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"Reverse" Intergovernmental Transfers Between Regions with Local Public Goods

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Abstract

We report on the nature of a utility optimizing transfer from one regional govenment to another when local public goods are present. Computer examples reveal that small differences in regional endowments result in large differences in equilibrium outcomes for two regions, under optimal transfers. The scale effect (lower tax charge per person for the same public good in more populous regions) leads to the small region generally providing transfers to the larger region.

- Key words: intergovenmental transfers, local public goods, inter-regional resource allocation
- JEL classification: H410, H770, R280

1 Introduction

We return to the matter of transfers from one regional government, with its own local public good, to another being carried out in order to counter sub-optimal, "free market" interregional resource allocation (Flatters, Henderson and Miezkowski [1974]); as for example, manifested by similar workers migrating to the "wrong" region. In fairly straightforward computer examples, we observe that a nominally "small", poorly endowed region can end up making a positive transfer to a large, seemingly well endowed region. Our result turns on two things: utility equalization by the costless migration of like workers can occur with considerable substitution of private consumption goods for local public goods across by individuals in distinct regions (in equilibrium, the per capita consumption of the local public good in one region can differ considerably in magnitude from that in another region) and secondly, the scale effect of reducing per capita charges for the local public good in a region by "adding" households can be large. This latter effect seems to drive our "reverse" transfer result (the "small", "poor" region making a transfer to the larger, seemingly better endowed region). Our transfers are made costlessly to each household in the receiving region.

There is no strategic play by local governments in our analysis. Hence our transfers do not neutralize potential strategic tax responses by a local government as in Kothenburger [2002] and Bucovetsky and Smart [2007]. And our transfers are not tailored to equalize per capita fiscal capacities as in Boadway[1] [2004; pp. 228-231]; nor are our transfers designed to equalize utility levels under the maximization of a Benthamite social welfare function (Hartwick [1980]). Our optimal transfer is not then a vehicle for compensating a region, poorly endowed with land, for its small endowment; rather the transfer is acting as an instrument for extending the scale effect in the provision of a local public good to a region with more taxpayers. The transfer follows from a large scale effect and also more welfare at the margin for our two region economy. We assume that free migration is equalizing utility levels for like migrants across regions and simply look into the size of transfers that will maximize the utility of a representative migrant. The optimal transfers do of course affect the equilibrium population distribution and allocation of $K-capital$ between any two regions and that is their role, as in Flatters,

[1] A sub-component of interprovincial transfers in Canada are equalization payments, flows to a provincial government designed to raise the per capita "fiscal capacity" of a lower income province. The equalization payments are funded with transfers from a pool of revenue based on taxes from all provinces but in fact turn out to be transfers from higher income provinces to lower income provinces. The payments are intended to assure "reasonably comparable levels" of health care, education, and welfare in all the provinces. In 2009-2010, the total amount of the program was roughly 14.2 billion Canadian dollars.

et. al. [1974][2]. Our contribution serves to emphasize that a system of inter-regional transfers can be tailored to meet a variety of objectives and these objectives may well conflict with each other. The fact that different objectives are in general associated with different designs of transfer schemes serves to alert us to the importance of being very clear in what each of us is doing: the real world has an abundance of systems of intergovernmental transfers and good design of a transfer system is itself a worthy objective.

Following Flatters, et. al. we invoke the Samuelson condition for determining the size of the public good in each region. We are of course able to "retrieve" the well-known optimal population condition: fund the local public good with local rentals. In our model however rentals include those from $K-capital$ as well as land and the classical optimal population condition needs to be supplemented with the optimal transfer condition in order for the classic funding result of Flatters et. al. to appear. For an "arbitrary" overall population, our optimal transfer condition is: the difference is total per capita rentals between regions must equal the difference in, net of per capita transfer, per capita tax charges between regions. In other words, the difference in per capita rentals between regions is capitalizing the difference in per capita net tax charges between regions. Relative fiscal capacity is showing up in the difference between per capita rentals in the two regions. In our simulations, our smaller region emerges with the "stronger" per capita fiscal capacity and ends up funding the transfer flowing to the larger region. We observe that the transfer is larger the more substitutability we allow in the production function and turns out to be larger, the smaller is the difference in the initial endowment of land to the two regions. In a purely private

[2] Mansoorian and Myers ([1993] and [1997]) have regions (governments are defined implicitly) play Nash strategies with respect to transfer setting and consider the question of a best resource allocation across regions. They do not have local public goods in their model.

goods two region economy (eg. Mansoorian and Myers [1993] and [1997]) we find that the characterization of the optimal transfer is very similar. Per capita liability for "taxes" is simply the transfer (there is no charge for a local public good) and per capita fiscal capacity continues to take the form of per capita rentals.

2 The Two Region Equilibrium

We have three inputs in each region: a fixed amount of labor to be spread over the two regions, a fixed amount of produced capital K to be spread over the two regions and a natural resource stock, say land, T^i, fixed in supply in each region. A superscript denotes a region. Assuming a transfer S given and free mobility of workers equalizing utility levels across the two regions, our equilibrium system is six equations:

$$U(\frac{F(N^1, K^1, T^1) - G^1 - S}{N^1}, G^1) = U(\frac{F(N^2, K^2, T^2) - G^2 + S}{N^2}, G^2), \tag{1}$$

where $U(.,.)$ is our positive-valued utility, increasing and concave in each argument, with $U(0, x) = U(y, 0) = 0$ for x and y positive values. When a person enters region i he or she immediately begins receiving an equal share of the local capital rent and land rent. This in a sense deprives current residents of some income. In addition we assume that the regional government in each region produces the respective government good in accord with the Samuelson public goods efficiency condition. That is, we have

$$U_{C^1} = N_1 U_{G^1} \tag{2}$$

$$U_{C^2} = N_2 U_{G^2} \tag{3}$$

where U_{C^1} indicates the derivative of the utility function with respect to the first argument, namely C^1; and so on for the other derivatives.

Produced capital is "spread" across the two regions to satisfy

$$F_{K^1} = F_{K^2}. \tag{4}$$

In addition we have input "exhaustion" relations

$$N^1 + N^2 = N \tag{5}$$

$$\text{and } K^1 + K^2 = K. \tag{6}$$

This is a six equation system in G^1, G^2, N^1, N^2, K^1 and K^2.

In each region, each person is maximizing her utility with budget constraint $w^1 + \frac{K^1 F_{K^1}}{N^1} + \frac{T^1 F_{T^1}}{N^1} - \frac{S}{N^1} = C^1 + \frac{G^1}{N^1}$ for region 1 and $w^2 + \frac{K^2 F_{K^2}}{N^2} + \frac{T^2 F_{T^2}}{N^2} + \frac{S}{N^2} = C^2 + \frac{G^2}{N^2}$. In region 1, $w^1 = F_{N^1}$ and in region 2, $w^2 = F_{N^2}$. An entrant to a region automatically loses possession of any capital income from the region she departed from and becomes the recipient of an equal per capita share of capital income $K^i F_{K^i} + T^i F_{T^i}$ for the region she is settling in. She also is liable for an equal per capita share of the transfer that her region is sending out. We comment more on the matter of the ownership of capital below.

3 The Utility Maximizing Value for S

Once we have solved our six equation system ((1) to (6)), we can in principle express each of G^1, G^2, N^1, N^2, K^1 and K^2 in terms of the current value of transfer S. One then asks for a value of S that results in each of $U(C^1, G^1)$ and $U(C^2, G^2)$ being a maximum, subject to $G^1(S), G^2(S), N^1(S), N^2(S), K^1(S)$ and $K^2(S)$ satisfying the equilibrium system. We simply maximize each of $U(C^1(S), G^1(S))$ and $U(C^2(S), G^2(S))$ with respect to S, using equilibrium conditions. The optimizations yield

$$\frac{U_{C^1}}{N^1}\{[F_{N^1} - C^1]\frac{dN^1}{dS} + F_{K^1}\frac{dK^1}{dS} - 1\} = 0 \tag{7}$$

$$\text{and } \frac{U_{C^2}}{N^2}\{[F_{N^2} - C^2]\frac{dN^2}{dS} + F_{K^2}\frac{dK^2}{dS} + 1\} = 0. \tag{8}$$

We can divide through by $\frac{U_{C^1}}{N^1}$ and $\frac{U_{C^2}}{N^2}$ in the respective equations. Also, since $\frac{dN^1}{dS} = -\frac{dN^2}{dS}$ and $\frac{dK^1}{dS} = -\frac{dK^2}{dS}$, we can substitute and sum the two expressions to get $[F_{N^1} - C^1] + F_{K^1}\frac{dK^1}{dS}\frac{dS}{dN^1} - \frac{dS}{dN^1} = [F_{N^2} - C^2] + F_{K^2}\frac{dK^1}{dS}\frac{dS}{dN^1} - \frac{dS}{dN^1}$. Since we have the equalibrium condition, $F_{K^1} = F_{K^2}$. This reduces to

$$[F_{N^1} - C^1] = [F_{N^2} - C^2] \tag{9}$$

(9) is the equation for the optimal value of transfer S. This suggests that free migration is not equalizing wages across the two regions, but a particular sort of net wage. We comment more on the absence of wage equalization below.

The sign of each difference in (9) will be the same and the sign turns on, for our purposes at least, the magnitude of N, aggregate population. Our new system of equations (1) to (6) plus (9) is now seven equations in G^1, G^2, N^1, N^2, K^1, K^2 and S. If we express $F(N^1, K^1, T^1)$ as $N^1F_{N^1} + K^1F_{K^1} + R^1$ ($R^1 = T^1F_{T^1}$ for the case of constant returns to scale) and $F(N^2, K^2, T^2)$ as $N^2F_{N^2} + K^2F_{K^2} + R^2$ ($R^2 = T^2F_{T^2}$ for the case of constant returns to scale) and recall that $C^1N^1 + G^1 + S = F(N^1, K^1, T^1)$ and $C^2N^2 + G^2 - S = F(N^2, K^2, T^2)$, then the condition for an optimal S in (9) implies that

$$[\frac{G^1 + S}{N^1}] - [\frac{G^2 - S}{N^2}] = \frac{[R^1 + K^1F_{K^1}]}{N^1} - \frac{[R^2 + K^2F_{K^2}]}{N^2}. \tag{10}$$

(10) indicates that the difference is net tax obligations for an individual in the two regions, namely $[\frac{G^1+S}{N^1}] - [\frac{G^2-S}{N^2}]$, equals the difference in per capita rentals, where the latter includes rent from $K - capital$.

In the absence of K capital in the model,[3] (9) is still $[F_{N^1} - C^1] = [F_{N^2} - C^2]$ but (10) reduces to

$$[\frac{G^1 + S}{N^1}] - [\frac{G^2 - S}{N^2}] = \frac{R^1}{N^1} - \frac{R^2}{N^2}. \tag{11}$$

[3] Our two production functions are $F(N^1, T^1)$ and $F(N^2, T^2)$ for this less complicated set up.

4 Simulation Results

We proceeded to solve our six equation system for N^1, K^1 and $U(C^1, G^1)$ $(= U(C^2, G^2))$ for various values of S, given a Cobb-Douglas specification for the utility function and a more general CES specification for the production function, the same, in each region. For $U(.,.) \equiv [C^i]^\alpha [G^i]^{1-\alpha}$ our six equation system (the "arbitrary" S case) reduces to two equations in N^1 and K^1. That is, for $N^2 = 20 - N^1$ and $K^2 = 8 - K^1$, our six equation system becomes

$$\frac{F(N^2, K^2, T^2) + S}{F(N^1, K^1, T^1) - S} = \left[\frac{N^2}{N^1}\right]^\alpha \text{ and } F_{K^1} = F_{K^2}$$

for $F(N^i, K^i, T^i) \equiv [\frac{2}{5}(N^i)^{-\theta} + \frac{1}{5}(K^i)^{-\theta} + \frac{2}{5}(T^i)^{-\theta}]^{-1/\theta}$.

For the case of S optimal, we had the option of varying S over a grid and selecting the value of S for utility a maximum or of solving the seven equation system with S endogenous. This seven equation system, with S endogenous, reduced to the three equation system

$$\frac{F(N^2, K^2, T^2) + S}{F(N^1, K^1, T^1) - S} = \left[\frac{N^2}{N^1}\right]^\alpha \text{ and } F_{K^1} = F_{K^2}$$
$$\text{and } F_{N^1} - C^1 = F_{N^2} - C^2.$$

For this three equation system, we could solve for S in the first equation and substitute in the third equation (recall that $C^1 = \frac{F(N^1,K^1,T^1)-S-G^1}{N^1}$ and $C^2 = \frac{F(N^2,K^2,T^2)+S-G^2}{N^2}$) to have two non-linear equations in two unknowns. Each of our two non-linear equation systems (one with S exogenous and one with S endogenous) solved readily with Matlab.

Our first case has the elasticity in the CES production function at 1/2 ($\theta = 1.0$). The production function is parameterized as $[\frac{2}{5}(N^i)^{-\theta} + \frac{1}{5}(K^i)^{-\theta} + \frac{2}{5}(T^i)^{-\theta}]^{-1/\theta}$ and the utility function is $(C^i)^{3/4}(G^i)^{1/4}$. Total population is given as $N = 20$ and total capital as $K = 8$. Region 1 has $T^1 = 4$ and region 2 has $T^2 = 2$.

Table 1

S	U	N^1	K^1	G^1	G^2	C^1	C^2	F_{N^1}	F_{N^2}
-0.05	0.4605	14.9653	5.5040	1.5456	0.6828	0.3098	0.4068	0.0672	0.1220
-0.2	0.46425	15.5985	5.5868	1.5985	0.6189	0.3074	0.4218	0.0631	0.1478
-0.3975	0.46503	16.5064	5.7347	1.6706	0.5213	0.3036	0.4477	0.0580	0.2020

The striking result for this simulation is that the "small" region (T^2 is smaller than T^1) is making a positive transfer to the larger region when the transfer has been optimized (at $S = -0.3975$). If we think here of per capita rentals signalling "fiscal capacity", we indeed observe that for S optimal the smaller region has per capita K rentals of 0.1558 compared to the value of 0.0835 for region 1 and again, the smaller region has per capita T rentals of 0.3529 compared to the value of 0.2393 for region 1. At S optimal, region 1 has attracted more than 80% of the labor and more than 70% of the mobile K. These large *ex post* "endowments" still do not yield a higher per capita land rent in region 1 or higher per capita rentals for $K - capital$. The wage in region 1 is less than 1/3 of that in region 2[4] and public good per capita in region 1 is more than five times the amount in region 2. For G^i/N^1 as per capita payment for the local public good, a person in region 1 has about 2/3 the payment of a person in region 2 (0.101 compared with 0.15). Region 1 is roughly speaking the low tax, public good abundant region.

We note that the values for per capita utility are close together for our various values of S. Hence for our parameters, a mis-setting of S is not a costly welfare mistake, provided the selected value is not too distant from the optimal value.

For our other experiments, the elasticity in the production function

[4]It is quite standard in spatial economics to have similar workers in different locations earn quite different wages. For example Ciccone and Hall [1996] analyze the very different labor productivity measures of similar workers across counties in the United States. They find that worker density works well in explaining the wide range of productivities. High wage cities are not only high productivity cities but they are also high housing cost cities. The high housing costs are capitalizing in land values the productivity premia exhibited by certain, typically large, cities. Glaeser and Mare [2001] analyze wage differences for similar workers across cities in the United States.

was varied toward more elastic. See Table 2. As θ was selected smaller, the size of the transfer from region 2 to region 1 increased and the labor and $K-capital$ resources in region 1 increased. There was a smooth transition in these "trends" over the elasticity at unity, $(\theta = 0)$.[5] Observe also that more elasticity in production (θ smaller) goes along with a higher utility level for people in the two regions.

Table 2

θ	S	N^1	K^1	Utility
0.5	-0.36489	17.1645	6.0389	0.497782
0.125	-0.39282	18.5400	6.6361	0.531090
-0.05	-0.39597	19.2811	7.0464	0.551595

The "reverse" transfer phenomenon (region with larger T^i is the receiver of the transfer) begs for further analysis. We proceeded to make the difference in the sizes of T^1 and T^2 small in order to see how relatively asymmetric our two regions emerged in equilibrium, under the optimal transfer. We proceeded to set $T^1 = 3.1$ and $T^2 = 2.9$ in place of our $T^1 = 4$ and $T^2 = 2$ above. Other parameters including the aggregate endowments of labor and $K-capital$ remained unchanged. A larger transfer from region 2 to region 1 emerged and the equilibrium exhibited considerable asymmetry, with noticeably more labor and $K-capital$ in region 1. See Table 3.

Table 3

θ	S	N^1	K^1	Utility	G^1	G^2	C^1	C^2
-0.05	-0.5152168	15.0564	5.1726	0.52643	1.7652	0.7657	0.35172	0.46464

Region 1 emerged as the place with abundant local public good and a smaller amount of per capita pricate goods consumption. Region 1 ended up with about 2/3 of the population and more than 5/8 of the $K-capital$. The large asymmetry in the equilibrium outcome "explains"

[5] Our computations were not satisfactory for cases of θ more negative than -0.05. We infer that numerical outputs with the constant elasticity production function "very elastic" do not compute in a straightforward way.

in part the need for a relatively large transfer. We infer that the transfer here is not providing "compensation" for the exogenous difference in land endowments but is rather reinforcing the scale effect associated with local public good provision in region 1. Roughly speaking "the economy" can generate extra welfare at the margin by taking advantage of the scale effect in the provision of local public goods, here in region 1, and the transfer provides an assist in generating the extra welfare.

As we observed earlier, per capita rentals in region 2 came out larger (in region 1 the per capita rentals for $K-capital$ and land were 0.08593 and 0.1675 compared with the respective values, 0.1431 and 0.2865 for region 2). The smaller region has more "fiscal capacity" measured by per capita rentals and does indeed end up supplying region 1 with a transfer. The wage in region 1 emerged lower (0.183 compared with 0.294). We infer that a small difference in local endowment T^i induces "extra" labor to the region with the larger T^i and this "extra labor effect" is magnified by the "scale economy" associated with the provision of the local public good. The more populous region can support the same G^i with lower per capita taxes and this induces an "extra" inflow of labor.

5 Optimal Population

Not surprising perhaps is the fact that familiar results from optimal population analyses fail to obtain in our model, with produced capital being "spread" across our two regions. We turn to the matter of overall population N being selected in order to maximize the utility of a representative household. Suppose then that we have solved our six equation system ((1) to (6)), given an interior solution for "arbitrary" values for S and N. We can in principle express each of the solved values G^1, G^2, N^1, N^2, K^1 and K^2 in terms of the current value of the aggregate population, N. One then asks for a value of N that results in each of $U(C^1, G^1)$ and $U(C^2, G^2)$ being a maximum, subject to $G^1(N), G^2(N), N^1(N), N^2(N), K^1(N)$ and $K^2(N)$ satisfying the equilibrium system. We simply maximize each of

$U(C^1(N), G^1(N))$ and $U(C^2(N), G^2(N))$ with respect to N, using equilibrium conditions. The optimizations yield

$$\frac{U_{C^1}}{N^1}\{[F_{N^1} - C^1]\frac{dN^1}{dN} + F_{K^1}\frac{dK^1}{dN}\} = 0$$
$$\text{and } \frac{U_{C^2}}{N^2}\{[F_{N^2} - C^2]\frac{dN^2}{dN} + F_{K^2}\frac{dK^2}{dN}\} = 0.$$

We can divide by $\frac{U_{C1}}{N^1}$ and $\frac{U_{C2}}{N^2}$ in each equation respectively. Since $\frac{dN^2}{dN} = 1 - \frac{dN^1}{dN}$ and $\frac{dK^1}{dN} = -\frac{dK^2}{dN}$, we can substitute and sum the two expressions to get $[F_{N^2} - C^2] + \{[F_{N^1} - C^1] + F_{K^1}\frac{dK^1}{dN}\frac{dN}{dN^1}\} = \{[F_{N^2} - C^2] + F_{K^2}\frac{dK^1}{dN}\frac{dN}{dN^1}\}$. Recall that $F_{K^1} = F_{K^2}$ and $[F_{N^1} - C^1] = [F_{N^2} - C^2]$ for the case of an optimal S. Hence with both S and N selected optimally, we infer that

$$[F_{N^2} - C^2] = 0. \tag{12}$$

And it follows that $[F_{N^1} - C^1] = 0$. We can then infer that

$$G^1 + S \ = R^1 + K^1 F_{K^1} \text{ and } [G^2 - S] = R^2 + K^2 F_{K^2}.$$

This generalizes the classic Henry George result to the case of $K-capital$ and S being optimal. I like to read this as total tax obligations in each region are covered by total rentals respectively. It follows of course that for the marginal mover from region i to region j, the difference in per capita tax obligations is precisely capitalized in the difference in per capita rentals.

If the model were set up without $K-capital$, then an optimal S and an optimal N would yield the precise Henry George result: land rent in each region equals the net aggregate tax payment in each region.

$$G^1 + S \ = R^1 \text{ and } [G^2 - S] = R^2. \tag{13}$$

6 Ownership Issues

Our people (households) are taken to be identical and thus besides ending up with the same utility in every place, one expects that their personal endowments would be similar. The tradition we are working in transfers the ownership of land and capital to each resident equally when a resident enters the region in question. This is not too bad an assumption in a multi-region model. When one changes regions, one gives up (possibly sells) one's holdings in the region being departed from and acquires (possibly buys) equal shares in the new region. The tradition we are working in treats the departer as walking away from its "owned" shares of land and produced capital and automatically acquiring equal shares per capita of land and produced capital in the region being moved to. This mechanical switching of ownership is thus not too bad a procedure for a multi-region model with all identical worker-households. Nevertheless one wonders about the implications of property rights being assigned at the outset and having these ownerships remain unchanged as say the marginal worker shifts her residence from one region to another. We will not take this "complete" ownership case up here because it is complicated[6] and what we have is fairly sensible. We can gain some insight into matters of not "symmetric" ownership by simply having some land in one region owned "abroad". This case "causes" new interesting input pricing issues to enter into the model.

Suppose then that fraction λ, between zero and unity, of land in region 1 is owned abroad. Let us work through with no produced capital, K in the model, to keep matters less cluttered. This implies that

$$C^1 = \frac{F(N^1,T^1) - G^1 - S - \lambda T^1 F_{T^1}}{N^1}.$$

We have a four equation system in G^1, G^2, N^1 and N^2. T^1 is assumed larger than T^2. The equal utility equation is now

$$U\left(\frac{F(N^1,T^1) - G^1 - S - \lambda T^1 F_{T^1}}{N^1}, G^1\right) = U\left(\frac{F(N^2,T^2) - G^2 + S}{N^2}, G^2\right).$$

We assume that we have an interior solution and that we have solved for G^1, G^2, N^1 and N^2. As before, we can express each solution value in

[6] When Wildasin [1986; pp. 66-70] exposits capitalization in cities of a fiscal benefit, he assumes explicitly that his relevant workers have only wage income.

terms of parameter S and proceed to solve for the utility maximizing value of S, a value contingent on the values of G^1, G^2, N^1 and N^2 being equilibrium values. The first order conditions for each region yield

$$F_{N^1} - C^1 - \lambda T^1 \frac{dF_{T^1}}{dS} = F_{N^2} - C^2.$$

There is a price effect $\lambda T^1 \frac{dF_{T1}}{dS}$ breaking the symmetry now. We can now express $F(N^1, T^1)$ as $N^1 F_{N^1} + R^1$ and $F(N^2, T^2)$ as $N^2 F_{N^2} + R^2$ and recall that $C^1 N^1 + G^1 + S + \lambda T^1 F_{T^1} = F(N^1, T^1)$ and $C^2 N^2 + G^2 - S = F(N^2, T^2)$. Then the condition for an optimal S implies that

$$\frac{[G^1 + S] - R^1 + \lambda T^1 [F_{T^1} - N^1 \frac{dF_{T1}}{dS}]}{N^1} = \frac{[G^2 - S] - R^2}{N^2}.$$

Novel now is the input price-effect term, $\lambda T^1 [F_{T^1} - N^1 \frac{dF_{T1}}{dS}]$. It is not clear what the sign of this term will be in general.

Our observation now is that "arbitrary" initial ownership arrangements, arrangements to maintain every household treated "equally", will introduce many price effects into our relatively simple equilibrium relationships when we consider the switch of a marginal household from one region to the other.[7] For the marginal mover in our two region model, we expect that the price effects caused by her move from one region to another will tend to cancel each other out, with losses from price effects in one region being offset by gains in the other region, on average over the two regions, household by household. Hence in a many region, many household model set up to treat all households relatively equally, "arbitrary" initial and unchanging ownership assignments should not cause large departures from the equilibrium conditions that we are obtaining. This observation gains strength if we always consider the movement of a marginal household from one region balanced with that of a marginal household from the other region.

[7] Mansoorian and Myers [1997] introduce a preference for a particular location in their workers and are able to work with a unique marginal worker for each region. We could proceed in their fashion without difficulty.

There is another way to think about somewhat heterogeneous ownership arrangements. There is nothing in our general analysis or in our digression into outside ownership issues which suggests that anything different from our capitalization thinking requires serious consideration. In particular there appears to be no opening for a consideration of free migration equalizing net fiscal benefits in models of the kind we are dealing with.

7 The Private Goods Case

We now have three inputs in each region: a fixed amount of labor to be spread over the two regions, a fixed amount of produced capital K to be spread over the two regions and a natural resource stock, say land, T^i, fixed in supply in each region. Assuming a transfer S given and free mobility equalizing utility levels of people across the two regions, our system is six equations:

$$U(\frac{F(N^1,K^1,T^1)-G^1-S}{N^1},\frac{G^1}{N^1})=U(\frac{F(N^2,K^2,T^2)-G^2+S}{N^2},\frac{G^2}{N^2}), \tag{14}$$

where $U(.,.)$ is our positive-valued utility, increasing and concave in each argument, with $U(0,x)=U(y,0)=0$ for x and y positive values. When a person enters region i he or she immediately begins receiving an equal share of the local capital rent and land rent. This in a sense deprives current residents of some income. Within each region, "prices" satisfy"

$$U_{C^1}=U_{(G^1/N^1)} \tag{15}$$

$$U_{C^2}=U_{(G^2/N_2)} \tag{16}$$

where U_{C^1} indicates the derivative of the utility function with respect to the first argument, namely C^1; and so on for the other derivatives. Produced capital is "spread" across the two regions to satisfy

$$F_{K^1}=F_{K^2}. \tag{17}$$

(17) indicates the equality of rentals for K across our two regions. In addition we have input "exhaustion" relations

$$N^1 + N^2 = N \tag{18}$$

$$\text{and } K^1 + K^2 = K. \tag{19}$$

This is a six equation system in G^1, G^2, N^1, N^2, K^1 and K^2.

Suppose that an interior solution exists and that we have solved our system. Each solution value can be expressed as a function of the parameter S as with $G^1(S), G^2(S), N^1(S), N^2(S), K^1(S)$ and $K^2(S)$ satisfying the equilibrium system. We now consider the value of the transfer S that maximizes the representative household's utility, region by region. For each region the first order condition is

$$\frac{U_{C^1}}{N^1}\{[F_{N^1} - C^1 - G^1/N^1]\frac{dN^1}{dS} + F_{K^1}\frac{dK^1}{dS} - 1\} = 0 \tag{20}$$

$$\text{and } \frac{U_{C^2}}{N^2}\{[F_{N^2} - C^2 - G^2/N^2]\frac{dN^2}{dS} + F_{K^2}\frac{dK^2}{dS} + 1\} = 0. \tag{21}$$

We can divide our respective equations through by $\frac{U_{C^1}}{N^1}$ and $\frac{U_{C^2}}{N^2}$. Recall that $F_{K^1} = F_{K^2}$. Since $\frac{dK^2}{dS} = -\frac{dK^1}{dS}$ and $\frac{dS}{dN^2} = -\frac{dS}{dN^1}$, the when we sum our two equations, we get

$$\{F_{N^1} - C^1 - \frac{G^1}{N^1}\} = \{F_{N^2} - C^2 - \frac{G^2}{N^2}\}. \tag{22}$$

Wage income net of current expenditure on consumption plus housing is equal in region 1 to the comparable value in region 2. Since $N^1F_{N^1} + K^1F_{K^1} + R^1 = N^1C^1 + G^1 + S$ and $N^2F_{N^2} + K^2F_{K^2} + R^2 = N^2C^2 + G^2 - S$ for our respective regions, condition (22) can be expressed as $[\frac{S}{N^1} - \frac{S}{N^2}] = \frac{[R^1+K^1F_{K^1}]}{N^1} - \frac{[R^2+K^2F_{K^2}]}{N^2}$. Hence the inference that the optimal transfer, S^* leads to the capitalization of per capita "tax" obligation differences between regions, $[\frac{S}{N^1} - \frac{S}{N^2}]$ in per capita rental differences, $\frac{[R^1+K^1F_{K^1}]}{N^1} - \frac{[R^2+K^2F_{K^2}]}{N^2}$.

The concept of an optimal population for a multi-region economy with purely private goods is not as straight forward as that for a multi-region economy, each region with its own local public good. We do not pursue the optimal population matter for the case of our economy with only private goods.

8 Concluding Remarks

The central contribution of Flatters, Henderson and Miezkowski [1974] was to make clear than free migration of workers among regions, each with a local public good, would lead to outcomes that could be improved upon by the introduction of transfers from one region to another. Here we characterize the utility optimizing transfer in a model with $K-capital$ in addition to land and observe in simulations the dramatic impact of the scale effect in the provision of local public goods. The scale effect turns on per capita "tax charges" being small for a given level of the local public good in a region with a large number of workers. The scale effect is sufficiently strong as to lead generally to the small region in our analysis providing a transfer to the larger region. Thus the optimal transfer turns out not to be a vehicle for compensating a "poor land" region for its small endowment, rather the transfer is acting as an instrument for extending the scale effect in the provision of a local public good. The transfer leads to a large scale effect and more welfare at the margin for our two region economy. Though the large population in the region with the "low per capita cost" for the public good makes for an abundance of the local public good in that region, it also leads to relatively low wages in that larger region. And the smaller region ends up with relatively large per capita rentals for land and $K - capital$ and it is these per capita rentals that contribute crucially to funding local tax charges including those for transfers from the high per capita rental region to the other region. Of interest then is our characterization of the optimal transfer; but of more interest are the curious equilibrium outcomes that we simulated for economies with optimized transfers.

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