Stock-based Compensation Plans and Employee Incentives

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Abstract

Standard principal-agent theory predicts that large firms should not use employee stock options and other stock-based compensation to provide incentives to non-executive employees. Yet, business practitioners appear to believe that stock-based compensation improves incentives, and mounting empirical evidence points to the same conclusion. This paper provides an explanation for why stock-based incentives can be effective. In the model of this paper, employee stock options complement individual measures of performance in inducing employees to invest in firm-specific knowledge. In some situations, a contract that only consists of options is more efficient than a contract based solely on individual performance.

Keywords: Stock-based Compensation, Employee Stock Options, Optimal Incentive Contracts, Firm-specific Knowledge

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1 Introduction

Employee stock options, restricted stock, stock purchase plans, and other equity-based pay represent a substantial part of compensation for millions of U.S. employees. The National Center for Employee Ownership estimates that as of 2013, there were almost 11,000 employee stock ownership plans and 10 million employees participated in plans that provide stock options or other equity based compensation to most or all employees.¹ Such equity plans for employees are quite common even in very large companies. Well known companies that use broad-based stock option plans include Apple, Intel, EDS, Microsoft, Oracle, AT&T, Merck, DuPont, PepsiCo, Procter & Gamble, Kimberly-Clark, and others.

Despite its prevalence in practice, economists find the use of equity-based compensation for rank-and-file employees puzzling. The received wisdom is that while stock and stock options may be good incentive tools for a firm’s top executives, they are not suitable for motivating lower level employees. This is because stock-based compensation imposes too much risk on workers, as it ties their rewards to the value of the whole company on which an individual worker in a large firm has negligible influence. If local or individual measures of performance are available, the argument goes, then it should be more efficient to provide incentives using these local and individual measures rather than relying on stock based compensation such as option grants.²

This argument is perhaps most strikingly presented in Oyer and Schaefer (2005). They calibrate a standard agency model to data on actual grants of stock options to middle-level employees, and conclude that if the option grants in the studied firms were indeed used for motivational purposes, the typical firm in their sample “would be paying each employee many thousands of dollars in risk premium in order to generate added effort that the employee values at less (often much less) than $100.” (p. 131). This appears to be a vastly inefficient way to provide incentives. Oyer and Schaefer therefore argue that firms provide stock-based compensation for reasons other than incentive provision, such as sorting and retention.³

As convincing as the above argument sounds, the incentive role of option grants seems intuitively appealing and the notion that equity-based plans motivate employees is common in the popular press and among business practitioners. For example, Andrew Grove, the then chairman of Intel, argued that “When you have a company where practically all employees . . . are stock option holders or stock owners, their motivation . . . is vectored closer to the

¹http://www.esop.org/
²Throughout the paper, the discussion will be couched mostly in terms of employee stock options, but the theory applies also to restricted stock grants, stock purchase plans, and other equity-based compensation.
³See also Oyer and Schaefer (2003).
interests of the company, and the whole organization works a lot better” (Kiechel, 2003). A similar sentiment is expressed by John Doerr, a partner at Kleiner Perkins Caufield & Byers (one of the largest venture capital firms in the world): “Awarding employees options motivates them, and aligns their interests with shareholders” (Marshall, 2003). This intuition also seems to be born out in a growing number of empirical studies on broad-based option plans, including Core and Guay (2001), Kedia and Mozumdar (2002), Sesil et al (2002), Ittner, Lambert, and Larcker (2003), Black and Lynch (2004), Hochberg and Lindsey (2010), and Bryson and Freeman (2013), who all document that option grants and subsidized stock purchase plans have incentive effects. For example, Hochberg and Lindsey (2010) find a positive, causal relationship between the implied per-employee incentives of non-executive options and subsequent firm operating performance.

This paper provides a theoretical foundation for the argument that broad-based stock option plans can provide meaningful incentives. The key idea is that certain kinds of effort are easier to motivate with stock options than with contracts based on individual performance. In particular, the standard conclusion about poor incentive properties of broad-based option plans is informed by models in which workers simply exert effort in the production process; a typical example would be a maintenance worker who needs to be motivated to sweep the floor or a sales person who needs incentives to sell the firm’s product. But in many firms, such as high-tech firms, it is not enough that an employee simply exerts effort in the direction pointed by her supervisor. Rather, in order to be productive and to be able to handle their jobs, these firms’ employees first need to learn intricate details about various aspects of their company’s operations.

As a concrete example, consider the employees of Intel’s product development department and suppose they are developing a new chip for which Apple is a potential customer. The Intel engineers could simply develop the chip without worrying about questions such as When will Apple transition to the next generation chips?, What are the characteristics of an ideal chip from Apple’s perspective?, and What volume will they need? But if they actually take the time to learn about the needs of their customers from Apple, the engineers can better time their development efforts, better target the properties of the new chip to Apple’s needs, and so on. All of this makes their efforts more valuable to Intel.

This paper takes it as a starting point that firms have a need to motivate this kind of firm-specific knowledge acquisition and shows that stock-based compensation plans are particularly suitable for this purpose. The idea is that the same knowledge that helps a worker to better perform her job can also help her to better gauge the firm’s future prospects.
This private information about the firm’s prospects in turn allows the worker to better time the exercise of her options and the sale of the shares, which provides her with powerful incentives to acquire the firm-specific information.

The mechanism described above is broadly consistent with the evidence on informed trading by firm insiders. In particular, there is overwhelming evidence that insider trading indeed takes place and that it is profitable. Betzer et al (forthcoming), for example, find that only 32.1% of the insider trades appear to be non-strategic. Bris (2005), Keown and Pinkerton (1981), Meulbroek (1992), Seyhun (1992), and others provide additional evidence on the prevalence of insider trading, both legal and illegal. Moreover, Roulstone (2003) documents that firms that impose restrictions on trading by their insiders pay higher total compensation (4% - 13%), which suggests not only that employees view insider trading as profitable but also that they are willing to take pay cuts to be able to engage in it. He also finds that the firms that limit insider trading use stronger incentives than the firms that do not restrict insider trading, which is consistent with this paper’s premise that insider trading provides incentives. Finally, the evidence shows that it is not just the top managers who benefit from insider trading. Using data on employee stock purchase plans (ESPPs) and on option exercises, a recent paper by Babenko and Sen (2014) documents that rank-and-file employees have private information about their firm’s future performance which has not been incorporated into the firm’s stock price. Specifically, Babenko and Sen show that higher purchases through ESPPs (which are typically open to all employees, often with the exception of top executives) tend to be followed by better stock price performance, whereas option exercises tend to be negatively related to future abnormal returns.\textsuperscript{4} This corroborates the earlier evidence in Huddart and Lang (2003), whose data show that stock option exercise decisions of relatively junior employees contain at least as much private information as the exercise decisions of more senior employees.

The model explored in this paper has two periods. In the first, a representative worker invests in specific knowledge that increases her productivity with her current employer, such as learning about the details of the production process, the customers’ needs, the suppliers’ possibilities, the competencies of the worker’s superiors and co-workers, and so on. In the second period, the worker takes an action that generates a positive expected return for the company if and only if the worker possesses specific knowledge. The specific knowledge also allows the worker to better assess the future prospects of the whole company, which in the

\textsuperscript{4}The potential returns are quite significant. Babenko and Sen (p. 29) report that “A trading strategy that goes long in the firms that are in the top quartile of employee stock purchases and short in the firms in the bottom quartile earns 10% in annual abnormal returns.”
model is captured by the assumption that the firm has assets in place that are not directly affected by the worker’s action, but whose value is apparent to an informed worker.

The contract that the firm offers to the worker consists of a stock-based component that depends on the value of the whole company, such as employee stock options, and of a bonus based on the revenue she generates for the company. As in the standard theories, the advantage of the bonus is that it ties the risk-averse worker’s pay to a measure of her individual performance, which may be less variable and more directly affected by the worker’s actions than the value of the whole company.

The central premise of this paper is that the advantage of stock-based compensation is that its value to the worker depends on how well she is informed about the company’s prospects, which gives her an incentive to invest in specific knowledge. Accordingly, the paper’s first main result is that it is optimal to include options in the worker’s incentive contract even if the firm has access to an individual measure of her performance that is a sufficient statistic for the firm’s market value with respect to the worker’s action. This result stands in contrast to the intuition (explained earlier) that one might have based on the textbook principal-agent model.

The paper also shows that stock option-based incentives are not useful just on the margin, but can be quite powerful — for some parameter values, a pure stock-option contract is much more efficient than a contract based solely on the individual measure of the worker’s performance. In fact, there are situations in which a pure bonus contract is so inefficient that the company prefers that the worker remains uninformed, whereas a pure option contract comes close to achieving the first best outcome, even in large firms.

**Related literature**

In the absence of a compelling incentive-based theory of employee stock-options, the literature has focused on alternative explanations. Among these, the most prominent is probably the retention argument, formalized by Oyer (2004). In Oyer’s theory, stock options help firms to retain employees when the employees’ outside opportunities vary with market conditions. Given that renegotiating a worker’s employment contract whenever the market conditions change may be costly and impractical, the firm tries to design the worker’s pay so as to automatically track her outside option. Oyer shows that an option grant may provide a relatively easy way to achieve this.

Although turnover concerns are not central to the present paper, the theory developed here does shed some light on the retention effects of stock options – in particular, similar to Oyer (2004), it predicts that a broad-based stock option plan will reduce subsequent
employee turnover. However, the mechanism that gives rise to this relationship differs from the one proposed in Oyer (2004). Rather than tracking the workers’ outside opportunities, options in the current model discourage turnover because they motivate accumulation of firm-specific knowledge, which in turn makes switching employers more costly. In contrast to Oyer’s theory, the current model suggests that the negative relationship between option grants and turnover should persist even after the options vest, as long as the accumulated specific knowledge remains relevant.

The widespread use of broad-based equity plans in real world firms suggests that there may be multiple reasons why firms find such compensation plans attractive. These include sorting (Lazear, 2004), tax benefits (Babenko and Tserlukevich, 2009), the possibility that firms use employees as a source of equity capital (Core and Guay, 2001; Michelacci and Quadrini, 2005 and 2009), that firms underestimate the true costs of employee stock options due to accounting rules that apply to options (Murphy, 2002 and 2003), and that firms use stock options to exploit boundedly rational employees (Bergman and Jenter, 2007). The aim of this paper is to bring back to the debate the incentive role of broad-based option plans by suggesting and formalizing a mechanism through which employee options can serve as an effective incentive device even in large firms.

The paper is also related to the theoretical literature on insider trading. This literature is vast and its most relevant strand for the present purposes is the study of insider trading as a part of an optimal incentive contract. This strand goes back to Manne (1966), who argued (without a formal model) that trading on inside information aligns a manager’s preferences with the firm’s interests because it allows the manager to capitalize on the increase in the firm value that is due to her managerial efforts. Formal models of the idea that a firm might optimally allow for insider trading when designing managerial incentive contracts can be found in Dye (1984), Noe (1997), and Laux (2010). Dye (1986) shows that if a manager’s pay can be made conditional on her insider trading, then insider trading can signal to the firm’s owners the manager’s private information about the firm’s future earnings and this can improve the risk-sharing properties of the manager’s contract. Noe (1997) models insider trading by a firm’s risk-neutral manager who is protected by limited liability. He shows that if the manager plays a mixed strategy with respect to her effort, then allowing her to profit from insider trading can help the firm owners to extract the manager’s limited liability rent.

Knez and Simester (2001) document significant incentive effects of a firm-wide bonus plan in a company with 35,000 employees (Continental Airlines) and attribute these effects to mutual monitoring by workers within autonomous teams. In principle, firms that use employee stock options may be hoping to induce such mutual monitoring, but I am not aware of a formalization of this argument.
This mechanism, however, requires that the manager’s effort have a meaningful effect on the firm’s market value; it is therefore unlikely to be of practical importance for explaining option grants to lower level employees. Laux (2010) shows that allowing a firm’s CEO to time the exercise of her options makes her more willing to terminate projects that are unprofitable. This benefit of insider trading only applies to senior executives with decision-making authority over the firm’s projects. Furthermore, none of the above papers shares with this paper its main focus, which is to investigate the usefulness of stock-based compensation when an individual measure of performance exists that is a sufficient statistic for the firm’s market value with respect to the agent’s action.

The rest of the paper proceeds as follows. Section 2 describes the details of the model. Section 3 sets up the firm’s optimization problem and shows that the optimal contract always includes stock options. Section 4 analyzes the efficiency properties of option grants in relation to the efficiency properties of individual bonuses. Section 5 discusses the model’s main empirical implications. Section 6 provides a discussion of some of the modeling assumptions and Section 7 concludes. All proofs are in the Appendix.

2 Model

Basic setup. A firm employs a representative worker/manager for two periods, delineated by three dates: $t = 0, 1, 2$. In the first period, the worker can invest in acquiring firm-specific knowledge, as detailed below; in the second period, he can then increase the firm’s value by taking an action $a \in \mathbb{R}$. Which of the worker’s actions most enhances the firm’s value depends on the conditions in the firm’s product and input markets, on the productive capabilities of the firm’s competitors and suppliers, and on the firm’s own productive capabilities, where the latter encompasses things such as the firm’s technology, organizational structure, skill composition of its employees, and so on. A worker who is well informed about these things will be able to take the optimal action, while a worker who is ignorant about them will not.

To capture this idea in a simple way, assume that all of the above internal and external characteristics of the firm’s environment are reflected in the state of the world, $s \in [0, \bar{s}]$, and the worker’s action is productive if and only if $a = s$. If his action is productive ($a = s$), the worker’s contribution to firm value, denoted $y$, is

$$
 y = \begin{cases} 
 Y > 0 & \text{with probability } p \in (0, 1) \\
 0 & \text{with probability } 1 - p.
\end{cases}
$$
In contrast, if \( a \neq s \), then \( y = 0 \) always.

Taking any given action is costless to the worker; hence, if the worker is informed, he is willing to take the action preferred by the firm (i.e., set \( a = s \)).

**Assets in place.** The firm has assets in place whose value is uncertain and depends on all of the characteristics of the competitive environment that determine the state of the world \( s \). Specifically, the value of these assets, \( x \), is

\[
x = \begin{cases} 
  H & \text{with probability } q \in (0, 1) \\
  L < H & \text{with probability } 1 - q.
\end{cases}
\]

For example, if \( s \) is uniform on \([0, \bar{s}]\), then the above specification obtains if \( x = L \) for \( s \leq (1 - q)\bar{s} \) and \( x = H \) otherwise. With some modifications, one could alternatively think of \( x \) as representing the aggregate output of all the other employees in this firm.

Define \( \triangle \equiv H - L \). It will be assumed that \((1 - p)Y < q\triangle\), which holds if an individual worker’s contribution to the firm value (\( Y \)) is smaller than the expected value increase due to the firm’s assets. This is a realistic assumption, with the additional benefit that it reduces the number of cases that need to be analyzed.

**Information acquisition and contracting.** Initially, the worker is uninformed about the state of the world (and about the value of the firm’s assets), but before taking an action, he can learn the state by incurring a private cost \( c \). It is not possible to directly contract on whether the worker got informed, but the contract can be contingent on both \( x \) and \( y \) and can include call options on the firm’s stock. A general form of such a contingent contract includes a base salary, an option grant, and three bonuses – one for a high realization of \( x \), one for a high realization of \( y \), and one for a joint high realization of both \( x \) and \( y \).

In the contracting stage, the firm has all the bargaining power; the worker only needs to receive his reservation utility, which will be normalized to zero.

**Preferences.** The firm is risk neutral and its owners maximize expected profit. The worker’s utility function \( u(w) \) is strictly concave and increasing in income \( w \), with \( u(0) = 0 \), \( \lim_{w \to -\infty} u'(w) = \infty \), and \( \lim_{w \to -\infty} u(w) = \bar{u} \), where \( \bar{u} > c/q \) but finite. Neither the firm nor the worker discount their future payoffs.

**Trading.** The following assumptions will streamline the analysis of the option contract:

(i) All of the worker’s options have to be exercised at once, either at \( t = 1 \) or at \( t = 2 \).
(ii) The worker cannot purchase additional shares of the firm’s stock in the open market, or trade in the derivatives of the firm’s stock.
(iii) The investors cannot observe whether or when the worker exercised his options and the share price does not react to worker’s trades.

(iv) The loss from trading against the informed worker is incurred by the firm’s original owners and the share price reflects the expected loss.

Assumption (i) limits the worker’s possible trading strategies, which weakens the incentive value of the options and stacks the model against the results to be derived. This assumption is therefore not essential. The first part of Assumption (ii) is somewhat restrictive because employees are typically free to purchase shares of the company’s stock, but will be relaxed in Section 6. The second part is based on insider trading policies of real world firms, which often prohibit employees from trading in put options and other derivatives of the firm’s stock.

Assumption (iii) magnifies the effects that drive the main result, but is realistic given the paper’s focus on a low level employee in a large firm. Low-level employees are not required to report their trades to the SEC, giving them ample opportunity to trade strategically. As mentioned in the Introduction, a number of empirical studies document that insider trading indeed takes place and that it is profitable.6

An alternative to assumption (iv), frequently used in the literature, would be to posit that the loss from trading against an informed worker is absorbed by liquidity traders who are different from the original investors. This alternative setting would make employee options even more attractive to the owners and hence strengthen the paper’s conclusion that in the present model options are an integral part of the optimal incentive contract.

**Timing.** The model unfolds in three stages, as illustrated in Figure 1 below. (In the figure, E stands for “employee.”)

At date 0, the firm and the worker sign an employment contract. Subsequently, the state of the world s is realized and the worker decides whether to invest c in learning the state and through it also the value of the firm’s assets.

At date 1, an informed worker chooses action a = s whereas an uninformed worker chooses an action at random. The option grant vests and the worker can choose to exercise his options (and sell the stock).

At date 2, both the value of the firm’s assets, x, and the realization of the worker’s contribution to the firm’s value, y, are publicly revealed. The worker is paid his contingent bonuses and decides whether to exercise his options if he has not done so at date 1.

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6Assumption (iii) could be relaxed by introducing noise trading a la Kyle (1985), but this would only complicate the analysis without changing the paper’s main insights. Alternatively, one could bypass the complications due to informed trading altogether, by assuming that the gains from option exercises are settled in cash, as in Laux (2010).
3 Analysis of the contracting problem

3.1 The first best outcome

It is helpful to start by considering the first-best benchmark. Because the worker is risk-averse, the efficient contract consists of a fixed wage. Furthermore, Pareto efficiency requires that the worker becomes informed if and only if \( pY - w^I \geq -w^U \), where \( w^I \) and \( w^U \) denote the worker’s respective wages if he gets informed and if he remains uninformed. These wages are determined by the participation constraint \( u(w^I) - c = 0 \), respectively \( u(w^U) = 0 \). Using \( u(0) = 0 \), the above condition implies that it is efficient for the worker to invest in information acquisition if and only if there exists a wage such that \( pY - w \geq 0 \) (the firm is willing to employ the worker) and \( u(w) = c \) (the worker is willing to accept the job). A \( w \) such that these two conditions are met exists if and only if \( u(pY) \geq c \), which will be assumed from now on.

3.2 Contract details

Note first that the worker has no control over \( x \). A contract that would directly condition his pay on \( x \) (say, via firm value \( x + y \)) would therefore only increase the risk to which he is exposed, without improving incentives. For a risk-averse worker, such a contract would
be dominated by a contract that is independent of \( x \). This is the essence of the arguments against the use of employee stock options for incentive purposes.\(^7\)

As will be shown below, in the current framework this argument does not imply that using options to provide incentives is sub-optimal. It does imply, though, that the optimal contract will not include a bonus directly contingent on \( x \). It is therefore enough to restrict attention to contracts that consist of (i) a base salary \( w_0 \), (ii) a bonus \( b \) contingent on \( y \):

\[
b = \begin{cases} 
B & \text{when } y = Y \\
0 & \text{when } y = 0,
\end{cases}
\]

and (iii) a stock option grant on \( \alpha \) shares of the firm, \( 0 \leq \alpha \leq 1 \), with strike price \( K \), where the options vest at date \( t = 1 \).

### 3.3 Informed worker

Starting with the worker’s decisions at \( t = 2 \), consider an informed worker who has not exercised his options at \( t = 1 \). In the absence of options, the firm’s value (net of the worker’s pay) at date 2 would be \( x + y - w_0 - b \). If the worker exercises his options at \( t = 2 \), then the firm gets an additional revenue \( \alpha K \). Thus, the income of an informed worker who exercises his options at \( t = 2 \) is \( W(x, y) = w_0 + b + \alpha(x + y + \alpha K - w_0 - b - K)^+ \), where, for any variable \( z \), \( z^+ \) stands for \( \max\{0, z\} \). Define the possible realizations of this income as

\[
\begin{align*}
W_1 & \equiv W(H, Y) = w_0 + B + \alpha(H + Y + \alpha K - w_0 - B - K)^+; \\
W_2 & \equiv W(H, 0) = w_0 + \alpha(H + \alpha K - w_0 - K)^+; \\
\Omega_1 & \equiv W(L, Y) = w_0 + B + \alpha(L + Y + \alpha K - w_0 - B - K)^+; \\
\Omega_2 & \equiv W(L, 0) = w_0 + \alpha(L + \alpha K - w_0 - K)^+. 
\end{align*}
\]

\(^7\)This argument is straightforward in the present model because the effect of the worker’s action on \( y \) is a sufficient statistic for its effect on the firm’s total value \( x + y \). If this were not true, however, then Holmström’s (1979) informativeness principle would imply that the worker’s pay should in fact optimally depend on \( x \). In such a case, employee stock options might be valuable even in a large firm. The arguments that large firms should not use employee stock options for incentive purposes are therefore based on the implicit assumption that either a less risky measure of the worker’s performance exists that is a sufficient statistic for the worker’s impact on firm value or that any additional incentives from conditioning the worker’s pay on firm value are too small to offset the cost of administering an option-grant scheme.
After he learns the value of the firm’s assets, this worker’s expected utility is thus

\[
EU_{2}^{\text{Inf}}(H) = pu(W_1) + (1 - p)u(W_2) \quad \text{if} \quad x = H
\]

and

\[
EU_{2}^{\text{Inf}}(L) = pu(\Omega_1) + (1 - p)u(\Omega_2) \quad \text{if} \quad x = L.
\]

What would the worker’s expected utility be if he instead exercised his options at \(t = 1\) (or not at all)? Suppose the investors believe the worker is informed. Then they expect the firm’s revenue to be \(L + q\Delta + pY + \delta\alpha S\), where \(\delta\) is their belief about the probability with which the worker will exercise his options at some point. Although the investors do not observe the worker’s trading choices, they realize that he will trade based on his private information, which will result in an expected loss to them of \(\ell(\alpha)\), the exact expression for which will be determined shortly. The \((1 - \alpha)\) shares retained by the investors are therefore worth to them

\[
E\pi = (1 - \delta) [L + q\Delta + p(Y - B) - w_0] + \delta (1 - \alpha) [L + q\Delta + p(Y - B) - w_0 + \alpha K] - \ell(\alpha).
\]

(5)

Thus, if the worker exercises his options at \(t = 1\), he can sell each share for the price of

\[
P_1 = \frac{E\pi}{1 - \alpha}.
\]

(6)

The income of an informed worker who exercises his options at \(t = 1\) is therefore \(w_0 + b + \alpha (P_1 - K)^+\) and his expected utility is

\[
EU_{1}^{\text{Inf}} = pu(W_3) + (1 - p)u(W_4)
\]

for both \(x = H\) and \(x = L\), where

\[
W_3 \equiv w_0 + B + \alpha (P_1 - K)^+ \quad \text{and} \quad W_4 \equiv w_0 + \alpha (P_1 - K)^+.
\]

(7)

The following proposition describes the informed worker’s optimal trading strategy.

**Proposition 1.** Let \(C^* \equiv (w_0^*, B^*, \alpha^*, K^*)\) denote an optimal contract and suppose it includes employee stock options (i.e., \(\alpha^* > 0\)). Under \(C^*\), an informed worker exercises his options as follows: When \(x = L\), the worker exercises his options at \(t = 1\). When

\(^{8}\)After exercising his options at \(t = 1\), the worker could hold on to his shares for a period and then sell them in \(t = 2\). This strategy, however, is payoff equivalent to exercising the options at \(t = 2\) and will therefore be ignored.
\[ x = H \text{ and } y = Y, \text{ he exercises his options at } t = 2. \text{ When } x = H \text{ and } y = 0, \text{ he may or may not exercise his options, but if he does, he does so at } t = 2. \]

The logic behind Proposition 1 is that because options impose risk on the worker, it makes sense to include them in the contract only if they provide incentives; otherwise, a bonus contract without an option grant would provide the same incentives at a lower expected wage cost. But options provide incentives only if the worker conditions their exercise on the realization of \( x \); otherwise, an uninformed worker could replicate an informed worker’s trading strategy, which would eliminate the worker’s incentive to invest in information acquisition. Hence, the option grant must be structured so that when \( x = L \), the worker wants to take advantage of his private information and sell his stock at \( t = 1 \), before other investors learn that the value of the firm’s assets is low. When \( x = H \), the worker waits until \( t = 2 \), when the good news gets incorporated in the price of the shares.

Given the worker’s trading strategy described in Proposition 1, the expected utility of an informed worker under an optimal contract that includes an option grant can be written as

\[
EU_{\text{Inf}} = qEU_{2,\text{Inf}}(H) + (1 - q)EU_{1,\text{Inf}} \\
= q[pu(W_1) + (1 - p)u(W_2)] + (1 - q)[pu(W_3) + (1 - p)u(W_4)]
\]

### 3.4 Firm’s expected payoff

The arguments behind Proposition 1, which rely on the option grant being valuable only if it provides incentives, are not enough to restrict the worker’s decision regarding the exercise of his options when \( x = H \) and \( y = 0 \). To streamline the exposition, the subsequent analysis will focus on the case where \( K \) is sufficiently low so that exercising the options is always optimal, even when \( y = 0 \). Given the goals of the analysis, this restriction is harmless – if the strike price in the option grant studied here is suboptimal but the principal nevertheless finds it profitable to include the grant in the worker’s contract, then she would surely find it optimal to include an option grant that is structured optimally. Similarly, any desirable efficiency properties of a potentially suboptimal option grant must also carry over to the case of an optimally designed option grant.

Proceeding under the assumption that the worker always exercises his options (\( \delta = 1 \))
the investors’ expected payoff (5) becomes

\[
E \pi = q (1 - \alpha) [H + p (Y - B) + \alpha K - w_0] + (1 - q) [L + p (Y - B) + \alpha K - w_0 - \alpha P_1] \\
= (1 - \alpha) [L + q \Delta + \alpha K + p (Y - B) - w_0] \\
- \alpha (1 - q) [P_1 - (L + \alpha K + p (Y - B) - w_0)].
\]  

(8)

This expression is simple to understand. The second line says that the investors keep the share \(1 - \alpha\) of the firm’s expected revenues net of the worker’s wage and expected bonus, while the term \(\ell(\alpha) \equiv \alpha (1 - q) [P_1 - (L + p (Y - B) + \alpha K - w_0)]\) in the third line represents the expected loss the firm’s owners incur because the worker trades on his private information. Of course, while \(\ell(\alpha)\) is a loss for the investors, it represents a gain for the worker and the prospect of this gain motivates him to become informed. The initial owners understand this and purposefully incorporate this gain/loss into the contract in the design stage.

Returning to the first period stock price, plugging (8) into (6) yields

\[
P_1 = L + q \Delta + \alpha K + p (Y - B) - w_0 - \frac{\alpha}{1 - \alpha} (1 - q) [P_1 - (L + \alpha K + p (Y - B) - w_0)],
\]

from which one can solve for the price as

\[
P_1 = L + q \Delta + \alpha K + p (Y - B) - w_0 - \frac{\alpha q (1 - q)}{1 - \alpha q} \Delta.
\]  

(9)

The first period price is thus equal to the firm’s expected value, \(L + q \Delta + \alpha K + p (Y - B) - w_0\), minus the expected loss per share, \(\frac{\alpha q (1 - q)}{1 - \alpha q} \Delta\), that is due to the worker’s informed trading at \(t = 1\) when \(x = L\). The worker takes advantage of his private information by exercising his options early when he learns that the state is bad, \(x = L\). Intuitively, the investors’ loss is thus proportional to the number of options held by the worker (\(\alpha\)), to the difference between the expected value of the firm’s assets and their true (low) value when the worker takes advantage of his private information \((L + q \Delta - L)\), and to the probability that the worker will get a chance to take advantage of his private information \((1 - q)\).

### 3.5 Uninformed worker

Now consider an uninformed worker. The probability that the worker correctly guesses the exact state \(s\) is zero; accordingly, an uninformed worker never receives the bonus \(B\). Will this worker exercise his options at date 1 or at date 2? By assumption, the optimal contract
motivates the worker to become informed. Investors will therefore expect the worker to be informed and hence, at \( t = 1 \), they will value the firm’s stock at \( P_1 \). Therefore, if the uninformed worker exercises his options at date 1, his payoff is

\[
EU_{1\text{Uninf}} = u(w_0 + \alpha(P_1 - K)^+) = u(W_4). \tag{10}
\]

If, on the other hand, he exercises his options at \( t = 2 \), after \( x \) and \( y \) have been publicly observed, his expected utility is

\[
EU_{2\text{Uninf}} = qu(w_0 + \alpha(H + \alpha K - w_0 - K)^+) + (1 - q)u(w_0 + \alpha(L + \alpha K - w_0 - K)^+)
= qu(W_2) + (1 - q)u(\Omega_2) \tag{11}
\]

Now, by Proposition 1, an informed worker who observes \( x = L \) is better off exercising his options at \( t = 1 \) than not exercising them at all. It therefore has to be \( P_1 > K \) under the optimal contract. Thus, an uninformed worker, too, can benefit from exercising his options at \( t = 1 \). The question is whether the worker prefers to wait until \( t = 2 \) or to exercise at \( t = 1 \). Unfortunately, a priori, it is not possible to rule out either of these trading strategies. If the first-period stock price \( P_1 \) is sufficiently depressed by the per-share loss \( \frac{aq(1-q)}{1-\alpha q} \\Delta \) expected by the investors, then the worker is better off exercising his options at \( t = 1 \); otherwise, waiting until \( t = 2 \) is optimal. Thus, in general, an uninformed worker exercises his options at \( t = 1 \) if \( EU_{1\text{Uninf}} \geq EU_{2\text{Uninf}} \) and at \( t = 2 \) if the opposite is true, and the analysis will have to account for both possibilities.

### 3.6 Firm’s problem

If the firm decides to induce information acquisition, its optimal contract solves

\[
\max_{w_0, B \geq 0, \alpha \geq 0, K \geq 0} E\pi \tag{P}
\]

subject to

\[
EU_{\text{Inf}} - c \geq \max\{EU_{1\text{Uninf}}, EU_{2\text{Uninf}}\}; \tag{12}
\]

\[
EU_{\text{Inf}} - c \geq 0, \tag{13}
\]

where (12) and (13) are the worker’s incentive compatibility and participation constraints respectively.

The following proposition offers a partial characterization of the optimal option grant,
which will be helpful in the subsequent analysis of the owners’ optimal contracting problem and in the characterization of the efficiency properties of stock options in motivating information acquisition.

**Proposition 2.** *If the contract includes a stock option grant under which the worker always exercises his options, then it is optimal to set the strike price such that* \( \frac{L-x_0}{1-\alpha} \leq K < P_1 \). *

Proposition 2 narrows down the range of possible strike prices that need to be considered in the analysis of the optimal option grant. In particular, it shows that at the time of the grant, the options will be in the money \( (K < P_1) \), which we have already argued has to hold, and that a grant of shares \( (K = 0) \) is weakly dominated by an option grant with a positive strike price \( (K \geq \frac{L-x_0}{1-\alpha}) \).

The next proposition contains the first main result of this paper.

**Proposition 3.** *If the principal finds it strictly profitable to induce information acquisition, then the optimal contract includes stock options, i.e., \( \alpha^* > 0 \).*

Proposition 3 demonstrates that in the present model, it is always optimal to motivate the worker by conditioning his pay on the value of the whole firm via a stock option grant. This is true even though the worker’s contribution to firm value is observed and can be contracted upon and even though the component of the firm value that is not affected by the worker’s action is risky and does not contain any additional information about the worker’s action. Note that these are precisely the conditions under which stock based compensation has been considered puzzling.

The intuition for this result is as discussed earlier: An option contract induces the worker to “get involved” in the firm by acquiring information that allows him to better estimate the firm’s future value, which helps him to time his trading. The firm is willing to tolerate the worker’s insider trading because his investment in firm-specific information makes him more productive and thus more valuable to the firm.

### 4 Efficiency properties of the option contract

How efficient are options in motivating the worker? We have seen that the optimal incentive contract will always include some options, but one may wonder whether the incentives that come from the option part of the contract are of first order importance. In other words, wouldn’t the option part disappear if administering an option grant entailed a small fixed
cost? This section shows that this is not the case; in fact, we will see that in some situations options provide much more effective incentives than bonuses based on the worker’s individual contribution to the firm value.

Under the standard theory, employee stock options are especially puzzling in large firms. This is because in large firms the component of the firm’s value over which the employee has no control is large and imposes on him large risk that is unrelated to his performance. The purpose of the analysis below is to show that this logic does not apply in the present model.

In the model of this paper, the part of the variation in firm value that the worker cannot affect is captured by the variance of the value of the firm’s assets, which is given by \( q(1-q)\Delta^2 \). A natural way to capture both the firm’s size and the magnitude of the risk unrelated to the worker’s performance is thus through \( \Delta \): For any given \( q \), both the variance \( q(1-q)\Delta^2 \) and the expected value of the assets, \( L + q\Delta \), are large if \( \Delta \) is large.\(^9\) The first result of this section will be that, contrary to the intuition based on the standard logic, in the present framework stock options are especially efficient when \( \Delta \) is large. To demonstrate this as simply as possible, the analysis will focus on the efficiency properties of a pure bonus contract and compare them with those of a pure option contract.

### 4.1 A pure bonus contract

A **pure bonus contract** is a contract that includes a bonus for \( y = H \) but does not contain any options: \( B > 0 \) and \( \alpha = 0 \). Given the assumption that the firm’s initial owners prefer to induce the worker to get informed, their profit is maximized when information acquisition is induced at the lowest possible wage cost. Thus, under a pure bonus contract the optimization problem (P) can be written as

\[
\min_{w_0,B \geq 0} \ (w_0 + pB)
\]

subject to

\[
pu(w_0 + B) + (1-p)u(w_0) - c \geq u(w_0);
\]

\[
pu(w_0 + B) + (1-p)u(w_0) - c \geq 0.
\]

The usual argument implies that both constraints must bind, so that \( u(w_0) = 0 \) (from which \( w_0 = 0 \)) and \( pu(B) = c \), or \( B^* = u^{-1}(c/p) \). The firm’s expected wage bill is then

\[
EW^{\text{Bonus}} = pu^{-1}(c/p).
\]  

\(^9\)The effect of variance would be better isolated if the mean \( L + q\Delta \) were kept constant. This, however, would necessitate allowing for a negative \( L \), which would violate the limited liability property of stock.
4.2 A pure option contract

A pure option contract includes options but no bonus: $\alpha > 0$ and $B = 0$. The analysis of this contract is more complicated than the analysis of the bonus contract, and the details are relegated to the proof of Proposition 4, but the core of the argument can be explained relatively easily for the case where $EU^{Uninf}_1 \geq EU^{Uninf}_2$ at the optimum. In this case, the relevant incentive compatibility constraint in (12) is $EU^{Inf} - c \geq EU^{Uninf}_1$. Using $W_3 = W_4$ implied by $B = 0$, the firm’s optimization problem becomes

$$
\min_{w_0, \alpha \geq 0} q [pW_1 + (1-p)W_2] + (1-q) W_3
$$
subject to

$$
q [pu(W_1) + (1-p)u(W_2)] + (1-q)u(W_3) - c \geq u(W_3); \quad (15)
$$
$$
q [pu(W_1) + (1-p)u(W_2)] + (1-q)u(W_3) - c \geq 0. \quad (16)
$$

Again, both of the constraints must bind, which implies $u(W_3) = W_3 = 0$. The two constraints thus collapse into a single constraint

$$
q [pu(W_1) + (1-p)u(W_2)] = c. \quad (17)
$$

The firm’s expected wage bill under a pure option contract is then

$$
EW^{Options} = q [pW_1 + (1-p)W_2].
$$
Recalling from (1) and (2) that $W_1 = W_2 + \alpha Y$ and $W_2 = w_0 + \alpha (H + \alpha K - w_0 - K)$, this can be written as

$$
EW^{Options} = qW_2 + q\alpha Y. \quad (18)
$$

4.3 A comparison

A pure option contract is more efficient than a pure bonus contract if $EW^{Options} < EW^{Bonus}$. Now, to focus on large $\Delta$, let $\Delta \to \infty$. It will be shown in the proof of Proposition 4 that $\alpha$ must then converge to zero, so that both $W_1$ and $W_2$ must approach some $\tilde{W}$. Condition (17) therefore converges to

$$
qu(\tilde{W}) = c,
$$
and the firm’s expected wage bill under a pure option contract (18) converges to

\[ EW_{\text{Options}} = q\bar{W} = qu^{-1}(c/q). \]  

(19)

Observe that (19) has the same form as the bonus wage bill (14), with \( p \) replaced by \( q \). This makes sense: A pure bonus contract is conditioned on the realization of \( y \), which is high with probability \( p \). In contrast, a pure option contract is conditioned predominantly on the realization of \( x \) when \( \Delta \) is very large (because \( y \) is negligible compared to \( x \)), which is high with probability \( q \).

Now, the expression \( zu^{-1}(c/z) \) decreases in \( z \). For large \( \Delta \), a pure option contract is therefore more efficient than a pure bonus contract \((EW_{\text{Options}} < EW_{\text{Bonus}})\) if and only if \( q > p \). The proof of Proposition 4 demonstrates that this conclusion continues to hold when one allows for the possibility that \( EU_{1}^{\text{Uninf}} \leq EU_{2}^{\text{Uninf}} \) at the optimum. The result is summarized as follows.

**Proposition 4.** For any \( L \) and any \( q > p \), there exists a finite \( \hat{\Delta} \) such that \( EW_{\text{Options}} < EW_{\text{Bonus}} \) for all \( \Delta \geq \hat{\Delta} \). That is, if the variance and the expected value of the firm’s assets are sufficiently large, a pure option contract exists that is more efficient than the optimal pure bonus contract.

Proposition 4 says that a pure option contract is often more efficient than a pure bonus contract, especially in large firms. But how much more efficient? The next result shows that the efficiency difference between these two contracts can be vast. Because the worker is risk averse and the firm’s value is risky, there of course exists no contract that can achieve the first best outcome. Nevertheless, there are situations in which the outcome under an option contract is arbitrarily close to the first best outcome, while a pure bonus contract is so inefficient that the firm is better off not using it and letting the agent remain uninformed.

Let \( V^{\text{FB}} \) denote the owners’ expected payoff under the first best scenario, i.e., when there is no moral hazard problem. Similarly, define \( V^{\text{Options}} \) to be the owners’ expected payoff under the optimal pure option contract.

**Proposition 5.** (i) Consider a pure bonus contract. For any \( p \), there exist \( Y_{1} \) and \( Y_{2} \) such that when \( Y \in (Y_{1}, Y_{2}) \), the optimal contract sets \( B^{*} = 0 \). Under this contract, the worker remains uninformed, even though efficiency requires that he get informed.

(ii) Consider a pure option contract. For any \( \varepsilon > 0 \), there exist \( \bar{q} < 1 \) and \( \bar{\Delta} < \infty \) such that if \( q \geq \bar{q} \) and \( \Delta \geq \bar{\Delta} \), the efficiency loss under the optimal contract is less than
$\varepsilon$, i.e., $V_{\text{FB}} - V_{\text{Options}} < \varepsilon$. In other words, for $\Delta$ and $q$ large the option contract approaches the first best outcome.

The result in part (i) of Proposition 5 is standard. Given the moral hazard problem, a second best contract cannot achieve the first-best outcome. Furthermore, in situations where the first-best surplus is relatively small to start with (which is the case when $Y < Y_2$), a second-best contract may not be efficient enough to generate any surplus.

Part (ii) is more interesting. It says that a pure option contract can be very efficient; almost as efficient as a first-best contract. Moreover, this efficiency result does not depend on the exact value of $Y$ — it only requires that the potential increase in the value of the firm’s assets, $\Delta$, and the probability $q$ that the asset value is high are sufficiently large. Thus, there are situations when an option grant is vastly more efficient than a contract based on the worker’s individual performance, even in a large firm.

5 Empirical implications

The theory developed above has implications for the kind of firms we should expect to use employee stock options, for the relationship between firm size and the use of stock option grants, for the optimal way to structure the vesting period of an option grant, and for the relationship between option grants and employee turnover.

First, the model predicts that the incentive effects of employee stock options should be valuable in firms that need to encourage their employees to invest in information, especially firm-specific information. This prediction is consistent with the evidence that both the likelihood and the intensity of the use of non-executive stock options are higher in firms in which human capital is a relatively more important factor of production (Core and Guay, 2001; Kroumova and Sesil, 2006; Jones et al, 2006). It is also consistent with the evidence that stock options are widespread in start-up firms: By default, all employees in a start-up are new to the job and therefore unlikely to possess firm-specific human capital. Start-up firms therefore tend to be in more need of motivating their employees to invest in firm-specific information than are established and mature firms. Moreover, in mature firms, new employees may need less of an incentive to invest in firm-specific knowledge because at least some of the required knowledge will seep to them from their more senior co-workers and from their supervisors. Young firms, not having workers with long tenure, cannot rely on this “automatic” mechanism of information dissemination.
Second, firm size has an ambiguous effect on whether employee stock options are useful for providing incentives. The usual free rider argument favors their use in small firms, but benefits to informed trading may be easier to realize in large firms, in which a trade by any single (lower level) employee is less likely to affect the stock price. Although the paper does not directly model this tradeoff, it is not hard to see that the trade-off could be resolved in favor of any firm size: If investment in firm-specific information is not very important relative to the direct effect of the workers’ effort on firm value, then in line with the traditional logic a large firm will not use stock options. If, however, investment in firm-specific knowledge is important and hiding informed trading becomes easier the larger is the firm, then the incentive effects of employee stock options could well be most useful in medium to large sized firms. Indeed, there is some evidence that larger firms are more likely to adopt broad based share plans (Jones et al, 2006).

Third, the theory offers a new perspective on how the options’ vesting period affects their incentive properties. While a proper investigation of this topic would require a more dynamic model than the one studied in this paper, the basic issues that are at play here are relatively transparent. Specifically, the length of the vesting period will influence the type of information the employee will be willing to acquire. Options with a long vesting period, for example, will not be effective in motivating the acquisition of short-lived information whose relevance expires before the options vest. On the other hand, when the vesting period is short, the option grant may not work well if information acquisition is important in the time period after the options vest because by that time the worker may have exercised the options, either for liquidity reasons or due to diversification reasons.

Finally, similar to Oyer (2004), the model has implications for the relationship between stock options’ use and employee turnover. In Oyer’s theory, the main role of options is to prevent employee turnover; accordingly, we should expect that in the years following a broad-based stock option grant, employee turnover falls at the granting firm. The theory developed here has a similar implication: Since in the present model options serve to induce firm-specific investments, they also affect the probability of turnover negatively, for the reasons well known from the theory of firm-specific human capital. In this respect, both models are consistent with the available empirical evidence, which indeed documents a negative relationship between stock option grants and subsequent worker turnover (Aldatmaz, Ouimet and Van Wesep, 2012).

There is one important aspect though in which the implications here differ from those of Oyer (2004): Whereas in Oyer’s model the relationship between options and turnover
disappears after the vesting period, in the present model the relationship between vesting a
turnover is more complicated: On the one hand, some of the workers who were planning to
move may wait long enough for their options to vest, and then quit shortly afterwards. This
would increase the probability of turnover right after the vesting period. On the other hand,
as long as the firm-specific knowledge acquired by the workers remains relevant, it should
provide a disincentive to move even after the options vest. Thus, suppose we compare three
firms, A, B, and C. Firms A and B each adopt a broad-based option plan that vests, say,
in three years. In firm A, the options are used solely to manage turnover whereas in firm
B they serve to motivate investments in firm-specific knowledge. Firm C does not grant
options. In all other respects, the three firms are identical. Then comparing the aggregate
levels of turnover in the three firms over the period of, say, five years, we should see little
difference between firms A and C, whereas firm B (the one whose workers invest in firm-
specific knowledge), should exhibit a lower level of turnover over this period.

The above prediction is consistent with the findings in Bryson and Freeman (2013) who
use survey data from the UK establishments of a multinational firm to examine the effects of
a company subsidized employee stock purchase plan. Bryson and Freeman document that the
workers who choose to participate in the plan are subsequently less likely to leave or search
for a different job than those who do not participate in the plan. The UK tax laws offer a
substantial tax break for holding the shares for at least five years, so similar to the effects
of a vesting period for option grants, any negative relationship between plan participation
and turnover should disappear if the plan only affects the workers’ participation constraints
but not their incentives, as in Oyer (2004). However, Bryson and Freeman show that even
the workers who have been participating in the plan for more than five years exhibit lower
probabilities of turnover. Bryson and Freeman’s interpretation is that the workers view the
subsidized shares as a gift and reciprocate it with better performance and a lower probability
of quitting. The present paper offers an alternative explanation, according to which the
share purchase plan motivates firm-specific investments by the workers, which in turn makes
it more costly for them to switch employers.

6 Discussion of assumptions

The model developed above assumes that the employment contract cannot be based on
messages from the worker to the firm and that the worker cannot trade company securities on
his own in the financial markets. This section provides a discussion of these two assumptions.
6.1 Contracting on messages

In the model of this paper, an option contract provides incentives because it conditions the worker’s pay on his information about the state of the world. In principle, this could be done more directly: at $t = 1$, the worker could make an announcement $m \in \{L, H\}$ about the future value of the firm’s assets and then receive a bonus if $x = m$. Being a more general version of the option contract, such a message-based contract could always do at least as well as the option contract. An option contract, however, may have advantages over a contract based on messages:

First, the messages would have to be publicly verifiable. But for various strategic reasons, a firm might not want its employees to make public predictions about the future of their firm, especially if the news is negative. The firm could try and keep the predictions hidden from the public eye, but given that someone would have to overlook such a scheme, it would not be easy to ensure that the predictions are kept secret. Moreover, the scheme would be enforceable only if a compliance with it could be verified in the court of law, which would make secrecy even harder to achieve. In short, administering such a scheme seems impractical and could well be costlier than administering an option grant.

Second, employees might free ride on each other’s predictions. They could, of course, also free ride on each other’s trading strategies, but informed trading may be easier to hide – and the workers would have an incentive to do so, in order to prevent imitators from moving the stock price. Furthermore, workers may trade for reasons other than to take advantage of private information; in particular, they may exercise their options to meet their liquidity and consumption needs. If employees cannot distinguish between a co-worker’s motives for trading, this would limit their benefits from free riding.

To sum up, compared to an option contract, a contract based on messages might be more costly to implement, would leak more information to the firm’s competitors and investors, and would require that the owners have more information about the details of the environment. An option contract might thus be more practical, especially when, as shown in Proposition 5, its efficiency is close to the first best benchmark.

6.2 Open market trading

The analysis of the model was conducted under the assumption that the worker cannot purchase his company’s shares, options, and other derivatives in the open market. As mentioned earlier, companies often prohibit their employees from short selling company stock and from
engaging in trades involving derivatives, such as put options on the firm’s stock.\(^{10}\) Nevertheless, workers are typically free to purchase shares of the company stock. Thus, in principle, rather than granting him options, the firm could simply let the worker trade the firm’s stock in the open market, which could be a profitable strategy for an informed worker.

There are at least two reasons firms may offer employee stock options even if open market trading is feasible. First, although inside information may induce a risk-averse worker to purchase more of his own company’s stock than suggested by traditional hedging arguments (Van Nieuwerburgh and Veldkamp, 2006), it may not be enough from the firm’s perspective. This is because the worker only takes into account his private benefit from getting informed and does not internalize the resulting increase in the firm’s profit. Proposition 6 below formalizes this logic.

**Proposition 6.** Suppose that at \(t = 1\) the worker can purchase the firm’s shares on the open market. There exist \(q^+ < 1\), \(\Delta^+ < \infty\), and a non-empty interval \((Y^-, Y^+)\) such that if \(q \geq q^+\), \(\Delta \geq \Delta^+\), and \(Y \in (Y^-, Y^+)\), the optimal contract includes an option grant (i.e., \(\alpha^* > 0\)).

Proposition 6 says that the firm may find it optimal to include an option grant in the worker’s incentive contract even if the worker is able to trade in the financial markets. Although the result is likely to hold for a broader range of parameter values, for tractability purposes the proposition focuses on parameter values such that \(q\) and \(\Delta\) are large and \(Y\) is in an intermediate range. These are also the parameter values for which the intuition behind the result is most transparent: We know from Proposition 5 that when \(Y \in (Y_1, Y_2)\), incentives provided by a pure bonus contract (with no open market trading) are so costly that the firm opts to provide no incentives even though efficiency requires that the worker get informed. The proof of Proposition 6 shows that when \(q\) is close to 1, allowing for open market trading does not alter this qualitative conclusion: There again exists a range of \(Y\), given by the interval \((Y^-, Y^+)\), such that in the absence of an option grant the worker does not (always) get informed.\(^{11}\) This is because the stock price reflects the expected average asset value \(qH + (1 - q)L\), which is close to \(H\) when \(q\) is close to 1. The potential gain from

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\(^{10}\) Benefits to such restrictions are outside of this model, but statements of real world firms’ insider trading policies indicate that firms implement the restrictions because they worry an employee who shorts the company stock, or holds puts on it, would benefit from a decrease in the stock price, which might provide her with perverse incentives. Note that employee stock options do not suffer from this problem.

In addition, firms worry that trading in puts or other hedging instruments might be interpreted as a negative signal to the market that the employee has no confidence in the company’s prospects. This also applies to exercising call options, but to a lesser extent.

\(^{11}\) The worker may choose to randomize in his decision to get informed because the positive probability
buying the firm’s stock when the asset value is $H$ is therefore minuscule, so that it would be worth getting informed only if the worker expected to purchase a relatively large number of shares. However, a risk-averse worker is reluctant to buy a large number of shares because of the risk associated with the realization of his marginal contribution $y$. Thus, in this case open market trading is not enough to always induce the worker to get informed, even if combined with a bonus contract. On the other hand, Proposition 5 tells us that when $q$ is close to 1 and $\Delta$ is large, an option contract is almost fully efficient. Thus, the firm would want to motivate the worker with an option grant.

The second reason why open market trading may not provide sufficient incentives is that the worker may face a wealth constraint and therefore may not be able to purchase (enough) shares of the firm’s stock. This assumption is often adopted in the literature on insider trading (e.g., Dye, 1984; Noe, 1997; Laux, 2010). Of course, a wealth constraint changes other aspects of the analysis as well; in particular, it makes it harder for the firm to extract from the worker his expected gains from informed trading. It is therefore not immediate that an option grant would remain optimal if the worker were liquidity constrained. The next proposition establishes that the conclusion of Proposition 3 continues to hold even when the worker is liquidity constrained and hence unable to trade in the open market.

**Proposition 7.** Suppose the worker is liquidity constrained in the sense that his pay cannot be negative. Then if the principal finds it strictly profitable to induce information acquisition, the optimal contract always includes stock options, i.e., $\alpha^* > 0$.

To sum up, although open market trading could provide the worker with some incentives to get informed, in general it is not a good substitute for including an option grant in the worker’s contract.

7 Conclusion

This paper has argued that, contrary to the prevalent view among economists, the observation that large firms suffer from severe free rider problems is not sufficient to rule out stock option plans as effective incentive tools for lower level employees. The free rider critique is predicated upon the assumption that a worker can be motivated by stock-based compensation only to the extent that her actions increase the value of the firm’s shares, an effect that the worker is uninformed depresses the equilibrium stock price $P_1$ and increases the worker’s trading gain when he does get informed. The proof of the proposition allows for such a mixed strategy.
that for a typical worker is indeed minuscule in large firms. However, by the very nature of stock options, a worker can increase their value not only by increasing the value of the firm but also by simply getting informed about the details of the firm’s operations. Unlike the standard free rider problem, this effect is not directly related to the size of the firm and therefore can be of significant magnitude even in large firms. Consequently, the paper demonstrates that if the firm’s goal is to encourage investments by workers in firm-specific information, broad-based stock option grants may be an effective way to do this. Moreover, under some conditions, options can provide significantly more efficient incentives than even performance contracts based on the worker’s individual contribution to the firm value.

The theory developed in this paper has potentially testable empirical implications. For example, the use of employee stock options for incentive purposes should be observed mainly in firms that need to encourage their employees to accumulate firm-specific knowledge, such as high tech firms and start-ups. Also, firms that use broad based stock option grants should exhibit less employee turnover, even over periods that exceed the duration of the vesting period.

The basic model examined in this paper could be extended in several directions. Perhaps the most promising avenue for further study would be to endogenize the vesting period. Such a framework could then be used to address the open question of what determines the length of the optimal vesting period and to study the relationship between an optimal vesting policy and the nature of the specific information (e.g., long-lived vs short-lived) that the firm wants the workers to accumulate.

A Appendix: Proofs

Proof of Proposition 1. Step 1. Observe first that if \( \alpha^* > 0 \), then the options must provide incentives, i.e., it must be that under the contract \( C^* \) the worker gets informed, whereas under the contract \( C^0 \equiv (w_0^*, B^*, \alpha = 0) \) he would find it optimal to stay uninformed. If this were not the case, then there would be an alternative contract \( C' = (w_0, B^*, \alpha = 0) \) that would dominate \( C^* \), because \( C' \) could provide the same level of incentives as \( C^* \) without imposing on the worker the additional risk due to the variations in \( x \).

Step 2. Given that the stock option part of \( C^* \) provides incentives, it must be that an informed worker conditions his trading on his private information. Moreover, it cannot be that he never exercises his options at \( t = 1 \), regardless of his private information. If this were the case, the options would not provide any incentives because the worker could replicate
this trading strategy even without getting informed — he could simply wait until the firm’s revenue is publicly observed at \( t = 2 \) and then exercise his options optimally. Similarly, it cannot be that in equilibrium an informed worker never exercises his options at \( t = 2 \). In such a case, the worker would exercise his options at \( t = 1 \) iff \( K < P_1 \). But this rule does not depend on the exact realization of \( x \), so an uninformed worker could replicate this trading strategy. Once again, the options would not provide any incentive to get informed.

Step 3. The previous step leaves three possible cases for an informed worker’s equilibrium trading strategy:

(i) If \( x = L \), exercise at \( t = 1 \); if \( x = H \), exercise at \( t = 2 \), either for \( y = Y \), or for both realizations of \( y \).

(ii) If \( x = H \), exercise at \( t = 1 \); if \( x = L \), then exercise at \( t = 2 \), but only if \( y = Y \).

(iii) If \( x = H \), exercise at \( t = 1 \); if \( x = L \), exercise at \( t = 2 \) for both realizations of \( y \).

The proof is concluded by ruling out (ii) and (iii). Start with (iii): If at \( t = 2 \) it is publicly revealed that \( x = L \), the stock price must (in expectation with respect to \( y \)) decrease from what it was at \( t = 1 \). Hence, if the worker finds it optimal to exercise his options at \( t = 2 \) when \( x = L \), it must be even better to exercise them at \( t = 1 \), because not only is his payoff higher in expectation (due to the higher stock price), but it is also less variable (due to the realization of \( y \) in \( t = 2 \)), which the risk-averse worker likes. As for case (ii), note that when \( x = L \), any realization of the stock price at \( t = 2 \) must always be lower than the stock price at \( t = 1 \). The reason is that at \( t = 1 \), the price reflects the expected value of \( x + y \) as given by \( L + q\Delta + pY \), but when it is revealed at \( t = 2 \) that \( x = L \), the price must fall because, by assumption, \( q\Delta > (1 - p)Y \). Thus, when \( x = L \), the worker strictly prefers exercising his options at \( t = 1 \) to exercising them at \( t = 2 \), which rules out case (iii). Q.E.D.

Proof of Proposition 2. As pointed out in the text, the fact that \( K < P_1 \) follows from Proposition 1, which says that the informed worker always finds it optimal to exercise his options at \( t = 1 \) if \( x = L \). Thus, it remains to prove that \( K \geq \frac{L - qw_0}{1 - \alpha} \).

Given the assumption that the firm’s initial investors prefer to induce the worker to get informed, their optimization problem (P) can be reduced to the problem of minimizing the expected cost of inducing the worker to get informed:

\[
\min_{w_0, \alpha \geq 0, B \geq 0, K \geq 0} q \left[ pW_1 + (1 - p)W_2 \right] + (1 - q) \left[ pW_3 + (1 - p)W_4 \right]
\]

\(^{12}\)The cases where the worker exercises his options in \( t = 2 \) when \( y = 0 \) but not when \( y = Y \) are ruled out immediately.
s.t. \[ EU^{\text{Inf}} \equiv q [pu(W_1) + (1 - p)u(W_2)] + (1 - q) [pu(W_3) + (1 - p)u(W_4)] - c \geq qu(W_2) + (1 - q)u(\Omega_2) \equiv EU_2^{\text{Uninf}}, \]

\[ EU^{\text{Inf}} \equiv q [pu(W_1) + (1 - p)u(W_2)] + (1 - q) [pu(W_3) + (1 - p)u(W_4)] - c \geq u(W_4) \equiv EU_1^{\text{Uninf}}, \]

and \[ EU^{\text{Inf}} \equiv q [pu(W_1) + (1 - p)u(W_2)] + (1 - q) [pu(W_3) + (1 - p)u(W_4)] - c = 0. \]

Suppose \( K < \frac{L - w_0}{1 - \alpha} \). Given that the firm’s value at \( t = 2 \) is never less than \( L + \alpha K - w_0 \), under this strike price the worker always exercises his options. Thus, the worker’s possible incomes (originally given by (1)-(4) and (7)) and the first period price (9) can be written as

\[
W_1 = \Omega_2 + B + \alpha(\Delta + Y - B); \tag{A1}
\]

\[
W_2 = \Omega_2 + \alpha \Delta; \tag{A2}
\]

\[
W_3 = \Omega_2 + B + \alpha \left( q\Delta + p(Y - B) - \frac{\alpha q (1 - q)}{1 - \alpha q} \Delta \right); \tag{A3}
\]

\[
W_4 = \Omega_2 + \alpha \left( q\Delta + p(Y - B) - \frac{\alpha q (1 - q)}{1 - \alpha q} \Delta \right); \tag{A4}
\]

\[
\Omega_2 = \alpha L + (1 - \alpha) (w_0 - \alpha K); \tag{A5}
\]

\[
P_1 = L + q\Delta + \alpha K + p(Y - B) - w_0 - \frac{\alpha q (1 - q)}{1 - \alpha q} \Delta. \tag{A6}
\]

Now, consider the effects of an increase in \( K \) to \( K' \) accompanied by an increase in \( w_0 \) to \( w'_0 \) so that \( \Omega_2 \) remains unchanged and \( K' \leq \frac{L - w_0}{1 - \alpha} \). Given that \( K \) and \( w_0 \) enter \( \Omega_2 \) only through the term \( (w_0 - \alpha K) \), such an offsetting change in \( K \) and \( w_0 \) is always possible. Moreover, the worker’s income vector \((W_1, W_2, W_3, W_4)\) depends on \( K \) and \( w_0 \) only through \( \Omega_2 \). This increase in \( K \) therefore has no effect on \((W_1, W_2, W_3, W_4)\) and hence no effect on the optimization problem. Consequently, \( K' = \frac{L - w_0}{1 - \alpha} \) must be at least as profitable for the owners as any \( K < \frac{L - w_0}{1 - \alpha} \). Q.E.D.

**Proof of Proposition 3.** Given the result of Proposition 2, it is without loss of generality to restrict attention to strike prices such that \( \frac{L - w_0}{1 - \alpha} \leq K < P_1 \). Furthermore, for the purposes of this proposition, one can set \( K = \frac{L - w_0}{1 - \alpha} \) because if the optimal contract includes stock options under this strike price, then it must also include stock options under the optimal strike price, even if the optimal strike price differs from \( \frac{L - w_0}{1 - \alpha} \).

Thus, setting \( K = \frac{L - w_0}{1 - \alpha} \), the optimization problem specified in the proof of Proposition
2 simplifies to
\[
\min_{w_0, \alpha \geq 0, B \geq 0} q [pW_1 + (1 - p)W_2] + (1 - q) [pW_3 + (1 - p)W_4]
\]
s.t. \[EU^{\text{Inf}} \equiv q [pu(W_1) + (1 - p)u(W_2)] + (1 - q) [pu(W_3) + (1 - p)u(W_4)] - c \geq qu(W_2) + (1 - q)u(w_0) \equiv EU^{\text{Uninf}}_2; \]
\[EU^{\text{Inf}} \equiv q [pu(W_1) + (1 - p)u(W_2)] + (1 - q) [pu(W_3) + (1 - p)u(W_4)] - c \geq u(W_4) \equiv EU^{\text{Uninf}}_1; \]
and \[EU^{\text{Inf}} \equiv q [pu(W_1) + (1 - p)u(W_2)] + (1 - q) [pu(W_3) + (1 - p)u(W_4)] - c = 0, \]
where
\[
W_1 = w_0 + B + \alpha(\triangle + Y - B);
W_2 = w_0 + \alpha \triangle;
W_3 = w_0 + B + \alpha \left( q\triangle + p(Y - B) - \frac{\alpha q (1 - q)}{1 - \alpha q} \triangle \right);
W_4 = w_0 + \alpha \left( q\triangle + p(Y - B) - \frac{\alpha q (1 - q)}{1 - \alpha q} \triangle \right).
\]

Suppose, contrary to the claim in the proposition, that \( \alpha^* = 0 \). Then the worker’s incentives must come solely from the bonus \( B \). Moreover, it must be \( B < Y \), because otherwise the principal would not find inducing information acquisition strictly profitable.

Now, from (10) and (11), we have \( EU^{\text{Uninf}}_1(\alpha = 0) = EU^{\text{Uninf}}_2(\alpha = 0) = u(w_0) \). Furthermore, using \( K = \frac{L - w_0}{1 - \alpha} \), we have
\[
\frac{\partial EU^{\text{Uninf}}_1}{\partial \alpha}_{|\alpha = 0} = u'(W_4(\alpha = 0)) \frac{\partial W_4}{\partial \alpha}_{|\alpha = 0} = u'(w_0) [q\triangle + p(Y - B)]
\]
and
\[
\frac{\partial EU^{\text{Uninf}}_2}{\partial \alpha}_{|\alpha = 0} = qu'(W_2(\alpha = 0)) \frac{\partial W_2}{\partial \alpha}_{|\alpha = 0} = u'(w_0)q\triangle,
\]
from which \( \frac{\partial EU^{\text{Uninf}}_1}{\partial \alpha}_{|\alpha = 0} > \frac{\partial EU^{\text{Uninf}}_2}{\partial \alpha}_{|\alpha = 0} \). This implies that there exists an \( \hat{\alpha} > 0 \) such that \( EU^{\text{Uninf}}_1 > EU^{\text{Uninf}}_2 \) for all \( \alpha \in (0, \hat{\alpha}] \). For all such \( \alpha \), an uninformed worker exercises his options at \( t = 1 \). His expected utility is therefore given by \( EU^{\text{Uninf}}_1 \) and his IC constraint (12) becomes \( EU^{\text{Inf}} - c \geq EU^{\text{Uninf}}_1 \). Furthermore, the standard argument implies that this constraint has to hold with equality at the optimum, which combined with the participation constraint (13) implies \( EU^{\text{Uninf}}_1 = u(W_4) = 0 \). This in turn yields \( W_4 = 0 \) and \( W_3 = B \).
Restricting attention to \( \alpha \leq \hat{\alpha} \), the firm’s optimization problem can thus be written as

\[
\min_{w_0, \alpha \geq 0, B \geq 0} q \left[ pW_1 + (1 - p)W_2 \right] + (1 - q) pB \\
\text{s.t.} \quad q \left[ pu(W_1) + (1 - p)u(W_2) \right] + (1 - q) pu(B) - c = 0; \quad (A7) \\
w_0 + \alpha \left( q\Delta + p(Y - B) - \frac{\alpha q (1 - q)}{1 - \alpha q} \Delta \right) = 0, \quad (A8)
\]

where

\[
W_1 = w_0 + B + \alpha (\Delta + Y - B); \\
W_2 = w_0 + \alpha \Delta; \\
W_3 = w_0 + B + \alpha \left( q\Delta + p(Y - B) - \frac{\alpha q (1 - q)}{1 - \alpha q} \Delta \right).
\]

Let \( \lambda \) and \( \mu \) be the Lagrange multipliers that go with constraints (A7) and (A8) respectively. The corresponding first order conditions are then

\[
\alpha : \quad q p(\Delta + Y - B) [1 + \lambda u'(W_1)] + q(1 - p) \Delta [1 + \lambda u'(W_2)] \\
\quad + \mu \left[ q\Delta + p(Y - B) - \frac{\alpha q (1 - q)}{1 - \alpha q} \Delta \right] \geq 0; \\
B : \quad q p \frac{\partial W_1}{\partial B} + (1 - q) p + \lambda \left[ qpu'(W_1) \frac{\partial W_1}{\partial B} + (1 - q) pu'(B) \right] - \mu \alpha p \geq 0; \\
w_0 : \quad q + \lambda [qpu'(W_1) + q(1 - p)u'(W_2)] + \mu = 0.
\]

Suppose \( B = \alpha = 0 \). Then the contract provides no incentives, which means that (12) cannot hold. Hence, if \( \alpha = 0 \), then \( B > 0 \). Consequently, FOC(B) must hold with equality.

Now, evaluating at \( \alpha = 0 \), we get \( W_1 = W_3 = w_0 + B \) and \( W_2 = W_4 = w_0 \), and, from (A8), \( w_0 = 0 \), so that the above first order conditions reduce to

\[
\alpha : \quad q p(\Delta + Y - B) [1 + \lambda u'(B)] + q(1 - p) \Delta [1 + \lambda u'(0)] + \mu [q\Delta + p(Y - B)] \geq 0; \quad (A9) \\
B : \quad 1 + \lambda u'(B) = 0; \quad (A10) \\
w_0 : \quad q [1 + \lambda [pu'(B) + (1 - p)u'(0)]] + \mu = 0. \quad (A11)
\]

Using (A10), condition (A9) becomes

\[
\alpha : \quad q(1 - p) \Delta [1 + \lambda u'(0)] + \mu [q\Delta + p(Y - B)] \geq 0. \quad (A12)
\]
Now, (A10) implies \( \lambda < 0 \). Given that \( u'(0) > u'(B) \) (by concavity of \( u \)), (A10) yields \( 1 + \lambda u'(0) < 0 \). Similarly, \( pu'(B) + (1 - p)u'(0) < u'(B) \) together with (A10) yields \( 1 + \lambda [pu'(B) + (1 - p)u'(0)] > 0 \), which combined with (A11) implies \( \mu < 0 \). It must therefore be \( LHS(A12) < 0 \), which contradicts (A12). Hence, it cannot be that \( \alpha^* = 0 \), which implies \( \alpha^* > 0 \). Q.E.D.

**Proof of Proposition 4.** The analysis of the pure bonus contract was completed in the text, so consider a pure option contract.

**Step 1.** Let \( B = 0 \) and set \( K = \frac{L - w_0}{1 - \alpha} \) (which, as already argued earlier, can be done without loss of generality given the goals of the analysis). (A1) - (A6) then yield

\[
\begin{align*}
W_1 & = w_0 + \alpha(\Delta + Y); \\
W_2 & = w_0 + \alpha \Delta; \\
W_3 & = W_4 = w_0 + \alpha \left( pY + \frac{1 - \alpha}{1 - \alpha q} q \Delta \right); \\
\Omega_2 & = w_0.
\end{align*}
\]

Using (9), the expected utilities (10) and (11) can be written as

\[
\begin{align*}
EU_{1Uninf}^{Uninf} & = u \left( w_0 + \alpha \left( q \alpha + pY - \frac{\alpha q (1 - q)}{1 - \alpha q} \Delta \right) \right); \\
EU_{2Uninf}^{Uninf} & = qu(w_0 + \alpha \Delta) + (1 - q)u(w_0).
\end{align*}
\]

Start with the case where \( EU_{1Uninf}^{Uninf} \geq EU_{2Uninf}^{Uninf} \) at the optimum, so that the relevant incentive compatibility constraint is \( EU_{1Inf}^{Inf} - c \geq EU_{1Uninf}^{Uninf} \). As mentioned in the text, constraints (15) and (16) must both bind, from which \( u(W_3) = W_3 = 0 \). This holds if

\[
w_0 = -\alpha \left( pY + \frac{1 - \alpha}{1 - \alpha q} q \Delta \right).
\]

It then follows that the option grant \( \alpha \) and the equilibrium incomes \( W_1 \) and \( W_2 \) satisfy

\[
\begin{align*}
W_1 & = \alpha \left[ \frac{1 - q}{1 - \alpha q} \Delta + (1 - p) Y \right], \quad (A13) \\
W_2 & = \alpha \left( \frac{1 - q}{1 - \alpha q} \Delta - pY \right), \quad (A14) \\
q [pu(W_1) + (1 - p)u(W_2)] & = c, \quad (A15)
\end{align*}
\]

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and the firm’s expected compensation cost is

\[ EW^{\text{Options}} = q [pW_1 + (1 - p)W_2] = \frac{q\alpha}{1 - \alpha q} (1 - q) \Delta. \]

Let \( \alpha^\Delta \) denote the \( \alpha \) that solves (A13)-(A15) for a given \( \Delta \). Because at \( \alpha = 0 \) we have \( \text{LHS}(17) = q [pu(0) + (1 - p)u(0)] = 0 < c \), it must be that \( \alpha^\Delta > 0 \). Next, let \( \alpha = 1 \). Then \( \text{LHS}(17) = q [pu(\Delta + (1 - p)Y) + (1 - p)u(\Delta - pY)] \), which is greater than \( c \) if \( \Delta \) is sufficiently large because \( \lim_{u \to \infty} u(w) > c/q \) by assumption. Hence, \( \alpha^\Delta \in (0, 1) \).

**Step 2.** It must be \( \alpha^\infty \equiv \lim_{\Delta \to \infty} \alpha^\Delta = 0 \). To see this, assume to the contrary that this equality does not hold. Then there is an \( \varepsilon > 0 \) such that for any given \( M > 0 \), there must exist a \( \Delta > M \) such that \( \alpha^\Delta \geq \varepsilon \). Hence, it must be that, for any \( M \),

\[
\text{LHS}(17) = q [pu(W_1) + (1 - p)u(W_2)] \\
\geq q \left[ pu \left( \varepsilon \left[ \frac{1 - q}{1 - \varepsilon q} M + (1 - p)Y \right] \right) + (1 - p)u \left( \varepsilon \left[ \frac{1 - q}{1 - \varepsilon q} M - pY \right] \right) \right].
\]

From this,

\[
\lim_{\Delta \to \infty} \text{LHS}(17) \geq \lim_{M \to \infty} q \left[ pu \left( \varepsilon \left[ \frac{1 - q}{1 - \varepsilon q} M + (1 - p)Y \right] \right) + (1 - p)u \left( \varepsilon \left[ \frac{1 - q}{1 - \varepsilon q} M - pY \right] \right) \right] \\
= q \lim_{w \to \infty} u(w) > c,
\]

where the last inequality follows from the assumption \( \lim_{w \to \infty} u(w) > c/q \). But the inequality contradicts (17); hence, it must be \( \alpha^\infty = 0 \), as claimed.

**Step 3.** It must be \( \lim_{\Delta \to \infty} W_1 = \lim_{\Delta \to \infty} W_2 \). This follows immediately from \( \alpha^\infty = 0 \) applied to (A13) and (A14):

\[
\lim_{\Delta \to \infty} W_1 = \lim_{\Delta \to \infty} \alpha^\Delta \left[ (1 - p)Y + \frac{(1 - q)}{1 - \alpha^\Delta q} \Delta \right] = \lim_{\Delta \to \infty} \alpha^\Delta (1 - q) \Delta \\
= \lim_{\Delta \to \infty} \alpha^\Delta \left[ \frac{(1 - q)}{1 - \alpha^\Delta q} \Delta - pY \right] = \lim_{\Delta \to \infty} W_2.
\]

Denoting this limit by \( W^\infty \), we get that equality (17) converges to \( qu(W^\infty) = c \). When \( \Delta \to \infty \), a pure option contract therefore yields the expected wage bill

\[
EW^{\text{Options}} = qW^\infty = qu^{-1}(c/q).
\]

**Step 4.** A pure option contract is more efficient than a pure bonus contract if \( EW^{\text{Options}} <
For \( q > p \) and large \( \Delta \), a comparison of (A16) and (14) reveals that this condition holds if \( zu^{-1}(c/z) \) decreases in \( z \). Using \( \frac{\partial u^{-1}(z)}{\partial z} = \frac{1}{u'(u^{-1}(z))} \), this is true if and only if

\[
u^{-1}(c/z)u'(u^{-1}(c/z)) < \frac{c}{z}.
\]

Let \( u^{-1}(c/z) = W \). Then the above condition can be rewritten as \( W u'(W) < u(W) \), which always holds because \( u(.) \) is strictly concave. Hence, in this case \( EW^{\text{Options}} < EW^{\text{Bonus}} \) if \( q > p \) and if \( \Delta \) is sufficiently large.

**Step 5.** If \( EU_1^{\text{Uninf}} \leq EU_2^{\text{Uninf}} \) at the optimum, it must be \( W_3 \leq 0 \). To see this, note that in this case (12) becomes \( EU^{\text{Inf}} - c \geq EU_2^{\text{Uninf}} \), so that the firm’s constraints are

\[
q [pu(W_1) + (1-p)u(W_2)] + (1-q)u(W_3) - c \geq qu(W_2) + (1-q)u(w_0);
q [pu(W_1) + (1-p)u(W_2)] + (1-q)u(W_3) - c \geq 0.
\]

Again, both of these constraints must bind, which implies

\[
EU_2^{\text{Uninf}} = qu(W_2) + (1-q)u(w_0) = 0;
EU^{\text{Inf}} = q [pu(W_1) + (1-p)u(W_2)] + (1-q)u(W_3) = c.
\]

Now, using (10) and \( W_3 = W_4 \) (from Step 1), we have \( EU_1^{\text{Uninf}} = u(W_3) \). Thus, \( EU_1^{\text{Uninf}} \leq EU_2^{\text{Uninf}} \) combined with (A17) yields \( u(W_3) \leq 0 \), which implies \( W_3 \leq 0 \).

**Step 6.** It must be that \( \lim_{\Delta \to \infty} \alpha^* = 0 \). To obtain this claim, rewrite the firm’s optimization problem using the expressions for \( W_1, W_2, W_3 \), and \( \Omega_2 \) introduced in Step 1:

\[
\min_{w_0 \geq 0, \alpha \geq 0} w_0 + p\alpha Y + q\alpha \Delta + q (1-q) \alpha \frac{1 - \alpha}{1 - \alpha q} \Delta
\]

subject to

\[
qu(w_0 + \alpha \Delta) + (1-q)u(w_0) = 0;
\]

and

\[
q [pu(w_0 + \alpha(\Delta + Y)) + (1-p)u(w_0 + \alpha \Delta)]
+ (1-q)u \left( w_0 + \alpha \left( pY + \frac{1 - \alpha}{1 - \alpha q} q\Delta \right) \right) = c.
\]

Observe first that it must be \( \lim_{\Delta \to \infty} w_0 > -\infty \). Otherwise, given that \( \lim_{w \to \infty} u(w) = \bar{u} < \infty \) and \( \lim_{w \to -\infty} u(w) = -\infty \) (where the latter is implied by the assumption that \( \lim_{w \to -\infty} u'(w) = \infty \)), the left hand side of (A19) would converge to \(-\infty \) and (A19) would
be violated. The conclusion that \( \lim_{\Delta \to \infty} \alpha^* = 0 \) then follows from the fact that the objective function would otherwise increase without bounds as \( \Delta \to \infty \), which cannot be optimal.

**Step 7.** Now suppose \( \lim_{\Delta \to \infty} \alpha^* \Delta = 0 \). Then \((A19)\) implies \( w_0^\infty \equiv \lim_{\Delta \to \infty} w_0 = 0 \) and applying the limit to the \( LHS(A20) \), we get

\[
\lim_{\Delta \to \infty} LHS(A20) = u(0) < c,
\]

which violates \((A20)\). Hence, it must be that \( N \equiv \lim_{\Delta \to \infty} \alpha^* \Delta > 0 \). The limits of \((10)\) and \((11)\) are then

\[
\lim_{\Delta \to \infty} EU_1^{Uninf} = u(w_0^\infty + qN), \quad \text{and} \quad \lim_{\Delta \to \infty} EU_2^{Uninf} = qu(w_0^\infty + N) + (1 - q)u(\Omega_2^\infty) = 0,
\]

so that \( \lim_{\Delta \to \infty} EU_2^{Uninf} \geq \lim_{\Delta \to \infty} EU_1^{Uninf} \) iff

\[
qu(w_0^\infty + N) + (1 - q)u(w_0^\infty) \geq u(w_0^\infty + qN).
\]

This, however, contradicts the assumption that \( u(.) \) is strictly concave. Hence, for large \( \Delta \), it must be that \( EU_1^{Uninf} > EU_2^{Uninf} \) in equilibrium, so that \( EW^{Options} < EW^{Bonus} \) for all \( q > p \), as established in Step 4. Q.E.D.

**Proof of Proposition 5.** (i) Recall that it is efficient for the worker to provide effort whenever \( Y > u^{-1}(c)/p \). According to the analysis leading to Proposition 4, under a pure bonus contract the optimal bonus \( B^* \) is given by \( pu(B^*) = c \). The firm finds this contract worth using only if \( B^* \leq Y \). Thus, when \( Y < u^{-1}(c)/p \), the firm prefers that the worker remains uninformed. This means that when \( Y \in (Y_1, Y_2) \), where \( Y_1 \equiv u^{-1}(c)/p \) and \( Y_2 \equiv u^{-1}(c/p) \), the worker remains uninformed even though getting informed would be efficient. Note that the interval \((Y_1, Y_2)\) is non-empty because the expression \( pu^{-1}(c/p) \) decreases in \( p \) (as shown in the proof of Proposition 4), which implies \( u^{-1}(c) < pu^{-1}(c/p) \) for all \( p < 1 \).

(ii) As shown in the proof of Proposition 4, when \( \Delta \to \infty \) the expected wage bill under an optimal pure option contract converges to

\[
EW^{Options} = qu^{-1}(c/q).
\]

In the absence of moral hazard, the agent would only need to be compensated for his effort
cost (through a fixed wage). The first-best expected wage bill is therefore

\[ W^{FB} = u^{-1}(c). \]

Thus, \( \lim_{q \to 1} (EW^{\text{Options}} - W^{FB}) = 0 \), which proves the claim. Q.E.D.

Proof of Proposition 6. Consider a pure bonus contract (\( \alpha = 0 \) and \( B > 0 \)) and assume that at \( t = 1 \) the worker can purchase shares of the firm’s stock in the open market. Suppose the investors expect the worker to get informed with probability \( \eta \). If the worker’s trading were not reflected in the stock price at \( t = 1 \), then the stock price would be given by \( \hat{P}_1 = L + q\Delta + pq(\gamma - B) - w_0 \). Given that part of the \( t = 1 \) demand for shares comes from an informed worker who has observed \( x = H \), and given that a worker who has observed \( x = L \) does not purchase any shares at \( t = 1 \), the actual \( t = 1 \) price would be higher than \( \hat{P}_1 \). Nevertheless, it is without loss of generality to proceed under the assumption that \( P_1 = \hat{P}_1 \), because the worker’s trading gains, and hence his incentives to get informed, are higher the lower is \( P_1 \) and the goal of the proof is to show that open market trading does not provide sufficient incentives for the worker to get informed.

Thus, set \( P_1 = \hat{P}_1 \) and consider an informed worker who observes \( x = H \). The worker expects the stock to be ultimately worth \( P_2(y = Y, H) = H + Y - B - w_0 \) with probability \( p \) and \( P_2(y = 0, H) = H - w_0 \) with probability \( 1 - p \). The worker’s \( t = 1 \) expected utility from purchasing \( \beta \) shares is therefore

\[
EU_{2}^{\text{Inf}}(H, \beta, \eta) = pu(w_0 + B + \beta (P_2(y = Y, H) - P_1)) + (1 - p) u(w_0 + \beta (P_2(y = 0, H) - P_1))
\]

\[
= pu(w_0 + B + \beta ((1 - q) \Delta + (1 - pq) (Y - B))) \\
+ (1 - p) u(w_0 + \beta ((1 - q) \Delta - pq (Y - B))).
\]

The worker’s incentive compatibility and participation constraints are then

\[
qEU_2^{\text{Inf}}(H, \beta, \eta) + (1 - q) [pu(w_0 + B) + (1 - p) u(w_0)] - c = u(w_0);
\]

\[
qEU_2^{\text{Inf}}(H, \beta, \eta) + (1 - q) [pu(w_0 + B) + (1 - p) u(w_0)] - c = 0.
\]

This implies \( u(w_0) = 0 \) and \( w_0 = 0 \), so that the above two constraints collapse into

\[
qEU_2^{\text{Inf}}(H, \beta, \eta) + (1 - q)pu(B) - c = 0,
\]

or

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\[
q [pu (B + \beta (((1 - q) \triangle + (1 - p\eta) (Y - B)))
+ (1 - p) u (\beta ((1 - q) \triangle - p\eta (Y - B)))] + (1 - q)pu(B) = c. \quad \text{(A21)}
\]

Denote the optimal number of shares that the informed worker purchases at \(t = 1\) by \(\beta^*\) and define \(\hat{\beta} \equiv \lim_{q \to 1} \beta^*\). Similarly, define \(\hat{w}_0 \equiv \lim_{q \to 1} w_0^*\), \(\hat{B} \equiv \lim_{q \to 1} B^*\), and \(\hat{\eta} \equiv \lim_{q \to 1} \eta^*\). Taking the limit as \(q \to 1\), condition (A21) implies

\[
pu \left( \hat{B} + \hat{\beta} (1 - p\hat{\eta}) \left( Y - \hat{B} \right) \right) + (1 - p) u \left( -\hat{\beta}p\hat{\eta} \left( Y - \hat{B} \right) \right) = c. \quad \text{(A22)}
\]

Now, assume \(\hat{\eta} = 1\). Then (A22) becomes

\[
pu \left( \hat{B} + \hat{\beta} (1 - p) \left( Y - \hat{B} \right) \right) + (1 - p) u \left( -\hat{\beta}p \left( Y - \hat{B} \right) \right) = c \quad \text{(A23)}
\]

and the firm’s expected compensation bill is

\[
p \left[ \hat{B} + \hat{\beta} (1 - p) \left( Y - \hat{B} \right) \right] - (1 - p) \hat{\beta}p \left( Y - \hat{B} \right)
= p\hat{B}.
\]

From (A23), we have \(\frac{d\hat{B}}{d\hat{\beta}} > 0\). Thus, when \(q\) is close to 1, the expected compensation is (weakly) higher under open market trading than it would be under pure bonus contract with no trading. From Proposition 5 we know that, holding all other parameters fixed, there is a \(Y_2\) such that when \(Y \leq Y_2\) the firm prefers to set \(B^* = 0\) when \(\beta = 0\). Given that \(p\hat{B}\) increases in \(\hat{\beta}\), this conclusion has to hold also when trading is feasible. But if \(\hat{B} = 0\), then (A23) cannot hold, due to the concavity of \(u(\cdot)\).

Thus, it must be \(\hat{\eta} < 1\). That is, for \(q\) close to 1, the probability that the worker gets informed has to be bounded away from 1, which in turn implies that the efficiency of any incentive contract that lacks options is bounded away from the first best. Given that by part (ii) of Proposition 5 for \(q\) close to one and \(\triangle\) large \(\varepsilon\)-efficiency can be achieved by a pure option contract, for these parameter values the owners must find it optimal to include an option grant in the worker’s contract. Q.E.D.

**Proof of Proposition 7.** If the limited liability constraint does not bind, then Proposition 3 applies and we are done. Thus, assume the constraint binds, which immediately implies \(w_0 = 0\). Now, the result of Proposition 2 applies even under limited liability; thus, it is again

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without loss of generality to set $K = \frac{L-u_0}{1-\alpha} = \frac{L}{1-\alpha}$. The firm’s optimization problem is then

$$\min_{\alpha \geq 0, B \geq 0} q \left[pW_1 + (1-p)W_2\right] + (1-q) \left[pW_3 + (1-p)W_4\right]$$

s.t. $EU^{\text{Inf}} \equiv q \left[pu(W_1) + (1-p)u(W_2)\right] + (1-q) \left[pu(W_3) + (1-p)u(W_4)\right] - c \\
\geq qu(W_2) + (1-q)u(w_0) \equiv EU^{\text{Uninf}}_2; \\
EU^{\text{Inf}} \equiv q \left[pu(W_1) + (1-p)u(W_2)\right] + (1-q) \left[pu(W_3) + (1-p)u(W_4)\right] - c \geq u(W_4) \equiv EU^{\text{Uninf}}_1; \\
EU^{\text{Inf}} \equiv q \left[pu(W_1) + (1-p)u(W_2)\right] + (1-q) \left[pu(W_3) + (1-p)u(W_4)\right] - c = 0.$

Suppose, contrary to the claim in the proposition, that $\alpha^* = 0$. The same argument as in the proof of Proposition 3 implies that it must be $B \in (0,Y)$, because otherwise inducing information acquisition cannot be strictly profitable. Similarly, an argument almost identical to the one in the proof of Proposition 3 (with the exception that $w_0 = 0$) yields that for all $\alpha \in (0,\hat{\alpha}]$ the worker’s IC constraint (12) is given by $EU^{\text{Inf}} - c = EU^{\text{Uninf}}_1$. Furthermore, given that $w_0 = 0$, the participation constraint (13) must hold whenever the IC constraint holds.

Restricting attention to $\alpha \leq \hat{\alpha}$, the firm’s optimization problem can thus be written as

$$\min_{\alpha \geq 0, B \geq 0} q \left[pW_1 + (1-p)W_2\right] + (1-q) \left[pW_3 + (1-p)W_4\right]$$

s.t. $q \left[pu(W_1) + (1-p)u(W_2)\right] + (1-q) \left[pu(W_3) + (1-p)u(W_4)\right] - c = u(W_4),$

where

$$W_1 = B + \alpha(\triangle + Y-B);$$
$$W_2 = \alpha \triangle;$$
$$W_3 = B + W_4;$$
$$W_4 = \alpha \left(p(Y-B) + \frac{1-\alpha}{1-\alpha q}q\triangle\right).$$

Let $\lambda$ be the Lagrange multiplier associated with the constraint. The corresponding first
order conditions are then

\[ \alpha : \quad qp(\triangle + Y - B)[1 + \lambda u'(W_1)] + q(1-p)\triangle [1 + \lambda u'(W_2)] + [p(1-q)][1 + \lambda u'(W_3)] \\
+ (1-p)(1-q)[1 + \lambda u'(W_4)] \left[ p(Y - B) + \frac{1 - 2\alpha + q\alpha^2}{(1 - \alpha q)^2} q\triangle \right] \geq 0; \]

\[ B : \quad qp(1 - \alpha)[1 + \lambda u'(W_1)] + p(1-q)[1 + \lambda u'(W_3)](1 - \alpha) - (1-p)(1-q)\alpha p[1 + \lambda u'(W_4)] = 0, \]

where the equality in FOC(B) follows from the fact that if \( \alpha = 0 \) then (12) can hold only if \( B > 0 \).

Now, evaluating at \( \alpha = 0 \), we get \( W_1 = W_3 = B \) and \( W_2 = W_4 = 0 \), so that the above first order conditions reduce to

\[ \alpha : \quad qp(\triangle + Y - B)[1 + \lambda u'(B)] + q(1-p)\triangle [1 + \lambda u'(0)] + [p(1-q)][1 + \lambda u'(B)] \\
+ (1-p)(1-q)[1 + \lambda u'(0)] [p(Y - B) + q\triangle] \geq 0; \]

\[ B : \quad p[1 + \lambda u'(B)] = 0. \]

The second condition implies \( 1 + \lambda u'(B) = 0 \). Plugging this into FOC(\( \alpha \)) yields

\[ [1 + \lambda u'(0)] [q(1-p)\triangle + (1-p)(1-q)p(Y - B) + q\triangle] \geq 0. \]

Given that \( B > 0 \), we have \( u'(0) > u'(B) \), which implies \( 1 + \lambda u'(0) < 0 \), so that the above yields

\[ q(1-p)\triangle + (1-p)(1-q)p(Y - B) + q\triangle \leq 0. \]

This cannot hold because all the terms on the left hand side are strictly positive. The assumption that \( \alpha^* = 0 \) thus yields a contradiction and it must be that \( \alpha^* > 0 \). Q.E.D.
References


