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# Optimal Redistributive Pensions with Temptation and Costly Self-Control

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# Optimal redistributive pensions and the cost of self-control

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**Abstract**

We examine how the introduction of self-control preferences influences the trade-off between two fundamental components of a public pension system: the contribution rate and its degree of redistribution. The pension regime affects individuals' welfare by altering how yielding to temptation (i.e. not saving, or saving less) is attractive. We show that proportional taxation increases the cost of self-control, and that this adverse effect is more acute when public pensions become more redistributive.

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# 1 Introduction

In this paper, we examine how temptation and costly self-control influence the trade-off between two fundamental components of a public pension system: the contribution rate and its degree of redistribution. The pension regime affects individuals' welfare by altering how attractive it is to yield to temptation - i.e. not saving, or saving less. We show that proportional taxation increases the cost of self-control and reduces voluntary savings, and that this adverse effect is more acute when public pensions become more redistributive.

Our analysis is conducted for individuals who have self-control preferences (Gul & Pesendorfer, 2001, 2004). With this utility representation, decision makers exhibit a preference for commitment mechanisms that would shelter them from sources of temptation. In a world stripped of all sources of temptation, they would save money so as to smooth consumption over the life cycle. Unfortunately for them, saving necessitates cognitive self-control that imposes an immediate cost. As a consequence, they tentatively accommodate two competing desires: smoothing consumption and seeking immediate gratification.

The mental cost of self-control arises because an individual who saves remains aware of the immediate gratification that *could have* been had by consuming all his available liquidity. Suppose, for instance, that someone with $1,000 available to spend needs to save $100 now in order to smooth consumption over time. By not saving anything at all, that person could afford a thousand-dollar vacation right now. Thus, saving entails immediately depriving oneself from the vacation, which creates mental suffering. This person has a preference for commitment because it would be better to live in a world without vacations, which would make saving the $100 effortless. Absent such an unlikely commitment device, the optimal decision will be to compromise between the no-temptation ideal (saving $100) and yielding to temptation (spending $1,000). This person may therefore save $50 and spend for $950.

Partly succumbing to immediate temptation leaves that same individual with less cash on hand in the future, a source of liquidity that would have become an eventual source of temptation. Assume that the same individual receives a paycheck of $1 000 each month. By perfectly resisting temptation and saving $100 in January, the person is left with $1,100 in February. If savings is delayed and only $50 is saved in January, there will be only $1,050 left in February. Thus, resisting temptation now increases future costs of resisting it later. Hence, individuals delay savings to mitigate both immediate and future costs of self-control.

In our model, a retiree's income flows from two sources: public pension benefits and personal savings. One's pension benefit consists of both a contributory-based (Bismarckian) payment, and a lump-sum (Beveridgian) transfer that everyone receives independently of their past income or contributions. The former part of the pension scheme consists, therefore, of forced savings, whereas the latter redistributes income across retirees. The forced-saving and redistributive roles of public pensions conflict with each other. To balance its budget constraint, a government that wants to make public pensions incrementally more redistributive must give up on forcing individuals to save by the same amount.

One the one hand, we find that forcing individuals to save provides them with a commitment device, which induces optimal public pensions to be more Bismarckian when the share of individuals with self-control problems in the economy increases, or when temptation become more intense. On the other hand, introducing such a commitment mechanism tends to make it more costly for individuals to save by themselves. This undesirable welfare effect of Bismarckian pensions can be mitigated by making the optimal pension plan more redistributive. Therefore, when more individuals have self-control problems, governments must consider that adverse marginal effect on the cost of self-control and should focus more on the redistributive aspect of the scheme. Although simple, this underlying intuition has not yet been pointed out, possibly because no traditional optimal pensions exercise has yet been done with self-control preferences.[1]

[1]Krussel et al. (2010) studied a Ramsey tax problem with linear taxes in a single-agent model, and advocated late consumption and savings subsidies.

The remainder of the paper is structured as follows. In Section 2 we review the related literature and we introduce the self-control preferences. A special emphasis is put on their suitable properties for conducting standard welfare analysis. Section 3 presents our social security model. We introduce the public pension scheme, show the individuals' maximization problem and derive optimal pension formulas. We complete our analysis by providing numerical examples of our results in subsection 4.2. Section 5 concludes.

# 2 Literature and the rationale for self-control preferences

After its introduction by Samuelson (1937), the discounted-utility model (DU) has become the standard way to account for intertemporal utility.[2] Within that framework, one's perception of one's own welfare is $\sum_{t=0}^{T} \beta^t u(c_t)$ where $u(c_t)$ is the cardinal instantaneous utility at $t$, and $\beta^t$ is a geometric discount function that captures the relative weight one attaches to one's own well-being in period $t$. Attractive by its simplicity, the model condenses all motives for time-preferences into a personal discount factor $\beta$. A consequence of using a geometric discounting function is that the individual's sequence of decisions is time-consistent (Strotz, 1955). For instance, a typical consumption-saving plan will satisfy $u'(c_t)/u'(c_{t+1}) = \beta(1+r)$ for all periods, no matter when decisions are made.[3] Marginal rates of substitution between consumption for any two periods do not depend on time and, accordingly, individuals would not change their plans if they were given the opportunity to do so.[4] In the context of an optimal policy problem, the government's assessment of what is preferred by any individual coincides with the preferences that the individual reveals. Thus, policy design can be

[2]See Frederick et al. (2002) for a historical survey of time-discounting.

[3]For simplicity, here we assume that individuals face no binding liquidity constraints and the absence of risks.

[4]Two of the standard assumptions of the standard decision-utility model are therefore generated by the geometric discount function: the *fixed lifetime preferences* condition and the *no-mistake* property (Bernheim & Rangel, 2007).

conducted on a nonpaternalistic basis.

On empirical grounds, the DU model has been criticized on the basis of its low predictive accuracy. First, it does not account for drastic increases in savings just before retirement or for sudden drops in consumption just after (Bernheim et al., 2001). Second, the model seems incapable of explaining why individuals value commitment: they are willing to pay to have some consumption opportunities removed from their future choice sets (Ashraf et al., 2006).[5] Third, the DU model is incompatible with preference reversals documented in experiments: an earlier reward is often preferred when it offers an immediate payoff, whereas a later reward is preferred when both are delayed (Kirby & Herrenstein, 1995).

These anomalies, a psychologist would argue, can be explained by the existence of self-control problems. Immediate gratification is often tempting, and resisting it requires willpower, which may be especially difficult to do in the face of visceral temptations (Frederick et al., 2002; Baumeister, 2002). Attempts to account for these issues have often come through modifications in individuals discount functions. One prevalent example is the quasi-hyperbolic discounting model (Phelps & Pollak, 1968; Laibson, 1997; Angeletos et al., 2001) in which the individual's discount function is time-dependent. A quasi-hyperbolic discounter applies a higher utility discount rate to the tradeoff between $t$ and $t+1$, but expects to behave like a geometric discounter in decisions involving later periods. In a consumption-saving context, the marginal rate of substitution between immediate and delayed consumption satisfies $u'(c_t)/u'(c_{t+1}) = \delta\beta(1+r)$, whereas that between two delayed periods is $u'(c_{t+1})/u'(c_{t+2}) = \beta(1+r)$.

Since they give a special discounting treatment to immediate utility, quasi-hyperbolic individuals reveal different preferences in every period. In response to these so-called preference reversals, policy recommendations must involve violations of the principle of revealed

[5] In particular, credit card and debt puzzles raised the issue that many households simultaneously hold high-interest credit card debts, while otherwise saving through devices that yield significantly lower returns (Laibson et al., 2003; Haliassos & Reiter, 2005).

preferences. In other words, the social planner must choose in what period the individual reveals his "true" preferences. For example, Cremer et al. (2008) and Cremer & Pestieau (2011) design an optimal forced savings scheme when some individuals are myopic ($\delta = 0$). Forced savings are optimal because the social planner disagrees with individuals' rate of time discounting. In the macroeconomics literature, Imrohoroglu et al. (2003) and Fehr et al. (2008) evaluate whether social security may improve paternalistic social welfare.

Unfortunately, modifications in discount functions do not account for several pieces of evidence. The first issue relates to individuals being aware of their self-control problems while, at the same time, yielding to temptation. First, Ameriks et al. (2007) show that individuals understand their self-control problem, and that they know how it affect their choices.[6] Loewenstein (1996) also finds that tempted individuals feel out of control at the moment when temptation is felt. One important consequence is that individuals value commitment, because they know what their ideal consumption choice would be if resisting temptation was costless. Since it is not, they exert some self-control, but not enough to attain their ideal solution (Wertenbroch, 1998; Ariely & Wertenbroch, 2002). In an intertemporal setup, this means that individuals delay savings in order to reduce the cost of self-control.

Gul & Pesendorfer (2001, 2004) accounted for these empirical regularities by developing the self-control preference representation. Instead of altering discount rates, they modify the domain of preferences so that experienced utility depends on both the actual consumption choice, and on a tempting available option that is not chosen. The intuition is that someone might be better off if some particularly tempting option were not available, even if that option were never chosen. The self-control utility representation is

$$\sum_{t=0}^{T} \beta^t \left( u(c_t) + \psi(v(c_t) - \max_{c_t} v(c_t)) \right). \tag{1}$$

[6]In Ameriks et al. (2007), subjects were asked to allocate a prize over time. They were also questioned about their ideal plan, and about whether they expected to deviate from it. The authors used these data to construct an index called the "ideal-expected gap", which was found to be correlated with present-biased behavior. So individuals act in full knowledge that they do not follow their ideal plan.

Instantaneous utility at $t$ depends on two elements. First, $u(\cdot)$ is the standard instantaneous utility function that the individual would maximize if he was not subject to temptation or, equivalently, if he could pre-commit at no cost. The negative term $\psi(v(c_t) - \max v(c_t))$ is the utility penalty of exerting self-control at $t$ by not consuming the most tempting option $\hat{c}_t \equiv \arg\max_{c_t} v(c_t)$. It is generally called the "cost of self-control." In consumption-saving problems, $\hat{c}_t$ represents the immediate consumption level of someone who yields to temptation, and who consumes all available cash on hand. The function $\psi v(\cdot)$, which is increasing in its argument, is called the temptation function.

Since instantaneous utility is discounted geometrically, the self-control preferences meet the conditions for time-consistency as enounced in Strotz (1955). The representative Euler equation for a consumption-saving problem,

$$\frac{u'(c_t) + \psi v'(c_t)}{u'(c_{t+1}) + \psi v'(c_{t+1}) - \psi v'(\hat{c}_{t+1})} = \beta(1 + r), \ \forall t. \tag{2}$$

shows that the marginal rate of substitution between consumption at $t$ and $t + 1$ does not depend on time, which is a by-product of geometric discounting. This means that welfare analyses can be conducted without violating the principle of revealed preferences. Therefore, the cost of self-control is fully experienced by individuals, and it must be accounted for when calculating individual welfare.

Equation (2) also shows how the cost of self-control influences intertemporal allocation of resources. The term $\psi v'(c_t)$ in the numerator, and $\psi v'(c_{t+1})$, respectively in the numerator and in the denominator, capture a static effect that is significant in inducing delayed savings when the cost of self-control is driven to zero at $t + 1$. The most important mechanism is dynamic and is captured by $-v'(\hat{c}_{t+1}))$ in the denominator. Because saving more at $t$ increases cash on hand tomorrow, it changes the size of the most tempting option at $t + 1$. Thus, individuals can simultaneously reduce their immediate and future cost of self-control by consuming more today. According to recent research, both the static and the dynamic effects are shown to be empirically relevant (Bucciol, 2012; Huang et al., 2013).

## Self-control preferences and social security

One consequence of the self-control preferences is that a public pension policy may be useful, even if individuals do not regret their past decisions. In particular, governments dispose of fiscal instruments that change cash on hand and, therefore, the cost of self-control. In the case of public pensions, forced savings through taxation has two effects. Taxes collected at $t$ reduce the size of the most tempting option at $t$ without modifying the size of that at $t+1$ by deferring consumption to retirement age. Thus, forced savings can increase individual welfare, by making individuals save at no cost. However, forced savings change the marginal cost of exerting self-control for those who intend to complement their pension wealth with personal savings.

Despite their appealing features, the self-control preferences have largely been ignored in the normative taxation literature, but its implications for social security policy has been briefly treated in the macroeconomic literature. Kumru & Thanopoulos (2008) studied numerically the welfare effect of social security as compared to a benchmark economy populated by identical time-inconsistent agents. Self-control preferences then mitigate the adverse welfare costs of social security, focusing on the case of a very convex temptation function. Kumru & Thanopoulos (2011) show that the elimination of social security may not be optimal when the intensity of the self-control problem is high, still with a convex temptation ranking. Bucciol (2011) allowed households to allocate time between labor and leisure, but with a pension system that has no redistributive objective. Still solely focusing on a convex temptation ranking, he concludes that social security can be welfare improving, also obtaining the special case that payroll taxation can reduce the mental cost of self-control.

We innovate by rigorous analytical study of the impact of self-control preferences on the optimal design of a redistributive pension scheme. We provide a characterization using tax formulas. In particular, we find that providing commitment saving through proportional taxation increases the cost of self-control. As compared to a policy that is based on paternalism, as in Cremer et al. (2008), it is optimal for the government to put more emphasis on the redistributive aspect of the social security scheme. This result remains robust, no matter whether the temptation function is convex or concave.

# 3 The model

The economy consists of $2N$ types of individuals. They differ with respect to their productivity (indexed by $i$) and the intensity of their self-control problems (indexed by $j$). Heterogeneity in productivity is captured by the existence of $N$ exogenously given wage rates, which are denoted by $w_1 < \cdots < w_i < \cdots < w_N$. For each wage type, individuals are distinguished by the intensity of their self-control problem, which we denote $\lambda_j$. We call those who face a positive self-control problem *tempted individuals*, ($\lambda_{j=1} = \psi > 0$), and we refer to those who have no self-control problem as *untempted individuals*, ($\lambda_{j=0} = 0$).

The total mass of these $2N$ individuals is normalized to one. For each proportion $p(w_i)$ of individuals with productivity $w_i$, there is a fixed share $\pi$ of tempted individuals and the remainder $(1 - \pi)$ are untempted individuals. Thus, the mass of individuals of type $ij$ is given by $\pi p(w_i)$ if they face temptation and $(1-\pi)p(w_i)$ if not and we use $\pi_{ij}$ as a shorthand for all these proportions. The government does not observe the type of each individual, but it does know their proportions.

There are three periods that we index by $t = 0, 1, 2$. In the first two periods, individuals supply labor ($L_{ijt}$) and save ($s_{ijt}$). Individuals are liquidity constrained, so $s_{ijt} \geq 0$. This ensures that public pension claims cannot be used as a collateral to obtain consumption credit (Lindbeck & Persson, 2003). Moreover, individuals start their active lives without liquidities, and there are no bequests. In period $t = 2$, individuals do not work and consume their savings and their pension benefits $b_{ij}$.

### Redistributive pensions

Pension benefits depend on a proportional tax $\tau$ on labor income, which finances the pension, and a redistributive parameter $\alpha$, which determines the proportion of benefits that depends on one's own contribution. The tax rate is the same for all individuals, since the government cannot observe the types directly. Both private and public savings are capitalized at an exogenous interest rate $r$. Denote one's lifetime labor earnings in value at $t = 2$ by $Y_{ij} \equiv \sum_{t=0}^{1} w_i L_{ijt}(1+r)^{2-t}$. Thus, when an individual reaches retirement time, the lifetime capitalized contributions to the pension fund are $\tau Y_{ij}$. Denote further the (capitalized) lifetime earnings collected in the economy by $E[Y] = \sum_{ij} \pi_{ij} Y_{ij}$. The pension benefit of an individual $ij$ is a linear combination of his own contributions and those of the average:

$$b_{ij} = \alpha \tau Y_{ij} + (1-\alpha)\tau E[Y] \tag{3}$$

The first component $\alpha \tau Y_{ij}$ is the contributory/Bismarckian benefit. It is a forced saving device. The second component $(1-\alpha)E[Y]$ is the lump-sum/Beveridgian one and is redistributive, since everyone receives it independently of their past income or contributions. Increasing $\alpha$ forces people to save more, a policy objective that conflicts with its redistributive counterpart.[7] A purely Bismarckian system has $\alpha = 1$ whereas a Beveridgian system has $\alpha = 0$. A pension system featuring $\alpha < 0$ is targeted since individuals are implicitly taxed for their contributions. This system is very redistributive, but also highly very distortionary.

### Consumption and savings decisions

We denote by $c_{ijt}$ the consumption level of an individual $ij$ at time $t$. Consumption is expressed net of disutility of labor, which is captured by a strictly convex cost function

[7] Note that the pension plan's budget is always balanced by definition, as is typical in linear-progressive tax models.

$\varphi(L_{ijt})$.[8] Hence, budget constraints governing choices are given by:

$$c_{ijt} = (1-\tau)w_i L_{ijt} - \varphi(L_{ijt}) + (1+r)s_{ijt-1} - s_{ijt} \quad t = 0, 1, \tag{4}$$

$$c_{ij2} = (1+r)s_{ij1} + b_{ij}. \tag{5}$$

Individuals decide on their consumption and savings allocation according to:

$$V^*_{ij}(w_i, \lambda_j; \alpha, \tau) \equiv \max_{\{L_{ijt}, s_{ijt}\}_{t=0}^{1}} \sum_{t=0}^{1} \beta^t \left[ u(c_{ijt}) + \lambda_j \left( v(c_{ijt}) - \max_{L_{ijt}, s_{ijt}} v(c_{ijt}) \right) \right] + \beta^2 u(c_{ij2}). \tag{6}$$

subject to (4), (5) and to the liquidity constraints $s_{ijt} > 0$.

Throughout the paper, we append variables with a star (for example, $c^*_{ijt}$) to refer to the optimal allocation chosen by the individual, given $\tau$ and $\alpha$.. The term $u(c^*_{ijt})$ is the standard utility level one gets from consumption and is the only term that matters if agents are untempted ($\lambda_j = 0$). The function $u(\cdot)$ is just a typical utility function, or a“commitment ranking” which is strictly concave and meets Inada’s requirements.

If, however, an individual is tempted, she has self-control preferences (Gul & Pesendorfer, 2001, 2004) and the function $\psi v(\cdot)$ is a temptation ranking that captures the welfare effect of temptation. The most tempting option at time $t$ is given by

$$\max_{L_{ijt}, s_{ijt}} v\left((1-\tau)w_i - \varphi(L_{ijt}) - s_{ijt}\right) \tag{7}$$

and we will denote $\hat{c}_{ijt}$ as this most tempting allocation. Likewise, labor supply under the most tempting option is denoted by $\hat{L}_{ijt}$. It is implicitly given by $\varphi'(\hat{L}_{ijt}) = (1-\tau)w_i$ because savings are null.

This individual derives immediate utility $u(c_{ijt}) + \psi\left(v(c_{ijt}) - v(\hat{c}_{ijt})\right)$, where $\hat{c}_{ijt} > c^*_{ijt}$. It is typical to call $-\psi(v(c_{ijt}) - v(\hat{c}_{ijt})) > 0$ the cost of self-control one imposes on oneself in

[8] It satisfies $\varphi(0) = 0,\ \varphi'(0) = 0,\ \varphi'(L_{ijt}) > 0\ \forall L_{ijt} > 0$ and $\varphi''(L_{ijt}) > 0\ \forall L_{ijt} > 0$.

a given period, a cost which is triggered by saving money in the face of temptation.

As is typical with linear-progressive taxation models, individuals do not internalize the effect of their own labor supply decisions on the lump-sum (Beveridgian) part of the pension plan. If one inserts equations directly - (4) and (5) in (6) - individual decisions are given by the following first-order conditions:

$$
\begin{aligned}
(L_{ij0}): &\quad [((1-\tau)w_i - \varphi'(L_{ij0})][u'(c_{ij0}) + \lambda_j v'(c_{ij0})] + \alpha\tau w_i u'(c_{ij2}) = 0, && (8)\\
(L_{ij1}): &\quad [((1-\tau)w_i - \varphi'(L_{ij1})][u'(c_{ij1}) + \lambda_j v'(c_{ij1})] + \alpha\tau w_i u'(c_{ij2}) = 0, && (9)\\
(s_{ij0}): &\quad -[u'(c_{ij0}) + \lambda_j v'(c_{ij0})] + [u'(c_{ij1}) + \lambda_j v'(c_{ij1})] - \lambda_j v'(\hat{c}_{ij1}) \leq 0, && (10)\\
(s_{ij1}): &\quad -[u'(c_{ij1}) + \lambda_j v'(c_{ij1})] + u'(c_{ij2}) \leq 0, && (11)
\end{aligned}
$$

where labor supply always find interior solutions, but where the two last conditions strictly equal zero only when the liquidity constraints are not binding. From (10) and (11) above, one can readily see that all individuals with self-control issues delay savings.

The assumptions on the disutility of labor $\varphi(\cdot)$ ensure interior solutions for individual labor supply. We get

$$
\varphi'(L_{ijt}) = (1-\tau)w_i + \alpha\tau w_i \frac{u'(c_{ij2})}{u'(c_{ijt}) + \lambda_j v'(c_{ijt})}, \ t = 0, 1. \tag{12}
$$

Equation (12) exhibits the good incentive properties of a system that is Bismarckian. When $\alpha > 0$ individuals reduce their labor supply when taxed more, but the distortion is partly alleviated because they are aware that a share of their contribution will be paid back to them at $t = 2$. This implies that the labor supply satisfies $L^*_{ijt} > \hat{L}_{ijt}$ if and only if $\alpha > 0$.

### Skills levels and the cost of self-control

Before designing an optimal pension plan, it is instructive to refer to the laissez-faire solution to assess the economic significance of the shape of the temptation ranking $v(\cdot)$. Over one's lifetime, the total *cost of self-control* that will be experienced is

$$\gamma_{ij} = \lambda_j \sum_{t=0}^{1} \beta^t [v(\hat{c}_{ijt}) - v(c^*_{ijt})] > 0. \tag{13}$$

Proposition 1 shows that the cost of self-control increases in $w_i$ if the temptation-ranking is convex, and that it decreases with $w_i$ otherwise.

**Proposition 1.** *In a laissez-faire equilibrium (with $\tau = 0$ and $\alpha$ undetermined), commitment-ranking utility is strictly increasing with respect to $w_i$. However, the cost of self-control is strictly increasing in $w_i$ if $v''(\cdot) > 0$, and strictly decreasing in $w_i$ if $v''(\cdot) < 0$.*

**Proof:** Using the envelope theorem, differentiating one's indirect utility function with respect to $w_i$ yields:

$$\frac{\partial V^*_{ij}(w_i, \lambda_j; \alpha, 0)}{\partial w_i} = \underbrace{\sum_{t=0}^{1} \beta^t L_{ijt} u'(c^*_{ijt})}_{\Delta\text{commitment utility}} - \underbrace{\sum_{t=0}^{1} \beta^t L_{ijt} \lambda_j (v'(\hat{c}_{ijt}) - v'(c^*_{ijt}))}_{\Delta\text{cost self-control}}. \tag{14}$$

Since $u$ is strictly increasing in its argument, the first term within brackets implies that commitment utility is strictly increasing in $w_i$ as well. For individuals with problems of self-control, since marginal tax rates are null, $L^*_{ijt} = \hat{L}_{ijt}$ for all $i$. By the definition of the maximization problem, $\hat{c}_{ijt} \geq c^*_{ijt}$ holds with strict equality if savings are positive in at least one period. Because $u$ satisfies the Inada conditions, $s^*_{ij1} > 0$ and $\gamma_{ij} > 0$. Thus, the net effect of $w_i$ on the cost of self-control relies solely on the sign of $v'(\hat{c}_{ijt}) - v'(c^*_{ijt})$. $\square$

Proposition 1 has important economic features, and has significant consequences for potential policy involvements. Absent any public intervention, the effect of $w_i$ on the cognitive cost of self-control depends on the difference between the marginal temptation-utility of actual consumption and that of the most tempting consumption level. The difference between both can take either sign, depending on the shape of temptation, which is itself characterized by the sign of $v''(\cdot)$. It is noteworthy that the axioms underlying the self-control preferences allow the function $v(\cdot)$ to be either concave or convex.[9]

The curvature of the temptation ranking $v(\cdot)$ turns out to be relevant. If $v(\cdot)$ is strictly concave, the cost of temptation is more significant for poorer individuals whereas self-control is costlier for the rich when it is convex. Both possibilities have been explored in other papers. In Kumru & Thanopoulos Kumru & Thanopoulos (2008, 2011), they examine reforms to social security when individuals face both convex and concave-shaped temptation. However, some empirical literature suggests that individuals who make decisions in contexts of scarcity find it more difficult to exert self-control. For example, Mullainathan & Banerjee (2010) show that the poor may be more likely to exhibit a hands-to-mouth type of behavior when fulfilling basic needs is involved. Spears (2011), Bernheim et al. (2012) and Shah et al. (2012) also reached similar conclusions. Others have argued that living in a context of scarcity taxes individuals' mental resources and reduce one's ability to resist temptation (Mani et al., 2013). As we seek a complete analysis, we look at these two possibilities and include them in our paper.

# 4 Optimal redistributive pensions

We can derive expressions for the effect of an exogenous change in the policy parameters, $\alpha$ and $\tau$, on one's welfare. We do so by using both the envelope theorem and the fact that individuals take the Beveridgian component of public pensions as given when making decisions.

[9] The only requirement is that the problem must be globally concave, or that $u''(c_{ijt})+\lambda_j v''(c_{ijt}) < 0$, $\forall ij$.

A marginal increase in $\alpha$ has the following effect on one's welfare:

$$\frac{\partial V_{ij}^*(w_i,\lambda_j;\alpha,\tau)}{\partial\alpha} = u'(c_{ij2})\tau\left[Y_{ij} - E(Y) + (1-\alpha)\frac{\partial E(Y)}{\partial\alpha}\right]. \tag{15}$$

Making the system more Bismarckian increases the welfare of those whose lifetime income is larger than average, by making them benefit more from their own contributions. By the same token, it penalizes retirees whose lifetime income was lower than the average. The second effect, this time via the tax-base, is beneficial to all since $E(Y)$ is increasing in $\alpha$. Making the system more contributory has a positive effect on labor supply, thereby reducing the distortions entailed by income taxes.[10]

The effect of a marginal increase in $\tau$ is somewhat more complex. Taking the derivative of one's indirect utility function with respect to the tax rate and reorganizing terms yields:

$$\begin{aligned}\frac{\partial V_{ij}^*(w_i,\lambda_j;\alpha,\tau)}{\partial\tau} =& \overbrace{\sum_{t=0}^{1}\beta^t[u'(c_{ij2}^*) - u'(c_{ijt}^*)]w_i L_{ijt}^*}^{\text{Consumption smoothing}} + \overbrace{w_i\lambda_j\sum_{t=0}^{1}[\hat{L}_{ijt}v'(\hat{c}_{ijt}) - L_{ijt}v'(c_{ijt}^*)]}^{\text{Cost of self-control}} \\ &+ \underbrace{(1-\alpha)u'(c_{ij2}^*)(E(Y) - Y_{ij})}_{\text{Equity}} + \underbrace{(1-\alpha)\tau u'(c_{ij2}^*)\frac{\partial E(Y)}{\partial\tau}}_{\text{Efficiency}}.\end{aligned} \tag{16}$$

The right-hand side of (16) clarifies the four effects of taxation on individuals' welfare. The first term, $\sum_{t=0}^{1}\beta^t[u'(c_{ij2}^*) - u'(c_{ijt}^*)]w_i L_{ijt}^*$, is the consumption-smoothing benefit of taxation. By displacing consumption from early periods to retirement, it increases the value of one's commitment ranking unless it induces $c_{ijt}^* < c_{ij2}^*$ for some liquidity-constrained agents. The second term, which is of high interest to us, is the effect of taxation on the cost of self-control $\gamma_{ij}$ for $j = 1$. Its sign, which is not fully characterizable analytically without imposing functional forms, is analyzed in proposition 2:

[10]That $\partial E(Y)/\partial\alpha > 0$ can be observed from the first-order conditions with respect to $L_{ijt}$, although the comparative statics is highly intractable in our three-period model.

**Proposition 2.** ***Effect of taxation on the cost of self-control.*** *The net effect of payroll taxation is to* ***increase*** *the cost of self-control for all individuals with $j = 1$ if $v'' < 0$ and $\alpha \geq 0$. It* ***reduces*** *it when $v'' > 0$ and $\alpha \leq 0$. The effect is ambiguous otherwise.*

*Proof.* The net effect of taxation on the cost of self-control is $\phi_{ij} \equiv w_i \lambda \sum_{t=0}^{1} [\hat{L}_{ijt} v'(\hat{c}_{ijt}) - L^*_{ijt} v'(c^*_{ijt})]$. First recall that $\hat{c}_{ijt} \geq c^*_{ijt}$ for $t = 0, 1$. Also, by the first-order conditions of the individuals with self-control problems, $(L^*_{ijt} - \hat{L}_{ijt})$ takes the same sign as $\alpha$. It ensues that whenever that $\phi_{ij} < 0$ $\alpha \geq 0$ and $v''(\cdot) < 0$, and that $\phi_{ij} > 0$ whenever $\alpha \leq 0$ and $v''(\cdot) \geq 0$. The sign of $\phi_{ij}$ can take either sign otherwise and depends on the numerical specification of the model. □

With public pensions in the economy, the effect of taxation on the cost of self-control depends on both the shape of $v(\cdot)$ but also on $\alpha$, the extent to which the social security is dedicated to forcing individuals to save. Our result can be explained in a simple heuristic way. In period $t$, the effect of taxation on individual welfare via the cost of self-control is

$$w_i \hat{L}_{ijt} v'(\hat{c}_{ijt}) - L^*_{ijt} v'(c^*_{ijt}) \equiv w_i \hat{L}_{ijt}(v'(\hat{c}_{ijt}) - v'(c^*_{ijt})) + w_i(\hat{L}_{ijt} - L^*_{ijt}) v'(c^*_{ijt}). \tag{17}$$

The first term on the right-hand side of (17) depends only on the sign of $v''(\cdot)$ : it is negative when $v'' < 0$ and positive if $v'' > 0$. The rightmost term depends on the sign of $\alpha$. When $\alpha > 0$ then $\hat{L}_{ijt} < L^*_{ijt}$, which further reduces utility. By the same token, a very redistributive system with $\alpha < 0$ has the opposite effect. As one will see, this is one of the reasons why a social planner who feels concerned about self-control will tend to make the system more Bismarckian. The result is also summarized in the following table. In particular, the table shows that the only situation in which public pensions unambiguously reduce one's cost of self-control is when social security is extremely redistributive ($\alpha < 0$) joint with a strictly convex temptation ranking $v(\cdot)$.

[Table A about here]

## 4.1 Problem of the government and tax formulas

The government's problem is to choose the optimal values for $\tau$ and $\alpha$ that maximize its social objective. We continue assuming that the social welfare function is weighted utilitarian and that it assigns a weight $\omega_{ij}$ on each type-$ij$ individual. Recall that individuals with self-control problems do not face preference reversal problems. Thus, the optimal tax problem is rather straightforward, given the nonpaternalistic nature of the policy. Individuals' perception of their own welfare is equivalent to their indirect utility functions $V_{ij}^*(w_i, \lambda_j; \alpha, \tau)$ for all $ij$, which is aggregated as by the social welfare function.[11] The government solves

$$(\alpha^*, \tau^*) \equiv \arg\max_{\alpha,\tau} \sum_{t=0}^{2} \sum_{ij} \pi_{ij}\omega_{ij} V_{ij}^*(w_i, \lambda_j; \tau, \alpha) \tag{18}$$

where $\alpha^*$ and $\tau^*$ denote the solution to the maximization problem, and where the pension plans' balanced budget constraint is implicitly included in the indirect utility functions of individuals. Given that the problem is globally concave, interior solutions for the policy parameters are characterized by the first-order conditions

$$\sum_{t=0}^{2} \sum_{ij} \pi_{ij}\omega_{ij} \frac{\partial V_{ij}^*(w_i, \lambda_j; \alpha, \tau)}{\partial \alpha} = 0 \tag{19}$$

$$\sum_{t=0}^{2} \sum_{ij} \pi_{ij}\omega_{ij} \frac{\partial V_{ij}^*(w_i, \lambda_j; \alpha, \tau)}{\partial \tau} = 0 \tag{20}$$

where the partial derivatives of the indirect utility functions with respect to $\alpha$ and $\tau$ are respectively expressed in (15) and (16). Reorganizing the first-order condition allows us to obtain implicit tax formulas. We denote by $\xi_{ij} \equiv \omega_{ij} u'(c_{ij2}^*)$ the marginal social value of an increase in old-age revenue of a type-$ij$ individual. Given decreasing marginal utility, $\xi_{ij}$ is decreasing with retirement income and increasing with the welfare weights. The implicit

[11] This contrasts with paternalistic objectives found in models of quasi-hyperbolic discounting (as previously discussed) in which the government must choose how to interpret the preference reversal of individuals.

policy formula for $\alpha^*$ is characterized by

$$\operatorname{cov}(\xi, Y) + (1-\alpha)E(\xi)\frac{\partial E(Y)}{\partial \alpha^*} = 0. \tag{21}$$

Equation (21) reflects the typical equity-efficiency tradeoff in optimal taxation. A negative covariance term between $Y_{ij}$ and the marginal utilities of retirement consumption strengthen the case for more redistribution (lower $\alpha$). However, the desirable effect of making the system more contributory-based counterbalances equity concerns.

The behavioral role of the pension system must figure in the implicit tax formula, which is the following:

$$\tau^* = \frac{\overbrace{\operatorname{cov}(\xi, Y)}^{\text{Equity}}}{E(\xi)\frac{\partial E(Y)}{\partial \tau^*}} + \dots$$
$$\dots \frac{\overbrace{\sum_{t=0}^{1}\beta^t E[\omega_{ij} w_i L^*_{ijt}(u'(c^*_{ij2}) - u'(c^*_{ijt}))]}^{\text{consumption smoothing effect}} + \overbrace{\sum_{t=0}^{1}\beta^t E[\omega_{ij}\lambda_j(w_i \hat{L}_{ijt} v'(\hat{c}_{ijt}) - w_i L^*_{ijt} v'(c^*_{ijt}))]}^{\text{self-control effect} \lesseqgtr 0}}{\underbrace{-(1-\alpha)E(\xi)\frac{\partial E(Y)}{\partial \tau^*}}_{\text{Distortions} > 0}}. \tag{22}$$

To clarify how the cost of self-control affects the optimal tax rate, we have divided the right-hand side of (22) into two parts. The first part captures the traditional equity (numerator) and efficiency (denominator) tradeoff that we find in linear-progressive optimum tax models.

The rightmost term, which has the labor-market distortion in the denominator, contains two components in the numerator. These are the terms of importance here, because they capture the two roles of public pensions that we want to emphasize, namely consumption-smoothing (commitment effect) and its effect on the mental cost of exerting self-control. For clarity, both of them are textually identified in (22).

The social consumption-smoothing benefits are due to forced savings, which helps satisfy individuals' commitment rankings. It is generally positive in the presence of individuals with problems of self-control, unless the society consists of a large number of liquidity constrained individuals who end up consuming more during their retirement years than when they are younger.[12] Accordingly, the consumption-smoothing benefit of taxation seems to justify higher tax rates.

However, that commitment benefit may conflict with the effect of an increase in taxes on individuals' costs of self-control. If taxation reduces someone's cost of exerting self-control, that individual will be induced to save more by himself. In this case, the consumption-smoothing and self-control effects of taxation go in the same direction in the implicit tax formulas. We find that for a significant family of cases, increasing taxes also increases individuals' costs of self-control, thereby offsetting the consumption-smoothing benefits of taxation.

## 4.2 Taxation and redistribution increase the cost of self-control: numerical examples

We provide a numerical illustration of how the forced-savings (commitment) role of the pension system may conflict with its effect on the aggregate mental costs of self-control in the economy. The results that are reported are a representative and nuanced subset of the several simulated experiments that we ran with the model.

Our simulations are of the same type as those of Cremer et al. (2008) and Cremer &

[12]Note that $\alpha = 1$ means that the sole role of the pension system is to force individuals to save. It can only happen when all individuals are identical in all respects, in which case pensions are perfect substitutes for savings. It does not happen here, since the distribution of wages is a motive for redistribution.

Pestieau (2011). The government maximizes a utilitarian utility function, so $\omega_{ij} = 1$ for all $ij$. Wages are distributed according to a beta(2,4) distribution, discretized on the domain [1,4], which induces income inequality. The commitment ranking is logarithmic with $u(c) = \log(c)$ and the temptation ranking, which is allowed to be both convex or concave, takes the CRRA form $v(c) = c^{1-\rho}/(1-\rho)$. The interest rate and discount factors satisfy $\beta = 1/(1+r)$ where $r = 0$, and none of our results hinge on this number. We conduct simulations with a strictly concave temptation ranking where $\rho = 0.5$ and with a strictly convex one, where $\rho = -0.5$.

Tables B and C report the optimal policies when $v''(\cdot) < 0$ and when the intensity of the self-control problem is $\psi = 0.1$. As $\pi$ denotes the proportion of tempted individuals, the optimal policy in an economy where all individuals are tempted is reported on the first line of the tables, whereas that in an economy populated with untempted individuals figures only on the very last line[13]. For several values of $\pi$, we show the optimal policy $(\alpha^*, \tau^*)$. Additionally, we report what we call the "marginal behavioral welfare effects of pension taxation", evaluated at the optimal policy. These are the consumption-smoothing benefits due to forced savings:

$$\sum_{t=0}^{1} \beta^t E[w_i L^*_{ijt}(u'(c^*_{ij2}) - u'(c^*_{ijt}))], \tag{23}$$

and the marginal social welfare effect of taxation due to it affecting costs of self-control

$$\sum_{t=0}^{1} \beta^t E[\psi(w_i \hat{L}_{ijt} v'(\hat{c}_{ijt}) - w_i L^*_{ijt} v'(c^*_{ijt}))]. \tag{24}$$

Recall that both of these terms are identified in the tax formula (22).

Generally, we find that the tax rates and the extent to which the pension plan is contributory increase with both the proportion of individuals with self-control problem and with the intensity of self-control problems. This should seem intuitive, since when the intensity of

[13] In our simulations, all agents have positive savings when $\pi = 0$. As a consequence, the first-order conditions for labor depend solely on $(1-\alpha)\tau$ altogether. There is thus a degree of freedom in choosing the pair $(\alpha^*, \tau^*)$. We report in this last line the value $(0, \tau_0^*)$ that satisfy the first-order condition. Any other pair satisfying $(1-\alpha)\tau = \tau_0^*$ would work.

self-control problems $\psi$ increases, individuals with these problems displace more consumption toward early periods. Thus, when a high proportion of individuals have self-control issues, the forced-saving role of pensions is important and a large portion (around one-third) of pension benefits are Bismarckian.

However, as our theoretical results show, optimal tax rate and Bismarckian factors increase with $\psi$ and $\pi$ because they provide self-control individuals with a forced savings device, but not because the pension system reduces their marginal cost of exerting self-control. Thus, our numerical analysis shows that the rationale for forced savings arises because it provides commitment, but generally not by reducing the cost of self-control. Thus, our policy conclusions go in the same direction as Cremer & Pestieau (2011).

To clearly observe this, refer to the rightmost columns in tables B and C, which gives how the marginal social welfare changes because of the cost of self-control. In both cases, one can see that this effect is always negative: taxation *increases* the costs of self-control, and partly offsets the forced savings role of the pension system. As a logical consequence, we see that when $\psi$ goes from 0.1 to 0.25 (i.e. when passing from table B to table C), the marginal increase in the cost of self-control induced by taxation roughly doubles. Thus, if $\alpha$ and $\tau$ increase with the intensity of self-control, it is simply because the forced-savings benefits of the pension system increase accordingly. In this regard, the roles of social security with self-control preferences is comparable to that under a time-inconsistent, paternalistic policy.

It is no surprise, however, that the negative social welfare effect of taxation generally does not fully offset the positive consumption-smoothing effect of the pension system. If it were the case, the only role of public pensions would be to redistribute income. We would then have a purely Beveridgian or a targeted system, even when a large proportion of individuals has a self-control problem.

[Table B about here]

[Table C about here]

Finally, in table D we consider the case where all individuals have self-control problems ($\pi = 1$), and we provide the optimal policy for some very large values of $\psi$. One can then observe that for reasonably low intensities of self-control problems a larger $\psi$ is associated to more forced savings. However, when $\psi$ becomes outstandingly large, the negative self-control effect of taxation tends to drive $\alpha$ down and the optimal Bismarkian term $\alpha^*$ starts decreasing with $\psi$.

[Table D about here]

Let us now consider the case where $v''(\cdot) > 0$. As before, the optimal $\tau$ and $\alpha$ both increase when the intensity of self-control problem increases, and when a larger share of the population is subject to it. In all the simulations that we have run we have found that, for high values of $\psi$, a similar tradeoff as with the concave temptation ranking operates. However, the effect on the cost of self-control is welfare-enhancing when a small share of the population has self-control problems, which induces an optimal policy where $\alpha$ becomes small (redistribution becomes dominant), and taxes become small as well. This result should nonetheless be nuanced: in the optimum, taxation increasingly reduces the marginal cost of self-control as government gradually gives up on forced savings and focuses only on its normative objective (redistribution). So, one can hardly think of pension taxation as a useful device to reduce mental costs of self-control that are either very severe, or highly prevalent in the economy.

# 5 Concluding comments

This paper analyzed an optimal public pension scheme where individuals' well-being is characterized by self-control preferences. We focus on the effect of taxation on the costs of exerting the required self-control to voluntarily save. We study cases in which that cost decreases with productivity levels and where it increases with productivity. We find that the commitment benefits of pension taxation can be offset by increasing the cost of self-control. Thus, in a nonnegligible and realistic set of situations, the joint presence of temptation and self-control weakens the rationale for forced savings.

Deriving an optimal-linear pension scheme allowed us to find simple tax formulas and to characterize the possibly competing commitment and self-control effects of taxation. One possible criticism is its partial equilibrium nature, and the fact that the only source of distortions comes from the noncontributory part of the pension benefit formula (as is typically the case with static linear-progressive taxation models). The important element for our results is that self-control preferences induce a wedge between marginal temptation-utility of actual consumption and that of the most tempting option. One should note that this effect would be robust to a more complex environment, including an overlapping-generation model with endogenous capital accumulation. A next step in this line of research is to study non-linear pension schemes, with which this wedge may be relaxed.

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| | $v'' < 0$ | $v'' > 0$ |
|---|---|---|
| $\alpha > 0$ | $> 0$ | $\gtrless 0$ |
| $\alpha = 0$ | $> 0$ | $< 0$ |
| $\alpha < 0$ | $\gtrless 0$ | $< 0$ |

A positive sign indicates an increase in the c.s.c and, thus, a reduction in utility.

Table A: Effect of $\tau$ on cost of self-control

Table B: Policy and marginal behavioral effects: $v''(x) < 0$ and $\psi = 0.1$

| | Optimal policy | | Marginal behavioral effects | |
|---|---|---|---|---|
| $\pi$ | $\alpha^*$ | $\tau^*$ | Cons. Smooth. | Self-Control |
| 1.0 | 0.3245 | 0.1592 | 0.1640 | -0.0483 |
| 0.9 | 0.3090 | 0.1573 | 0.1363 | -0.0427 |
| 0.8 | 0.2849 | 0.1545 | 0.1201 | -0.0370 |
| 0.7 | 0.2607 | 0.1517 | 0.0968 | -0.0315 |
| 0.6 | 0.2121 | 0.1461 | 0.0623 | -0.0262 |
| 0.5 | 0.1565 | 0.1399 | 0.0462 | -0.0211 |
| 0.4 | 0.0875 | 0.1325 | 0.0316 | -0.0162 |
| 0.3 | 0.0500 | 0.1271 | -0.0103 | -0.0123 |
| 0.2 | 0.0500 | 0.1250 | -0.0631 | -0.0085 |
| 0.1 | -0.3267 | 0.0978 | 0.0017 | -0.0036 |
| 0.0 | — | 0.0281 | — | — |

Note: We report the pair $(0, \tau^*)$ when $\pi = 0$. See footnote 13 for details.

Table C: Policy and marginal behavioral effects: $v''(x) < 0$ and $\psi = 0.25$

| | Optimal policy | | Marginal behavioral effects | |
|---|---|---|---|---|
| $\pi$ | $\alpha^*$ | $\tau^*$ | Cons. Smooth. | Self-Control |
| 1.0 | 0.3763 | 0.1671 | 0.4957 | -0.1020 |
| 0.9 | 0.3794 | 0.1666 | 0.4136 | -0.0925 |
| 0.8 | 0.3620 | 0.1646 | 0.3538 | -0.0794 |
| 0.7 | 0.3475 | 0.1625 | 0.2849 | -0.0676 |
| 0.6 | 0.3250 | 0.1597 | 0.2612 | -0.0549 |
| 0.5 | 0.2863 | 0.1552 | 0.1790 | -0.0430 |
| 0.4 | 0.2350 | 0.1493 | 0.1714 | -0.0304 |
| 0.3 | 0.1850 | 0.1426 | 0.0951 | -0.0210 |
| 0.2 | 0.0950 | 0.1338 | 0.0172 | -0.0118 |
| 0.1 | 0.0500 | 0.1251 | -0.0495 | -0.0056 |
| 0.0 | — | 0.0281 | — | — |

Note: We report the pair $(0, \tau^*)$ when $\pi = 0$. See footnote 13 for details.

Table D: Policy and marginal behavioral effects: $v''(x) < 0$ and $\pi = 1$

| | Optimal policy | | Marginal behavioral effects | |
|---|---|---|---|---|
| $\psi$ | $\alpha^*$ | $\tau^*$ | Cons. Smooth. | Self-Control |
| 5 | 0.1158 | 0.1661 | 1.0481 | -0.1172 |
| 3 | 0.1610 | 0.1668 | 0.9523 | -0.1482 |
| 1 | 0.2839 | 0.1671 | 0.7413 | -0.1807 |
| 0.35 | 0.3601 | 0.1671 | 0.5576 | -0.1247 |
| 0.25 | 0.3763 | 0.1671 | 0.4957 | -0.1020 |
| 0.10 | 0.3245 | 0.1592 | 0.1640 | -0.0483 |