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Information, Commitment, and Separation in Illiquid Housing Markets

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(Job Market Paper)

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Abstract

I propose a model of the housing market using a search framework with asymmetric information in which sellers are unable to commit to asking prices announced ex ante. Relaxing the commitment assumption prevents sellers from using price posting as a signalling device to direct buyers' search. Adverse selection and inefficient entry on the demand side then contribute to housing market illiquidity. Real estate agents that can improve the expected quality of a match can segment the market and alleviate information frictions. Even if one endorses the view that real estate agents provide no technological advantage in the matching process, incentive compatible listing contracts are implementable as long as housing is not already sufficiently liquid. The theoretical implications are qualitatively consistent with the empirical observations of real estate brokerage: platform differentiation, endogenous sorting, and listing contract features that reinforce incentive compatibility.

JEL classification: D40, D44, D83, R31

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1 Introduction

In this paper, I develop a search-theoretic model of the housing market that (i) employs a method of price determination that accounts for the strategic interaction between buyers and sellers; (ii) incorporates the documented heterogeneity in seller motivation and asymmetry of information; and (iii) provides insight about the role of real estate agents and intuition for the seemingly puzzling structure of listing contracts. I first show that satisfying the first two requirements leads to an equilibrium with adverse selection and inefficient entry of buyers. I then focus on the potential role of real estate agents in overcoming information frictions and improving market efficiency.

Extensive empirical work has established several stylized facts about housing market prices and selling times.[1] The correlation between prices and liquidity and the observed price dispersion in housing markets point to search theory as an appropriate modelling technique. While existing search models of the housing market can account for a wide range of the empirical trends, I argue that off-the-shelf search frameworks are not consistent with casual observations of the real estate market. For instance, some of these models do not allow for multiple offers by competing bidders, while others ignore the possibility of renegotiating offers announced ex ante when there are ex post incentives to do so. I show that accounting for these phenomena in the pricing protocol of a search and matching model has implications for liquidity and efficiency, and introduces the informational role of agency in illiquid markets.

There is good reason to suspect that sellers of identical houses differ in terms of their reservation price. Glower, Haurin, and Hendershott (1998) conduct a survey of home sellers and find substantial heterogeneity in terms of motivation to sell: some sellers have a strong desire to sell quickly, while other sellers are much more patient. A seller's degree of patience can be a reflection of a job opportunity elsewhere or the seller's arrangement to purchase her next home (i.e., the seller might have already bought a new home, and wants to sell the

[1] See for example, Glower, Haurin, and Hendershott (1998), Merlo and Ortalo-Magné (2004), Krainer (2001), and Leung, Leong, and Wong (2006).

first home quickly in order to avoid double mortgage payments). Accordingly, I introduce heterogeneity on the seller side of the market to reflect differences in reservation values. Importantly, the seller's willingness to sell is unobservable to the buyer. Market participants would benefit if this information could be credibly conveyed, for example, by means of list prices. I show that the inability to commit to a list price prevents sellers from using price posting as a signalling device. Instead, patient sellers mimic impatient sellers in order to drive up the final sale price by increasing the probability of a bidding war. Consequently, illiquidity in the housing market is rendered more severe because of adverse selection and inefficient entry on the demand side.

I extend the model to include real estate agents as service providers that can improve the expected quality of a match. In North American housing markets, sellers typically hire real estate agents to provide expert advice about setting a list price, market their house, and negotiate on their behalf. By modelling the listing contract between a seller and her agent, I find that in some circumstances, real estate agents can offer incentive compatible contracts to segment the market by seller type. This alleviates the information problem and increases liquidity in the housing market. Even if real estate agents provide no technological advantage in the matching process and offer no direct benefit to the seller, incentive compatible listing contracts are implementable as long as housing is sufficiently illiquid; i.e., a house is not readily saleable due to search and information frictions.

In the theory, incentive compatibility is not the result of exogenously imposed assumptions on preferences or technologies to satisfy a Spence-Mirrlees sorting condition, since the direct costs and benefits of real estate services are independent of a seller's type. Instead, the housing market is characterized by a directed search environment, in which real estate agents play the role of market makers as in Mortensen and Wright (2002). Designing a new real estate listing agreement creates a new submarket in the search framework that can potentially attract sellers and buyers. Sellers respond differently to changes in the arrival rate of buyers, which in turn is related to the endogenous composition of sellers. Anxious sellers might be willing to over-invest in real estate services if it allows them to distinguish them-

selves from relaxed sellers and attract more potential buyers. Market separation is therefore the result of a sorting condition that arises endogenously because of the beliefs and equilibrium search strategies of buyers. These theoretical predictions are consistent with the recent empirical evidence of endogenous sorting and service differentiation between full-commission full-service realtors, and low-cost limited-service agents (Bernheim and Meer, 2008; Levitt and Syverson, 2008a; Hendel, Nevo, and Ortalo-Magné, 2009): sellers represented by full-commission agents tend to exhibit characteristics consistent with high motivation to sell, and consequently experience shorter selling times and a higher probability of sale.

This paper is related to the recent literature that applies search theory to model the housing market (Wheaton, 1990; Arnold, 1999; Krainer, 2001; Albrecht et al., 2007; Díaz and Jerez, 2010; Head, Lloyd-Ellis, and Sun, 2010). My approach differs from these papers in that I develop a process of price determination that reflects the following stylized facts: sometimes the terms of sale are determined through bilateral bargaining, other times the house is sold in an auction with multiple bidders. Moving away from Nash bargaining and non-negotiable price posting towards a setting that more closely resembles the pricing mechanism observed in North American real estate markets has important implications for housing liquidity and market efficiency.

The model presented here is perhaps closest to Albrecht, Gautier, and Vroman (2010). They also depart from benchmark search models and allow for multilateral matches with terms of trade determined through auctions. Their framework imposes commitment to sell when a buyer offers the list price. I demonstrate the importance of this type of assumption for achieving a fully separating equilibrium and constrained efficiency. In Canada and the U.S., there is no legal obligation associated with the list price that compels a seller to accept an offer. By relaxing the commitment assumption, I show that the equilibrium is necessarily pooling. The housing market is then plagued by illiquidity as a result of adverse selection and inefficient entry of buyers relative to the solution to a social planner's problem. By introducing real estate agents in a realistic manner, I investigate when separation can be restored and the implications for liquidity and efficiency in the housing market. The results

are robust to changes in the fee structure of real estate listing agreements. In particular, market separation remains feasible and asking prices become part of the signalling game when real estate fees are expressed as a percentage of the sale price and the listing agreement contains a clause that entitles the agent to the commission upon receipt of an offer greater than or equal to the list price. In general, sellers signal their willingness to sell via their real estate agent. Some agents represent anxious sellers and as a result attract more buyers, while relaxed sellers are more likely to sell without the assistance of an agent, or with limited-service discount realtors.

This paper also contributes to the search literature, and in particular the study of markets with search frictions and private information. With only a few exceptions, most theories rely on strong commitment assumptions. Guerrieri, Shimer, and Wright (2010) present a search environment with adverse selection and show that screening can at least partly alleviate the symptoms of private information in a competitive search environment when the uninformed party can commit to a take-it-or-leave-it trading mechanism. Delacroix and Shi (2007) study a model with adverse selection where sellers can post non-negotiable prices as a means of directing search, and also as a signal of the quality of their asset. In contrast, relaxing the assumption of full commitment to the announced terms of trade is an important element in this paper.

Kim (2009) shows that non-binding messages can generate a partially separating equilibrium in a decentralized asset market when there is private information about the quality of the asset. Sustaining endogenous market segmentation requires the condition that the seller's type affects the buyer's value. Here, the hidden information is the seller's motivation, which is independent of the buyer's valuation. Menzio (2007) relaxes the commitment assumption in a model of the labour market and shows that cheap talk can sometimes credibly convey information when wages are determined through bilateral bargaining. In essence, incentive feasible market separation is a consequence of an inflexible process of wage determination. The rigidity of the bilateral bargaining game with asymmetric information limits the share of the surplus that can be extracted by a deviating firm. In my environment, the trans-

action price increases with the number of buyers in a match.[2] This generates an incentive to exploit ex post opportunities, which hinders truthful information revelation and unravels market separation in the version of the model without real estate agents.

The next section presents the model of the housing market with heterogeneity in seller motivation but without real estate agents. A comparison of the market equilibrium with the constrained efficient allocation leads to a discussion of how information frictions give rise to housing illiquidity. Real estate agents are introduced in Section 3. Section 4 concludes.

2 The Model

There is a fixed measure S of sellers, and a measure B of buyers determined by free entry. Buyers pay a cost κ to enter the market for housing and visit a home listed for sale. Buyers are ex ante homogeneous. Upon visiting a house, however, the value v that they assign to home ownership is a match specific random variable, depending on the idiosyncratic quality of the match. The random variable is designed to capture the fact that houses are "inspection" goods, or "search" goods as in Nelson (1970). The subtle differences between units that are only observable by visiting and inspecting a house result in variation in buyers' ex post valuations. For convenience, a buyer visits a home and values it at v_L with probability $1-q$ (he likes the house) and $v_H > v_L$ with probability q (he loves the house). In Section 3, I introduce real estate agents that can use their expert knowledge of the market and provide marketing services to increase the probability that a potential buyer's valuation is high.

Heterogeneity on the seller side reflects differences in willingness to sell. Consistent with the evidence documented by Glower, Haurin, and Hendershott (1998), some sellers are desperate to sell quickly, while other sellers are more relaxed.[3] In a dynamic setting, preferences

[2]Julien, Kennes, and King (2006) highlight the implications of this type of setup for residual price dispersion in a theory of the labour market with full information.

[3]For instance, a seller moving to another city to start a new job is likely willing to sell at a low price if it means a shorter time on the market. On the other hand, a seller hoping to move to a different neighbourhood in the same town is more inclined to hold out for a higher sale price. The fact that most sellers are also buyers in the housing market is likely another source of heterogeneity in seller motivation. Some sellers

over price and liquidity would affect the discount rate. In a static setting, heterogeneity in reservation values is sufficient for capturing this phenomenon. A fraction σ_0 of sellers are anxious or impatient sellers with a low reservation value, c_A. The remaining $1-\sigma_0$ of sellers are relaxed/patient, with a high reservation value, $c_R \in (c_A, v_L)$. Differences in sellers' willingness to sell is an important source of asymmetric information in the housing market, since reservation values are unobservable to buyers.

If a buyer meets a seller and a transaction takes place at price p, the payoff to the buyer is $v-p$, and the payoff to the seller is $p-c$, where $v \in \{v_L, v_H\}$ refers to the quality of the match between the buyer and the house, and $c \in \{c_A, c_R\}$ is the reservation value of the seller. Buyers are unable to coordinate their search activities, which generates both an unsold stock of housing and bidding wars in equilibrium. The matching process of buyers and sellers is governed by the urn-ball matching function. Let $\theta = B/S$ denote the ratio of buyers to sellers, or market tightness. The probability that a seller is matched with exactly k buyers follows a Poisson distribution,[4]

$$e^{-\theta} \cdot \frac{\theta^k}{k!}, \qquad k = 0, 1, 2, ...$$

I depart from the price determination mechanisms typically used in off-the-shelf search models. Nash bargaining is inappropriate for modelling the interaction between buyers and sellers in housing markets with multilateral matches (i.e., when several buyers visit the same house), especially in settings with private information. Price posting by sellers requires commitment, even though ex post there are incentives for sellers to allow buyers to bid the price up above the list price. Instead, I propose a different mechanism to reflect these important dimensions of house price determination. In a bilateral match, the buyer negotiates directly with the seller, but if other buyers are interested in the same house, they

might have already submitted offers to purchase another home. Illiquidity in the housing market means that they may either find themselves servicing two mortgages, or have the purchase fall through if it was a conditional-on-sale offer.

[4]These matching probabilities are calculated for a large market with $B, S \to \infty$ and $B/S = \theta$. Search frictions therefore arise because of a lack of coordination among buyers (see Burdett, Shi, and Wright, 2001).

bid competitively for the purchase.

2.1 Buyers' Bidding Strategies

Consider a housing market characterized by the buyer-seller ratio θ, and the fraction of highly motivated sellers σ. If a buyer is the only one to visit a particular house (a bilateral match), he is free to make an offer without worrying about competing bidders. In such cases, the buyer is a monopsonist, and makes a take-it-or-leave-it offer of either c_A or c_R, whichever yields the highest expected payoff. If $\sigma(v - c_A) > v - c_R$, there is a selection problem, and a monopsonist offers c_A, knowing that if the seller is of type R, the offer is rejected and there is no transaction. Otherwise, the monopsonist makes a safer offer of c_R, and trade will occur regardless of the seller's type. When more than one buyer arrives (a multilateral match), they compete for the house in a private value sealed bid auction.[5][6] Type L buyers compete à la Bertrand and bid v_L. If all bidders are of type L, the seller randomly selects among the buyers bidding v_L, so that each bidder has an equal probability of purchasing the home. Type H buyers implement a mixed bidding strategy, conditional on the number of other bidders, k. The bidding strategy can be represented by a distribution function $F_k : (v_L, \bar{b}_k] \to [0, 1]$, where $\bar{b}_k \leq v_H$ is the endogenously determined highest bid. Since all

[5]A potential buyer can observe the number of competing bidders, which is consistent with a survey of recent home buyers, conducted by Genesove and Han (2011). A seller has a vested interest in disclosing this information, since the presence of other buyers bids up the price of her house. Sellers/real estate agents have strategic ways to credibly convey this information to competing bidders. For example, a home listing can specify a date and time when offers will be accepted and reviewed. This leads to a scenario with competing bidders in the same location at the same time, where buyers can condition their bidding strategy on the number of other buyers interested in the same house. Alternatively, sellers can inform potential buyers after the initial offer submission that there are k competing offers and provide opportunity to resubmit. Intermediation by real estate agents adhering to a code of ethics would prevent sellers from being untruthful about the existence of competing offers. Instead, permitting buyers to submit bids with escalator clauses would circumvent the issue of truthful disclosure regarding the participation of other bidders (see footnote 6).

[6]The theoretical results in this paper are robust to perturbations of the process of price determination. For example, it is straightforward to show that the expected payoff functions are unaltered when buyers are permitted to submit bids with escalator clauses, or when sellers run simultaneous multiple round auctions. Incorporating a more sophisticated bilateral bargaining game instead of a take-it-or-leave-it offer, such as the one studied by Grossman and Perry (1986) and used by Menzio (2007), does not change the theoretical implications of the model.

type H buyers are identical, I assume symmetric bidding strategies. Bidders are indifferent between bids in $(v_L, \bar{b}_k]$. A type H buyer bidding an infinitesimal amount more than v_L wins the auction if none of the other k bidders draw v_H. A higher bid increases the probability of winning the auction if other bidders are of type H, but reduces the payoff if the bidder is successful in winning the auction.

2.2 Expected Payoffs and Free Entry

The expected payoff to a buyer depends on the match specific quality, v_L or v_H. For a low quality match, the expected payoff to the buyer is

$$U_L(\sigma,\theta) = e^{-\theta}\max\left\{\sigma(v_L - c_A), v_L - c_R\right\} = \begin{cases} e^{-\theta}\sigma(v_L - c_A) & \text{if} \quad \sigma > \frac{v_L - c_R}{v_L - c_A} \\ e^{-\theta}(v_L - c_R) & \text{if} \quad \sigma \leq \frac{v_L - c_R}{v_L - c_A} \end{cases} \tag{1}$$

This is just the payoff in the monopsony case, which occurs with probability $e^{-\theta}$. The expected payoff in a multilateral match with $k = 1, 2, \ldots$ other buyers is zero for a type L buyer since the equilibrium bid is v_L. The expected payoff of visiting a house as a type H buyer with bidding strategies $\{b_k \sim F_k\}_{k=1}^{\infty}$ is

$$U_H(\sigma,\theta) = e^{-\theta}\max\left\{\sigma(v_H - c_A), v_H - c_R\right\} + e^{-\theta}\sum_{k=1}^{\infty}\frac{\theta^k}{k!}\sum_{j=0}^{k} p(j;k,q)F_k(b_k)^j(v_H - b_k) \tag{2}$$

where $p(j;k,q) = \binom{k}{j}q^j(1-q)^{k-j}$ is the probability mass function for the binomial distribution with parameters k and q. It is the probability that exactly j out of the k other bidders are of type H when q is the probability of a high quality match each time a buyer visits a house on the market. Then

$$\begin{aligned} U_H(\sigma,\theta) &= e^{-\theta}\max\left\{\sigma(v_H - c_A), v_H - c_R\right\} + e^{-\theta}\sum_{k=1}^{\infty}\frac{\theta^k}{k!}\sum_{j=0}^{k}\binom{k}{j}q^j(1-q)^{k-j}F_k(b_k)^j(v_H - b_k) \\ &= e^{-\theta q}(v_H - v_L) + \begin{cases} e^{-\theta}\left[\sigma(v_H - c_A) - (v_H - v_L)\right] & \text{if} \quad \sigma > \frac{v_H - c_R}{v_H - c_A} \\ e^{-\theta}(v_L - c_R) & \text{if} \quad \sigma \leq \frac{v_H - c_R}{v_H - c_A} \end{cases} \end{aligned} \tag{3}$$

The final expression makes use of the binomial theorem, imposes the property that mixed bidding strategies require indifference over bids, then recognizes the McLaurin series of the exponential function. Indifference over bids means that submitting any $b_k \in (v_L, \bar{b}_k]$ has to yield the same expected payoff as submitting a bid infinitesimally greater than v_L, which is just enough to win the auction if every other potential buyer is of type L. In a match with k other bidders, equating these expected payoffs and solving for $F_k(b)$ uncovers the bidding strategy:

$$F_k(b) = \frac{1-q}{q}\left[\left(\frac{v_H - v_L}{v_H - b}\right)^{\frac{1}{k}} - 1\right], \qquad b \in (v_L, \bar{b}_k] \tag{4}$$

The highest bid $\bar{b}_k$, defined by $F_k(\bar{b}_k) = 1$, is therefore

$$\bar{b}_k = (1-q)^k v_L + \left[1 - (1-q)^k\right] v_H \tag{5}$$

The expected payoff of entering the housing market as a potential buyer reflects the two possible match qualities: $U(\sigma, \theta) = qU_H(\sigma, \theta) + (1-q)U_L(\sigma, \theta)$, or

$$\begin{aligned} U(\sigma,\theta) = {} & qe^{-\theta q}(v_H - v_L) \\ & + \begin{cases} e^{-\theta}\left[\sigma(v_L - c_A) - q(1-\sigma)(v_H - v_L)\right] & \text{if} \quad \sigma > \frac{v_H - c_R}{v_H - c_A} \\ e^{-\theta}\left[q(v_L - c_R) + \sigma(1-q)(v_L - c_A)\right] & \text{if} \quad \frac{v_L - c_R}{v_L - c_A} < \sigma \leq \frac{v_H - c_R}{v_H - c_A} \\ e^{-\theta}(v_L - c_R) & \text{if} \quad \sigma \leq \frac{v_L - c_R}{v_L - c_A} \end{cases} \end{aligned} \tag{6}$$

The three separate cases, defined by the fraction of anxious sellers, arise because the cut-off for offering c_A in a bilateral match is different depending on the quality of the match. In a type H match, the adverse selection problem must be relatively more severe before the buyer risks offering c_A. In such cases, no transaction will occur if the seller happens to be the relaxed type, since the offer is below her reservation value, c_R. The expected payoff function (6) and the free entry of buyers, $U(\sigma, \theta) = \kappa$, determine the equilibrium buyer-seller ratio, θ.

The expected payoff to a relaxed seller is

$$\begin{aligned} V_R(\sigma,\theta) &= e^{-\theta}\sum_{k=2}^{\infty}\frac{\theta^k}{k!}\left[(v_L-c_R)+\sum_{j=0}^{k}p(j;k,q)\int_{v_L}^{\bar{b}_{k-1}}(b-v_L)dF_{k-1}^{j}(b)\right] \\ &= \left[1-(1+\theta)e^{-\theta}\right](v_L-c_R) \\ &\qquad + e^{-\theta}\sum_{k=2}^{\infty}\frac{\theta^k}{k!}\sum_{j=0}^{k}\binom{k}{j}q^j(1-q)^{k-j}\int_{v_L}^{\bar{b}_{k-1}}(b-v_L)dF_{k-1}^{j}(b) \end{aligned} \tag{7}$$

Using the bidding strategies $\{F_k\}_{k=1}^{\infty}$, the highest bids $\{\bar{b}_k\}_{k=1}^{\infty}$, and solving yields the following expression for the expected payoff to a relaxed seller:

$$V_R(\sigma,\theta) = \left[1-(1+\theta)e^{-\theta}\right](v_L-c_R)+\left[1-(1+\theta q)e^{-\theta q}\right](v_H-v_L) \tag{8}$$

A complete derivation of $V_R(\sigma,\theta)$ is provided in Appendix A. The simplicity of this expression arises because the payoff to a type R seller in a bilateral match is zero regardless of whether or not a transaction takes place. A motivated seller, on the other hand, has the following expected payoff:

$$\begin{aligned} V_A(\sigma,\theta) &= \left[1-(1+\theta)e^{-\theta}\right](v_L-c_A)+\left[1-(1+\theta q)e^{-\theta q}\right](v_H-v_L) \\ &\quad + \begin{cases} 0 & \text{if} \quad \sigma > \frac{v_H-c_R}{v_H-c_A} \\ q\theta e^{-\theta}(c_R-c_A) & \text{if} \quad \frac{v_L-c_R}{v_L-c_A} < \sigma \leq \frac{v_H-c_R}{v_H-c_A} \\ \theta e^{-\theta}(c_R-c_A) & \text{if} \quad \sigma \leq \frac{v_L-c_R}{v_L-c_A} \end{cases} \end{aligned} \tag{9}$$

The last term reflects the positive surplus for a type A seller in a bilateral match whenever the buyer offers $c_R > c_A$. If $\sigma \leq (v_L-c_R)/(v_L-c_A)$, anxious sellers get the c_R-c_A bonus in a bilateral match, and if $\sigma \in ((v_L-c_R)/(v_L-c_A),(v_H-c_R)/(v_H-c_A)]$, they get the bilateral bonus only if the buyer draws v_H.

2.3 Full Information Benchmark

If sellers' reservation values were observable, buyers could condition their search strategy and bilateral offers on the seller's willingness to sell. The expected payoffs to sellers in a housing market with observable c_A and c_R, according to (8) and (9), are

$$V_A(1,\theta_A) = \left[1-(1+\theta_A)e^{-\theta_A}\right](v_L - c_A) + \left[1-(1+\theta_A q)e^{-\theta_A q}\right](v_H - v_L) \tag{10}$$

and

$$V_R(0,\theta_R) = \left[1-(1+\theta_R)e^{-\theta_R}\right](v_L - c_R) + \left[1-(1+\theta_R q)e^{-\theta_R q}\right](v_H - v_L) \tag{11}$$

with $\{\theta_A, \theta_R\}$ determined by the free entry conditions according to (6):

$$U(1,\theta_A) = e^{-\theta_A}(v_L - c_A) + qe^{-\theta_A q}(v_H - v_L) = \kappa \tag{12}$$

$$U(0,\theta_R) = e^{-\theta_R}(v_L - c_R) + qe^{-\theta_R q}(v_H - v_L) = \kappa \tag{13}$$

This full information separating equilibrium is constrained efficient. The pricing mechanism is efficient in the sense that a house is always transferred to the highest bidder, and no buyer-seller match leaves positive surplus on the table. Efficiency of the separating equilibrium further requires that θ_R and θ_A maximize social surplus. To show that buyer entry is optimal, denote by Π_A the social surplus from putting a house on the market when the seller has reservation value c_A. As long as one or more potential buyers show up, the surplus is $v_H - c_A$ if at least one of them draws a high quality match, and $v_L - c_A$ otherwise.

$$\begin{aligned}\Pi_A(\theta) &= \sum_{k=1}^{\infty} \frac{\theta^k}{k!e^{\theta}} \left[(v_H - c_A) + (1-q)^k(v_L - v_H)\right] \\ &= (1-e^{-\theta})(v_L - c_A) + (1-e^{-\theta q})(v_H - v_L)\end{aligned} \tag{14}$$

Define Π_R in the analogous manner for houses available for purchase from relaxed sellers. Constrained efficiency means the social planner is also subject to the same coordination frictions faced by market participants. Taking the measures of sellers as given, the social

planner has only to choose the measures of buyers visiting sellers of each type to maximize total social surplus less entry costs. Equivalently, the social planner can choose θ_A and θ_R to maximize the average social surplus per house.

$$\max_{\theta_A,\theta_R} \sigma_0 \left[\Pi_A(\theta_A) - \kappa\theta_A\right] + (1-\sigma_0)\left[\Pi_R(\theta_R) - \kappa\theta_R\right] \tag{15}$$

After substituting for Π_A using the definition in equation (14) and likewise for Π_R, the first order conditions for the planner's problem are

$$e^{-\theta_A}(v_L - c_A) + qe^{-\theta_A q}(v_H - v_L) = \kappa \tag{16}$$

$$e^{-\theta_R}(v_L - c_R) + qe^{-\theta_R q}(v_H - v_L) = \kappa \tag{17}$$

These are the same equations as the free entry conditions for buyers in the full information benchmark housing market, equations (12) and (13). When sellers' reservation values are observable, the equilibrium free entry conditions imply that the arrival rates of buyers are efficient. The intuition for this result is as follows: Buyers are the ones paying the search cost, κ. With take-it-or-leave-it offers in bilateral matches, buyers are also the ones reaping the benefits of search. Finally, since house prices are bid higher in multilateral matches, buyers also bear the cost of congestion. Since buyers face undistorted incentives in searching for a house, their entry decisions are consistent with the solution to the constrained planner's problem.

2.4 Equilibrium and Efficiency Under Asymmetric Information

In contrast to the full information equilibrium, the equilibrium of this model with unobservable reservation values is a random search equilibrium with both types of sellers attracting buyers in a single market. Equilibrium payoffs are given by (6), (8), and (9) with θ determined by a single free entry condition, and the share of anxious sellers in the market equal to the aggregate fraction of motivated sellers, σ_0. The information problem generates illiquidity

in the housing market due to adverse selection and inefficient entry. Figure 1 illustrates the liquidity of housing (the average probability of a transaction) in the housing market equilibrium relative to the full information benchmark in terms of the composition of sellers. When σ_0 is high ($\sigma_0 > (v_H - c_R)/(v_H - c_A)$), the adverse selection problem is severe in the sense that buyers make take-it-or-leave-it offers in bilateral matches that get rejected whenever the seller is less motivated to sell. Failure to trade in a match even when the surplus is positive reduces the number of transactions in the real estate market relative to the efficient allocation. Even when σ_0 is low ($\sigma_0 \leq (v_L - c_R)/(v_L - c_A)$), the private information about the seller's motivation makes houses less liquid. When buyers offer $c_R > c_A$ in a bilateral match and their share of the surplus in a transaction with an impatient seller is reduced, fewer buyers find it worthwhile to participate in the housing market. This is an implication of the free entry condition. Finally, for intermediate values of σ_0, both issues arise: fewer buyers enter the market because the buyers' payoff is less in a bilateral type H match, and unconsummated matches occur between a type R seller and a type L buyer.

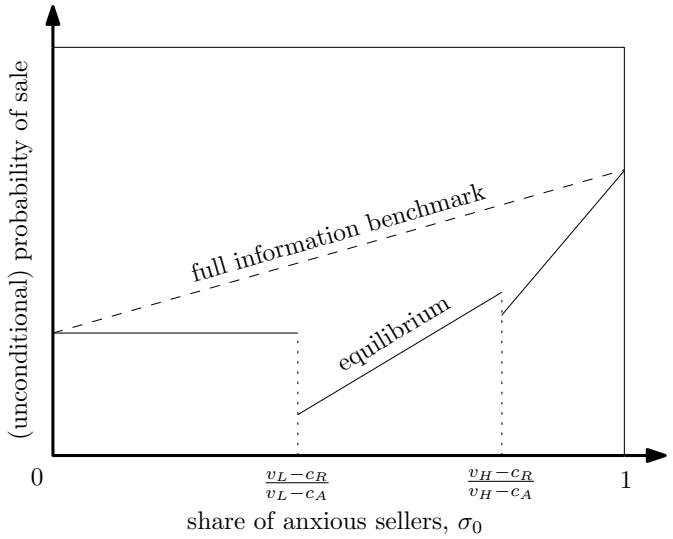

Figure 1: Housing liquidity in equilibrium relative to the full information benchmark.

The full information equilibrium and solution to the social planner's problem establish that it is efficient for sellers with different reservation values to be distinguishable. With

c_A and c_R unobservable, there could be efficiency gains associated with a mechanism that allows sellers to reveal their type. If sellers can differentiate themselves, buyers can direct their search. More buyers will visit the impatient sellers, knowing that a lower offer will be accepted in a bilateral match. Past studies have proposed the list price as a means of signalling private information (Albrecht, Gautier, and Vroman, 2010; Delacroix and Shi, 2007). Menzio (2007) shows that non-contractual messages in job listings can sometimes credibly convey information when wages are determined through bilateral bargaining. In my framework, the list price is not a credible signalling device: Type R sellers will list their house at a low price, mimicking the type A sellers in order to attract more buyers. This increases the probability that a bidding war will drive the selling price upward. Unlike in Menzio's (2007) model of partially directed search, the process of price determination is not rigid enough to discourage such mimicking. In the event of a bilateral match, a type R seller gets a payoff of zero regardless of whether the buyer offers c_R (leaving the seller with none of the surplus) or c_A (in which case the seller simply rejects the offer and gets a payoff of zero). This result is stated formally in Proposition 2.1. All proofs are relegated to Appendix B.

Proposition 2.1 *Suppose sellers can costlessly communicate with buyers through negotiable list prices. A correlation between the list price and the seller's reservation value is unsustainable, and the equilibrium reduces to random search with uninformative list prices.*

With the inability to commit to list prices, market separation violates incentive compatibility. The housing market equilibrium is inefficient, and housing units are illiquid relative to the full information benchmark. Even with asymmetric information, however, the separating allocation is implementable by the social planner as long as the planner can commit not to alter the trading mechanism ex post. That is, the planner can design a mechanism to achieve market separation, increase social surplus, and circumvent both the welfare loss of unconsummated matches generated by the adverse selection problem and the inefficient entry resulting from information asymmetry. Implementing the separating allocation is accomplished, for example, using auctions with publicly observable and binding reserve bids. The planner therefore imposes a commitment to ex ante announcements which is absent

in the market equilibrium. Submitting appropriate reserve bids is incentive compatible for sellers, and the endogenous arrival rates of buyers to sellers of either type are then efficient. These results are summarized in Proposition 2.2.

Proposition 2.2 *Consider the following price-posting game: a seller sets a list price, and the planner sells the home by sealed bid auction using the posted price as an unsealed reserve bid. Then, sellers' optimal list prices are* $\{p_A, p_R\} = \{c_A, c_R\}$*, and buyers' search and bidding strategies are identical to those in the full information benchmark. The constrained efficient allocation is therefore implementable even when reservation values are unobservable.*

This is similar to the efficient equilibrium in Albrecht, Gautier, and Vroman (2010), which imposes partial commitment to posted prices as part of the environment. In their housing market model, sellers are forced to sell whenever a buyer offers her asking price, even in the decentralized equilibrium. They suggest that the commitment to sell when a *bona fide* offer arrives could be part of the contract with a real estate agent, although real estate agents are not explicitly part of their model. In the next section, I investigate whether agency can fulfil the role of a signalling mechanism in the housing market. I derive conditions that permit real estate agents to offer distinct incentive compatible listing agreements to segment the market, allow buyers to direct their search, and help overcome the problem of asymmetric information. It turns out that in some cases, the type of real estate contract that is often observed in housing markets is conducive to market separation.

3 Real Estate Agents

I add real estate agents to the model as a way of endogenizing q: the likelihood of a high quality match when a buyer visits a house for sale. Intuitively, real estate agents (REAs) have access to more detailed information about the characteristics of houses and the preferences of prospective buyers. Acquiring and using this knowledge by offering marketing services improves the average quality of a match. In addition, REAs can work with a seller to increase

the probability that a potential buyer assigns a high value to the house by decluttering, painting, repairing, renovating, decorating, and staging the home. Let $a \in [0, \infty)$ denote the level of services supplied by a REA, and let the probability of a high quality match be an increasing function of a, $q : [0, \infty) \to [0, 1]$, with $q(0) = 0$ and $\lim_{a \to \infty} q(a) = 1$. Of course, providing services to increase q is costly. Let $\phi : [0, \infty) \to [0, \infty)$ be the cost function associated with supplying a seller with service level a. The cost function satisfies the following properties: $\phi(0) = 0$, $\phi'(a) > 0$ for all $a \in [0, \infty)$, and $\lim_{a \to \infty} \phi(a) = \infty$.

Assume that the market for REAs is frictionless and perfectly competitive. While this assumption may seem implausible given the allegations in the report by the Federal Trade Commission and U.S. Department of Justice (2007), there is evidence that barriers to entry in the real estate brokerage industry are minute (Barwick and Pathak, 2011). A REA offers a contract $(a, z) \in \mathbb{C}$: a is the extent of the REA's marketing efforts, which can also be expressed in terms of q (the quality of service expressed as the probability that each prospective buyer values the house at v_H rather than v_L); z is the REA's commission, expressed as an upfront non-refundable fee; and $\mathbb{C} = [0, \infty)^2$ is the set of all possible contracts. The flat fee assumption is made for tractability, and is sufficient for deriving results that are robust to changes in the structure of the REA's commission. A fixed rate commission structure would better reflect the listing contracts commonly observed in residential real estate markets. Most REAs in large U.S. cities charge a commission rate between 5 and 7 percent of the sale price (Hsieh and Moretti, 2003; Federal Trade Commission and U.S. Department of Justice, 2007).[7] I return to fixed rate contracts in Section 3.4 and show that features common in real world listing contracts are important for incentive compatibility.

I study the equilibria of the following two stage game: in the first stage, REAs enter the housing market by posting contracts; in the second stage, sellers sort themselves by selecting a contract/REA, and buyers enter submarkets which are identifiable by the supply

[7]Most, but not all real estate brokers adopt fixed-rate fee structures. There appears to be an emergence of flat-fee, limited-service brokers in real estate markets (Hendel, Nevo, and Ortalo-Magné, 2009; Levitt and Syverson, 2008a). Moreover, even REAs with an ostensible fixed-rate commission structure will demand that most of it be paid as an upfront non-refundable fee, effectively transforming the contract into a flat-fee contract.

of real estate services, a. When buyers match with sellers, they implement competitive bidding strategies to purchase the house. Equilibria are constructed by solving backward. An equilibrium of the second stage subgame takes as given the set of real estate contracts. This pins down the arrival rate of buyers and the expected number of sellers of each type attracted to a particular contract. In the first stage, REAs correctly anticipate the search behaviour of buyers and sellers in the second stage subgame. Taking as given the contracts posted by other agents, a REA enters the market and posts contract (a, z) if it is profitable to do so.

Adding REAs to the model in this manner introduces several more layers of analytical complexity. A useful intermediate step is to imagine that the services provided by REAs are completely valueless but observable by other market participants. A straightforward way to impose such an environment is to set $v_L = v_H$. Increasing q has no direct benefit to the seller, but with a observable it becomes feasible for sellers to spend resources on REAs as a means of signalling their type. I proceed by investigating when even ineffective REAs play a role in the housing market. The intuition developed from the analytical results derived in this simpler environment carry through to the version of the model with $v_L < v_H$.

3.1 Real Estate Agents in an Environment with $v_L = v_H$

Let v denote the common value of a house to all potential buyers. The probability q is meaningless in this environment, and the level of real estate services, a, has no economic interpretation except that it can act as an observable market signal and affect beliefs about the buyer-seller ratio, θ, and the composition of sellers, σ. The fraction of anxious sellers is an important submarket characteristic because it affects the take-it-or-leave-it offer in a bilateral match. In particular, the equilibrium offer in a bilateral match is $\max\{\sigma(v - c_A), v - c_R\}$. In a multilaterial match, Bertrand competition drives the sale price up to buyers' valuation,

v. A buyer's expected payoff in a submarket with beliefs (σ, θ) is

$$U(\sigma,\theta) = \begin{cases} e^{-\theta}\sigma(v-c_A) & \text{if} \quad \sigma > \frac{v-c_R}{v-c_A} \\ e^{-\theta}(v-c_R) & \text{if} \quad \sigma \leq \frac{v-c_R}{v-c_A} \end{cases} \tag{18}$$

The free entry condition for buyers is therefore $U(\sigma, \theta) = \kappa$, which implicitly defines the buyer-seller ratio, θ. When the real estate fee is z, the value functions for sellers are

$$V_R(z,\sigma,\theta) = [1-(1+\theta)e^{-\theta}](v-c_R) - z \tag{19}$$

and

$$V_A(z,\sigma,\theta) = [1-(1+\theta)e^{-\theta}](v-c_A) - z + \begin{cases} 0 & \text{if} \quad \sigma > \frac{v-c_R}{v-c_A} \\ \theta e^{-\theta}(c_R-c_A) & \text{if} \quad \sigma \leq \frac{v-c_R}{v-c_A} \end{cases} \tag{20}$$

The real estate market can be characterized by a competitive search framework. REAs post contracts, effectively creating submarkets that can be distinguished by the observable real estate services, a. Buyers and sellers then direct their search to the different submarkets. What follows is a formal definition of the second stage equilibrium of the housing market model, taking as given a set of real estate contracts, $\mathbb{C}_P$. Definition 3.1 already takes into account the optimal bidding strategies of buyers and the optimal accept/reject decisions of sellers and focuses instead on equilibrium search behaviour. Next, a definition of an equilibrium at the first stage determines the optimal set of contracts, $\mathbb{C}_P$.

Definition 3.1 Given a set of real estate contracts $\mathbb{C}_P$, a second stage equilibrium of the housing market is a distribution of buyers Γ on $\mathbb{C}$ with support $\mathbb{C}_P$, buyer-seller ratios $\{\theta_a\}$, and compositions of sellers $\{\sigma_a\}$ across submarkets satisfying the following:

1. Buyers' optimal entry: $U(\sigma_a, \theta_a) = \kappa$ for all $(a, z) \in \mathbb{C}_P$.

2. Sellers' optimal search:

   (i) If $\sigma_a > 0$ for some $(a, z) \in \mathbb{C}_P$, then $V_A(z, \sigma_a, \theta_a) = \max_{(a',z') \in \mathbb{C}_P} V_A(z', \sigma_{a'}, \theta_{a'})$.

(ii) If $\sigma_a < 1$ for some $(a, z) \in \mathbb{C}_P$, then $V_R(z, \sigma_a, \theta_a) = \max_{(a',z') \in \mathbb{C}_P} V_R(z', \sigma_{a'}, \theta_{a'})$.

3. Market clearing:

$$\int_{\mathbb{C}_P} \frac{\sigma_a}{\theta_a} d\Gamma(a, z) = \sigma_0 S \quad \text{and} \quad \int_{\mathbb{C}_P} \frac{1 - \sigma_a}{\theta_a} d\Gamma(a, z) = (1 - \sigma_0) S$$

The first two parts of Definition 3.1 specify optimal search behaviour on the part of buyers and sellers. For instance, 2(i) requires that anxious sellers do not enter a submarket unless it enables them to achieve their highest possible payoff. Part (ii) is the analogous requirement for type R sellers. The final part of Definition 3.1 ensures that every seller enters a submarket.[8]

What is missing from Definition 3.1 is the equilibrium behaviour of REAs. In any equilibrium, perfect competition and free entry in the market for REAs ensure that commission fees will be bid down to earn zero profit. Let $\mathbb{C}_0$ denote the set of zero profit contracts:

$$\mathbb{C}_0 = \big\{(a, z) | a \geq 0,\ z = \phi(a)\big\} \tag{21}$$

The zero profit fee schedule result is stated formally in the following Lemma.

Lemma 1 *With perfect competition and free entry in the market for REAs, every real estate contract posted in equilibrium must earn zero profit,* $\mathbb{C}_P \subset \mathbb{C}_0$.

While Lemma 1 restricts the set of contracts that REAs can post in equilibrium, further restrictions are needed to characterize the set of zero profit equilibrium contracts. REAs play a market-making role, creating submarkets by constructing new listing agreements. An equilibrium set of contracts must be such that no other contract can be introduced to earn positive profit. This restriction requires specifying the beliefs about submarket tightness, θ, the composition of sellers, σ, and the commission fee, z, for real estate contracts that are

[8] Definition 3.1 ignores the possibility that a REA posts a contract that attracts neither buyers nor sellers. An implicit assumption is that sellers find it worthwhile to list their house for sale in at least one of the submarkets.

not offered in equilibrium. An equilibrium at stage one is such that no REA can offer an out-of-equilibrium listing contract and earn a positive profit given the equilibrium behaviour of buyers and sellers in the stage two subgame. An equivalent characterization of equilibrium at stage one rules out a candidate set of contracts $\mathbb{C}_P$ if there exists a zero profit deviation that can improve the expected payoffs to sellers participating in the new submarket.[9]

Definition 3.2 A stage one equilibrium in the housing market with REAs is a set of real estate contracts $\mathbb{C}_P$ with $(0,0) \in \mathbb{C}_P$, a distribution of buyers Γ on $\mathbb{C}$ with support $\mathbb{C}_P$, a function $\theta : [0,\infty) \to [0,\infty]$, and a function $\sigma : [0,\infty) \to [0,1]$ satisfying the following:

1. REAs offer zero profit contracts: $\mathbb{C}_P \subset \mathbb{C}_0$; and $\{\Gamma, \theta, \sigma\}$ satisfy Definition 3.1 given the set of contracts $\mathbb{C}_P$.

2. Let $\{\overline{V}_A, \overline{V}_R\}$ denote a pair of seller values associated with an equilibrium:

$$\overline{V}_A = \max_{(a,z)\in\mathbb{C}_P} V_A(z, \sigma(a), \theta(a)) \quad \text{and} \quad \overline{V}_R = \max_{(a,z)\in\mathbb{C}_P} V_R(z, \sigma(a), \theta(a)) \tag{22}$$

   For any $(a', z') \in \mathbb{C}_0 \setminus \mathbb{C}_P$,

$$V_A(z', \sigma(a'), \theta(a')) \leq \overline{V}_A \quad \text{and} \quad V_R(z', \sigma(a'), \theta(a')) \leq \overline{V}_R \tag{23}$$

   where $\{\Gamma, \sigma, \theta\}$ satisfy Definition 3.1 given $\mathbb{C}_P \cup (a', q')$.[10]

First note that $(0,0) \in \mathbb{C}_P$, which means that sellers always have the option not to hire a REA: the for-sale-by-owner option. Part 1 of the definition then states that the entry

[9] Equivalence follows from the following argument: a listing contract that attracts some sellers and makes them strictly better off can be restructured to divide the extra surplus between the seller and the agent. Inversely, if a profitable deviation is possible, the real estate agent could instead pass some of the surplus on to his clients. This equivalent characterization is applied here in order to avoid introducing extra notation for beliefs regarding submarkets with real estate contracts that earn strictly positive profit. The assumption is maintained that upon observing a, a prospective buyer deduces that the commission charged to the seller is $\phi(a)$.

[10] For completeness, part 2 of Definition 3.2 should also require the following: If $V_A(z', \sigma(a'), \theta(a')) < 0$ and $\sigma(a') > 0$, then $\theta(a') = \infty$. If $V_R(z', \sigma(a'), \theta(a')) < 0$ and $\sigma(a') < 1$, then $\theta(a') = \infty$. This allows REAs to consider contracts that would not attract any sellers in stage two.

and search behaviour of buyers and sellers is a second stage equilibrium given the posted set of zero profit listing agreements. Part 2 states that no out-of-equilibrium contract can benefit sellers. This requires beliefs about θ and σ for out-of-equilibrium submarkets to be consistent with the search behaviour of buyers and sellers in the subgame that includes the additional deviation under consideration. The resulting buyer-seller ratio, $\theta(a')$, has to be consistent with the free entry condition for buyers. Similarly, the resulting composition of sellers, $\sigma(a')$, must reflect the equilibrium search strategies of sellers following the posting of contract $(a', z') \in \mathbb{C}_0$.

There is a local single-crossing property that can arise endogenously which introduces the possibility of signalling. Paying for ineffective real estate services is not a traditional sorting variable, as it directly affects both types of sellers in the identical manner. In other words, the REA technology does not satisfy a Spence-Mirrlees single crossing property for exogenous reasons. Instead, the endogenous composition of sellers and buyer-seller ratio can initiate sorting. To see this, consider a pooling equilibrium without real estate agents, and imagine a real estate agent deciding to enter the housing market and offer a listing agreement (a, z) with zero profit commission $z = \phi(a) > 0$. The payoff functions for buyers in the new submarket would be

$$V_R(\phi(a), \sigma(a), \theta(a)) = [1 - (1 + \theta(a))e^{-\theta(a)}](v - c_R) - \phi(a) \tag{24}$$

$$V_A(\phi(a), \sigma(a), \theta(a)) = [1 - (1 + \theta(a))e^{-\theta(a)}](v - c_A) - \phi(a) \tag{25}$$

where I have assumed $\sigma(a) > (v - c_R)/(v - c_A)$. Differentiating the payoff functions with respect to a yields

$$\frac{dV_R}{da} = \theta e^{-\theta}(v - c_R)\frac{d\theta}{da} - \frac{d\phi}{da} \quad \text{and} \quad \frac{dV_A}{da} = \theta e^{-\theta}(v - c_A)\frac{d\theta}{da} - \frac{d\phi}{da} \tag{26}$$

Consider the following conceptual adjustment process after the new contract is introduced. Relaxed sellers have no signalling incentive and are initially uninterested in the new listing agreement. The real estate agent therefore expects anxious sellers to be the first to accept

the listing agreement in an attempt to signal their type and attract a high number of buyers. With $\sigma(a) = 1$ and a high buyer-seller ratio, relaxed sellers might thereafter find it worthwhile to mimic the anxious types by entering the hotter submarket and signing the new listing agreement. Type R sellers continue to flow into the new submarket, and $\sigma(a)$ continues to decline until relaxed sellers are indifferent between the two markets. The type R indifference condition is

$$\begin{aligned} V_R(\phi(0), \sigma(0), \theta(0)) &= [1 - (1 + \theta(0))e^{-\theta(0)}](v - c_R) \\ &= [1 - (1 + \theta(a))e^{-\theta(a)}](v - c_R) - \phi(a) = V_R(\phi(a), \sigma(a), \theta(a)) \end{aligned} \tag{27}$$

Differentiation yields

$$\theta e^{-\theta}(v - c_R)\frac{d\theta}{da} - \frac{d\phi}{da} = 0 \tag{28}$$

which can be substituted into (26) to obtain

$$\frac{dV_R}{da} = 0 \quad \text{and} \quad \frac{dV_A}{da} = \theta e^{-\theta}(c_R - c_A)\frac{d\theta}{da} = \left(\frac{c_R - c_A}{v - c_R}\right)\frac{d\phi}{da} > 0 \tag{29}$$

Therefore, the endogenously determined composition of sellers and arrival rate of buyers generate a single crossing property: the expected payoff to a type A seller is increasing in a, while type R sellers remain indifferent between the two submarkets.

The piecewise nature of the payoff function for type A sellers in (20) introduces another complication. If $\sigma_0 \leq (v - c_R)/(v - c_A)$, type A sellers get a positive payoff even in a bilateral match because buyers are making cautious take-it-or-leave-it offers to ensure the purchase of a home regardless of the seller's motivation. Thus, even though V_A is locally increasing in a, they might still prefer the original pooling submarket because of the bilateral bonus. Even if $\sigma_0 > (v - c_R)/(v - c_A)$, a fully separating equilibrium might not be feasible. There are two offsetting effects. First, type A sellers are attracted to a submarket with real estate fees because σ is increasing in a and therefore so is θ. A higher buyer-seller ratio improves the likelihood of a multilateral match and a payoff of $v - c_A$. On the other hand, the bilateral bonus of $c_R - c_A$ in a type R submarket is appealing to an anxious seller. A fully separating

equilibrium is only achievable if the first effect dominates. This occurs whenever the buyer-seller ratios are sufficiently low (i.e., if housing is sufficiently illiquid) that the benefit from an increase in market tightness, θ, is large. When θ is too high, the benefit of further increasing market tightness is inadequate to offset the appeal of the bilateral bonus in the type R market. The parameter most directly (but inversely) related to market tightness is κ, the entry cost for buyers. When κ is high, buyers are scarce and the potential benefit from signalling a high motivation to sell is sizeable. I proceed by characterizing the housing market equilibrium in terms of the parameters κ and σ_0.

Lemma 2 *Type R sellers select the for-sale-by-owner contract* $(a_R, z_R) = (0, 0)$.

Lemma 3 *A pair of fully separating submarkets with contracts* $(a_R, z_R) = (0, 0)$ *and* (a_A, z_A) *is incentive feasible if and only if*

$$\kappa \geq (v - c_A) \exp\left(\frac{c_A - c_R}{v - c_R}\right) \equiv \underline{\kappa} \tag{30}$$

where (a_A, z_A) *is the zero profit real estate contract that binds the type R incentive compatibility constraint,* $V_R(0, 0, \theta_R) = V_R(z_A, 1, \theta_A)$.

Suppose $\kappa \geq \underline{\kappa}$, so that condition (30) is satisfied, and let (a_A, z_A) denote the zero profit contract that binds the type R incentive compatibility constraint for full separation. Lemmas 2 and 3 specify the condition under which that listing contracts $(a_R, z_R) = (0, 0)$ and (a_A, z_A) induce search behaviour by buyers and sellers that is consistent with a fully separating equilibrium. The final criterion for a stage one equilibrium is to determine the appropriate conditions under which no other real estate contract can generate better expected payoffs to sellers deviating to the new submarket. The deviation of interest is a full pooling contract. The following Lemma characterizes the conditions necessary and sufficient for sellers to prefer a full pooling submarket over the pair of fully separating submarkets.

Lemma 4 *Assume the parameters of the model satisfy* (30) *so that the pair of fully separating contracts is incentive feasible. A full pooling contract* $(0, 0)$ *can increase the expected payoffs for both types of sellers (strict for at least one type) if and only if* $\sigma_0 \in [\sigma^*, 1]$ *and*

$\kappa \in [\underline{\kappa}, \overline{\kappa}(\sigma_0)]$, *where*

$$\overline{\kappa}(\sigma_0) \equiv \exp\left(\frac{(v-c_A)[1+\log(\sigma_0(v-c_A))]-\sigma_0(v-c_A)[1+\log(v-c_R)]-\sigma_0(c_R-c_A)[1+\log(v-c_A)]}{(1-\sigma_0)(v-c_A)-\sigma_0(c_R-c_A)}\right) \tag{31}$$

and σ^* *is the unique solution to* $\overline{\kappa}(\sigma^*) = \underline{\kappa}$.

When $\kappa \geq \underline{\kappa}$, the single crossing property precludes a pooling equilibrium. If the conditions of Lemma 4 are satisfied, a pooling contract can nonetheless be welfare improving. This leads to the typical equilibrium non-existence problem as in Rothschild and Stiglitz (1976). Lemmas 2, 3, and 4 combine to form the necessary and sufficient conditions for a fully separating equilibrium in the housing market with REAs, which are stated in the following Proposition.

Proposition 3.1 *The pair of incentive feasible contracts,* $(a_R, z_R) = (0, 0)$ *and* (a_A, z_A), *constitute a fully separating equilibrium if and only if*

$$\kappa \geq \begin{cases} \underline{\kappa} & \text{if } \sigma_0 < \sigma^* \\ \overline{\kappa}(\sigma_0) & \text{if } \sigma_0 \geq \sigma^* \end{cases} \tag{32}$$

Proposition 3.1 is consistent with the intuition developed earlier. The ratio of buyers to sellers in the housing market must be low in order for anxious sellers to engage in costly signalling by accepting real estate agreements with positive commission fees. When the entry cost κ is low, the buyer-seller ratios are sufficiently high that the benefit of signalling is not enough to provide anxious sellers with the incentive to give up the bilateral bonus. Proposition 3.1 also points to a relationship between the aggregate composition of sellers, σ_0, and the existence of a fully separating equilibrium. When most sellers are anxious to sell, the full pooling submarket resembles the separating type A submarket: market tightness is high, and buyers make low offers of c_A in the event of a bilateral match. Therefore, as the population of sellers becomes relatively homogeneous, it becomes harder to justify paying agency fees to achieve full market segmentation.

Proposition 3.1 is reminiscent of the endogenous market segmentation result in Fang (2001). In Fang's paper, social culture is a seemingly irrelevant activity that can be used as an endogenous signalling device to partially overcome an information problem in the labour market. Here, if the parameters are conducive to separation, the hiring of irrelevant but costly real estate agents is used to signal type. Buyers form different beliefs about the composition of sellers in each separate submarket. Given these beliefs, anxious and relaxed sellers face different incentives to join a particular submarket. The advantage of listing a house with a costly REA is a higher arrival rate of buyers, which results in a higher probability of trade. Because sellers differ in their reservation values, (a_A, z_A) can be carefully chosen by REAs so that relaxed sellers are just indifferent between the two submarkets, while anxious sellers strictly prefer the one with REAs. Embedding Fang's (2001) result in a search framework with profit maximizing REAs thus rules out Pareto inferior signalling equilibria.

With parameters that violate (32), an incentive feasible contract (a_A, z_A) can no longer be constructed. When $\sigma_0 > \sigma^*$ and $\kappa \in (\underline{\kappa}, \overline{\kappa}(\sigma_0))$, the entire group of anxious sellers prefer to enter the submarket with $a = 0$, along with the relaxed sellers. It is of interest to know what happens in the housing market when condition (30) is violated (i.e., when $\kappa < \underline{\kappa}$). For example, under what parameter restrictions is there a full pooling equilibrium? Proposition 3.2 fills in the details, and Figure 2 provides a graphical representation.

Proposition 3.2 *Suppose $\kappa < \underline{\kappa}$. Then,*

1. *if $\sigma_0 > \underline{\sigma}$, the model has no equilibrium; and*

2. *if $\sigma_0 \leq \underline{\sigma}$, there exists a full pooling equilibrium.*

If $\sigma_0 > \underline{\sigma}$ and $\kappa < \underline{\kappa}$, a pooling contract does not constitute an equilibrium because a deviating REA can offer a listing agreement with a positive commission to attract only the anxious sellers. Once the anxious sellers exit the pooling submarket, buyers alter their bidding strategy and offer c_R instead of c_A in a bilateral match. This change in buyers' behaviour affects the expected payoffs such that anxious sellers' search behaviour is no longer

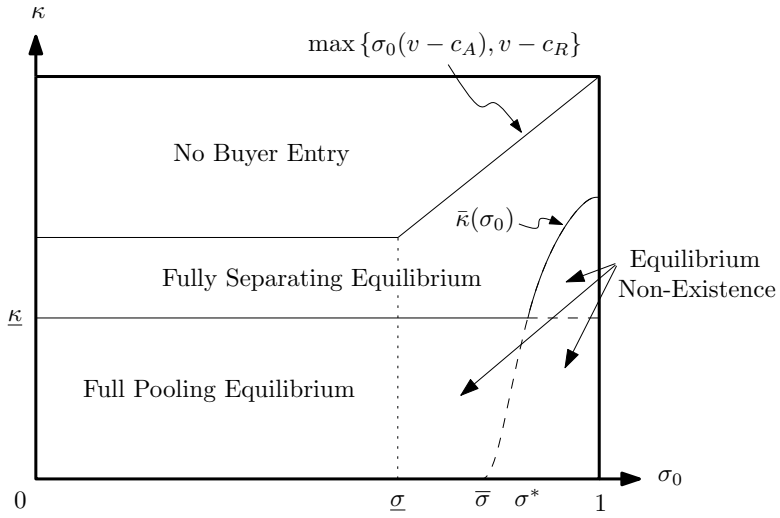

Figure 2: Graphical characterization of the housing market equilibrium with REAs.

optimal. This explains the equilibrium non-existence problem in part 1 of Proposition 3.2. When $\sigma_0 \leq \underline{\sigma}$, the share of anxious sellers is low enough that buyers cautiously offer c_R in a bilateral match even in a pooling submarket in order to guarantee a successful home purchase. If $\kappa < \underline{\kappa}$, there is no deviation that will attract only the motivated sellers.

3.2 Real Estate Agents in an Environment with $v_L < v_H$

The intuition developed in the previous section is still relevant when the economic importance of real estate services, a, is derived from the monotonic relationship with q: the probability that a buyer draws the high value $v_H > v_L$. For notational convenience, the signalling role of real estate services a and the direct economic benefit of increasing q via a can be collapsed by imagining that q itself is observable. One can therefore consider REA contracts of the form (q, z). Let $\psi(q)$ denote implicit cost function REAs face when supplying the level of service required to increase the probability of a high value match to q.

When $v_L < v_H$, the quality of REA services, q, enters the sellers' payoff functions directly. As in the environment with $v_L = v_H$, REA contracts do not automatically generate market

segmentation because q is not a traditional sorting variable. The extra payoff to a seller when $q > 0$ is $\left[1 - (1 + \theta q)e^{-\theta q}\right](v_H - v_L)$, which is independent of the seller's reservation value. As before, endogenous market segmentation can arise because sellers respond differently to changes in market tightness in a given submarket. One important difference from the case with $v_L = v_H$, however, is the possibility of real estate service differentiation. Since the incentive to select a particular listing agreement depends on both cost and quality, selecting a particular REA to signal a high willingness to sell is no longer straightforward.

The marginal benefit to a seller of increasing q is always positive. Whether anxious or relaxed sellers benefit more from an increase in q depends on the effect of q on market tightness, θ. In a submarket with only type A sellers, a buyer's expected payoff is initially increasing in q because real estate services improve the probability of a high quality match. If the number of buyers relative to sellers is high enough, eventually bidding wars become too competitive and a buyer's expected payoff begins to decline with q. The free entry condition for buyers therefore implies a non-monotonic relationship between q an θ, where θ is initially increasing but eventually decreasing in q. Since the seller's share of the surplus in a multilateral match is higher when the seller's reservation price is low, anxious sellers respond more favourably to an increase in the arrival rate of buyers. The optimal deviation is therefore the one that increases θ. If θ is low and q is relatively costly, anxious sellers will seek to signal their type by over-investing in real estate services. If θ is high and q is affordable, anxious sellers will under-invest in REAs.

The first case seems more plausible and empirically justified. It arises when sellers face a trade-off between increasing θ and paying REA fees. Equilibrium listing agreements specify fees that exceed the direct benefit to the seller, $\psi(q) > [1 - (1 + \theta q)e^{-\theta q}](v_H - v_L)$, because sellers benefit indirectly from a higher arrival rate of buyers. Consequently, anxious sellers tolerate high commission fees. This environment is an appropriate fit for the North American housing markets if the marketing efforts of a realtor do not directly warrant compensation between 5 and 7 percent of the price. Hiring a full service agent can still be worthwhile for motivated sellers because listing the house on the Multiple Listing Service (MLS) signals a

high willingness to sell and generates additional visits from potential buyers.

This provides a theoretical foundation for the empirical results of Hendel, Nevo, and Ortalo-Magné (2009). They compare housing market transactions on two different marketing platforms: the MLS and the newly established low cost FSBO Madison. They find that after controlling for observable house characteristics, the precommission sale prices are similar between the two platforms, but that homes listed with a traditional real estate broker have shorter times on the market and are more likely to ultimately result in a transaction. They also find evidence of endogenous sorting and report that impatient sellers are more likely to list with the high commission, high service option. These findings are consistent with the main theme of this paper. A higher buyer-seller ratio for houses listed on the MLS and the higher level of services provided by full-commission REAs lead to a higher probability of a sale. Further, the similarity in precommission prices is not inconsistent with the model. The average transaction price in a submarket with identical sellers is

$$\mathrm{E}\left[p|q,\theta\right] = \frac{\theta e^{-\theta} c + \left[1-(1+\theta)e^{-\theta}\right] v_L + \left[1-(1+\theta q)e^{-\theta q}\right](v_H - v_L)}{1-e^{-\theta}} \tag{33}$$

The lower take-it-or-leave-it offer in type A bilateral match ($c_A < c_R$) reduces the average sale price, but higher tightness ($\theta_A > \theta_R$) and superior marketing services ($q_A > q_R$) have the opposite effect. Depending on the parameters of the model, precommission prices $\mathrm{E}[p_A|q_A,\theta_A]$ and $\mathrm{E}[p_R|q_R,\theta_R]$ can be indistinguishable, despite distinct transaction price distributions.

Levitt and Syverson (2008a) similarly compare limited-service and full-service REAs. Time on the market is longer for houses sold with the assistance of less costly realtors, but sale prices are not significantly different. Bernheim and Meer (2008) study Stanford Housing listings and find that sellers realize similar prices but sell less quickly when they select not to hire an agent.[11] These empirical observations and the predictions of the theory point to

[11]In contrast, Johnson, Springer, and Brockman (2005) compare house prices when the seller's REA decides not to advertise the listing with the MLS. They calculate the average sale price of a house to be more than 6 percent higher for homes that are marketed without the MLS, after controlling for the documented characteristics of homes. Unfortunately, they do not present any results related to transaction probabilities or time on the market. Nevertheless, their main finding is consistent with the idea that separate bundles of real estate services are offered to attract different types of sellers.

REAs and the MLS as primarily fulfilling a liquidity role in the housing market, rather than directly affecting the expected sale price of a home.

3.3 Constrained Efficiency with Real Estate Agents

With endogenous real estate services, the constrained planner chooses market tightness, θ, and service quality q to maximize the (per seller) social surplus:

$$\max_{\theta,q}[1-e^{-\theta}](v_L-c)+[1-e^{-\theta q}](v_H-v_L)-\theta\kappa-\psi(q) \tag{34}$$

The first order conditions with respect to θ and q are

$$e^{-\theta}(v_L-c)+qe^{-\theta q}(v_H-v_L)=\kappa \tag{35}$$

$$\theta e^{-\theta q}(v_H-v_L)=\psi'(q) \tag{36}$$

The first condition pins down the optimal buyer-seller ratio by equating the cost of entering the market, κ, with the marginal social surplus of having an additional buyer searching for a house. The additional condition stemming from the optimal choice of q equates the marginal benefit of real estate services (the marginal increase in the probability of a type H match, $\theta e^{-\theta q}$, times the additional surplus, $v_H - v_L$) with the marginal cost, $\psi'(q)$. Interestingly, the optimal provision of real estate services is independent of the seller's reservation utility. Consequently, the efficient level of q is the same for both type R and type A sellers. The efficient allocation with heterogeneous sellers is simply the separating allocation described in Section 2.4 with the additional restriction that q satisfy equation (36).

In a fully separating equilibrium with REAs, the type R submarket achieves the efficient level of real estate services and efficient buyer entry. The type A submarket, on the other hand, involves excess spending on REAs, which is the signalling cost required to induce efficient buyer entry. Full separation in an equilibrium with REAs is welfare improving, but does not achieve the solution to the constrained planner's problem because of the inefficien-

cies required to make the type A real estate contract incentive compatible. These welfare results contrast those of Albrecht, Gautier, and Vroman (2010). In their paper, partial commitment to the list price yields an efficient separating equilibrium without the efficiency loss from costly signalling. They conjecture that the contract between a seller and her agent leads to market segmentation. I have shown that while separation is possible under certain parameters, the first best allocation remains unattainable.

3.4 Fixed Rate Commissions

The analysis thus far deals with flat fee commissions charged by REAs. In practice, however, a fixed rate commission structure is more common: real estate contracts in North America typically specify a commission of 5 to 7 percent of the sale price (Hsieh and Moretti, 2003; Federal Trade Commission and U.S. Department of Justice, 2007). From a principal-agent perspective,[12] a real estate fee that increases with the sale price is more likely to induce effort on the part of the real estate agent, whereas upfront non-refundable fees are least effective at motivating the agent. While I abstract from principal-agent matters in this paper, it is important to check the robustness of the results when listing contracts are modelled to reflect the type of contract commonly observed between a seller and her agent.

Restricting the analysis to fixed rate contracts introduces two additional effects that further hinder full market separation. First, when the commission is specified as a fraction of the sale price, a buyer has to increase his take-it-or-leave-it offer in a bilateral match so that the seller deems it acceptable after real estate fees are deducted. More specifically, when the commission is z percent and the seller is willing to accept c, the offer must be at least

[12]Many theoretical models of real estate agents focus on the principal-agent relationship between the seller (the principal) and the realtor (the agent) (Zorn and Larsen, 1986; Anglin and Arnott, 1991; Geltner, Kluger, and Miller, 1991). Yavaş (1992) and Yavaş and Yang (1995) analyze the search effort of the real estate agent, while Arnold (1992) considers the incentives for conveying truthful information about the conditions of the real estate market to aid with setting an appropriate list price. The attention of empiricists has also been aimed at the principal-agent problem in the market for real estate services. Levitt and Syverson (2008b) and Rutherford, Springer, and Yavaş (2005) find evidence to support the hypothesis that sellers' and their agents' incentives are misaligned by comparing the selling prices and duration on the market in transactions when the real estate agent is a third party and when the agent is also the owner of the home.

$c/(1-z)$. This reduces the payoff to a buyer in a bilateral match and implies that buyer entry is affected by the commission rate. Higher fees result in fewer buyers. Second, fixed rate contracts affect the incentive for relaxed sellers to mimic because payment to REAs is contingent on a transaction. To see why this is important, compute the expected real estate fee to be paid by an anxious seller in a type A submarket with commission rate is z_A:

$$\theta_A e^{-\theta_A} \frac{z_A c_A}{1-z_A} + \left[1-(1+\theta_A)^{-\theta_A}\right] z_A v_L + \left[1-(1+\theta_A q_A)e^{-\theta_A q_A}\right] z_A (v_H - v_L) \tag{37}$$

The first term is the commission paid on the take-it-or-leave-it offer of $c_A/(1-z_A)$ in a bilateral match, the second term is the minimum commission paid when two or more buyers arrive, and the last term is the additional commission in the event of high quality matches. When a relaxed seller accepts the (q_A, z_A) contract and lists her home in the type A submarket, the expected commission fee is only

$$\left[1-(1+\theta_A)^{-\theta_A}\right] z_A v_L + \left[1-(1+\theta_A q_A)e^{-\theta_A q_A}\right] z_A (v_H - v_L) \tag{38}$$

An offer is rejected by a mimicker in a bilateral match since $c_R > c_A$, and the REA only collects the commission when two or more buyers visit a relaxed seller's house. Since both types of sellers receive zero payoff in a bilateral match, this does not affect the incentive compatibility constraint directly. Instead, the zero profit conditions in the market for REAs imply that mimickers can essentially free ride on the commissions paid by anxious sellers. This makes the type A submarket relatively more appealing compared to the type R submarket, where relaxed sellers bear the full burden of real estate marketing costs. The two effects just described work against incentive compatibility and full market separation. However, the analysis is not fundamentally altered when fixed rate contracts are imposed: it merely implies that a smaller parameter space generates a fully separating equilibrium.

Listing agreements typically specify a list price. What if the REA's commission can be made contingent on procuring a "ready, willing, and able" buyer (i.e., contingent on receiving an offer at or above the list price)? This form of contract is often observed in

North American real estate markets.[13] Even if the seller rejects an offer equal to or above the list price, it is considered that the REA has provided the agreed upon services and the seller must still pay the commission. The "ready, willing, and able" clause (hereinafter, the RWA clause) is useful for generating a separating equilibrium. This structure of real estate contract dissuades patient sellers from mimicking impatient ones and entering the market with the higher buyer-seller ratio. The contract introduces a cost to rejecting a take-it-or-leave-it offer in a bilateral match.

Proposition 3.3 *Adding the list price and a RWA clause to the real estate contract tightens the incentive compatibility constraint for relaxed sellers. In other words, it becomes more costly for relaxed sellers to mimic, and hence less costly for anxious sellers to signal their type.*

Consider a listing agreement designed for type A sellers with the list price $p_A = c_A/(1 - z_A)$. The RWA clause has no effect on type A sellers' payoff since they are willing to accept an offer of $c_A/(1 - z_A)$ regardless. Type R sellers, on the other hand, now pay a cost in a bilateral match if they choose to list their house at p_A. The extra cost to mimickers makes it easier for REAs to offer incentive compatible contracts that separate sellers by type. If list prices are determined strategically, it is possible that an anxious seller's expected payoff can be further enhanced. While a list price in $[c_A, c_A/(1 - z_A))$ does adversely affect even anxious sellers, it might sting less than the direct cost of the agency fee. In other words, there is the possibility that simultaneously lowering z_A and p_A improves the expected payoff to type A sellers without attracting type R sellers. Thus, RWA clauses effectively mitigate both of the unfavourable incentive effects associated with fixed rate commissions.

[13]For example, a listing agreement with the Toronto Real Estate Board stipulates that "the Seller agrees to pay the Listing Brokerage a commission of .....% of the sale price of the Property or ..... for any valid offer to purchase the Property from any source whatsoever obtained during the Listing Period and on the terms and conditions set out in this Agreement."

4 Concluding Remarks

In this paper I present a model of the market for housing using a search framework that captures the realistic and strategic interaction between buyers and sellers in determining transaction prices. The model reflects differences in sellers' willingness to make a sale. Private information about a seller's motivation leads to an inefficient equilibrium with illiquidity in the real estate market. Some buyer-seller matches fail to result in a transaction despite the positive gains from trade. Reduced entry of buyers further impacts the volume of trade in the housing market. By introducing real estate agents into the model, there is a potential for housing market segmentation to alleviate the information problem and increase housing market efficiency. When the adverse selection problem is too severe, or the equilibrium buyer-seller ratio is too high, market segmentation can break down and situations arise wherein patient sellers mimic impatient sellers in order to drive up the final sale price by increasing the probability of a bidding war.

The model can qualitatively account for many of the observed realtor facts in residential real estate markets. For instance, 88 percent of home sellers choose to enlist the services of a real estate agent according to a 2010 survey conducted by the National Association of Realtors. This percentage has remained high in recent years, despite evidence suggesting that the value of the services provided by real estate agents, measured in terms of transaction prices and time on the market, is not enough to justify a high commission rate between five and seven percent. With seller heterogeneity and incomplete information, the theory sheds light on the demand for real estate services. Realtors not only provide valuable marketing/matching services, but can also structure their contracts in a way that offers a potential solution to the adverse selection problem. A seller can select a particular listing agreement as a means of signalling a high willingness to sell, thus attracting more buyers. In fact, I show that the demand for agency can, in some circumstances, be maintained even when the services offered by agents provide no direct benefit to the seller.

The listing agreement between a seller and her real estate agent typically specifies the

commission as a fixed percentage of the sale price, and outlines the broker's right to earn a commission when a suitable buyer is found. In the real estate agent literature, the listing contract is often studied from the perspective of a principal-agent relationship. Many have noted the apparent inefficiency in applying a fee structure that fails to completely align the incentives of the seller and the agent. A better commission structure in a simple (linear) model of incentive compensation would require a lower intercept and a steeper slope. Nevertheless, real estate contracts with a constant percentage commission have persisted for decades. In this paper, I analyze the listing contract from a different perspective and offer an alternative explanation for the structure of real estate contracts. I highlight the "ready, willing, and able" clause as a mechanism to induce sellers to truthfully reveal their willingness to sell. Interestingly, the clause would be less effective if the fee structure was altered according to the solution to an agency problem.

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A Deriving the Seller's Expected Payoff

The expected payoff to a relaxed seller is

$$\begin{aligned} V_R(\sigma,\theta) &= e^{-\theta}\sum_{k=2}^{\infty}\frac{\theta^k}{k!}\left[(v_L-c_R)+\sum_{j=0}^{k}p(j;k,q)\int_{v_L}^{\bar{b}_{k-1}}(b-v_L)dF_{k-1}^{j}(b)\right] \\ &= \left[1-(1+\theta)e^{-\theta}\right](v_L-c_R) \\ &\qquad + e^{-\theta}\sum_{k=2}^{\infty}\frac{\theta^k}{k!}\sum_{j=0}^{k}\binom{k}{j}q^j(1-q)^{k-j}\int_{v_L}^{\bar{b}_{k-1}}(b-v_L)dF_{k-1}^{j}(b) \qquad \text{(A.1)} \end{aligned}$$

A closed form solution can be obtained by solving (A.1) using the bidding strategies $\{F_k\}_{k=1}^{\infty}$ in equation (4), with the highest bids $\{\bar{b}_k\}_{k=1}^{\infty}$ from equation (5) in the main text. While this approach yields the correct expression for $V_R(\sigma,\theta)$, it is algebraically cumbersome. In this section, I derive $V_R(\sigma,\theta)$ using a different and much simpler approach. First define $\Pi_R(\sigma,\theta)$ as the expected surplus associated with putting a relaxed seller's house on the market. As long as $k \geq 1$ potential buyers show up, the surplus is $v_H - c_R$ if at least one buyer draws v_H, and $v_L - c_R$ if all visiting buyers draw a low quality match. Therefore,

$$\begin{aligned} \Pi_R(\sigma,\theta) &= e^{-\theta}\sum_{k=1}^{\infty}\frac{\theta^k}{k!}\left[(v_H-c_R)+(1-q)^k(v_L-v_H)\right] \\ &= (1-e^{-\theta})(v_L-c_R)+(1-e^{-\theta q})(v_H-v_L) \qquad \text{(A.2)} \end{aligned}$$

Denote by $L_R(\sigma,\theta)$ the deadweight loss resulting from adverse selection. $L_R(\sigma,\theta)$ is the forgone surplus when bilateral matches result in failure to trade.

$$L_R(\sigma,\theta) = \begin{cases} \theta e^{-\theta}\left[q(v_H-c_R)+(1-q)(v_L-c_R)\right] & \text{if} \quad \sigma > \frac{v_H-c_R}{v_H-c_A} \\ \theta e^{-\theta}(1-q)(v_L-c_R) & \text{if} \quad \frac{v_L-c_R}{v_L-c_A} < \sigma \leq \frac{v_H-c_R}{v_H-c_A} \\ 0 & \text{if} \quad \sigma \leq \frac{v_L-c_R}{v_L-c_A} \end{cases} \qquad \text{(A.3)}$$

For the stock of housing sold by relaxed sellers, the total expected gains from trade is $\Pi_R(\sigma,\theta) - L_R(\sigma,\theta)$ multiplied by the housing stock owned by relaxed sellers, $(1-\sigma)S$. The

amount of surplus appropriated by buyers is number of buyers matched with relaxed sellers, $(1-\sigma)B$, times a buyer's expected payoff conditional on matching with a relaxed seller:

$$U_R(\sigma,\theta) = qe^{-\theta q}(v_H - v_L) + \begin{cases} qe^{-\theta}(v_H - v_L) & \text{if} \quad \sigma > \frac{v_H - c_R}{v_H - c_A} \\ qe^{-\theta}(v_L - c_R) & \text{if} \quad \frac{v_L - c_R}{v_L - c_A} < \sigma \leq \frac{v_H - c_R}{v_H - c_A} \\ e^{-\theta}(v_L - c_R) & \text{if} \quad \sigma \leq \frac{v_L - c_R}{v_L - c_A} \end{cases} \tag{A.4}$$

The remaining surplus, $(1-\sigma)S\left[\Pi_R(\sigma,\theta) - L_R(\sigma,\theta)\right] - (1-\sigma)B \cdot U_R(\sigma,\theta)$, must be attributed to the sellers. Dividing by $(1-\sigma)S$ yields the expected payoff to an individual type R seller,

$$\begin{aligned} V_R(\sigma,\theta) &= \Pi_R(\sigma,\theta) - L_R(\sigma,\theta) - \theta \cdot U_R(\sigma,\theta) \\ &= \left[1 - (1+\theta)e^{-\theta}\right](v_L - c) + \left[1 - (1+\theta q)e^{-\theta q}\right](v_H - v_L) \end{aligned} \tag{A.5}$$

which is exactly equation (8) in the text. A similar process yields the payoff function (9) for anxious sellers. In the event of a match, homes sold by anxious sellers always result in a transaction, since buyers never offer below c_A. An anxious seller's payoff can thus be expressed $V_A(\sigma,\theta) = \Pi_A(\sigma,\theta) - \theta \cdot U_A(\sigma,\theta)$, where

$$\Pi_A(\sigma,\theta) = (1 - e^{-\theta})(v_L - c_A) + (1 - e^{-\theta q})(v_H - v_L) \tag{A.6}$$

and

$$\begin{aligned} U_A(\sigma,\theta) &= qe^{-\theta q}(v_H - v_L) \\ &\quad + \begin{cases} e^{-\theta}(v_L - c_A) & \text{if} \quad \sigma > \frac{v_H - c_R}{v_H - c_A} \\ e^{-\theta}\left[v_L - qc_R - (1-q)c_A\right] & \text{if} \quad \frac{v_L - c_R}{v_L - c_A} < \sigma \leq \frac{v_H - c_R}{v_H - c_A} \\ e^{-\theta}(v_L - c_R) & \text{if} \quad \sigma \leq \frac{v_L - c_R}{v_L - c_A} \end{cases} \end{aligned} \tag{A.7}$$

B Omitted Proofs

Proof of Proposition 2.1. Suppose (for the sake of contradiction) that list prices are informative. The housing market can then be characterized by two submarkets: One submarket for sellers with list prices in $\mathbb{P}_1$, and another submarket for sellers with list prices in $\mathbb{P}_2$, $\mathbb{P}_1 \cap \mathbb{P}_2 = \varnothing$.[14]

Consider a type R seller's expected payoff in a submarket with $\sigma \in [0,1]$, and θ determined by the free entry condition, $U(\sigma,\theta) = \kappa$, with

$$\begin{aligned} U(\sigma,\theta) &= qe^{-\theta q}(v_H - v_L) \\ &+ \begin{cases} e^{-\theta}\big[\sigma(v_L - c_A) - q(1-\sigma)(v_H - v_L)\big] & \text{if} \quad \sigma > \frac{v_H - c_R}{v_H - c_A} \\ e^{-\theta}\big[q(v_L - c_R) + \sigma(1-q)(v_L - c_A)\big] & \text{if} \quad \frac{v_L - c_R}{v_L - c_A} < \sigma \leq \frac{v_H - c_R}{v_H - c_A} \\ e^{-\theta}(v_L - c_R) & \text{if} \quad \sigma \leq \frac{v_L - c_R}{v_L - c_A} \end{cases} \end{aligned} \tag{B.1}$$

The seller's payoff would be

$$V_R(\sigma,\theta) = [1 - (1+\theta)e^{-\theta}](v_L - c_R) + [1 - (1+\theta q)e^{-\theta q}](v_H - v_L) \tag{B.2}$$

Differentiating yields

$$\frac{dV_R}{d\sigma} = \big[\theta e^{-\theta}(v_L - c_R) + \theta q e^{-\theta q}(v_H - v_L)\big]\frac{d\theta}{d\sigma} \tag{B.3}$$

The derivative $d\theta/d\sigma$ can be obtained by differentiating the free entry condition and rearranging.

$$\frac{d\theta}{d\sigma} = \begin{cases} \frac{e^{-\theta}[v_L - c_A + q(v_H - v_L)]}{e^{-\theta}[\sigma(v_L - c_A) - q(1-\sigma)(v_H - v_L)] + q^2 e^{-\theta q}(v_H - v_L)} > 0 & \text{if} \quad \sigma > \frac{v_H - c_R}{v_H - c_A} \\ \frac{e^{-\theta}(1-q)(v_L - c_A)}{e^{-\theta}[q(v_L - c_R) + \sigma(1-q)(v_L - c_A)] + q^2 e^{-\theta q}(v_H - v_L)} > 0 & \text{if} \quad \frac{v_L - c_R}{v_L - c_A} < \sigma \leq \frac{v_H - c_R}{v_H - c_A} \\ 0 & \text{if} \quad \sigma \leq \frac{v_L - c_R}{v_L - c_A} \end{cases} \tag{B.4}$$

[14] Here, a submarket represents a group of sellers with list prices within a certain interval as opposed to identical list prices. The generality reflects the fact that a list price is merely a cheap talk message, and does not connote a contractual obligation on the part of the seller.

Therefore,

$$\frac{dV_R}{d\sigma}\begin{cases} > 0 & \text{if} \quad \sigma > \frac{v_L - c_R}{v_L - c_A} \\ = 0 & \text{if} \quad \sigma \leq \frac{v_L - c_R}{v_L - c_A} \end{cases} \tag{B.5}$$

Relaxed sellers prefer the submarket with the highest buyer-seller ratio. According the free entry conditions, the submarket with the highest share of anxious sellers will have the highest buyer-seller ratio. This rules out a fully separating equilibrium since $V_R(0, \theta_R) > V_R(1, \theta_A)$ requires $\theta_R > \theta_A$, while the free entry conditions imply $\theta_A > \theta_R$. It also rules out partial pooling equilibria with informative list prices, since two distinct submarkets can only be an equilibrium if $\sigma_1, \sigma_2 < (v_L - c_R)/(v_L - c_A)$. Otherwise, the relaxed sellers have an incentive switch to the hotter submarket. Even in such cases that $\sigma_1, \sigma_2 < (v_L - c_R)/(v_L - c_A)$, buyer entry and bidding strategies are such that list prices are meaningless and the equilibrium resembles random search, since $\theta_1 = \theta_2$. □

Proof of Proposition 2.2. Assuming full separation, the expected payoff to a seller with reservation value c submitting a reserve bid of $p \in [0, v_L]$ is

$$V(p, \theta) = \theta e^{-\theta}(p - c) + [1 - (1 + \theta)e^{-\theta}](v_L - c) + [1 - (1 + \theta q)e^{-\theta q}](v_H - v_L) \tag{B.6}$$

with bidders arriving according to the free entry condition

$$U(p, \theta) = e^{-\theta}(v_L - p) + qe^{-\theta q}(v_H - v_L) = \kappa \tag{B.7}$$

The optimal reserve bid is uniquely defined by the first order condition:

$$\frac{dV}{dp} = \theta e^{-\theta} + e^{-\theta}(p - c)\frac{d\theta}{dp} + [\theta e^{-\theta}(v_L - p) + \theta q^2 e^{-\theta q}(v_H - v_L)]\frac{d\theta}{dp} = 0 \tag{B.8}$$

The expression for $d\theta/dp$ can be obtained by differentiating the free entry condition. After substituting this into the first order condition it becomes

$$\frac{dV}{dp} = -\frac{e^{-\theta}(p - c)}{(v_L - p) + q^2 e^{\theta(1-q)}(v_H - v_L)} = 0 \tag{B.9}$$

The optimal reserve bid is therefore equal to the seller's true reservation value, $p = c$. Since the arrival rate of buyers is a function of the reserve bid and not the seller's type, there is no reason for any seller to deviate from their optimal reserve bid. In other words, relaxed sellers submit the reserve bid $p_R = c_R$ in order to optimally trade-off transaction probability and expected price. Anxious sellers are more concerned about the probability of trade and so prefer to submit a lower reserve bid $p_A = c_A$. A separating allocation is thus implementable by the social planner. Moreover, the free entry conditions ((B.7) with $\{p_A, p_R\} = \{c_A, c_R\}$) are the same as equations (12) and (13), which implies that the arrival rates of buyers are efficient. The constrained efficient allocation described in Section 2.3 is thus implementable by a social planner even when reservation values are unobservable. □

Proof of Lemma 1. Suppose that some REA offering (a, z) earns positive profit. Free entry of REAs implies that a new REA can offer $(a, z - \varepsilon)$ with $\varepsilon > 0$. With perfect competition in the market for REAs, every seller in submarket a will then choose the new contract over the original one. Moreover, a is unchanged so buyers' beliefs about seller types and submarket tightness remain the same. Finally, since ε can be arbitrarily small, it can be chosen so that the new real estate agent earns positive profit. REA entry remains profitable until $z = \phi(a)$. □

Proof of Lemma 2. Suppose (for the sake of contradiction) that $a_R > 0$. By Lemma 1, $z_R = \phi(a_R)$. Consider a new contract with $a'_R < a_R$. If $z'_R < z_R$, it attracts (at least) all the sellers that were attracted to the original contract. Moreover, the free entry condition for buyers imply that $\theta'_R \geq \theta_R$ (with equality if the same set of sellers accept the new contract). The REA can set $z'_R < z_R$ close enough to z_R that it earns a positive profit: a contradiction by Lemma 1. □

Proof of Lemma 3. The relevant expected payoffs, assuming a separating equilibrium, are

$$V_A(0,0,\theta_R) = [1-(1+\theta_R)e^{-\theta_R}](v-c_A) + \theta_R e^{-\theta_R}(c_R - c_A) \tag{B.10}$$

$$V_R(0,0,\theta_R) = [1-(1+\theta_R)e^{-\theta_R}](v-c_R) \tag{B.11}$$

$$V_A(z_A,1,\theta_A) = [1-(1+\theta_A)e^{-\theta_A}](v-c_A) - z_A \tag{B.12}$$

$$V_R(z_A,1,\theta_A) = [1-(1+\theta_A)e^{-\theta_A}](v-c_R) - z_A \tag{B.13}$$

The incentive compatibility constraint for relaxed sellers pins down the optimal commission, z_A:

$$\begin{aligned} V_R(0,0,\theta_R) &= [1-(1+\theta_R)e^{-\theta_R}](v-c_R) \\ &= [1-(1+\theta_A)e^{-\theta_A}](v-c_R) - z_A = V_R(z_A,1,\theta_A) \end{aligned} \tag{B.14}$$

Substituting the payoff functions from above, the incentive compatibility constraint can be rewritten as

$$V_A(0,0,\theta_R) - [1-e^{-\theta_R}](c_R - c_A) = V_A(z_A,1,\theta_A) - [1-(1+\theta_A)e^{-\theta_A}](c_R - c_A) \tag{B.15}$$

The fully separating market arrangement is incentive compatible for anxious sellers if $V_A(z_A,1,\theta_A) \geq V_A(0,0,\theta_R)$. Using the condition derived above, a separating equilibrium requires

$$[e^{-\theta_R} - (1+\theta_A)e^{-\theta_A}](c_R - c_A) > 0 \quad \iff \quad e^{\theta_A - \theta_R} \geq 1 + \theta_A \tag{B.16}$$

The free entry conditions can be used to to solve for θ_A and θ_R explicitly.

$$U(0,\theta_R) = e^{-\theta_R}(v-c_R) = \kappa \quad \iff \quad \theta_R = \log\left(\frac{v-c_R}{\kappa}\right) \tag{B.17}$$

$$U(1,\theta_A) = e^{-\theta_A}(v-c_A) = \kappa \quad \iff \quad \theta_A = \log\left(\frac{v-c_A}{\kappa}\right) \tag{B.18}$$

The inequality above reduces to

$$\kappa \geq \exp\left(-\frac{c_R - c_A}{v - c_R}\right)(v - c_A) \tag{B.19}$$

As long as (30) is satisfied, the fully separating submarkets are incentive compatible for both types. □

Proof of Lemma 4. First consider the case where $\sigma_0 \leq (v - c_R)/(v - c_A) \equiv \underline{\sigma}$. Since buyer entry and bidding strategies are identical in both the type R submarket and the pooling submarket, relaxed sellers are indifferent between the two: $V_R(0, 0, \theta_R) = V_R(0, \sigma_0, \theta_P)$. Moreover, a full pooling contract cannot strictly benefit anxious sellers because $V_A(z_A, 1, \theta_A) \geq V_A(0, 0, \theta_R) = V_A(0, \sigma_0, \theta_P)$ by Lemma 3. Since neither seller type can achieve a strictly better expected payoff, (30) is a sufficient condition to rule out the possibility that sellers can benefit from pooling when $\sigma_0 \leq \underline{\sigma}$.

Next consider the case where $\sigma_0 > (v - c_R)/(v - c_A) = \underline{\sigma}$. The relevant expected payoffs, assuming a separating equilibrium, are (B.10), (B.11), (B.12), (B.13), and

$$V_A(0, \sigma_0, \theta_P) = [1 - (1 + \theta_P)e^{-\theta_P}](v - c_A) \tag{B.20}$$

$$V_R(0, \sigma_0, \theta_P) = [1 - (1 + \theta_P)e^{-\theta_P}](v - c_R) \tag{B.21}$$

With $\sigma_0 > \underline{\sigma}$, relaxed sellers prefer the pooling submarket because of the higher buyer-seller ratio: $V_R(0, \sigma_0, \theta_P) > V_R(0, 0, \theta_R)$. Whether anxious sellers prefer the pooling submarket depends on κ and σ_0.

The incentive compatibility constraint for relaxed sellers pins down the optimal commission rate for full separation, z_A:

$$\begin{aligned} V_R(0, 0, \theta_R) &= [1 - (1 + \theta_R)e^{-\theta_R}](v - c_R) \\ &= [1 - (1 + \theta_A)e^{-\theta_A}](v - c_R) - z_A = V_R(z_A, 1, \theta_A) \end{aligned} \tag{B.22}$$

Substituting the pooling payoff (B.21) from above, the constraint becomes

$$\begin{aligned} V_R(0,\sigma_0,\theta_P) + [(1+\theta_P)e^{-\theta_P} - (1+\theta_R)e^{-\theta_R}](v-c_R) \\ = [1-(1+\theta_A)e^{-\theta_A}](v-c_R) - z_A \end{aligned} \tag{B.23}$$

Substituting the type A payoff functions (B.12) and (B.20) from above, the constraint can be rewritten and rearranged to obtain

$$\begin{aligned} V_A(0,\sigma_0,\theta_P) - V_A(z_A,1,\theta_A) = [(1+\theta_R)e^{-\theta_R} - (1+\theta_A)e^{-\theta_A}](v-c_R) \\ + [(1+\theta_A)e^{-\theta_A} - (1+\theta_P)e^{-\theta_P}](v-c_A) \end{aligned} \tag{B.24}$$

The preference for pooling among anxious sellers, $V_A(0,\sigma_0,\theta_P) \geq V_A(z_A,1,\theta_A)$, therefore requires

$$\frac{v-c_R}{v-c_A} \geq \frac{(1+\theta_P)e^{-\theta_P} - (1+\theta_A)e^{-\theta_A}}{(1+\theta_R)e^{-\theta_R} - (1+\theta_A)e^{-\theta_A}} \tag{B.25}$$

Using the free entry conditions, we can substitute for the buyer-seller ratios to obtain

$$\frac{v-c_R}{v-c_A} \geq \frac{\left[1+\log\left(\frac{\sigma_0(v-c_A)}{\kappa}\right)\right]\frac{\kappa}{\sigma_0(v-c_A)} - \left[1+\log\left(\frac{v-c_A}{\kappa}\right)\right]\frac{\kappa}{v-c_A}}{\left[1+\log\left(\frac{v-c_R}{\kappa}\right)\right]\frac{\kappa}{v-c_R} - \left[1+\log\left(\frac{v-c_A}{\kappa}\right)\right]\frac{\kappa}{v-c_A}} \tag{B.26}$$

Rearranging to isolate κ yields

$$\begin{aligned} &\big[(1-\sigma_0)(v-c_A) - \sigma_0(c_R-c_A)\big]\log\kappa \\ &\quad \geq \big[(1-\sigma_0)(v-c_A) - \sigma_0(c_R-c_A)\big] - \sigma_0(c_R-c_A)\log(v-c_A) \\ &\qquad + (v-c_A)\big[\log\left(\sigma_0(v-c_A)\right) - \sigma_0\log\left(v-c_R\right)\big] \end{aligned} \tag{B.27}$$

Dividing both sides by the multiplier $[(1-\sigma)(v-c_A) - \sigma(c_R-c_A)]$ affects the inequality depending on its sign: If $\sigma_0 < \overline{\sigma}$, the condition is $\kappa \geq \overline{\kappa}$, where $\overline{\sigma}$ and $\overline{\kappa}$ are defined as

$$\overline{\sigma} \equiv \left[1+\tfrac{c_R-c_A}{v-c_A}\right]^{-1} \tag{B.28}$$

and

$$\overline{\kappa} \equiv \exp\left(1 + \frac{(v-c_A)[\log(\sigma_0(v-c_A)) - \sigma_0 \log(v-c_R)] - \sigma_0(c_R-c_A)\log(v-c_A)}{(1-\sigma_0)(v-c_A) - \sigma_0(c_R-c_A)}\right) \tag{B.29}$$

Otherwise, if $\sigma_0 > \overline{\sigma}$, the inequality is reversed, $\kappa \leq \overline{\kappa}$. I proceed by showing that in the first case, the inequality can never be satisfied because it would imply an entry cost that prohibits buyer entry.

Claim 1: If $\sigma_0 \in (\underline{\sigma}, \overline{\sigma})$, $\kappa \geq \overline{\kappa}$ implies $\kappa > \sigma_0(v - c_A)$.

Proof of Claim 1. The condition that $\overline{\kappa} > \sigma_0(v - c_A)$ can be written

$$1 + \frac{(v-c_A)[\log(\sigma_0(v-c_A)) - \sigma_0 \log(v-c_R)] - \sigma_0(c_R-c_A)\log(v-c_A)}{(1-\sigma_0)(v-c_A) - \sigma_0(c_R-c_A)} > \log\left(\sigma_0(v - c_A)\right) \tag{B.30}$$

Condition (B.30) can be rearranged so that σ_0 appears only on one side of the inequality:

$$v - c_A > \sigma_0(v - c_A)\left[1 + \log\left(\frac{v-c_R}{\sigma_0(v-c_A)}\right)\right] + \sigma_0(c_R - c_A)\left[1 + \log\left(\frac{v-c_A}{\sigma_0(v-c_A)}\right)\right] \tag{B.31}$$

Claim 1 requires that the right hand side of (B.31) remains strictly less than the left hand side when evaluated at any $\sigma_0 \in (\underline{\sigma}, \overline{\sigma})$. The right hand side is concave in σ_0:

$$\frac{\partial^2 \text{RHS}}{\partial \sigma_0^2} = -\big[(v - c_A) + (c_R - c_A)\big]\frac{1}{\sigma_0} < 0 \tag{B.32}$$

Moreover, it is straightforward to show that the right hand side attains a maximum at some $\sigma' \in (\underline{\sigma}, \overline{\sigma})$. At σ',

$$\left.\frac{\partial \text{RHS}}{\partial \sigma_0}\right|_{\sigma_0 = \sigma'} = (v - c_A)\log\left(\frac{v - c_R}{(v - c_A)}\right) - \big[(v - c_A) + (c_R - c_A)\big]\log(\sigma') = 0 \tag{B.33}$$

Evaluating condition (B.31) at σ' yields

$$(1 - \sigma')(v - c_A) - \sigma'(c_R - c_A) > 0 \tag{B.34}$$

which is true by the fact that $\sigma' < \overline{\sigma}$. This proves that $\kappa \geq \overline{\kappa}$ can never hold when $\sigma \in (\underline{\sigma}, \overline{\sigma})$

without the entry cost prohibiting buyers from participating in the full pooling submarket altogether. □

Claim 1 proves that a pooling contract cannot improve the expected payoff to anxious sellers when $\sigma_0 \in (\underline{\sigma}, \overline{\sigma})$. When $\sigma \in (\overline{\sigma}, 1]$ on the other hand, the condition becomes $\kappa \leq \overline{\kappa}$, which can be satisfied depending on the parameters of the model. To gain further insight, I first prove the following Claim about the properties of the threshold $\overline{\kappa}$.

Claim 2: When $\sigma_0 \in (\overline{\sigma}, 1]$*, the threshold* $\overline{\kappa}(\sigma_0)$ *exhibits the following properties:* (1) $\partial\overline{\kappa}/\partial\sigma_0 > 0$*,* (2) $\lim_{\sigma_0 \to \overline{\sigma}^+} \overline{\kappa}(\sigma_0) = 0$*, and* (3) $\overline{\kappa}(1) > \underline{\kappa}$*.*

Proof of Claim 2. To prove part 1, simply differentiate $\overline{\kappa}$ with respect to σ_0:

$$\frac{\partial\overline{\kappa}}{\partial\sigma_0} = \overline{\kappa}(v - c_A)\left[\frac{(v-c_A)-\sigma_0(v-c_A)\left[1+\log\left(\frac{v-c_R}{\sigma_0(v-c_A)}\right)\right]-\sigma_0(c_R-c_A)\left[1+\log\left(\frac{v-c_A}{\sigma_0(v-c_A)}\right)\right]}{\sigma_0\left[(1-\sigma_0)(v-c_A)-\sigma_0(c_R-c_A)\right]^2}\right] \tag{B.35}$$

which is positive if and only if

$$v - c_A > \sigma_0(v - c_A)\left[1 + \log\left(\frac{v-c_R}{\sigma_0(v-c_A)}\right)\right] + \sigma_0(c_R - c_A)\left[1 + \log\left(\frac{v-c_A}{\sigma_0(v-c_A)}\right)\right] \tag{B.36}$$

This is the same condition as (B.31). The concavity in σ_0 of the right hand side and the proof of Claim 1 therefore establish that (B.36) is satisfied for all $\sigma_0 \in (\overline{\sigma}, 1]$, and therefore $\overline{\kappa}$ is increasing in σ_0.

To prove part 2, recall the expression for $\overline{\kappa}(\sigma_0)$:

$$\exp\left(\frac{(v-c_A)[1+\log(\sigma_0(v-c_A))]-\sigma_0(v-c_A)[1+\log(v-c_R)]-\sigma_0(c_R-c_A)[1+\log(v-c_A)]}{(1-\sigma_0)(v-c_A)-\sigma_0(c_R-c_A)}\right) \tag{B.37}$$

For $\sigma_0 > \overline{\sigma}$, the denominator is negative, but as $\sigma_0 \to \overline{\sigma}^+$, the denominator approaches zero. Property 2 then requires that the numerator remain positive as $\sigma_0 \to \overline{\sigma}^+$. The numerator at $\overline{\sigma}$ is

$$(v - c_A)\log\left(\overline{\sigma}(v - c_A)\right) - \overline{\sigma}(v - c_A)\log(v - c_R) - \overline{\sigma}(c_R - c_A)\log(v - c_A) \tag{B.38}$$

Dividing by $(v - c_A)$ and using the definition of $\overline{\sigma}$, the inequality becomes

$$\log\left(\overline{\sigma}(v - c_A)\right) - \overline{\sigma}\log(v - c_R) - (1 - \overline{\sigma})\log(v - c_A) > 0 \tag{B.39}$$

Applying Jensen's inequality yields

$$\begin{aligned}\overline{\sigma}\log(v - c_R) + (1 - \overline{\sigma})\log(v - c_A) &< \log\left(v - c_A - \overline{\sigma}(c_R - c_A)\right) \\ &= \log\left(\overline{\sigma}(v - c_A)\right)\end{aligned} \tag{B.40}$$

where the equality follows from the definition of $\overline{\sigma}$.

The proof of part 3 requires an expression for $\overline{\kappa}(1)$:

$$\overline{\kappa}(1) = \exp\left(1 + \log(v - c_A) - \frac{v - c_A}{c_R - c_A}\log\left(\frac{v - c_A}{v - c_R}\right)\right) \tag{B.41}$$

Part 3 of Claim 2 therefore states that

$$\exp\left(1 + \log(v - c_A) - \frac{v - c_A}{c_R - c_A}\log\left(\frac{v - c_A}{v - c_R}\right)\right) > \exp\left(-\frac{c_R - c_A}{v - c_R}\right)(v - c_A) \tag{B.42}$$

Taking the logarithms of both sides and simplifying yields

$$\frac{c_R - c_A}{v - c_R} > \log\left(1 + \frac{c_R - c_A}{v - c_R}\right) \tag{B.43}$$

which must hold because $(c_R - c_A)/(v - c_R) > 0$. □

Claim 2 states that as σ increases over the interval $(\overline{\sigma}, 1]$, $\overline{\kappa}$ increases from 0 to a value above $\underline{\kappa}$. Therefore, there exists a unique $\sigma^* \in (\overline{\sigma}, 1)$ such that $\overline{\kappa}(\sigma^*) = \underline{\kappa}$. For any $\sigma_0 \in (\overline{\sigma}, \sigma^*)$, the incentive feasible pair of submarkets dominate a full pooling submarket; and for any $\sigma_0 \in [\sigma^*, 1]$, a full pooling submarket dominates the pair of fully separating contracts if and only if $\kappa \in [\underline{\kappa}, \overline{\kappa}]$.

Claim 3: The $\sigma^* \in (\overline{\sigma}, 1)$ satisfying $\overline{\kappa}(\sigma^*) = \underline{\kappa}$ is given by the following expression:[15]

$$\sigma^* = \exp\left(-\tfrac{v-c_A}{v-c_R}\right)\exp\left(-W_{-1}\left(\exp\left(-\tfrac{v-c_A}{v-c_R}\right)\left[\log\left(\tfrac{v-c_A}{v-c_R}\right) - \tfrac{(v-c_A)+(c_R-c_A)}{v-c_R}\right]\right)\right) \tag{B.44}$$

Proof of Claim 3. The proof of Claim 3 consists of setting $\overline{\kappa}(\sigma^*) = \underline{\kappa}$ and solving for σ^*. □

Claims 1, 2, and 3 complete the proof of Lemma 4. □

Proof of Proposition 3.1. There are no possible deviations that attract only type R sellers, since they refrain from paying real estate fees in the type R submarket. Moreover, there are no out-of-equilibrium contracts that can attract only type A sellers, since their commission rate z_A is as low as possible without attracting some relaxed sellers. Existence of a fully separating equilibrium can therefore only be compromised by a full pooling or partial pooling submarket. By Definition 3.2, this pooling or partial pooling submarket must offer greater expected payoffs to both types of sellers, and a strictly greater payoff to at least one type. Type R sellers benefit from pooling, but type A sellers do not benefit as long as

$$V_A(z_A, 1, \theta_A) \geq V_A(0, \sigma, \theta), \quad \text{for all } \sigma \in [0, \sigma_0] \tag{B.45}$$

Potential submarkets with $\sigma > \sigma_0$ can be ignored because the endogenous sorting condition guarantees that the expected payoff to anxious sellers is increasing in the level of real estate services, a, even when relaxed sellers are indifferent, which makes (a_A, z_A) the most preferred incentive compatible contract by anxious sellers.

Given the properties of $V_A(z, \sigma(a), \theta(a))$, conditions (B.45) reduce to

$$V_A(z_A, 1, \theta_A) \geq \max\{V_A(0, 0, \theta_R), V_A(0, \sigma_0, \theta_P)\} \tag{B.46}$$

The proof therefore follows from Lemma 4. □

[15] W_{-1} is the -1 branch of the Lambert W function.

Proof of Proposition 3.2. Proof of part 1: $\kappa < \underline{\kappa}$ rules out a fully separating equilibrium. Therefore, a candidate equilibrium must involve a pooling submarket. Moreover, $\sigma_0 > \underline{\sigma}$ and $\kappa < \underline{\kappa}$ require that there be a pooling submarket with $\sigma_P > \underline{\sigma}$. The endogenous sorting condition ensures that the expected payoff to anxious sellers is increasing in a when relaxed sellers are indifferent. Consequently, when $V_R(\phi(a), 1, \theta(a)) = V_R(0, \sigma_P, \theta_P)$, it must be that $V_A(\phi(a), 1, \theta(a)) > V_A(0, \sigma_P, \theta_P)$. In other words, an incentive compatible zero profit contract with $a > 0$ can be designed to attract the type A sellers without attracting the type R sellers. This rules out a equilibria with pooling when $\sigma_0 > \underline{\sigma}$. This proves non-existence of equilibria, since an equilibrium must involve either separation or pooling and both are incompatible with $\kappa < \underline{\kappa}$ and $\sigma_0 > \underline{\sigma}$.

Proof of part 2: Take as given a full pooling equilibrium $(a_P, z_P) = (0, 0)$ with σ_0 and θ_0, and consider a deviation: a submarket (a, z), with $a > 0$, $\sigma \equiv \sigma(a) > \sigma_0$, and $\theta \equiv \theta(a) > 0$. If $\sigma \leq (v - c_R)/(v - c_A)$, the new submarket does not attract any sellers. If $\sigma > (v - c_R)/(v - c_A)$, the relevant payoff functions are

$$V_A(0, \sigma_0, \theta_0) = [1 - (1 + \theta_0)e^{-\theta_0}](v - c_A) + \theta_0 e^{-\theta_0}(c_R - c_A) \tag{B.47}$$

$$V_R(0, \sigma_0, \theta_0) = [1 - (1 + \theta_0)e^{-\theta_0}](v - c_R) \tag{B.48}$$

$$V_A(z, \sigma, \theta) = [1 - (1 + \theta)e^{-\theta}](v - c_A) - z \tag{B.49}$$

$$V_R(z, \sigma, \theta) = [1 - (1 + \theta)e^{-\theta}](v - c_R) - z \tag{B.50}$$

Type R sellers enter the new submarket as long as $V_R(z, \sigma, \theta) \geq V_R(0, \sigma_0, \theta_0)$. Similarly, the flow of type A sellers into the new submarket continues as long as $V_A(z, \sigma, \theta) \geq V_A(0, \sigma_0, \theta_0)$. Using the same approach as in the proof of Lemma 3, we can obtain the parameter restrictions under which a deviation violates incentive feasibility. Starting with the binding incentive compatibility constraint for relaxed sellers and substituting for the anxious sellers' expected payoffs yields

$$V_A(z, \sigma, \theta) > V_A(0, \sigma_0, \theta_0) \quad \Longleftrightarrow \quad e^{\theta - \theta_0} > 1 + \theta \tag{B.51}$$

The free entry conditions $e^{-\theta_0}(v - c_R) = \kappa$ and $e^{-\theta}\sigma(v - c_A) = \kappa$ can be used to substitute

for θ_0 and θ in the condition above. This yields

$$\kappa > \sigma(v - c_A) \exp\left(\frac{v - c_R - \sigma(v - c_A)}{v - c_R}\right) \tag{B.52}$$

The deviation (a, z) therefore attracts both types of sellers and violates the equilibrium conditions of Definition 3.2 if

$$\kappa \in \left(\sigma(v - c_A) \exp\left(\frac{v - c_R - \sigma(v - c_A)}{v - c_R}\right), (v - c_A) \exp\left(-\frac{c_R - c_A}{v - c_R}\right)\right) \tag{B.53}$$

A contradiction obtains if this interval is empty; that is, if

$$\begin{aligned} &\sigma(v - c_A) \exp\left(\frac{v - c_R - \sigma(v - c_A)}{v - c_R}\right) \geq (v - c_A) \exp\left(-\frac{c_R - c_A}{v - c_R}\right) \\ \iff \quad &\frac{v - c_A}{v - c_R} - \log\left(\frac{v - c_A}{v - c_R}\right) \geq \frac{\sigma(v - c_A)}{v - c_R} - \log\left(\frac{\sigma(v - c_A)}{v - c_R}\right) \end{aligned} \tag{B.54}$$

Given that $(v - c_A)/(v - c_R) > 1$ and $\sigma(v - c_A)/(v - c_R) > 1$, the condition above leads to the conclusion that the interval is empty because $\sigma \leq 1$. Intuitively, this result means that if a fully separating submarket is not incentive feasible, then neither is any other possible deviation when $\sigma_0 < (v - c_R)/(v - c_A)$. Thus, the full pooling submarket constitutes an equilibrium. □

Proof of Proposition 3.3. I prove Proposition 3.3 for the model with $v_L = v_H$. The analysis is similar when $v_L < v_H$, except with the payoff functions from Section 2.

Consider a listing agreement designed for type A sellers with the list price $p_A = c_A/(1 - z_A)$. The RWA clause has no effect on buyers' bidding strategies in a type A submarket, and does not impact anxious sellers' expected payoff because they are willing to accept an offer of $c_A/(1 - z_A)$ regardless of the clause. Type R sellers, on the other hand, now pay a cost in a bilateral match if they choose to mimic type A sellers by entering the type A submarket. The cost is $\min\{z_A c_A/(1 - z_A), c_R - c_A\}$, where the minimization operator allows mimicking sellers to choose between rejecting the offer but paying the REA's commission, $z_A c_A/(1 - z_A)$,

or going through with the transaction at the low price in a bilateral match, resulting in a negative payoff. Since commissions must be paid upon receiving an offer at or above p_A regardless of whether a transaction takes place, a mimicking seller would optimally choose to refuse the offer and pay her REA. This is true as long as $c_A/(1-z_A) < c_R$, which must be the case for type A sellers to find signalling worthwhile.[16]

Recall the type R incentive compatibility constraint in the absence of a RWA clause:

$$\begin{aligned} V_R(0,0,\theta_R) &= \left[1-(1+\theta_R)e^{-\theta_R}\right](v-c_R) \\ &= \left[1-(1+\theta_A)e^{-\theta_A}\right][(1-z_A)v-c_R] = V_R(z_A,1,\theta_A) \end{aligned} \tag{B.55}$$

With the RWA clause, this same condition becomes

$$\begin{aligned} V_R(0,0,\theta_R) &= \left[1-(1+\theta_R)e^{-\theta_R}\right](v-c_R) \\ &> \left[1-(1+\theta_A)e^{-\theta_A}\right][(1-z_A)v-c_R] - \theta_A e^{-\theta_A}\frac{z_A c_A}{1-z_A} = V_R(z_A,1,\theta_A) \end{aligned} \tag{B.56}$$

The extra term on the right hand side implies that the inequality is no longer binding. A lower commission rate than z_A will therefore induce market separation, which directly benefits anxious sellers in a fully separating equilibrium. □

[16] When $v_L < v_H$ and both types of sellers choose to hire REAs such that $q_A, q_R > 0$, it is not always the case that a mimicking seller will opt not to transact in a bilateral match. Nevertheless, the RWA clause tightens the incentive compatibility constraint because the cost to mimicking is positive: $c_R - c_A > 0$. Then, type A contracts need not be so distorted to achieve type R incentive compatibility.