A fractionally cointegrated VAR analysis of economic voting and political support

Maggie E.C. Jones  Morten Årregaard Nielsen
Queen’s University  Queen’s University and CREATEs

Michal Ksawery Popiel
Queen’s University

Department of Economics
Queen’s University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

6-2014
A fractionally cointegrated VAR analysis of economic voting and political support*

Maggie E. C. Jones  Morten Ørregaard Nielsen†
Queen’s University  Queen’s University and CREATEES
Michał Ksawery Popiel
Queen’s University
July 8, 2014

Abstract

We use a fractionally cointegrated vector autoregressive model to examine the relationship between Canadian political support and macroeconomic conditions. This model is well suited for the analysis because it allows multiple fractional time series and admits simple asymptotic inference for the model parameters and tests of the hypotheses of interest. In the long-run equilibrium, we find that support for the Progressive Conservative Party was higher during periods of high interest rates and low unemployment, while support for the Liberal Party was higher during periods of low interest rates and high unemployment. We also test and reject the notion that party support is driven only by relative (to the United States) economic performance. Indeed, our findings suggest that US macroeconomic variables do not enter the long-run equilibrium of Canadian economic voting (political opinion poll support) at all.

JEL Codes: C32, D72.
Keywords: Economic voting, fractional cointegration, political economy, vector autoregressive model.

1 Introduction

Economic voting is a term used to describe the relationship between political preference and economic conditions. Since the 1950s, several theories in political science and economics...
have been proposed that attempt to characterize this relationship. However, empirical tests of these theories have been inconclusive; to date, the literature has not reached an agreement on which economic variables matter, whether we should be looking at actual or perceived macroeconomic conditions, or how many lags to include when modelling the time series components (Lewis-Beck and Stegmaier, 2000). Due at least in part to the considerable amount of disagreement over model specification, some studies have found little or no relationship between economic conditions and political preference, while many others have found evidence that confirms the economic voting hypothesis (for a review, see Duch and Stevenson (2008)). One possible reason for the ambiguity surrounding this question is that perhaps relative economic performance plays a role in individuals’ political preference. For instance, voters may not show dissatisfaction with their current government in poor economic times if other countries are also experiencing lower than average economic performance and, in contrast, voters may be especially disconcerted when their country’s economic climate is poor relative to other countries.

In this paper, we investigate the relationship between Canadian political support, as measured by public opinion polls, and economic performance. Our analysis explores the dynamics of partisanship for the two leading political parties and the economy, rather than how support for the incumbent government responds to changing economic conditions, which is also common in the literature. In particular, we examine support for the Progressive Conservative Party (PC) and the Liberal Party and their response to interest rates (as measured by the yield on 3-month treasury bills) and seasonally adjusted unemployment. We find that during the time period of our sample, support for the PC party was higher during periods of high interest rates and low unemployment and support for the Liberals was higher during low interest rates and high unemployment. These finding are consistent with other papers that discuss the divide in support between left- versus right-leaning parties; see, for example, Hibbs Jr (1977) and Quinn and Shapiro (1991).

We also study how the relative economic performance between Canada and the United States affects the popularity of political parties in Canada. Our results suggest that US economic variables do not enter the long-run equilibrium of Canadian political opinion poll support. This implies that, in the aggregate, Canadian voters not only disregard relative performance but in fact are not swayed by movements in US variables altogether.

For our analysis, we compile a dataset that merges monthly public opinion poll questionnaires from Gallup Canada Incorporated with macroeconomic variables from the Organization for Economic Development and Cooperation. The Gallup surveys contain monthly data on aggregate political support from September, 1974 until December, 2000. Previous research using opinion poll data of this type has shown that aggregate political support is well modelled as a fractional (or fractionally integrated or long memory) time series, e.g. Box-Steffensmeier and Smith (1996) and Byers et al. (1997), wherein an event that affects political support in one period continues to do so for many periods in the future. Furthermore, the macroeconomic variables that we consider have also been shown to exhibit this property (e.g., Sowell 1992, Crato and Rothman 1994, Baillie 1996, Tkacz 2001, Gil-Alana 2002, Mikhail et al. 2006). As such, appropriate considerations must be given when using these variables in any type of estimation procedure. A natural methodology for the analysis of multiple fractional time series variables is the fractionally cointegrated vector autoregressive (FCVAR) model of Johansen (2008) and Johansen and Nielsen (2012), which generalizes the
well-known cointegrated VAR model of Johansen (1995) to fractional time series. We apply this model to our dataset to investigate both the relationship between economic performance and political support as well as relative economic performance and political support.

The FCVAR model has many advantages when estimating a system of fractional time series variables. The flexibility of the model permits one to determine the cointegrating rank, or number of equilibrium relations, via statistical tests and to jointly estimate the adjustment coefficients and the cointegrating relations, while accounting for the short-run dynamics. Each of these features are relevant to the research questions in our analysis. For example, the cointegrating rank is the number of long-run equilibria that exist between the political and economic variables, and the cointegrating relations themselves are the linear combinations of these variables that form a stationary equilibrium. The adjustment coefficients tell us which variables adjust to changes in the equilibrium and at what rate. For instance, we find that political support and the economic variables are cointegrated and that, when a shock moves the system away from equilibrium, political support and unemployment tend to adjust to the change whereas interest rates behave exogenously in the long run.

The asymptotic theory for estimation and inference in the FCVAR model was developed recently by Johansen and Nielsen (2010, 2012, 2014). They derive the asymptotic distributions of the maximum likelihood estimators and of the likelihood ratio tests for cointegration rank. Furthermore, MacKinnon and Nielsen (2014) provide accompanying computer programs for calculation of $P$ values and critical values for the cointegration rank tests, and Nielsen and Morin (2014) provide a Matlab package for calculation of estimators and test statistics. Taken together, this means that inference and estimation within the FCVAR framework is now possible such that these models can be fully applied empirically.

The contribution of this paper is thus twofold. First, we explore the empirical capabilities of the recently developed FCVAR model and thereby outline a general procedure for estimation and inference within the FCVAR framework that can be used to conduct empirical analyses using this methodology. Second, we add to the existing literature on economic voting and we also test the idea that only relative economic performance matters. We provide new empirical results regarding economic voting in the Canadian context. Specifically, we show that from 1974 to 2000, support for the two main political parties together with interest rates and unemployment formed a long-run equilibrium relationship, wherein economic fluctuations lead to opposite changes in support for the Liberal and Progressive Conservative parties.

The remainder of our paper is structured as follows. In Section 2 we introduce economic voting and in Section 3 we discuss fractional integration and its relevance for polling data. In Section 4 we describe the FCVAR model and briefly discuss some of its relevant properties. Section 5 contains the empirical analysis and provides a description of our dataset, a discussion of political cycles, the hypotheses of interest, and the empirical results from estimation of the FCVAR model. We conclude in Section 6 and some additional plots and robustness results are given in two appendices.

2 Economic voting

Research on economic voting is plentiful. For a more complete description of the many studies that have shaped this body of literature, see Lewis-Beck and Stegmaier (2000) or Duch and Stevenson (2008). Early theoretical research on economic voting discusses a rational
agent who chooses to vote in a way that maximizes expected utility (Downs 1957; Kramer 1971; Fair 1978), but exactly how the economy enters the agent’s utility function is still debated: are voters retrospective and do they punish the incumbent government for poor past performance, or are they prospective and do they vote based on their future expectations of government competency? The retrospective voter model is also known as the sanctioning or moral hazard model. If voters have the ability to reward or punish the government then they have an obligation to sanction the government for poor economic performance, because otherwise they risk signalling to the incumbents that poor performance will be tolerated (Barro 1973; Ferejohn 1986). Subsequent research moved away from the retrospective voting model and towards a prospective model of rational expectations, where agents do not make systematic mistakes when forecasting the future (Rogoff and Sibert 1988; Cukierman and Meltzer 1989).

Our empirical results seem to be in line with the forward-looking voter theories, where agents choose to vote in such a way that reflects their expectations of the government given the current economic climate. In the context of our model, if the current unemployment rate is high, then voters believe the Liberal party will be able to help them through more challenging economic times, which results in a rise in support for the Liberal party. On the other hand, in a low unemployment environment voters prefer a more conservative government and we see an increase in support for the PC party.

The empirical literature has largely confirmed what the early theoretical work predicted, namely that the economy is important to voters. Goodhart and Bhansali (1970) provide some of the first empirical work examining the relationship between the economy and voting behaviour. They analyze the relationship between unemployment, inflation and party support in the UK and find that support for the Labour party increases with unemployment and inflation, and support for the Conservative party decreases with unemployment and inflation. Powell and Whitten (1993) conduct a cross-national analysis of economic voting and also find that unemployment and inflation are significant factors considered by voters in the international context. In addition, they consider economic performance of one country relative to the performance of an international baseline, but their results are inconclusive.

More recently, Kayser and Peress (2012) expand on this idea of “benchmarking”: placing the domestic economy in an international context, so that domestic economic conditions are considered relative to the global economy. They focus on the post-2008 recession time period in the United States and find effects of relative economic growth on vote share but do not to find any statistically significant results for the effect of unemployment on vote share. Several other papers have attempted to describe the relationship between government support and the economy by estimating equations known as popularity or vote functions, e.g. Pickup (2006), Brückner and Grüner (2010), and De Neve (2013). Typically, macroeconomic variables like unemployment, inflation, GDP, personal income and interest rates are examined as economic factors that voters take into consideration when evaluating the economy.

Empirical analyses of economic voting in Canada have found varying results. Early work by Monroe and Erickson (1986) found no evidence that fluctuations in inflation and unemployment affect support for either the governing or opposing party and this finding was confirmed by Clarke and Zuk (1987). However, both studies focussed on the years prior to 1979. In contrast, Archer and Johnson (1988) found that unemployment in 1984 was a big contributor to dissatisfaction with the incumbent government, but again found that inflation
was not a factor in determining voter support. Johnston (1999) finds that the political business cycle theory — the idea that governments tend to manipulate economic conditions immediately before an election in order to boost their popularity — is not applicable in the Canadian case. In addition, he reports that worsening economic conditions are bad for both the Liberal and Progressive Conservative parties. Similarly, Monroe and Erickson (1986) reiterate a claim, originally suggested by Downs (1957), that the Liberals and Progressive Conservatives differ little in terms of economic policy and so voters would respond primarily to the incumbent party performance.

3 Fractional integration and polling data

As econometric modelling has become more sophisticated, researchers have been able to analyze data of an increasingly complicated nature. Box-Steffensmeier and Smith (1996) and Byers et al. (1997) show that political popularity, as measured by public opinion polls, can be modelled as fractional time series processes. The fractional (or fractionally integrated or just integrated) time series models are based on the fractional difference operator,

$$\Delta^d X_t = \sum_{n=0}^{\infty} \pi_n(-d)X_{t-n}, \quad (1)$$

where the fractional coefficients $\pi_n(u)$ are defined in terms of the binomial expansion $(1 - z)^{-u} = \sum_{n=0}^{\infty} \pi_n(u)z^n$, i.e.,

$$\pi_n(u) = \frac{u(u+1)\cdots(u+n-1)}{n!}. \quad (2)$$

For details and for many intermediate results regarding this expansion and the fractional coefficients, see, e.g., Johansen and Nielsen (2014, Appendix A). Efficient calculation of fractional differences, which we apply in our estimation, is discussed in Jensen and Nielsen (2014).

With the definition of the fractional difference operator in (1), a time series $X_t$ is said to be fractional of order $d$, denoted $X_t \in I(d)$, if $\Delta^d X_t$ is fractional of order zero, i.e. if $\Delta^d X_t \in I(0)$. The latter property can be defined in the frequency domain as having spectral density that is finite and non-zero near the origin or in terms of the linear representation coefficients if the sum of these is non-zero and finite, see, e.g., Johansen and Nielsen (2012). An example of a process that is fractional of order zero is the stationary and invertible ARMA model.

The standard reasoning for political opinion poll series being fractional relies on Granger’s (1980) aggregation argument, and can briefly be described as follows. Suppose individual level (typically binary) voting or polling behavior is governed by the autoregressive process

$$x_{i,t} = \delta_{i,1} + \delta_{i,2} x_{i,t-1} + u_{i,t}, \quad (3)$$

where $i = 1, \ldots, N$ denotes individuals and $t = 1, 2, \ldots$ as usual denotes time. The important point here is that the autoregressive coefficients $\delta_{i,2}$ differ across individuals. Some individuals have coefficients $\delta_{i,2} \approx 0$ and are referred to as "floating" voters, whereas others have coefficients $\delta_{i,2} \approx 1$ and are referred to as "committed" voters.\footnote{"Floating" voters are defined as those who do not have a strong alliance to one party and may be more easily swayed by current events, media, etc., and "committed" voters, on the other hand, are those who consistently vote for a particular party and are less inclined to change their voting preference.} If it is assumed that
the distribution of $\delta_{i,2}$ across individuals follows a Beta($u, v$) distribution, then the aggregate vote share or polling share $X_t = N^{-1} \sum_{i=1}^{N} x_{i,t}$ is fractionally integrated of order $d = 1 - v$ when $N$ is large, i.e., $X_t \in I(1 - v)$. For more details, see Box-Steffensmeier and Smith (1996) or Byers et al. (1997).

The above theoretical argument in favor of modelling opinion poll data as fractional time series has been supported by empirical work by a number of authors. For example, Box-Steffensmeier and Smith (1996) estimate fractional models for US data, Byers et al. (1997) and Dolado et al. (2002) analyze UK data, and Dolado et al. (2003) analyze Spanish data. All find strong evidence in support of fractional integration with estimates of $d$ around 0.6-0.8. In addition, Byers et al. (2007) analyze an updated version of the sample in Byers et al. (1997) and show that the change to phone interviews had no effect on estimates of $d$ and did not appear to constitute a structural break.

In addition to the developments in modelling political opinion poll data as fractional processes, similar results have been obtained in the macroeconometric literature. For example, Crato and Rothman (1994) re-examine the Nelson and Plosser (1982) data set and find that many macroeconomic time series can be modelled as fractional processes. Tkacz (2001) analyzes the same interest rate series that we use in our empirical analysis and finds estimates of $d$ that are very similar to the ones that we obtain at around 0.7 to 0.8. For unemployment, the observation of persistence in unemployment has been interpreted as indicative of hysteresis: the theory that high unemployment can be self-perpetuating because of skill depreciation and the stigma attached to being out of the work force. In empirical work, Dolado et al. (2002) find unemployment in the Nelson and Plosser (1982) data set to be fractional of order 0.8, while Koustas and Veloce (1996), Gil-Alana (2002), and Mikhail, Eberwein, and Handa (2006) find evidence that unemployment in Canada is fractionally integrated with estimates of $d$ around 0.8.

Much of the previous economic voting research has ignored the fractional feature of both political and macroeconomic variables. This can result in model misspecification and spurious findings. Davidson (2005) is one of the first papers to address both the fractional integration of opinion polls and economic factors together in a system. Davidson analyzed government popularity and economic performance in the UK by estimating an FCVAR-like system for which no asymptotic properties were developed, but found only very little evidence of a relationship between the economy and political support. Of course, Davidson’s (2005) study predates the development of the FCVAR model, which we introduce next.

4 FCVAR methodology

Our empirical analysis uses the fractionally cointegrated vector autoregressive (FCVAR) model, see Johansen (2008) and Johansen and Nielsen (2010, 2012, 2014). The model is a generalization of Johansen’s (1995) cointegrated vector autoregressive (CVAR) model to allow for fractional processes of order $d$ that cointegrate to order $d - b$. To introduce the FCVAR model, we begin with the well-known, non-fractional, CVAR model. Let $Y_t, t = 1, \ldots, T$ be a $p$-dimensional $I(1)$ time series. Then the CVAR model is

$$\Delta Y_t = \alpha \beta' Y_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta Y_{t-i} + \varepsilon_t = \alpha \beta' L Y_t + \sum_{i=1}^{k} \Gamma_i L^i Y_t + \varepsilon_t. \quad (4)$$

The simplest way to derive the FCVAR model is to replace the difference and lag operators
\( \Delta \) and \( L \) in (4) by their fractional counterparts, \( \Delta^b \) and \( L_b = 1 - \Delta^b \), respectively. We then obtain
\[
\Delta^b Y_t = \alpha \beta' L_b Y_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L^i_b Y_t + \varepsilon_t, \quad (5)
\]
which we apply to \( Y_t = \Delta^{d-b} X_t \) such that
\[
\Delta^d X_t = \alpha \beta' L^d X_t + \sum_{i=1}^{k} \Gamma_i \Delta^d L^i_d X_t + \varepsilon_t. \quad (6)
\]

Here, as usual, \( \varepsilon_t \) is \( p \)-dimensional independent and identically distributed with mean zero and covariance matrix \( \Omega \).

The parameters have the usual interpretations known from the CVAR model. In particular, \( \alpha \) and \( \beta \) are \( p \times r \) matrices, where \( 0 \leq r \leq p \). The columns of \( \beta \) are the cointegrating relationships in the system, and the elements of \( \beta' X_t \) are the stationary combinations of the variables in the system, i.e. the long-run equilibria. The coefficients in \( \alpha \) represent the speed of adjustment towards equilibrium for each of the variables. The parameters \( \Gamma_i \) govern the short-run behaviour of the variables.

Thus, the FCVAR model enables simultaneous modelling of the long-run equilibria (with tests to determine how many such equilibria exist), the adjustment responses to deviations from the equilibria, and the short-run dynamics of the system. In addition, the FCVAR model makes it possible to evaluate model fit, i.e. whether the assumptions underlying the asymptotic distribution theory are likely satisfied, by examining the model residuals using, for instance, tests for serial correlation.

As an intermediate step towards the final model, consider a version of model (6) with \( d = b \) and a constant mean term for the cointegrating relations. That is,
\[
\Delta^d X_t = \alpha \beta' L^d X_t + \rho + \sum_{i=1}^{k} \Gamma_i \Delta^d L^i_d X_t + \varepsilon_t. \quad (7)
\]

Estimation and inference for the model is discussed in Johansen and Nielsen (2012) and Nielsen and Morin (2014), with the latter providing Matlab computer programs for the calculation of estimators and test statistics.

It is important at this point to note that the fractional difference in (1) is defined in terms of an infinite series, but that any observed sample will include only a finite number of observations, thus prohibiting calculation of the fractional differences as defined. An assumption that would allow calculation of the fractional differences is that \( X_t \) were zero before the start of our sample. However, in our case, as would often be the case, we cannot reasonably make such an assumption. The bias introduced by making such an assumption to allow calculation of the fractional differences is analyzed by Johansen and Nielsen (2014) using higher-order expansions in a simpler model. They also show, albeit in a simpler model, that this bias can be completely avoided by including a level parameter \( \mu \) that shifts each of the series by a constant. We follow this suggestion also in our analysis. Thus, the model we consider in the empirical section below is
\[
\Delta^d (X_t - \mu) = L_d \alpha \beta' (X_t - \mu) + \sum_{i=1}^{k} \Gamma_i \Delta^d L^i_d (X_t - \mu) + \varepsilon_t, \quad t = 1, \ldots, T. \quad (8)
\]
with each of the fractional differences truncated such that the summation in (1) terminates at \( n = t - 1 \). This model is estimated using a slight modification of the Matlab package of Nielsen and Morin (2014) to account for the additional parameter \( \mu \). Note that \( \beta'\mu = -\rho' \) denotes the mean of the stationary cointegrating relations.

The asymptotic analysis in Johansen and Nielsen (2012) shows that the maximum likelihood estimators of \((d, \alpha, \Gamma_1, \ldots, \Gamma_k)\) are asymptotically normal, while the maximum likelihood estimator of \((\beta, \rho)\) is asymptotically mixed normal when \( d_0 > 1/2 \) and asymptotically normal when \( d_0 < 1/2 \). The important implication is that asymptotic \( \chi^2 \)-inference can be conducted on the parameters \((d, \rho, \alpha, \beta, \Gamma_1, \ldots, \Gamma_k)\) using likelihood ratio (LR) tests. In general the asymptotic distribution of \( \mu \) is not known, however, as mentioned above, the inclusion of the level parameter \( \mu \) corrects for the fact that all initial values of \( X_t \) are not observed, and as such its asymptotic distribution is not particularly important.

We will test a number of interesting hypotheses on the model parameters in our empirical analysis. The general theory of hypothesis testing for the CVAR model (Johansen, 1995) carries over almost unchanged to the FCVAR model. In particular, the degrees of freedom is equal to the number of overidentifying restrictions under the null, and, although counting the degrees of freedom is non-standard because of the normalization required to separately identify \( \alpha \) and \( \beta \), this issue applies in exactly the same way for the CVAR model. To facilitate the discussion of hypotheses in the empirical analysis, we describe here the general framework for hypothesis testing on \( \alpha \) and \( \beta \).

Hypotheses on \( \beta \) can be formulated as

\[
\beta = H\varphi, \tag{9}
\]

where the known \( p \times s \) matrix \( H \) specifies the restriction(s) and \( \varphi \) is an \( s \times r \) matrix of freely varying parameters. In this case the same restriction is imposed on each cointegrating relation and the degrees of freedom of the test is given by \( df = (p - s)r \). If \( r > 1 \), different restrictions can be imposed on different columns of \( \beta \), i.e., \( \beta = (H_1\varphi_1, \ldots, H_r\varphi_r) \) for known \( p \times s_i \) matrices \( H_i \) and \( s_i \times 1 \) vectors \( \varphi_i \) containing the freely varying parameters in column \( i \) of \( \beta \). In this case the degrees of freedom of the test is \( df = \sum_{i=1}^r (p - r - s_i + 1) \).

Similarly, hypotheses on \( \alpha \) can be formulated as

\[
\alpha = A\psi, \tag{10}
\]

where the known matrix \( A \) is of dimension \( p \times m \) and the matrix of free parameters \( \psi \) is \( m \times r \) with \( m \geq r \). The degrees of freedom of the test is \( df = (p - m)r \).

In Sections 5.4 and 5.5 below we describe each of our hypotheses in detail, and relate them to the specification of \( H \) and \( A \) and give the relevant degrees of freedom.

### 5 Empirical analysis

In this section we present our empirical results and discuss the findings from several hypothesis tests that are relevant to the investigation of economic voting. Using the FCVAR model allows us to investigate the long-run equilibrium relationship of economic variables and political party support. Since all the variables are non-stationary, we look for linear combinations of the variables that cointegrate to produce stationary equilibria so that we can interpret the results of the estimation procedures. To motivate the equilibrium analysis, we begin with a description of the data and univariate estimation of the individual series.
5.1 Data description

The aggregate polling data is from Gallup Canada Incorporated and was retrieved from the Ontario Data Documentation, Extraction Service and Infrastructure (ODESI) repository. Our series are monthly and span the period from September, 1974 to December, 2000.

In each month survey respondents were asked the question: “If a federal election were held today, which party’s candidate do you think you would favour?” For each of the political parties, we define popularity by the percentage of decided voters who responded to this question with the specified political party, where a decided voter is one who is sure about their voting intentions. Our analysis concentrates on the two main Canadian political parties that span this time period: the Progressive Conservative (PC) Party and the Liberal Party of Canada. These parties were the only two parties to hold office and garnered the majority of support throughout our sample period.

The macroeconomic variables we use in our analysis are the Canadian and US interest rates (the 3-month treasury bill yield) and the seasonally adjusted Canadian and US unemployment rates. This data is obtained from the Main Economic Indicators Database of the Organization for Economic Development and Cooperation and spans the same time period as our political opinion poll data.

Our analysis proceeds after converting PC and Liberal support and unemployment to log-odds (for more details and background, see Byers et al. (1997)). This is done to map variables on the unit interval into the real line, in order to use error terms with unbounded support in our regressions. The log-odds (or logit) transformation for a variable \( Y_t \in (0, 1) \) is

\[
y_t = \log \left( \frac{Y_t}{1 - Y_t} \right),
\]

where \( Y_t \) is the original series and \( y_t \) is the log-odds transformed series with support \((-\infty, \infty)\).

Figure 1(a) shows aggregate support for the PC and Liberal parties over the span of our sample and Figure 1(b) shows interest and unemployment rates. PC and Liberal support and unemployment are in log-odds. Election dates have been included as vertical lines in Figures 1(a) and 1(b) for context. Throughout our sample period there were seven federal general elections in Canada, four of which were won by the Liberal party and three of which were won by the PC party.

Many scholars recognize three distinct periods in Canadian political history: the clientelistic (pre 1917), brokerage (1917-1957) and pan-Canadian, the end of which is disputed in the literature — some argue that we are still in the pan-Canadian period, while others contend that there exits a fourth political period in Canada. Furthermore, among those who

---

2Unfortunately, Gallup Inc. stopped selling its Canadian polling data to the ODESI repository at the end of 2000, and so we have been unable to obtain polling data past December, 2000.

3We thank Mark Pickup as well as James Davidson, David Byers and David Peel for their polling data which we used to replace some missing values in the ODESI time series.

4In some months, if a survey respondent answered the preferred political party question with “undecided” or “don’t know” they were then asked an additional question, “Although you may not have made up your mind, which federal party do you tend to lean towards at the present time?” To make the series consistent across all months, we disregard the undecided voter intentions when they are available, and thus our data points for all months are given as a percentage of decided voters.

5The Progressive Conservatives eventually merged with the Canadian Alliance Party to form the Conservative Party of Canada in 2003, but we do not have opinion poll data for this time period.
Figure 1: Time series plots for 1974-2000

(a) PC and Liberal support with election dates

(b) interest and unemployment rates

acknowledge the fourth period, there is additional disagreement over when it begins. Some consider the beginning of Brian Mulroney’s tenure as Prime Minister in 1984 as the end of the *pan-Canadian* era and others argue that the *pan-Canadian* period ended with the fall of the PCs in the 1993 election (Clarkson, 2005). The latter election was a historic turning point in Canadian politics as the PCs, who had already begun losing support in the months before, went from 156 seats in the House of Commons to a total of 2. The falling support for the PCs prior to the 1993 election has been attributed to a number of possible factors that rendered the PCs unfavourable among Canadian voters. In the late 80s and early 90s, unemployment was the highest since the Great Depression, the government had been running a high persistent deficit, and Canada found itself in the worst recession since World War II. The introduction of the Generalized Sales Tax in 1991 contributed to voter distain, and the failure of the Meech Lake Accord coupled with rumours of government corruption left voters feeling increasingly alienated by the government. The failure of the PCs to regain
voter support culminated in a series of attack ads against the Liberal leader, Jean Chrétien, leading up to the 1993 election.

In addition to the political dynamics, the early 90s were marked by several important policy changes that significantly affected the macroeconomic environment in Canada. Both the Canadian and US economies were adjusting to the Canada-US Free Trade Agreement (FTA), a precursor to the North American Free Trade Agreement, which aimed at reducing tariffs and facilitating trade between the two countries. During this time frame the Bank of Canada adopted an inflation targeting monetary policy regime with the intention of reducing inflation from approximately 5% in late 1990 to about 2% by the end of 1995 (Bank of Canada, 1991). These macroeconomic policies and the dramatic shift in PC support occurred simultaneously and may suggest a structural break in the time series of PC support as well as unemployment and interest rates. For this reason we re-estimate our models using a subsample of the data that is truncated at the 1993 election as a robustness check of our main empirical analysis (these results are available in Appendix B).

5.2 Political cycles

Before proceeding with the empirical analysis, it is customary in this literature to account for fluctuations in support that are associated with so-called political cycles. We follow the basic procedure outlined in Byers et al. (1997) and assume that the series contains a political cycle component $C_t$ and a long-run component, either $pc_t$ or $lib_t$, as in

$$\hat{pc}_t = C_t^p + pc_t, \quad (11)$$
$$\hat{lib}_t = C_t^l + lib_t, \quad (12)$$

where $\hat{pc}_t$ and $\hat{lib}_t$ are the (observed) log-odds transformed series. There are several ways to model the cyclical component and we consider two: a time-in-office cycle and a time-since-last-election cycle. The political cycle can be decomposed into three effects: the “in-power” effect, the “anticipation” effect, and the “honeymoon” effect. The “in-power” effect refers to the generally increased support while a party is in office and we account for this by using dummy variables, $D_{lib,t}$ and $D_{pc,t}$, that are coded as 1 if the corresponding party is in power. A party also normally experiences a gradual rise in support before it enters into power, referred to as the “anticipation” effect, which is usually followed by a decline in support over that party’s tenure in office, commonly referred to as the “honeymoon” effect. We capture this cyclical rise and fall in support by generating a time-since-election or a time-in-office count variable, $\tau_t$, and in the regression, we include this variable, as well as its squared value, interacted with the in-power dummy variables. The complete cyclical component is specified as

$$C_t^i = \gamma_1^i D_{pc,t} + \gamma_2^i D_{lib,t} + \gamma_3^i D_{pc,t} \tau_t + \gamma_4^i D_{lib,t} \tau_t + \gamma_5^i D_{pc,t} \tau_t^2 + \gamma_6^i D_{lib,t} \tau_t^2, \quad i \in p, l. \quad (13)$$

We estimate regressions (11) and (12) with the cycle specified by (13) and use the residuals to obtain our final time series of political support. As a robustness check we estimate the regressions with the variable $\tau_t$ counting both time-in-office and time-since-last-election. The results are given in Table 1. In order to show significant digits in the results, we have multiplied the dependent variable by a factor of one-hundred. The “in-power” effect is present in both specifications for both parties: Progressive Conservatives have higher support...
Table 1: Election and time-in-office cycle removal

<table>
<thead>
<tr>
<th></th>
<th>pc election</th>
<th>lib election</th>
<th>pc office</th>
<th>lib office</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{pc,t}$</td>
<td>3.556</td>
<td>-76.19</td>
<td>-15.20</td>
<td>-71.96</td>
</tr>
<tr>
<td></td>
<td>(15.54)</td>
<td>(8.637)</td>
<td>(14.46)</td>
<td>(8.167)</td>
</tr>
<tr>
<td>$D_{lib,t}$</td>
<td>-164.509</td>
<td>11.085</td>
<td>-170.211</td>
<td>7.220</td>
</tr>
<tr>
<td></td>
<td>(13.128)</td>
<td>(7.299)</td>
<td>(12.461)</td>
<td>(7.639)</td>
</tr>
<tr>
<td>$D_{pc,t} \times \tau_t$</td>
<td>-8.449</td>
<td>1.677</td>
<td>-2.598</td>
<td>0.585</td>
</tr>
<tr>
<td></td>
<td>(1.332)</td>
<td>(0.740)</td>
<td>(0.658)</td>
<td>(0.371)</td>
</tr>
<tr>
<td>$D_{pc,t} \times \tau_t^2$</td>
<td>0.124</td>
<td>-0.019</td>
<td>0.013</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.013)</td>
<td>(0.006)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$D_{lib,t} \times \tau_t$</td>
<td>1.348</td>
<td>-1.462</td>
<td>4.746</td>
<td>-2.030</td>
</tr>
<tr>
<td></td>
<td>(1.094)</td>
<td>(0.608)</td>
<td>(0.696)</td>
<td>(0.393)</td>
</tr>
<tr>
<td>$D_{lib,t} \times \tau_t^2$</td>
<td>0.015</td>
<td>0.010</td>
<td>-0.066</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.011)</td>
<td>(0.008)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$T$</td>
<td>316</td>
<td>316</td>
<td>316</td>
<td>316</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.778</td>
<td>0.530</td>
<td>0.787</td>
<td>0.535</td>
</tr>
<tr>
<td>$F$</td>
<td>180.988</td>
<td>58.172</td>
<td>191.214</td>
<td>59.418</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses.

when they are in power and so do Liberals. For example, this can be seen for the PCs by comparing the coefficient on $D_{pc,t}$, 3.556, to that on $D_{lib,t}$, -164.509, in the first column. Furthermore, the “honeymoon” effect is also present in all of the regressions; the coefficient on the party’s dummy interacting with the time variable is negative implying that support declines after entering office or after an election. Support also rises again before the next political change, which can be interpreted as the “anticipation” effect, and this is seen by a positive coefficient on the party’s own dummy interacting with the squared time variable in both of the time-in-office regressions and the election equation for liberals.

In Figures 2 and 3, we plot the time-in-office removal process for the PCs and Liberals, respectively. Panel (a) shows the political support for the party in log-odds, while the fitted values from regressions (11) and (12), i.e. the political cycles, are in Panel (b). The final series with the effects of the political cycle removed, i.e. the residuals from (11) and (12), are found in Panel (c). Panel (d) is a plot of political support with the time-in-office cycle removed that has been fractionally differenced by the estimate of $d$ from the FAR(0) model discussed in Section 5.3. As can be seen by the similarities between Panels (a) and (c), removing the political cycle does little to affect the time-dependence. Fractionally differencing the data, however, removes this dependence and gives the appearance of white noise in Panel (d) of both Figures 2 and 3.

Henceforth, we use the political support series with the time-in-office cycle removed. The series with time-since-last-election cycle removed, as shown in Figures 5 and 6, are used in our robustness analysis in Appendix B.

5.3 Univariate analysis

To verify that the FCVAR model is appropriate for our data, we examine each of our series individually before estimating the full multivariate system. As a preliminary check we plot the sample autocorrelations and the estimated spectral density of each series. One fea-
Figure 2: Time-in-office cycle removal for PC support

(a) support in log-odds

(b) time-in-office cycle

(c) support with time-in-office cycle removed

(d) fractionally differenced series from Panel 2(c) using estimate from the FAR(0) model: $\hat{d} = 0.803$
Figure 3: Time-in-office cycle removal for Liberal support

(a) support in log-odds

(b) time-in-office cycle

(c) support with time-in-office cycle removed

(d) fractionally differenced series from Panel 3(c) using estimate from the FAR(0) model: $\hat{d} = 0.689$
The autocorrelation and spectral density plots for PC and Liberal support can be found in Figure 4. A brief inspection shows that the autocorrelation and spectral density plots exhibit exactly those fractional time series characteristics mentioned above: autocorrelations decay hyperbolically and mass of the spectrum is concentrated near the zero frequency (note the logarithmic axis in the spectral density plots). The autocorrelation and spectral density plots for unemployment and interest rates are similar and can be found in Appendix A.

6 We display the figures for the time-in-office series, which we use in our main analysis. The autocorrelation and spectral density plots for the raw series are similar to those with the time-in-office cycle removed.
respectively. The remaining columns in Table 2 present FAR(\(k\)) tests. The sample size is \(T = 316\).

In general, if a time series rejects both stationarity tests and unit root tests, that would suggest that it is likely a fractional time series. Therefore, before obtaining estimates of \(d\) we perform standard Kwiatkowski-Phillips-Schmidt-Shin (KPSS) and augmented Dickey-Fuller (ADF) tests for stationarity and unit roots, respectively, as well as the “nearly efficient” likelihood ratio tests for unit roots (Jansson and Nielsen 2012) on each of our individual series. All series reject stationarity and the political series strongly reject the presence of a unit root.

Next, we proceed to estimation of the fractional parameter, \(d\), for each univariate series with results presented in Table 2. The first three columns are semiparametric log-periodogram regression estimates from the Geweke and Porter-Hudak (1983) and Robinson (1995), here labeled GPH, computed with bandwidths \(m = T^{0.4}\), \(m = T^{0.5}\), and \(m = T^{0.6}\), respectively. The remaining columns in Table 2 present FAR(\(k\)) estimates, which is the univariate version of \(\hat{\xi}\) with \(r = 0\) and \(k\) lags, see Johansen and Nielsen (2010). Results are shown for \(k = 0\), \(k = 1\), and \(k = 2\), and the associated Ljung-Box Q-test statistics, labeled \(Q_{k}\), for serial correlation up to lag 12 in the residuals are also given.

For the party support series, the estimates of \(d\) are mostly in the 0.6-0.8 range, which is consistent with previous studies by, e.g., Davidson et al. (2000), and Byers et al. (1997, 2000). For the macroeconomic series, the GPH estimates

\footnote{In fact estimate the fractional order of integration of Liberal party and PC party support to be 0.882 and 1.018, respectively. Their sample, however, ends in 1995.}
have large standard errors, making it difficult to draw any conclusions. For the interest rates, the FAR(0) models show evidence of serial correlation in the residuals. With $k = 1$ and $k = 2$, the residuals are well behaved\(^8\) and the estimates of $d$ are in line with those for the political support series. For unemployment we also find similar estimates of $d$ when $k = 2$. Furthermore, estimates of $d$ around 0.8 are in line with those found in Tkacz (2001) and Gil-Alana (2002) for Canadian interest rates and unemployment, respectively.

### 5.4 Model specification and hypotheses of interest

The main issue in model specification after choosing the variables for the system is to select the lag augmentation, $k$. We apply a general-to-specific testing strategy. Starting with a generous lag order, we test in each step for significance of the coefficient of the highest-order lag, i.e. significance of $\Gamma_k$, by an LR test. If the null hypothesis is not rejected, the highest-order lag is dropped and the model is re-estimated. This continues until the coefficient of the highest-order lag is significant. In each step, we check the residuals for serial correlation using a multivariate Ljung-Box Q-test, $\hat{Q}(h)$, with $h = 12$ lags because our data is monthly. If the null hypothesis that the residuals are serially uncorrelated is rejected, then we go back one step in the specification. In each model we begin testing with a maximum lag of $k = 3$ since previous studies using polling data, e.g. Byers et al. (1997) and Dolado et al. (2003), have found that very little lag augmentation is needed, which is also supported by the univariate analysis in Table 2.

After establishing the appropriate lag length we determine the rank of the system, which tells us the number of stationary cointegrating relations. Following standard practice in the cointegration literature (Johansen 1995), we select the rank based on a series of LR tests where we sequentially test $\mathcal{H}(r) : \text{rank} = r$ against $\mathcal{H}(p) : \text{rank} = p$ for $r = 0, 1, \ldots$. The “estimated” rank is then the first non-rejected value in the sequence of tests. The asymptotic distributions of these LR test statistics are non-standard and are derived in Johansen and Nielsen (2012). We use the $P$ values obtained from computer programs made available by MacKinnon and Nielsen (2014) based on their numerical distribution functions.

Once lag length and cointegration rank are established, we estimate the model and conduct inference on the parameters of the model. In the context of economic voting, there are several relevant hypotheses of interest. Each of the hypotheses that we use in our analysis are included in Table 3, which acts as a key to hypothesis testing throughout our empirical analysis. The first hypothesis is $\mathcal{H}_1$, which tests the null hypothesis that $d = 1$, i.e. that the model is a CVAR. The rest of the hypotheses can be conveniently segregated into tests on the cointegration vectors $\beta$ and tests for weak exogeneity of the variables on the $\alpha$ parameters.

The parameters in $\alpha$ and $\beta$ are not separately identified without additional normalization restrictions, see Johansen (1995). All our empirical results impose an identification restriction which normalizes the $\beta$ matrix with respect to the political variables. This seems to be the natural normalization in this context and has the added benefit in our analysis that it makes interpretation of the equilibrium relations easier and more intuitive in terms of analyzing the long-run dynamics of each political party separately.

Our primary interest in the cointegrating vectors is whether or not economic and political variables form a stationary long-run equilibrium. The hypotheses $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$ test

---

\(^8\)For US interest rates, we also conduct Lagrange Multiplier tests (unreported), which are robust to heteroskedasticity, and these do not reject the null hypothesis that the residuals are serially uncorrelated.
Table 3: Key for hypothesis tests

| $H^d_1$ | The fractional parameter, $d$, is equal to one. |
| $H^\beta_1$ | Political variables do not enter the cointegrating relation(s). |
| $H^\beta_2$ | Economic variables do not enter the cointegrating relation(s). |
| $H^\beta_3$ | One cointegrating relation contains only political variables and the other cointegrating relation contains only economic variables. |
| $H^\beta_4$ | US and Canadian macroeconomic variables enter the cointegrating relation(s) with equal magnitude and opposite signs. |
| $H^\beta_5$ | US macroeconomic variables do not enter the cointegrating relation(s). |
| $H^\alpha_1$ | Support for Progressive Conservatives is weakly exogenous. |
| $H^\alpha_2$ | Support for Liberals is weakly exogenous. |
| $H^\alpha_3$ | Canadian interest rates are weakly exogenous. |
| $H^\alpha_4$ | US interest rates are weakly exogenous. |
| $H^\alpha_5$ | Unemployment in Canada is weakly exogenous. |
| $H^\alpha_6$ | Unemployment in the US is weakly exogenous. |

this general hypothesis. Specifically, $H^\beta_1$ imposes zero restrictions on the $\beta$ coefficients associated with political support and $H^\beta_2$ does the same but for economic variables. When $r = 2$, i.e., in the case of two cointegrating vectors, we also test the hypothesis $H^\beta_3$ that one equilibrium relation consists of only political variables and the other one has only economic variables. If we reject these hypotheses, then we can conclude that there exists a long-run equilibrium consisting of both economic variables and political support.

In Section 5.6 we introduce US macroeconomic variables and test further interesting hypotheses on $\beta$. If voters respond only to relative economic performance, i.e., the stronger form of the “benchmarking” theory, then the coefficients on the Canadian and US economic variables will have equal and opposite signs in the cointegrating relations. This is hypothesis $H^\beta_4$. We also test that only Canadian economic variables enter the cointegrating relations, i.e., that US economic variables all have zero coefficients in the cointegrating relations. This hypothesis ($H^\beta_5$) can thus be seen as rejection of a weaker version of the “benchmarking” theory.

The matrix $\alpha$ measures the speed of adjustment for the variables in the system in response to disequilibrium errors. If the $j$’th row of $\alpha$ is zero, then variable $X_{j\ell}$ is exogenous — sometimes denoted weakly or long-run exogenous — with respect to the long-run parameters, $\alpha$ and $\beta$. That is, the variable does not respond at all to disequilibrium errors. The hypotheses $H^\alpha_1, \ldots, H^\alpha_6$ test that each of the variables are individually weakly exogenous.

We provide examples of how the hypotheses in Table 3 are formulated in terms of $\hat{\beta} = H\phi$ and $\alpha = A\psi$, see (9) and (10), in the following subsections as we describe each of the empirical models. Finally, because of the superconsistency of $\hat{\beta}$, see Johansen (1995) and Johansen and Nielsen (2012), we test hypotheses on $\beta$ first, and if any of them fail to reject, we leave them imposed when testing hypotheses on $\alpha$. 

18
5.5 Economic voting in Canada

We begin our analysis with an examination of the Canadian data. Due to the relatively short sample size for a fractional cointegration analysis, it seems natural to proceed by analyzing sub-models first to establish dynamics within a smaller system before estimating the full model. Thus, we first consider each of the parties separately and then estimate all of the variables together.

Model 1: Liberal support, interest rates and unemployment

We start with a system that includes Liberal support, the interest rate, and unemployment. The results are shown in Table 4. The first step in our estimation procedure is to select a lag order for the model, which is done using the general-to-specific testing procedure outlined in Section 5.4. For this model, we choose lag $k = 2$.

Once we have established the lag augmentation, we move on to selecting the rank. As shown in the rank tests subtable, we start with the null hypothesis that $r = 0$ and, since there are three variables in the system, test it against the alternative that $r = 3$. With a $P$ value of 0.04 we reject $r = 0$. Next, we test the null hypothesis that $r = 1$ against the same alternative hypothesis as before. This time the $P$ value is 0.82, so we do not reject the null hypothesis, and we conclude that $r = 1$.

The estimation results for $k = 2$ and $r = 1$ are shown in (14) with the equilibrium relation in (15). The estimate of $d$ is 0.569 with standard error 0.049. The residuals appear well-behaved with no evidence of serial correlation; the Ljung-Box Q-test has a $P$ value of 0.747 (reported in parenthesis below the test statistic). On the right-hand-side of (14), the estimated adjustment coefficients $\hat{\alpha}$ are shown in the vector preceding $\nu_t$, which is the stationary long-run equilibrium defined by $\nu_t = \hat{\beta}'(X_t - \hat{\mu})$. The coefficients that characterize this long-run equilibrium are normalized with respect to the political variable and presented in (15), with the constant term given by $\hat{\beta}'\hat{\mu}$. This relation suggests that liberal support is increasing in unemployment and decreasing in interest rates. The estimates of $\{\Gamma_i\}_{i=1}^2$ are suppressed since we are only concerned with long-run dynamics.

Next, we consider the relevant hypothesis tests for this model, the results of which are shown in the hypothesis tests subtable. First, as a type of model specification check, we test $H_d^1$, i.e. that $d = 1$, to see if the CVAR model is adequate. The hypothesis $H_d^1$ is very strongly rejected suggesting that the FCVAR model is a more appropriate specification than the alternative CVAR model.

We then test for the absence of economic voting, which is the hypothesis that imposes a zero restriction on the coefficient of Liberal support in the long-run equilibrium. This restriction is imposed as in (9) with

$$H_{p \times s} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$\phi_{s \times r} = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix},$$

where $(p, s, r) = (3, 2, 1)$ and $\varphi_1, \varphi_2$ are freely varying. The degrees of freedom for this test is given by $df = (p - s)r = (3 - 2)1 = 1$. The LR test statistic is 13.557 and we strongly reject the null hypothesis that liberal support is absent from the long-run equilibrium.

Since there are no other $\beta$ hypotheses that are relevant to this model, we move to tests of weak exogeneity on the $\alpha$ coefficients to determine whether or not the variables respond to

---

9Complete estimation results including estimates of $\{\Gamma_i\}_{i=1}^2$ are available upon request.
Table 4: FCVAR results for Model 1

Rank tests:

<table>
<thead>
<tr>
<th>Rank</th>
<th>( \hat{d} )</th>
<th>Log-likelihood</th>
<th>LR statistic</th>
<th>( P ) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.643</td>
<td>440.040</td>
<td>25.454</td>
<td>0.040</td>
</tr>
<tr>
<td>1</td>
<td>0.569</td>
<td>451.174</td>
<td>3.186</td>
<td>0.820</td>
</tr>
<tr>
<td>2</td>
<td>0.576</td>
<td>452.707</td>
<td>0.120</td>
<td>0.940</td>
</tr>
<tr>
<td>3</td>
<td>0.581</td>
<td>452.767</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Unrestricted model:

\[
\Delta \hat{d} \begin{bmatrix} \text{lib}_t \\ \text{ir}_t \\ \text{un}_t \end{bmatrix} - \begin{bmatrix} -0.345 \\ 11.481 \\ -2.872 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.180 \\ 0.167 \\ 0.037 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_i^j (X_t - \hat{\mu}) + \hat{\xi}_t \quad (14)
\]

\[
\hat{d} = 0.569, \quad Q_{\hat{\varepsilon}}(12) = 93.291, \quad \log(\mathcal{L}) = 451.174
\]

Equilibrium relation:

\[
\text{lib}_t = 1.624 - 0.111 \text{ir}_t + 0.240 \text{un}_t + \nu_t \quad (15)
\]

Hypothesis tests:

<table>
<thead>
<tr>
<th>( \mathcal{H}^d )</th>
<th>( \mathcal{H}^\beta )</th>
<th>( \mathcal{H}^\alpha_1 )</th>
<th>( \mathcal{H}^\alpha_2 )</th>
<th>( \mathcal{H}^\alpha_3 )</th>
<th>( \mathcal{H}^\alpha_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>df</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>LR</td>
<td>18.295</td>
<td>13.557</td>
<td>10.176</td>
<td>0.633</td>
<td>9.979</td>
</tr>
<tr>
<td>( P ) value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td><strong>0.426</strong></td>
<td>0.002</td>
</tr>
</tbody>
</table>

Restricted model:

\[
\Delta \hat{d} \begin{bmatrix} \text{lib}_t \\ \text{ir}_t \\ \text{un}_t \end{bmatrix} - \begin{bmatrix} -0.310 \\ 11.538 \\ -2.873 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.188 \\ 0.000 \\ 0.039 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_i^j (X_t - \hat{\mu}) + \hat{\xi}_t \quad (16)
\]

\[
\hat{d} = 0.575, \quad Q_{\hat{\varepsilon}}(12) = 93.125, \quad \log(\mathcal{L}) = 450.857
\]

Equilibrium relation:

\[
\text{lib}_t = 1.4344 - 0.106 \text{ir}_t + 0.182 \text{un}_t + \nu_t \quad (17)
\]

Notes: The table shows FCVAR estimation results, including cointegration rank tests and hypothesis tests. Standard errors are in parentheses below \( \hat{d} \) and \( P \) values are in parentheses below \( Q_{\hat{\varepsilon}} \). In the hypothesis tests subtable, \( P \) values in bold denote hypotheses that are imposed in the restricted model. The sample size is \( T = 316 \).
long-run disequilibrium errors. As an example, consider the test $H_{\alpha}^2$ that the first variable, in this case $\text{lib}_t$, is weakly exogenous. This null hypothesis is formulated in terms of (10) with

$$A_{p \times m} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \psi_{m \times r} = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix},$$

where $(p, m, r) = (3, 2, 1)$ and $\psi_1, \psi_2$ are freely varying. In general, the null of weak exogeneity of the $j$'th variable has $j$'th row of $\alpha$ equal to a zero row and the remaining rows equal to the identity matrix. The degrees of freedom for the test of $H_{\alpha}^2$ is $\text{df} = (p - m)r = (3 - 2)1 = 1$. We strongly reject this hypothesis as well as $H_{\alpha}^5$. However, we do not reject $H_{\alpha}^3$, which suggests that interest rates are long-run exogenous.

With the non-rejected hypotheses (only $H_{\alpha}^3$ in this case) imposed, we re-estimate the model and provide the results in (16) and (17). The signs and magnitudes of the estimated coefficients are similar to the ones obtained in the unrestricted model. Furthermore, the residuals appear to be white noise with a $P$ value of 0.39 for the Ljung-Box Q test. The interpretation of the long-run equilibrium is the same as in the unrestricted model. Namely, we find that support for the Liberal party rises in response to an increase in unemployment and declines with an increase in interest rates.

To interpret the $\alpha$ coefficients we consider an example of a shock to interest rates that pushes the system out of equilibrium. Ignoring the short-run dynamics in $\Gamma_i$ and holding everything else fixed, suppose that the interest rate increases by one percentage point. In (17), we see that the change in $ir_t$ results in an increase in $\nu_t$. In response to a fractionally lagged increase in $\nu_t$, the fractional change in Liberal support is given by its corresponding $\alpha$ coefficient, which is negative ($-0.188$), and hence pushes the system back towards equilibrium, given by $\nu_t = 0$. Likewise, the positive $\alpha$ coefficient ($0.039$) on unemployment causes it to increase and also push the system back towards equilibrium. The magnitudes imply that Liberal support moves towards equilibrium quicker than does unemployment. The weak exogeneity restriction on the interest rate implies that it does not respond to shocks to the long-run equilibrium. This latter result may suggest interest rates respond to the rate set by the the Bank of Canada rather than to movements in the long-run equilibrium between the variables in our system.

We further examine the robustness of these results in Appendix B. Table 10 reports estimates for the model using the alternative political cycle removal process and Table 11 considers a subsample of the data ending just prior to the 1993 election. In both cases, the hypothesis tests lead to similar conclusions and the final restricted models closely resemble the one shown in Table 4.

**Model 2: PC support, interest rates and unemployment**

We now perform the same analysis for the Progressive Conservative party. The results are reported in Table 5. Once again we choose $k = 2$ from the general-to-specific testing method. The rank tests subtable shows a $P$ value of 0.12 for the test that $r = 0$ against the alternative that the system is full rank and 0.75 for the null that $r = 1$. Because the cointegration rank tests can have low power in small samples, we proceed to estimate the model with rank $r = 1$ and present the results in (18) and the long-run equilibrium in (19). The estimate of $d$ is very similar to that obtained in the model with the Liberal party and again the residuals appear
Table 5: FCVAR results for Model 2

Rank tests:

<table>
<thead>
<tr>
<th>Rank</th>
<th>$\hat{d}$</th>
<th>Log-likelihood</th>
<th>LR statistic</th>
<th>$P$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.645</td>
<td>358.747</td>
<td>21.244</td>
<td>0.120</td>
</tr>
<tr>
<td>1</td>
<td>0.599</td>
<td>367.367</td>
<td>4.003</td>
<td>0.750</td>
</tr>
<tr>
<td>2</td>
<td>0.605</td>
<td>369.256</td>
<td>0.225</td>
<td>0.910</td>
</tr>
<tr>
<td>3</td>
<td>0.613</td>
<td>369.369</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Unrestricted model:

$$\Delta^{\hat{d}} \left( \begin{bmatrix} pc_t \\ ir_t \\ un_t \end{bmatrix} - \begin{bmatrix} 0.878 \\ 11.525 \\ -2.871 \end{bmatrix} \right) = L^{\hat{d}} \begin{bmatrix} -0.065 \\ 0.025 \\ -0.018 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta^{\hat{d}} L_i^{\hat{d}} (X_t - \hat{\mu}) + \hat{\varepsilon}_t$$

(18)

$\hat{d} = 0.599$, $Q_{\hat{\varepsilon}}(12) = 109.164$, $\log(\mathcal{L}) = 367.367$

Equilibrium relation:

$$pc_t = -3.787 + 0.230 ir_t - 0.702 un_t + \nu_t$$

(19)

Hypothesis tests:

<table>
<thead>
<tr>
<th>$\mathcal{H}_1^{d}$</th>
<th>$\mathcal{H}_1^{\beta}$</th>
<th>$\mathcal{H}_1^{\alpha}$</th>
<th>$\mathcal{H}_3^{\alpha}$</th>
<th>$\mathcal{H}_5^{\alpha}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>df</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>LR</td>
<td>9.973</td>
<td>8.619</td>
<td>3.568</td>
<td>0.063</td>
</tr>
<tr>
<td>$P$ value</td>
<td>0.002</td>
<td>0.003</td>
<td>0.059</td>
<td><strong>0.802</strong></td>
</tr>
</tbody>
</table>

Restricted model:

$$\Delta^{\hat{d}} \left( \begin{bmatrix} pc_t \\ ir_t \\ un_t \end{bmatrix} - \begin{bmatrix} 0.894 \\ 11.504 \\ -2.870 \end{bmatrix} \right) = L^{\hat{d}} \begin{bmatrix} -0.064 \\ 0.000 \\ -0.018 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta^{\hat{d}} L_i^{\hat{d}} (X_t - \hat{\mu}) + \hat{\varepsilon}_t$$

(20)

$\hat{d} = 0.599$, $Q_{\hat{\varepsilon}}(12) = 109.316$, $\log(\mathcal{L}) = 367.336$

Equilibrium relation:

$$pc_t = -3.884 + 0.235 ir_t - 0.721 un_t + \nu_t$$

(21)

Notes: The table shows FCVAR estimation results, including cointegration rank tests and hypothesis tests. Standard errors are in parentheses below $\hat{d}$ and $P$ values are in parentheses below $Q_{\hat{\varepsilon}}$. In the hypothesis tests subtable, $P$ values in bold denote hypotheses that are imposed in the restricted model. The sample size is $T = 316$. 
to be white noise. Compared to the previous model, the economic variables now enter the cointegrating relation with opposite signs in the equilibrium in (19). That is, support for the Progressive Conservative party decreases in unemployment and increases in the interest rate.

Hypothesis testing yields exactly the same results as in Table 4. The model is clearly fractional as indicated by $\mathcal{H}_1^d$, there is evidence of economic voting as confirmed by $\mathcal{H}_1^\beta$, and, once again, only interest rates are weakly exogenous.

The restricted model estimates are shown in (20) and (21). The estimate of $d$ and the equilibrium relation are very similar to those from the unrestricted model. The $P$ value for the statistic $Q_\hat{\varepsilon}$ is large, suggesting that the residuals are white noise. Both unemployment and PC support adjust towards equilibrium in response to shocks to the long-run equilibrium.

Once again, we check these results for robustness with the political cycle removed using time-since-last-election as well as using the 1974 to 1993 subsample. The results are in Tables 12 and 13, respectively. For the restricted models, the coefficient estimates are quite similar to the findings in Table 5. Most importantly, the qualitative result that PC support moves in the opposite direction to unemployment and in the same direction as interest rates in the long-run equilibrium is robust.

Model 3: PC support, Liberal support, interest rates and unemployment

Our next model combines the two previous models into one system to allow for more intricate dynamics. We include support for the PCs and Liberals, along with the interest and unemployment rates and report the results in Table 6. After general-to-specific testing for lag selection, we choose $k = 2$. For the rank test, we reject both $r = 0$ and $r = 1$ and estimate the model with $r = 2$, which fails to reject with a $P$ value of 0.71. The estimates of the unrestricted model are shown in (22), and the Ljung-Box Q-test shows no signs of serial correlation in the residuals. Because $r = 2$ there are two cointegrating relations, and with an appropriate normalization we can write them in terms of support for the PCs, see (23), and support for the Liberals, see (24). These equations show that support for the PCs increases with the interest rate and decreases with the unemployment rate and that the reverse is true for Liberal support, which is exactly what we found in the two previous models. To investigate these results further, we next test the relevant hypotheses for this model.

First, we test $\mathcal{H}_1^d$ and as expected, the null hypothesis is strongly rejected, with a $P$ value of 0.000. We thus move on to hypothesis testing involving the cointegration vectors in the columns of $\beta$. In order to determine that there exists a long-run relationship between macroeconomic conditions and political support we verify that it is not just political variables or economic variables entering each of the cointegrating relations separately. This is done by testing the hypotheses $\mathcal{H}_1^\beta$, $\mathcal{H}_2^\beta$, and $\mathcal{H}_3^\beta$. In the present model with two cointegration vectors, the formulation of $\mathcal{H}_1^\beta$ and $\mathcal{H}_2^\beta$ in terms of (9) has

$$H_{p \times s} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \varphi_{s \times r} = \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix} \quad \text{and} \quad H_{p \times s} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \varphi_{s \times r} = \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix},$$

where, in both cases, $(p, s, r) = (4, 2, 2)$ and the degrees of freedom is $df = (p - s)r = (4 - 2)2 = 4$. From the hypothesis tests subtable of Table 6 it is seen that $\mathcal{H}_1^\beta$, which
Table 6: FCVAR results for Model 3

Rank tests:

<table>
<thead>
<tr>
<th>Rank</th>
<th>( \hat{d} )</th>
<th>Log-likelihood</th>
<th>LR statistic</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.635</td>
<td>517.577</td>
<td>44.919</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>0.573</td>
<td>530.347</td>
<td>19.379</td>
<td>0.120</td>
</tr>
<tr>
<td>2</td>
<td>0.554</td>
<td>538.032</td>
<td>4.009</td>
<td>0.710</td>
</tr>
<tr>
<td>3</td>
<td>0.541</td>
<td>540.012</td>
<td>0.050</td>
<td>0.970</td>
</tr>
<tr>
<td>4</td>
<td>0.544</td>
<td>540.037</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Unrestricted model:

\[
\Delta \hat{d} \left( \begin{bmatrix} pc_t \\ \text{lib}_t \\ ir_t \\ \text{un}_t \end{bmatrix} - \begin{bmatrix} 0.836 \\ -0.340 \\ 11.372 \\ -2.872 \end{bmatrix} \right) = L_{\hat{d}} \begin{bmatrix} -0.186 & -0.198 \\ -0.040 & -0.276 \\ 0.406 & 0.830 \\ -0.015 & 0.015 \end{bmatrix} \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} + \sum_{i=1}^{2} \widehat{\Gamma}_i \Delta \hat{d} L_{\hat{d}}^i (X_t - \hat{\mu}) + (22)
\]

\( \hat{d} = 0.554, \quad Q_{\hat{e}}(12) = 168.266, \quad \log(\mathcal{L}) = 538.032 \)

Equilibrium relations:

\[
pc_t = -3.893 + 0.195 ir_t - 0.874 \text{un}_t + \nu_{1t} \quad (23)
\]

\[
\text{lib}_t = 1.615 - 0.108 ir_t + 0.251 \text{un}_t + \nu_{2t} \quad (24)
\]

Hypothesis tests:

\begin{align*}
\mathcal{H}_1^d & \quad \mathcal{H}_1^\beta & \quad \mathcal{H}_2^\beta & \quad \mathcal{H}_3^\beta & \quad \mathcal{H}_1^\alpha & \quad \mathcal{H}_2^\alpha & \quad \mathcal{H}_3^\alpha & \quad \mathcal{H}_5^\alpha \\
df & 1 & 4 & 4 & 2 & 2 & 2 & 2 & 2 \\
P value & 0.000 & 0.000 & 0.000 & 0.003 & 0.033 & 0.002 & \textbf{0.258} & 0.005
\end{align*}

Restricted model:

\[
\Delta \hat{d} \left( \begin{bmatrix} pc_t \\ \text{lib}_t \\ ir_t \\ \text{un}_t \end{bmatrix} - \begin{bmatrix} 0.920 \\ -0.268 \\ 11.427 \\ -2.868 \end{bmatrix} \right) = L_{\hat{d}} \begin{bmatrix} -0.155 & -0.178 \\ -0.064 & -0.345 \\ 0.000 & 0.000 \\ -0.019 & 0.007 \end{bmatrix} \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} + \sum_{i=1}^{2} \widehat{\Gamma}_i \Delta \hat{d} L_{\hat{d}}^i (X_t - \hat{\mu}) + (25)
\]

\( \hat{d} = 0.559, \quad Q_{\hat{e}}(12) = 169.568, \quad \log(\mathcal{L}) = 536.679 \)

Equilibrium relations:

\[
pc_t = -4.235 + 0.218 ir_t - 0.929 \text{un}_t + \nu_{1t} \quad (26)
\]

\[
\text{lib}_t = 1.265 - 0.099 ir_t + 0.139 \text{un}_t + \nu_{2t} \quad (27)
\]

Notes: The table shows FCVAR estimation results, including cointegration rank tests and hypothesis tests. Standard errors are in parentheses below \( \hat{d} \) and P values are in parentheses below \( Q_{\hat{e}} \). In the hypothesis tests subtable, P values in bold denote hypotheses that are imposed in the restricted model. The sample size is \( T = 316 \).
imposes zero restrictions on the political variables in the cointegration vectors, as well as $H^\beta_2$, which imposes zero restrictions on the economic variables in the cointegrating vectors, are both strongly rejected. Since we have two cointegrating relations in this specification, we also test $H^\beta_3$, i.e., the hypothesis that economic variables enter one cointegrating relation and political variables enter the other one. For this hypothesis, we impose different restrictions on each of the two columns of $\beta$, and in terms of (9), we impose

$$ \beta_1 = H_1 \varphi_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_{11} \\ \varphi_{21} \end{bmatrix} \quad \text{and} \quad \beta_2 = H_2 \varphi_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varphi_{12} \\ \varphi_{22} \end{bmatrix}, $$

where the subscript on $\beta$ corresponds to the column number. The degrees of freedom is $df = (p - r - s_1 + 1) + (p - r - s_2 + 1) = 2$. As seen in the hypothesis tests subtable, this restriction is also strongly rejected. These results confirm our earlier findings from Models 1 and 2 in Tables 4 and 5 that there is evidence of economic voting.

As for tests on the parameters of $\alpha$, with two cointegrating relations testing for weak exogeneity amounts to testing that an entire row of $\alpha$ is equal to zero. We find that PC support, Liberal support and unemployment are all endogenous in the system, meaning that they respond to shocks to the disequilibrium errors, but that interest rates are weakly exogenous. For example, the latter hypothesis, i.e. $H^\alpha_3$, is imposed by the formulation (10) with

$$ A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \psi = \begin{bmatrix} \psi_{11} & \psi_{21} \\ \psi_{12} & \psi_{22} \\ \psi_{13} & \psi_{23} \end{bmatrix}, $$

where $(p, m, r) = (4, 3, 2)$ and the degrees of freedom is $df = (p - m)r = (4 - 3)2 = 2$.

The restricted parameter estimates are shown in (25) with the equilibrium relations in (26) and (27). The signs on all of the variables are the same as in the unrestricted model and the coefficient magnitudes are also very similar. The estimate of $d$ is very close to that of the unrestricted model and the residuals appear serially uncorrelated. The restricted model results confirm the findings from Models 1 and 2. That is, a rise in unemployment shifts support away from the Progressive Conservative party and towards the Liberal party, whereas a rise in interest rates has the opposite effect.

Since there are two cointegrating vectors in this model, the adjustment dynamics are more complex. As an example, we consider again a one percentage point rise in the interest rate that pushes the system out of equilibrium and again we ignore the short-run dynamics in $\{\Gamma_t\}$. Holding everything else constant, the effect on the equilibrium errors is

$$ \nu_{1t} = -0.218 \quad \text{and} \quad \nu_{2t} = 0.099, $$

where the magnitudes correspond to the coefficient on $ir_t$ in each equilibrium relation. The adjustment associated with (fractional differences of) $pc_t$ is

$$ \alpha_{11} \nu_{1t} + \alpha_{12} \nu_{2t} = -0.155(-0.218) - 0.178(0.099) = 0.016, $$

which is positive and implies that the series moves the system back towards equilibrium, i.e. it pushes $\nu_{1t}$ back up towards zero. For $lib_t$, the adjustment is

$$ \alpha_{12} \nu_{1t} + \alpha_{22} \nu_{2t} = -0.064(-0.218) - 0.345(0.099) = -0.020, $$

25
so that \( \text{lib}_t \) moves back towards equilibrium, i.e. it pushes \( \nu_{2t} \) down towards zero. Finally, \( \text{un}_t \) adjusts with

\[
\alpha_{13}\nu_{1t} + \alpha_{23}\nu_{2t} = -0.019(-0.218) + 0.007(0.099) = 0.005,
\]

which has a positive impact on \( \nu_{1t} \) in (26) and a negative impact on \( \nu_{2t} \) in (27), thus pushing both cointegrating relations towards equilibrium.

This model is estimated also in Appendix B with results from the alternative specification of the political cycle given in Table 14 and results using the subsample from 1974 to 1993 given in Table 15. The final restricted model in Table 15 has very similar magnitudes and all of the same signs for the coefficients as presented here. The same is true for Table 14 with the exception of the \( \alpha \) coefficient for \( \text{pc}_t \) corresponding to the second cointegrating vector, which has the opposite sign of what we found in this model. Nevertheless, the coefficient is small enough in magnitude to ensure that support for the PC party adjusts towards equilibrium in response to a one percentage point increase in the interest rate.\(^{10}\)

The findings in regards to unemployment in this section appear quite intuitive in terms of Canadian politics. The Liberal party has historically been more left-leaning and socially oriented than the Progressive Conservatives (Azoulay, 1999), so when unemployment is high voters respond by supporting the political party that is more likely to provide social security. On the other hand, when unemployment is low voters shift support to a more conservative party. Thus, in the long-run equilibrium, low unemployment and also high interest rates are associated with increased support for the PCs, while high unemployment and low interest rates are associated with increased support for the Liberal party.

5.6 Relative economic performance

In this section we consider the economic performance of Canada relative to the United States and how this relationship affects political support, i.e. the “benchmarking” theory. We begin with a system containing support for the Liberal party as well as Canadian and US interest rates and unemployment rates. This model is presented in Table 7. The results from a system with support for the PC party and the same macroeconomic variables can be found in Table 8 and finally we present our results from the full system (Liberal support, PC support, Canadian and US interest rates, and Canadian and US unemployment rates) in Table 9.

Model 4: Liberal support, interest rates and unemployment in US and Canada

In the first model, which includes support for the Liberal party together with the macroeconomic variables, we choose two lags from the general-to-specific testing procedure. We also choose a rank of one, as the rank test at \( r = 0 \) rejects the null hypothesis that there is no cointegration between the variables in the system with a \( P \) value of 0.020. Results from estimating the unrestricted model are found in (28) with the cointegrating relation in (29). The Ljung-Box test shows no signs of serial correlation. Surprisingly, the cointegrating relation now implies that support for the Liberal party is decreasing in the Canadian unemployment rate, contradictory to our findings in Section 5.5. Fortunately, this oddity disappears after we perform relevant hypothesis testing and estimate the restricted model, see (30).

\(^{10}\)Note that \( \alpha_{11}\nu_{1t} + \alpha_{12}\nu_{2t} = -0.057(-0.316) + 0.003(0.112) = 0.0183, \) which is still positive.
Table 7: FCVAR results for Model 4

<table>
<thead>
<tr>
<th>Rank</th>
<th>( \hat{d} )</th>
<th>Log-likelihood</th>
<th>LR statistic</th>
<th>( P ) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.649</td>
<td>915.250</td>
<td>55.640</td>
<td>0.020</td>
</tr>
<tr>
<td>1</td>
<td>0.640</td>
<td>932.082</td>
<td>21.974</td>
<td>0.580</td>
</tr>
<tr>
<td>2</td>
<td>0.662</td>
<td>939.716</td>
<td>6.706</td>
<td>0.950</td>
</tr>
<tr>
<td>3</td>
<td>0.673</td>
<td>941.864</td>
<td>2.411</td>
<td>0.940</td>
</tr>
<tr>
<td>4</td>
<td>0.684</td>
<td>943.034</td>
<td>0.072</td>
<td>0.970</td>
</tr>
<tr>
<td>5</td>
<td>0.687</td>
<td>943.069</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Unrestricted model:

\[
\Delta^d \begin{bmatrix} \text{lib}_t \\ \text{ir}_t \\ \text{ir}_us_t \\ \text{un}_t \\ \text{un}_us_t \end{bmatrix} - \begin{bmatrix} -0.254 \\ 11.515 \\ 11.975 \\ -2.894 \\ -2.802 \end{bmatrix} = L_d \begin{bmatrix} -0.079 \\ -0.133 \\ 0.112 \\ 0.003 \\ 0.028 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta^d L_i^d (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (28)
\]

\[\hat{d} = 0.640, \quad Q_{\hat{\varepsilon}}(12) = 330.757, \quad \log(\mathcal{L}) = 932.082\]

Equilibrium relation:

\[\text{lib}_t = -0.093 \text{ir}_t - 0.055 \text{ir}_us_t - 0.194 \text{un}_t + 1.115 \text{un}_us_t + \nu_t \quad (29)\]

Hypothesis tests:

<table>
<thead>
<tr>
<th>( H_0^d )</th>
<th>( H_1 )</th>
<th>( H_4 )</th>
<th>( H_5 )</th>
<th>( H_6^a )</th>
<th>( H_7 )</th>
<th>( H_8 )</th>
<th>( H_9 )</th>
<th>( H_{10} )</th>
<th>( H_{11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>df</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>LR</td>
<td>28.457</td>
<td>8.461</td>
<td>18.481</td>
<td>5.754</td>
<td>5.584</td>
<td>0.082</td>
<td>0.547</td>
<td>1.350</td>
<td>14.147</td>
</tr>
<tr>
<td>( P ) value</td>
<td>0.000</td>
<td>0.004</td>
<td>0.000</td>
<td>0.056</td>
<td>0.018</td>
<td>0.774</td>
<td>0.234</td>
<td>0.245</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Restricted model:

\[
\Delta^d \begin{bmatrix} \text{lib}_t \\ \text{ir}_t \\ \text{ir}_us_t \\ \text{un}_t \\ \text{un}_us_t \end{bmatrix} - \begin{bmatrix} -0.249 \\ 11.530 \\ 11.907 \\ -2.886 \\ -2.803 \end{bmatrix} = L_d \begin{bmatrix} -0.103 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.030 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta^d L_i^d (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (30)
\]

\[\hat{d} = 0.684, \quad Q_{\hat{\varepsilon}}(12) = 328.644, \quad \log(\mathcal{L}) = 926.556, \quad \text{LR}(df=5) = 11.052 \quad (0.050)\]

Equilibrium relation:

\[\text{lib}_t = 1.666 - 0.098 \text{ir}_t + 0.272 \text{un}_t + \nu_t \quad (31)\]

Notes: The table shows FCVAR estimation results, including cointegration rank tests and hypothesis tests. Standard errors are in parentheses below \( \hat{d} \) and \( P \) values are in parentheses below \( Q_{\hat{\varepsilon}} \). In the hypothesis tests subtable, \( P \) values in bold denote hypotheses that are imposed in the restricted model. Sample size is \( T = 316 \).
It is important to note that this model and the rest of the models in this section have a large number of parameters to estimate. With $p = 5$ variables, $k = 2$ lags, and rank $r = 1$, there are 65 parameters in this model, without including the covariance matrix. Compared to the largest model in the previous section (with $p = 4$, $k = 2$, and $r = 2$), this is an additional ten parameters. We take this into consideration and use intuition from the preceding models, in addition to statistical significance, when interpreting the results in this section, in particular the results of the cointegration rank tests.

As usual, we first test the hypothesis that $d = 1$ and reject that the CVAR model is adequate in this setting. We also examine whether or not the cointegrating relation is comprised solely of economic variables, $H_β^β$, and again this hypothesis is strongly rejected.

The next two hypothesis tests on $β$ are particularly relevant to test the idea of relative economic performance, and are therefore unique to the models with both Canadian and US variables. Specifically, $H_4^β$ is the hypothesis that US and Canadian variables enter the cointegrating relation with coefficients of equal magnitude and opposite direction. Or, in other words, that it is purely the relative economic performance between the two countries that belongs in the long-run equilibrium. The restriction is imposed using (9) as

$$H = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1 \\
0 & 0 & -1
\end{bmatrix} \quad \text{and} \quad \varphi = \begin{bmatrix}
\varphi_1 \\
\varphi_2 \\
\varphi_3
\end{bmatrix},$$

where $\varphi_1$, $\varphi_2$, and $\varphi_3$ are freely varying. Even though we are not imposing exclusion restrictions, we still limit the number of freely varying parameters so the degrees of freedom is calculated as before. The degrees of freedom for this test is $df = (p - s)r = (5 - 3)1 = 2$, and the hypothesis is strongly rejected. The final hypothesis test we perform on the cointegrating relation is $H_5^β$, i.e. that US variables can in fact be excluded from the long-run equilibrium. The $P$ value for this test is 0.056 and is greater by several magnitudes when compared to the $P$ values for the other $β$ tests, and, as a result, we do not reject $H_5^β$.

As mentioned in Section 5.4, since we fail to reject $H_5^β$, we leave it imposed for the hypothesis tests on $α$. That is, the LR tests on $α$ are performed with $H_5^β$ as the unrestricted model. Besides this modification, the $α$ restriction testing proceeds the same way as in Section 5.5; we individually test whether each variable is exogenous to the disequilibrium errors. In this case we find that Canadian and US interest rates are weakly exogenous and that Canadian unemployment is weakly exogenous. The two variables that appear to respond to disequilibrium errors are support for the Liberal party and the US unemployment rate. While this result appears puzzling, it could be that the US unemployment rate is responding to fluctuations between the macroeconomic variables in the system and not necessarily political support in Canada.

Imposing all three $α$ restrictions as well as the $β$ restriction that US variables do not enter the cointegrating relation, we estimate the restricted model as shown in (30). Based on the $Q_ε$ statistic, it appears that the residuals are white noise. Once we impose all the

11Recall that $θ = \{d, α, β, Γ_1, \ldots, Γ_k, Ω, μ\}$, where $d$ is scalar, $α$ has five parameters, $β$ has four parameters after identification, $Γ_i$ has 25 parameters per lag, and $μ$ has five parameters.
restrictions we find that the cointegrating relation, presented in (31), is very similar to the Canada-only Model 1 in Table 4. The relation again implies that Liberal support is decreasing in Canadian interest rates and increasing in Canadian unemployment. Since we have imposed several restrictions we also provide an additional LR test statistic, reported below equation (30), for the joint null hypothesis that all restrictions should be imposed, the $P$ value of which is 0.050. The degrees of freedom are given by the summation of the degrees of freedom of the individual tests.

Robustness checks for this model are presented in Table 16 of Appendix B. Since the models with US variables include a minimum of five variables, which means a large number of parameters to estimate, we do not perform robustness checks using the 1974 to 1993 subsample as there would be too many parameters to estimate for such a small sample. The results in Table 16 are very similar to those in Table 7, with the exception of the cointegrating relation, see (71), in which Canadian and US unemployment enter in opposite directions from (29). However, this discrepancy disappears when we impose restrictions on $\alpha$ and $\beta$ and estimate the restricted model. The cointegrating relation from the restricted model is presented in (73) and is very close to the relation in (31).

Model 5: PC support, interest rates and unemployment in US and Canada

Next we replace Liberal support with support for the PC party and estimate the system including both US and Canadian macroeconomic variables. In Table 8 we display these results. Using the same lag-selection process as previously we find lag $k = 2$ and the rank test provides strong evidence of one cointegration vector, $r = 1$. The initial unrestricted results are in (32) and the cointegrating relation is in (33). The direction and magnitude of the Canadian variables in the cointegrating relation are consistent with the estimates from the Canada-only Model 2 in Section 5.5.

We perform all the same hypothesis testing as in the previous model with Liberal support. In this case we fail to reject the same $\beta$ hypothesis, namely $\mathcal{H}_5^\beta$ that the US variables do not enter the cointegrating relation, while all other $\beta$ hypotheses are rejected. In particular, the hypothesis $\mathcal{H}_4^\beta$ that the relative performance of US and Canadian economic variables enters the cointegrating relation is again strongly rejected.

Since we failed to reject hypothesis $\mathcal{H}_5^\beta$ we leave that restriction imposed and perform our usual tests on $\alpha$. Again, we fail to reject $\alpha$ restrictions of weak exogeneity of both US and Canadian interest rates and of Canadian unemployment. The $P$ value for the hypothesis test that PC support is weakly exogenous ($\mathcal{H}_1^\alpha$) is 0.115, but we do not impose it based on the findings from the previous models. The restricted model is shown in equations (34) and (35) and the results are very similar to equations (20) and (21) from the Canada-only Model 2. A potential issue with this model is that the Ljung-Box test shows some signs of serial correlation; in the restricted model the $P$ value for the $Q_\varepsilon$ test is 0.029. However, we also performed Lagrange Multiplier tests on each of the residuals individually and the associated $P$ values on these tests are all well over 0.100, with the exception of that on the residuals from the US interest rate equation, which is 0.094. This seems to be sufficient evidence to conclude that the residuals are not serially correlated and that the model appears to be well specified. Once again, since we have imposed several restrictions, we include the LR test of the model with all of the restrictions imposed against the unrestricted model. The test does not reject the restricted model and confirms that the restrictions are appropriate.
### Table 8: FCVAR results for Model 5

<table>
<thead>
<tr>
<th>Rank</th>
<th>( \hat{d} )</th>
<th>Log-likelihood</th>
<th>LR statistic</th>
<th>( P ) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.633</td>
<td>832.530</td>
<td>54.657</td>
<td>0.020</td>
</tr>
<tr>
<td>1</td>
<td>0.653</td>
<td>848.209</td>
<td>23.299</td>
<td>0.530</td>
</tr>
<tr>
<td>2</td>
<td>0.683</td>
<td>855.462</td>
<td>8.792</td>
<td>0.890</td>
</tr>
<tr>
<td>3</td>
<td>0.673</td>
<td>858.860</td>
<td>1.997</td>
<td>0.960</td>
</tr>
<tr>
<td>4</td>
<td>0.681</td>
<td>859.838</td>
<td>0.040</td>
<td>0.980</td>
</tr>
<tr>
<td>5</td>
<td>0.683</td>
<td>859.858</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Unrestricted model:

\[
\Delta^{\hat{d}} \begin{bmatrix} pc_t \\ ir_t \\ ir_{us_t} \\ un_t \\ un_{us_t} \end{bmatrix} - \begin{bmatrix} 0.835 \\ 11.542 \\ 12.120 \\ -2.895 \\ -2.798 \end{bmatrix} = L^{\hat{d}} \begin{bmatrix} -0.028 \\ 0.111 \\ 0.006 \\ -0.002 \\ -0.016 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}^{\hat{d}} L^{\hat{d}} (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (32)
\]

\[\hat{d} = 0.653, \quad Q_{\hat{d}}(12) = 334.363, \quad \log(\mathcal{L}) = 848.209\]

Equilibrium relation:

\[pc_t = -7.795 + 0.177 ir_t + 0.097 ir_{us_t} - 0.398 un_t - 1.522 un_{us_t} + \nu_t \quad (33)\]

Hypothesis tests:

\[
\begin{array}{cccccccccc}
\mathcal{H}^d_1 & \mathcal{H}^\beta_1 & \mathcal{H}^\beta_4 & \mathcal{H}^\beta_5 & \mathcal{H}^\alpha_1 & \mathcal{H}^\alpha_3 & \mathcal{H}^\alpha_4 & \mathcal{H}^\alpha_5 & \mathcal{H}^\alpha_6 & \mathcal{H}^\alpha_6 \\
\text{df} & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\
\text{LR} & 22.723 & 6.775 & 17.647 & 2.949 & 2.480 & 1.244 & 0.015 & 1.893 & 19.222 \\
\text{P value} & 0.000 & 0.009 & 0.000 & \textbf{0.229} & 0.115 & 0.265 & \textbf{0.903} & \textbf{0.169} & 0.000 \\
\end{array}
\]

Restricted model:

\[
\Delta^{\hat{d}} \begin{bmatrix} pc_t \\ ir_t \\ ir_{us_t} \\ un_t \\ un_{us_t} \end{bmatrix} - \begin{bmatrix} 0.873 \\ 11.564 \\ 12.024 \\ -2.884 \\ -2.796 \end{bmatrix} = L^{\hat{d}} \begin{bmatrix} -0.046 \\ 0.000 \\ 0.000 \\ -0.017 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}^{\hat{d}} L^{\hat{d}} (X_t - \hat{\mu}) + \hat{\varepsilon}_t \quad (34)
\]

\[\hat{d} = 0.705, \quad Q_{\hat{d}}(12) = 332.259, \quad \log(\mathcal{L}) = 844.621, \quad \text{LR(df=5)} = 7.176 \quad (0.208)\]

Equilibrium relation:

\[pc_t = -4.973 + 0.2065 ir_t - 1.120 un_t + \nu_t \quad (35)\]

Notes: The table shows FCVAR estimation results, including cointegration rank tests and hypothesis tests. Standard errors are in parentheses below \( \hat{d} \) and \( P \) values are in parentheses below \( Q_{\varepsilon} \). In the hypothesis tests subtable, \( P \) values in bold denote hypotheses that are imposed in the restricted model. Sample size is \( T = 316 \).
We present the results of the robustness check in Table 17 in Appendix B. The cointegrating equilibrium is similar to that in Table 8 and consistent with the models presented in Section 5.5, showing that the relationship between support for the PCs and the economic variables is robust.

Model 6: PC support, Liberal support, interest rates and unemployment in Canada and the US

Finally, we consider the system with both parties as well as economic variables from both countries. The results are presented in Table 9. We choose two lags from the general-to-specific procedure. The rank tests strongly reject rank of zero, while the $P$ value for rank one is quite large (0.260). However, due to the small sample size combined with the large number of parameters that we are estimating, we nevertheless estimate the system with rank two. This is in accordance with our prior evidence from the Canadian analysis in Model 3. Moreover, since we found one cointegrating vector in Table 7 and another one in Table 8 and neither were composed only of economic variables, then it should be the case that there are two cointegrating vectors when the variables are combined as in the current model.

The unrestricted estimation is in (36) and the equilibrium relations are shown in (37) and (38). As before, the Ljung-Box test shows no signs of serial correlation in the residuals. In the cointegrating relations for both parties, interest rates for the US and Canada enter with the same sign. Unemployment from the two countries enter (37) with the same sign, but that is not the case for the cointegrating relation (38) with the Liberal party, where now support is decreasing in Canadian unemployment and increasing in US unemployment.

As in the previous models, we strongly reject the hypothesis $H^d_1$ that the model is not fractional. We also reject the hypotheses that the cointegrating relations are made up entirely of either political or economic variables. Since the model has rank two, we also test and reject the hypothesis that one cointegrating relation is comprised entirely of political variables and the other one contains only economic variables. Thus, hypotheses $H^α_1$, $H^α_2$, and $H^α_3$ are all rejected as in the corresponding Canada-only Model 3.

Also, as in Models 4 and 5, we strongly reject that the difference in Canadian and US economic indicators impacts political support in Canada ($H^β_4$), but we do not reject that US economic variables do not enter the long-run equilibrium at all ($H^β_5$). The $P$ value for the latter hypothesis is 0.126.

Tests on $α$ restrictions are implemented after imposing the restriction that the US variables do not enter the cointegrating relations. The hypothesis tests on $α$ fail to reject weak exogeneity on all of the economic variables except for US unemployment and PC support, however, when we estimate the restricted model we do not include the restriction that support for the PC party is exogenous for the same reason as in Model 5. The final restricted model is shown in (39). The Ljung-Box Q-test fails to reject that there is no serial correlation in the residuals. The additional LR test for the final restricted model has a $P$ value of 0.128, confirming that the restrictions are appropriate.

The cointegrating relations from the restricted model, found in (40) and (41), are very similar to those from the analogous Canada-only Model 3 in Section 5.5. Not only are all the signs of the coefficients the same, but they are also close in magnitude.

Keeping in line with the rest of the analysis, we present the robustness check using the alternative time-since-election removal process in Table 18 of Appendix B. The restriction
Table 9: FCVAR results for Model 6

<table>
<thead>
<tr>
<th>Rank</th>
<th>( \hat{d} )</th>
<th>Log-likelihood</th>
<th>LR statistic</th>
<th>( P ) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.631</td>
<td>998.970</td>
<td>75.009</td>
<td>0.010</td>
</tr>
<tr>
<td>1</td>
<td>0.611</td>
<td>1016.729</td>
<td>39.492</td>
<td>0.260</td>
</tr>
<tr>
<td>2</td>
<td>0.611</td>
<td>1026.519</td>
<td>19.910</td>
<td>0.640</td>
</tr>
<tr>
<td>3</td>
<td>0.629</td>
<td>1033.213</td>
<td>6.522</td>
<td>0.940</td>
</tr>
<tr>
<td>4</td>
<td>0.624</td>
<td>1035.383</td>
<td>2.182</td>
<td>0.940</td>
</tr>
<tr>
<td>5</td>
<td>0.639</td>
<td>1036.437</td>
<td>0.075</td>
<td>0.970</td>
</tr>
<tr>
<td>6</td>
<td>0.636</td>
<td>1036.474</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Unrestricted model:

\[
\Delta \hat{d} \begin{pmatrix} pc_t \\ lib_t \\ ir_t \\ ir_us_t \\ un_t \\ un_us_t \end{pmatrix} - \begin{pmatrix} 0.748 \\ -0.279 \\ 11.468 \\ 12.069 \\ -2.897 \\ -2.800 \end{pmatrix} = L_{\hat{d}} \begin{pmatrix} \nu_{1t} \\ \nu_{2t} \end{pmatrix} + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d}_L(X_t - \hat{\mu}) \tag{36} \]

\( \hat{d} = 0.611, \quad Q_\varepsilon(12) = 445.156, \quad \log(\mathcal{L}) = 1026.519 \)

Equilibrium relations:

\[
\begin{align*}
    pc_t &= -7.2492 + 0.116 ir_t + 0.129 ir_us_t - 0.297 un_t - 1.517 un_us_t + \nu_{1t} \\
    lib_t &= 4.3270 - 0.102 ir_t - 0.047 ir_us_t - 0.144 un_t + 1.172 un_us_t + \nu_{2t}
\end{align*} \tag{37, 38} \]

Hypothesis tests:

<table>
<thead>
<tr>
<th>( \mathcal{H}_1^{\alpha} )</th>
<th>( \mathcal{H}_2^{\alpha} )</th>
<th>( \mathcal{H}_3^{\alpha} )</th>
<th>( \mathcal{H}_4^{\alpha} )</th>
<th>( \mathcal{H}_5^{\alpha} )</th>
<th>( \mathcal{H}_6^{\alpha} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>df</td>
<td>1</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( P ) value</td>
<td>0.000</td>
<td>0.002</td>
<td>0.000</td>
<td>0.026</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Restricted model:

\[
\Delta \hat{d} \begin{pmatrix} pc_t \\ lib_t \\ ir_t \\ ir_us_t \\ un_t \\ un_us_t \end{pmatrix} - \begin{pmatrix} 0.877 \\ -0.237 \\ 11.486 \\ 11.903 \\ -2.883 \\ -2.797 \end{pmatrix} = L_d \begin{pmatrix} \nu_{1t} \\ \nu_{2t} \end{pmatrix} + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d}_L(X_t - \hat{\mu}) \tag{39} \]

\( \hat{d} = 0.667, \quad Q_\varepsilon(12) = 444.234, \quad \log(\mathcal{L}) = 1018.959, \quad LR(df=10) = 15.121 \tag{39} \)

Equilibrium relations:

\[
\begin{align*}
    pc_t &= -4.952 + 0.202 ir_t - 1.218 un_t + \nu_{1t} \\
    lib_t &= 1.454 - 0.093 ir_t + 0.215 un_t + \nu_{2t}
\end{align*} \tag{40, 41} \]

Notes: The table shows FCVAR estimation results, including cointegration rank tests and hypothesis tests. Standard errors in parentheses below \( \hat{d} \) and \( P \) values are in parentheses below \( Q_\varepsilon \). In the hypothesis tests subtable, \( P \) values in bold denote hypotheses that are imposed in the restricted model. The sample size is \( T = 316 \).
that US variables do not enter the cointegrating relation is in fact rejected in this case and so the restricted model is somewhat different from the one in Table 9. In spite of this, however, all the signs on the Canadian variables of interest, except for interest rates in the Liberals equation, are as expected in the equilibrium relations.

The models estimated in this section have been very large, containing five or six variables. As a result, we have been cautious in the hypothesis testing and use intuition and prior evidence obtained from the Canada-only models, in addition to statistical significance, when deciding which restrictions to impose. Nevertheless, the model with both parties as well as economic variables from both the US and Canada yielded results that were consistent with our previous findings in Section 5.5 and also with the single party models presented earlier in this section. Furthermore, we were able to test two interesting hypotheses corresponding to the way Canadian voters evaluate their economic climate. We found strong evidence against the idea that Canadian voters benchmark their perceptions of the domestic economy on that of the United States. Indeed, we were able to conclude that US economic variables do not enter the long-run equilibrium of Canadian political support.

6 Concluding remarks

In this paper we analyze the concept of economic voting in Canada using a fractionally cointegrated vector autoregressive (FCVAR) model. The FCVAR framework is particularly well suited for investigating the relationship between economic polling data and macroeconomic variables because these appear to be fractionally integrated of similar orders. The flexibility of the FCVAR model allows us to test the nested CVAR model as well as interpret long-run relationships and establish some long-run causal effects. As far as we know, we are the first to consider a system estimation of economic and political variables in a fractional cointegration model with asymptotic theory supporting inference. Moreover, the FCVAR model is new to the literature and, consequently, this paper serves to outline an empirical procedure that can be used for future analyses of this type.

We find strong empirical evidence that the economy matters for public opinion. That is, from 1974 to 2000 there was a long-run stationary equilibrium between the two main parties in Canada, the Liberals and the Progressive Conservatives, and interest rates and unemployment. This relationship is robust over many specifications, including alternative election-cycle estimations and restricting our sample to the years prior to the historical 1993 election.

Within the long-run equilibrium we find that increasing interest rates and decreasing unemployment are associated with a rise in support for the PCs and a decline in support for the Liberals. In contrast, we see that Liberal support has the opposite relationship with the economic variables. Furthermore, the FCVAR model allows us to identify how the variables react to disequilibrium errors. In particular, we find that the interest rate is weakly exogenous, in the sense that it does not respond to deviations from the long-run equilibrium. This may suggest some evidence that interest rates react more to changes in the rate set by the Bank of Canada as opposed to the variables in our system.

Our findings related to the Canadian economy and political support are in line with studies from other countries that have shown a relationship between unemployment and aggregate party support [Goodhart and Bhansali 1970, Powell and Whitten 1993], but contrast Canadian studies that have found no evidence of an effect of unemployment on
voting behaviour (Monroe and Erickson, 1986; Clarke and Zuk, 1987; Johnston, 1999). Our analysis differs from some previous work in that we examine how support for the two main parties has evolved over time in correspondence with the economic climate, while some earlier studies have concentrated on support for the incumbent government.

Finally, we test the hypothesis that relative economic performance is important for voters and that public opinion in Canada can be swayed by changes in macroeconomic events in the US. We find this not to be the case. These results contrast some of the previous literature that has found relative performance to be an important component of political popularity (Kayser and Peress, 2012). However, it has been widely acknowledged that the degree of economic voting within a country varies substantially between countries (Duch and Stevenson, 2008), and so it is difficult to draw universal conclusions from a single-country study. Accordingly, an instinctive extension that emerges from our analysis is to consider a multi-country system of economic voting and political support.

References


Johansen, S. and M. Ø. Nielsen (2014). The role of initial values in nonstationary fractional time series models. QED working paper 1300, Queen’s University.


Nielsen, M. Ø. and L. Morin (2014). FCVARmodel.m: a Matlab software package for estimation and testing in the fractionally cointegrated VAR model. QED working paper 1273, Queen’s University.


A Additional plots

Figure 5: Election cycle removal for PC support

(a) support in log-odds

(b) election cycle

(c) support with election cycle removed

(d) fractionally differenced series from Panel 5(c) using estimate from the FAR(0) model: \( \hat{d} = 0.724 \)
Figure 6: Election cycle removal for Liberal support

(a) support in log-odds

(b) election cycle

(c) support with election cycle removed

(d) fractionally differenced series from Panel 6(c) using estimate from the FAR(0) model: $\hat{d} = 0.683$
Figure 7: Autocorrelation and spectral density plots (election cycle removed)

(a) autocorrelation function for PC support

(b) spectral density for PC support

(c) autocorrelation function for liberal support

(d) spectral density for liberal support
Figure 8: Autocorrelation and spectral density plots for Canadian unemployment and interest rates

(a) autocorrelation function for interest rates

(b) spectral density for interest rates

(c) autocorrelation function for unemployment

(d) spectral density for unemployment
B Robustness results

Table 10: FCVAR results for Model 1 (robustness)

Election Cycle Removed

Unrestricted model:
\[
\Delta \hat{d} \left( \begin{bmatrix} \text{lib}_t \\ \text{ir}_t \\ \text{un}_t \end{bmatrix} - \begin{bmatrix} -0.344 \\ 11.579 \\ -2.873 \end{bmatrix} \right) = L_{\hat{d}} \begin{bmatrix} -0.222 \\ 0.234 \\ 0.039 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d}_i L_{\hat{d}} (X_t - \hat{\mu}) + \hat{\epsilon}_t
\]
\[
\hat{d} = 0.553, \quad Q_{\hat{\epsilon}}(12) = 94.541, \quad \log(\mathcal{L}) = 450.649
\]

Equilibrium relation:
\[
\text{lib}_t = 1.620 - 0.116 \text{ir}_t + 0.214 \text{un}_t + \nu_t
\]

Restricted model:
\[
\Delta \hat{d} \left( \begin{bmatrix} \text{lib}_t \\ \text{ir}_t \\ \text{un}_t \end{bmatrix} - \begin{bmatrix} -0.300 \\ 11.642 \\ -2.875 \end{bmatrix} \right) = L_{\hat{d}} \begin{bmatrix} -0.231 \\ 0.000 \\ 0.040 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d}_i L_{\hat{d}} (X_t - \hat{\mu}) + \hat{\epsilon}_t
\]
\[
\hat{d} = 0.564, \quad Q_{\hat{\epsilon}}(12) = 94.275, \quad \log(\mathcal{L}) = 450.072, \quad LR(df=1) = 1.155
\]

Equilibrium relation:
\[
\text{lib}_t = 1.377 - 0.109 \text{ir}_t + 0.141 \text{un}_t + \nu_t
\]

Notes: The table shows FCVAR estimation results. The sample size is \( T = 316 \).
Table 11: FCVAR results for Model 1 (robustness)

Subsample: 1979m9-1993m7

Unrestricted model:

\[
\Delta^d \begin{bmatrix} \text{lib}_t \\ \text{ir}_t \\ \text{un}_t \end{bmatrix} - \begin{bmatrix} -0.216 \\ 11.824 \\ -2.881 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.220 \\ 0.085 \\ 0.037 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta^d L^i_{\hat{d}} (X_t - \hat{\mu}) + \hat{\epsilon}_t \tag{46}
\]

\(\hat{d} = 0.591, \quad Q_{\hat{\epsilon}}(12) = 102.732, \quad \log(\mathcal{L}) = 330.406\)

Equilibrium relation:

\[\text{lib}_t = 1.159 - 0.108 \text{ir}_t + 0.034 \text{un}_t + \nu_t \tag{47}\]

Restricted model:

\[
\Delta^d \begin{bmatrix} \text{lib}_t \\ \text{ir}_t \\ \text{un}_t \end{bmatrix} - \begin{bmatrix} -0.200 \\ 11.844 \\ -2.882 \end{bmatrix} = L_{\hat{d}} \begin{bmatrix} -0.220 \\ 0.000 \\ 0.037 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta^d L^i_{\hat{d}} (X_t - \hat{\mu}) + \hat{\epsilon}_t \tag{48}
\]

\(\hat{d} = 0.595, \quad Q_{\hat{\epsilon}}(12) = 102.196, \quad \log(\mathcal{L}) = 450.857, \quad \text{LR(df=1)} = 0.729 \tag{0.120}\)

Equilibrium relation:

\[\text{lib}_t = 1.087 - 0.107 \text{ir}_t - 0.008 \text{un}_t + \nu_t \tag{49}\]

Notes: The table shows FCVAR estimation results. The sample size is \(T = 227\).
Table 12: FCVAR results for Model 2 (robustness)

Election Cycle Removed

Unrestricted model:

\[
\Delta \hat{d} \left( \begin{bmatrix} pc_t \\ ir_t \\ un_t \end{bmatrix} - \begin{bmatrix} 0.851 \\ 11.563 \\ -2.870 \end{bmatrix} \right) = L_{\hat{d}} \begin{bmatrix} -0.051 \\ -0.030 \\ -0.012 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_{\hat{d}}^i (X_t - \hat{\mu}) + \hat{\epsilon}_t
\]

\[
\hat{d} = 0.628, \quad Q_{\hat{\epsilon}}(12) = 103.373, \quad \text{log}(L) = 386.282
\]

Equilibrium relation:

\[
\hat{d} = 0.628, \quad Q_{\hat{\epsilon}}(12) = 103.373, \quad \text{log}(L) = 386.282
\]

Restricted model:

\[
\Delta \hat{d} \left( \begin{bmatrix} pc_t \\ ir_t \\ un_t \end{bmatrix} - \begin{bmatrix} 0.834 \\ 11.597 \\ -2.871 \end{bmatrix} \right) = L_{\hat{d}} \begin{bmatrix} -0.051 \\ 0.000 \\ -0.012 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_{\hat{d}}^i (X_t - \hat{\mu}) + \hat{\epsilon}_t
\]

\[
\hat{d} = 0.632, \quad Q_{\hat{\epsilon}}(12) = 103.586, \quad \text{log}(L) = 386.180, \quad \text{LR(df=1)} = 0.204
\]

Equilibrium relation:

\[
\hat{d} = 0.632, \quad Q_{\hat{\epsilon}}(12) = 103.586, \quad \text{log}(L) = 386.180, \quad \text{LR(df=1)} = 0.204
\]

Notes: The table shows FCVAR estimation results. The sample size is \( T = 316 \).
Table 13: FCVAR results for Model 2 (robustness)

Subsample: 1979m9-1993m7

Unrestricted model:

\[
\Delta \hat{d} \begin{bmatrix} p_{c_t} \\ \hat{r}_t \\ u_n_t \end{bmatrix} - \begin{bmatrix} 0.760 \\ 11.738 \\ -2.881 \end{bmatrix} = L \hat{d} \begin{bmatrix} -0.057 \\ 0.072 \\ -0.020 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_i^d (X_t - \hat{\mu}) + \hat{\varepsilon}_t
\]

\( \hat{d} = 0.607, \quad Q(12) = 114.570, \quad \log(\mathcal{L}) = 294.879 \)

Equilibrium relation:

\[ p_{c_t} = -3.211 + 0.189 \hat{r}_t - 0.608 u_n_t + \nu_t \] (55)

Restricted model:

\[
\Delta \hat{d} \begin{bmatrix} p_{c_t} \\ \hat{r}_t \\ u_n_t \end{bmatrix} - \begin{bmatrix} 0.615 \\ -0.165 \end{bmatrix} = L \hat{d} \begin{bmatrix} -0.056 \\ 0.000 \\ -0.020 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_i^d (X_t - \hat{\mu}) + \hat{\varepsilon}_t
\]

\( \hat{d} = 0.606, \quad Q(12) = 115.044, \quad \log(\mathcal{L}) = 294.724, \quad LR(df=1) = 0.578 \) (0.309)

Equilibrium relation:

\[ p_{c_t} = -3.319 + 0.192 \hat{r}_t - 0.649 u_n_t + \nu_t \] (57)

Notes: The table shows FCVAR estimation results. The sample size is \( T = 227 \).
Table 14: FCVAR results for Model 3 (robustness)

**Election Cycle Removed**

Unrestricted model:

\[
\Delta \hat{d} \left( \begin{bmatrix} \text{pc}_t \\ \text{lib}_t \\ \text{ir}_t \\ \text{un}_t \end{bmatrix} - \begin{bmatrix} 0.903 \\ -0.267 \\ 11.482 \\ -2.866 \end{bmatrix} \right) = L_d \begin{bmatrix} -0.057 & -0.002 \\ -0.083 & -0.394 \\ -0.006 & 0.172 \\ -0.010 & 0.012 \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d}_i L_d^i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \tag{58} \]

\[\hat{d} = 0.567 \quad Q_{\varepsilon}(12) = 160.903, \quad \text{log}(\mathcal{L}) = 554.536\]

Equilibrium relations:

\[
\text{pc}_t = -4.453 + 0.332 \text{ir}_t - 0.538 \text{un}_t + \nu_{1t} \tag{59} \\
\text{lib}_t = 1.367 - 0.120 \text{ir}_t + 0.091 \text{un}_t + \nu_{2t} \tag{60} 
\]

Restricted model:

\[
\Delta \hat{d} \left( \begin{bmatrix} \text{pc}_t \\ \text{lib}_t \\ \text{ir}_t \\ \text{un}_t \end{bmatrix} - \begin{bmatrix} 0.874 \\ -0.235 \\ 11.54 \\ -2.868 \end{bmatrix} \right) = L_d \begin{bmatrix} -0.057 & 0.003 \\ -0.078 & -0.391 \\ 0.000 & 0.000 \\ -0.012 & 0.010 \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d}_i L_d^i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \tag{61} \]

\[\hat{d} = 0.575, \quad Q_{\varepsilon}(12) = 161.180, \quad \text{log}(\mathcal{L}) = 554.118, \quad \text{LR}(\text{df}=2) = 0.836 \tag{62} \]

Equilibrium relations:

\[
\text{pc}_t = -4.125 + 0.316 \text{ir}_t - 0.472 \text{un}_t + \nu_{1t} \tag{62} \\
\text{lib}_t = 1.167 - 0.112 \text{ir}_t + 0.037 \text{un}_t + \nu_{2t} \tag{63} 
\]

Notes: The table shows FCVAR estimation results. The sample size is \( T = 316 \).
Table 15: FCVAR results for Model 3 (robustness)

Subsample: 1974m9-1993m7

Unrestricted model:
\[ \Delta \hat{d} \left( \begin{array}{c} pc_t \\
lib_t \\
ir_t \\
um_t \\
\end{array} \right) - \left( \begin{array}{c} 0.723 \\
-0.314 \\
11.60 \\
-2.876 \\
\end{array} \right) = L\hat{d} \left( \begin{array}{c} -0.231 \\
-0.072 \\
0.514 \\
-0.014 \\
\end{array} \right) + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \] (64)

\[ \hat{d} = 0.494, \quad Q_{\hat{\varepsilon}}(12) = 175.205, \quad \log(\mathcal{L}) = 420.401 \]

Equilibrium relations:
\[ pc_t = -3.551 + 0.155 ir_t - 0.862 un_t + \nu_{1t} \] (65)
\[ lib_t = 1.997 - 0.127 ir_t + 0.291 un_t + \nu_{2t} \] (66)

Restricted model:
\[ \Delta \hat{d} \left( \begin{array}{c} pc_t \\
lib_t \\
ir_t \\
um_t \\
\end{array} \right) - \left( \begin{array}{c} 0.770 \\
-0.242 \\
11.71 \\
-2.878 \\
\end{array} \right) = L\hat{d} \left( \begin{array}{c} -0.168 \\
-0.081 \\
0.000 \\
-0.013 \\
\end{array} \right) + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_i (X_t - \hat{\mu}) + \hat{\varepsilon}_t \] (67)

\[ \hat{d} = 0.528, \quad Q_{\hat{\varepsilon}}(12) = 171.233, \quad \log(\mathcal{L}) = 419.015, \quad LR(df=1) = 2.772 \] (0.250)

Equilibrium relations:
\[ pc_t = -3.501 + 0.156 ir_t - 0.849 un_t + \nu_{1t} \] (68)
\[ lib_t = 1.544 - 0.1174 ir_t + 0.1431 un_t + \nu_{2t} \] (69)

Notes: The table shows FCVAR estimation results. The sample size is $T = 227$. 
Table 16: FCVAR results for Model 4 (robustness)

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Election Cycle Removed</td>
</tr>
</tbody>
</table>

**Unrestricted model:**

\[
\Delta \hat{d} \left( \begin{bmatrix} \text{lib}_t \\ \text{ir}_t \\ \text{ir}_\text{us}_t \\ \text{un}_t \\ \text{un}_\text{us}_t \end{bmatrix} - \begin{bmatrix} -0.274 \\ 11.546 \\ 11.891 \\ -2.884 \\ -2.802 \end{bmatrix} \right) = L \hat{d} \begin{bmatrix} -0.194 \\ -0.092 \\ 0.367 \\ 0.010 \\ 0.041 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_i (X_t - \hat{\mu}) + \hat{\epsilon}_t \tag{70} \]

\[
\hat{d} = 0.618, \quad Q_{\hat{\epsilon}}(12) = 327.246, \quad \log(L) = 930.543 \]

Equilibrium relations:

\[
\text{lib}_t = 1.140 - 0.050 \text{ir}_t - 0.055 \text{ir}_\text{us}_t + 0.069 \text{un}_t - 0.005 \text{un}_\text{us}_t + \nu_t \tag{71} \]

**Restricted model:**

\[
\Delta \hat{d} \left( \begin{bmatrix} \text{lib}_t \\ \text{ir}_t \\ \text{ir}_\text{us}_t \\ \text{un}_t \\ \text{un}_\text{us}_t \end{bmatrix} - \begin{bmatrix} -0.200 \\ 11.579 \\ 11.873 \\ -2.885 \\ -2.801 \end{bmatrix} \right) = L \hat{d} \begin{bmatrix} -0.160 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.036 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_i (X_t - \hat{\mu}) + \hat{\epsilon}_t \tag{72} \]

\[
\hat{d} = 0.640, \quad Q_{\hat{\epsilon}}(12) = 327.128, \quad \log(L) = 925.526, \quad \text{LR}(df=5) = 10.034 \tag{73} \]

Equilibrium relations:

\[
\text{lib}_t = 1.258 - 0.093 \text{ir}_t + 0.132 \text{un}_t + \nu_t \tag{73} \]

Notes: The table shows FCVAR estimation results. The sample size is \( T = 316 \).
Table 17: FCVAR results for Model 5 (robustness)

**Election Cycle Removed**

Unrestricted model:

\[
\Delta \hat{d} \left( \begin{array}{c}
pc_t \\
nr_t \\
ir_{us_t} \\
un_t \\
um_{us_t}
\end{array} \right) = L_{\hat{d}} \left( \begin{array}{c}
-0.154 \\
-0.063 \\
-0.376 \\
-0.014 \\
-0.030
\end{array} \right) \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_i (X_t - \hat{\mu}) + \hat{\varepsilon}_t
\]

\[
\hat{d} = 0.647, \quad Q_{\hat{\varepsilon}}(12) = 327.616, \quad \log(\mathcal{L}) = 870.704
\]

Equilibrium relations:

\[pc_t = 0.447 + 0.056nr_t + 0.104ir_{us_t} - 1.272un_t + 1.791un_{us_t} + \nu_t\]  

(75)

Restricted model:

\[
\Delta \hat{d} \left( \begin{array}{c}
pc_t \\
nr_t \\
ir_{us_t} \\
un_t \\
um_{us_t}
\end{array} \right) = L_{\hat{d}} \left( \begin{array}{c}
-0.026 \\
0.000 \\
0.000 \\
0.000 \\
-0.019
\end{array} \right) \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta \hat{d} L_i (X_t - \hat{\mu}) + \hat{\varepsilon}_t
\]

\[
\hat{d} = 0.661, \quad Q_{\hat{\varepsilon}}(12) = 332.647, \quad \log(\mathcal{L}) = 864.673, \quad LR(df=5) = 12.062
\]

Equilibrium relations:

\[pc_t = -4.230 + 0.210nr_t - 0.881un_t + \nu_t\]  

(77)

Notes: The table shows FCVAR estimation results. The sample size is \(T = 316\).
Table 18: FCVAR results for Model 6 (robustness)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>pc_t</td>
<td>-0.910</td>
<td>0.039</td>
<td>0.02</td>
</tr>
<tr>
<td>lib_t</td>
<td>-0.309</td>
<td>0.045</td>
<td>0.01</td>
</tr>
<tr>
<td>ir_t</td>
<td>11.494</td>
<td>-0.237</td>
<td>0.01</td>
</tr>
<tr>
<td>ir_us_t</td>
<td>11.771</td>
<td>-0.533</td>
<td>0.01</td>
</tr>
<tr>
<td>un_t</td>
<td>-2.890</td>
<td>-0.012</td>
<td>0.01</td>
</tr>
<tr>
<td>un_us_t</td>
<td>-2.807</td>
<td>0.006</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Equilibrium relations:

\[
\begin{align*}
pc_t &= 2.890 + 0.006 ir_t + 0.127 ir_us_t - 1.270 un_t + 2.569 un_us_t + \nu_{1t} \\
lib_t &= 2.470 - 0.073 ir_t - 0.040 ir_us_t + 0.109 un_t + 0.415 un_us_t + \nu_{2t}
\end{align*}
\]

Notes: The table shows FCVAR estimation results. The sample size is \( T = 316 \).