Exchange-Rate Discounting

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Abstract
Economists often describe nominal exchange rates as forward-looking, so that they reflect discounted, expected, future fundamentals. This study applies a method for identifying the discount rate involved, without knowing or measuring fundamentals. Identification arises from assumptions on the stochastic process followed by fundamentals, combined with nonlinearity arising from expected future regime changes. Two applications yield evidence against the present-value model in the form of discount rates which are negative and statistically significant.

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Economists often describe floating nominal exchange rates as reacting to news even if that news concerns future monetary and fiscal policy, and the exchange rate is widely viewed as discounting future fundamentals. The present-value model of exchange rates has been used by Mussa (1982), Flood and Garber (1983), Krugman (1991), Froot and Obstfeld (1991b) and many others, to study both floating exchange rates and target zones.

This paper describes and implements a simple method for estimating the discount rate in a present-discounted-value relationship for a nominal exchange rate. The method does not require knowledge or measurement of the fundamentals. Instead, identification arises from assumptions on the stochastic process followed by fundamentals, combined with nonlinearity arising from expected future regime changes. Two applications yield evidence against the present-value model in the form of discount rates which are negative and statistically significant.

Section I describes the identification scheme, which is similar to one of the methods used by Flood, Rose, and Mathieson (1991). Section II describes the data and historical episodes used in estimation. Section III presents the estimation results. Section IV contains conclusions.

I. Identifying Discount Rates.

The relationship under study is a linear, asset-pricing equation:

\[ e_t = f_t + \alpha E[(e_{t+1} - e_t)|\mathcal{F}_t], \quad \alpha > 0, \]  

(1)

where \( e_t \) is the log of the nominal exchange rate, \( f_t \) the fundamental, and \( \mathcal{F}_t \) a non-decreasing sequence of information sets to which \( f_t \) is adapted. This relation sometimes is derived from the monetary model of the exchange rate. Solving equation (1) with a transversality condition gives the present-value relationship:

\[ e_t = \frac{1}{1 + \alpha} \sum_{i=0}^{\infty} \left( \frac{\alpha}{1 + \alpha} \right)^i E[f_{t+i}|\mathcal{F}_t]. \]  

(2)

The discount rate is given by \( 1/\alpha \).

The first identifying assumption is that in the absence of intervention the fundamental follows:

\[ f_t = \mu + f_{t-1} + \epsilon_t, \]  

(3)
with \(E(\epsilon_t|\mathcal{F}_{t-1}) = 0\). Thus \(\{\epsilon_t\}\) is a martingale difference sequence with respect to \(\{\mathcal{F}_t\}\); its mean is unpredictable but it may have considerable serial dependence in higher moments.

Using (3) in (2) with the law of iterated expectations gives:

\[
e_t = \alpha\mu + f_t, \tag{4}
\]
in a pure float. Thus assumption (3) induces similar statistical properties in the log exchange rate: unpredictable changes (allowing for a constant drift) and possible heteroskedasticity. Numerous studies have established that floating exchange rates can be approximated with martingales and that their changes display persistence in variance. For example, Meese and Rogoff (1983) found that the random walk model was difficult to beat in a tournament of forecasting models. Diebold and Nason (1990) found that the random walk did as well as nonlinear models, estimated nonparametrically, in forecasting future exchange rates.

Equation (4) shows that \(\alpha\) and \(\mu\) cannot be separately identified without further information. One source for this information might be a measurement of \(f\); then both parameters could be identified from joint estimation of (4) and the law of motion (3). However, researchers have not found conclusive evidence which would allow one to choose \(f\) with confidence. This study implements a method for estimating \(\alpha\) which does not rely on such a choice.

The second identifying assumption is that future exchange-rate intervention is expected. That introduces a nonlinearity into the relationship between exchange rates and current fundamentals which allows identification of \(\mu\) and \(\alpha\).

For example, this nonlinearity arises in the state-dependent plan to peg the exchange rate described by Flood and Garber (1983). In that case when \(f\) reaches an absorbing barrier \(\bar{f}\) then \(e\) will be fixed at rate \(\bar{e}\). The rate prior to the peg follows an exponential path derived by Froot and Obstfeld (1991a) and Smith (1991). The planned return to the gold standard in the U.K. in the 1920s serves as an example of this process switch.

A second example arises with a time-dependent switch. Suppose that at time \(T > t\) the exchange rate will be pegged at \(\bar{e}\). Then the current rate follows an exponential path given by Obstfeld and Stockman (1985). Related examples have been studied by Miller
and Sutherland (1994). The plan to resume greenback convertibility in the U.S. in 1879 is an historical instance of this type of process switch.

In both of these examples a solution for the exchange rate is available, but using it for estimation involves either measuring \( f_t \), which is problematic, or estimation by simulation, which requires a complete specification of \( \{ \epsilon_t \} \). It may be preferable to use only the martingale property of fundamentals and exploit the existence of the nonlinearity but not its exact form. To do this, take first differences and expectations in (1), while using assumption (3):

\[
E(\Delta e_t | \mathcal{F}_{t-1}) = \mu + \alpha E(\Delta^2 e_{t+1} | \mathcal{F}_{t-1}),
\]

which relates the one-step forecast of returns to the two-step forecast of the change in returns. Taking differences and applying the law of iterated expectations again gives:

\[
E(\Delta^2 e_t | \mathcal{F}_{t-2}) = \alpha E(\Delta^3 e_{t+1} | \mathcal{F}_{t-2}).
\]

It is easy to show that in a pure float these moment conditions would not identify \( \alpha \). For example, from equation (4) \( E(\Delta e_t) = \mu \) and \( E(\Delta^2 e_t) = 0 \) so that \( \alpha \) cannot be identified from equation (5). But with a nonlinearity caused by expected future intervention equation (5) can be used to estimate \( \mu \) and \( \alpha \) with various instruments and equation (6) can supplement it to add precision. In section III, equations (5) and (6) are estimated by the generalized method of moments (GMM).

The proposed scheme uses non-linearity in the exchange-rate path, coupled with current linearity in the fundamental, in order to estimate the discount rate. The intuition is that the path of the exchange rate bends away from the pure-float path as gold standard parity nears, while fundamentals do not. The resulting nonlinearity in the relationship between \( e \) and \( f \) may allow identification.

The idea that episodes of intervention may help one learn about exchange-rate models certainly is not new. Meese and Rose (1990) studied the relationship between exchange rates and fundamentals using data from the target zones of the EMS, the Bretton Woods system, and the pre-1914 gold standard. They studied nonparametric regressions of \( e_t \) on \( e_{t-1} \) and of \( e_t \) on \( f_t \), with \( f_t \) given by the monetary model. They found very little evidence
of nonlinearity in either regression, though their simulations suggested the test based on
the univariate autoregression may not have been powerful.

A variety of studies have examined predictions of target-zone models while remaining
agnostic about the identity of the fundamental. Flood, Rose, and Mathieson (1991) ex-
amined data from target zones comprehensively for evidence concerning equation (1) and
other properties of target-zone models. They measured $E[(e_{t+1} - e_t)|F_t]$ with interest-rate
differentials (by UIP), and then backed out $f_t$ from (1) using various values for $\alpha$. They
also made assumption (3) and then estimated $\mu$ and $\alpha$ from (5) using both UIP and in-
strumental variables methods. They found little evidence in support of the present-value
model. Their instrumental-variables estimates ($p$ 25, $ft$ 28) for $\alpha$ were near $-1$.

Lewis (1991) suggested that some of Flood, Rose, and Mathieson’s negative findings
may have followed from their assuming (a) that all interventions occur at the edges of
a band and (b) that no realignments were expected in the episodes they studied. If
intramarginal intervention is important, then the fundamental may not be well described
by equation (3) and so (5) and (6) will be misspecified. And realignments may make
the relationship between $f$ and $e$ approximately linear within a target zone or may add a
further state variable.

This study briefly resumes the quest for evidence on the present-value model using
data from historical periods in which criticisms (a) and (b) seem unlikely to hold. Two
episodes which seem well-suited for this purpose are the temporary suspensions of the gold
standard in Britain during 1914–1925 and in the U.S. during 1862–1879. In neither case
was there much intervention prior to resumption of gold parity. And because suspension of
convertibility during war was part of the gold standard (see Bordo and Kydland, 1992)),
realignments were unlikely. At the same time future intervention – in the form of returning
to the gold standard after war – seems to have been very likely, and this expected future
intervention may allow identification of $\alpha$. Moreover, daily exchange-rate observations are
available for each period.

II. Data.

The first data set consists of observations on the spot exchange rate between the U.K.
and the U.S. for 1919–1925. The British restoration of pre-war parity in 1925 was widely expected and was the policy of successive governments after the report of the Cunliffe Committee in 1919. The data set contains 1627 daily observations on the noon buying rate for cable transfers in New York City from 1 December 1919 to 27 April 1925. The sources are Lawrie (1924) and *The Commercial and Financial Chronicle*.

The second data set consists of daily observations of the greenback price of a gold dollar, for the period 8 January 1875 to 17 December 1878. These can be thought of as observations on a floating exchange rate, because the U.S. suspended the gold standard from 1862 to 1879 while the U.K. did not. One might calculate the U.S. dollar price of sterling during the greenback period as the product of the greenback price of a gold dollar and $4.86 \frac{21}{32}$. Calculation of the dollar/sterling rate in this manner abstracts from fluctuations of the gold dollar price of sterling within the gold points. Exact measurements of the exchange rate (as described by Officer (1985), for example) use actual transactions in bills of exchange.

These data are taken from Mitchell’s (1908) appendix, Table 1, and are described by Smith and Smith (1993). Mitchell collected the prices from *American Gold, 1862–1878*, published by J.C. Mersereau, an official of the gold exchange in New York and from *The Commerical and Financial Chronicle*. I use the highest daily gold dollar price of the greenback and invert it to give the price of gold. Multiplication by the gold-sterling parity then also yields the lowest daily greenback price of sterling.

The sample period begins 8 January 1875, because the House of Representatives passed the Resumption Act on that day. The Act promised a resumption of the gold standard parity at 1 January 1879. The Senate had passed the Act on 22 December 1874, and President Grant’s approval was a formality. This sample ends 17 December 1878, when parity was permanently restored. It contains 1202 daily exchange rates.

In each case $e$ is the log of the price of sterling in U.S. dollars. This variable drifts up in the 1919–1925 data set and drifts down in the 1875–1879 data set.

### III. Estimation.

Estimation is by iterated GMM and uses a Newey-West weighting matrix with five
lags and damping parameter 1.0. This allows for heteroskedasticity of unknown form (arising from heterogeneity in the fundamental innovation $e_t$) and for the moving average induced by multi-step forecasts from the instrument set. In estimating equation (5) the instruments are a constant and $\Delta e_{t-1}, \ldots, \Delta e_{t-5}$. With two parameters and six moment restrictions the maximized value of the objective function is distributed asymptotically as $\chi^2(4)$ under the null.

For equation (6) the instruments are a constant and $\Delta e_{t-2}, \ldots, \Delta e_{t-6}$, so that with one parameter to estimate ($\alpha$) the J-statistic is asymptotically $\chi^2(5)$ under the null. In this case six lags are used in the Newey-West formula. Results were not sensitive to the instrument set or to the weighting matrix used. Results are given in Table 1. I have not pooled the estimates from equations (5) and (6) for each period, because the results from each are very similar.

There are two main findings. First, for the 1875–1879 period with both equations and for the 1919–1925 period with equation (6) the overidentifying restrictions are satisfied at the five percent significance level. The second and more striking result is that each equation and each data set yields a significant, negative value for $\hat{\alpha}$. The values are similar in the two historical periods, and are similar to those found by Flood, Rose, and Mathieson (1991). This evidence may be of interest because the applications seem ideal for estimation of (5) and (6). The daily data are numerous, realignment and intervention do not seem to have been important, and the process switches seem to have been expected.

A remaining possibility (also noted by Lewis (1991)) is that the negative estimates of $\alpha$ stem from misspecification of the law of motion for fundamentals (equation (3)). One interesting alternative specification involves a change in $\mu$ as gold-standard parity nears. In that case, misspecification of the forcing process might be expected to affect the stability of the estimates. Dividing both samples in half gives estimates (based on either equation) $\hat{\alpha}$ which are slightly lower in the second half-sample in each case. A formal test might reject the hypothesis of parameter instability but would not explain the significant, negative estimates in both halves.

An argument in favour of the martingale model is that that property holds approximately for floating exchange rates. There is little evidence of nonlinearity in structural
models of floating rates (see Meese and Rose (1991)) and so that martingale property must hold in fundamentals if the present-value model is accurate. However, another law of motion which it is natural to investigate is the Markov model with mean reversion. Perhaps when a future peg is planned the time series properties of \( f_t \) do differ from those in pure floats. To see whether mean reversion can explain the results, suppose specifically that the fundamental follows:

\[
f_t = \rho f_{t-1} + \epsilon_t, \tag{7}\]

with \( E(\epsilon_t|\mathcal{F}_{t-1}) = 0 \). This fundamental displays mean reversion for \( |\rho| < 1 \). In the historical episodes studied here it could be reverting towards the gold standard level, as Miller and Sutherland (1994) have suggested. Consider the case in which there is no additional process-switching effect, so that (7) and (2) give

\[
e_t = \frac{f_t}{1 + \alpha - \alpha \rho}. \tag{8}\]

With this description,

\[
E(\Delta f_t|\mathcal{F}_{t-1}) = (\rho - 1)f_{t-1},
\]

\[
E(\Delta^2 f_{t+1}|\mathcal{F}_{t-1}) = (\rho - 1)^2 f_{t-1}. \tag{9}\]

These population moments can be used (with (8)) directly in the population regression (5) to show that \( \text{plim} \hat{\alpha} = \rho - 1 \), when the instrument set is a constant and \( \Delta e_{t-1} \). The same result is given from equation (6) under this data-generating process.

Hence misspecifying the mean reversion (because the form of equation (5) assumes a martingale in fundamentals) can lead to negative estimates \( \hat{\alpha} \) when \( \rho < 1 \). But the actual estimates in Table 1 imply values for \( \rho \) in the range \((0.11, 0.24)\) which seems implausibly rapid mean reversion. Meanwhile, direct estimation of the first-order autocorrelation in \( e_t \) for the two data sets yields values very close to 1; so the combination of (7) and (1) cannot explain the results.

IV. Conclusion.

A linear asset-pricing model relates the log of the nominal exchange rate to its expected rate of change (with parameter \( \alpha \)) and to fundamentals. The parameter \( \alpha \) may be
estimated by GMM without observing fundamentals under two assumptions: (a) fundamentals follow a martingale; (b) a change in exchange-rate policy is expected. The key feature which allows identification of $\alpha$ is that the expected regime change induces an additional nonlinearity in the path of the exchange rate, beyond that present in the path of fundamentals. In two historical applications with daily data, $\alpha$ is identifiable, negative, and statistically significant.

Monte Carlo methods could be used to examine the properties of the GMM estimators under other combinations of stochastic processes for fundamentals (including non-Markov or nonlinear ones) and regime changes. And other estimates of expected, exchange-rate changes (and hence $\alpha$) might be made, based on survey data or uncovered interest parity (and comparing volatilities of exchange rates and interest differentials as in Svensson (1991)). But the simplest interpretation of the results is that the present-value relationship is not a useful simplification in describing movements in nominal exchange rates. It remains to be seen whether alternative models of exchange-rate dynamics can account for negative coefficients in the linear, asset-pricing model.

References


Table 1: Estimation Results

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<tr>
<th>Period</th>
<th>Equation</th>
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<th>$\hat{\alpha}$</th>
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<td></td>
<td>(se)</td>
<td>(se)</td>
<td>(p)</td>
</tr>
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