and errors and omissions are negligible. It is because of these necessarily strong assumptions, and the corresponding potential for substantial measurement error in this proxy variable, that we choose not to use it in our main analysis. The data we use is described in Table 4 of Appendix A.

Figure 6 reports the impulse response functions for a 25 bps expansionary monetary policy shock in Canada, with the different measure of the current account. As in Section 4 both the price level and output rise, but the initial exchange rate depreciation is no longer statistically significant. Figure 7 reports the Canadian impulse response functions for a 25 bps expansionary monetary policy shock in the US.[27] Again, the results are largely consistent with those in Section 4. Prices fall significantly on impact, then rise after a lag. Output rises after approximately a two year lag. Finally, the currency depreciates as the Canadian policy rate falls.

As with the impulse response functions, the counterfactual experiments using this alternative

[26] Miao and Pant (2012) note that similar approaches have been used by Forbes and Warnock (2012) and Reinhart and Reinhart (2008).

[27] Due to the block exogeneity assumption, the effect on US variables is identical to the main specification.

current account proxy variable produce results that are largely consistent with those presented in Section 4. These are reported in Figures 8 and 9 and Table 5. Canadian output would have been approximately 0.15 percent lower on average during the Canadian ZLB without unconventional monetary policies from the Bank of Canada and 1.28 percent lower on average had the Fed not enacted any unconventional monetary policies. The error bands from these estimates overlap the error bands from our main specification.

5.2 Government spending

Government stimulus can play an important role in economic recoveries, particularly so in the aftermath of the financial crisis. Since monetary policy easing often coincides with increased government spending, excluding this variable could lead to overestimating the effect of monetary policy on output. Therefore, in this section we adjust our main specification to control for fiscal policy, measured by federal government expenditures. Following the literature on government shocks (see *e.g.* Ramey, 2011, 2015), we assume that fiscal policy does not respond contemporaneously to any variables because of legislative constraints. We replace the Canadian current account balance variable with Canadian federal expenditures in the Canadian block, Y_t^{CAN}, and add US federal expenditures to the bottom of the US block, Y_t^{US}, of our original model.

Figures 10 and 11 contain the impulse response functions, which are generally consistent with the main specification. Following a Canadian expansionary shock, output and prices rise, the Canadian dollar depreciates on impact and later appreciates, and government spending rises on impact and falls after approximately 6 months. In response to a US expansionary shock, Canadian and US output rise significantly, although with a long delay and only after a small decline in the US case. Both US and Canadian spending rise initially, albeit not significantly, but then fall as output increases. This is consistent with both automatic stabilizers and discretionary expenditures falling as the economy improves.[28]

The results of the counterfactual experiments, reported in Figures 12 and 13 and Table 6, are similar to the findings in the main specification. There does appear to be some evidence, however, that controlling for fiscal expenditures reduces the degree to which monetary policy explains changes in the real variables. We estimate that Canadian output would have been 0.14 percent lower on

[28]Note: Add FEVD plot to show spending is driven (in large part) by output.

average during the ZLB without Canadian unconventional monetary policy. US output would have been 0.07 percent lower on average during the ZLB period, and Canadian output 0.56 percent lower on average without US unconventional monetary policy. Interestingly, without US unconventional monetary policy both Canadian and US fiscal policy would not have been any more expansionary.

6 Conclusion

This paper contributes to a growing literature on the effects of unconventional monetary policy by analyzing these policies in a small open economy. There are two main challenges for quantifying the effects of unconventional policy in this setting: controlling for the outside world and finding an appropriate policy variable. To deal with these issues, we use the recently proposed shadow rates (Wu and Xia, 2016) as a proxy for monetary policy at the ZLB and construct a block-exogenous B-SVAR model to allow for international policy spillovers. The choice of Canada as a small open economy allows us to assume that the US adequately controls for the outside world. Furthermore, given our model, we are also able to explore the effects of unconventional monetary policy spillovers from the US.

We find that over the 1994–2015 period both Canadian and US expansionary monetary policy shocks are associated with increased output and prices in Canada. This result is consistent with theoretical and empirical literature based on pre-global financial crisis data. To quantify the magnitude of unconventional monetary policies in both countries, we conduct two counterfactual experiments that restrict the policy rate separately in each country to the ZLB. We find that without the Bank of Canada's unconventional policies, output would have been 0.23 percent lower on average during the Canadian ZLB period. Although these policies had significant expansionary effects, we also show that without them, the Canadian economy would have eventually recovered to the path observed at the end of our sample. In contrast, without the Fed's unconventional monetary policies, both Canadian and US output would be on different paths. US policies had a much larger effect in general, boosting US and Canadian output by approximately 1.21 and 1.94 percent on average over the July 2007–2015 period. Our results are robust to alternative specifications, including a different proxy for the current account balance and controlling for government spending.

Our findings reveal that unconventional monetary policy in a small open economy is effective, but also underline the importance of favourable foreign monetary policy spillovers. Recent concerns about the large adverse cross-border spillovers of unconventional monetary policies have rekindled the discussion of international coordination of monetary policy and the need to acknowledge ones role in spillovers. However, as we demonstrate, in some cases these spillovers can have beneficial effects. In future work, it would be interesting to explore other small open economies and compare the effectiveness of their unconventional monetary policies as well as the impact of international spillovers.

A Data

Table 4: Data Sources and Definitions

Variable	Source	Definition
ca	CANSIM	Current account balance, seasonally adjusted, indexed at 2007=100. Quarterly series linearly interpolated to monthly frequency.
ca^{alt}	IFS, author's calculations	Difference between the monthly sum of Canadian exports (goods value of exports, free on board (FOB), US Dollars) and imports (goods, value of imports, FOB, national currency, converted to US dollars) and Canadian official reserve assets (US dollars), converted to Canadian dollars and indexed at 2007==100, Monthly.
g	CANSIM, author's calculations	General federal governments expenditure, seasonally adjusted at annual rates, indexed at 2007=100. Quarterly series linearly interpolated to monthly frequency.
g_{US}	FRED	Federal Reserve of St. Louis Economic Data (FRED) federal government expenditures, seasonally adjusted at annual rates, indexed at 2007=100. Quarterly series linearly interpolated to monthly frequency.
p	IFS	Consumer Price Index, All items, Index, Monthly.
p_{US}	IFS	Consumer Price Index, All items, Index, Monthly.
r	Bank of Canada, author's calculations	Bank of Canada target rate spliced with shadow rate at ZLB when shadow rate < target rate = 25 bps, Monthly.
r_{US}	FRED, Wu and Xia (2016)	Federal funds target rate spliced with Wu and Xia (2016) shadow rate at ZLB when SSR < FFR = 25bp, Monthly.
s	IFS	National Currency per U.S. Dollar, National Currency per US Dollar, Rate, Monthly average.
vix	CBOE	Chicago Board Options Exchange (CBOE) VIX index measuring market's expectation of 30-day volatility. Constructed using the implied volatilities of a range of S&P 500 index options, Monthly average.
wxp	IFS	Export Price, All Commodities, Index, Monthly average.
y	IFS	Industrial Production, Seasonally adjusted, Index, Monthly.
y_{US}	IFS	Industrial Production, Seasonally adjusted, Index, Monthly.

B Robustness results

B.1 Capital flows

Figure 6: Response to domestic (Canadian) expansionary monetary policy shock
(Jul 1994 – Jul 2015)
Alternative CA balance

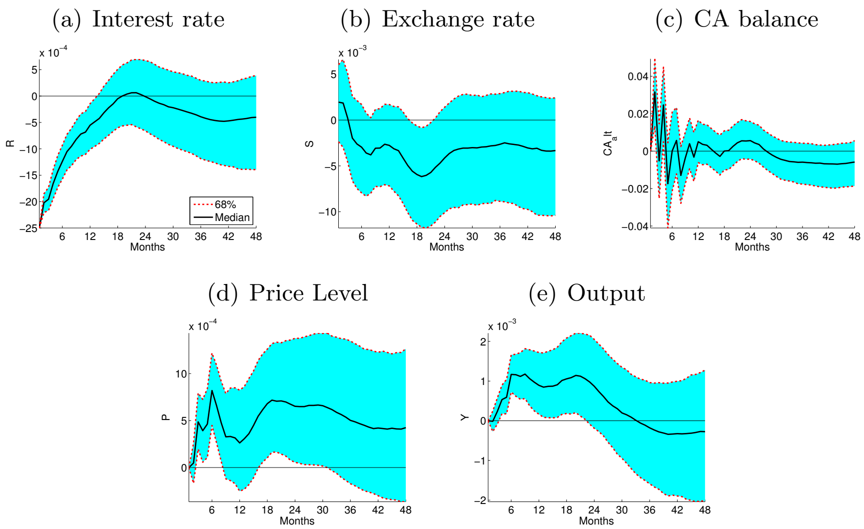
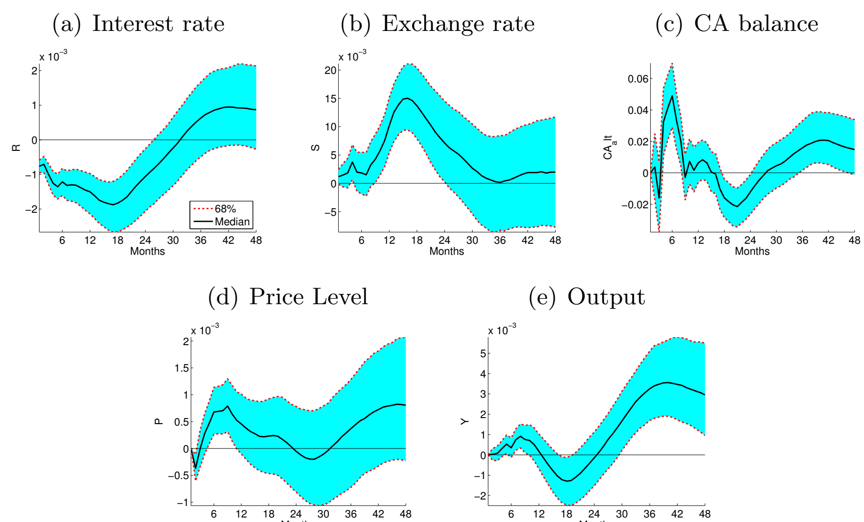
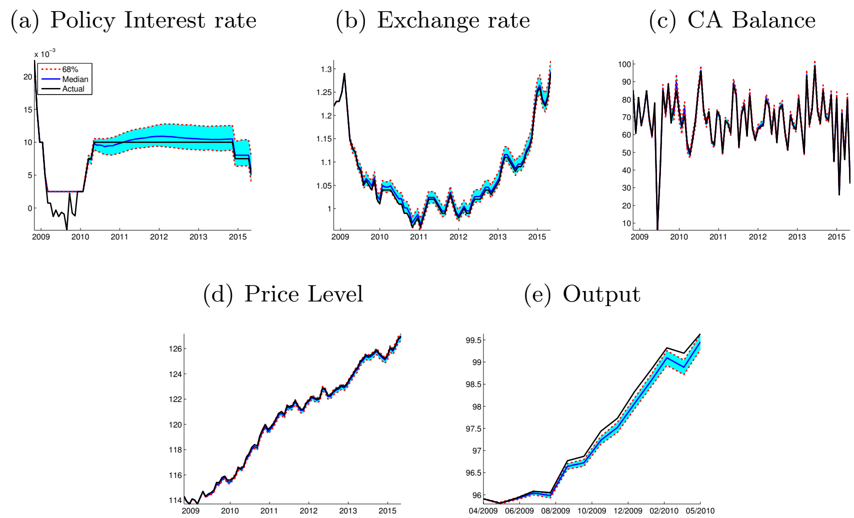


Figure 7: Response to foreign (US) expansionary monetary policy shock
(Jul 1994 – Jul 2015)
Alternative CA balance



Note: See figure 3 for U.S. impulse response functions, which are unchanged when we replace exports and imports with the current account balance.

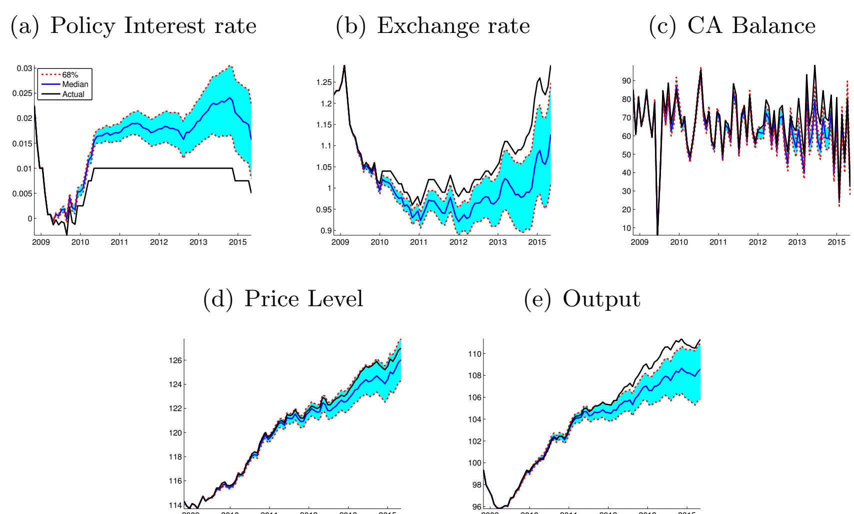
Figure 8: Counterfactual paths: Canadian ZLB imposed
(Jul 1994 – Jul 2015)
Alternative CA balance



(a) Policy Interest rate (b) Exchange rate (c) CA Balance

(d) Price Level (e) Output

Figure 9: Counterfactual paths: US ZLB imposed
(Jul 1994 – Jul 2015)
Alternative CA balance



(a) Policy Interest rate (b) Exchange rate (c) CA Balance

(d) Price Level (e) Output

Note: See figure 5 for U.S. impulse response functions, which are unchanged when we replace exports and imports with the current account balance.

Table 5: Average percent difference between data series and counterfactual path (full sample)

	Canadian ZLB Imposed			US ZLB Imposed		
Variable	Median	Max	Min	Median	Max	Min
Canadian Interest rate	0.00	0.00	0.00	0.01	0.01	0.00
Canadian Interest rate†	0.00	0.00	0.00			
Canadian CPI	-0.08	0.05	-0.23	-0.35	0.11	-0.81
Canadian CPI†	-0.07	-0.01	-0.14			
Canadian output	-0.03	0.25	-0.29	-0.85	-0.18	-1.66
Canadian output†	-0.14	-0.05	-0.23			

Note: This table contains the average percentage difference between the actual series and the counterfactual median as well as 68% bounds. The calculation uses the formula $\frac{1}{\tilde{T}}\sum_{t\in ZLB}\frac{y_t-y_t^{CF}}{y_t}$, where y_t is the series y at time t and y_t^{CF} is the same series under the counterfactual scenario, ZLB denotes the time period for the experiment and $\tilde{T}$ denotes the number of observations for the experiment. † indicates the estimates are for the Canadian ZLB period only. Also note that although the max and min bounds may include 0 this does not exclude the possibility that the paths were significantly different from zero for individual months during the experiment period.

B.2 Government spending

Figure 10: Response to domestic (Canadian) expansionary monetary policy shock
(Jul 1994 – Jul 2015)
Control for Fiscal Policy

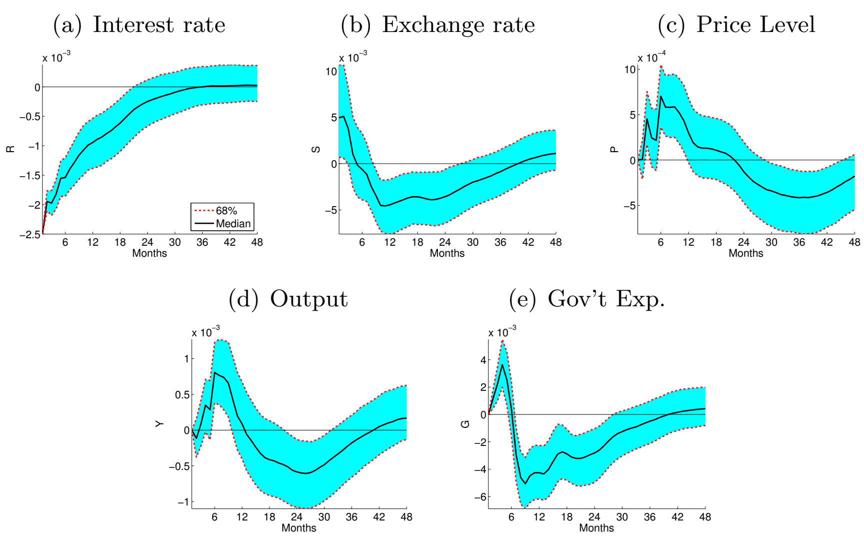


Figure 11: Response to foreign (US) expansionary monetary policy shock
(Jul 1994 – Jul 2015)
Control for Fiscal Policy

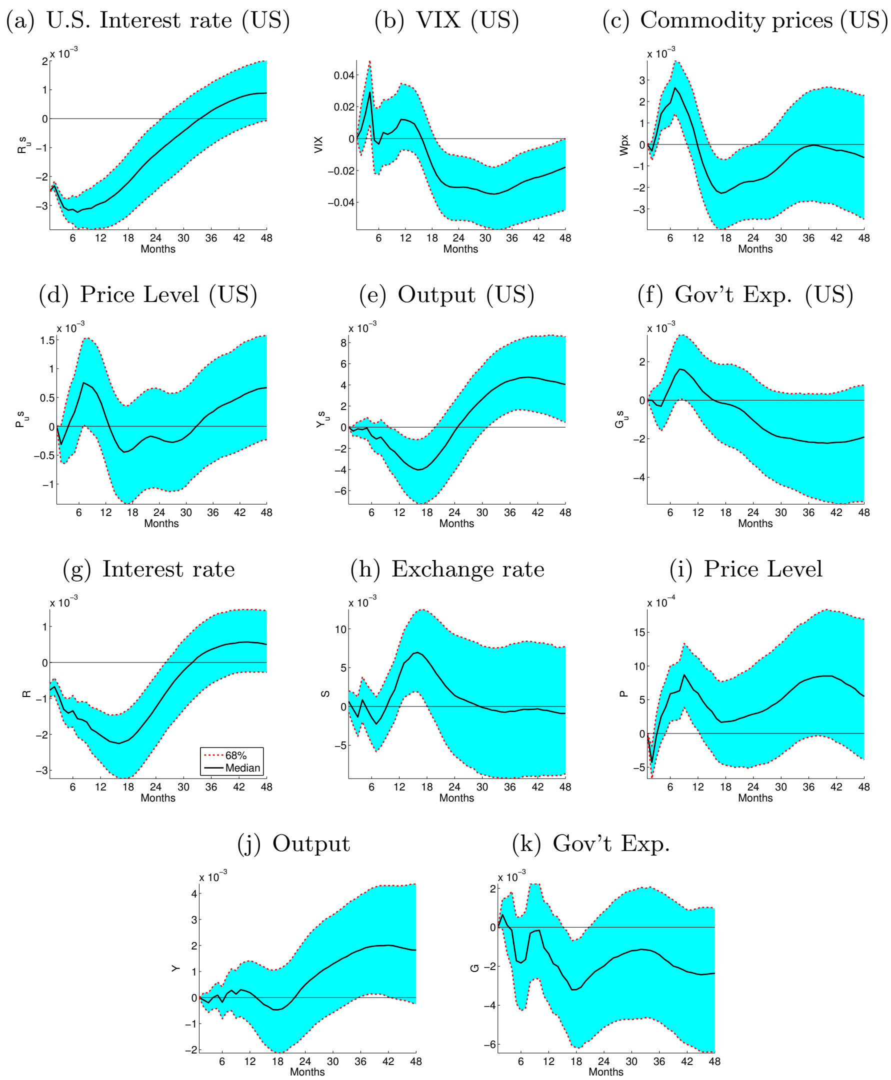
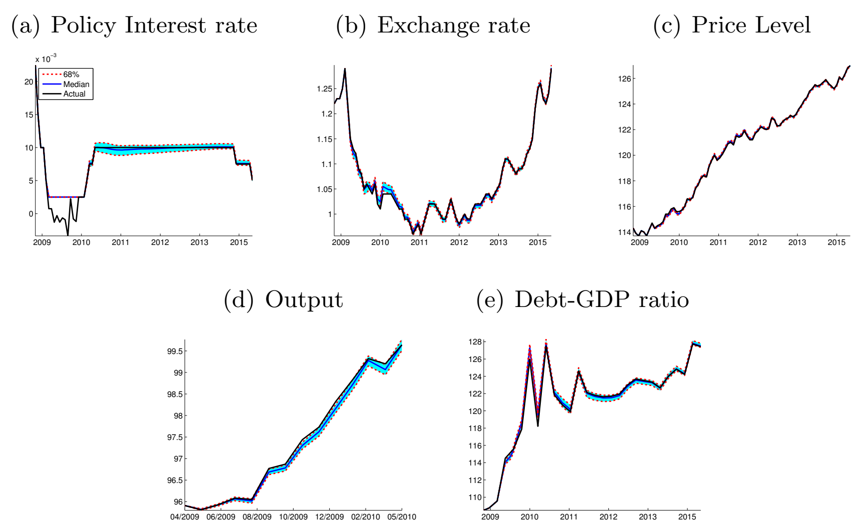


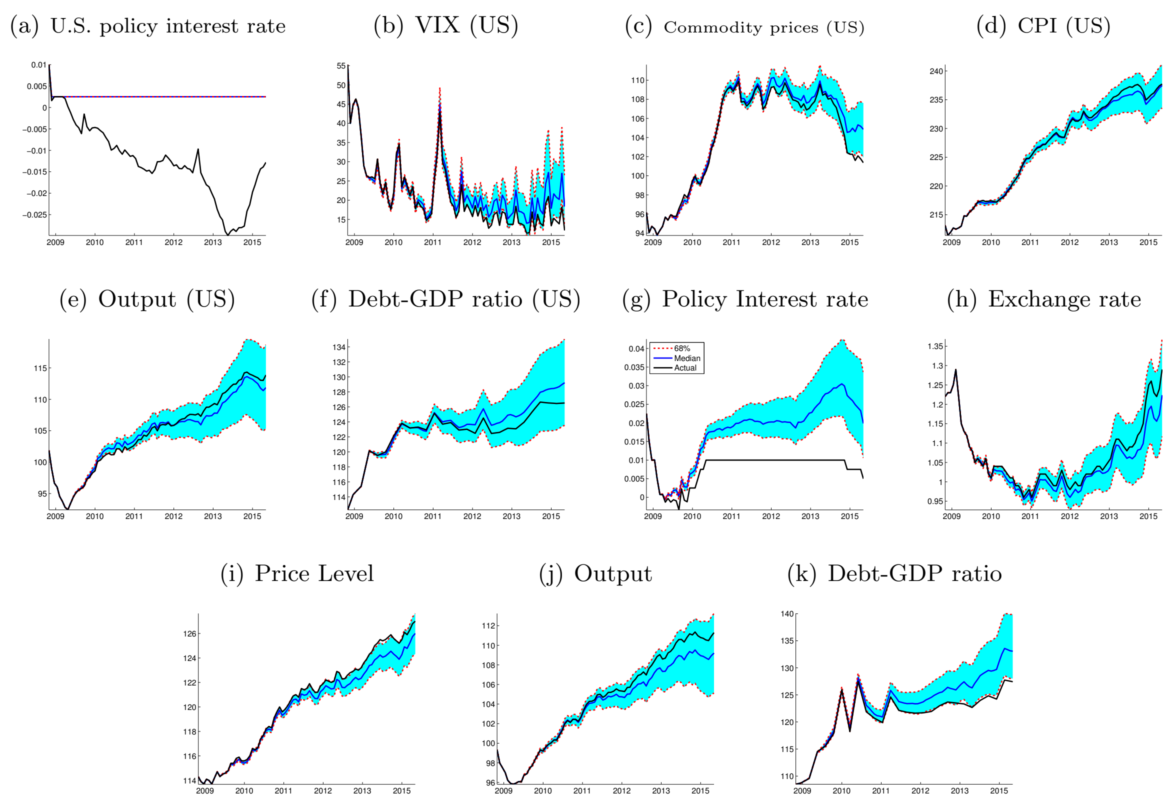
Figure 12: Counterfactual paths: Canadian ZLB imposed
(Jul 1994 – Jul 2015)
Control for Fiscal Policy



(a) Policy Interest rate (b) Exchange rate (c) Price Level

(d) Output (e) Debt-GDP ratio

Figure 13: Counterfactual paths: US ZLB imposed
(July 1994 – July 2015)
Control for Fiscal Policy



(a) U.S. policy interest rate (b) VIX (US) (c) Commodity prices (US) (d) CPI (US)

(e) Output (US) (f) Debt-GDP ratio (US) (g) Policy Interest rate (h) Exchange rate

(i) Price Level (j) Output (k) Debt-GDP ratio

Table 6: Average percent difference between data series and counterfactual path (full sample)

	Canadian ZLB Imposed			US ZLB Imposed		
Variable	Median	Max	Min	Median	Max	Min
Canadian Interest rate	0.00	0.00	0.00	0.01	0.03	0.01
Canadian Interest rate†	0.00	0.00	0.00			
Canadian CPI	0.00	0.06	-0.06	-0.43	0.01	-0.89
Canadian CPI†	-0.07	-0.02	-0.12			
Canadian output	0.00	0.07	-0.08	-0.61	0.59	-1.82
Canadian output†	-0.07	0.01	-0.14			
US interest rate				0.02	0.02	0.02
US CPI				-0.11	0.49	-0.73
US Output				-0.17	1.97	-2.31

Note: This table contains the average percentage difference between the actual series and the counterfactual median as well as 68% bounds. The calculation uses the formula $\frac{1}{\tilde{T}} \sum_{t \in ZLB} \frac{y_t - y_t^{CF}}{y_t}$, where y_t is the series y at time t and y_t^{CF} is the same series under the counterfactual scenario, ZLB denotes the time period for the experiment and $\tilde{T}$ denotes the number of observations for the experiment. † indicates the estimates are for the Canadian ZLB period only. Also note that although the max and min bounds may include 0 this does not exclude the possibility that the paths were significantly different from zero for individual months during the experiment period.

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