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The Signaling Effect and Optimal LOLR Policy

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Abstract

When a central bank implements the LOLR policy in a financial crisis, bank creditors often infer a bank's quality from whether or not it borrows from the central bank. We establish a formal model to study the optimal LOLR policy in the presence of this signaling effect, assuming that the central bank aims to encourage central bank borrowing to avoid inefficiencies caused by contagion. In our model, there are two types of banks: a high quality type with high expected asset returns and a low quality type with lower returns. Both types of banks need to roll over their short-term debts. A central bank offers to lend to both types of banks. After private creditors observe whether banks borrow from the central bank, banks try to borrow from the private market. We find that there may exist a separating equilibrium where only low quality banks borrow from the central bank; and two pooling equilibria where both types of banks do and do not borrow from the central bank. Our major results are as follows: (1) Considering the signaling effect, the central bank should set its lending rate lower than the prevailing market rate to induce both types of banks to borrow from the central bank. (2) Hiding the identity of banks borrowing from the central bank will encourage banks to borrow from the central bank. (3) The central bank may serve as a coordinator for the realization of its favored equilibrium.

Keywords: Signaling, Lender of Last Resort

JEL Classifications: E58 G28

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1 Introduction

The recent subprime mortgage crisis was full of drama. It is no doubt that one of its most dramatic moments occurred on October 13, 2008 when the crisis was at its peak.[1] On that day, the heads of nine major U.S. banks were invited to the Treasury department for a meeting with the Treasury Secretary Henry Paulson. In the meeting, the bankers were asked to accept a rescue package that combined several policies (e.g., a bank liability guarantee and capital purchase from the government). With some banks reluctant to accept the offer, Henry Paulson insisted that all the banks participate in the program. His reasons were: first to restore market confidence quickly; and second to prevent the market from drawing unfavorable conclusions about the quality of the participating banks, "with the strongest banks in the country taking the money to provide cover to the weaker banks that would follow suit."[2]

This event is a vivid example of when government assistance can serve an important signaling role. In particular, there is often a "stigma" associated with government assistance: the action of a bank accepting government assistance is often viewed as a negative signal about the bank's asset quality, increasing its financing costs on the market. As a result, banks are reluctant to seek assistance, and the government, trying to avoid contagion, has to force a "pooling" equilibrium where all the banks, strong and weak, accept assistance.

Another example of this signaling effect, where a government attempts to induce a pooling equilibrium, is the Fed Reserve's long history of hiding the identities of banks using its discount window. As explained by Courtois and Ennis (2010), the Fed used to regularly disclose only the total amount of discount window borrowing without disclosing the identities of borrowers. Since the subprime mortgage crisis, where the Fed provided large loans to financial institutions, it has been under increasing public pressure to disclose more detailed information about discount window lending. As a result, for discount window loans extended after July 21, 2010, the Fed is required to disclose more information (e.g., the identity of the borrowers) in accordance with the provisions of the Dodd-Frank Wall Street Reform and Consumer Protection Act. However, this information is required

[1]The following description of the event largely follows chapter 20 of Sorkin (2009), and chapter 6 of Johnson and Kwak (2010).

[2]See page 524 of Sorkin (2009).

to be disclosed generally about *two years* after the loan has been extended.[3] During the subprime mortgage crisis, the Fed even created the Term Auction Facility(TAF) where Fed loans are auctioned by participating financial institutions anonymously to better hide the identities of loan recipients.

The above examples raise an interesting question: When a central bank attempts to induce a pooling equilibrium, where all the banks seek central bank loans, what will be the optimal LOLR policy in the presence of this signaling effect? Note that in this paper we focus on a situation where the central bank aims to encourage central bank borrowing from all the banks, including the weak ones. This situation is most likely to happen in the time of crises, when the failure of weak banks may lead to the collapse of the whole financial system through the contagion effect. In such a situation, the social cost caused by the failure of weak banks is so large that the central bank cannot afford to let them become bankrupt, regardless of any concerns about moral hazard. A typical example of such a situation is the one cited at the beginning of this paper, where after the Lehman Brothers' bankruptcy, the U.S. financial system was feared to be on the edge of collapse.

We establish a formal signaling game to provide theoretical guidance for a central bank facing such a situation. In our model, there are two types of banks, a high quality type with higher expected asset returns and a low quality type with lower returns. They both need to roll over their short-term debts. The banks can choose either to borrow only from the private market, or to borrow from the central bank first, and then borrow the remaining funds from the private market. Banks' type is private information and can be observed by creditors on the market with a certain probability: A higher probability implies that creditors have more precise information about banks' quality. We also assume that if a bank's type is observed by creditors, they will never lend to a low quality bank, but will charge a riskless rate of zero to a high quality bank.

We explore possible equilibria in such a game, and how different factors affect equilibrium outcomes. We find that there may exist three equilibria: a separating equilibrium where only low quality banks borrow from the central bank and high quality banks do not; a pooling equilibrium with both types of banks not borrowing from the central bank (PNB hereafter); and a pooling equilibrium with both types of banks borrowing from the

[3]See "Changes to the Federal Reserve's Practices Regarding Disclosure of Discount Window Lending Information" on www.frbdiscountwindow.org.

central bank (PBB hereafter). A comparative statics analysis reveals the following major results:

First, when central bank loans are offered with more attractive terms (e.g., a larger amount and lower rate), both types of banks find central bank borrowing more attractive. As a result, PBB and the separating equilibrium become more likely, but PNB becomes less likely.

Second, if creditors on the market have more precise information about the banks' quality, low quality banks will face tougher borrowing terms on the market. As a result, they find central bank loans more attractive than market loans. For high quality banks, we have the opposite result. Thus, with more precise information, the separating equilibrium is more likely, but both pooling equilibria are less likely.

Third, in the two pooling equilibria, the equilibrium outcomes are affected by creditors' belief off the equilibrium path, which is the probability of being high quality type that creditors assign to a bank deviating from the equilibrium strategy. Both pooling equilibria will become less likely with a more optimistic belief off the equilibrium path, because it improves the market borrowing terms for a deviating bank.

We then explore the optimal LOLR policy in our game. We focus on the situation where the central bank aims to induce PBB. Our major results on central bank LOLR interest rate policy are as follows:

First, if the central bank wants both types of banks to borrow, then its lending rate must be lower than the prevailing market rate. This is because the market rate is a break-even rate that gives risk-neutral creditors an expected riskless return when the creditors cannot observe a bank's type. A high quality bank's average market borrowing rate is below this break-even rate, because with a positive probability, a high quality bank has its type observed by creditors and is charged a riskless rate. Only when a high quality bank does not have its type observed by creditors is it charged the break-even rate. Thus in order to induce a high quality bank to borrow from the central bank, the central bank lending rate must be below the break-even rate. This result provides insight into the long-standing LOLR policy debate over the central bank lending rate.

Second, there is a non-monotonic relationship between the central bank's lending rate and its net revenue.[4] More specifically, its net revenue increases initially in the lending rate, but has a downward jump when the lending rate reaches a threshold level. This downward jump is caused by two effects: an adverse selection effect and a signaling effect. The adverse selection effect exists because high quality banks will stop borrowing, while low quality banks will continue borrowing as the lending rate increases. The signaling effect exists because when the central bank's lending rate is sufficiently high, it signals that all the banks borrowing from the central bank are low quality ones. As a result, these banks face tougher borrowing terms on the market, weakening their ability to repay central bank loans, and decreasing the central bank's net revenue.

We find this signaling effect particularly interesting because it is missing in most of the existing literature on the LOLR policy. Our model reveals that it can play an important role in determining the equilibrium outcome and the central bank should take this effect into account when setting its policy. Since a higher lending rate not only deters high quality banks from borrowing, but may also cause a severe liquidity problem for low quality banks due to its signaling effect, the central bank may want to set a low rate, if it aims to alleviate a liquidity shortage in the whole financial system.

Moreover, we examine the identity hiding policy in which the central bank commits to hiding the information on whether or not a bank has borrowed from it. We find that with this policy, the central bank is more likely to attain its favored pooling equilibrium (PBB) (and avoid its unwanted pooling equilibrium (PNB)). This result is obtained under the assumption that creditors believe that a bank not borrowing from the central bank must be more likely to be high quality type than a bank borrowing from the central bank. This assumption is reasonable because low quality banks have a stronger incentive to borrow from the central bank due to tougher market borrowing terms that they face. With the identity hiding policy, the strategy of not borrowing from the central bank cannot be observed and signal that a bank is more likely to be high quality type. Similarly, the strategy of borrowing from the central bank cannot be observed and signal that the bank is more likely to be low quality type. As a result, the strategy of borrowing from the central bank becomes more attractive, making PBB more likely and PNB less likely.

Finally, we explore other policies that may help attain PBB. In general, we find that any policy that increases banks' payoff from borrowing central bank loans and decreases

[4] The central bank's net revenue is defined as its lending revenue minus the amount of its loans.

their payoff from not borrowing will help attain PBB. Moreover, when multiple equilibria exist, the central bank may serve as a coordinator to direct banks toward PBB.

Our paper contributes to several strands of literature. It first contributes to the literature on the LOLR policy. The early literature on the LOLR policy dates from Thornton (1802) and Bagehot (1873). The more recent literature includes Goodfriend and King (1988), Goodhart and Huang (1999), Freixas, Parigi, and Rochet (2004), Rochet and Vives (2004), Pritsker (2013), and Li, Milne and Qiu (2015, forthcoming). In particular, our paper is closely related to Ennis and Weinberg (2013). They build a sequential game to explain why banks may choose to borrow on the interbank market even when the interbank rate is higher than the rate available from the central bank discount window. In their model, illiquid banks with different asset quality meet their liquidity need first through borrowing either from a central bank or from the interbank market, and next through selling their assets to investors who have imperfect information about their type. Because interbank loan lenders have perfect information about the type of illiquid banks, illiquid banks with low asset quality are more likely to borrow from the central bank. As a result, investors who buy illiquid banks' assets will infer the asset quality of an illiquid bank from observing whether or not it borrows from the central bank. Thus similar to our paper, central bank borrowing in their model serves a signaling role. Our paper differs from theirs in two aspects. First, their paper does not contain a welfare analysis and policy implications in the presence of the signaling effect. Our paper complements theirs by focusing on how various central bank policies will induce PBB, the socially optimal equilibrium. As a result, our model produces several policy implications. Second, our model has a different structure. For example, because we wish to explore optimal polices, we abstract from the interbank market and an asset market where investors buy banks' assets. Instead, we simplify the bank's source of private funds into a single private loan market. In addition, in their model (by construction) investors form a unique belief about a bank's type using Bayes' rule. While we build a signaling game in which creditors may have various beliefs off the equilibrium path in the case of pooling equilibria. Thus, given different beliefs off the equilibrium path, multiple equilibria may exist. We find this feature of multiple equilibria particularly interesting because empirical evidence reveals that in the time of crises, different countries might indeed reach different equilibria, which could be potentially explained by creditors' different beliefs off the equilibrium path.

Acharya and Sundaram (2009) provide a non-technical introduction to the main U.S. policy tools adopted during the subprime mortgage crisis. These included the loan guarantee scheme administered by the Federal Deposit Insurance Corporation (FDIC) under which the FDIC guarantees newly issued senior unsecured debts of banks. They compare the loan guarantee policies in the U.S. and U.K., observing that "by adopting a one-size-fits-all pricing scheme that is set at too low a level relative to the market rate" and "by offering very little in terms of optionality in participation, the U.S. loan guarantee scheme is effectively forced on all banks, giving rise to a pooling outcome." The U.K. scheme used a market-based fee structure and provided more optionality in participation, which would appear to induce a separating equilibrium, allowing strong banks to opt out while weaker banks choose to stay in the plan. Although our paper models central bank lending instead of loan guarantee policies, the intuition behind the pooling and separating equilibrium in their discussion is quite similar to that in our paper. Thus our paper provides a formal model consistent with their informal policy observations.

Our paper is also related to the literature on adverse selection and signaling in financial markets (e.g. Stiglitz and Weiss (1981), Leland and Pyle (1977)). In particular, in Stiglitz and Weiss (1981), a high bank lending rate could drive away less risky borrowers because the interest rate exceeds the maximum rate that they can afford. As a result, only more risky borrowers with lower expected project returns will borrow, leading to a lower expected profit for the lending bank. In our model, the central bank faces a similar adverse selection problem when its higher lending rate drives away high quality banks, although this time it is not because the borrowers cannot afford the rate but because they have a better option of borrowing on the private market. However, in our model, the central bank's net revenue is also affected by a signaling effect, which is missing in the Stiglitz and Weiss (1981) analysis.

The rest of the paper is organized as follows. Section 2 describes the environment of the model. Section 3 characterizes the equilibrium outcomes and conducts comparative statics to examine how different factors affect the equilibrium outcomes. Section 4 examines central bank policies based on our model. Section 5 concludes.

2 The environment of the model

2.1 The main events

This is a two-period model with three dates denoted by $t = 0$, 1 and 2. There is a continuum of banks in the economy, whose total size is normalized to 1. There also exists a central bank. On date $t = 0$, every bank has an exogenously given identical balance sheet as follows.

Banks' balance sheet on date 0	
Long-term Assets: A	Short-term Debts : D_0
	Equity: e_0

The long-term assets will mature on date 2. On date 0, these assets are expected to yield a certain gross return rate of $R_H > 1$ on date 2. If the assets are liquidated prematurely on date 1, then there will be a liquidation cost, which we will specify later. The short-term debts D_0 will mature on date 1. We assume that short-term creditors are atomically small. Additionally, we assume that creditors are risk-neutral and will charge a net riskless rate of zero on date 0. The banks will need to roll over the debts on date 1.

At the beginning of $t = 1$, an unexpected aggregate shock hits the economy. As a result, proportion λ of the banks will not be affected by the shock, and their long-term assets, if mature on date 2, will still yield a return of R_H. We call these banks H-type. The remaining proportion $1 - \lambda$ of the banks will be affected by the shock. Their long-term assets, if mature on date 2, will have a return rate as follows: with probability p, an up state is realized and the return rate is R_H. With probability $1 - p$, a down state is realized and the return rate is $R_L < 1$. We call these banks L-type. The return rate of every L-type bank is independently and identically distributed. The type of every bank is private information: Every bank knows its own type, but this information is not publicly known. However, the aggregate shock is public information: Market participants know the proportion of both types of banks and the return rate for each type of bank. Figure 1 gives the timeline of this model.

After the shock, every bank needs to roll over its short-term debts of D_0. That is, every bank has a liquidity need of D_0 on date 1. Banks can raise funds through three ways: borrowing central bank loans, borrowing on the market from short-term creditors,

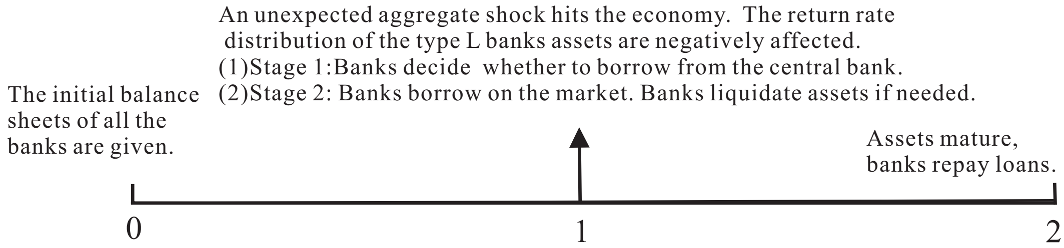


Figure 1: The timeline

and liquidating long-term assets.

We assume that banks raise funds on date 1 in two stages. In the first stage, the central bank offers a central bank loan of $L_{CB} < D_0$ to every bank at a net rate of $r_{CB} \geq 0$. Note that r_{CB} cannot exceed the highest interest rate that banks may be charged on the market. Otherwise, banks will not borrow central bank loans. Every bank decides whether to borrow central bank loans or not, and its decision is publicly observed. For simplicity, here we consider only the case where a bank either does not borrow or borrow the full amount of L_{CB} from the central bank.

In the second stage, every bank raises funds from short-term creditors on the market. At the beginning of this stage before the market rate is determined, we assume that, for every bank, with probability q, there will be a public signal which perfectly reveals its type. With probability $1 - q$, there will be no such a signal. The probability for the incidence of a signal for a bank is i.i.d among all the banks. Thus, for a bank with a signal, because the signal perfectly reveals its type, creditors will ignore all the previous information. However, for a bank without a signal, creditors will form their beliefs about its type based on all the information available, which we will specify below.

Creditors lend to the banks in the following steps: after observing a bank's signal (or no signal), creditors decide whether to lend to it or not, and if yes, the rate to charge. Since creditors are risk neutral, they will aim at an expected riskless rate of zero. Given creditors' offers, every bank then decides how many loans to borrow, and if necessary, how many long-term assets to liquidate. A creditor cannot make his offer contingent on the actions taken by the bank, such as the quantity of short-term loans that the bank borrows and the quantity of long-term assets that the bank liquidates. The liquidation technology is as follows. For H-type and L-type banks, each unit of the long-term assets liquidated on date 1 will yield γ_H and γ_L units of liquid funds on date 1, respectively. We assume that $\gamma_L \leq \gamma_H < \frac{D_0}{A} < 1$. That is, liquidation is so costly that both types of banks become insolvent if they are forced to liquidate their assets on date 1. In addition, we assume that $\gamma_L < pR_H + (1-p)R_L$. That is, it is socially costly for an L-type bank to liquidate its assets.[5]

The liquidity need for every bank on date 1 can be summarized as follows. If the bank does not liquidate any long-term assets, it will need to borrow a loan of D_0. More specifically, if the bank has borrowed from the central bank, it will need to borrow a loan of $D_0 - L_{CB}$ on the market. Otherwise, it will need to borrow a loan of D_0 on the market. If the bank liquidates its assets to repay part of its debts, then its liquidity need on date 1 will be reduced by the cash proceeds from liquidation.

Finally, on date 2, for banks that survive date 1 (i.e., their assets are not completely liquidated on date 1), the return of their remaining assets will be realized. Banks repay their loans, and keep the remaining assets as their equity. We assume the following rules for loan repayment. If the bank's total asset value equals or exceeds its total debts (which equal the principal plus accrued interests), then all the debts will be fully repaid. If its asset value is lower than its total principal, then its asset will be allocated proportionally to the principal level of every creditor.[6]

2.2 Additional assumptions for banks' decisions

This section specifies additional assumptions related to banks' decisions on date 1. First, we assume that every bank aims to maximize its expected equity. This assumption specifies banks' objective. Denote a bank's net asset value as NV, and denote its equity value as e. Note that $e = \max(NV, 0)$. Thus, when a bank's net asset value is negative, its equity level is defined as zero.

Second, we assume that $AR_L < D_0$, implying that an L-type bank's date 2 asset value in the down state is lower than D_0, the principal of its original debts, even if it does not liquidate any long-term assets on date 1. Given this assumption, we arrive at the following result.

[5]A more detailed explanation about this assumption will be offered in Section 4.

[6]The case where a bank's total asset is between its principal and total debts does not apply to our model.

Proposition 1. *In equilibrium, the net asset value of an L-type bank in the down state is negative, and its equity in the down state is zero. As a result, an L-type bank aims to maximize its equity value in the up state.*

Proof: See the Appendix. ■

Note that for an H-type bank the up state is realized with probability 1. Thus both types of banks share the same objective function.

Let $r_{m,i}$ denote the interest rate that an individual bank, say bank i, faces when it borrows on the market. We have the following result about banks' borrowing decisions.

Proposition 2. *If $1 + r_{m,i} \leq \frac{R_H}{\gamma_H}$, an H-type bank will borrow on the market and will not liquidate assets. If $1 + r_{m,i} > \frac{R_H}{\gamma_H}$, an H-type bank will not borrow on the market and will liquidate assets to repay its debts. The similar rule applies to L-type banks, with $\frac{R_H}{\gamma_H}$ replaced by $\frac{R_H}{\gamma_L}$.*

Proof: See the Appendix. ■

The intuition of Proposition 2 is that banks will borrow on the market only when the borrowing cost is lower than the cost of liquidating long-term assets. If an H-type bank liquidates its assets, for every 1 unit of funds on date 1, it needs to sacrifice $\frac{R_H}{\gamma_H}$ units of goods on date 2. Similarly, an L-type bank needs to sacrifice $\frac{R_H}{\gamma_L}$ units of goods in the up state on date 2. Therefore, if the borrowing cost is lower than the liquidation cost, banks will borrow on the market. Otherwise, banks will choose to liquidate assets to repay their debts.

Third, we assume that creditors' gross expected return rate from lending to an L-type bank is lower than the gross riskless rate of one. This assumption implies that creditors will never lend to an L-type bank if they know its type, which greatly simplifies our analysis. The detailed sufficient conditions for this assumption are given in the Appendix.

3 Equilibrium outcomes

This section characterizes all the possible pure strategy equilibria in this game. It turns out that there are three pure strategy equilibria: a separating equilibrium in which L-type banks borrow central bank loans and H-type banks do not, PNB in which both types of banks do not borrow central bank loans, and PBB in which both types of banks borrow central bank loans.

3.1 The separating equilibrium

We first analyze the following pure strategy equilibrium: H-type banks signal their type by not borrowing central bank loans, and L-type banks borrow central bank loans. The payoffs for each type of bank under the equilibrium and deviating strategies are as follows.

When an H-type bank follows the equilibrium strategy of not borrowing central bank loans, with probability q, there is a public signal that reveals its type. With probability $1-q$, there is no such a signal. Since this is a separating equilibrium, creditors know for sure that it is H-type. As a result, with or without the signal, the bank will be able to borrow on the market at the riskless rate of zero. Hence, its date 2 equity value will be

$$e_{H,NCB} = AR_H - D_0, \tag{1}$$

where the subscript "NCB" means "no central bank loans".

It is straightforward to see that an H-type bank has no incentive to deviate because it is charged the lowest possible interest rate and attains the highest possible payoff under the equilibrium strategy. If it deviates by borrowing first from the central bank and then on the market, it would be charged an interest rate no lower than zero. Furthermore, with a positive probability of no signal revealing its type, the bank will be viewed as L-type. As a result, it will not be able to borrow additional loans on the market and be forced to liquidate its assets, which is costly.

Next, we examine the condition for an L-type bank not to deviate from its equilibrium strategy. Similar to the H-type bank case, when an L-type bank chooses the equilibrium strategy of borrowing central bank loans, with or without a public signal that reveals its type, creditors will believe that the bank is L-type. According to our assumptions, creditors will not lend to it, and the bank can borrow only a central bank loan of L_{CB}. Note that this equilibrium exists only when an L-type bank will not have to liquidate all the assets after borrowing central bank loans, and its date 2 equity value in the up state is positive, because otherwise the bank has no incentive to continue operating or to borrow central bank loans.

Because $D_0 > L_{CB}$, the bank will have to liquidate $l_L = \frac{D_0 - L_{CB}}{\gamma_L}$ units of assets on date 1 to repay its debts of $D_0 - L_{CB}$. On date 2, the bank's equity in the up state is

$$e^u_{L,CB} = (A - l_L)R_H - L_{CB}(1 + r_{CB}) = AR_H - L_{CB}(1 + r_{CB}) - R_H \frac{D_0 - L_{CB}}{\gamma_L}. \quad (2)$$

Note that when $e^u_{L,CB} \leq 0$, this separating equilibrium does not exist. We need only to consider the bank's equity in the up state, because, as proved previously, the bank's equity in the down state is zero, and the bank aims to maximize its equity in the up state.

If an L-type bank deviates and does not borrow central bank loans, then with probability $1 - q$, there will be no signal that reveals its type. Thus it will be viewed as H-type and borrow a loan of D_0 on the market at the zero rate. In this case, with probability p, the up state is realized and its date 2 equity will be the same as that of an H-type bank:

$$e^u_{L,NCB,s_i=\emptyset} = AR_H - D_0, \quad (3)$$

where $s_i = \emptyset$ means "no signal". With probability q, there will be a signal. In this case, the bank cannot borrow any loans on the market. In addition, because we assume that $\gamma_L A < D$, the bank will have to liquidate all of its assets, and gain zero equity.

Thus, the no-deviation condition for an L-type bank is that borrowing central bank loans yields higher expected equity than not borrowing, that is,

$$pe^u_{L,CB} + (1 - p) \times 0 \geq q \times 0 + (1 - q)[pe^u_{L,NCB,s_i=\emptyset} + (1 - p) \times 0], \quad (4)$$

which can be simplified to

$$e^u_{L,CB} \geq (1 - q)e^u_{L,NCB,s_i=\emptyset}. \quad (5)$$

We thus arrive at the following results:

Proposition 3. *(1) The no-deviation condition for this separating equilibrium is $e^u_{L,CB} \geq (1 - q)e^u_{L,NCB,s_i=\emptyset}$. (2) $e^u_{L,CB} - (1 - q)e^u_{L,NCB,s_i=\emptyset}$ is increasing in L_{CB}, decreasing in r_{CB}, and increasing in q. Hence the equilibrium is more likely to exist when L_{CB} is higher, r_{CB} is lower, and q is higher.*

Proof: See the Appendix. ■

The intuition for result (2) is that, higher L_{CB} and lower r_{CB} will make borrowing central bank loans more attractive to L-type banks. Thus they will be less likely to deviate. A higher q will make the equilibrium more likely to exist because when q is higher, the market is more likely to observe a bank's true quality, and an L-type bank will more likely face a market freeze if it deviates. Hence, it has a weaker incentive to mimic an H-type bank. As a result, if q is high enough, an L-type bank would rather borrow central bank loans, even when doing so will signal its type to the market.

Note that a separating equilibrium in which H-type banks borrow central bank loans and L-type banks do not borrow cannot exist, because in this case L-type banks are always better off by imitating H-type ones and will therefore always deviate.

3.2 PNB: both types of banks do not borrow central bank loans

3.2.1 The equilibrium market rate

In equilibrium, both types of banks choose not to borrow central bank loans. In stage 2 on date 1, banks borrow on the market. More specifically, banks with public signals will borrow according to their true types, and the remaining banks without signals will be charged the same market rate, which we denote as r_M. Because this is a pooling equilibrium, r_M is based on creditors' prior belief that a bank is H-type with probability λ.

Here we need to first offer an explanation about the existence of a market rate. An equilibrium market rate exists only when: (1) it can actually be paid by H-type banks and L-type banks in the up state; (2) both types of banks are willing to borrow at this rate; and (3) it will give creditors an expected riskless rate of zero, since we assume that creditors are risk neutral. Note that if λ or R_L is too low, there can be a market freeze where creditors stop lending. This is because for the interest rates that banks are willing to borrow, creditors may gain an expected rate lower than the riskless rate of zero due to their expected loss from lending to L-type banks. Therefore, the market freezes. Here, we focus on the case where an equilibrium market rate exists, because otherwise, H-type banks will always be better off by deviating to borrowing central bank loans, and this pooling equilibrium cannot exist.

Using the condition that creditors must attain an expected riskless rate of zero, we arrive at the following result:

Lemma 1. *In PNB, an equilibrium market rate r_M should satisfy:*

$$1 = \lambda(1 + r_M) + (1 - \lambda) \left[p(1 + r_M) + (1 - p)\frac{AR_L}{D_0}\right], \tag{6}$$

or equivalently

$$r_M = \left(1 - \frac{AR_L}{D_0}\right) \left(\frac{1}{\lambda + (1 - \lambda)p} - 1\right) \tag{7}$$

The intuition is as follows. When a creditor offers to lend, the bank is H-type with probability λ, in which case the bank will pay $1 + r_M$. With probability $1 - \lambda$, the bank is L-type. With probability p, the up state happens, and the bank will pay $1 + r_M$. With probability $1 - p$, the down state happens, and the bank will go bankrupt and pay a recovery rate of $\frac{AR_L}{D_0}$.

The above formula for r_M satisfies only the third condition for the existence of an equilibrium market rate. We must impose the additional assumption that

$$1 + \left(1 - \frac{AR_L}{D_0}\right) \left(\frac{1}{\lambda + (1 - \lambda)p} - 1\right) < \min\{\frac{R_H}{\gamma_H}, \frac{AR_H}{D_0}\} \tag{8}$$

such that the first and second conditions for the existence of an equilibrium market rate hold. A detailed explanation about this assumption is provided in the Appendix.

3.2.2 Banks' payoffs under the equilibrium and deviating strategies

For an H-type bank, with probability q, there is a signal indicating its type, which we denote as $s_i = s_H$. With probability $1 - q$, there is no signal, which we denote as $s_i = \emptyset$. When there is a signal, the bank can get the funds it needs at the riskless rate. Thus, its date 2 equity value will be

$$e_{H,NCB,s_i=s_H} = AR_H - D_0 \tag{9}$$

When $s_i = \emptyset$, the creditors will charge r_M and the date 2 net value of the bank will be

$$e_{H,NCB,s_i=\emptyset} = AR_H - D_0(1 + r_M) \tag{10}$$

The bank's expected equity is

$$Ee_{H,NCB} = qe_{H,NCB,s_i=s_H} + (1 - q)e_{H,NCB,s_i=\emptyset} \tag{11}$$

Table 1: The outcome of different strategies in PNB

H-type banks		
Do not borrow L_{CB}	$s_i = s_H$	Borrow D_0 on the market at a zero rate.
	$s_i = \emptyset$	Borrow D_0 on the market at r_M.
Borrow L_{CB} at r_{CB}	$s_i = s_H$	Borrow $D_0 - L_{CB}$ on the market at a zero rate.
	$s_i = \emptyset$	Borrow $D_0 - L_{CB}$ on the market at $\tilde{r}_{M,CB}$.
L-type banks		
Do not borrow L_{CB}	$s_i = s_L$	Cannot borrow any loans. Liquidate all the assets.
	$s_i = \emptyset$	Borrow D_0 on the market at r_M.
		Positive equity if R_H is realized.
Borrow L_{CB} at r_{CB}	$s_i = s_L$	Cannot borrow additional loans.
		Liquidate assets to repay $D_0 - L_{CB}$.
	$s_i = \emptyset$	Borrow $D_0 - L_{CB}$ on the market at $\tilde{r}_{M,CB}$.
		Positive equity if R_H is realized.

For an L-type bank, with probability q, there is a signal and the bank cannot roll over any loans. Because $\gamma_L A < D_0$, the bank will be forced to liquidate all the assets and gain zero equity. With probability $1 - q$, it can borrow at r_M. In this case, its date 2 equity in the up state is the same as that of an H-type bank,

$$e^u_{L,NCB,s_i=\emptyset} = e_{H,NCB,s_i=\emptyset} = AR_H - D_0(1 + r_M), \tag{12}$$

and its equity in the down state is zero. As a result, its expected equity is

$$Ee_{L,NCB} = (1 - q)pe^u_{L,NCB,s_i=\emptyset} \tag{13}$$

Next, we derive a bank's payoff when it deviates from the equilibrium strategy and chooses to borrow central bank loans. Then we need to specify creditors' belief off the equilibrium path. Let $\tilde{\lambda}$ denote the probability that creditors assign to a bank being H-type when observing it borrow central bank loans.[7] We impose the additional assumption that $\tilde{\lambda} \leq \lambda$, which means that if creditors observed a bank borrow central bank loans,

[7] We use ˜ to denote the variables under the deviating strategy.

their belief of the bank being H-type would be at most as high as their prior belief. We impose this assumption because an H-type bank benefits relatively less from borrowing central bank loans than an L-type bank.

After borrowing L_{CB} from the central bank, the bank needs to borrow the remaining funds of $D_0 - L_{CB}$ on the market. With probability q, the bank's true quality will be revealed. If the bank is H-type, it will be able to borrow the remaining funds at the zero rate. If the bank is L-type, it will not be able to borrow any funds on the market and will have to liquidate its assets to repay the remaining debt of $D_0 - L_{CB}$. With probability $1-q$, the bank's type is not revealed, and the bank will borrow at a market rate of $\tilde{r}_{M,CB}$,[8] if such a rate exists. Since the bank's type is not revealed, creditors will decide $\tilde{r}_{M,CB}$ based on their belief off the equilibrium path, $\tilde{\lambda}$. It turns out that when $\tilde{\lambda}$ is sufficiently high such that

$$1+\left(1-\frac{AR_L}{D_0}\right)\left(\frac{1}{\tilde{\lambda}+(1-\tilde{\lambda})p}-1\right) \leq \min\left\{\frac{AR_H - L_{CB}(1+r_{CB})}{D_0 - L_{CB}}, \frac{R_H}{\gamma_H}\right\}, \qquad (14)$$

$\tilde{r}_{M,CB}$ exists and equals $\left(1-\frac{AR_L}{D_0}\right)\left(\frac{1}{\tilde{\lambda}+(1-\tilde{\lambda})p}-1\right)$, which is similar to r_M in Eq. (7) except that λ is replaced by $\tilde{\lambda}$. The derivations of $\tilde{r}_{M,CB}$ and the payoffs when banks deviate are provided in the Appendix. A summary of the outcomes under the equilibrium and deviating strategies is provided in Table 1.

3.2.3 The no-deviation conditions

The no-deviation conditions are $Ee_{H,NCB} \geq E\widetilde{e}_{H,CB}$ for H-type banks and $Ee_{L,NCB} \geq E\widetilde{e}_{L,CB}$ for L-type banks. We have the following results:

Proposition 4. *(1) Provided that $Ee_{L,NCB} - E\widetilde{e}_{L,CB} \geq 0$,*

$$Ee_{H,NCB} - E\widetilde{e}_{H,CB} \geq Ee_{L,NCB} - E\widetilde{e}_{L,CB}, \qquad (15)$$

implying that as long as an L-type bank does not deviate, an H-type bank will not deviate either. The above condition holds without equality if we reasonably assume $\tilde{\lambda} < 1$.[9]

[8] CB means that this is a rate after borrowing central bank loans.

[9] $\tilde{\lambda} = 1$ means that creditors will view a bank deviating to the strategy of borrowing central bank loans as H-type for sure.

(2) For an L-type bank, $Ee_{L,NCB} \geq E\widetilde{e}_{L,CB}$ implies that

$$
\begin{aligned}
&(1-q)\left[D_0(\tilde{r}_{M,CB}-r_M)+L_{CB}(r_{CB}-\tilde{r}_{M,CB})\right] \\
&-q\max\left\{0,\left[(A-\frac{D_0-L_{CB}}{\gamma_L})R_H-L_{CB}(1+r_{CB})\right]\right\}\geq 0
\end{aligned}
\tag{16}
$$

(3) PNB is more likely to exist with a lower L_{CB}, q, $\tilde{\lambda}$, and higher r_{CB}.

Proof: See the Appendix. ■

Condition (15) means that once the no-deviation condition for an L-type bank is satisfied, the no-deviation condition for an H-type bank must also be satisfied. The intuition is that an L-type bank will benefit more from borrowing central bank loans than an H-type bank. This is because when there is a signal, an H-type bank can borrow at a zero market rate and will not benefit from borrowing L_{CB} of central bank loans at $r_{CB} \geq 0$; while an L-type bank cannot borrow any loans on the market and borrowing L_{CB} of central bank loans can lower its asset liquidation. Thus, an L-type bank benefits more when deviating to the strategy of borrowing central bank loans than an H-type bank. As a result, the essential no-deviation condition for this equilibrium to exist is the one for L-type banks.

Our previous analysis focuses on the case where an equilibrium market rate, $\tilde{r}_{M,CB}$, exists when banks deviate. When $\tilde{\lambda}$ is sufficiently low and condition (14) does not hold, $\tilde{r}_{M,CB}$ will not exist . In this case, if a bank deviates to borrowing central bank loans, it will not be able to borrow any loans on the market without a signal. This will greatly reduce banks' incentive to deviate, because when $\tilde{r}_{M,CB}$ exists, the bank without a signal can at least borrow at $\tilde{r}_{M,CB}$ and does not need to liquidate any assets. It can be shown that the no-deviation condition will be satisfied more easily than in the case where $\tilde{r}_{M,CB}$ exists. We omit the detailed derivations because the intuition for this result is straightforward.

3.3 PBB: both types of banks borrow central bank loans

In this equilibrium, both types of banks first borrow a central bank loan of L_{CB}, then try to borrow the remaining funds on the market. We first find the payoffs for both types of banks under the equilibrium and deviating strategies. Let $r_{M,CB}$ denote the market rate for banks without a signal to borrow on the market under the equilibrium strategy.

It is decided in the same way as $\tilde{r}_{M,CB}$, with $\tilde{\lambda}$ in $\tilde{r}_{M,CB}$ replaced with λ. Thus we find that $r_{M,CB} = r_M$, where r_M is the equilibrium rate in PNB. This is mainly because both interest rates are based on creditors' prior belief λ. Again we focus on the case where

$$1 + \left(1 - \frac{AR_L}{D_0}\right)\left(\frac{1}{\lambda + (1-\lambda)p} - 1\right) \leq \min\left\{\frac{AR_H - L_{CB}(1 + r_{CB})}{D_0 - L_{CB}}, \frac{R_H}{\gamma_H}\right\} \tag{17}$$

such that an equilibrium market rate exists.

Similarly, let $\hat{\lambda}$ denote creditors' belief off the equilibrium path when they observe a bank not borrow central bank loans.[10] Let $\hat{r}_M$ denote the market rate when a bank deviates to the strategy of not borrowing central bank loans. Then we find that $\hat{r}_M$ takes the same form as r_M, with λ replaced by $\hat{\lambda}$. We impose the additional assumption that $\hat{\lambda} \geq \lambda$, which means that if creditors observed a bank not borrow central bank loans, their belief of the bank being H-type would be at least as high as their prior belief. We impose this assumption because an H-type bank benefits relatively less from borrowing central bank loans than an L-type bank. The derivations of the banks' payoffs under the equilibrium and deviating strategies are provided in the Appendix.

The no-deviation conditions are $Ee_{H,CB} \geq E\hat{e}_{H,NCB}$ for H-type banks and $Ee_{L,CB} \geq E\hat{e}_{L,NCB}$ for L-type banks, where $Ee_{H,CB}(Ee_{L,CB})$ denotes the expected equity of an H-type (L-type) bank under the equilibrium strategy, and $E\hat{e}_{H,NCB}(E\hat{e}_{L,NCB})$ denotes the expected equity of an H-type (L-type) bank under the deviating strategy. Similar to PNB, we find that an L-type bank benefits more from the strategy of borrowing central bank loans. As a result, in PBB, L-type banks will never deviate to the strategy of not borrowing central bank loans as long as H-type banks do not deviate. Therefore, for the equilibrium to exist, the essential no-deviation condition is the one for H-type banks. Proposition 5 summarizes the results.

Proposition 5. *(1) In PBB, L-type banks will never deviate as long as H-type banks do not deviate. Thus, for this equilibrium to exist, the no-deviation condition for H-type banks, $Ee_{H,CB} \geq E\hat{e}_{H,NCB}$, which is given as follows,*

$$(1-q)D_0[\hat{r}_M - r_{M,CB}] + L_{CB}[(1-q)r_{M,CB} - r_{CB}] \geq 0 \tag{18}$$

must be satisfied.

(2) PBB is more likely to exist with a higher L_{CB} and a lower r_{CB}, q and $\hat{\lambda}$.

[10] Here we use ˆ denote the variables under the deviating strategy.

Proof: See the Appendix. ■

The above analysis is for the case where an equilibrium rate off the equilibrium path, $\hat{r}_M$, exists. If $\hat{\lambda}$ is sufficiently low, an equilibrium rate may not exist and the market freezes when a bank without a signal deviates to the strategy of not borrowing central bank loans. In this case banks will be less likely to deviate than in the case without a market freeze, because banks without a signal will not be able to borrow any loans on the market. The detailed analysis about this case is omitted.

3.4 Possible multiple equilibria

Depending on parameter values, the no-deviation conditions for more than one equilibrium can be satisfied simultaneously. For example, the conditions for the two pooling equilibria to exist can be satisfied simultaneously. Thus these two equilibria can co-exist. Note that although the no-deviation conditions for the two equilibria seem to be opposite to each other, they depend on two exogenously given beliefs off the equilibrium path, $\tilde{\lambda}$ and $\hat{\lambda}$. Different values for these two parameters make it possible for the two equilibria to co-exist.

In addition, the separating equilibrium and PBB may co-exist. Note that these two equilibria differ in that H-type banks choose different strategies. In the separating equilibrium, H-type banks do not borrow central bank loans, because they can get the best rate when borrowing on the market. In PBB, H-type banks instead borrow central bank loans. They may not want to deviate because deviating to the strategy of not borrowing central bank loans will no longer give them a zero rate for sure. With probability $1-q$, they will be charged the market rate $\hat{r}_M$ which is determined by creditors' belief off the equilibrium path, $\hat{\lambda}$. If $\hat{\lambda}$ is low enough, H-type banks will not deviate. As a result, these two equilibria do not exclude each other.

However, the no-deviation conditions for the separating equilibrium and PNB cannot be satisfied simultaneously, and these two equilibria cannot co-exist.

Proposition 6. *If the no-deviation condition for the separating equilibrium is satisfied, then PNB cannot exist.*

Proof: See the Appendix. ■

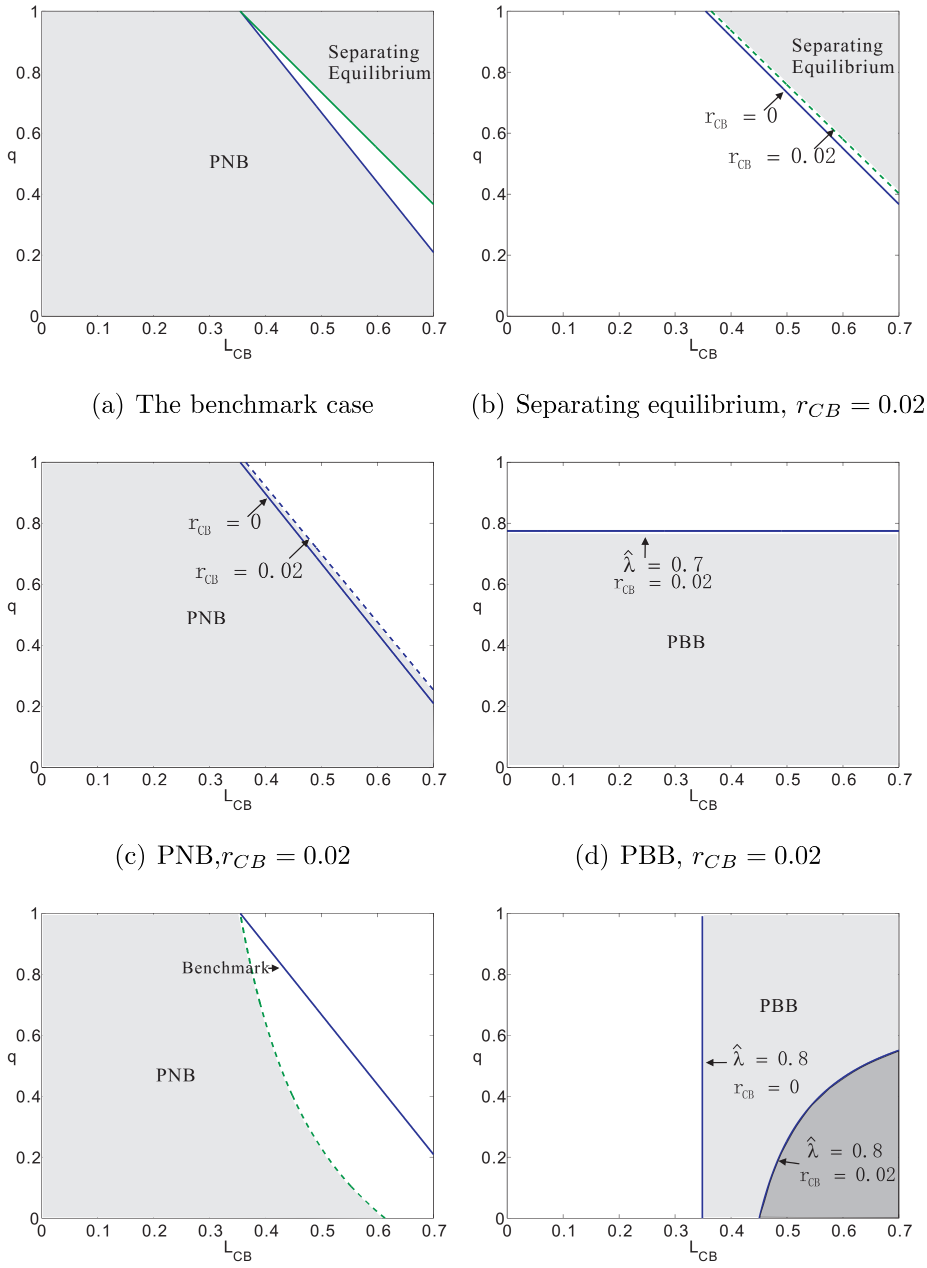
The intuition is as follows. When the no-deviation condition for the separating equilibrium is satisfied, an L-type bank's expected payoff difference from borrowing and not borrowing central bank loans must equal or exceed zero. It implies that the same payoff difference for an L-type bank in PNB must exceed zero. As a result, an L-type bank will always deviate to borrowing central bank loans. Too see this, note that in PNB, an L-type bank's expected payoff from borrowing central bank loans is higher than in the separating equilibrium, because in the pooling equilibrium, an L-type bank may be able to borrow more funds on the market after borrowing central bank loans, while in the separating equilibrium, it cannot borrow any funds on the market due to a perfect revelation of its type. On the other hand, an L-type bank's expected payoff from not borrowing central bank loans is lower than in the separating equilibrium, because in the pooling equilibrium, the market rate is $r_M > 0$, while in the separating equilibrium, this rate is zero.

3.5 Numerical Examples

In this section, we use numerical examples to illustrate the qualitative results in our model, such as how the amount of central bank loans, L_{CB}, central bank's lending rate, r_{CB}, and creditors' beliefs off the equilibrium path, $\tilde{\lambda}$ and $\hat{\lambda}$, will affect the existence of different types of equilibria. Here we do not intend to calibrate the data in reality to provide any quantitative results.

We first specify the parameter values in the benchmark case as follows. We assume that $R_H = 1.3$, $R_L = 0.6$, $p = 0.3$, $\gamma_H = 0.8$, $\gamma_L = 0.75$, and $\lambda = 0.7$. The asset level is normalized to 1, that is, $A = 1$. In addition, $D_0 = 0.9$, so that the initial equity level is $e_0 = 0.1$. As a result, $e_0/A = 0.1$, which is slightly higher than the capital adequacy ratio of 8% required by the Basel Accord. The central bank lending rate is set at zero, that is, $r_{CB} = 0$. These parameter values satisfy the assumptions given in the environment of the model. In addition, in PNB, we assume that $\tilde{\lambda} = 0$, which makes the existence of the equilibrium most likely. For PBB, recall that we assume that $\hat{\lambda} \geq \lambda$. So we assume that $\hat{\lambda} = \lambda = 0.7$ in the benchmark case, which makes the existence of this equilibrium most likely. Given the above parameter values, the equilibrium market rate, $r_M = r_{M,CB} = 0.0886$.

Figure 2 shows the results from the benchmark case and comparative statics exercises on various parameters.



(a) The benchmark case

(b) Separating equilibrium, $r_{CB} = 0.02$

(c) PNB, $r_{CB} = 0.02$

(d) PBB, $r_{CB} = 0.02$

(e) PNB, $\tilde{\lambda} = 0.35$

(f) PBB, $\hat{\lambda} = 0.8$.

Figure 2: Examples of three equilibria.

Figure 2(a) shows the results of the benchmark case. The separating equilibrium exists for high values of L_{CB} and q. As explained previously, this is because high values of L_{CB} and q induce a stronger incentive for L-type banks to borrow central bank loans. A higher L_{CB} makes borrowing central bank loans more attractive. A higher q makes deviating to not borrowing central bank loans less attractive for an L-type bank, because its type is more likely to be revealed to creditors.

For similar reasons, PNB exists for low values of q or L_{CB}. Note that the upper bound for PNB is lower than the lower bound for the separating equilibrium. This is because banks that do not borrow central bank loans are believed to be H-type with probability $\lambda < 1$ in PNB, but with probability one in the separating equilibrium. Therefore, in PNB, banks have a stronger incentive to deviate from the strategy of not borrowing central bank loans.

It turns out that given parameter values in the benchmark case, PBB can exist for all the values of q and L_{CB}.[11] The reason is that, given $r_{CB} = 0$ and $\hat{\lambda} = \lambda$, H-type banks are strictly better off from borrowing central bank loans than from not borrowing. Thus H-type banks will never deviate.[12] In addition, as explained previously, L-type banks have an even weaker incentive to deviate and will never deviate either.

As we can see, for certain values of L_{CB} and q, no-deviation conditions for PBB and the separating equilibrium (or PNB) can be satisfied simultaneously. But PNB and the separating equilibrium cannot co-exist.

The remaining panels of Figure 2 show the results of comparative statics on the central bank loan rate, r_{CB}, and creditors beliefs off the equilibrium path, $\tilde{\lambda}$ and $\hat{\lambda}$.

Figure 2(b) shows how a higher r_{CB} affects the separating equilibrium. When r_{CB} is increased from 0 to 0.02, borrowing central bank loans becomes less attractive. Thus L-type banks have a stronger incentive to deviate to not borrowing central bank loans.

[11] We hence do not indicate the region for this equilibrium to exist in the figure.

[12] When an H-type bank borrows central bank loans, it first borrows L_{CB} at a zero rate. Then if there is a signal that reveals its type, it will borrow the remaining loans on the market at a zero interest rate. If there is no signal, it will borrow the remaining loans on the market at $r_{M,CB}$. In the case where the bank borrows only from the market, if there is a signal, it will borrow all the loans on the market at a zero rate, and its payoff is the same as when borrowing central bank loans. If there is no signal, given that $\hat{\lambda} = \lambda$, it will borrow all the loans at $\hat{r}_M = r_{M,CB}$, and its payoff is strictly lower than that from borrowing central bank loans.

As a result, the lower bound for the existence of the separating equilibrium shifts up.

Figure 2(c) shows how a higher r_{CB} affects PNB. A higher r_{CB} reduces banks' payoff from borrowing central bank loans, inducing them a weaker incentive to deviate to borrowing central bank loans. Therefore, the region for PNB to exist expands.

Figure 2(d) shows how PBB is affected when we increase r_{CB}. When r_{CB} is higher, borrowing central bank loans becomes less attractive. Given that $\hat{\lambda} = 0.7$ is unchanged, with r_{CB} raised to 0.02, H-type banks will deviate to the no-central-bank-loans strategy when q is high. This is because with a high q, H-type banks are more likely to borrow at a zero rate on the market when they deviate.

Figure 2(e) shows how a higher $\tilde{\lambda}$ affects PNB. A higher $\tilde{\lambda}$ will lower the interest rate that banks are charged on the market after borrowing central bank loans, inducing a stronger incentive for L-type banks to deviate to borrowing central bank loans. As a result, the region for PNB to exist shrinks.

Figure 2(f) shows how PBB is affected when creditors' belief off the equilibrium path, $\hat{\lambda}$, is increased to 0.8 in the benchmark case. A higher $\hat{\lambda}$ will lower the market rate at which banks borrow on the market when banks deviate to not borrowing central bank loans, and makes the deviating strategy more attractive. Thus, H-type banks will now deviate when L_{CB} is low enough, rather than never deviate for all the values of L_{CB} as in the benchmark case. The figure also shows that when both $\hat{\lambda}$ and r_{CB} are raised, the region for PBB to exist will shrink further. The equilibrium will exist only in the region with rather high values of L_{CB} and low values of q. This is because a higher r_{CB} makes the equilibrium strategy of borrowing central bank loans less attractive, while a higher $\hat{\lambda}$ makes the deviating strategy more attractive.

In sum, the above numerical examples reveal that a higher L_{CB} and lower r_{CB} will make the strategy of borrowing central bank loans more attractive for both types of banks. A higher q will induce L-type banks a stronger incentive to borrow central bank loans, but will induce a weaker incentive for H-type banks to do so, because they will have a higher payoff from the strategy of not borrowing central bank loans, due to a larger chance for them to get loans on the market at a zero rate. In addition, a higher $\tilde{\lambda}$, creditors' belief off the equilibrium path in PNB, will make the deviating strategy of borrowing central bank loans more attractive for both types of banks, while a higher $\hat{\lambda}$, creditors' belief off the equilibrium path in PBB, will make the deviating strategy of not borrowing central bank loans more attractive for both types of banks.

4 Central bank policies to induce borrowing

4.1 Why the central bank may prefer banks to borrow

In this section, we will analyze in more detail central bank policies to induce banks to borrow central bank loans. But first, we need to explain why the central bank may prefer banks, including L-type banks, to borrow central bank loans. To understand it, first note that in our model PBB is socially optimal. This is because in our model, by assumption, asset liquidation, including L-type banks' asset liquidation, is socially costly. More specifically, for an H-type bank, one unit of assets will produce R_H unit of goods on date 2. If the asset is liquidated on date 1, it will become γ_H unit of goods. Because $\gamma_H < R_H$, the social cost of one unit asset liquidation is $R_H - \gamma_H$. Similarly, for an L-type bank, on unit of asset will on average produce $pR_H + (1-p)R_L$ unit of goods on date 2. If liquidated, the asset will produce γ_L unit of goods on date 1. We have assumed that $\gamma_L < pR_H + (1-p)R_L$. As a result, liquidation for L-type banks is also socially costly. Thus if we ignore the distributional effect and measure social welfare by the aggregate output level in the economy, the equilibrium that produces the least asset liquidation will be socially optimal. It is straightforward to see that PBB leads to the least asset liquidation and is socially optimal. We must point out that our model ignores moral hazard caused by the LOLR policy because in our model banks cannot choose the type of their assets ex ante.

In essence, our assumption that liquidation of L-type banks is socially costly is a short-cut way to model a situation where the social cost associated with weak banks' bankruptcy is so huge in the time of crisis that the central bank cannot afford not offering emergency liquidity assistance to them, despite its concern for moral hazard caused by such a policy. This social cost could be collapse of a whole financial system caused by the contagion effect, which has been studied by a large body of literature.[13] In the recent subprime mortgage crisis, such a situation occurred in the U.S., which explained why

[13]The related works include Allen and Gale (2000), Chen (1999), Kiyotaki, and Moore (1997, 2002), Li, Milne, and Qiu (forthcoming), and Pritsker (2013) among many others.

the U.S. government took various measures to induce all the banks to accept financial assistance from the government. Our model aims to examine what types of policies will better serve this purpose. However, it is possible that the central bank places moral hazard as its priority and intends to lend only to healthy banks, which tends to happen when the crisis is less severe and the failure of distressed financial institutions will not cause a systematic collapse of the financial system. Such a situation happened in the U.K. in the recent subprime crisis, which is not considered in our model.

Note that the central bank plays a role that cannot be replaced by creditors on the market in lowering banks's asset liquidation. The reason is as follows. After the negative shock, an L-type bank can no longer fully repay its original creditors and at the same time pay new creditors an expected return rate of the riskless rate. However, an individual original creditor still requires the bank to fully repay his loans by using newly borrowed loans or proceeds from liquidating assets, even when doing so will lead to costly liquidation of the bank, and reducing the bank's ability to repay new creditors. Facing an expected return rate below the riskless rate, new creditors will refuse to lend, which will in turn reduce the actual payment that original creditors will receive. Therefore, there is coordination failure among creditors in our model. Original creditors lending on date 0 have no incentive to internalize banks' liquidation cost and the payoff of new creditors lending to banks on date 1. As a result, an L-type bank may fail to get loans and be forced to liquidate its asset even when it is social welfare improving for its project to continue until date 2.

4.2 Interest rate policy and its signaling effect

In this section, we explore in more detail the central bank's interest rate policy. We will focus on three aspects: (1) the relationship between the central bank lending rate and the prevailing market rate; (2) the relationship between the central bank's net revenue and the interest rate it charges; (3) the signaling effect of the central bank's interest rate policy.

First, let us examine the relationship between the central bank lending rate and the market rate. From our previous analysis, we know that if the central bank wants both types of banks to borrow from it, then its lending rate must be low enough to attract

H-type banks. Now we will prove that the central bank lending rate must be lower than the market rate to induce PBB.

Recall that in PBB, for banks with no signal, creditors on the market will offer the rate of $r_{M,CB}$ based on their prior belief, λ. Let us call this rate as a break-even rate, since it gives creditors an expected riskless rate of zero. This break-even rate is shared also by the central bank, because the central bank shares the same belief as creditors. However, in order to induce PBB, the central bank must offer a rate lower than this break-even rate. Proposition 7 summarizes the result.

Proposition 7. *Given the assumption that $\hat{\lambda} \geq \lambda$, in order to attain PBB, r_{CB} must be lower than the break-even rate (or the market rate) based on creditors' prior belief, λ.*

Proof: See the Appendix. ■

The key intuition here is that, because an H-type bank has a stronger incentive to deviate from the strategy of borrowing central bank loans, the interest rate policy must be attractive enough not only to both types of banks on average, but also to H-type banks. Because the break-even rate is determined based on the expected returns from both types of banks, it takes into account the relatively lower expected returns from L-type banks. H-type banks, on the other hand, will be able to borrow on the market at a rate lower than the break-even rate when its type is revealed. As a result, the central bank lending rate must be lower than the break-even rate to induce H-type banks to borrow.

In reality, central banks sometimes indeed offer loans at a rate lower than the prevailing market rate. The idea that the central bank provides cheap loans to private banks contradicts the famous Bagehot's rule that argues that central bank loans should be lent at a rate higher than the prevailing market rate. Our model justifies such a policy in the case where the central bank intends to induce all the types of banks to participate in an emergency lending project.

The above result has further implications on the central bank's net revenue, which is defined as the central bank's lending revenue deducted by the amount of central bank loans. In particular, we find that the central bank's net revenue is non-monotonic in the interest rate it charges. It is possible that when the central bank charges a lower rate to induce all the banks to borrow, its net revenue is actually higher (or its loss is lower). To see this, note that the central bank faces a typical adverse selection problem studied in

Stiglitz and Weiss (1981). When the rate is low, both types of banks will borrow, and the central bank's net revenue is increasing in the rate. However, when the rate exceeds a certain threshold level, H-type banks will stop lending, leading to a lower net revenue for the central bank. Below, we use numerical examples to illustrate this result.[14]

Figure 3 shows how the central bank's net revenue changes in r_{CB} when $L_{CB} = 0.6$, $q = 0.6$, and $\hat{\lambda} = 0.8$ ($\hat{r}_M = 0.0543$). The remaining parameter values are the same as in the benchmark case. The market rate after banks borrow central bank loans is still $r_{M,CB} = 0.0886$ as in the previous numerical examples. It turns out that both PBB and the separating equilibrium exist when $r_{CB} \leq 0.0148$. Assume that the central bank can always coordinate all the banks toward PBB as long as it exists. Then we find that the central bank's net revenue is increasing in r_{CB} when $r_{CB} \leq 0.0148$: Given a constant loan level of L_{CB}, a higher r_{CB} will yield a higher net revenue, because when R_H is realized, the central bank is fully repaid and receives a higher interest income with a higher R_{CB}. Note that because r_{CB} is below the break-even rate, the central bank's net revenue is negative.[15]

When $r_{CB} > 0.0148$, H-type banks will deviate to not borrowing central bank loans. As a result, PBB cannot exist and only the separating equilibrium exists, in which only L-type banks borrow central bank loans. In this case, because H-type banks stop borrowing, there is a downward jump in the central bank's net revenue level at $r_{CB} = 0.0148$. Note that this downward jump is caused by two reasons: (1) H-type banks stop borrowing, who would have bring a relatively higher net revenue for the central bank if they borrowed; (2) The equilibrium switches from PBB to the separating one. In the separating equilibrium, L-type banks have their type fully revealed to creditors and face a market freeze. They are hence forced to liquidate their assets and their repayment to the central bank is lower than in the PBB case without a market freeze. Thus we find that the downward jump is caused by two effects: an adverse selection effect and a signaling effect caused by a high central bank lending rate, which leads to the perfect revelation of the type of banks who borrow central bank loans.

[14] An analytical derivation of the central bank's net revenue is provided in the Appendix.

[15] The central bank's net revenue can better be thought of as a relative one to the fair profit earned by the market. A negative central bank net revenue is due to the assumption that the riskless rate equals zero. As a result, a break-even rate will yield a zero profit. If we set the riskless rate at a higher level so that a break-even rate earns a positive profit, then the central bank's net revenue could become positive.

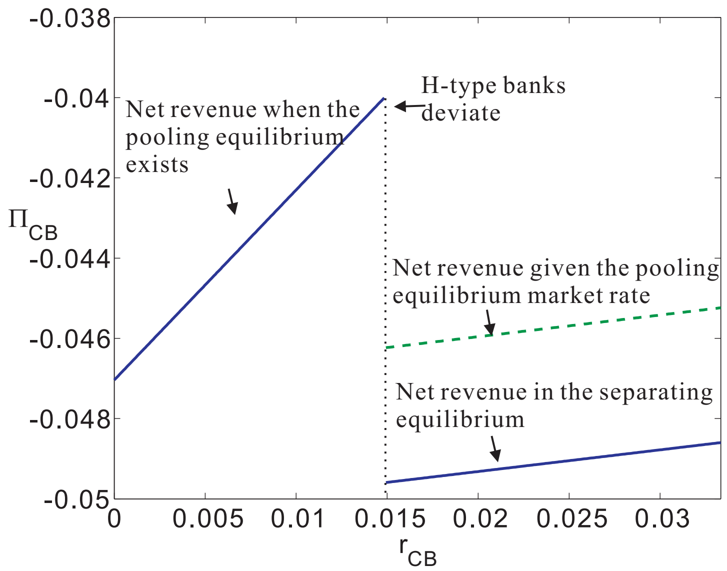


Figure 3: The central bank's net revenue. $L_{CB} = 0.6$, $q = 0.6$, $\hat{\lambda} = 0.8$.

When r_{CB} increases further from 0.0148, our numerical example shows that the separating equilibrium continues existing when $r_{CB} \leq 0.0333$. Thus when r_{CB} falls in this range, the central bank's net revenue starts to increase again, because the central bank can now receive a higher repayment from L-type banks when the up state is realized. In order to separate the adverse selection effect from the signaling effect, we also calculate the central bank's net revenue if the creditors continue charging a rate based on their prior belief, λ, which is given by the dash line.

The above analysis highlights the importance of the signaling effect of the interest rate policy. In our model, the central bank lending rate does not only affects banks' payoffs directly through changing banks' central bank loan costs, but also affects creditors' beliefs about the types of banks who borrow central bank loans and subsequently their borrowing costs on the market. Creditors can infer from the central bank lending rate the borrowing banks' type. If the central bank's lending term is tough, then banks that borrow central bank loans may also face a tough term on the market, because creditors on the market will believe that banks borrowing central bank loans despite the tough term are likely to be L-type. Thus the central bank should take this effect into account when implementing the LOLR policy.

In the above example, at the level r_{CB} where H-type banks deviate, the separating equilibrium exists so that L-type banks will continue borrowing central bank loans. With alternative parameter values, it is also possible that PNB exists instead in which L-type banks will also switch to not borrowing central bank loans. For example, suppose that L_{CB} decreases to 0.4, with all the other parameter values unchanged. Then H-type banks will switch at $r_{CB} = 0.0045$. In this case, provided that $\tilde{\lambda}$ is low enough, PNB rather than the separating equilibrium exists. Thus in equilibrium both types of banks will not borrow central bank loans. That is, when H-type banks deviate, L-type banks may choose to mimic H-type banks to hide its type.

4.3 Identity hiding policy

In the above analysis, we assume that the banks' action of borrowing central bank loans can be observed by creditors on the market. In this section, we examine the case where the central bank can hide the identities of the borrowers. For example, the central bank can commit to not revealing to the public the identities of the banks borrowing central bank loans. In this case, creditors can no longer observe whether a bank has borrowed from the central bank or not. Intuitively, banks will be more likely to borrow, because borrowing central bank loans will no longer reveal negative information about banks' quality to creditors. We arrive at the following results.

Proposition 8. *With the identity hiding policy, there may exist two pooling equilibria, PNB and PBB.*

(1) In both equilibria, all the banks without a signal revealing their type will be charged the same market rate, r_m, no matter they have borrowed from the central bank or not. Here $r_m = r_M = r_{M,CB}$, where r_M and $r_{M,CB}$ are derived previously in the no identity hiding policy case. In addition, PNB exists when

$$p \max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right) R_H - L_{CB}(1 + r_{CB})\right\} + (1-q)pL_{CB}(r_m - r_{CB}) \leq 0 \quad (19)$$

PBB exists when $r_{CB} < (1-q)r_m$.[16]

[16]Here we assume that when banks are indifferent between borrowing or not borrowing central bank loans, they choose not to borrow.

(2) *Compared to the no identity hiding policy case, the identity hiding policy makes PBB where banks choose to borrow central bank loans more likely, provided that $\hat{\lambda} > \lambda$. It also makes PNB where banks do not borrow central bank loans less likely, provided that $\tilde{\lambda} < \lambda$.*

Proof: See the Appendix. ■

The main intuition of Proposition 8 is as follows. With the new LOLR policy, creditors on the market will no longer observe whether a bank has borrowed from the central bank or not. Therefore, they will not be able to charge a different rate on banks that have and have not borrowed from the central bank. Because L_{CB} is not large enough to meet the banks' entire liquidity needs, all the banks will need to borrow loans on the market no matter they have borrowed from the central bank or not. As a result, for any bank that borrows on the market, creditors' belief about it being H-type is simply the prior belief, λ, and the market rate, r_m, will be the same as r_M and $r_{M,CB}$, which are derived before in PNB and PBB, respectively. Because L-type banks have a stronger incentive to borrow central bank loans than H-type banks, the no-deviation condition for PNB is given by L-type banks' no-deviation condition, which is Condition (19). While the no-deviation condition for PBB is given by H-type banks' no-deviation condition, $r_{CB} < (1-q)r_m$.

Result (2) means that banks have a stronger incentive to borrow central bank loans in both PNB and PBB with the identity hiding policy than without the identity hiding policy. Too see this, note that PNB and PBB with the identity hiding policy can be thought as a special case of PNB and PBB without the identity hiding policy, where the beliefs off the equilibrium path, $\hat{\lambda}$ and $\tilde{\lambda}$, are assumed to equal the prior belief, λ. It means that creditors on the market do not infer the type of a bank from its action of borrowing central bank loans. In PBB, if $\hat{\lambda} > \lambda$, then creditors will raise their belief about the bank being H-type over their prior belief when they observe a bank not to borrow central bank loans, which will induce banks a stronger incentive not to borrow central bank loans. As a result, the identity hiding policy makes banks more likely to borrow central bank loans. Similarly, in PNB, if $\tilde{\lambda} < \lambda$, then creditors will lower their belief about a bank being H-type over their prior belief when they observe it borrow central bank loans. The identity hiding policy will then again make banks more likely to borrow central bank loans. As shown before, L-type banks benefit more from borrowing central bank loans than H-type banks. As a result, it is reasonable to believe that creditors' beliefs off the equilibrium path in the real world satisfy $\hat{\lambda} > \lambda$ and $\tilde{\lambda} < \lambda$, and, consequently, the identity hiding policy will make banks more likely to borrow central bank loans.

Proposition 9. *With the identity hiding policy, there may exist a separating equilibrium where only L-type banks borrow from the central bank. In this equilibrium, all the banks without a signal revealing their type will be charged the same market rate, r_m^s, which is given by*

$$r_m^s = \left(1 - \frac{AR_L}{D_0}\right)\left(\frac{1}{\lambda^s + (1-\lambda^s)p} - 1\right) \tag{20}$$

where $\lambda^s = \frac{\lambda D_0}{\lambda D_0 + (1-\lambda)(D_0 - L_{CB})}$. In addition, $r_m^s < r_m$. The equilibrium exists when

$$p \max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right) R_H - L_{CB}(1 + r_{CB})\right\} + (1-q)pL_{CB}(r_m^s - r_{CB}) > 0 \tag{21}$$

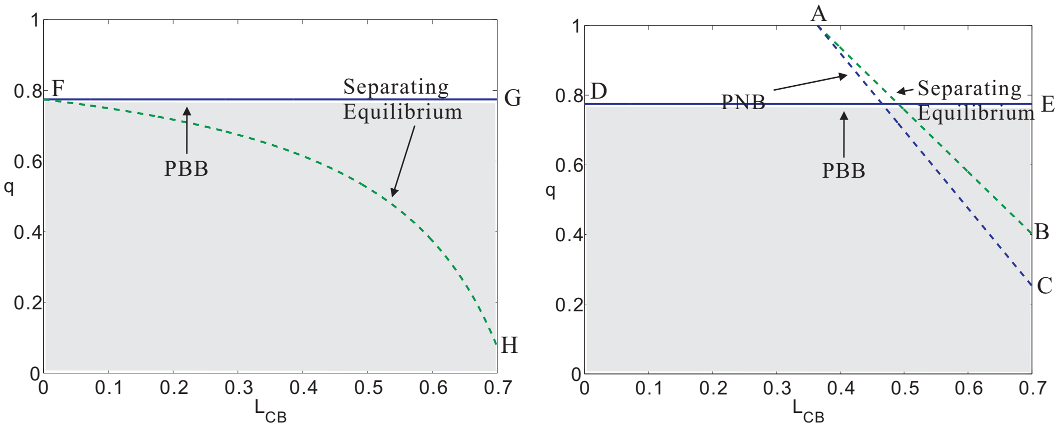
and $r_{CB} \geq (1-q)r_m^s$.

Proof: See the Appendix. ■

The intuition of Proposition 9 is as follows. In the separate equilibrium, with the identity hiding policy, creditors cannot identify banks' types by observing whether or not they borrow from the central bank. However, creditors know the proportion of each type of banks. In addition, creditors know that H-type banks will borrow D_0 on the market, while L-type banks will borrow only $D_0 - L_{CB}$ in this equilibrium. Since we assume that each creditor is atomically small, his probability to be approached by an H-type bank is given by $\lambda^s = \frac{\lambda D_0}{\lambda D_0 + (1-\lambda)(D_0 - L_{CB})}$. It is straightforward to see that $\lambda^s > \lambda$, that is, creditors have a more optimistic belief than their prior belief, because in equilibrium H-type banks will borrow more than L-type banks. As a result, $r_m^s < r_m$, because r_m is calculated based on the prior belief.

Figure 4 compares the results with and without identity hiding, using a numerical example with $r_{CB} = 0.02$. The values for the remaining parameters are the same as in the benchmark case. Note that in this example, $r_{CB} < r_m = r_M = 0.0886$. Figure 4(a) gives the results with identity hiding, while Figure 4(b) gives the result without identity hiding, with $\lambda = \tilde{\lambda} = \hat{\lambda} = 0.7$.

First, note that in Figure 4(b), PNB exists below the line AC, while in Figure 4(a), PNB does not exist. This is because in this example, $r_{CB} < r_m$. Given this condition, the



(a) The identity hiding case (b) The no identity hiding case

Figure 4: Equilibrium outcomes when $r_{CB=0.02}$.

no-deviation condition for PNB (Condition (19)) cannot be satisfied. This is consistent with the result that in the identity hiding case, PNB is less likely to exist. Second, in Figure 4(a), PBB exists below the line FG, while in Figure 4(b), PBB exists below the line DE. These two lines are identical, because in the identity hiding case, the market rate is determined by creditors' prior belief, λ, while in the no identity hiding case, the market rate for banks not borrowing central bank loans is determined by $\hat{\lambda}$, which is assumed to be λ in this particular example. Since the market rates in these two cases are the same, the no-deviation conditions that determine the lines FG and DE are also the same. If we use a more reasonable assumption that $\hat{\lambda} > \lambda$, then the region for PBB in Figure 4(b) will shrink, because banks will be more likely to deviate to not borrowing central bank loans. This is consistent with the result that given $\hat{\lambda} > \lambda$, PBB is more likely to exist with identity hiding. Finally, in Figure 4(b), the separating equilibrium exists above the line AB, while in Figure 4(a), the separating equilibrium exists above the line FH. Thus with identity hiding, as L_{CB} increases, the separating equilibrium can exist with a lower q. As we explained before, this is because a higher L_{CB} means that L-type banks will borrow less loans on the market. As a result, the probability for creditors to lend to an H-type bank is higher, resulting in a lower market rate. Hence, H-type banks will have a stronger incentive to borrow from the market and will choose the strategy of not borrowing central bank loans starting from a lower q. Note that in some regions where only PNB can exist in the no identity hiding case (such as the regions left to AC

and above DE in Figure 4(b)), the separating equilibrium can also exist with identity hiding. This is because identity hiding gives L-type banks a stronger incentive to borrow from the central bank in those regions.

Note that the identity hiding policy has its limits. For example, when the U.S. Treasury and the Federal Reserve provided loans to the major U.S. commercial and investment banks after the failure of Lehman Brothers, then politically, it is difficult to hide the borrowers' identity and the size of the loans.

4.4 Other policies

In our model, the central bank aims to induce PBB where both types of banks borrow from the central bank. Generally speaking, any policies that make PBB more likely to exist will be favored. Our previous analysis has revealed that a higher L_{CB} and low r_{CB} will serve this purpose. Meanwhile, we find that a higher q makes this equilibrium less likely to exist. Thus our model presents an interesting result: in a severe financial crisis when the government cannot afford failing distressed financial institutions due to the fear of a systematic collapse of the financial system, more ambiguity in the market about banks' quality can actually facilitate the implementation of the LOLR policy.

On the other hand, any policies that make other equilibria less likely to exist will also help the central bank achieve its goal, since it will reduce the strategic uncertainty about which equilibrium will be realized by reducing the chance for PBB to co-exist with other equilibria. A higher L_{CB} and low r_{CB} will make PNB less likely, but will make the separating equilibrium more likely. Similarly, a higher q will make PNB less likely, but will make the separating equilibrium more likely.

In our model, creditors' beliefs off the equilibrium path in PNB and PBB, $\tilde{\lambda}$ and $\hat{\lambda}$, are exogenously given. Suppose that the government could use certain means such as media to affect the public opinion. Then we find that a higher $\tilde{\lambda}$ or lower $\hat{\lambda}$ will help the government achieve its goal. That is, when creditors observe a bank borrow from the central bank while others do not, they believe that this bank is not necessarily an L-type. As a result, PNB is less likely to exist. On the other hand, when creditors observe a bank not borrow from the central bank while others do, they believe that this bank is not necessarily an H-type. As a result, PBB is more likely to exist.

The central bank can also serve as a coordination tool to induce the realization of PBB. Our model reveals that even when PBB exists, it can co-exist with either PNB or the separating equilibrium. There is still strategic uncertainty about which equilibrium will be realized. Thus the central bank can play a role in coordinating all the banks toward PBB, such as ordering all the banks to borrow central bank loans, or bringing all the banks together to facilitate their cooperation.

5 Conclusions

This paper studies the optimal LOLR policy when banks' borrowing from the central bank is used as a signal about their quality. In particular, we focus on a situation where a central bank aims to induce central bank borrowing from all types of banks, strong and weak. This situation can occur in a financial crisis, when the whole financial system is in danger of a systematic breakdown due to contagion induced by the failure of weak banks. In such a situation, the central bank may decide to rescue weak banks to avoid a systematic breakdown, regardless of concerns for moral hazard. Thus our paper focuses on crisis management without considering the ex ante choices of banks induced by this LOLR policy. In a companion paper, Li, Milne, and Qiu (2015) study the optimal LOLR policy to prevent a crisis. In that paper, we focus on a situation where the central bank aims to lend only to strong banks to discourage risk-taking behavior of banks ex ante.

The model in this paper shows that there are three types of equilibria: a separating equilibrium where only low quality banks borrow from the central bank and two pooling equilibria where both types of banks borrow and do not borrow from the central bank. The separating equilibrium and the pooling equilibrium (where both types of banks borrow from the central bank) cannot co-exist. Nevertheless, multiple equilibria are possible. Since we focus on the case where the central bank aims to boost the liquidity of all the financial institutions, including the distressed ones, the pooling equilibrium (where both types of banks borrow from the central bank) is favored by the central bank. We find that given the signaling effect, the central bank should offer a lending rate lower than the prevailing market rate. Meanwhile, hiding the identities of the banks borrowing from the central bank will encourage banks to borrow from the central bank. Finally, when there are multiple equilibria, the central bank could serve as a coordinator, steering all the banks toward an equilibrium where all the banks borrow from the central bank.

Appendix

A Proofs

A.1 Proof of Proposition 1

Proof: In the down state, an L-type bank's maximum asset on date 2 will be AR_L. This is because by assumption, any long-term asset liquidation is costly, and the bank's maximum asset value is achieved at zero asset liquidation. On the other hand, an L-type bank's minimum liability on date 2 is D_0. Since by assumption, $AR_L < D_0$, an L-type bank's maximum net asset value must be negative in the down state. Hence its equity in the down state is always zero. Since banks maximize their expected equity, L-type banks will essentially aim to maximize their equity value in the up state. ■

A.2 Proof of Proposition 2

Proof: First, consider the case where an H-type bank does not borrow central bank loans. We have

$$NV_H = (A - l_H)R_H - (D_0 - \gamma_H l_H)(1 + r_{m,i}) \tag{22}$$

The bank will choose l_H to maximize its net asset value. It is easy to see that if $\gamma_H(1 + r_{m,i}) - R_H > 0$, NV_H is strictly increasing in l_H. Thus the bank will choose to liquidate all its asset to repay as many debts as possible, and will become bankrupt on date 1 (because $\gamma_H A < D_0$). If $\gamma_H(1 + r_{m,i}) - R_H < 0$, NV_H is strictly decreasing in l_H. The bank will choose to borrow on the market and will not liquidate any asset. If $\gamma_H(1+r_{m,i}) = R_H$,the bank is indifferent between liquidating assets and borrowing on the market. In this case, we assume the bank will choose to borrow on the market.

Similarly, when an H-type bank borrows central bank loans, we have

$$NV_H = (A - l_H)R_H - (D_0 - \gamma_H l_H - L_{CB})(1 + r_{m,i}) - L_{CB}(1 + r_{CB}) \tag{23}$$

Again, when $\gamma_H(1 + r_{m,i}) - R_H > 0$, NV_H is strictly increasing in l_H. The bank will liquidate assets until $D_0 - \gamma_H l_H - L_{CB} = 0$. That is, the bank will never borrow on the market. When $\gamma_H(1 + r_{m,i}) - R_H \le 0$, NV_H is strictly decreasing in l_H. The bank will choose to borrow on the market and will not liquidate any assets. Note that the bank will borrow positive loans on the market because $L_{CB} < D_0$, and borrowing central bank loans is not enough to repay all the debts.

The case for L-type banks can be proved in the same way, with γ_H being replaced by γ_L. ■

A.3 The sufficient conditions for creditors not to lend to L-type banks

We assume that creditors are unwilling to lend to an L-type bank when its type is perfectly revealed. We now examine the sufficient conditions for this assumption to hold. First, we have shown that when an L-type bank is willing to borrow on the market only when creditors charge a rate lower than $\frac{R_H}{\gamma_L}$, and in this case the bank will not liquidate any assets. Second, provided that an L-type bank is willing to borrow on the market, we first calculate the maximum possible rate it can repay creditors in the up state. If the bank does not borrow central bank loans, then the maximum rate is $\frac{AR_H}{D_0}$. If the bank borrows from the central bank at the lowest possible interest rate of $r_{CB} = 0$, it will repay the principal of L_{CB} to the central bank, and the maximum rate for private loans is $\frac{AR_H - L_{CB}}{D_0 - L_{CB}}$. Because $AR_H > D_0$, it is straightforward to see that $\frac{AR_H - L_{CB}}{D_0 - L_{CB}} > \frac{AR_H}{D_0}$. As a result, the maximum rate that an L-type bank can repay in the up state is $\frac{AR_H - L_{CB}}{D_0 - L_{CB}}$. Considering the maximum rate at which an L-type bank is willing to borrow, we find that the actual maximum rate at which creditors can receive from L-type banks in the up state is given by $R_{max,U} = \min\{\frac{R_H}{\gamma_L}, \frac{AR_H - L_{CB}}{D_0 - L_{CB}}\}$.

In the down state, an L-type bank's total asset is $AR_L < D_0$, implying that the bank cannot repay its total principal fully. In this case, the bank's assets are shared by creditors proportionally to their principals, and the recovery rate for creditors is $\frac{AR_L}{D_0}$.

We impose the condition that

$$pR_{max,U} + (1 - p)\frac{AR_L}{D_0} < 1 \tag{24}$$

When this condition holds, creditors will never lend to an L-type bank if its type is perfectly revealed.

A.4 Proof of Proposition 3

Proof: Using Eqs. (2) and (3), Eq. (5) becomes

$$AR_H + L_{CB}\left[\frac{R_H}{\gamma_L} - (1 + r_{CB})\right] - \frac{R_H}{\gamma_L}D_0 \geq (1 - q)(AR_H - D_0) \tag{25}$$

Result (1) can be derived from Eqs. (5) and (25). Result (2) can also be derived from these two equations. Recall that the bank will not borrow if the interest rate is higher than $\frac{R_H}{\gamma_L}$. So we can reasonably assume that $\frac{R_H}{\gamma_L} > (1 + r_{CB})$. Thus, the LHS of Eq. (25) is increasing in the level of L_{CB}, and decreasing in r_{CB}. The RHS is decreasing in q because $AR_H > D_0$. ■

A.5 Derivation for Condition 8

First, we argue that an equilibrium market rate must be lower than $\frac{R_H}{\gamma_H}$. This is because when the market rate exceeds $\frac{R_H}{\gamma_H}$, H-type banks will stop lending on the market as proved previously. As a result, creditors know for sure that any borrower on the market at the ongoing rate is L-type, and by assumption, creditors will never lend to an L-type bank when its type is perfectly revealed. Second, we argue that an equilibrium rate cannot exceed the maximum return rate an H-type bank or an L-type bank in the up state can afford, $\frac{AR_H}{D_0}$. This is because when the market rate exceeds this level, the actual return rate that creditors receive will be $\frac{AR_H}{D_0}$, implying that creditors' expected return rate will be below the riskless rate of zero. In this case creditors will refuse to lend. Thus we derive condition 8.

A.6 Deviations for $\tilde{r}_{M,CB}$ and banks' payoffs in PNB

We first derive $\tilde{r}_{M,CB}$, which should satisfy

$$1 \;=\; \tilde{\lambda}(1 + \tilde{r}_{M,CB}) + (1 - \tilde{\lambda})\left[p(1 + \tilde{r}_{M,CB}) + (1 - p)\frac{AR_L}{D_0}\right] \tag{26}$$

Note that in the down state, the recovery rate for private loans is $\frac{AR_L}{D_0}$ as explained previously. Hence, we get $\tilde{r}_{M,CB} = \left(1 - \frac{AR_L}{D_0}\right)\left(\frac{1}{\tilde{\lambda}+(1-\tilde{\lambda})p} - 1\right)$. Following our previous analysis about the existence of an equilibrium market rate, for this equilibrium market rate to exist, we must have

$$1+\left(1-\frac{AR_L}{D_0}\right)\left(\frac{1}{\tilde{\lambda}+(1-\tilde{\lambda})p}-1\right)\leq \min\left\{\frac{AR_H-L_{CB}(1+r_{CB})}{D_0-L_{CB}},\frac{R_H}{\gamma_H}\right\} \tag{27}$$

Here $\frac{AR_H-L_{CB}(1+r_{CB})}{D_0-L_{CB}}$ is the maximum return rate an H-type bank can afford after borrowing central bank loans. It is straightforward to see that the LHS is strictly decreasing in $\tilde{\lambda}$. Thus an equilibrium market rate exists when $\tilde{\lambda}$ exceeds a threshold level.

Banks' payoffs are derived as follows. For an H-type bank, after it borrows a central bank loan of L_{CB}, with probability q, its type is revealed and it can borrow the remaining funds of D_0-L_{CB} at a zero market rate. Its date 2 equity will be

$$\widetilde{e}_{H,CB,s_i=s_H} \;=\; AR_H-L_{CB}(1+r_{CB})-(D_0-L_{CB}) \tag{28}$$

With probability $1-q$, there is no signal revealing its type, and the bank borrows the remaining funds at the market rate of $\tilde{r}_{M,CB}$. Its equity will be

$$\widetilde{e}_{H,CB,s_i=\emptyset}=AR_H-L_{CB}(1+r_{CB})-(D_0-L_{CB})(1+\tilde{r}_{M,CB}) \tag{29}$$

Thus, its expected equity is

$$\begin{aligned} E\widetilde{e}_{H,CB} &= q\widetilde{e}_{H,CB,s_i=s_H}+(1-q)\widetilde{e}_{H,CB,s_i=\emptyset} \\ &= AR_H-L_{CB}(1+r_{CB})-[q(D_0-L_{CB})+(1-q)(D_0-L_{CB})(1+\tilde{r}_{M,CB})] \end{aligned} \tag{30}$$

For an L-type bank, after it borrows L_{CB} of central bank loans, with probability $1-q$ there is no signal that reveals its type. Thus it can borrow D_0-L_{CB} of funds on the market at the rate of $\tilde{r}_{M,CB}$. If the up state is realized, its equity will be same as that of an H-type bank, i.e., $\widetilde{e}^u_{L,CB,s_i=\emptyset}=\widetilde{e}_{H,CB,s_i=\emptyset}$. With probability q, its type is revealed and it cannot borrow any additional loans on the market. As a result, the bank has to liquidate assets to repay the debts of D_0-L_{CB}. The amount of assets to be liquidated is $l_L=\frac{D_0-L_{CB}}{\gamma_L}$. If $l_L\geq A$, then the bank's equity will be zero. If $l_L<A$, then on date 2 the bank's equity in the up state will be

$$\widetilde{e}^u_{L,CB,s_i=s_L}=\max\left\{0,(A-l_L)R_H-L_{CB}(1+r_{CB})\right\} \tag{31}$$

In other words, even if the up state is realized, the bank's equity will be positive only when its asset value exceeds its liabilities to the central bank. Thus, the bank's expected equity is

$$
\begin{aligned}
& E\tilde{e}_{L,CB} = qp\widetilde{e}^{u}_{L,CB,s_i=s_L} + (1-q)p\widetilde{e}^{u}_{L,CB,s_i=\emptyset} && (32)\\
= \; & qp\max\left\{0, \left[(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H - L_{CB}(1+r_{CB})\right]\right\} \\
& +(1-q)p\left[AR_H - L_{CB}(1+r_{CB}) - (D_0 - L_{CB})(1+\tilde{r}_{M,CB})\right] && (33)
\end{aligned}
$$

A.7 Proof of Proposition 4

For an H-type bank,

$$
\begin{aligned}
& Ee_{H,NCB} - E\widetilde{e}_{H,CB} \\
= \; & [AR_H - D_0\left[q + (1-q)(1+r_M)\right]] - \\
& [AR_H - L_{CB}(1+r_{CB}) - [q(D_0 - L_{CB}) + (1-q)(D_0 - L_{CB})(1+\tilde{r}_{M,CB})]] \\
= \; & (1-q)D_0[\tilde{r}_{M,CB} - r_M] + L_{CB}[r_{CB} - (1-q)\tilde{r}_{M,CB}] && (34)
\end{aligned}
$$

For an L-type bank,

$$
\begin{aligned}
& Ee_{L,NCB} - E\widetilde{e}_{L,CB} \\
= \; & (1-q)p[AR_H - D_0(1+r_M)] - qp\max\left\{0, \left[(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H - L_{CB}(1+r_{CB})\right]\right\} \\
& -(1-q)p\left[AR_H - L_{CB}(1+r_{CB}) - (D_0 - L_{CB})(1+\tilde{r}_{M,CB})\right] \\
= \; & (1-q)p\left[D_0(\tilde{r}_{M,CB} - r_M) + L_{CB}(r_{CB} - \tilde{r}_{M,CB})\right] - \\
& qp\max\left\{0, \left[(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H - L_{CB}(1+r_{CB})\right]\right\} && (35)
\end{aligned}
$$

Thus $Ee_{H,NCB} - E\widetilde{e}_{H,CB} - (Ee_{L,NCB} - E\widetilde{e}_{L,CB})$ equals

$$
\begin{aligned}
& (1-p)(1-q)\left[D_0(\tilde{r}_{M,CB} - r_M) + L_{CB}(r_{CB} - \tilde{r}_{M,CB})\right] + qL_{CB}\tilde{r}_{M,CB} + \\
& qp\max\left\{0, \left[(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H - L_{CB}(1+r_{CB})\right]\right\} && (36)
\end{aligned}
$$

Provided that $Ee_{L,NCB} - E\widetilde{e}_{L,CB} \geq 0$, $[D_0(\tilde{r}_{M,CB} - r_M) + L_{CB}(r_{CB} - \tilde{r}_{M,CB})] \geq 0$. Thus $Ee_{H,NCB} - E\widetilde{e}_{H,CB} - (Ee_{L,NCB} - E\widetilde{e}_{L,CB}) \geq 0$. In fact, it can be reasonably assumed that $\tilde{\lambda} < 1$ (creditors will not believe that any bank that borrows from the central bank must be H-type), then $\tilde{r}_{M,CB} > 0$ so that $Ee_{H,NCB} - E\widetilde{e}_{H,CB}$ is strictly greater than $Ee_{L,NCB} - E\widetilde{e}_{L,CB}$. Thus we prove Results (1) and (2).

In order to prove Result (3), we need to find the first order derivatives of $Ee_{L,NCB} - E\widetilde{e}_{L,CB}$ with respect to L_{CB}, r_{CB}, q and $\tilde{\lambda}$ at $Ee_{L,NCB} - E\widetilde{e}_{L,CB} = 0$. Let $\tilde{\Phi} = Ee_{L,NCB} - E\widetilde{e}_{L,CB}$ (Eq. (35)). We find that

$$\left.\frac{\partial\tilde{\Phi}}{\partial L_{CB}}\right|_{\tilde{\Phi}=0} = (1-q)p(r_{CB} - \tilde{r}_{M,CB}) - qp[\frac{R_H}{\gamma_L} - (1+r_{CB})] \tag{37}$$

when $\left[(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H - L_{CB}(1+r_{CB})\right] \geq 0$ and

$$\left.\frac{\partial\tilde{\Phi}}{\partial L_{CB}}\right|_{\tilde{\Phi}=0} = (1-q)p(r_{CB} - \tilde{r}_{M,CB}) \tag{38}$$

when $\left[(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H - L_{CB}(1+r_{CB})\right] < 0$. Because $\tilde{\lambda} \leq \lambda$ by assumption, $r_{CB} \leq \tilde{r}_{M,CB}$. Because we assume that the central bank lending rate is below the prevailing market rate and the market rate must not exceed $\frac{R_H}{\gamma_H} \leq \frac{R_H}{\gamma_L}$, $qp[\frac{R_H}{\gamma_L} - (1+r_{CB})] > 0$. Thus in both cases we find that $\left.\frac{\partial\tilde{\Phi}}{\partial L_{CB}}\right|_{\tilde{\Phi}=0} \leq 0$. That is, PNB is more likely to exist with a lower L_{CB}.

We find that

$$\left.\frac{\partial\tilde{\Phi}}{\partial r_{CB}}\right|_{\tilde{\Phi}=0} = (1-q)pL_{CB} + qpL_{CB} \tag{39}$$

when $\left[(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H - L_{CB}(1+r_{CB})\right] \geq 0$ and

$$\left.\frac{\partial\tilde{\Phi}}{\partial r_{CB}}\right|_{\tilde{\Phi}=0} = (1-q)pL_{CB} \tag{40}$$

when $\left[(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H - L_{CB}(1+r_{CB})\right] < 0$. Thus in both cases $\left.\frac{\partial\tilde{\Phi}}{\partial r_{CB}}\right|_{\tilde{\Phi}=0} > 0$. That is, PNB is more likely to exist with a higher r_{CB}.

We find that

$$\begin{aligned}\left.\frac{\partial\tilde{\Phi}}{\partial q}\right|_{\tilde{\Phi}=0} &= -p\left[D_0(\tilde{r}_{M,CB} - r_M) + L_{CB}(r_{CB} - \tilde{r}_{M,CB})\right] - \\ &\quad p\max\left\{0, \left[(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H - L_{CB}(1+r_{CB})\right]\right\}\end{aligned} \tag{41}$$

Eq. (35) implies that at $\tilde{\Phi} = 0$, $[D_0(\tilde{r}_{M,CB} - r_M) + L_{CB}(r_{CB} - \tilde{r}_{M,CB})] \geq 0$. Thus $\left.\frac{\partial\tilde{\Phi}}{\partial q}\right|_{\tilde{\Phi}=0} \leq 0$. That is, PNB is more likely to exist with a lower q.

Finally, we find that

$$\left.\frac{\partial\tilde{\Phi}}{\partial\tilde{r}_{M,CB}}\right|_{\tilde{\Phi}=0} = (1-q)p(D_0 - L_{CB}) > 0 \tag{42}$$

Note that $\frac{\partial\tilde{r}_{M,CB}}{\partial\tilde{\lambda}} < 0$ because a more optimistic belief off the equilibrium path will lower the market rate off the equilibrium path. Thus we find that $\left.\frac{\partial\tilde{\Phi}}{\partial\tilde{\lambda}}\right|_{\tilde{\Phi}=0} < 0$. That is, PNB is more likely to exist with a lower $\tilde{\lambda}$. Thus we prove Result (3). ■

A.8 Banks' payoffs in PBB

Banks' payoffs under the equilibrium strategy are as follows. For an H-type bank, similar to Eqs.(28), (29), and (30), we get

$$e_{H,CB,s_i=s_H} = AR_H - L_{CB}(1+r_{CB}) - (D_0 - L_{CB}), \tag{43}$$

$$e_{H,CB,s_i=\emptyset} = AR_H - L_{CB}(1+r_{CB}) - (D_0 - L_{CB})(1+r_{M,CB}), \tag{44}$$

and

$$\begin{aligned} & Ee_{H,CB} = qe_{H,CB,s_i=s_H} + (1-q)e_{H,CB,s_i=\emptyset} \\ = \; & AR_H - L_{CB}(1+r_{CB}) - [q(D_0 - L_{CB}) + (1-q)(D_0 - L_{CB})(1+r_{M,CB})] \end{aligned} \tag{45}$$

For an L-type bank, we have $e^u_{L,CB,s_i=\emptyset} = e_{H,CB,s_i=\emptyset}$, and similar to Eqs. (31) and (32), we get

$$e^u_{L,CB,s_i=s_L} = \max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right) R_H - L_{CB}(1+r_{CB})\right\} \tag{46}$$

and

$$Ee_{L,CB} = qpe^u_{L,CB,s_i=s_L} + (1-q)pe^u_{L,CB,s_i=\emptyset} \tag{47}$$

Banks' payoffs under the deviating strategy are as follows. For an H-type bank, similar to Eqs. (9), (10) and (11), its payoffs are

$$\hat{e}_{H,NCB,s_i=s_H} = AR_H - D_0 \tag{48}$$

$$\hat{e}_{H,NCB,s_i=\emptyset} = AR_H - D_0(1+\hat{r}_M) \tag{49}$$

and

$$\begin{aligned} E\hat{e}_{H,NCB} &= q\hat{e}_{H,NCB,s_i=s_H} + (1-q)\hat{e}_{H,NCB,s_i=\emptyset} \\ &= AR_H - D_0\left[q + (1-q)(1+\hat{r}_M)\right] \end{aligned} \tag{50}$$

For an L-type bank, similar to Eqs. (12) and (13), we get

$$\hat{e}^u_{L,NCB,s_i=\emptyset} = \hat{e}_{H,NCB,s_i=\emptyset} = AR_H - D_0(1+\hat{r}_M) \tag{51}$$

$$E\hat{e}_{L,NCB} = (1-q)p\hat{e}^u_{L,NCB,s_i=\emptyset} = (1-q)p[AR_H - D_0(1+\hat{r}_M)] \tag{52}$$

A.9 Proof of Proposition 5

Proof: Using Eqs. (45) and (50), we get

$$\begin{aligned} & Ee_{H,CB} - E\hat{e}_{H,NCB} \\ = \; & AR_H - L_{CB}(1+r_{CB}) - [q(D_0 - L_{CB}) + (1-q)(D_0 - L_{CB})(1+r_{M,CB})] \\ & -AR_H + D_0\left[q + (1-q)(1+\hat{r}_M)\right] \\ = \; & (1-q)D_0[\hat{r}_M - r_{M,CB}] + L_{CB}[(1-q)r_{M,CB} - r_{CB}] \end{aligned} \tag{53}$$

Thus we have derived Condition (18).

On the other hand, using Eqs. (47) and (52), we find that

$$\begin{aligned} & Ee_{L,CB} - E\hat{e}_{L,NCB} \\ = \; & qp\max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right)R_H - L_{CB}(1+r_{CB})\right\} \\ & +(1-q)p[AR_H - L_{CB}(1+r_{CB}) - (D_0 - L_{CB})(1+r_{M,CB})] \\ & -(1-q)p[AR_H - D_0(1+\hat{r}_M)] \\ = \; & qp\max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right)R_H - L_{CB}(1+r_{CB})\right\} \\ & +(1-q)p[D_0(\hat{r}_M - r_{M,CB}) + L_{CB}(r_{M,CB} - r_{CB})] \end{aligned} \tag{54}$$

Provided that $Ee_{H,CB} - E\hat{e}_{H,NCB} \geq 0$, we find that

$$\begin{aligned}
& Ee_{L,CB} - E\hat{e}_{L,NCB} \\
= \; & p(1-q)D_0[\hat{r}_M - r_{M,CB}] + pL_{CB}[(1-q)r_{M,CB} - r_{CB}] + pqL_{CB}r_{CB} + \\
& qp \max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right) R_H - L_{CB}(1 + r_{CB})\right\} \\
= \; & p(Ee_{H,CB} - E\hat{e}_{H,NCB}) + pqL_{CB}r_{CB} + \\
& qp \max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right) R_H - L_{CB}(1 + r_{CB})\right\} > 0 \qquad (55)
\end{aligned}$$

Thus we prove that as long as H-type banks' no-deviation condition is satisfied, L-type banks' no-deviation condition must be satisfied too. Thus we prove Result (1).

In order to prove Result (2), we need to find the first order derivatives of $Ee_{H,CB} - E\hat{e}_{H,NCB}$ with respect to L_{CB}, r_{CB}, q and $\hat{\lambda}$ at $Ee_{H,CB} - E\hat{e}_{H,NCB} = 0$. Let $\hat{\Phi} = Ee_{H,CB} - E\hat{e}_{H,NCB}$ (Eq. (53)). We find that

$$\left.\frac{\partial \hat{\Phi}}{\partial L_{CB}}\right|_{\hat{\Phi}=0} = (1-q)r_{M,CB} - r_{CB} \qquad (56)$$

Eq. (53) implies that at $\hat{\Phi} = 0$, $(1-q)r_{M,CB} - r_{CB} \geq 0$ because $\hat{r}_M \leq r_{M,CB}$ due to the assumption of $\hat{\lambda} \geq \lambda$. Thus we find that $\left.\frac{\partial \hat{\Phi}}{\partial L_{CB}}\right|_{\hat{\Phi}=0} \geq 0$. That is, PBB is more likely to exist with a higher L_{CB}.

We also find that $\left.\frac{\partial \hat{\Phi}}{\partial r_{CB}}\right|_{\hat{\Phi}=0} = -L_{CB} < 0$. That is, PBB is more likely to exist with a lower r_{CB}.

In addition, $\left.\frac{\partial \hat{\Phi}}{\partial q}\right|_{\hat{\Phi}=0} = D_0(r_{M,CB} - \hat{r}_M) - L_{CB}r_{M,CB}$. Using Eq. (53), we can see that at $\hat{\Phi} = 0$, since $r_{CB} \geq 0$, $L_{CB}r_{M,CB} \geq D_0(r_{M,CB} - \hat{r}_M)$(strictly larger if $r_{CB} > 0$). Thus we find that $\left.\frac{\partial \hat{\Phi}}{\partial q}\right|_{\hat{\Phi}=0} \leq 0$. That is, PBB is more likely to exist with a lower q.

Finally, $\left.\frac{\partial \hat{\Phi}}{\partial \hat{r}_M}\right|_{\hat{\Phi}=0} = (1-q)D_0 > 0$. Note that $\frac{\partial \hat{r}_M}{\partial \hat{\lambda}} < 0$ because a more optimistic belief off the equilibrium path will lower the market rate off the equilibrium path. Thus $\left.\frac{\partial \hat{\Phi}}{\partial \hat{\lambda}}\right|_{\hat{\Phi}=0<0}$. That is, PBB is more likely to exist with a lower $\hat{\lambda}$. Thus we prove Result (2). ■

A.10 Proof of Proposition 6

Proof: In the separating equilibrium, an L-type bank cannot borrow any more funds on the market after borrowing L_{CB}. Its expected payoff from borrowing central bank loans is $pe^u_{L,CB}$, with $e^u_{L,CB}$ given by Eq. (2). In PNB, when an L-type bank deviates to borrowing central bank loans, it can at least borrow L_{CB}. If r_M exists, the bank will be able to borrow $D_0 - L_{CB}$ on the market at the rate of $\tilde{r}_{M,CB}$ with probability $1-q$. Its expected payoff $E\tilde{e}_{L,CB}$ is given by Eq.(32). If $\tilde{r}_{M,CB}$ does not exist, which includes the extreme case when creditors believe that the bank is L-type for sure so that $\tilde{\lambda} = 0$, the bank can borrow only L_{CB}, which will result in the same payoff as that in the separating equilibrium. As a result, $E\tilde{e}_{L,CB}$ cannot be worse than $pe^u_{L,CB}$, i.e., $E\tilde{e}_{L,CB} \geq pe^u_{L,CB}$. Thus an L-type bank's payoff from borrowing central bank loans is higher in PNB than in the separating equilibrium.

In the separating equilibrium, if an L-type bank pretends to be H-type and do not borrow central bank loans, it can borrow D_0 with probability $1-q$ at a zero rate, and will not be able to borrow any funds with probability q. Its expected payoff is $(1-q)e^u_{L,NCB,s_i=\emptyset}$, with $e^u_{L,NCB,s_i=\emptyset}$ given in Eq.(3). In PNB, an L-type bank who does not borrow central bank loans can borrow D_0 at the rate of $r_M > 0$ with probability $1-q$ and will not be able to borrow any funds with probability q. Its expected payoff $Ee_{L,NCB}$ is given by Eq.(13). Because $r_M > 0$, $Ee_{L,NCB} < (1-q)e^u_{L,NCB,s_i=\emptyset}$. Thus an L-type bank's payoff from not borrowing central bank loans is strictly lower in PNB than in the separating equilibrium.

The no-deviation condition for the separating equilibrium requires that $pe^u_{L,CB} > (1-q)e^u_{L,NCB,s_i=\emptyset}$. Hence, we get $E\tilde{e}_{L,CB} > Ee_{L,NCB}$, implying that an L-type bank will always deviate in PNB, even when $\tilde{\lambda} = 0$. ■

A.11 Proof of Proposition 7

Proof: In PBB, an H-type bank's expected payoff under the equilibrium strategy is

$$Ee_{H,CB} = AR_H - L_{CB}r_{CB} - D_0 - (1-q)(D_0 - L_{CB})r_{M,CB} \tag{57}$$

and its expected payoff under the deviating strategy is

$$E\hat{e}_{H,NCB} = AR_H - D_0 - D_0(1-q)\hat{r}_M \tag{58}$$

Given our assumption that $\hat{\lambda} \geq \lambda$, we have $\hat{r}_M \leq r_{M,CB}$. If the central bank sets $r_{CB} = r_{M,CB}$, then

$$\begin{aligned} & Ee_{H,CB} - E\hat{e}_{H,NCB} \\ = \; & -L_{CB}r_{CB} - [(1-q)(D_0 - L_{CB})r_{M,CB}] + D_0(1-q)\hat{r}_M \\ = \; & (1-q)L_{CB}r_{M,CB} - L_{CB}r_{CB} + D_0(1-q)(\hat{r}_M - r_{M,CB}) < 0 \end{aligned} \tag{59}$$

as long as $q < 1$ (when $q = 1$, a pooling equilibrium cannot exist). As a result, if r_{CB} reaches the fair market rate $r_{M,CB}$, an H-type bank will deviate for sure. If fact, an H-type bank will deviate before r_{CB} reaches r_M. ■

A.12 Proof of Proposition 8

Proof: We first consider no-deviation conditions for PNB and PBB. Because $L_{CB} < D_0$, all the banks will borrow on the market no matter they have borrowed a central bank loan of L_{CB} or not. With the identity hiding policy, creditors cannot infer any information about banks' type from their action of whether or not to borrow central bank loans. As a result, the only information that creditors have about the type of a bank without a signal is their prior belief, λ. In PNB and PBB, both types of the banks will borrow the same amount of funds on the market. Therefore, the market rate, r_m, equals r_M and $r_{M,CB}$ which are also based on creditors' prior belief, λ. We assume that $1 + \left(1 - \frac{AR_L}{D_0}\right)\left(\frac{1}{\lambda+(1-\lambda)p} - 1\right) < \min\{\frac{R_H}{\gamma_H}, \frac{AR_H}{D_0}, \frac{AR_H - L_{CB}(1+r_{CB})}{D_0 - L_{CB}}\}$ such that this equilibrium rate exists.

Banks will decide whether to borrow central bank loans or not by comparing their payoffs under the two strategies. Let us first analyze an H-type bank's decision. Its expected payoff from borrowing central bank loans is

$$\begin{aligned} & Ee_{H,CB} = qe_{H,CB,s_i=s_H} + (1-q)e_{H,CB,s_i=\emptyset} \\ = \; & q[AR_H - L_{CB}r_{CB} - D_0] + (1-q)[AR_H + L_{CB}(r_m - r_{CB}) - D_0(1+r_m)] \end{aligned} \tag{60}$$

Its expected payoff from not borrowing central bank loans is

$$\begin{aligned} Ee_{H,NCB} &= qe_{H,NCB,s_i=s_H} + (1-q)e_{H,NCB,s_i=s_H} \\ &= q[AR_H - D_0] + (1-q)[AR_H - D_0(1+r_m)] \end{aligned} \tag{61}$$

Its payoff difference between borrowing and not borrowing central bank loans is

$$
\begin{aligned}
Ee_{H,CB} - Ee_{H,NCB} &= -qL_{CB}r_{CB} + (1-q)L_{CB}(r_m - r_{CB}) && (62)\\
&= -L_{CB}r_{CB} + (1-q)L_{CB}r_m && (63)
\end{aligned}
$$

The intuition of Eq.(62) is that, an H-type bank with a signal can borrow on the market at a zero rate, and borrowing central bank loans will lead to a higher interest payment of $L_{CB}r_{CB}$. If there is no signal, then the bank's interest difference between borrowing on the market and from the central bank is $L_{CB}(r_m - r_{CB})$. An H-type bank will borrow central bank loans if Eq.(63) is positive, or $r_{CB} < (1-q)r_m$.

Similarly, when an L-type bank borrows from the central bank,

$$
\begin{aligned}
Ee_{L,CB} &= qpe^u_{L,CB,s_i=s_L} + (1-q)pe^u_{L,CB,s_i=\emptyset}\\
&= qp\max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right)R_H - L_{CB}(1+r_{CB})\right\}\\
&\quad +(1-q)p[AR_H + L_{CB}(r_m - r_{CB}) - D_0(1+r_m)] && (64)
\end{aligned}
$$

When it does not borrow central bank loans,

$$
Ee_{L,NCB} = (1-q)p[AR_H - D_0(1+r_m)] \tag{65}
$$

Thus,

$$
\begin{aligned}
Ee_{L,CB} - Ee_{L,NCB} &= qp\max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right)R_H - L_{CB}(1+r_{CB})\right\}\\
&\quad +(1-q)pL_{CB}(r_m - r_{CB}) && (66)
\end{aligned}
$$

The bank will borrow central bank loans only when $Ee_{L,CB} - Ee_{L,NCB}$ is positive.

Similar to the no identity hiding case, we find that L-type banks have a stronger incentive to borrow central bank loans than H-type ones. More specifically, we find that provided that $Ee_{H,CB} - Ee_{H,NCB} > 0$, $Ee_{L,CB} - Ee_{L,NCB}$ must be positive too, which we prove as follows.

$$
\begin{aligned}
Ee_{L,CB} - Ee_{L,NCB} &= qp\max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right)R_H - L_{CB}(1+r_{CB})\right\}\\
&\quad +pL_{CB}((1-q)r_m - r_{CB} + qr_{CB})\\
&= qp\max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right)R_H - L_{CB}(1+r_{CB})\right\}\\
&\quad +p(Ee_{H,CB} - Ee_{H,NCB}) + pqL_{CB}r_{CB} > 0 && (67)
\end{aligned}
$$

On the other hand, the above result implies that provided that $Ee_{L,CB} - Ee_{L,NCB} \leq 0$,

$$
\begin{aligned}
Ee_{H,CB} - Ee_{H,NCB} &= -q \max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right) R_H - L_{CB}(1 + r_{CB})\right\} \\
&\quad + \frac{1}{p}(Ee_{L,CB} - Ee_{L,NCB}) - qL_{CB}r_{CB} < 0 \qquad (68)
\end{aligned}
$$

Thus PNB exists when L-type banks do not borrow from the central bank, or $qp \max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right) R_H - L_{CB}(1 + r_{CB})\right\} + (1-q)pL_{CB}(r_m - r_{CB}) \leq 0$. PBB exists when H-type banks borrow from the central bank, or $r_{CB} < (1 - q)r_m$. Thus we have proved Result (1).

Compared to the case without identity hiding, we find that banks' payoff in PBB with identity hiding differs in that when they do not borrow central bank loans, they are charged the rate of r_m instead of $\hat{r}_M$. Recall that $\hat{r}_M$ is determined based on $\hat{\lambda}$. If $\hat{\lambda} > \lambda$, then $\hat{r}_M < r_m$. Thus banks have a stronger incentive to borrow central bank loans with the identity hiding policy. In other words, as long as not borrowing central bank loan can be used as a signal to boost creditors' belief about a bank being H-type, banks have a stronger incentive to choose not to borrow central bank loans, than in the case with identity hiding.

Similarly, in PNB, if $\tilde{\lambda} < \lambda$, then borrowing central bank loans will lower creditors' belief about a bank being H-type. As a result, $\tilde{r}_{M,CB} > r_{M,CB}$. In this case, the hiding identity policy induces banks a stronger incentive to switch from not borrowing central bank loans to borrowing central bank loans, because now the market rate $r_m = r_{M,CB}$. Thus we have proved Result (2). ■

A.13 Proof of Proposition 9

Proof: In the separating equilibrium where only L-type banks borrow from the central bank, each L-type bank will borrow $D_0 - L_{CB}$ on the market, while each H-type bank will borrow D_0 on the market. As a result, the probability of a creditor lending to an H-type bank is given by $\lambda^s = \frac{\lambda D_0}{\lambda D_0 + (1-\lambda)(D_0 - L_{CB})}$, and the probability of lending to an L-type bank is given by $1 - \lambda^s$. Accordingly, the market rate that creditors charge is given by Eq. (20). It is straightforward to see that $\lambda^s > \lambda$ because $D_0 > D_0 - L_{CB}$. As a result, $r_m^s < r_m$.

In addition, the no-deviation condition for an H-type bank is given by

$$Ee_{H,CB} - Ee_{H,NCB} = -qL_{CB}r_{CB} + (1-q)L_{CB}(r_m^s - r_{CB}) \quad (69)$$

$$= -L_{CB}r_{CB} + (1-q)L_{CB}r_m^s \leq 0 \quad (70)$$

The no-deviation condition for an L-type bank is given by

$$Ee_{L,CB} - Ee_{L,NCB} = qp\max\left\{0, \left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right)R_H - L_{CB}(1+r_{CB})\right\} + (1-q)pL_{CB}(r_m^s - r_{CB}) > 0 \quad (71)$$

The separating equilibrium exists when the no-deviation conditions for both types of banks are satisfied. Thus we have proved Proposition 9. ■

B Deriving the central bank's net revenue

When both types of banks borrow central bank loans, the central bank's net revenue can be written as

$$\Pi_{CB} = \lambda E\Pi_H + (1-\lambda)E\Pi_L \quad (72)$$

where $E\Pi_H$ is the central bank's expected net revenue earned from an H-type bank, and $E\Pi_L$ is its expected net revenue earned from an L-type bank.

If H-type banks do not borrow central bank loans and only L-type banks borrow, then

$$\Pi_{CB} = (1-\lambda)E\Pi_L \quad (73)$$

In PBB, we focus on the case where the equilibrium market rate, $r_{M,CB}$, exists. For such an equilibrium market rate to exist, we must have: (1) Creditors are willing to lend at the rate, that is, their expected return rate equals the riskless rate of zero. (2) Banks are willing to borrow at the equilibrium rate. We require that when $r_{M,CB}$ exits, it can actually be paid by H-type banks. This also means that H-type banks will have enough assets to repay all the loans (including the interests) fully. As a result, the central bank's net revenue from lending to an H-type bank is

$$E\Pi_H = L_{CB}r_{CB} \quad (74)$$

For L-type banks, we first consider the case where the market rate, $r_{M,CB}$, is taken as given. That is, creditors are not smart enough to use the interest charged by the central bank to infer whether a borrowing bank is L-type or not. Instead, they use their prior belief of λ.

With probability $1 - q$, there is no signal for an L-type bank. Thus it can borrow $D_0 - L_{CB}$ on the market at the rate of $r_{M,CB}$ on date 1. Then on date 2, with probability p, the up state is realized. In this case, the bank is identical to an H-type one, and will be able to repay all debt. With probability $1-p$, the down state is realized. In this case, the bank's asset is $AR_L < D_0$, implying that the bank's assets are below its liabilities. Therefore, its assets are shared by the central bank and creditors proportionally to their principals, and the assets paid to the central bank are $\frac{L_{CB}}{D_0}AR_L$. The recovery rate for the central bank is $\frac{AR_L}{D_0}L_{CB}$. Thus, the central bank's expected return from lending to an L-type bank without a signal is

$$E\Pi_{L,s_i=\emptyset} = p(L_{CB}r_{CB}) + (1-p)\left(\frac{AR_L}{D_0}L_{CB} - L_{CB}\right) \tag{75}$$

With probability q, there is a signal for an L-type bank. Thus after borrowing central bank loans, the L-type bank will not be able to borrow any additional loans on the market, and will be forced to liquidate its assets. If after liquidation, there is no asset left (i.e., if $A - \frac{D_0 - L_{CB}}{\gamma_L} \leq 0$, then the central bank will get nothing back. In this case,

$$E\Pi_{L,s_i=s_L} = 0 - L_{CB} = -L_{CB} \tag{76}$$

If there are some assets left, then the remaining assets are $\left(A - \frac{D_0 - L_{CB}}{\gamma_L}\right) R_H$ in the up state. If the assets are more than enough to repay central bank loans, then the central bank will be paid $L_{CB}(1 + r_{CB})$. Otherwise, all the assets will be taken by the central bank. So the central bank will get $\min\{(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H, L_{CB}(1+r_{CB})\}$. Similarly, in the down state, the central bank will get $\min\{(A - \frac{D_0 - L_{CB}}{\gamma_L})R_L, L_{CB}(1 + r_{CB})\}$. As a result, the central bank's expected return from lending to an L-type bank with a signal will be

$$\begin{aligned} E\Pi_{L,s_i=s_L} &= p \min\left\{(A - \frac{D_0 - L_{CB}}{\gamma_L})R_H, L_{CB}(1 + r_{CB})\right\} \\ &\quad + \min\left\{(A - \frac{D_0 - L_{CB}}{\gamma_L})R_L, L_{CB}(1 + r_{CB})\right\} \end{aligned} \tag{77}$$

The central bank's expected net revenue from lending to an L-type bank is thus given by

$$E\Pi_L = (1-q)E\Pi_{L,s_i=\emptyset} + qE\Pi_{L,s_i=s_L} \tag{78}$$

Now, let us consider the case where the market will infer whether a borrowing bank is H-type or not from the interest rate charged by the central bank. If the market knows that when this rate is too high, H-type banks will deviate, then there will be two possible outcomes. The first is a separating equilibrium, where the market knows that a borrowing bank must be L-type , and L-type banks are still willing to borrow central bank loans. The second possible outcome is that L-type banks will deviate to pooling with H-type banks by not borrowing central bank loans.

If a separating equilibrium occurs, then the market will not lend to L-type banks, the outcome is similar to the one in the case where there is a signal for an L-type bank, which we just analyzed. That is,

$$E\Pi_L = E\Pi_{L,s_i=s_L} \tag{79}$$

where $E\Pi_{L,s_i=s_L}$ is specified above. If L-type banks deviate to not borrowing central bank loans, then the central bank's net revenue is zero.

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