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# Search, Liquidity and the Dynamics of House Prices and Construction

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# Search, Liquidity and the Dynamics of House Prices and Construction

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**Abstract**

We characterize the dynamics of relative house prices, housing sales, construction rates and population growth in response to city–specific income shocks for 106 US cities. We then develop a dynamic search model of the housing market in which construction, the entry of buyers, house prices and sales are endogenously determined in equilibrium. Our theory generates dynamics that are qualitatively consistent with our empirical observations and a version of the economy calibrated to match long-run features of the housing market in U.S. cities offers a substantial quantitative improvement over similar models with no search. In particular, variation in the time it takes to sell a house (*i.e.* the house's *liquidity*) induces house values and transaction prices to exhibit *momentum*, or serially correlated growth.

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# 1 Introduction

We explore the consequences of time-consuming search and matching for the dynamics of house prices, sales, and construction. First, we characterize the impact of city-specific income shocks on the short-run dynamics of average house prices, home sales, construction and population growth for a panel of U.S. cities. We then develop a model in which the entry of new buyers and the construction of new houses in response to such shocks are endogenously determined. Our theory generates short term *momentum* (*i.e.* serially correlated growth rates) for house prices and construction, even if income is strictly mean-reverting following shocks. When calibrated to data on U.S. cities our model accounts for over 80% of the variance of house price movements driven by city-specific income shocks and a smaller, but nevertheless substantial share of the observed autocorrelation of house price growth.

In our empirical analysis, we estimate a structural panel VAR with city-level observations and use it to isolate the joint dynamics of relative prices, housing sales, construction and population growth in response to persistent shocks to local income. We focus on prices and conditions at the city level because, as a number of authors have noted, a major share of the time-series variation in house prices is local in nature, rather than national.[1] This has motivated several researchers to use local factors such as income, regulations and construction costs to account for price movements.

We find that housing market dynamics in U.S. cities can be characterized by several key facts, some of which are consistent with those documented by other authors using different methods and data. Firstly, house prices are volatile relative to *per capita* incomes. Moreover, at the city-level house price appreciation is *much* more volatile than a standard asset pricing model would predict for a simple claim to local *per capita* income. Secondly, there is strong positive serial autocorrelation in house price appreciation over the short term, but mean reversion in prices over longer periods.[2] Thirdly, sales growth is volatile relative to income and is positively autocorrelated with, but lags, population growth. Fourthly, population growth rates are more volatile than construction rates, especially in the short run. Finally, construction rates are more persistent than population growth rates and both are substantially more persistent than income fluctuations.

Some of these observations have been documented previously, but never to our knowledge

[1] This has been documented for the U.S. by Abraham and Hendershott (1996), Del Negro and Otrok (2006) and Glaeser et al. (2010), and for Canada by Allen, Amano, Byrne and Gregory (2007).

[2] See also Abraham and Hendershott (1996), Capozza, Mack and Mayer (1997), Malpezzi (1999) and Meen (2002).

in a unified study of city-level data. In any case, it has been noted, for example by Capozza, Hendershott and Mack (2004), that a behavioral theory which accounts for them has proved difficult to construct. In particular, the fact that house price appreciation exhibits substantial autocorrelation appears to be inconsistent with an asset-pricing approach in which houses are treated as simple claims to local incomes and/or rents. For example, Case and Shiller (1989) argue that serial correlation in rents cannot explain momentum in price changes (see also Cutler, Poterba and Summers, 1991). Glaeser *et al.* (2010) find that while a dynamic rational expectations model of housing with endogenous construction can generate long-run mean reversion, it "fails utterly at explaining high frequency positive serial correlation of price changes."

Several authors have argued that there are good reasons to suspect that search and matching play important roles in housing markets. For example, both the observed positive aggregate co-movement of prices and sales (Rios-Rull and Sanchez-Marcos, 2007) and the fact that prices and sales are negatively correlated with average time on the market (Krainer, 2008) are broadly consistent with search theories of housing markets. Moreover, as first noted by Peach (1983) and more recently documented by Caplin and Leahy (2008), there is significant negative correlation between vacancies and price appreciation. Diaz and Jerez (2012) suggest that movements in the division of surplus between buyers and sellers driven by changes in the tightness of housing markets (as is predicted by competitive search theory) may be a significant source of fluctuations in house prices.

In light of the above literature, we construct a framework that introduces search and matching into a housing market, in which both the entry of new buyers and the construction of new houses evolve endogenously. The value of living in a particular city is determined by an exogenous housing dividend in the form of the income that can be earned locally relative to that in other locations. New buyers enter a city whenever the expected value of doing so exceeds their next best alternative. These entrants require housing. They initially rent, but many then search for houses to buy. This process takes time as agents must find the "right" house which will yield utility to them individually. Exchange in the market for residential housing is characterized by random search with entry of both buyers and sellers. New houses are constructed and offered either for sale or for rent by profit-maximizing development firms. Resident home-owners may also put their houses up for sale or rent due to idiosyncratic shocks that either render them dissatisfied with their current house or cause them to exit the city altogether. In our environment, we establish the existence of a unique stationary growth path characterized by constant rates of population growth and construction.

We study the implications of city-specific income shocks by calibrating our model to data on U.S. cities. The theory generates short-term price momentum in equilibrium even in the absence of persistent income growth (*i.e.* even if income follows a first-order auto-regressive process). In the model, an increase in the value of living in a city spurs an immediate increase in house search activity as households enter the city. It takes time, however, for these buyers to find a house because of the matching friction, as well as for construction of new housing to respond. To meet the immediate housing demands of new entrants, some existing vacant houses are thus shifted to the rental market. As a result, the matching rate for individual house buyers initially declines, while both sales and the rate at which houses sell rise immediately. Therefore, although the value of house search begins to decline after just one period (due to mean reversion in income), the tightness of the housing market (*i.e.* the ratio of buyers to sellers) continues to rise for several more.

A tighter market results in houses selling more quickly. This ongoing increase in housing market *liquidity* causes the expected re-sale value of housing to grow. Because the value of houses in part reflects this, current transaction prices grow in anticipation of easier future re-sale. Over time, as income reverts to its long-run relative level, the stock of buyers declines as entry slows and residents become home-owners. Higher home values induce increased construction so that the decline in vacancies slows and is eventually reversed. The buyer–seller ratio, and hence housing liquidity, eventually fall. Therefore, after initially growing for several periods, the house price falls in anticipation of a less tight market (and slower re-sale) in the future, and eventually revert to its steady-state level.

Although a number of other researchers have studied the role of search and matching in housing markets (e.g. Wheaton, 1990; Krainer, 2001; Albrecht at al., 2007; and Head and Lloyd-Ellis, 2012), they have generally treated the aggregate housing stock as fixed, and/or considered only steady-states. Caplin and Leahy (2008) consider the non-steady-state implications of their model, assuming a fixed housing stock. In contrast, we focus on the role of transitional dynamics of prices and construction of new homes in response to shocks. Furthermore, we allow for the turnover of existing homes, which turns out to be crucial for both the qualitative and quantitative nature of price and investment dynamics. Finally, models of housing investment and construction (e.g. Davis and Heathcote, 2007) generally abstract from search and matching in the market for houses in order to focus on supply-side factors. In contrast, in this paper we bring together aspects of these different literatures in a unified framework.

Our analysis is most closely related to those of Glaeser et al. (2010) and Diaz and Jerez (2012). Glaeser et al. (2010) also study short-term dynamics driven by an estimated process for city-level incomes in a model in which house prices reflect the interaction between local supply conditions and the willingness of households to pay to live in particular location.[3] They do not, however, considers the role of housing liquidity associated with search and matching in the housing market. The possibility of short-term price momentum and mean reversion in prices and construction arises in Glaeser et al. (2010) only because of the observed "hump-shaped" pattern of relative incomes.[4] Their model is to a significant extent successful in accounting for short term volatility in prices and construction but fails to generate short-term momentum in price appreciation and cannot account for volatility over a longer time horizon.

Diaz and Jerez (2012) develop and calibrate a search model in which trade in houses takes place because home-owners experience taste shocks which render them unsatisfied with their current house (Wheaton, 1990). They study the impact of aggregate shocks to the rate at which preferences change. They do not, however, consider endogenous construction responses or either the entry or exit of buyers, all of which play important roles in our results.[5] In their model, they find that competitive search can magnify the effects of exogenous changes on house prices due to movements in the shares of surplus accruing to buyers and sellers. In the context of our model, we find that allowing for this effect, by considering competitive search and a matching function similar to theirs, does increase relative volatility, but only at the expense of a substantial reduction in momentum.

Our empirical strategy isolates the effects of *city-specific* income shocks. We explicitly remove aggregate effects from the analysis by controlling for time-fixed effects. In accordance with this, we focus on a model with a large number of cities where the impact of shocks in one has negligible effects on others. Although several recent papers develop models that allow for cross-city interactions they limit their analyses in other ways. Van Nieuwerburgh and Weill (2010) study the long-run implications of rising wage dispersion for housing price dispersion in a model with cross-city migration and endogenous contruction. However, their model does not feature search dynamics (and hence no distinction between owning and renting),

[3]In this respect these models build on the urban tradition of Alonso (1962), Rosen (1979) and Roback (1982).

[4]Glaeser et al. (2010) also assume utility is decreasing in local population size which has a dampening effect on prices. In their calibration, however, this effect is tiny so, in fact, that the shock process drives everything.

[5]Diaz and Jerez (2012) quantitatively evaluate their model using aggregate, economy-wide data. In contrast we study the role of city-specific shocks.

they do not study short-term price movements and they do not consider the implications of their model for the empirical counterparts of construction, population growth and sales growth at the city level. Karahan and Rhee (2012) study the interaction between illiquid housing markets and labour market outcomes during the recent recession, in a model with two representative cities. Again this paper does not try to account for city-level, short and medium term movements in house prices, construction, population growth, sales, rents, etc.

Section 2 documents empirical features of the dynamics of housing markets at the city level. Section 3 develops the basic model structure. In Section 4, a search equilibrium is characterized and a deterministic steady-state derived. Section 5 presents both a baseline calibration for our search economy and an alternative economy without search. Section 6 considers the dynamic implications of income shocks in the theory. Section 7 checks the robustness of our main findings by examining three modifications to the baseline search economy. Section 8 concludes. Details regarding the data may be found in Appendix A and all proofs and extended derivations are contained in Appendix B.[6]

# 2 Empirical properties of MSA housing markets

In this section, we characterize the joint dynamics of city-level income, house prices, house sales, construction rates and population growth for a sample of 106 U.S. metropolitan statistical areas (MSA's). Our data is annual and runs from 1981 through 2008. We provide details on the data, including sources, in Appendix A. We focus on five variables: *per capita* income at the city level, the level of house prices, growth in the sales of existing houses, the growth rate of the housing stock, and population growth.[7] Since we are interested in the dynamics of city-level measures relative to those in other cities, we first transform the data by removing common time effects. That is, we estimate a panel regression for each series with time dummies and study the residual components.

## 2.1 A Structural Panel VAR

Relative movements in house prices, house sales, construction and city populations are likely affected by many factors. Here we seek to isolate the dynamics that result from changes

[6] There are also two not-for-publication appendices, C and D.

[7] Our focus on these variables is largely driven by the availability of a reasonably long time series for a large number of cities. Subsequently, we will consider the predictions of our model for other variables. As we will show, the quantitative relationship between these five variables is robust to the inclusion of other variables in a panel VAR.

to *income* at the city level, which we interpret as a dividend to residence in the city. We focus on such dynamics so as to be consistent with the theory we present in the next section. In our theoretical model, when income in a particular city rises relative to the average, the city's rate of population growth also rises as households from outside move into the city to take advantage of either the higher income itself or the factor(s) that caused it. Higher entry drives housing demand above trend, puts upward pressure on house prices, and over time spurs construction. As increased construction pushes costs higher and local income reverts toward its trend, entry slows relative to construction. Eventually, house prices decline and the city returns to its long-run trend.

Given that this is the theory we have in mind, we estimate a panel vector auto-regression (VAR) model of the following form:

$$\mathbf{B}\mathbf{X}_{ct} = \sum_{i=1}^{T} \mathbf{A}_i \mathbf{X}_{ct-i} + \mathbf{F}_c + \boldsymbol{\varepsilon}_{ct}, \tag{1}$$

where $\mathbf{X}_{ct} = [Y_{ct}, P_{ct}, g^S_{ct}, g^H_{ct}, g^N_{ct}]'$ denotes the vector of the log of income *per capita*, the log of house prices, sales growth, the growth rate of the stock of houses (cumulative permits) and population growth in each city at each date.[8] Here $\mathbf{B}$ and $\mathbf{A}_i$ are matrices of parameters, $\mathbf{F}_c$ is a vector of city fixed–effects and $\boldsymbol{\varepsilon}_{ct} = [\varepsilon^Y_{ct}, \varepsilon^P_{ct}, \varepsilon^S_{ct}, \varepsilon^H_{ct}, \varepsilon^N_{ct}]'$ contains the structural shocks.

To estimate the structural parameters of this model, we must impose a set of identifying restrictions. Specifically, we assume that the shocks are orthogonal and adopt a Cholesky decomposition with the ordering indicated by the definition of $\boldsymbol{\varepsilon}_c$ above. In particular, income does not depend contemporaneously on any of the other variables and house prices depend contemporaneously only on income. This ordering, which is consistent with the theory we develop below, emphasizes the importance of shocks to fundamentals that affect current and future income, as well as house prices, sales, construction, and population growth contemporaneously. It also allows for shocks to prices that have no *contemporaneous* effect on income, but can affect sales, construction, population growth, and *future* income.[9] Like our theory, this ordering rules out shocks to current and future income that have no contemporaneous effect on house prices.

[8] The fact that we use log levels of incomes and prices and growth rates of population and construction is consistent with our theoretical analysis. However, in Appendix B we report the results of using growth rates of all variables in the panel VAR. With this specification, the nature of the results remain unchanged.

[9] Note that $g^H_{ct}$ and $g^N_{ct}$ are growth rates going forward (i.e. $g^H_{ct} = \ln H_{t+1} - \ln H_t$). Thus it seems reasonable that these variables are able to respond to time $t$ shocks to in income and prices. The relative ordering of $g^H_{ct}$ and $g^N_{ct}$ in the system makes little difference to our results.

We estimate equation (1) for $T = 2$ using the system GMM estimator proposed by Arellano and Bover (1995) and Blundell and Bond (1998).[10] This estimator is asymptotically consistent when the number of panels becomes large for a given time-dimension, thereby avoiding the incidental parameters problem associated with fixed-effects estimators (Nickell, 1981). There are several reasons for which we choose to focus on results using this estimator rather than others. Firstly, it is generally found to outperform other standard GMM estimators such as that of Arellano and Bond (1991) when the endogenous variables are persistent.[11] Secondly, its asymptotic properties are well understood and it has been extended to the context of panel VARs by Binder, Hsiao and Pesaran (2005). Finally, the standard fixed–effect estimator has been found to exhibit a significant finite-sample bias for samples with similar dimensions to ours (i.e. moderately large time and panel dimension; see Judson and Owen, 1999). There are, however, some potential pitfalls in using the system GMM estimator in finite samples. We discuss these in more detail in Appendix C and compare our estimates to those generated using other methods.

A full set of parameter estimates from our baseline panel VAR is reported in Appendix C. Here, Table 1 contains overall summary statistics. The first column shows the average standard deviation of each series relative to that of the growth in *per capita* income. The second column shows the correlation with *per capita* income growth and the third shows the correlation with price growth. The remaining columns show the first four coefficients of autocorrelation.[12]

Several observations can be made based on Table 1. First, house prices and sales growth are much more volatile than city-level incomes. Secondly, price changes are more persistent than those of income growth, with a first-order autocorrelation of 0.56 as compared to 0.27. Moreover, the *level* of house prices exhibits momentum. Thirdly, population growth rates are more volatile on average than construction rates. Fourthly, construction rates are more persistent than population growth rates and both, like price changes, exhibit substantially more persistence than income. Finally, growth in the sales of existing houses at the city

[10] We have also estimated the system with more than two lags, although the results show little difference from the two-lag case.

[11] Essentially, the system GMM estimator instruments the endogenous variables using lagged differences. In fact, we adopt an alternative, but asymptotically equivalent approach which has been found to perform better in finite samples. Specifically, we instrument levels with lagged deviations from the forward mean of the remaining sample (see Arellano and Bover, 1995). As such the model is "just identified" (i.e. the number of regressors equals the number of instruments), so that system GMM is numerically equivalent to equation-by-equation 2SLS.

[12] In the table we report statistics for price growth, while the model is estimated using the *level* of house prices.

level are negatively correlated with price growth. This contrasts with others findings at the aggregate economy-wide level.

**Table 1: Moments from Panel VAR — all shocks**

| | Relative Std. Dev. | Corr. with Inc. growth | Corr. with Price growth | Autocorrelations year 1 | year 2 | year 3 | year 4 |
|---|---|---|---|---|---|---|---|
| Income growth | 1.00 | 1.00 | 0.43 | 0.27 | 0.03 | -0.06 | -0.09 |
| Price growth | 2.75 | 0.43 | 1.00 | 0.56 | 0.24 | 0.02 | -0.11 |
| Sales growth | 2.42 | 0.30 | -0.26 | 0.63 | 0.37 | 0.23 | 0.15 |
| Construction | 0.28 | 0.26 | 0.34 | 0.75 | 0.49 | 0.28 | 0.14 |
| Pop. Growth | 0.49 | 0.27 | 0.10 | 0.39 | 0.31 | 0.17 | 0.10 |

Figure 1 depicts the implied impulse response functions for a shock to relative local income together with the associated 95% confidence intervals.[13] In response to the shock, local income exhibits positively auto-correlated growth, peaking after one year, and is quite persistent. The resulting movement in the relative house price exhibits considerably more momentum, continuing to rise for three years before starting to revert to its mean. Mean reversion is, however, more rapid overall for house prices than for income. After an initial peak, sales growth slows quickly before rising again in the medium term. Population growth responds immediately to the shock then slows down, whereas the construction rate responds more sluggishly, peaking after two years. A key consequence of the latter is that the ratio of city population to the housing stock rises and remains persistently high following a shock to income.

**Table 2: Moments from Panel VAR — Income Shock only**

| | Relative Std. Dev. | Corr. with Inc. growth | Corr. with price growth | Autocorrelations year 1 | year 2 | year 3 | year 4 |
|---|---|---|---|---|---|---|---|
| Income growth | 1.00 | 1.00 | 0.76 | 0.28 | 0.03 | -0.06 | -0.09 |
| Price growth | 1.60 | 0.76 | 1.00 | 0.75 | 0.36 | 0.05 | -0.15 |
| Sales growth | 1.32 | 0.56 | 0.01 | 0.59 | 0.39 | 0.35 | 0.35 |
| Construction | 0.11 | 0.55 | 0.80 | 0.88 | 0.61 | 0.33 | 0.11 |
| Pop. growth | 0.17 | 0.75 | 0.60 | 0.67 | 0.40 | 0.19 | 0.09 |

[13]Confidence intervals are computed using a Monte Carlo simulation for panel VAR provided by Love and Zicchino (2006).

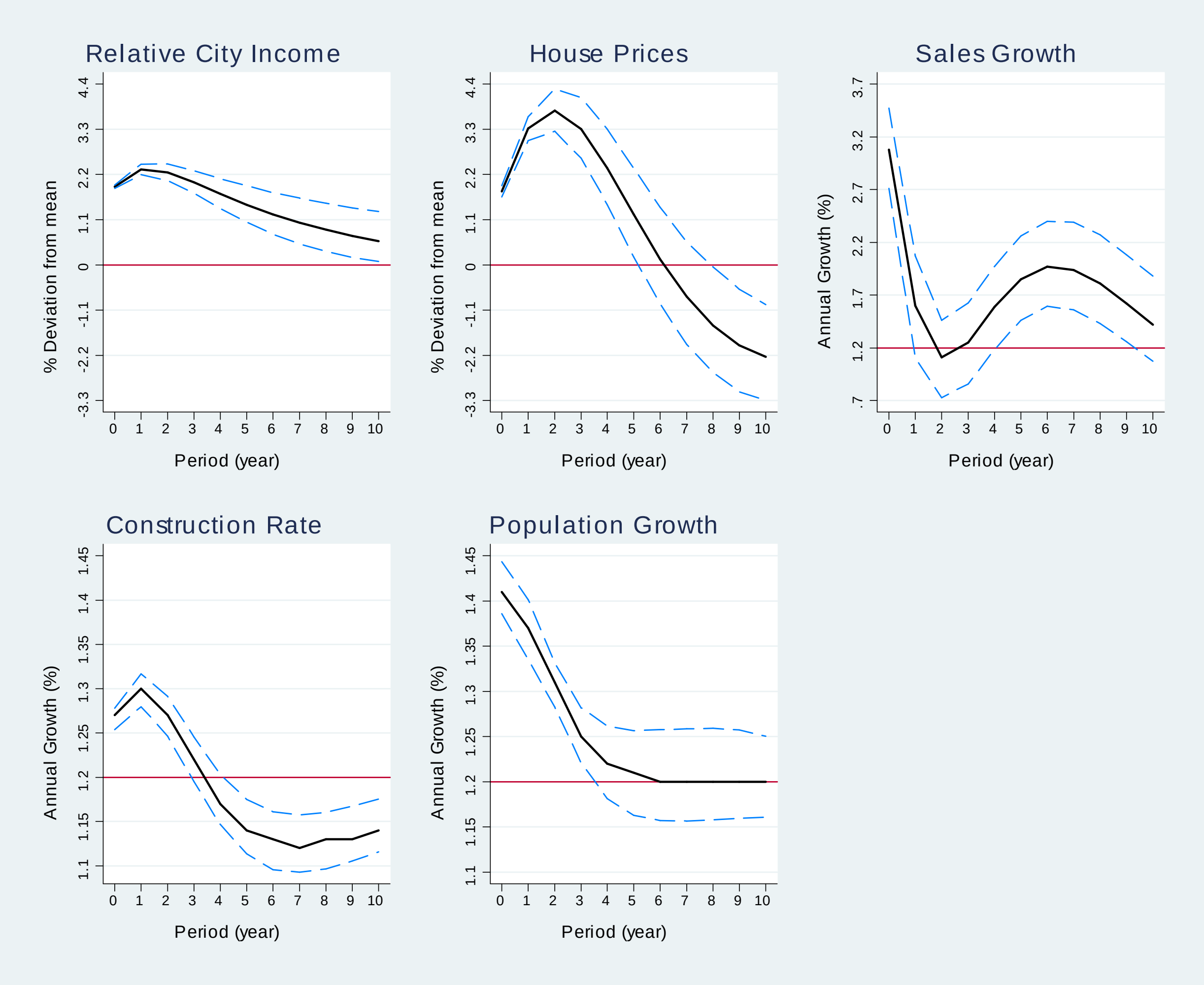


Figure 1: Estimated Impulse Response Functions: Income Shock

These observations are quantified in Table 2, which reports statistics associated with the effect of income shocks implied by the panel VAR. The unconditional standard deviation of house prices generated by income shocks alone is more than half that observed in the data overall (see Table 1 ). The persistence of house price movements generated by local income shocks is, however, somewhat larger than what they exhibit overall. Similar statements hold for sales, construction and population growth rates. In response to a shock to local income, house price growth and sales growth are respectively about 60% and 32% more volatile than income. In contrast, the construction rate exhibits only 11% as much volatility as income, and population growth 17% as much. All four variables, however, exhibit substantially more momentum than income in response to a shock to relative local income. Finally, the induced correlation in price growth and sales growth is close to zero.

In Appendix C, we report the results of re-estimating the panel VAR model for three regional sub-samples. These regional groupings, corresponding to those considered by Glaeser et al. (2010), are coastal cities within 100kms of a coastline, (non-coastal) sun-belt cities and the remaining interior cities. While the parameter estimates naturally vary across samples, the general picture provided by the statistics in Table 2, is unchanged (see Table C2). Moreover, the effects of a shock to local income, as depicted by the impulse responses in Figure 1, are similar qualitatively for all sub-samples considered.

We also report the result of several alternative specifications of the panel VAR. We consider alternative definitions of the key variables and alternative estimators. While parameter estimates and moments vary, the general picture that emerges is robust across specifications. Interestingly, when we restrict the VAR so that income is assumed to follow a univariate AR(2) process, the results are hardly changed. This suggests that feedback effects of the other variables to *per capita* income are not very large and that, to a first approximation, it may be reasonable to think of *per capita* income as an exogenous process.

## 2.2 Pricing a claim to local income

In Section 3 we develop a theory to determine the extent to which search and matching in the housing market can account for volatility and momentum in house prices as well as persistent movements in sales growth, construction rates and population growth, in response to shocks to relative local income. Before doing so, however, it is useful to reflect on the dynamics of the price of a simple claim to local income, as this may serve as a useful benchmark for evaluating the importance of the particular characteristics of *houses* in our theory and the way we model the market for them.

If agents' utility is linear in consumption (as it will be in our theory below), and if claims to local *per capita* income, $y_t$, are traded in a frictionless Walrasian market, then their price, $P_t^L$, will equal the present discounted value of local income:

$$P_t^L = E_t \sum_{i=1}^{\infty} \beta^i y_{t+i}, \tag{2}$$

where $\beta$ is the market discount factor. Imposing the transversality condition, $\lim_{T\to\infty} \beta^T E_t P_{t+T}^L = 0$, setting, in order to be specific, $\beta = 0.96$, and assuming that $\ln(y_t)$ follows the univariate AR(2) process described above, we generate the implied moments for $P^L$. Table 3 compares these moments with those documented for house prices in U.S. cities in Table 2. As may be seen the relative volatility of the price of a claim to local income is less than a third of that houses in the data. Moreover, despite the fact that income growth exhibits serial correlation, this does not translate into any momentum in $P^L$, at all. Indeed, for an income process to generate momentum in the price of such an asset, extremely high and persistent serial correlation in income growth would be required. The reason for this is, of course, that movements in the price of these claims immediately capitalize future income fluctuations.

**Table 3: Prices of a claim to local income – income shock**

| Moment | Houses in U.S. MSA's | Claims to MSA incomes |
|---|---|---|
| $\sigma_p/\sigma_y$ | 1.60 | 0.38 |
| $\sigma_{py}$ | 0.76 | 0.95 |
| $\rho_1^p$ | 0.75 | -0.04 |
| $\rho_2^p$ | 0.36 | -0.04 |
| $\rho_3^p$ | 0.05 | -0.04 |
| $\rho_4^p$ | -0.15 | -0.04 |

# 3 The model environment

Time is discrete and indexed by $t$. We consider an economy characterized by a single housing market (which we refer to as the *city*) and a largely unmodeled "rest of the world".[14] The *aggregate* economy is populated by measure $Q_t$ of *ex ante* identical households, which grows exogenously at net rate $\mu$. Each period, new households enter the city through a process described below. All households living in the city require housing, and they each may either own or rent a house to live in. On entry to the city, households are randomly and permanently differentiated into two types, those who may derive utility from owning the house in which they live and those that do not. All households discount the future at rate $\beta \in (0, 1)$.

Each household is infinitely-lived and is endowed with two types of labour: general labour and construction labour. At each date $t$, a household supplies one unit of general labour inelastically and $l_t$ units of construction labour endogenously, taking the construction wage $w_t$ as given.[15] General labour earns $y_t$ per unit supplied, where $y_t$ is city-specific, exogenous

[14] The sole concern of agents residing in the rest of the world at the beginning of each period is whether or not to enter the city.

[15] It is straightforward to further generalize preferences so that general labour is also supplied endogenously. This, however, would make no difference to our analysis, provided that agents' preferences are separable in the disutility of supplying each type of labour.

and follows a stationary stochastic process in log-levels. We parameterize specific processes for $y_t$ below in Section 6.

At date $t$, preferences over consumption $c_t$, construction labour $l_t$ and housing $z_t$ are given by:

$$U_t(c_t, l_t, z_t) = c_t - v(l_t) + z_t, \tag{3}$$

where

$$v(l_t) = \frac{l_t^{1+\frac{1}{\eta}}}{\zeta^{\frac{1}{\eta}}\left(1+\frac{1}{\eta}\right)} \tag{4}$$

and $\eta$ and $\zeta$ are constants. The variable $z_t$ denotes a utility premium derived from owning the house in which the household lives. In particular, $z_t = z^H$ if the household likes the house it owns, and $z_t = 0$ either if the household does not like the house it owns or is renting. We assume that $z^H$ represents a constant service flow and reflects the owner's personal preference for his/her house. It is constant over time because any depreciation resulting from occupancy is assumed to be offset by maintenance. We let $m$ denote the cost of maintenance incurred by the owner.

In period $t$, the city has a stock $H_t$ of housing units, which are either occupied by a resident owner, rented to a resident, or vacant and offered for sale. The measure of resident home-owners is denoted by $N_t$, and that of renters by $B_t + F_t$. Here $B_t$ is the measure of renters who would like to own a house (and so are currently searching for one to buy) and $F_t$ is that of renters that are not interested in owning.[16] A measure $S_t$ of houses are for sale, where $S_t = H_t - N_t - B_t - F_t$. Houses for sale include both newly built ones that are currently owned by developers, and houses put up for sale by resident owners who either do not want them anymore or are moving elsewhere. At the beginning of each period, a house that is not currently owner-occupied can either be rented or listed for sale. Let $H_t^R$ denote the stock of houses available for rent. A rented house earns rent $r_t$ less the constant maintenance cost $m$. The rental market is competitive.

In the city, there are a large number of *developers* who behave competitively and operate a technology for the construction of new housing units. Each new house requires one unit of land, which can be purchased in a competitive market at unit price $q_t$, and $1/\phi$ units of construction labour. Houses constructed at time $t$ become available either for sale or for rent at time $t+1$ and do not depreciate over time. The stock of houses thus evolves over time

[16] These permanent renters do not play a crucial role in our analysis. We allow for them in order to capture the fact that not all renters are actively searching for a home to own.

according to

$$H_{t+1} - H_t = \phi L_t, \tag{5}$$

and the labor market clears so that

$$(N_t + B_t + F_t)l_t = L_t. \tag{6}$$

Newly built houses are identical to pre-existing ones. Developers can either rent them out or designate them for sale, in which case they remain vacant for at least one period and have exactly the same value as existing vacant houses. Only houses that are occupied require maintenance to offset depreciation.

We assume that land which is potentially available for residential use, $\hat{H}_t$, grows at an exogenous rate equal to the growth rate of the population, $\mu$.[17] To convert land to residential use, however, a cost must be incurred. The cost of converting a particular parcel of land is stochastic, and is represented by a draw $c$ from a stationary distribution $\Lambda(\cdot)$ with support $[0, \bar{c}]$. Here we have in mind that costs vary depending on the specific nature of the new land (*e.g.* with topography, etc.). Only those parcels with $c < q_t$ will be converted in a given period. It follows that the supply of actual residential land for housing, $H_t$, evolves according to

$$\Delta H_t = \Lambda(q_t)\mu\hat{H}_t. \tag{7}$$

As an example, note that with the distribution of conversion cost given by $\Lambda(c) = \left(\frac{c}{\bar{c}}\right)^{\xi}$, the market price of land satisfies

$$q_t = \bar{c}\left(\frac{\Delta H_t}{\mu\hat{H}_t}\right)^{\frac{1}{\xi}}. \tag{8}$$

Here $\xi$ represents the elasticity of new land supply with respect to its price. In principle, this elasticity depends on many factors including physical characteristics, land regulations and local politics (see Saiz, 2010).

At the beginning of period $t$, measure $\mu Q_{t-1}$ of new households arrive in the economy. Each of these households has a best alternative to entering the city, which we denote $\varepsilon$. Here $\varepsilon$ is distributed across the new households according to a stationary distribution function

[17] This assumption is necessary to ensure the existence of a balanced growth path. A similar assumption is made by Davis and Heathcote (2005).

$G(\varepsilon)$ with support $[0, \bar{\varepsilon}]$.[18] There exists a critical alternative value $\varepsilon_t^c$, at which a new household is just indifferent to entering the city:

$$\varepsilon_t^c = \bar{W}_t, \tag{9}$$

where $\bar{W}_t$ is the value of being a new entrant to the city. All non-resident households with $\varepsilon \leq \varepsilon_t^c$ enter the city and are immediately separated into two types. A fraction $\psi$ of the new entrants derive utility from owning their own home *per se* and become potential buyers, while the rest do not and become perpetual renters.[19] Let $W_t$ denote the value of being a potential buyer and $W_t^f$ the value of being a perpetual renter. It follows that

$$\bar{W}_t = \psi W_t + (1 - \psi) W_t^f. \tag{10}$$

Searching for a house to own takes at least one period, and during this time potential buyers also rent. At the end of each period, perpetual renters may, with probability $\pi_f \in (0, 1)$, experience an exogenous shock that induces them to leave the city. On receiving this shock they move out immediately and receive a continuation value $Z$. Otherwise they remain as renters in the next period.

Home-owners are subject to two exogenous shocks. With probability $\pi_h \in (0, 1)$ owners receive a shock that causes them to want to leave the city. Like renters, upon receiving this shock they move out immediately and receive $Z$. They now also have a vacant house which they either rent or hold vacant for sale. With probability $\theta \in (0, 1)$ the remaining $(1 - \pi_h)N_t$ of owners at date $t$ will find that they no longer derive the utility premium $z^H$ from owning their current house. Such "mismatched" owners immediately move out of their current house, put it up for sale, and rent while searching for a new one.[20]

We assume that capital markets are perfect and that the gross interest rate is $1/\beta$. Because capital markets are complete and there is free entry into construction, households do not have any interest in owning houses either as a means of saving or for speculative purposes. As such, it makes no difference whether or not we allow for the trading of vacant

[18] Appendix D describes an interpretation of $G(\cdot)$ based on a multi-city model in which agents realized different amenity values from residence in any of the many cities. The key assumption here is that income, $y$, is truly *city-specific*. That is, that fluctuations in it affect *only* the attractiveness of our representative city to potential entrants.

[19] We could also allow a fraction of renters to become buyers each period. Once calibrated appropriately, it turns out that this makes little difference to our results.

[20] As we shall see later, in equilibrium mis-matched owners are indifferent between moving out and remaining in their own house while searching. Assuming some or all mis-matched owners remain in their current home while searching yields almost identical results. In any case, a mis-matched owner who moves out has a vacant house, which in each period may be either rented or held vacant for sale.

houses in a Walrasian market. It is, however, important that in order to receive utility $z^H$ from owning, households must search for the right house through a time-consuming process that depends on the measures of buyers and vacant houses/sellers in the market.

The market for residential houses is characterized by random search. The measure of matches is determined by the matching function $\mathcal{M}(B_t, S_t)$, which is increasing in both arguments and exhibits constant returns to scale. It follows that a buyer will find a vacant house in the current period with probability

$$\lambda_t = \frac{\mathcal{M}(B_t, S_t)}{B_t} = \lambda(\omega_t). \tag{11}$$

where $\omega_t = B_t/S_t$ is the *tightness* of the housing market, *i.e.* the ratio of searching buyers to sellers. Similarly, each period a seller finds a buyer with probability

$$\gamma_t = \frac{\mathcal{M}(B_t, S_t)}{S_t} = \gamma(\omega_t) = \omega_t\ \lambda(\omega_t). \tag{12}$$

All buyers and sellers take the matching probabilities $\lambda$ and $\gamma$ as given, when they search in the market. We impose the following assumption on the matching function:

**Assumption 1.** *There exists an interval* $(\underline{\omega}, \bar{\omega})$, *with* $0 < \underline{\omega} < \bar{\omega} < \infty$, *such that for all* $\omega \in (\underline{\omega}, \bar{\omega})$: *(i)* $\lambda(\omega) \in [0,1]$, $\gamma(\omega) \in [0,1]$, $\lim_{\omega \to \bar{\omega}} \lambda(\omega) = \lim_{\omega \to \underline{\omega}} \gamma(\omega) \geq 0$ *and* $\lim_{\omega \to \underline{\omega}} \lambda(\omega) = \lim_{\omega \to \bar{\omega}} \gamma(\omega) = 1$; *(ii)* $\lambda'(\omega) < 0$, $\gamma'(\omega) > 0$.

We associate the rate at which houses sell, $\gamma_t$, with their *liquidity*. When this rate increases (decreases), houses become more (less) liquid, by which we mean they sell more (less) quickly. We parameterize a specific matching function as part of our calibration in Section 5. Once matched, the price is determined according to a simple Nash bargaining scheme in which $\chi$ denotes the share of the total match surplus received by the buyer.

Our model has much in common with standard search and matching models used, for example, in the labour search literature. There are, however, several key differences which play a crucial role in our characterization of housing market dynamics. Firstly, every potential home buyer eventually becomes a seller. This implies that expected future market conditions (which determine capital gains) play a role in determining a potential buyer's value of owning a home and hence the current transaction price. Secondly, new vacant homes are produced at a unit cost which varies with supply and demand. It is through this avenue that housing price adjustments ultimately occur. Finally, the rental market plays a key role in absorbing new potential buyers into the city while allowing some substitution of non-owned housing between the rental and the vacant-for-sale pools.

# 4 Equilibrium

Linearity of preferences in consumption, together with the perfect capital markets, implies that households are indifferent with regard to the timing of their consumption. The optimal construction labour supply decision is effectively static and yields the labour supply function

$$l\left(w_t\right) = \zeta w_t^{\eta}. \tag{13}$$

We denote the net benefit of supplying labour as

$$x(w_t) = w_t l_t - v(l_t) = \frac{\zeta w_t^{1+\eta}}{1+\eta}. \tag{14}$$

Perpetual renters never choose to search for a house and remain as renters until they exogenously move to another city. It follows that the value of being such a renter is

$$W_t^f = u_t^R + \pi_f \beta Z + \left(1 - \pi_f\right) \beta E_t W_{t+1}^f, \tag{15}$$

where

$$u_t^R = y_t + x(w_t) - r_t. \tag{16}$$

The stock of perpetual renters evolves according to

$$F_t = (1-\pi_f)F_{t-1} + (1-\psi)G(\varepsilon_t^c)\mu Q_{t-1}. \tag{17}$$

The value of being a home-owner $J_t$ is given by

$$\begin{aligned} J_t \;=\;& u_t^H + \pi_n \beta \left(Z + E_t \tilde{V}_{t+1}\right) + (1-\pi_n)\theta\beta \left(E_t W_{t+1} + E_t \tilde{V}_{t+1}\right) \\ & + (1-\pi_n)(1-\theta)\beta E_t J_{t+1}, \end{aligned} \tag{18}$$

where

$$u_t^H = y_t + x(w_t) + z^H - m. \tag{19}$$

At the beginning of each period, a house that is not currently occupied by the owner can be either rented or listed for sale. Thus, the value of a house which is not owner–occupied is given by

$$\tilde{V}_t = \max\left[r_t - m + \beta E_t \tilde{V}_{t+1},\ V_t\right], \tag{20}$$

where $V_t$ is the value of a house designated for sale.

An agent with a vacant house is free to enter the market as a seller at no cost and matches with a prospective buyer with probability $\gamma(\omega_t)$. Within a match, the house purchase price, $P_t$, solves the Nash bargaining problem, so that:

$$P_t = (1-\chi)\,\beta\,(E_t J_{t+1} - E_t W_{t+1}) + \chi\beta E_t V_{t+1}, \tag{21}$$

where we interpret $\chi$ as the seller's bargaining weight. In our analysis, we focus on situations in which the total trade surplus in the housing market is strictly positive, *i.e.*

$$E_t\left[J_{t+1} - W_{t+1} - \tilde{V}_{t+1}\right] > 0. \tag{22}$$

As we demonstrate below, condition (22) must hold in a steady–state equilibrium.

If a house held vacant for sale is not sold in the current period, the seller keeps it and receives the value of a house that is not currently owner-occupied at the beginning of the next period. It follows that the value of a vacant house for sale satisfies

$$V_t = \gamma(\omega_t)\,P_t + [1-\gamma(\omega_t)]\,\beta E_t \tilde{V}_{t+1}. \tag{23}$$

Given (23), a seller is willing to enter the market if $P_t \geq \beta E_t \tilde{V}_{t+1}$.

A buyer who is successfully matched pays the price $P_t$, and becomes a home-owner in the next period, receiving value $J_{t+1}$. One who remains unmatched continues to search in the next period. Buyers, who are by definition searching for a house, are subject to neither separation nor preference shocks. Recall that they are currently renting, and thus receive the renter utility $u_t^R$ for the current period. The value of being a buyer $W_t$ is therefore given by

$$W_t = u_t^R + \lambda(\omega_t(P_t))\,(\beta E_t J_{t+1} - P_t) + [1-\lambda(\omega_t(P_t))]\,\beta E_t W_{t+1}. \tag{24}$$

It is clear that a buyer is willing to enter the market if and only if $P_t \leq \beta\,(E_t J_{t+1} - E_t W_{t+1})$.

Given, however, that there is an active market of houses for sale, the stock of buyers at date $t$ is given by:

$$B_t = \theta(1-\pi_h)N_{t-1} + \psi G(\varepsilon_t^c)\mu Q_{t-1} + (1-\lambda(\omega_{t-1}))B_{t-1}, \tag{25}$$

and the stock of home-owners evolves via

$$N_t = (1-\pi_n)(1-\theta)N_{t-1} + \lambda(\omega_{t-1})\,B_{t-1}. \tag{26}$$

Overall, at time $t$, the *state*, $s_t$, of the economy is given by level of income in the city, $y_t$, and the measures of buyers, $B_t$, home-owners, $N_t$, permanent renters, $F_t$, and houses, $H_t$. The state evolves via (5), (17), (25), (26), and the stochastic process for local income, $y_t$.

**Definition.** A *search equilibrium* is a collection of functions of the state, $s_t$. The relevant functions are the values of houses vacant for sale, $V_t$, home-ownership, $J_t$, new entrants, $\overline{W}_t$, searchers, $W_t$, permanent renters, $W_t^f$, the entry value cutoff, $\varepsilon_t^c$, The price of houses, $P_t$, rent, $r_t$, wage, $w_t$, the number of houses for rent, $H_t^R$, the measures $B_t$, $N_t$, $F_t$, $H_t$, and housing market tightness, $\omega_t$. These functions satisfy:

**i.** New households enter the market optimally so that (9) and (25) are satisfied;

**ii.** The values of home-ownership, vacant houses, search (*i.e.* being a buyer), permanent renters, and new entrants satisfy (10), (15), (18), (23) and (24), respectively.

**iii.** The owner of a vacant house is indifferent between renting the unit and holding it vacant for sale:

$$\tilde{V}_t = r_t - m + \beta E_t \tilde{V}_{t+1} = V_t; \tag{27}$$

**iv.** The house price, $P_t$, satisfies (21).

**v.** The market for rental housing clears:

$$H_t^R = B_t + F_t; \tag{28}$$

**vi.** Given house prices and construction costs, there is free entry into construction:

$$\beta E_t \tilde{V}_{t+1} \leq \frac{w_t}{\phi} + q_t, \qquad H_{t+1} \geq H_t, \tag{29}$$

where the two inequalities hold with complementary slackness;

**vii.** The construction wage, $w_t$, clears the market for construction labour.

**viii.** The value of home-ownership, $J_t$, is bounded, ruling out bubbles: $\lim_{T\to\infty} \beta^T E_t J_{t+T} = 0$.

## 4.1 The equilibrium dynamic system

In an equilibrium with an active housing market (*i.e.* in which (22) holds) the return to renting a house for a period equals the expected gain from holding it vacant for sale. Combining (23) and (27), we have:

$$r_t - m = \gamma_t \left(P_t - \beta E_t V_{t+1}\right). \tag{30}$$

We focus on equilibria in which construction of houses is always positive, that is, $H_{t+1} > H_t$.[21] It then follows from (5), (27) and (29) that the quantity of new housing constructed in period $t$ is given by

$$H_{t+1} - H_t = \zeta\phi^{1+\eta}\left(N_t + B_t + F_t\right)\left(\beta E_t V_{t+1} - q_t\right)^{\eta}. \tag{31}$$

To obtain a stationary representation of the economy, we normalize the state variables (other than $y_t$) by the total population $Q_t$. We use lower case letters to represent *per capita* values. Given (17), (25), (26) and (31), the laws of motion for renters, buyers, owners and houses, *per capita,* respectively, can be written as

$$\begin{aligned}
(1+\mu)f_t &= (1-\psi)\,\mu G\left(\bar{W}_t\right) + (1-\pi_f)f_{t-1} && (32)\\
(1+\mu)b_t &= \psi\mu G\left(\bar{W}_t\right) + \left[1-\lambda(\omega_{t-1})\right]b_{t-1} + \theta(1-\pi_n)n_{t-1} && (33)\\
(1+\mu)n_t &= (1-\theta)(1-\pi_n)n_{t-1} + \lambda(\omega_{t-1})b_{t-1} && (34)\\
(1+\mu)h_{t+1} &= h_t + \zeta\phi^{1+\eta}\left(n_t + b_t + f_t\right)\left(\beta E_t V_{t+1} - q_t\right)^{\eta}. && (35)
\end{aligned}$$

By definition, the tightness of the housing market is given by

$$\omega_t = \frac{b_t}{h_t - b_t - f_t - n_t}. \tag{36}$$

Moreover, market-clearing in the rental market implies

$$h_t^R = b_t + f_t. \tag{37}$$

## 4.2 The deterministic steady-state

We now consider a steady-state in which non-construction income *per capita* is constant and normalized to unity: $y_t = 1$. In this setting all normalized quantities and values are constant and their steady-state values are indicated with an asterix. The steady-state conditions are laid out in full in Appendix B.

**Lemma 1.** *In the steady-state, the surplus from a match in the owned housing market must be positive and is given by*

$$J^* - W^* - V^* = \frac{z^H}{1 - \beta(1-\pi_n)\left(1-\theta\right) + \left(\frac{1-\beta+\pi_n\beta}{1-\beta}\right)\beta\lambda(\omega^*)\chi}. \tag{38}$$

[21] It is straightforward to show that this will be the case in any search equilibrium for an economy with sufficient population growth.

Intuitively, the fact that this surplus is always positive follows from the fact that matched buyers strictly prefer owning to renting and there is no cost to selling. As a result, housing transactions always take place as long as the matching rate is positive.

Using the steady-state conditions, it is straightforward to derive a relationship between market tightness and the value of a house for sale required to induce developers to produce:

**Lemma 2.** *In the steady-state, there exists a negative "supply-side" relationship between the value of a house for sale and market tightness:*

$$V^* = V^S(\omega^*) = \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\left(1+\frac{\psi(\mu+\pi_f)}{A\gamma(\omega^*)+B\omega^*}\right)\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta}, \tag{39}$$

*where* $A = \frac{\mu+\psi\pi_f+(1-\psi)\pi_n}{\mu+\pi_n+\theta(1-\pi_n)}$, $B = \mu+\psi\pi_f$ *and* $\bar{q} = \bar{c}F^{-1}(h^*/\hat{h})$.

The relationship described in (39) can be interpreted as follows. As the value of vacant housing rises, new construction is stimulated and more houses become available for sale. This drives down the market tightness $\omega$, *i.e.* the ratio of buyers to houses for sale. Note that (39) implies that houses will always be built in steady state so long as $\mu > 0$ since this ensures that $\beta V > \bar{q}$.

The steady-state conditions also yield another relationship between the value of houses for sale to buyers and market tightness:

**Lemma 3.** *In a steady-state, there exists a positive "demand–side" relationship between the value of a house for sale and market tightness:*

$$V^* = V^D(\omega^*) = \frac{\gamma(\omega^*)(1-\chi)\beta z^H}{(1-\beta)[1-\beta(1-\theta)(1-\pi_n)] + (1-\beta+\pi_n\beta)\beta\lambda(\omega^*)\chi}. \tag{40}$$

Intuitively, a higher ratio of buyers to sellers, *i.e.* a tighter market, has two effects. Firstly, it increases the rate at which houses sell, $\gamma$. For a given selling price, this drives up the value of a vacant house. Secondly, it *lowers* the rate at which buyers find houses, which increases the gain from becoming an owner. This raises the transaction price of houses, and so the value of a house for sale.

Combining lemmas 2 and 3 yields the following proposition:

**Proposition 1.** *Let parameters be such that* $V^D(\underline{\omega}) < V^S(\underline{\omega})$ *and* $V^D(\bar{\omega}) < V^S(\bar{\omega})$. *Then, there exists a unique interior steady-state equilibrium in which* $\omega^* \in (\underline{\omega}, \bar{\omega})$.

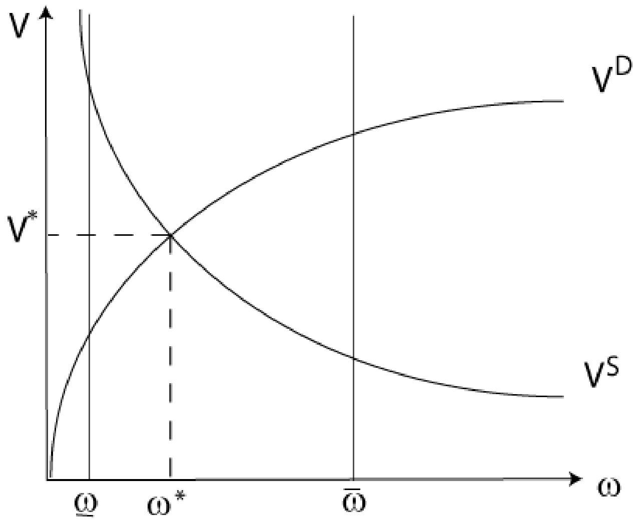


Figure 2: Steady State Equilibrium

Figure 2 depicts the existence of a steady-state equilibrium (SSE) at the intersection of (39) and (40) such that $\omega^* \in [\underline{\omega}, \bar{\omega}]$. This SSE is interior in the sense that the probabilities of buying and selling are strictly less than unity. If the point of intersection were to occur at $\omega^* < \underline{\omega}$, the economy would be driven to a corner SSE in which the probability of buying is given by $\lambda = 1$. Conversely, if the point of intersection were to occur at $\omega^* > \bar{\omega}$, the economy would be driven to a corner SSE in which the probability of selling is given by $\gamma = 1$. The conditions under which an interior SSE exists are parametric, with the values of $\underline{\omega}$ and $\bar{\omega}$ depending on the specific matching function (see Appendix B, equations (79) and (80)). In our baseline calibration below, and in all of the experiments and robustness checks that we consider, the steady-state is always interior. Although it is straightforward, we therefore omit a full analysis of corner SSE for the sake of brevity.[22]

# 5 Calibration

In order to study the dynamics of the model, we linearize the dynamic system for a calibrated version of the economy in a neighborhood of its unique deterministic steady-state. In all cases that we consider, the resulting systems of first-order linear difference equations satisfy the conditions for saddle-path stability. We solve numerically for the implied local dynamics

[22] For some matching functions (e.g. urn-ball) the SSE is always interior.

driven by stochastic movements in $y_t$ using a first-order perturbation method.[23]

## 5.1 Baseline parameterization

We begin with baseline parameterization. To start, we let the matching function take the Cobb-Douglas form:

$$M = \kappa B_t^{\delta} S_t^{1-\delta}, \tag{41}$$

where $\kappa > 0$ and $\delta \in (0, 1)$.

Table 4 gives the parameter values for the baseline. Numbers above the line are set to match the indicated targets directly. Values below the line are set jointly so that the specified steady-state values generated by the model match the given targets. For illustrative purposes, however, in the table we associate these parameters with a specific target for which it is particularly relevant.

We define a period to equal one quarter. We set $\beta$ to reflect an annual interest rate of 4% and $\mu$ is chosen to match annual population growth during the 1990s.[24] We normalize $\bar{y} = 1$. Thus, present values and prices are all measured relative to the steady-state *per capita* income.

We set $\pi_f$ to match the annual fraction of renters that move between counties, which is about 12% on average according to the Census Bureau. Similarly, $\pi_n$ is set to match the annual fraction of home-owners that move between counties (3.2%) and $\theta$ is set so to match the fraction of owners that, conditionally on moving, do not change counties (60%). Dieleman, Clark and Deurloo (2000) estimate an overall housing turnover rate of 8% annually (see also Caplin and Leahy, 2008), which is consistent with our quarterly value of $\pi_n + (1 - \pi_n)\theta \simeq 0.02$. We set $Z = \bar{u}^R/(1-\beta)$ so that the value of exiting is equal to the steady-state value of being perpetual renter.

The parameter $\phi$ represents the labor productivity of the construction sector. The ratio of permits issued in the U.S. each quarter to the numbers of employees in residential construction is approximately 0.1 on average. If the average working week is 35 hours (or roughly 400 hours per quarter), then the number of permits produced per hour worked equals approximately 0.00025 (this amounts to 4,000 man-hours per house).

A related parameter is the price elasticity of land supply, $\xi$. Saiz (2010) studies the relationship between house prices and the stock of housing based on a long difference esti-

[23] We have also solved it using a second-order perturbation method in Dynare. The results are identical.

[24] Population growth has slowed somewhat in recent years.

mation between 1970 and 2000 for 95 U.S. cities.[25] In particular, by instrumenting using new measures of regulatory restrictions and geographical constraints, he is able to infer city level price elasticities that vary due to natural and man-made land constraints. His supply elasticity estimates vary from 0.60 to 5.45 with a population–weighted average of 1.75 (2.5 unweighted). We therefore set $\xi = 1.75$ in our baseline. In any case, below we consider the sensitivity of our results to variations in this parameter.

The steady-state unit price of land $\bar{q}$ is set so that the relative share of land in the price of housing is 30% (see Davis and Palumbo, 2008, and Saiz, 2010). The average price of a house is approximately 3.2 times annual income or 12.8 times quarterly income. This implies a ratio of the land price to income of $0.3 \times 12.8 = 3.84$.

**Table 4: Baseline Calibration Parameters: Steady State**

| **Parameter** | **Value** | **Target** |
|---|---|---|
| $\beta$ | 0.99 | Annual real interest rate = 4% |
| $\mu$ | 0.003 | Annual population growth rate = 1.2% |
| $\pi_f$ | 0.030 | Annual mobility of renters = 12% |
| $\pi_n$ | 0.008 | Annual mobility of owners = 3.2% |
| $\theta$ | 0.012 | Fraction of moving owners that stay local = 60% |
| $\phi$ | $2.5 \times 10^{-4}$ | Quarterly permits/construction employment (hours) |
| $\xi$ | 1.75 | Median price-elasticity of land supply = 1.75 |
| $\overline{q}$ | 3.84 | Average land price-income ratio |
| $\psi$ | 0.43 | Fraction of households that rent = 32% |
| $m$ | 0.0125 | Average rent to average income ratio, $r^* = 0.137$ |
| $z^H$ | 0.028 | Zero net-of-maintenance depreciation |
| $\kappa$ | 0.76 | Vacancy rate = 2% |
| $\chi$ | 0.0916 | Months to sell = months to buy |
| $\zeta$ | 0.038 | $P^* = 12.8$ |

We choose the remaining parameters so that several key steady-state statistics match their average counterparts in U.S. data. In particular, the value of $\psi$ is calibrated so that the average fraction of households that rent in the steady-state $(b + f)/(n + b + f)$ is 32%. The maintenance cost $m$ is chosen so that the rent is 13.7% of median income. Note that the income of the average renter in the U.S. is less than half of that of the average owner, reflecting the fact that the characteristics of owners and renters differ systematically. On average, a renter in the U.S. allocates 24% of his after-tax income to rent (see Davis and

[25] In this sense, the estimated relationship picks up long term dynamics associated with $\xi$. In contrast, the estimates of Green, Malpezzi and Mayo (2005) relate to short run dynamics associated with $\eta$.

Ortalo-Magne, 2011). Since in our model all agents are homogeneous, we target the ratio of rent to the median income of owners and renters, which is somewhat lower (see Head and Lloyd-Ellis, 2012 for details).

As described earlier, we assumed that the maintenance cost is just enough to offset depreciation. Let $d$ denote the rate of depreciation. Then, under a simple optimal maintenance program, the implicit steady-state flow utility derived from owning a house would be given by[26]

$$z^H = \left(1 + \frac{1-\beta}{\beta d}\right) m. \tag{42}$$

Harding et al. (2007, p. 212) estimate the gross maintenance rate of depreciation for a house of median age in the U.S. to be about 3% annually.[27] It follows from (42) that $z^h = 0.028$. Again, we consider below the sensitivity of our results to alternative values.

We assume that, in steady-state, the time taken to sell a house is equal to the time taken to buy, so that $\omega^* = 1$. This is approximately true in the long run according to surveys conducted by the National Association of Realtors (NAR). Given the other parameters of the model, it follows that the matching function parameter, $\kappa$, determines the steady-state value of the vacancy rate. Average vacancy rates for the U.S. economy and by MSA are available from the Census Bureau's Housing Vacancy Survey (HVS). In our model, houses that are vacant in equilibrium are designated for sale. The HVS distinguishes the category "vacant units which are for sale only". In 2000, for example, this category constituted 1% of the overall housing stock. Since owned homes constituted approximately two-thirds of the housing stock, this corresponds to a home-owner vacancy rate of about 1.5%.[28] Housing units that are in the category "vacant units for rent" actually consist, however, of vacant units offered for rent only and those offered *both* for rent and sale. In 2000, for example, houses both for rent and sale constituted a further 2.6% of the overall housing stock.[29] In our model, vacant units are technically available for rent in the subsequent period, so it makes

[26]Suppose $z_t^H = z^H(Q_t)$ where $Q_t$ denotes the quality of a house. The optimal maintenance program of a home–owner can be expressed as

$$\begin{aligned} V(Q_t) &= \max_{\{m_t, Q_t\}} z^H(Q_t) - m_t + \beta V(Q_{t+1}) \\ \text{s.t. } Q_{t+1} &= (1-d)Q_t + m_t. \end{aligned}$$

If $z^H(Q_t)$ is approximately linear, then the steady-state solution to this program implies (42).

[27]The resulting actual depreciation rate is rather less than 1% precisely because maintenance is undertaken.

[28]This number is close to the average over the period 1980-2008. However, more recently homeowner vacancy rates have exceeded 2.5%.

[29]Again, since rental units constitute about a third of the housing stock, this corresponds to a rental vacancy rate of about 8%.

sense to include some of those vacant units offered for both rent and sale in our measure of vacancies. For this reason, we assume an additional 1% of the housing stock is vacant and for sale, so that $v = 0.02$. Again, we consider the sensitivity of our results to alternative values of $v$.

Given the values of $\mu$, $\theta$ and $\pi_n$ from Table 4 and our targets for $\psi$ and $v$, the implied steady-state probability of sale each period is $\gamma^* = 0.76$, leading to an average time for a house to be on the market of just under 4 months. This may seem somewhat high given that according to the NAR, the time taken to sell a typical house is about 2 months.[30]

Given the other parameters, $\phi$ is chosen so that the price of a house is 3.2 times annual income or 12.8 times quarterly income. Note that the value of $\chi$ required to hit the targets implies that just over 90% of the surplus from housing transactions goes to the seller.

## 5.2 The earnings process

Parameterization of the process for local non-construction earnings, $y_t$, is complicated by mismatch between the frequency of available city-level income data and the period length assumed in our calibrated model. The income data is available annually, whereas the baseline calibration assumes that each period is a quarter. While the period length could be increased to one year in the model, this would be restrictive as it would require that houses for sale remain vacant for at least one year, which is clearly counter-factual.

Instead, we derive a quarterly process for income that shares key properties at annual frequencies with the process estimated in our panel VAR in Section 2. Specifically, we assume that the quarterly income process is given by

$$\ln y_t = a \ln y_{t-1} + b \ln y_{t-4} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon). \tag{43}$$

We set the parameters $a = 1.05$, $b = -0.0375$ and $\sigma_\varepsilon = 0.011$, so that the implied annual income process matches the volatility of income growth, $\sigma_y$, the first-order autocorrelation coefficient, $\rho_1$, and the sum of the second-, third- and fourth-order autocorrelation coefficients $(\rho_2 + \rho_3 + \rho_4)$ in the data. In Table 5 we report these moments for both the estimated and constructed processes.[31]

[30] There are varying estimates of the time to buy and the time to sell. Diaz and Jerez (2012) use 2 months based on a report from the National Association of Realtors. Piazzesi and Schneider (2009) suggest using 6 months. Anglin and Arnott (1999) report estimates of up to 4 months. The NARs estimate of "time on the market" is somewhat misleading because houses are sometimes strategically de-listed and quickly re-listed in order to reset the "days on market" field in the MLS listing (see Levitt and Syverson, 2008).

[31] The translation from an annual to a quarterly process is discussed more formally in Appendix C.

**Table 5: Implied moments for annual income growth process**

| | $\sigma_y$ | $\rho_1$ | $\rho_2$ | $\rho_3$ | $\rho_4$ |
|---|---|---|---|---|---|
| Data | 0.02 | 0.26 | 0.02 | -0.05 | -0.07 |
| Artificial | 0.02 | 0.26 | -0.03 | -0.03 | -0.04 |

Using the constructed process for earnings and the linearized model, we generate sample paths for the key variables of the model and use these to construct "annual" time series for the economy.[32]

## 5.3 Three elasticities

The dynamics of the model depend crucially on three elasticities:

• The elasticity of the distribution of alternative values, $G(\cdot)$, in the vicinity of steady-state $\varepsilon^c$. This is given by

$$\alpha = \frac{\varepsilon^c G'(\varepsilon^c)}{G(\varepsilon^c)} \tag{44}$$

and determines the responsiveness of the entry of potential buyers to changes in the value of search.[33] In the steady state, all that depends on $G(\cdot)$ is the measure of searching households per capita, $b^*$. This is not something that is directly observed, and so the parameters determining the relevant characteristics of $G(\cdot)$ cannot be identified in this way.

• The elasticity of the matching function with respect to the number of buyers, $\delta$. Since $\omega^* = 1$, the elasticity of the matching function does not affect the steady-state. Out of the steady-state, however, it determines in part the relationship between sales and prices.

• The wage elasticity of construction labor supply, $\eta$. This determines the responsiveness of construction to the value of housing. In a model with no frictions, this elasticity equals that of new housing construction to the sales price. Recent estimates of this using annual data range from about 1.6 to 5 at the national level (see Topel and Rosen (1988), Poterba (1991) and Blackley (1999)). For a sample of 45 cities, Green, Malpezzi and Mayo (2005) estimate a median elasticity of about 5. In our model with search frictions, however, the value of a newly constructed house does not correspond directly to the transactions price, but also depends on the endogenous absorption rate.

Our approach is to use our estimates of the relative standard deviations of population growth, construction and sales growth in response to income shocks from Table 2 to jointly

[32] We have also experimented with an ARMA(1,4) process where the shock's direct effect on income is divided over four quarters. This has no impact on our results.

[33] Since we log-linearize the model around the steady state, it is not necessary to specify the entire function $G(\cdot)$.

calibrate the values of $\alpha$, $\delta$ and $\eta$. Specifically, we choose these parameters so that, given the earnings process, the stochastic output from our model generates the relative standard deviations, $\sigma_n/\sigma_y$, $\sigma_h/\sigma_y$ and $\sigma_s/\sigma_y$, which exactly match those in the data.[34]

Several observations should be made here. Firstly, it is not necessarily possible *a priori* to match all three moments exactly. In a model with no frictions, for example, $\sigma_n = \sigma_h$ by construction and there is no parameter $\delta$. Secondly, the calibration of these parameters depends crucially on the pricing protocol in the housing market. For this reason any calibration or estimation must be specific to the model being assumed. We cannot use independent estimates from the literature which are inferred using models with no frictions. Finally, pinning down the elasticities in this way does not, by itself, neither imposes any particular process for house prices, nor does it determine the autocorrelation or comovement of these variables. Once again, however, we consider the sensitivity of our results to variations in the calibrated parameters in Section 6.

**Table 6: Baseline Calibration Parameters: Non Steady-State**

| Parameter | Value | Target |
|---|---|---|
| $\alpha$ | 7.00 | Relative volatility of population growth = 0.17 |
| $\delta$ | 0.67 | Relative volatility of sales growth = 1.32 |
| $\eta$ | 1.60 | Relative volatility of construction rate = 0.11 |

## 5.4 An economy with no search

A version of the economy with no search is described in Appendix B. We think of this economy as comparable in many ways to that studied by Glaeser *et. al.* (2010), although there are some important differences and these are discussed in the appendix. In this economy, houses are effectively identical. That is, an agent who is not a permanent renter realizes the utility gain from home-ownership, $z^H$, from living in *any* house. This eliminates the need for time-consuming matching between buyers and sellers.

The parameters of this economy are set as in Table 4 except for $\psi$, $m$, $z^H$ and $\phi$, These are adjusted so that the steady-state again matches the relevant targets.[35] Without search, in the deterministic steady-state the house price is given by

$$P^* = \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta}. \tag{45}$$

[34] When computing sales of existing houses in our model, we assume that new and existing houses which are not owner occupied are equally likely to be rented each period.

[35] Obviously, the parameters of the matching function $\kappa$ and $\delta$ are not relevant in this case.

We use (45) to derive the value of $\phi$ such that $P^* = 12.8$. In the model with no search the stock of housing must equal the number of households. It follows (as noted above) that it is not possible to match the different volatilities of population growth and construction. In this case, we fixed $\alpha$ and set $\eta = 2.1$, so that the implied relative volatility of the construction rate, $\sigma_h/\sigma_y$, matched that in the data.[36]

## 5.5 Steady-state implications

Although we are primarily concerned with the model's dynamics, here we briefly consider the steady-state implications of search frictions for market tightness, house prices and welfare. In particular, we ask what happens as we vary the "productivity" of the matching function, which in the Cobb-Douglas case is given by $\kappa$. The measure of welfare that we use is the steady-state value of entering the city, $\bar{W}^*$. This measure gives equal weight to the welfare of prospective owners and renters and takes into account the likelihood of buying, selling and exiting the city in the future.[37] Figure 3 illustrates how house prices and welfare, repectively, vary with market tightness as we change $\kappa$ between 0 and 1.

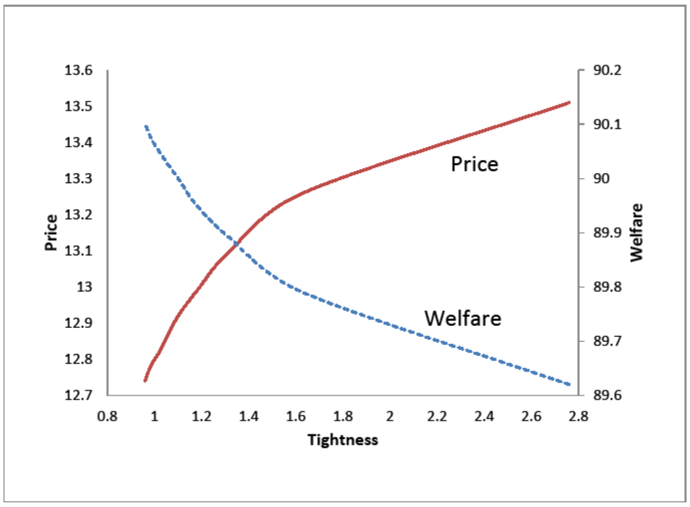


Figure 3: Varying the productivity of matching in steady state

[36] Alternatively, we could have fixed $\eta$ and calibrated $\alpha$. We tried this, and produced similar results.
[37] As such it is akin to the expected utility of a newborn in an overlapping generations model. For this reason, the measure reflects the utility of current buyers rather than current sellers.

As $\kappa$ is increased, the rate at which houses sell for a given buyer-seller ratio, $\gamma(\cdot)$, increases as well. This causes the value of unoccupied homes to their owners to increase and the unit value required by developers to decrease. These two factors induce market tightness to decline in the steady state (Figure 2), as the supply of new housing increases faster than the demand. The impact on the value of vacant houses and hence on transaction prices is, in general, ambiguous and depends on the relative elasticities of entry and housing supply. In our calibration, steady-state housing prices rise with market tightness. It should not, therefore, be surprising that as $\kappa$ is increased, the welfare of new entrants rises; it becomes easier to find a house and prices fall.

# 6 Equilibrium Dynamics

## 6.1 Qualitative implications of a shock to local income

We first describe the qualitative implications of the model. We note at the outset that the model's dynamics are not driven by the hump-shaped dynamics of income observed in the data. That is, a sufficiently persistent AR(1) process for income generates impulse response functions that are essentially identical to those reported here.

The implied impulse response functions (IRF's) following a shock to local income are depicted in Figures 4 and 5.[38] The five panels of Figure 4 depict the IRFs for income, house prices, sales growth (of existing houses), construction rates, and population growth relative to trend for the economies with and without search. In both economies, the shock to income induces entry and population growth rises. Although the responses of city population growth in the two cases are qualitatively similar, entry is initially much more rapid in the search economy. The responses of house prices, growth in sales of existing houses and the construction rate, however, differ qualitatively across the two economies. The search model generates momentum in both prices and construction which is qualitatively similar to the dynamics of the empirical model illustrated in Figure 1. Moreover, as in the empirical characterization, sales growth spikes quickly and then declines sharply before rising again in the long run. In contrast, the economy with no search generates no momentum in either prices or construction rates, despite generating substantial momentum in the housing stock, and sales growth rises and then returns monotonically to trend.

The force which generates serial correlation in both house price appreciation and the

[38] Although the model time period is a quarter, we have provided annualized versions of the IRFs to allow an easier comparison with Figure 1.

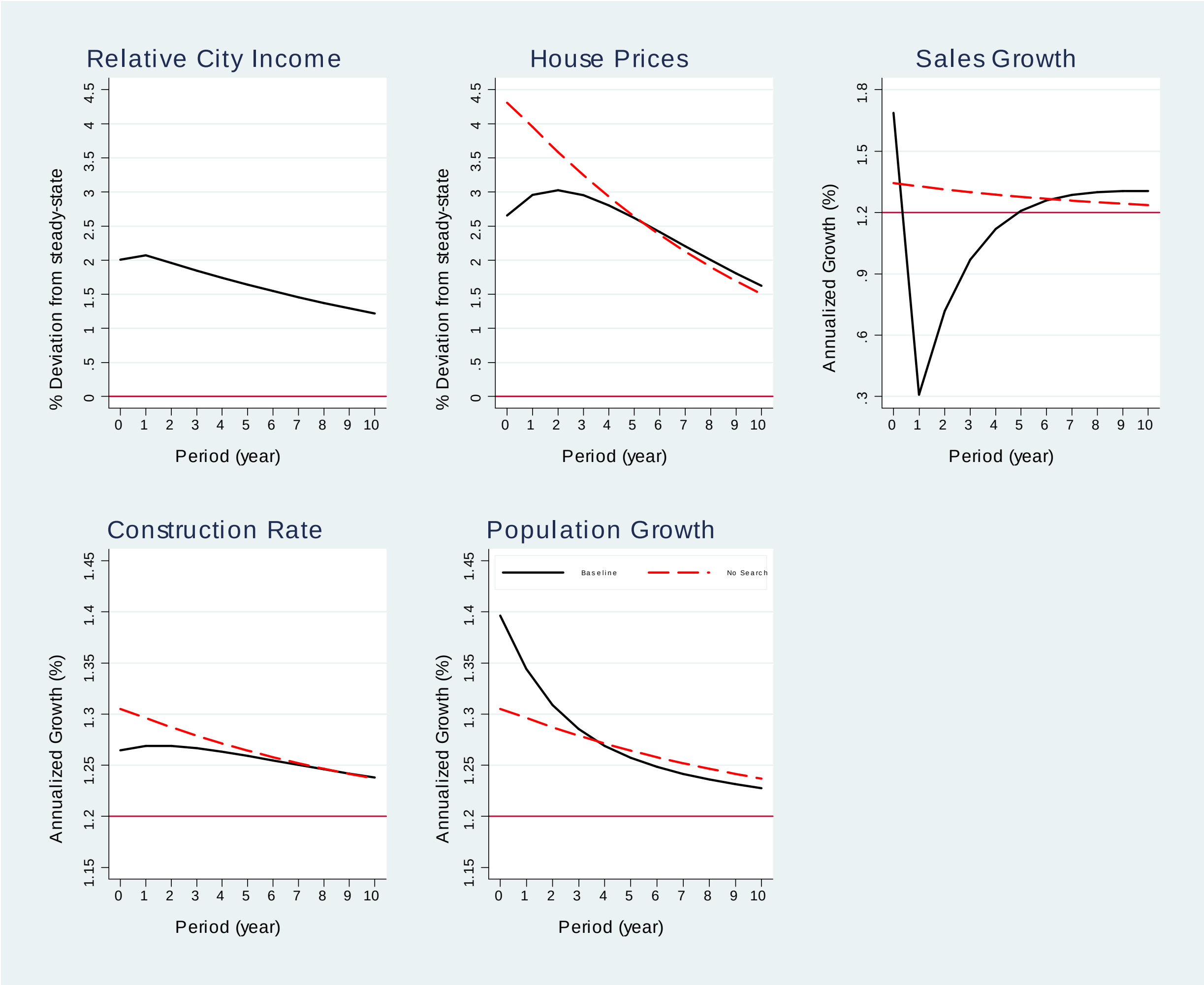


Figure 4: Impulse responses with and without search (1)

construction rate and the movements in sales growth in the search economy is the *illiquidity* of housing. To see this, consider Figure 5, which depicts in its five panels vacancies and market tightness, $\omega$, for the search economy, and rent, $r_t$, construction wages and construction employment for both economies.

Initially, an increase in the value of living in the city (due here to the income shock) generates an immediate increase in search activity as households enter and some begin searching for a house. Ignoring, for now, any response of the measure of vacant houses for sale, the ratio of buyers to sellers (*i.e.* tightness) increases, reducing the rate at which buyers find homes through the matching process. Although the growth in sales of existing homes jumps initially, the increase in tightness causes it to slow again almost immediately. The price of a house reflects in part its future resale value (as home-owners expect to sell the house eventually, when they leave the city). Thus, an increase in tightness, by raising the per-period probability of sale, increases both the value of a vacant house, and the transaction price.

Because newly-entering buyers are not all immediately matched with sellers, and because entry is persistent (owing to the persistence of the income shock), unmatched buyers "build up" in the market over time, generating future increases in both tightness and the rate at which houses sell in the search economy (see the two upper panels of Figure 5). As houses become more liquid over time and sell more quickly, their value increases and transaction prices increase further as well, resulting in persistent house price appreciation in response to the income shock.

Both the overall supply of housing and the allocation of houses between the rental market and vacancies for sale, respond to the income shock in ways affected by movements in housing liquidity. New entrants to the city require housing immediately. This causes the shifting of vacant houses into the rental market because the overall stock of housing units cannot respond instantaneously. Even if the equilibrium rent does not initially rise, or rises only slightly, owners of vacant houses are compensated for the temporary renting of their housing units by the return on houses associated with both expected future increases in house sale prices and lower future time on the market. An increased relative supply of rental housing keeps the rental rate from rising too rapidly and reinforces the continued entry of buyers which drives the subsequent price appreciation. This effect supports the underlying momentum in house prices. The increased value of a vacant house induces developers to build houses and leads to an increase in the housing stock. Moreover, momentum in the construction rate is generated through the same mechanism as momentum in the housing price; persistent growth of market tightness and thus reductions in the time required for a resident owner to sell a house.

Eventually, as *per capita* income reverts to its steady-state level, entry slows and the population growth rate returns to its trend. Increased construction lowers market tightness and causes both the value of a vacant house and the transaction price to return to their steady-state values. As may be seen, one failure of the model is that it generates too much persistence in the deviation of all the variables from their steady state paths when compared to the data. Once we approach ten years out, however, the confidence intervals associated with the empirical IRFs become quite large.

Momentum in both the construction rate and the house price is thus generated by movements in the liquidity of housing associated with the population flows that are caused by


Figure 5: Impulse response functions with and without search (2)

the income shock. The economy with no search exhibits no such liquidity effect and thus has very different dynamics. There, increased entry simply leads to a higher house price and increases in both the construction rate and house sales. All of these variables track the population growth rate and thus exhibit no momentum. In addition, rent (which here is paid only by those agents who do not derive utility from owning) behaves very differently in the two economies. Because in the no-search economy there are no vacant houses that can be shifted into the rental market, new entrants who do not want to buy houses bid up the rent immediately as this is necessary to induce developers to produce new rental housing. In contrast, for our baseline model with search, the rent actually falls initially because the anticipated growth in prices temporarily induces houses that were previously vacant-for-sale to be supplied to the rental market. Eventually, however, as permanent renters move in, growing demand counteracts this effect causing rents to rise.

## 6.2 Quantitative implications

As described above, using the constructed process for income and the linearized model, we generate sample paths for the key variables of the model and use these to construct annualized time series for the economy.[39] Moments form these series, along with the corresponding moments for the U.S. economy, are presented in Tables 7.1 and 7.2.

**Table 7.1: Volatilities and co-movements (models and 106 cities, 1981-2008)**

| Moment | US Cities | Baseline | No search |
|---|---|---|---|
| $\sigma_p/\sigma_y$ | 1.60 | 1.45 | 2.10 |
| $\sigma_s/\sigma_y$ | 1.32 | 1.32* | 0.70 |
| $\sigma_h/\sigma_y$ | 0.11 | 0.11* | 0.11* |
| $\sigma_n/\sigma_y$ | 0.17 | 0.17* | 0.11 |
| $\sigma_{py}$ | 0.76 | 0.98 | 0.98 |
| $\sigma_{sy}$ | 0.56 | 0.11 | 0.28 |
| $\sigma_{hy}$ | 0.54 | 0.26 | 0.36 |
| $\sigma_{ny}$ | 0.76 | 0.50 | 0.36 |
| $\sigma_{sp}$ | 0.02 | 0.07 | 0.28 |

Note: Values marked with a * are calibrated targets

First consider the moments in Table 7.1, which contains both the standard deviations of house price appreciation, housing stock growth, and population growth relative to that of local per capita income growth, and the correlations of those variables with local income growth. In the table, the first column reports the numbers from our empirical analysis in Section 2. The second column reports the results for our search model with the baseline calibration, and the third column results for the economy with no search. Table 7.2 contains the first four autocorrelation coefficients for price appreciation, sales growth, housing growth and population growth for the data and in the model.

The calibrated search model generates price volatility relative to income that is more than 80% of what is observed in the data. In contrast, the volatility generated by the model with

[39] Along a small fraction of sample paths, the buyer's matching probability, $\lambda$, exceeded unity for several periods. To address this we ran the simulation both with no constraints and with the constraint that $\lambda \leq 1$. This made no difference to our results.

no search is much higher. The search model is also able to account for a considerable amount of serial correlation in price appreciation, almost half, and most of that in sales growth. The model with search also does relatively well in terms of the rankings of volatility, correlation with income growth and serial correlation for the four variables. That is, it is consistent with the observation that price appreciation is the most volatile and most correlated with income growth, followed by population growth and then by construction rates and sales growth, whereas for persistence they are ranked in the opposite order.[40] The search model does, however, substantially understate the correlations of sale growth, construction and population growth with income growth and overstate the persistence of both population growth and construction. Finally, in contrast to the model with no search, the search economy correctly predicts the very low correlation of price appreciation and sales growth generated by city-specific income shocks.

**Table 7.2: Autocorrelations (106 cities, 1981-2008)**

| | Price Appreciation | | | | Sales Growth | | | |
|---|---|---|---|---|---|---|---|---|
| Moment | $\rho_1^p$ | $\rho_2^p$ | $\rho_3^p$ | $\rho_4^p$ | $\rho_1^s$ | $\rho_2^s$ | $\rho_3^s$ | $\rho_4^s$ |
| US Cities | 0.75 | 0.37 | 0.05 | -0.15 | 0.59 | 0.39 | 0.35 | 0.35 |
| Baseline | 0.34 | 0.07 | 0.01 | -0.02 | 0.52 | 0.18 | 0.04 | -0.03 |
| | Population Growth | | | | Construction Rate | | | |
| Moment | $\rho_1^n$ | $\rho_2^n$ | $\rho_3^n$ | $\rho_4^n$ | $\rho_1^h$ | $\rho_2^h$ | $\rho_3^h$ | $\rho_4^h$ |
| US Cities | 0.67 | 0.40 | 0.19 | 0.09 | 0.88 | 0.61 | 0.33 | 0.11 |
| Baseline | 0.84 | 0.66 | 0.54 | 0.45 | 0.96 | 0.90 | 0.84 | 0.77 |

In summary, it is clear that the baseline calibration understates somewhat both the relative volatility of house prices and its serial correlation observed in the data. Search, however, seems to be necessary to generate any price momentum at all. Moreover, the model with no search generates too much volatility in price appreciation and too little volatility in sales growth.

## 6.3 Additional quantitative implications

Our estimated VAR was limited to certain key variables largely because of the availability of data for as large a number of MSA's as possible for a reasonably long time span. Our model,

[40]For the model with no search, population growth and the construction rate are equal, since there are no vacancies.

however, also makes predictions for several other variables on which we have more limited data. Here we compare the predictions of the model for city-level wages and employment in the construction sector and for rents with their counterparts in the data. In each case data availability requires that we reduce the sample of cities and/or time periods relative to that on which we estimated the VAR. It turns out, however, that the inclusion of these other measures in VAR's estimated over more limited samples has little or no effect on the estimated properties of income, house prices, sales growth, construction, and population growth.

### 6.3.1 Construction Wages and Employment

Construction labour data is not available on a consistent basis for all the cities in our sample. We therefore drop the 10% of cities for which there are missing values and re-estimate the panel VAR with the inclusion of construction wages and employment. As the implications for co-movements among the other five variables remain largely unchanged, we focus here only on the construction variables. The upper two panels of Figure 6 depicts the impulse response function resulting from a shock to income, with the associated confidence intervals.[41] Interestingly and in accordance with the theory, construction wages follow a very similar pattern to house prices in Figure 1 while the IRF for construction labour looks very much like that for the construction rate.

**Table 8: Construction wages and employment (98 cities, 1981-2008)**

| | Construction Wage | | | | | | Construction Employment | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Moment | $\sigma_w/\sigma_y$ | $\sigma_{wy}$ | $\rho_1^w$ | $\rho_2^w$ | $\rho_3^w$ | $\rho_4^w$ | $\sigma_l/\sigma_y$ | $\sigma_{ly}$ | $\rho_1^l$ | $\rho_2^l$ | $\rho_3^l$ | $\rho_4^l$ |
| US Cities | 0.58 | 0.96 | 0.41 | 0.04 | -0.19 | -0.23 | 1.41 | 0.79 | 0.60 | 0.18 | -0.15 | -0.34 |
| Baseline | 1.63 | 0.97 | 0.33 | 0.06 | 0.01 | -0.02 | 2.61 | 0.97 | 0.33 | 0.06 | 0.01 | -0.02 |

Table 8 documents key moments for growth in the construction wage and employment in the data and the model. As before, the moments from the model are based on annual statistics generated by a quarterly simulation that uses a linearized version of the model. Clearly, the model overstates the volatility of both wages and employment. However, both in the data and the model, the volatility of employment exceeds the volatility of wages. This is consistent with the fact that the labour supply elasticity exceeds unity. The correlations

[41] It made no significant difference whether we subtracted construction earnings from overall income.

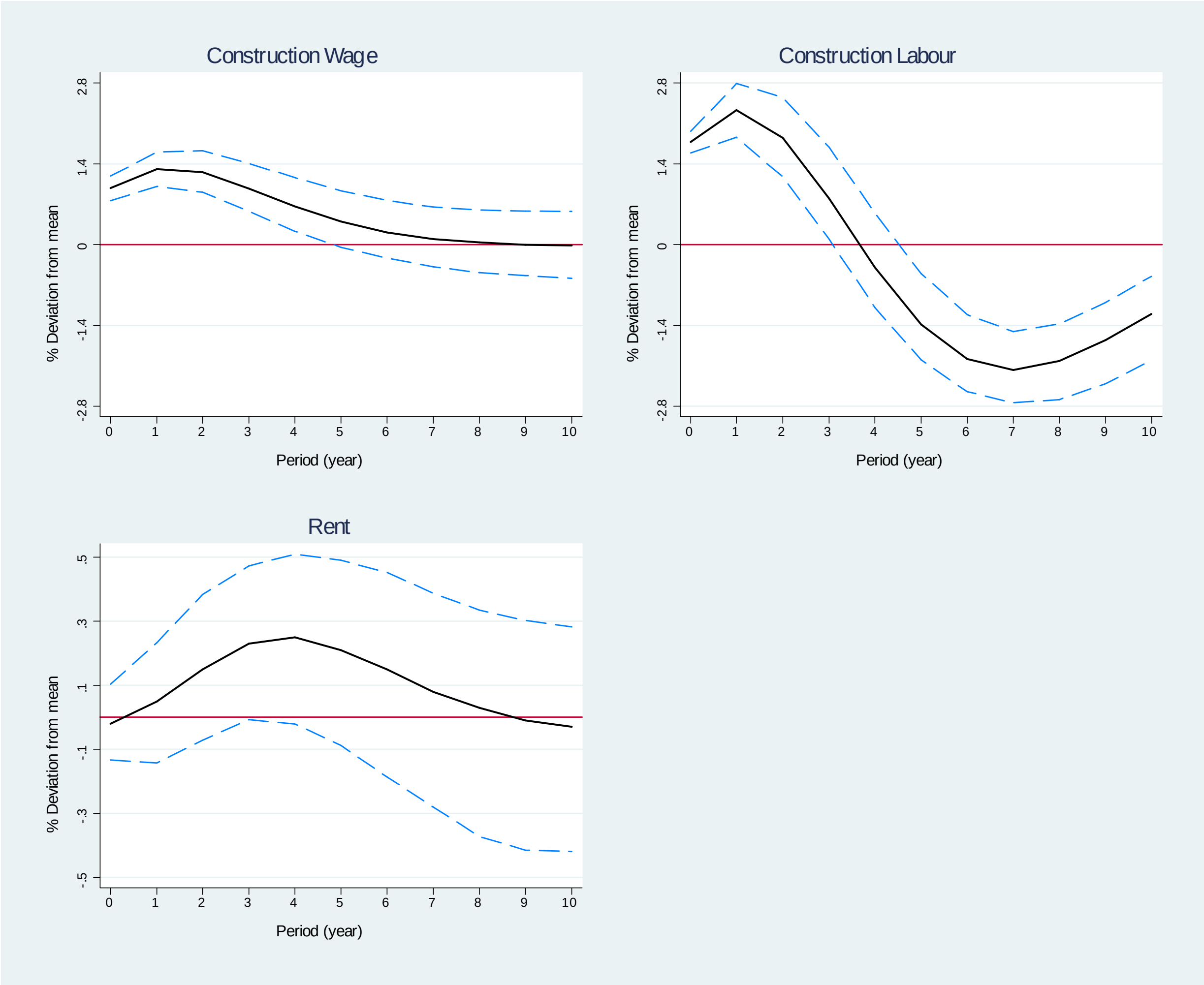


Figure 6: Empirical impulse response functions

of wages and employment with income and their autocorrelations are equal to each other in the model, by construction. The model understates the persistence in both variables, but captures their high correlation with income.

### 6.3.2 Rents

Panel data for MSA averages of (quality-controlled) rents over a reasonable time period appear to be unavailable. There is, however, data on "fair market rents" by MSA which is available on an annual basis going back to 1985 for the 106 cities in our sample. Here we use the adjusted data constructed by van Nieuwerbugh and Weill (2010) (see Appendix A). We re-estimate the panel VAR over the shorter time period with the inclusion of rents. One issue that must be dealt with is the fact that rents are commonly set for a year and may be difficult to adjust immediately in response to shocks. If we were to order rents before income in the VAR, however, this would effectively "force" there to be no initial response in rents to the income shock. Instead, we include rents at time $t+1$ ordered after income in the panel VAR and document the implications. Once again the results for the other variables are robust to these changes, so we focus on the implications for the rent variable.

The lower panel of Figure 6 illustrates the impulse response for rent together with the implied confidence interval. As predicted by the model, rents initially decline following the shock and then subsequently rise. The initial decline in rents in the data is, however, much smaller than that predicted by the model and is not statistically significant. Rents subsequently rise quite slowly for 4 years after the shock before mean-reverting, but again the confidence interval is very wide and includes zero.

In contrast to the construction labour market variables, the quantitative predictions of the model for rent do not match the data well. Although the persistence documented in Table 9 is similar, both the volatility of rent growth and its correlation with income growth are an order of magnitude lower in the data then is predicted by the model. Of course, the correspondence is even worse for the model with no search: volatility is much higher and the correlation with income is perfect. Moreover, the persistence is much lower.

**Table 9: Growth in Fair Market Rents (106 cities, 1985-2008)**

| Moment | $\sigma_r/\sigma_y$ | $\sigma_{ry}$ | $\rho_1^r$ | $\rho_2^r$ | $\rho_3^r$ | $\rho_4^r$ |
|---|---|---|---|---|---|---|
| US Cities | 0.09 | 0.01 | 0.75 | 0.28 | -0.15 | -0.39 |
| Baseline | 5.50 | 0.79 | 0.68 | 0.35 | 0.17 | 0.06 |

It is hard to know what to make of the comparison of rents between the model and the data. The fair market rent data may not provide an accurate measure of actual rents in a given city. Moreover, the model assumes that all rents are re-set every quarter, whereas actual rents are almost certainly adjusted less frequently. In many cases rents are adjusted only annually, say when a new lease is signed, or when a new tenant moves in. In this sense the average rent measured in the data may move much more sluggishly than the marginal rent paid by a new tenant, and this is what is picked up as rent in the model.

Note that the insensitivity of rents to city-level income movements in the short run is consistent with the empirical results of Saiz (2007). Based on an instrumental variables approach, he finds that MSA level incomes have no significant impact on rents, whereas their impact on house prices is much larger and significant. In the longer run he finds a stronger relationship between rents and incomes suggesting that rents adjust only very sluggishly in response to income shocks.

#### 6.3.3 Demographic shocks

It is straightforward to derive the implications of direct shocks to the population of cities (i.e. movements that are not driven by income) from a our panel VAR. The interpretation of such a shock, however, depends on exactly how it is modeled. For example, we could think of shocks to the $G(\cdot)$ function as driving entry. Alternatively, we could think of an unobserved shock to the utility associated with a particular city that induces entry endogenously (e.g. amenities). For the sake of brevity, it is useful to observe that the impulse response functions associated with "population shocks" coming from our panel VAR are qualitatively similar to those generated by income shocks. Therefore if we were to introduce an additive utility shocks process in to the model with features similar to the current income shocks process, we could generate reasonably similar IRFs. Of course such an exercise would be undisciplined since the utility shock process is unobserved. This is why we have focussed on the role of income shocks, the properties of which can be directly estimated.

### 6.4 Alternative calibrations

We now depart from the baseline calibration of Section 5 and consider the sensitivity of our results to changes in the values of several parameters. Table 8 reports the implications of alternative choices of the specified parameters for the relative volatility and the first-order autocorrelation of price appreciation . In each case the targets listed in Table 4 remain fixed. Thus, other parameters (i.e. $\psi$, $m$, $z^h$, $\delta$, $\kappa$, $\nu$ and $\alpha$) are adjusted in each case to match the targets. Each case in Table 10 therefore represents an alternative calibration of the model.

Perhaps not surprisingly, increasing either the elasticity of new construction supply or the elasticity of land supply results in a decrease in price volatility and an increase in momentum. We have chosen the alternative values of these parameters to be at the extremes of the range of typical estimates. As may be seen within this range, these moments are much more sensitive to new construction supply elasticity than land supply elasticity.[42] Indeed for low

[42] This is true even though the range of labour supply elasticities considered is proportionately larger than that of land supply elasticities.

values of $\eta$ we obtain price volatilities that equal or exceed those observed in the data. This, however, comes at the expense of a reduction in momentum, although it remains positive.

The trade-off between volatility and momentum may be seen for all the parameter changes considered in Table 10. Directly increasing the elasticity of entry, for example, implies a greater responsiveness of new entrants to current market conditions in the city and correspondingly less of a lag in entry. Consequently, prices become increasingly volatile and price momentum declines. Of course, when we adjust this parameter we no longer match the relative volatility of population growth.

**Table 10: Volatility and Persistence of Price Appreciation: Sensitivity Results**

| Moment | Baseline Calibration | New housing supply elasticity | | Entry (demand) elasticity | | Matching Elasticity | |
|---|---|---|---|---|---|---|---|
| | | $\eta = 0.1$ | $\eta = 10$ | $\alpha = 3$ | $\alpha = 20$ | $\delta = .5$ | $\delta = .9$ |
| $\sigma_p/\sigma_y$ | 1.45 | 2.92 | 0.83 | 0.48 | 2.63 | 1.59 | 1.28 |
| $\rho_1^p$ | 0.34 | 0.27 | 0.44 | 0.76 | 0.24 | 0.32 | 0.38 |

| Moment | Land supply elasticity | | Vacancy Rate | | Homeowner exit probability | | Housing Utility | |
|---|---|---|---|---|---|---|---|---|
| | $\xi = .5$ | $\xi = 5$ | $v = .01$ | $v = .03$ | $\pi_n = .004$ | $\pi_n = .012$ | $z^H = .01$ | $z^H = .04$ |
| $\sigma_p/\sigma_y$ | 1.83 | 1.30 | 1.75 | 1.22 | 1.46 | 1.39 | 0.89 | 1.56 |
| $\rho_1^p$ | 0.31 | 0.36 | 0.29 | 0.39 | 0.29 | 0.40 | 0.66 | 0.31 |

Targeting a higher steady-state vacancy rate of 3% implies a less effective matching process in the housing market, $\kappa$. As a result, it takes longer for households to find houses and market tightness grows to a higher level as buyers exit the pool of searchers more slowly. This tends to generate more momentum and less volatility in prices. When homeowners are more likely to exit the city (i.e. when $\pi_n$ increases), they put more weight on future market conditions. If these conditions are expected to improve due to gradual entry, the persistence of current price growth increases in anticipation.

When we divorce housing utility, $z^H$, from the cost of maintenance, so that (42) no longer holds, we find that decreasing $z^H$ raises price momentum and lowers volatility. The reason is that, in order to maintain the targets for the steady-state house price and vacancy rate, the decrease in $z^H$ necessitates a reduction in the buyers' share of the surplus in house transactions. As a consequence, the house price becomes increasingly sensitive to future market conditions and the slow increase in the likelihood of sale translates into more gradual price growth.

# 7 Robustness

We now consider three alternative environments to assess further the robustness of our findings. Qualitatively, none of these affect our results significantly and in this sense we find our results to be very robust. Quantitatively, we find that our findings with respect to both volatility and the co-movements among the variables that we have considered are very robust. With regard to the degree of price momentum, however, our results are to some extent sensitive to fluctuations in the share of the surplus accruing to buyers and sellers in housing transactions.

## 7.1 Mismatched owners remain in their houses

In our basic model, we assumed that mismatched owners put their houses up for sale immediately and become renters. This is, however, a stark choice in the following sense:

**Proposition 2.** *In equilibrium, mismatched owners are indifferent between the following two arrangements:*

*(1) putting up their house for sale or rent immediately and renting while searching;*

*(2) remaining in their current house while searching, then putting their vacant house up for sale once they are matched with a new one.*

Suppose instead that they remain in their houses until they find a new house, then put their vacant house up for sale. Since owners who become mismatched are indifferent between the two alternatives, their values remain unchanged. [43] Let $\tilde{n}_t$ denote mismatched owners who remain in their (owned) homes. Then the flows of households between states is now described by (32) and

$$
\begin{aligned}
(1+\mu)\tilde{n}_t &= \theta(1-\pi_n)n_{t-1} + [1-\lambda(\omega_{t-1})]\,\tilde{n}_{t-1} && (46)\\
(1+\mu)b_t &= \psi\mu G\left(\bar{W}_t\right) + [1-\lambda(\omega_{t-1})]\,b_{t-1} && (47)\\
(1+\mu)n_t &= (1-\theta)(1-\pi_n)n_{t-1} + \lambda(\omega_{t-1})\left(b_{t-1}+\tilde{n}_{t-1}\right) && (48)
\end{aligned}
$$

Market tightness is given by

$$
\omega_t = \frac{b_t + \tilde{n}_t}{h_t - b_t - \tilde{n}_t - f_t - n_t} \tag{49}
$$

[43] If there were a small moving cost involved in temporarily moving into a rented home, mismatched owners would strictly prefer to remain in their own homes while searching.

and the housing stock evolves according to

$$(1+\mu)h_{t+1} = h_t + \phi^{\eta}\left(n_t + \tilde{n}_t + b_t + f_t\right)\left(\beta E_t V_{t+1} - \bar{q}\right)^{\eta}. \tag{50}$$

When we change the model in this way and retain the same calibration targets as before we find that our results hardly change. If the parameters of the model were held constant, this change would lead to a somewhat tighter housing market. In this case, however, the implied steady-state fraction of renters would be too low compared to the data. To match this target, we would therefore increase the fraction of entering households that become perpetual renters rather than searchers (i.e. decrease $\psi$).[44] Following this this re-calibration, market tightness remains much as in our baseline case.

## 7.2 Endogenous exit

In the baseline model, we assume the exit rate of home-owners is exogenous. We now consider the implications of allowing the exit rate to be determined and vary endogenously. Specifically, suppose those homeowners who have the opportunity to exit the city receive a random draw from a distribution of continuation values. Only those who receive draws that exceed their value of staying, $J_t$, actually choose to exit. We suppose that these draws come from a uniform distribution of continuation values with support $[0, \bar{Z}]$. The implied value of being a homeowner is then given by

$$\begin{aligned} J_t &= u_t^H + \beta\pi_n E_t\left(Z_{t+1}^* + e_{t+1}V_{t+1}\right) + \theta\beta E_t(1-\pi_n e_{t+1})\left(W_{t+1} + V_{t+1}\right) \\ &\quad + \beta(1-\theta)E_t(1-\pi_n e_{t+1})J_{t+1}, \end{aligned} \tag{51}$$

where $Z_t^* = \left(\bar{Z}^2 - J_t^2\right)/2\bar{Z}$ and $e_t = 1 - J_t/\bar{Z}$ is the exit probability conditional on having an opportunity. The extended model is set up and calibrated so that the steady-state is the same as before. This includes calibrating $\bar{Z}$ so that the expected exit value is the same as it was in the baseline model. The relevant elasticities are re-calibrated to match the volatilities of population, construction and sales growth.

In this case, the resulting dynamics remain qualitatively unchanged. Quantitatively, price volatility is slightly higher and momentum slightly lower than in baseline case. This reflects the fact that in response to a positive income shock, the exit rate initially declines slightly, then gradually returns to its steady-state. The overall effect, however, is small. This example is somewhat arbitrary in the sense that a uniform distribution of continuation

[44] Equivalently, we could lower the rate at which renters become searchers (see footnote 19).

values is assumed. Nevertheless, it does illustrate that it is possible to extend the model in this way without changing the main results.

### 7.3 Competitive search

Under competitive search we suppose that the housing market consists of a variety of sub-markets. Each sub-market is characterized by a pair, $(\omega_t, P_t)$, where $P_t$ is the price of a house in the sub-market and $\omega_t$ is the *tightness* of the sub-market. Search is competitive in the sense that all buyers and sellers take the prices and tightness of all sub-markets as given, and then decide which *single* sub-market to enter and search for a trade. There is no cost to entering any sub-market.

By entering sub-market $(\omega_t, P_t)$, a seller sells a house at $P_t$ with probability $\gamma(\omega_t)$. The seller chooses to enter a sub-market that maximizes his/her expected return. It follows that the value of a vacant house for sale satisfies

$$V_t = \max_{(\omega_t, P_t)} \left\{ \gamma(\omega_t) P_t + [1 - \gamma(\omega_t)] \beta E_t \tilde{V}_{t+1} \right\}. \tag{52}$$

Free entry of sellers implies that all active sub-markets (*i.e.* sub-markets with $\lambda, \gamma \in (0,1)$) in equilibrium must offer the sellers the same payoff $V_t$, although $(\omega_t, P_t)$ varies across sub-markets. It follows that the relationship between the listed price and market tightness that must be satisfied by all active sub-markets:

$$\gamma(\omega_t(P_t)) = \frac{V_t - \beta E_t \tilde{V}_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}}. \tag{53}$$

Thus, it is sufficient to index sub-markets by the posted price $P_t$ alone.

Buyers also decide in each period which sub-market to enter. The value of being a buyer $W_t$ is therefore given by

$$W_t = u_t^R + \max_{P_t} \left\{ \lambda(\omega_t(P_t)) (\beta E_t J_{t+1} - P_t) + [1 - \lambda(\omega_t(P_t))] \beta E_t W_{t+1} \right\}. \tag{54}$$

In equilibrium, the set of active sub-markets is *complete* in the sense that there is no other sub-market that could improve the welfare of any buyer or seller.

Let $\epsilon(\omega_t)$ denote the elasticity of the measure of matches with respect to the measure of buyers. We then have the following proposition:

**Proposition 3.** *In a competitive search equilibrium, there is only one active sub-market. In this market, the share of the surplus from house transactions that accrues to the buyer is* *equal to the elasticity of the measure of matches with respect to the measure of buyers:*[45]

$$\chi(\omega_t) = \epsilon(\omega_t). \tag{55}$$

Therefore, the implication of competitive search in this model is that the respective shares of the trade surplus accruing to buyers and sellers in a transaction depends on the tightness of the market. In the case of a Cobb-Douglas matching function, this amounts to imposing the restriction that $\delta = \chi$. When we impose this restriction in our calibration scheme, it is no longer possible to match the volatility of sales growth. Moreover, we are unable to to find a combination of $\alpha$ and $\eta$ such that we match both the relative volatilities of construction and population growth. We therefore set $\eta = 1.6$ as in our baseline calibration and choose $\alpha = 5.5$ to match $\sigma_h/\sigma_y$. Under this calibration scheme, the competitive search model generates qualitatively similar price dynamics to the baseline case. Quantitatively, however, house prices are more volatile and exhibit less momentum.

In contrast to the random search model with Nash Bargaining, the nature of the matching function now matters because the bargaining parameter depends on market tightness. To explore this issue, we consider an alternative matching function for which the equilibrium shares of the surplus received by the buyers and sellers are not constant. Specifically, consider

$$\mathcal{M}(B, S) = S\upsilon(1 - e^{-\tau \frac{B}{S}}). \tag{56}$$

If $\tau = 1$, the matching probabilities are equivalent to the "urn-ball" matching process assumed by Diaz and Jerez (2012). Here we consider a somewhat more general form in order to calibrate the model to the same targets as for our baseline calibration above. This generalization could be motivated along the lines of Albrecht, Gauthier, and Vroman (2003), where $\tau$ denotes the average number of applications to purchase made per period and $\upsilon$ indexes the effort required to process each application. Given the other parameters of our baseline calibration, the matching function parameter values needed to achieve the same targets as above are $\upsilon = 0.78$ and $\tau = 3.73$.

The surplus accruing to the buyer for this matching function is

$$\chi(\omega) = \epsilon(\omega) = \frac{\tau\omega}{e^{\tau\omega} - 1}, \tag{57}$$

which is decreasing in market tightness $\omega$. That is, as the ratio of buyers to sellers increases, the share received by buyers falls.

[45] This result is a special case of that derived in Moen (1997).

Figure 7 illustrates the effect of a shock to general earnings on house prices for each of the matching functions. Clearly, the form of the matching function has a significant effect on price momentum, and this can be traced to the effect of an increase in local earnings on the initial response of prices and the extent of entry. In the urn-ball matching case, the share of the surplus received by the buyer falls as tightness rises. Thus, the initial price increase in prices is greater, and this discourages entry as can be seen in the response of tightness. Since tightness responds less, prices peak earlier and return to their steady-state level faster than with Cobb-Douglas matching. As a result, price volatility is higher and momentum lower with the urn-ball matching function.

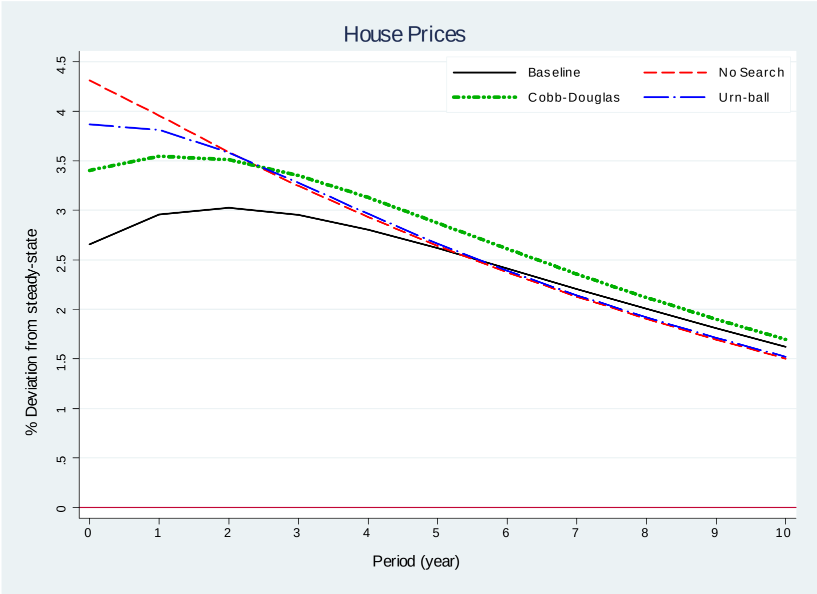


Figure 7: IRFs for Price under Competitive Search with Alternative Matching Functions

# 8 Concluding Remarks

This paper makes two main contributions. First, we provide a parsimonious characterization of the impact of relative income shocks across U.S. cities on the short-run dynamics of average house prices, sales of existing homes, construction and population growth. Specifically we estimate a panel VAR with city-level fixed effects and use it to isolate the impact of relative income shocks by making structural assumptions consistent with our theory. In particular, our estimates are consistent with previous findings (*e.g.* those of Glaeser et al. 2010) that house price appreciation exhibits substantial serial correlation in the short term and long-run mean reversion. Moreover, we find that the volatility of house price movements that occur in response to income shocks is high relative to the volatility of local incomes. City level population growth responds quite quickly in response to income shocks, whereas construction rates tend to be sluggish initially. Sales growth is volatile, slowing quickly after an initial peak before rising again in the medium term.

Our second main contribution is to build a model that helps to understand the joint dynamics of income, house prices, sales, construction and population growth. To do this, we introduce time-consuming search and matching into a dynamic model of housing markets with endogenous entry and construction. Three key features of the model are (1) that it takes time for potential buyers to match with a house they want, and the length of this time depends on market conditions; (2) that home buyers foresee that they will eventually sell; and (3) that unoccupied housing can be rented temporarily to new entrants who are searching for a home to own.

In response to a persistent, relative increase in local income, entry into the market rises persistently. Because it takes time to match and new houses take time to build, the ratio of potential buyers to houses for sale rise slowly over time. Initially, therefore, the liquidity of houses is expected to rise over time. Since this causes the re-sale value of housing to grow and because current transactions prices in part reflect this, the current transaction price grows in anticipation. Higher home values induce increased construction so that the buyer–seller ratio, and hence housing liquidity, eventually fall. After initially growing for several periods, therefore, current house prices fall.

The fact that households can rent while searching implies that they can obtain the relative gains from living in the city without buying a house. Although an increase in local income therefore also increases the demand for rental housing, owners of unoccupied housing have an incentive to rent out their houses and delay selling them, if they expect prices to rise. Consequently entry continues to grow and is not immediately stemmed by a sharp rise in rents.

We calibrate the model so that its steady-state matches several long-run averages in U.S. data and allow the volatilities of population growth, construction, and sales growth determine three key elasticities which are indeterminate in the steady-state. We find that the calibrated version of our model captures qualitatively the observed dynamics of both house prices quite well. In particular, the model generates serial correlation in price appreciation and relatively high volatility in the long run. Quantitatively, the calibrated model accounts for more than

80% of the variance of house prices associated with local income shocks, and nearly half of the first-order autocorrelation of price growth. Moreover, the search model improves both qualitatively and quantitatively on one with no search along multiple dimensions.

Our objective in this paper has been to assess the extent to which search and matching in the housing market can help us to understand the relationship between city-level housing price dynamics and city-level fundamentals. The modelling choices we have made are intended to allow us to focus on these issues in a tractable and parsimonious fashion and to permit us to quantify the key mechanisms at play. Our model could, however, be generalized in various directions to study a number of other issues associated with housing markets. In particular, one could allow for heterogeneity both in incomes and housing quality and consider the role of income inequality in determining housing price dispersion. Alternatively, one could allow for heterogeneity in housing supply conditions across cities and assess the performance of the model on a city-by-city basis. We leave these issues for future research.

# Appendix A: Data appendix

This appendix provides details on data sources, definitions and calculations. Our unit of observation is a core-based statistical area (metropolitan statistical area or MSA). We use the 2006 MSA definitions. Our sample consists of 106 MSAs from 1980 to 2008.

**Local incomes:** We define local incomes as the total income from all sources. Our MSA level data are from the Regional Economic Accounts compiled by the Bureau of Economic Analysis (BEA, Table CA01). We could have subtracted construction earnings because they are endogenous in our model. Because of missing values in the construction earnings data for some cities, however, this would have reduced our panel size by about 10%. For the remaining cities, whether or not we make this adjustment makes no significant difference to the empirical results. We could also have defined local income as earnings before taxes and transfers. We chose not to because we would expect the incentives to move to a given location to depend on total income. However, when we estimate the panel VAR using this definition instead, the results were qualitatively similar.

**House prices:** Following van Niewenburgh and Weil (2010), we form a time series of home prices for each city by combining level information from the 2000 Census with time series information from the FHFA. From the 2000 Census, we use nominal home values for the median single-family home. From the FHFA we use the Home Price Index (HPI) from 1980 to 2008. The HPI is a repeat-sales index for single family properties purchased or refinanced with a mortgage below the conforming loan limit. As a repeat-sale index, it is a constant quality house price index. In contrast to Van Niewenburgh and Weil (2010), we combine prices for MSA divisions into those for MSAs by using population–weighted averages of the division level prices. We need to do this because the housing stock data (described below) can only be constructed using permits at the MSA level.

**Sales of existing houses:** We obtained quarterly estimates of the sales of existing houses for each city from Moody's Analytics (www.economy.com). Annual sales were computed as the sum of quarterly sales over the year.

**Populations:** City populations are taken from the BEAs Regional Economic Accounts (Table C02). Throughout we assume that city populations are proportional to the number of households. Although there has been a general decline in people per household in the U.S., this is an economy–wide trend that is removed after controlling for time–fixed effects.

**Housing Stocks:** We form a time series for housing stocks for each city by combining information from the 2000 Census with times series information from the U.S. Department of Housing and Urban Development (HUD). From the 2000 Census, we use the estimated number of housing units. This data was only available at the county level, so we summed across the counties within the relevant MSAs. From HUD we used annual permits issued for each city from 1980 to 2008. According to the U.S. Census Bureau, approximately 97.5% of permits issued each year translate into housing starts, 96% of which are completed. We therefore constructed housing stocks $H_t$ according to $H_{t+1} = H_t + 0.936 \times \text{Permits}_t$.

**Construction Employment:** Construction employment is taken from the BEAs Regional Economic Accounts (Table C25).

**Construction Wages:** These are computed as construction earnings divided by construction employment. The earnings data is taken from the BEAs Regional Economic Accounts (Table C06).

**Rents:** Rent data is taken from the data set constructed by van Niewerburgh and Weill (2010). They start with rental data from the Fair Market Rents database (FMR), published annually by HUD. The FMR are gross rents, including utilities, and are used to determine payment amounts in various government housing subsidy programs. The FMR reports the 40th (or sometimes the 45th or 50th) percentile of the housing rent distribution, the dollar amount below which 40 percent of the standard-quality rental housing units are rented in a given area. The 40th percentile rent is drawn from the distribution of rents of all units occupied by recent movers, who moved to their present residence within the past 15 months. The FMR data are reported for finer regions than the metropolitan areas used by the BEA. Van Niewerburgh and Weill (2010) aggregate to the MSA level using population weighted averages. Van Niewerburgh and Weill (2010) also adjust for the fact that the reported rent percentile actually changes over time (see their appendix D.3 for details).

# Appendix B: Math appendix

**The household's optimization problem:** This can be expressed as

$$\max_{c_t, l_t} E_t \sum_{t=0}^{\infty} \beta^t U_t(c_t, l_t, z_t) \qquad \text{s.t.} \qquad E_t \sum_{t=0}^{\infty} \beta^t c_t \le E_t \sum_{t=0}^{\infty} \beta^t \left[y_t + w_t l_t - \Omega_t\right] \tag{58}$$

where $\Omega_t$ denotes the net value of all housing transactions. It follows their dynamic optimization problem is equivalent to

$$\max_{l_t} E_t \sum_{t=0}^{\infty} \beta^t \left[y_t + w_t l_t - v(l_t) + z_t - \Omega_t\right] \tag{59}$$

The solution to the (static) household construction labour supply problem yields (13). Hence,

$$w_t l(w_t) - v\left(l(w_t)\right) = \zeta w_t^{1+\eta} - \frac{\zeta^{\frac{1+\eta}{\eta}} w_t^{1+\eta}}{\zeta^{\frac{1}{\eta}}\left(1 + \frac{1}{\eta}\right)} = \frac{\zeta w_t^{1+\eta}}{1+\eta}. \tag{60}$$

**The deterministic steady-state:** In a steady-state (32) implies that the normalized measure of renters is

$$f^* = \frac{(1-\psi)\mu G(\bar{W}^*)}{\mu + \pi_f}. \tag{61}$$

Similarly, from (33) the measure of buyers each period satisfies

$$b^* = \frac{\psi \mu G(\bar{W}^*)}{\mu + \lambda(\omega^*) - \frac{\theta(1-\pi_n)\lambda(\omega^*)}{\mu + \pi_n + \theta(1-\pi_n)}}. \tag{62}$$

Equation (34) implies that the steady-state fraction of the total population located in the city is

$$n^* = \frac{\lambda(\omega^*)}{\mu + \pi_n + \theta(1-\pi_n)} b^*, \tag{63}$$

and (35) yields that the housing stock per capita satisfies

$$h^* = \frac{\phi^\eta \left(n^* + b^* + f^*\right)}{\mu} \left(\beta V^* - \bar{q}\right)^\eta. \tag{64}$$

where $\bar{q} = \bar{c} F^{-1}(h^*/\hat{h})$.

In the steady-state, the values of owners, buyers and vacant houses, the house price and rent, must satisfy the following set of equations:

$$\begin{aligned}
J^* &= \bar{u}^H + \pi_n \beta Z + \pi_n \beta V^* + (1-\pi_n)\theta\beta\left(W^* + V^*\right) + (1-\pi_n)(1-\theta)\beta J^* && (65)\\
W^* &= \bar{u}^R + \lambda(\omega^*)(\beta J^* - P^*) + (1-\lambda(\omega^*))\beta W^* && (66)\\
V^* &= \gamma(\omega^*) P^* + (1-\gamma(\omega^*))\beta V^* && (67)\\
P^* &= (1-\chi)\beta\left(J^* - W^*\right) + \chi\beta V^* && (68)\\
r^* &= m + \gamma(\omega^*)\left(P^* - \beta V^*\right) && (69)\\
W_f^* &= \bar{u}^R + \pi_f \beta Z + (1-\pi_f)\beta W_f^* && (70)\\
\bar{W}^* &= \psi W^* + (1-\psi) W_f^*, && (71)
\end{aligned}$$

where $\bar{u}^H = \bar{y} + x(w^*) + z^H - m$ and $\bar{u}^R = \bar{y} + x(w^*) - r^*$ and we assume that when a household exits the city, its expected continuation value is the steady-state value of being a renter: $Z = \bar{u}_R/(1-\beta)$. The first five equations of this system can be solved for $J^*$, $W^*$, $V^*$, $P^*$ and $r^*$. Then, the last two equations can be used to determine $\bar{W}^*$ and $W_f^*$.

**Proof of Lemma 1.** Re-arranging (68) yields

$$P^* = (1-s)\beta\left(J^* - W^* - V^*\right) + \beta V^*. \tag{72}$$

Substituting the above into (65), (66), (67) and (69) yield

$$\begin{aligned}
V^* &= \gamma(\omega^*)(1-s)\beta\left(J^* - W^* - V^*\right) + \beta V^* && (73)\\
W^* &= Z + \frac{\lambda(\omega^*)s}{1-\beta}\beta\left(J^* - W^* - V^*\right) && (74)\\
r^* &= m + \gamma(\omega^*)(1-s)\beta\left(J^* - W^* - V^*\right) && (75)
\end{aligned}$$

and

$$\begin{aligned}
J^* - W^* - V^* = {} & \bar{u}^H + \pi_n \beta Z + \pi_n \beta V^* \\
& + (1-\pi_n)\theta\beta\left(W^* + V^*\right) + (1-\pi_n)(1-\theta)\beta J^* \\
& - \bar{u}^R - \lambda(\omega^*)s\beta\left(J^* - W^* - V^*\right) - \beta W \\
& - \gamma(\omega^*)(1-\chi)\beta\left(J^* - W^* - V^*\right) - \beta V^*. && (76)
\end{aligned}$$

Given (16), (19) and (75), we have

$$\bar{u}^H - \bar{u}^R = z^H + \gamma(\omega^*)(1-\chi)\beta\left(J^* - W^* - V^*\right). \tag{77}$$

The above two equations yield

$$J^* - W^* - V^* = z^H - \pi_n \beta (W^* - Z) + [(1-\pi_n)(1-\theta) - \lambda(\omega^*)\chi] \beta (J^* - W^* - V^*).$$

Using (74) to eliminate $(W^* - Z)$ in the right-hand side of the above equation yields

$$J^* - W^* - V^* = z^H + \left[(1-\pi_n)(1-\theta) - \left(\frac{1-\beta+\pi_n\beta}{1-\beta}\right)\lambda(\omega^*)\chi\right] \beta (J^* - W^* - V^*),$$

which implies (38). **QED**

**Proof of Lemma 2.** First use (36) and (37) to derive

$$h^* = \frac{b^*}{\omega_t} + b^* + n^* + f^*. \tag{78}$$

Then use the above and (64) to eliminate $h^*$:

$$\frac{b^*}{\omega_t} + b^* + n^* + f^* = \frac{\phi^\eta (n^* + b^* + f)}{\mu} (\beta V^* - \bar{q})^\eta. \tag{79}$$

It follows that (39) can be obtained by substituting (61), (62) and (63) into the above. It follows that $V^S(\omega^*)$ is strictly decreasing in $\omega^*$ because $\gamma'(\omega) > 0$ from Assumption 1. **QED**

**Proof of Lemma 3.** Substituting (38) into (73) yields (40). Also recall from Assumption 1 that $\lambda'(\omega) < 0$ and $\gamma'(\omega) > 0$. It follows that the right-hand side of (40) is increasing in $\omega^*$. **QED**

**Proof of Proposition 1.** Because $V^S(\omega)$ is decreasing in $\omega$ and $V^D(\omega)$ is increasing in $\omega^*$, a SSE must be unique if it exists. Existence of an interior SSE basically requires that the curves intersect at a value of $\omega \in (\underline{\omega}, \bar{\omega})$. That is if $V^D(\underline{\omega}) < V^S(\underline{\omega})$ and $V^D(\bar{\omega}) > V^S(\bar{\omega})$. Recall from Assumption 1 that $\lambda(\bar{\omega}) \geq 0$, $\gamma(\underline{\omega}) \geq 0$, $\lambda(\underline{\omega}) = \gamma(\bar{\omega}) = 1$, $\lambda'(\omega) < 0$ and $\gamma'(\omega) > 0$. It follows that necessary and sufficient conditions on the parameters are that

$$\frac{\gamma(\underline{\omega})(1-\chi)\beta z^H}{(1-\beta)[1-\beta(1-\theta)(1-\pi_n)] + (1-\beta+\pi_n\beta)\beta s} < \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\left(1+\frac{\psi(\mu+\pi_f)}{A\gamma(\underline{\omega}) + B\underline{\omega}}\right)\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta} \tag{80}$$

and

$$\frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\left(1+\frac{\psi(\mu+\pi_f)}{A + B\bar{\omega}}\right)\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta} > \frac{(1-\chi)\beta z^H}{(1-\beta)[1-\beta(1-\theta)(1-\pi_n)] + (1-\beta+\pi_n\beta)\beta\lambda(\bar{\omega})}. \tag{81}$$

In the Cobb-Douglas case, $\underline{\omega} = \kappa^{\frac{1}{1-\delta}}$ and $\bar{\omega} = \kappa^{-\frac{1}{\delta}}$, so that $\gamma\left(\underline{\omega}\right) = \kappa^{\frac{1}{1-\delta}}$ and $\lambda\left(\bar{\omega}\right) = \kappa^{\frac{1}{\delta}}$. **QED**

**Proof of Proposition 2.** The value of being a mis-matched owner who remains in their house while they search for a new one is given by

$$\tilde{J}_t = y_t + x_t - m + \lambda(\omega_t)\left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) + (1-\lambda(\omega_t))\beta E_t \tilde{J}_{t+1}. \tag{82}$$

The value of becoming a renter immediately and putting the vacant house up for sale is given by

$$\begin{aligned} W_t + V_t &= u_t^R + \lambda(\omega_t)\left(\beta E_t J_{t+1} - P_t\right) \\ &\quad + (1-\lambda(\omega_t))\beta E_t W_{t+1} + \gamma(\omega_t) P_t + (1-\gamma(\omega_t))\beta E_t V_{t+1} \\ &= u_t^R + \lambda(\omega_t)\left(\beta E_t J_{t+1} - P_t\right) \\ &\quad + (1-\lambda(\omega_t))\beta E_t W_{t+1} + \gamma(\omega_t)(1-\chi)\beta E_t\left(J_{t+1} - W_{t+1}\right) \\ &\quad + \gamma(\omega_t)\chi\beta E_t V_{t+1} + (1-\gamma(\omega_t))\beta E_t V_{t+1} \\ &= y_t + x_t - r_t + \lambda(\omega_t)\left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) \\ &\quad + \gamma(\omega_t)(1-\chi))\beta E_t\left(J_{t+1} - W_{t+1} - V_{t+1}\right) \\ &\quad + (1-\lambda(\omega_t))\beta\left(E_t W_{t+1} + E_t V_{t+1}\right). \end{aligned} \tag{83}$$

Given (30), the above implies that

$$\begin{aligned} W_t + V_t &= y_t + x_t - m + \lambda(\omega_t)\left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) \\ &\quad + (1-\lambda(\omega_t))\beta E_t\left[W_{t+1} + V_{t+1}\right]. \end{aligned} \tag{84}$$

Since $\lim_{T\to\infty}\beta^T E_t \tilde{J}_{t+T} = \lim_{T\to\infty}\beta^T E_t\left[W_{T+1} + V_{T+1}\right] = 0$, solving forwards implies that

$$\tilde{J}_t = W_t + V_t. \tag{85}$$

**QED**

**Proof of Proposition 3.** The first-order condition to the optimization problem in (54) yields

$$\lambda'(\omega_t)\omega_t'(P_t)\left(\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}\right) - \lambda\left(\omega_t(P_t)\right) = 0, \tag{86}$$

where $\omega_t(P_t)$ and $\omega_t'(P_t)$ are implicitly determined by (53). This implies

$$\frac{\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}} = -\frac{\lambda(\omega_t(P_t))/\lambda'(\omega_t(P_t))}{\gamma\left(\omega_t(P_t)\right)/\gamma'(\omega_t(P_t))}, \tag{87}$$

which can be used together with (53) to solve for $P_t$, given the values $E_t J_{t+1}$, $E_t W_{t+1}$ and $E_t \tilde{V}_{t+1}$. Then one can solve for $\omega_t$ from (53). Note that (53) implies that $\omega_t'(P_t) < 0$ given $\gamma'(\omega) > 0$ from Assumption 1.

The trade surplus in the housing market is strictly positive. Given the boundary condition $\lim_{T\to\infty} \beta^T E_t J_{t+T} = 0$, it is clear that the household's equilibrium values are bounded, which implies that the trade surplus is also bounded. Thus $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} \in (0, \infty)$, where we have incorporated that $V = \tilde{V}$ in the equilibrium. Recall condition (ix) of the equilibrium definition that $\gamma(\omega_t)$, $\lambda(\omega_t) \in (0,1)$ for all active sub-markets. Also recall from part (ii) of Assumption 1 that $\lambda'(\omega) < 0$, $\gamma'(\omega) > 0$. These conditions imply that $\epsilon(\omega) \in (0,1)$ where

$$\epsilon(\omega_t) = \frac{B_t}{\mathcal{M}} \cdot \frac{\partial \mathcal{M}}{\partial B_t} = \frac{1}{1 - \frac{\gamma(\omega_t)/\gamma'(\omega_t)}{\lambda(\omega_t)/\lambda'(\omega_t)}}. \tag{88}$$

Define $LHS(P_t)$ as the left-hand side of (87) and $RHS(P_t)$ the right-hand side. Given (88), it is clear that

$$RHS(P_t) = \frac{\epsilon(\omega_t(P_t))}{1 - \epsilon(\omega_t(P_t))}. \tag{89}$$

Because $\epsilon(\omega) \in (0,1)$, we have $RHS(P_t) \in (0,\infty)$ for all $P_t$. Moreover, recall $\omega_t'(P_t) < 0$ from (53) and $\epsilon'(\omega) \leq 0$ from Assumption 1. Thus $RHS'(P_t) \geq 0$.

For any given $V_t$, $J_t$, $W_t$, one can verify that $LHS'(P_t) < 0$ because $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} > 0$. Recall from (23) and (24) that the price in an active sub-market satisfies

$$\beta E_t V_{t+1} \leq P_t \leq \beta E_t J_{t+1} - \beta E_t W_{t+1}. \tag{90}$$

It follows that

$$\begin{aligned} LHS(P_t = \beta E_t V_{t+1}) &= \infty > RHS(P_t = \beta E_t V_{t+1}) & (91) \\ LHS(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}) &= 0 < RHS(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}), & (92) \end{aligned}$$

where the two inequalities are because $RHS(P_t) \in (0,\infty)$ for all $P_t$. The above results imply a unique $P_t^* \in (\beta E_t V_{t+1},\ \beta E_t J_{t+1} - \beta E_t W_{t+1})$ that satisfies

$$\frac{\beta E_t J_{t+1} - P_t^* - \beta E_t W_{t+1}}{P_t^* - \beta E_t V_{t+1}} = -\frac{\lambda(\omega_t^*(P_t^*))/\lambda'(\omega_t^*(P_t^*))}{\gamma(\omega_t^*(P_t^*))/\gamma'(\omega_t^*(P_t^*))}, \tag{93}$$

and a unique $\omega_t^*(P_t^*)$ that satisfies

$$\omega_t^*(P_t^*) = \gamma^{-1}\left(\frac{V_t - \beta E_t V_{t+1}}{P_t^* - \beta E_t V_{t+1}}\right). \tag{94}$$

Thus, there is a single active sub-market in the directed search equilibrium.

Equation (93) may be written as

$$\frac{\chi(\omega)}{1-\chi(\omega)} = \frac{\epsilon(\omega)}{1-\epsilon(\omega)}, \tag{95}$$

where $\chi(\omega)$ denotes the buyer's share of the surplus in a sub-market with tightness $\omega$. The right-hand side of the above is the ratio of the elasticities of the number of matches with respect to the numbers of buyers and sellers. It follows that $\chi(\omega) = \epsilon(\omega)$. **QED**

**An alternative economy without search:** New entrants can either rent or purchase a house immediately and move in. Since households derive more utility from owning and construction costs are the same, only pure renters will choose to rent in equilibrium. In this case, the dynamic system is given by

$$\begin{aligned}
(1+\mu)f_t &= (1-\pi_f)f_{t-1} + (1-\psi)\,\mu G\left(\bar{W}_t\right) && (96)\\
(1+\mu)n_t &= (1-\pi_n)n_{t-1} + \psi\mu G\left(\bar{W}_t\right) && (97)\\
(1+\mu)h_{t+1} &= h_t + \zeta\phi^{1+\eta}\,(n_t+f_t)\,(\beta E_t P_{t+1} - q_t)^{\eta} && (98)\\
h_t &= n_t + f_t && (99)\\
J_t &= u_t^H + \beta\pi_n\,(Z + E_t P_{t+1}) + \beta(1-\pi_n)E_t J_{t+1} && (100)\\
W_t^f &= u_t^R + \pi_f\beta Z + (1-\pi_f)\,\beta E_t W_{t+1}^f && (101)\\
\bar{W}_t &= \psi\,(J_t - P_t) + (1-\psi)W_t^f && (102)\\
r_t &= m + P_t - \beta E_t P_{t+1} && (103)
\end{aligned}$$

An important difference to the model studied by Glaeser et al. (2010) is that, in their paper, the alternative to living in the city yields a homogeneous payoff so that the elasticity of entry is effectively infinite. In response to a shock, this implies immediate entry of buyers until the price of housing adjusts to keep the value of entering constant. This tends to generate high variance in both prices and construction in response to income shocks. In our model there is a distribution of alternatives, so that as households enter, the critical outside value rises. This determines the flow of additional households into the city.

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# On-line Appendix C: Empirical Results

**Full Panel VAR Results**

Table C1 documents the parameter estimates for the baseline estimation of the Panel VAR discussed in Section 2. Estimating a panel VAR raises a number of econometric issues. A basic problem in dynamic panel data models with fixed effects is that the lagged dependent variables are, by construction, correlated with the individual effects. This renders the least squares estimator biased and inconsistent. Consistent estimation requires some transformation to eliminate fixed effects. A within transformation wipes out the individual effects by taking deviations from sample means, but the resulting within-group estimator is inconsistent when the number of panels becomes large for a given time-dimension (Nickell, 1981).

**Table C1: System GMM (2SLS) estimates**

| | $Y$ | | $P$ | | $g^S$ | | $g^H$ | | $g^N$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| $Y(-1)$ | 1.23 | (0.05) | 0.45 | (0.09) | 0.38 | (0.23) | 0.01 | (0.01) | 0.06 | (0.02) |
| $P(-1)$ | -0.01 | (0.01) | 1.37 | (0.04) | -0.53 | (0.08) | 0.01 | (0.00) | -0.02 | (0.00) |
| $g^S(-1)$ | 0.00 | (0.00) | -0.06 | (0.01) | 0.07 | (0.03) | 0.00 | (0.00) | 0.01 | (0.00) |
| $g^H(-1)$ | 0.74 | (0.12) | 1.28 | (0.26) | 0.06 | (0.83) | 0.74 | (0.04) | 0.39 | (0.09) |
| $g^N(-1)$ | -0.23 | (0.17) | 0.15 | (0.28) | 2.23 | (1.23) | 0.08 | (0.04) | 0.24 | (0.19) |
| $Y(-2)$ | -0.33 | (0.05) | -0.61 | (0.08) | -0.47 | (0.22) | -0.02 | (0.01) | -0.06 | (0.03) |
| $P(-2)$ | 0.02 | (0.01) | -0.46 | (0.05) | 0.63 | (0.08) | -0.01 | (0.00) | 0.02 | (0.00) |
| $g^S(-2)$ | -0.00 | (0.00) | -0.01 | (0.01) | 0.04 | (0.03) | -0.00 | (0.00) | 0.00 | (0.00) |
| $g^H(-2)$ | -0.67 | (0.09) | -1.30 | (0.19) | -2.77 | (0.60) | -0.14 | (0.03) | -0.21 | (0.04) |
| $g^N(-2)$ | 0.16 | (0.06) | 0.53 | (0.12) | 1.02 | (0.38) | 0.03 | (0.01) | 0.16 | (0.05) |

Standard errors in parenthesis. No. of observations = 2438

Given this inconsistency, the literature focuses mainly on a first-difference transformation to eliminate the individual effect while handling the remaining correlation with the (transformed) error term using instrumental variables and GMM estimators (e.g. Arellano and Bond, 1991). However, the Arellano-Bond estimator is known to suffer from a weak instruments problem when the relevant time series are highly persistent, as they are in our case. As Blundell and Bond (1998) demonstrate this can result in large finite-sample biases. In our baseline estimation we use the system GMM estimator proposed by Arellano and Bover (1995) and Blundell and Bond (1998). This estimator is consistent when the number of panels becomes large for a given time-dimension and is less likely to suffer from the weak instruments problem. Another reason for focussing on this estimator is that its properties are fairly well understood and it has been studied in the context of panel VARs by Binder, Hsiao and Pesaren (2005).

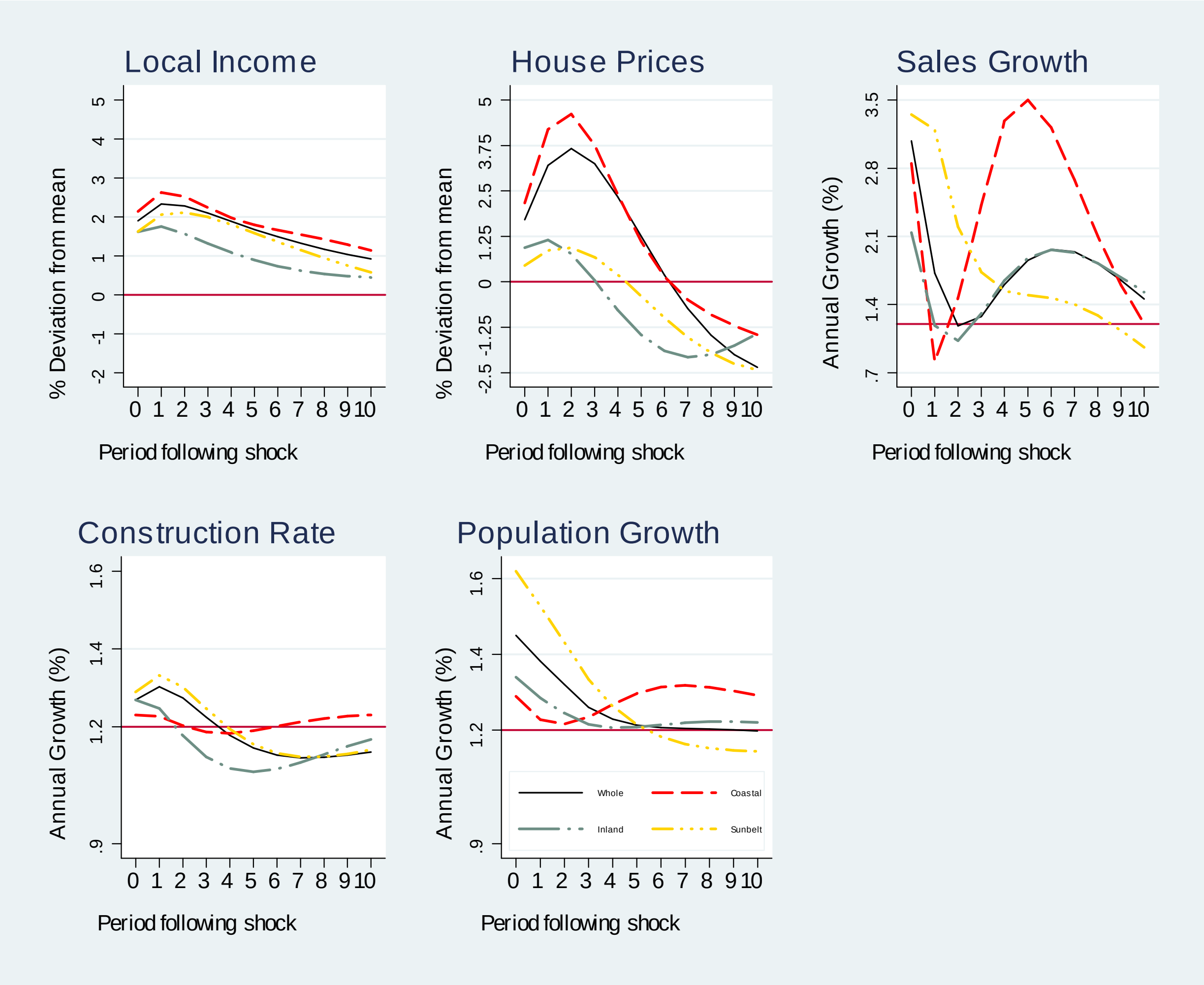
**Analysis of Regional Sub-samples**

**Table C2: Moments from system GMM estimation for regional sub-samples – income shock**

| | Region | Relative Std. Dev. | Corr. with Inc. Growth | Corr. with Price App. | Autocorrelation year 1 | year 2 | year 3 | year 4 |
|---|---|---|---|---|---|---|---|---|
| Income Growth | Coastal | 1.00 | 1.00 | 0.76 | 0.27 | 0.01 | -0.07 | -0.06 |
| | Interior | 1.00 | 1.00 | 0.68 | 0.14 | -0.05 | -0.10 | -0.10 |
| | Sunbelt | 1.00 | 1.00 | 0.51 | 0.34 | 0.12 | 0.02 | -0.04 |
| Price Appreciation | Coastal | 1.71 | 0.76 | 1.00 | 0.69 | 0.20 | -0.12 | -0.23 |
| | Interior | 1.10 | 0.68 | 1.00 | 0.69 | 0.31 | -0.02 | -0.25 |
| | Sunbelt | 0.89 | 0.51 | 1.00 | 0.89 | 0.66 | 0.42 | 0.20 |
| Sales Growth (existing) | Coastal | 3.11 | 0.19 | -0.33 | 0.90 | 0.83 | 0.74 | 0.65 |
| | Interior | 1.11 | 0.30 | -0.01 | 0.69 | 0.54 | 0.47 | 0.42 |
| | Sunbelt | 2.21 | 0.66 | -0.09 | 0.81 | 0.58 | 0.45 | 0.38 |
| Construction Rate | Coastal | 0.05 | 0.10 | 0.24 | 0.92 | 0.79 | 0.68 | 0.61 |
| | Interior | 0.15 | 0.52 | 0.66 | 0.88 | 0.66 | 0.45 | 0.27 |
| | Sunbelt | 0.15 | 0.63 | 0.65 | 0.86 | 0.57 | 0.27 | 0.02 |
| Population Growth | Coastal | 0.15 | 0.04 | -0.30 | 0.93 | 0.88 | 0.83 | 0.77 |
| | Interior | 0.11 | 0.70 | 0.39 | 0.60 | 0.32 | 0.16 | 0.13 |
| | Sunbelt | 0.35 | 0.79 | 0.26 | 0.73 | 0.48 | 0.26 | 0.11 |

We now consider the results of estimating the panel VAR model on various sub-samples of both cities and time periods. Table C2 provides key moments for local earnings, house prices, construction rates, and ratios of housing stocks to city population based on shocks to local income in the panel VAR for each of the three sub-samples. Several, key observations are apparent. The standard deviation of house prices is roughly equal to that of local earnings in the full sample. Both construction rates and housing stock-population ratios are much less volatile than local earnings. House prices, construction rates and housing stock-population ratios are all strongly positively correlated with local earnings, although for inland cities these correlations are somewhat weaker. The higher and more persistent autocorrelation in both house price appreciation and population growth relative to earnings growth can also be observed in all the sub-samples.

Certain features of these moments and impulse response functions in Figure 8 conform to *a priori* expectations regarding population and price movements. In particular, coastal cities typically have more inelastic land supply than sunbelt cities. Accordingly, in response to demand shocks, price volatility tends to be higher and population and construction volatility tend to be lower in the coastal cities.



**Alternative estimators**

There are several potential problems with using the system GMM estimator for a sample with the dimensions considered here. While it is usually thought to be suitable for typical microeconometric panels, with only a few waves but a large number of individuals, here we have moderately large number of cities and a moderately long time series. Moreover, GMM estimators tend to have a larger standard error compared to the within-group estimator and may suffer from a finite sample bias due to weak instruments. Here we address these issues by comparing our estimates with those of two alternative estimators: OLS with no fixed effects and a standard within-groups estimator (WGE). Although the WGE is inconsistent as the number of panels becomes large, this should be less of a problem given the dimensions of our sample.

For the sake of brevity we do not report here all of the estimation results for each estimator. Instead Table B3 reports only the sum of the coefficients on the lagged dependent variables for each equation under each estimator, as suggested by Blundell and Bond (1998). As may be seem, the OLS estimates yields the most persistent processes for each variable. This reflects the upward bias due to the fact any fixed effect is attributed to persistent effects of the shocks. The WGE etimates yield the least persistent processes, which reflects the downward bias. The system GMM (2SLS) estimator implies persistence that lie between these two extremes.

**Table C3: Implied persistence: sum of coefficients on lagged dependent variable**

| Equation | $WGE$ | $2SLS$ | $OLS$ |
|---|---|---|---|
| $Y$ | 0.90 | 0.90 | 0.99 |
| $P$ | 0.87 | 0.92 | 0.98 |
| $g^S$ | -0.12 | -0.11 | -0.03 |
| $g^H$ | 0.60 | 0.60 | 0.72 |
| $g^N$ | 0.06 | 0.40 | 0.45 |

Table C4 documents the same set of moments as we have previously considered, for each of the estimators. While there are clearly some differences across estimators, the same broad pattern emerges as that depicted in Table 2. The biggest outliers come from those based on OLS estimation. This is because the omission of city level fixed effects forces any permament differences to show up as high persistence. The system GMM (2SLS) estimator implies a price appreciation response that is the most volatile and the least persistent.

**Table C4: Moments from estimation using alternative estimators – income shocks**

| | Estimator | Relative Std. Dev. | Corr. with Inc. Growth | Corr. with Price App. | Autocorrelation year 1 | year 2 | year 3 | year 4 |
|---|---|---|---|---|---|---|---|---|
| Income Growth | WGE | 1.00 | 1.00 | 0.67 | 0.24 | 0.04 | -0.04 | -0.09 |
| | 2SLS | 1.00 | 1.00 | 0.76 | 0.28 | 0.03 | -0.06 | -0.09 |
| | OLS | 1.00 | 1.00 | 0.47 | 0.21 | 0.08 | 0.02 | -0.00 |
| Price Growth | WGE | 1.90 | 0.67 | 1.00 | 0.81 | 0.48 | 0.12 | -0.19 |
| | 2SLS | 1.60 | 0.76 | 1.00 | 0.75 | 0.36 | 0.05 | -0.15 |
| | OLS | 1.22 | 0.47 | 1.00 | 0.88 | 0.60 | 0.31 | 0.09 |
| Sales Growth | WGE | 1.75 | 0.60 | 0.02 | 0.57 | 0.11 | -0.15 | 0.27 |
| | 2SLS | 1.32 | 0.56 | 0.01 | 0.59 | 0.39 | 0.35 | 0.35 |
| | OLS | 1.98 | 0.59 | 0.12 | 0.71 | 0.37 | 0.25 | 0.25 |
| Cons. Rate | WGE | 0.12 | 0.41 | 0.90 | 0.84 | 0.47 | 0.09 | -0.20 |
| | 2SLS | 0.11 | 0.55 | 0.80 | 0.88 | 0.61 | 0.33 | 0.11 |
| | OLS | 0.40 | 0.19 | 0.44 | 0.98 | 0.95 | 0.91 | 0.88 |
| Pop. Growth | WGE | 0.16 | 0.89 | 0.84 | 0.57 | 0.20 | -0.07 | -0.25 |
| | 2SLS | 0.17 | 0.75 | 0.60 | 0.67 | 0.40 | 0.19 | 0.09 |
| | OLS | 0.38 | 0.47 | 0.55 | 0.91 | 0.84 | 0.77 | 0.73 |

## Alternative Specifications

Table C5 documents the relevant moments due to income shocks from the panel VAR for two alternative specifications.[46] The first specification, labelled "AR(2) Income", restricts the equation for income so that income depends only on its own lagged values. The specification labelled "All growth" uses growth rates of per capita incomes and prices in the VAR rather than levels. As may be seen by comparing to Table 2, restricting the income process to be univariate has negligible effects. This suggest thats lagged feedback effects of prices and population on per capita income are of second order importance. Specifying the VAR so that incomes and prices are in growth rates rather than in log levels has somewhat larger effects on our results, but does not change the broad conclusions. Note that, by construction, the level of relative income under this specification is permanently high following a shock. However, this has little impact on the moments that we consider here.

[46] We have considered others including alternative definitions of the construction rate and other definitions of income. Similar patterns emerge in all cases.

**Table C5: Moments from system GMM estimation for alternative specifications – income shocks**

| | Specification | Relative Std. Dev. | Corr. with Inc. Growth | Corr. with Price Growth | Autocorrelation year 1 | year 2 | year 3 | year 4 |
|---|---|---|---|---|---|---|---|---|
| Income Growth | AR(2) Income | 1.00 | 1.00 | 0.75 | 0.26 | 0.02 | -0.05 | -0.07 |
| | All growth | 1.00 | 1.00 | 0.47 | 0.23 | 0.07 | 0.03 | 0.01 |
| Price Growth | AR(2) Income | 1.62 | 0.74 | 1.00 | 0.76 | 0.38 | 0.07 | -0.12 |
| | All growth | 1.10 | 0.47 | 1.00 | 0.87 | 0.58 | 0.30 | 0.10 |
| Sales Growth | AR(2) Income | 1.40 | 0.58 | -0.02 | 0.62 | 0.44 | 0.40 | 0.41 |
| | All growth | 1.34 | 0.83 | 0.16 | 0.44 | -0.08 | -0.23 | -0.26 |
| Cons. Rate | AR(2) Income | 0.13 | 0.52 | 0.74 | 0.90 | 0.68 | 0.45 | 0.26 |
| | All growth | 0.13 | 0.44 | 0.98 | 0.90 | 0.67 | 0.42 | 0.23 |
| Pop. Growth | AR(2) Income | 0.17 | 0.77 | 0.57 | 0.68 | 0.42 | 0.22 | 0.12 |
| | All growth | 0.18 | 0.85 | 0.85 | 0.69 | 0.40 | 0.20 | 0.07 |

## Conversion to quarterly income shock process

If we now think of a period as a quarter, we can write an annual AR(2) process as

$$x_t = b_1 x_{t-4} + b_2 x_{t-8} + \varepsilon_t. \tag{104}$$

Let $y_t = x_{t-4}$. Then we can write this as a stacked system given by

$$X_t = \mathbf{B} X_{t-4} + \mathbf{e}_t \tag{105}$$

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{t-4} \\ y_{t-4} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix}. \tag{106}$$

Now consider a VAR(1) given by

$$X_t = \mathbf{A} X_{t-1} + \mathbf{v}_t, \tag{107}$$

where $\mathbf{v}_t = [v_t \quad 0]'$. Iterating on this yields

$$X_t = \mathbf{A}^4 X_{t-4} + \mathbf{A}^3 \mathbf{v}_{t-3} + \mathbf{A}^2 \mathbf{v}_{t-2} + \mathbf{A} \mathbf{v}_{t-1} + \mathbf{v}_t. \tag{108}$$

It follows that $\mathbf{A} = \mathbf{B}^{\frac{1}{4}}$ and $\mathbf{e}_t = \mathbf{A}^3 \mathbf{v}_{t-3} + \mathbf{A}^2 \mathbf{v}_{t-2} + \mathbf{A} \mathbf{v}_{t-1} + \mathbf{v}_t$. We can decompose the VAR(1) as

$$x_t = a_{11} x_{t-1} + a_{12} y_{t-1} + v_t \tag{109}$$

$$y_t = a_{21} x_{t-1} + a_{22} y_{t-1}. \tag{110}$$

Since $y_t = x_{t-4}$, it follows that

$$x_t = a_{11}x_{t-1} + a_{12}x_{t-5} + v_t \tag{111}$$

$$x_{t-4} = a_{21}x_{t-1} + a_{22}x_{t-5}. \tag{112}$$

Substituting out $x_{t-5}$ yields

$$x_t = a_{11}x_{t-1} + \frac{a_{12}}{a_{22}}\left(x_{t-4} - a_{21}x_{t-1} - v_{2t}\right) + v_t \tag{113}$$

$$x_t = \left(a_{11} - \frac{a_{12}a_{21}}{a_{22}}\right)x_{t-1} + \frac{a_{12}}{a_{22}}x_{t-4} + v_t. \tag{114}$$

Thus the AR(2) process at the annual frequency translates into a particular AR(4) process at the quarterly frequency. There is, of course, a loss of information.

**Full results of sensitivity analysis for elasticities**

**Table C6: Volatilities and Comovements: Sensitivity Results**

| | | New construction supply elasticity | | Entry (demand) elasticity | | Matching Elasticity | | Land supply elasticity | |
|---|---|---|---|---|---|---|---|---|---|
| Moment | Baseline | $\eta = 0.1$ | $\eta = 10$ | $\alpha = 3$ | $\alpha = 20$ | $\delta = .5$ | $\delta = .9$ | $\xi = .5$ | $\xi = 5$ |
| $\sigma_p/\sigma_y$ | 1.45 | 2.92 | 0.83 | 0.48 | 2.63 | 1.59 | 1.28 | 1.83 | 1.30 |
| $\sigma_s/\sigma_y$ | 1.32 | 0.83 | 1.67 | 1.12 | 1.59 | 0.87 | 2.04 | 1.04 | 1.45 |
| $\sigma_h/\sigma_y$ | 0.11 | 0.02 | 0.17 | 0.06 | 0.17 | 0.12 | 0.11 | 0.08 | 0.13 |
| $\sigma_n/\sigma_y$ | 0.17 | 0.08 | 0.23 | 0.12 | 0.23 | 0.17 | 0.18 | 0.13 | 0.19 |
| $\sigma_{py}$ | 0.98 | 0.96 | 0.97 | 0.67 | 0.98 | 0.99 | 0.98 | 0.99 | 0.98 |
| $\sigma_{sy}$ | 0.11 | 0.33 | 0.02 | 0.22 | 0.01 | -0.25 | 0.28 | 0.17 | 0.08 |
| $\sigma_{hy}$ | 0.26 | 0.15 | 0.29 | 0.14 | 0.26 | 0.27 | 0.24 | 0.25 | 0.26 |
| $\sigma_{ny}$ | 0.50 | 0.60 | 0.48 | 0.52 | 0.44 | 0.48 | 0.52 | 0.52 | 0.49 |
| $\sigma_{sp}$ | 0.07 | 0.33 | -0.10 | -0.39 | 0.09 | -0.25 | 0.20 | 0.16 | 0.02 |

Table C7: Autocorrelations: Sensitivity Results

| Moment | Baseline | Labour supply elasticity | | Entry (demand) elasticity | | Matching Elasticity | | Land supply elasticity | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\eta = .1$ | $\eta = 10$ | $\alpha = 3$ | $\alpha = 20$ | $\delta = .5$ | $\delta = .9$ | $\xi = .5$ | $\xi = 5$ |
| $\rho_1^p$ | 0.34 | 0.27 | 0.44 | 0.76 | 0.24 | 0.32 | 0.38 | 0.31 | 0.36 |
| $\rho_2^p$ | 0.07 | 0.02 | 0.13 | 0.45 | -0.01 | 0.06 | 0.11 | 0.05 | 0.09 |
| $\rho_3^p$ | 0.01 | 0.00 | 0.03 | 0.24 | -0.03 | 0.00 | 0.04 | 0.01 | 0.02 |
| $\rho_4^p$ | -0.02 | -0.02 | -0.03 | 0.10 | -0.03 | -0.02 | -0.01 | -0.02 | -0.02 |
| $\rho_1^s$ | 0.52 | 0.34 | 0.52 | 0.49 | 0.46 | 0.55 | 0.43 | 0.48 | 0.53 |
| $\rho_2^s$ | 0.18 | 0.07 | 0.15 | 0.20 | 0.09 | 0.21 | 0.09 | 0.16 | 0.18 |
| $\rho_3^s$ | 0.04 | 0.01 | 0.01 | 0.08 | -0.03 | 0.06 | -0.01 | 0.04 | 0.03 |
| $\rho_4^s$ | -0.03 | -0.02 | -0.06 | 0.00 | -0.07 | -0.02 | -0.05 | -0.03 | -0.05 |
| $\rho_1^h$ | 0.96 | 0.98 | 0.96 | 0.98 | 0.96 | 0.96 | 0.97 | 0.96 | 0.96 |
| $\rho_2^h$ | 0.90 | 0.94 | 0.89 | 0.94 | 0.89 | 0.90 | 0.91 | 0.90 | 0.90 |
| $\rho_3^h$ | 0.84 | 0.89 | 0.82 | 0.88 | 0.82 | 0.83 | 0.84 | 0.84 | 0.83 |
| $\rho_4^h$ | 0.77 | 0.84 | 0.74 | 0.81 | 0.76 | 0.76 | 0.78 | 0.78 | 0.76 |
| $\rho_1^n$ | 0.84 | 0.74 | 0.86 | 0.85 | 0.83 | 0.84 | 0.83 | 0.82 | 0.85 |
| $\rho_2^n$ | 0.66 | 0.46 | 0.71 | 0.67 | 0.67 | 0.67 | 0.65 | 0.63 | 0.68 |
| $\rho_3^n$ | 0.54 | 0.29 | 0.59 | 0.53 | 0.57 | 0.54 | 0.52 | 0.49 | 0.56 |
| $\rho_4^n$ | 0.45 | 0.17 | 0.50 | 0.43 | 0.50 | 0.46 | 0.43 | 0.40 | 0.47 |

# On-line Appendix D: A Multi-City Environment

There are $M$ symmetric cities, indexed by $i = 1, \ldots, M$, where $M$ is finite, but *large* in the sense that no individual city has a significant effect on aggregate quantities. The cities can be of identical or different sizes; what is important is that they all be small in this sense. We will focus on City 1, which will correspond to the *representative* or *average* city that was considered in the text.

Each city can be described as in Section 3, except that here city-specific quantities are indexed by $i$. In particular, at each point in time, income in City $i$ is denoted $\widetilde{y}_i$. Define average income across all cities by $\overline{y} = \sum_{i=1}^{M} \widetilde{y}_i$, and let $y_i \equiv \frac{\widetilde{y}_i}{\overline{y}}$. We will assume that in the steady-state, $y_i = 1$ for all $i$. We will think of the deviation of City 1 income from the average, $y_1$, as following a stochastic process just as in the text. This is straightforward under the assumption that City 1 is small so that fluctuations in $\widetilde{y}_1$ have no effect on $\overline{y}$. Alternatively, we can dispense with $y_i$ and consider fluctuations in the *level* of City $i$ income, $\widetilde{y}_i$. What is important in what follows is that the shocks considered be truly *city-specific.*

That is, that fluctuations in either $y_1$ or $\widetilde{y}_1$, have negligible effects on income and/or housing market conditions in all other cities.

As in the text, the population of the economy is given by $Q_t$, and grows at gross rate $1+\mu$. Every period, each new household that enters the economy draws $M$ potential amenity values, $a_i \in [0, \overline{a}]$ (one for each city), from distributions $F_i(a)$, for $i = 1, \ldots M$. Here for simplicity we will assume $F_i(\cdot) = F(\cdot)$ for all $i$., and that $\overline{a}$ is sufficiently large that a positive measure of households chooses to enter all cities in each time period. Amenity values are in utils, and like both consumption and housing services enter households' utility linearly. Utility from amenities, also like that from income, is realized only when the household chooses a particular city in which to live, and locates there.

For each new household $j$, let $\overline{W}_{ij}$ denote the value of being a new entrant to City $i$, defined just as in (10). Since new entrants to any city are identical, variation in $\overline{W}_{ij}$ across households is induced solely by variation in the amenity value, $a_{ij}$; in particular, $F_i(\overline{W}(a)) = F_i(a) = F(a)$ where $a$ is the amenity value that generates $\overline{W}(a)$ given all other city attributes (income, house prices, the housing market tightness, etc.). Let $\varepsilon_j \equiv \max[\overline{W}_{2j}, \ldots, \overline{W}_{Mj}]$. That is, $\varepsilon_j$ denotes the highest alternative value to entering City 1 for each new household $j$. Since $M$ is finite, $\varepsilon$ exists for all new households and identifies a single best alternative with probability 1. Similarly, the probability that $\varepsilon_j = \overline{W}_{1j}$, that is that a household is indifferent between entering City 1 and some other city, approaches zero as $M$ becomes large.

Let $G(\varepsilon)$ denote the distribution of the highest alternative value, $\varepsilon$, across households. In a situation in which all cities other than City 1 are identical, $\varepsilon_j$is the value of entering that city for which household $j$ has the highest amenity value. Thus, $G(\cdot)$ satisfies:

$$G(\varepsilon) = [F(a^*)]^{M-1} \tag{115}$$

where $a^* \in [0, \overline{a}]$ is the amenity value which generates the maximum value $\varepsilon$. Note, however, that for $G(\varepsilon)$ to be well-defined, it is not required for all cities other than City 1 to be identical. Finally, note that the entry cutoff, $\varepsilon_t^c$, in this case satisfies $\varepsilon_t^c = \overline{W}_{1t}$, just as in (9). That is, any household with a maximum alternative value below $\overline{W}_{1t}$ enters City 1.

When a household leaves their city of residence due to the realization of a relocation shock (which happens with probability $\pi$ for both home-owners and permanent renters), we assume that they are effectively in the same situation as a new household who has just arrived in the economy. That is, they re-draw, *in the current period*, from the amenity distribution for each city, and choose the city which yields the highest value. The expected continuation value following a relocation shock for any household currently resident in *any* city is thus given by

$$Z = \overline{\varepsilon} \equiv \int G(\varepsilon) d\varepsilon. \tag{116}$$

From (116) it is clear that $Z$ depends only on the distribution of amenity values, $F(\cdot)$. Also, note that since City 1 is small, the probability that a household which exits it due to a relocation shock returns immediately is negligible.

Let $POP_t$ denote the population of City 1 in period $t$. The population evolves via

$$POP_{t+1} = POP_t + \underbrace{\mu G(\overline{W}_{1t})Q_t + \pi \mathbb{Z}_t G(\overline{W}_{1t})}_{\text{new entrants}} - \underbrace{\pi[N_t + F_t]}_{\text{exiters}}, \tag{117}$$

where $\mathbb{Z}_t$ denotes the measure of agents that exit *all other* cities in period $t$, and is assumed to be unaffected by conditions in City 1. On the balanced growth path, we assume first that all cities are symmetric, so that $G(\overline{W}_{1t}) = 1/M$ for all $t$. Similarly, $\mathbb{Z}_t = M(N_t + F_t)$. Thus, from (117) $POP_{t+1} = (1+\mu)POP_t$.

Finally, note that it is not important that we model the shock to City 1's income as being relative to the average. Any stable distribution of income across cities will give rise to a well-defined distribution of alternative values for City 1 (although (115) will no longer apply). A direct increase in City 1 income, $\widetilde{y}_1$, will thus lead to entry for the same reasons as before. Again, the magnitude of the response will be determined by the properties of $G(\cdot)$ in a neighborhold of $\varepsilon = \widetilde{y}_1$ along the balanced growth path.

Suppose now that the economy is subject to aggregate income shocks which affect all cities symmetrically. Because utility is linear, adding a common component, $y_{ct}$, to city-level income of the form,

$$\widetilde{Y}_{it} = \widetilde{y}_{it} + y_{ct} \qquad i = 1, \ldots, N \tag{118}$$

will have no effect whatsoever, as it affects neither the ranking of cities by new entrants or relocaters nor the demand for housing. Common shocks to construction costs and/or population growth will affect housing markets within each city, but will have no effect on mobility as they will not change the ranking of cities across prospective entrants as this is determined only by the amenity distribution, $F(\cdot)$.