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THE MEANING AND MEASUREMENT OF AGGREGATE TECHNICAL CHANGE

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The Meaning and Measurement of Aggregate Technical Change[1]

Statistics of aggregate technical change have proved to be disconcertingly sensitive to adjustments in the measures of labour, capital and output. Estimates made in the late nineteen-fifties put the rate of aggregate technical change in the United States over the years 1900 to 1950 at about 1.6% per year.[2] This figure implies that most of the observed economic growth in this century is due to technical change rather than capital formation, and it suggests that there may be substantial externalities from research and education, and it presents us with an encouraging prospect for economic growth in the future as long as trends continue. New evidence culminating in an article by Jorgenson and Griliches alters the picture substantially,[3] and all but eliminates technical change as an explanation of observed trends. By correcting the data for errors in the pricing of capital goods, for changes in the skill composition of the labour force, for relative utilization of factors of production, and for errors in aggregation of capital services of labour services and of income, Jorgenson and Griliches reduce the estimate of technical change between 1945 and 1965 from 1.6% to virtually nothing (0.1%). Though the calculation is for only one period of time in one country, Jorgenson and Griliches' work is in the nature of a pilot study, and it is likely that their results could be duplicated for other times and places.

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(1) I am indebted to Mr. Brian Near and to Mr. Alan Coombs for doing the calculations, and to the Arts Research Council of Queen's University for financial support.

(2) R. M. Solow "Technical Change and the Aggregate Production Function" Review of Economics and Statistics, 1957. See also J. W. Kendrick, Productivity Trends in the United States, Princeton University Press, 1961.

(3) D. W. Jorgenson and Z. Griliches "The Explanation of Productivity Change". Review of Economic Studies, 1967.

Jorgenson and Griliches' findings have profound implications about the nature and prospects of economic development. First, there is no residual growth of income over and above what can be explained by the growth of labour and capital. The 1965 national income could have been generated out of the 1900 production function using a labour force equal in man-hours to that of 1965 and a stock of human and non-human capital no greater than could have been purchased at 1900 prices with the cumulated savings between 1900 and 1965. The commonly held belief that technical change is an indispensible component of prosperity would seem to be an error caused by misspecification of statistics and misuse of index number formulae. Second there is no trace in the statisitics of externalities to research and education, and "discrepancies between private and social returns to investment in physical capital may play a relatively minor role in explaining economic growth".(4) Third, and most disturbing of all, the absence of technical change implies that economic growth cannot continue indefinitely. Jorgenson and Griliches have fitted to the American economy a production function which is similar in relevant respects to a Cobb-Douglas production function with depreciation and without technical change. It is a well-known property of this function that it entails a maximum value of consumption per head, and that at any rate of saving the economy represented by this function will in time settle into a stationary state from which no further growth is possible.

In this paper I propose to show that Jorgenson and Griliches' evidence does not bear these implications, and that the inferences drawn from the earlier studies were largely correct.(5) I think that the

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(4) Jorgenson and Griliches, Ibid., p. 274.

(5) For a critical examination of the detail of Jorgenson and Griliches' calculation see E.F. Denison "Some Major Issues in Productivity Analysis An Examination of Estimates by Jorgenson and Griliches "Survey of Current Business, May, 1969, Office of Business Economics, United States Department of Commerce.

confusion over the magnitude of aggregate technical change occurs because the conventional formula for measuring technical change diverges in some respects from the meaning of the phrase as understood by most economists, economic historians, civil servants, businessmen etc.; and that when the confusion over the meaning of the term is cleared up, many of the problems of measurement disappear.

Jorgenson and Griliches do not describe the object of their measurement as technical change. They describe it as productivity change which may not be the same thing. Productivity change is defined conventionally as the rate of growth of output per unit of input where output is the national income and input is labour and capital divided into as many subgroups as the data allow and aggregated by factor prices in the base period or by a chain-link index number formula. If income, capital and labour can be thought of as homogeneous, the rate of productivity change is essentially the value of r in the equation

$$Y_R = A\, e^{rt}\, f\,(L,K) \qquad (1)$$

where Y_R is real income, K is the capital stock and L is the labour force. In computing the rate of productivity change, capital is measured in efficiency units. Two tractors that can do the same job are counted as equal amounts of capital no matter when they were made or how much consumption was foregone in making them.

In assessing the suitability of "the rate of growth of output per unit of input" as a measure of technical change it is important to recognize that capital, which is an ingredient of both input and output, is in the long run an intermediate rather than a final product. Capital is linked twice to consumption in the sequence:

$$C \rightarrow K \rightarrow C$$

Capital first appears as part of output and its relative price depends on the amount of consumption that must be foregone to create the capital. Then capital appears as part of input, combined with labour to produce new consumption goods and new capital goods. Technical change as the term is commonly understood may occur at either link in the chain. Technical change in the first link is the cheapening of capital goods relative to consumption goods. Technical change in the second link is an increase in the productivity of capital in making consumption goods. The sources of technical change are fully substitutable in the mind of a rational consumer. In an economy without labour, in which the consumption good is apples and the capital good is axes, a rational man is indifferent in the long run between a technical change that reduces by half the number of apples that need be foregone to create one new axe, and a technical change that doubles the number of apples that can be produced with a given stock of axes.

Because the sources of technical change are substitutes, it is important in constructing productivity statistics that the measure of capital as an input be no larger than the cumulated value of capital formation as part of output (reduced appropriately to account for depreciation), for otherwise the productivity measure excludes an essential component of technical change. Similarly if it is discovered that the growth of the capital series has been underestimated, corrections must be applied both to the capital stock and to new capital formation. Jorgenson and Griliches recognize this point in principle but I do not think they make full allowance for it in their computations.

The relation between capital as input and capital as output is reflected correctly in Jorgenson and Griliches' recomputation of the stock of capital in roads. The real stock of roads is estimated in the national accounts as the value of constant prices of the inputs to road-building, labour cement, tar, machines etc. Evidence is presented that the growth of the quantity of roads is understated in the accounts because, over time, engineers have learned to convert a given mix of labour and materials into an ever-larger quantity of roads of a standard quality and design. Jorgenson and Griliches make the appropriate increases both to the growth of the stock of roads as part of capital stock and to the growth of capital formation in roads. Similar alterations are made for other types of capital equipment.

Other changes in the measurement of the capital stock were not treated in the same way. For instance, the largest reduction in measured productivity change is the result of corrections for the skill composition in the labour force. Capital as a factor of production was augmented to allow for the influence of the growth of human capital, but the time-series of new capital formation appearing as a component of income was not augmented accordingly. Part of the wage differential among workers might be attributed to education which is treated in income as consumption when it ought to be treated as investment. The part due to on-the-job training, industrial experience, or higher than normal returns to education is counted in capital as input and excluded from capital as output.

In addition to the accounting problem of ensuring that there is a correspondence between measures of capital and of capital formation, there is a problem of aggregation in combining rates of technical change in the C→K link and the K→C link into one measure of technical change for the economy as a whole. An economy must consist of at least two industries, one to produce consumption goods and another to produce capital goods. Rates of productivity change in these industries must differ if the relative price of capital and consumption is changing over time.(6) The conventional measure of aggregate productivity change automatically averages rates of change in the two industries by shares of expenditure, a procedure that would be correct if new capital formation could be thought of as a second type of consumption good, but is incorrect for aggregating rates of technical change in an industry producing a final product and an industry producing an intermediate product. Other aggregation procedures discussed below are more appropriate for dealing with capital considered as an intermediate product in the long run.

The original factors of production in an economy are not labour and real capital, but labour and accumulated consumption foregone. Certainly accumulated consumption foregone is a kind of capital, but its natural dimension is consumption units rather than efficiency units.(7) Capital as

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(6) It is assumed that the capital-labour ratio is constant.

(7) The history of the distinction between efficiency units and units of consumption foregone in the measurement of capital is discussed in Professor T.K. Rymes as yet unpublished essay "Professor Hicks and Concepts of Capital and the Production Function". When we speak in this paper of consumption foregone we refer not to the amount of consumption that must be foregone today to create a piece of capital but to the consumption that was actually foregone when the capital was made. Two identical pieces of equipment made at different times would be counted as different amounts of capital if the price of capital changed in the interval. But no attempt would be made to say how much capital resides in any single piece of equipment. Capital in units of consumption foregone is a measure of accumulated sacrifice of consumption.

accumulated consumption foregone is measured as a stock of apples rather than as a stock of axes, for the size of the stock of axes may in part be a consequence of the rate of technical change in the $C \rightarrow K$ link. A case can be made for measuring capital as a component of productivity change in consumption units instead of in efficiency units. To do so is to look upon the economy as if output were a homogeneous stuff that could be consumed or accumulated as capital, and as if technical change occurred exclusively in the $K \rightarrow C$ link of the chain between consumption foregone and consumption created. There could be no problem of aggregation because there would be only one source of technical change. The accounting problem of co-ordinating measures of capital as input and as output also disappears. Computations are simplified enormously because it is easier to cumulate and depreciate consumption foregone than to measure the real capital stock in efficiency units.

Simplicity is undoubtedly an advantage but the final test of whether a statistic is good or bad, useful or not, is the correspondence between the statistic and what the statistic is supposed to measure. In assessing candidates for the measure of technical change we cannot escape asking what characteristic of the economy it is that the measure is expected to describe. We need a specification of aggregate technical change that is not initially an index number and against which competing index-numbers may be evaluated. I suggest that the defining characteristic of technical change is that without it there can be no permanent growth of consumption per head. Capital formation may increase consumption per head for a time, but the diminishing marginal product of capital causes each additional bit of capital to have a progressively smaller effect on income per head, and if the labour force is growing or if capital depreciates there is a maximum capital - labour ratio beyond which additional capital decreases rather than increases the availability of consumption goods. I submit as a definition of technical change "the rate of growth of consumption per man employed that can be sustained permanently at a constant rate of saving." The definition pertains to the growth of consumption rather than to the growth of income because it is consumption rather than income that is the object of economic activity.

As we are considering two definitions of technical change, it is convenient to refer to them as I and II.

Definition I is: technical change is the rate of growth of total factor productivity, or the rate of productivity change as defined by Jorgenson and Griliches.

Definition II is: technical change is the rate of growth of consumption per man employed that can be sustained at a constant rate of saving.

The manner in which definition II leads to a procedure for aggregating technical change in the $C \rightarrow K$ link and in the $K \rightarrow C$ link will be illustrated by means of a simple growth model. Then some estimates of technical change will be presented, and it will be shown the rate of technical change according to definition II (and according to definition I when measures of capital and capital formation are kept rigidly in line) is more or less what Kendrick and Solow claimed it to be. Finally some weaknesses of definition II will be discussed.

A Model of Technical Change as a Compound of Productivity Growth and the Cheapening of Capital Goods

To show how the definition as aggregate technical change as sustainable growth of consumption per man employed (definition II) picks up technical changes at both links of the chain $C \rightarrow K \rightarrow C$, a simple model is constructed in which technical change can occur either as disembodied productivity change similar but not identical to the r in equation 1 or as the cheapening of new capital goods relative to consumption goods. It is supposed that technical changes at both sources occur at a constant rate.

The model contains six equations: There is a production function with technical change embodied in labour

$$Y = F\,(Le^{\beta t}, K) \qquad (2)$$

where Y is income measured in apples

K is capital measured in axes

L is labour measured as a number of men

β is the rate of growth of productivity embodied in labour, and the usual assumptions are made about the signs of the first and second derivatives. There is an investment function

$$I = sY \qquad (3)$$

where I is investment measured in apples per year and s a constant marginal propensity to save. The rate of change of the capital stock, K, is indicated by the equation

$$P\dot{K} = I \qquad (4)$$

where P is the price of capital, measured as apples per axe.

The price of capital changes cver time as indicated by the equation

$$P = P_o e^{-\delta t} \qquad (5)$$

where P_o is the price of capital at time $t = 0$

and δ is the rate of decrease of P. The labour force is

$$L = L_o e^{nt} \qquad (6)$$

where L_o is the supply of labour at time $t = 0$

and n is the rate of growth of the labour force. The variables Y, I, L, K, $\dot{K}$ and P are functions of time, and all other terms are constants.

It is a simple matter to work out the rate of steady-state growth of consumption per man in this system. Following Fellner's usage[(8)] the rate of growth of a variable x will be written as G_X. In steady growth, the growth rate of capital must be constant. From equations 3 and 4, this rate is seen to be

$$G_K = \frac{\dot{K}}{K} = \frac{sY}{PK} \qquad (7)$$

Since G_K is constant, the growth rates of the components of G_K, names s, Y, P and K, must satisfy the equation

$$G_s + G_Y - G_P - G_K = 0 \qquad (8)$$

or since s is constant

$$G_Y = G_K + G_P = G_K - \delta \qquad (9)$$

Differentiating Equation 2 with respect to t and dividing the result by Y, it follows that

$$G_Y = \beta\alpha + \alpha G_L + (1 - \alpha) G_K \qquad (10)$$

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(8) W. Fellner, "Technological Progress and Recent Growth Theories", American Economic Review, 1967.

where α is defined as $^{LF}L/_Y$ (labour's share of total output) and is constant in steady-state growth. From Equation 9 and 10 it follows that the rate of aggregate technical change according to definition II is

$$G_{\frac{C}{L}} = G_{\frac{Y}{L}} = \frac{\alpha\beta + (1-\alpha)\delta}{\alpha} \qquad (11)$$

It is a weighted sum of productivity growth, β , and the rate of cheapening of capital goods, δ. Neither productivity growth, β , nor the rate of cheapening of capital goods, δ , is by itself a sufficient indicator of technical change, because a rational man is indifferent to the size of β and to the size of the capital stock, if δ is adjusted to hold $G_{\frac{C}{L}}$ constant, but any change in $G_{\frac{C}{L}}$ has a material effect of well-being.(9) Suppose that without any change in the history of the growth of real income, or the growth of the labour force, or the percentage of income invested, it is discovered that capital in roads has grown faster than had been believed; the quantity of new roads each year had been estimated as the value of constant prices of the inputs to road building, and it is learned that given amounts of labour, capital, and raw materials in road-building

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(9) For convenience, depreciation has been excluded from the model, but the inclusion of a fixed rate of depreciation would not affect the estimate of the rate of technical change. To include depreciation at a rate d, Equation 4 would be changed to $\dot{K} = {}^{I}/_{P} - dK$. The rate of growth of capital would be $G_K = sY/_{PK} - d$ and the rest of the derivationof proceeds unchanged.

It is a simple matter to prove that the equilibrium rate of growth of consumption per man, $G_{\frac{C}{L}}$, is unique and stable.

are transformed into an ever larger quantity of roads as time goes on. Undoubtedly real capital is larger than had been supposed, and the growth of total factor productivity is correspondingly less, but the estimate sustainable growth of consumption per man is unchanged because the higher growth of capital coupled with an unchanged growth of real income and an unchanged rate of saving implies an increase in the value of δ just sufficient to offset the fall of β in the formula

$$\frac{\alpha\beta + (1 - \alpha)\ \delta}{\alpha}$$

Though there are no restrictions on the form of the production function, F in this model, other than the usual ones on the signs of its first and second derivatives, it is essential that technical change be assumed labour-augmenting for otherwise the system will not eventually enter a golden-age and there may be no stable value of $G_{\frac{C}{L}}$.(10) In practice this is not a serious limitation for the rate of technical change can be measured as if technical change were labour augmenting. Suppose we are given time series of income, the price level of consumption goods, labour, capital, the share of income accruing to labour, and the rate of saving. A slight manipulation of Equation 10 above yields an estimate of β:

$$\beta = \frac{G_Y}{\alpha} - G_L - \frac{1 - \alpha}{\alpha} G_K \qquad (10')$$

where Y represents income valued in units of consumption goods (money income divided by the consumer price index) and all of the variables are measured at year t. (For example, G_K is not the equilibrium growth rate of

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(10) This proposition, due originally to H. Uzawa, is proved in E.S. Phelps, Golden Rules of Economic Growth, P. 18. However, any form of technical change, embodied or disembodied, is acceptable if F is Cobb-Douglas because, with this function, it is possible to convert disembodied technical change or technical change embodied in capital into technical change embodied in labour.

K but the actual rate of growth during the year t.) Since $P = {}^{sY}/_{\Delta K}$,

$$(1+\delta) = \frac{P_t}{P_{t-1}} = \frac{s_t}{s_{t-1}} \frac{Y_t}{Y_{t-1}} \div \frac{\Delta K_t}{\Delta K_{t-1}} \tag{12}$$

The parameter, δ , can be estimated from Equation 12 because all of the terms on the right hand side of the equation refer to available statistics. Once β and δ are estimated in this way they may be combined in accordance to formula 11 to yield a measure of the rate of aggregate technical change.

Nothing in these derivations requires that the values of β and δ remain constant over time. Equations 11 and 12 yield estimates of these values in the year t. The estimated rate of technical change is the growth of income per man that would be sustained if β and δ remained constant. There is no contradiction or inconsistency in supposing that values of β , δ and the rate of aggregate technical change vary over time.

If $\delta = 0$, the rate of technical change is simply β which in a Cobb-Douglas production function is $\frac{r}{\alpha}$ where r is the rate of growth of total factor productivity in Equation 1. In this case definition I and definition II are virtually the same, and definition II can be thought of as a generalization of definition I. The parameter β from Equation 2 differs from $\frac{r}{\alpha}$ if the production function is not Cobb-Douglas or of $\delta \neq 0$. If $\delta \neq 0$, Equations 1 and 2 differ in their dimensions. The term Y_R in Equation 1 is real income, an index number in which consumption and capital formation are valued at constant prices. The term Y in Equation 2 is income measured in consumption units.

A model in which the price of capital falls over time is a vintage model in disguise. Recall that Solow[11] defined a vintage model as one in which new capital measured in consumption units becomes more efficient over time. To say that new capital measured in consumption units becomes more efficient is to say that new capital measured in efficiency units becomes cheaper - precisely the situation described by Equation 5 in our model. Setting $\beta = 0$, it is a simple matter to derive our model from Solow's vintage model, and vice versa. Our model is somewhat more general than Solow's vintage model because technical can be either disembodied at a rate $\alpha\beta$ or embodied in new capital at a rate δ .

Our model can also be looked upon as a two-sector model. If Equation 2 is given a Cobb-Douglas form

$$Y = A\,(Le^{\beta t})\;K^{1-\alpha} \qquad (2')$$

it is easily shown that our model is equivalent to

$$C = A_c\, e^{\lambda_c t}\, L_c^{\alpha}\; K_c^{\;1-\alpha} \qquad (13)$$

$$\dot{K} = A_I\, e^{\lambda_I t}\; L_I^{\alpha}\; K_I^{\;1-\alpha} \qquad (14)$$

where $K_I + K_c = K$, $L_I + L_c = L$, $L = L_o E^{nt}$, $Y = C + P\dot{K}$, and the relative price of capital goods and consumption goods is

$$P \equiv \frac{\partial C}{\partial \dot{K}} \qquad (15)$$

This is not a full-fledged two-sector model because factor shares are the same in both sectors and the production functions differ only in the rates of technical change. It is what might be called a

(11) R.M. Solow "Investment and Technical Progress" in K. Arrow, S. Karlin and P. Suppes, Mathematical Models in the Social Sciences, Stanford University Press, 1960.

one and one-half sector model; it is like a one-sector model in its usage of factors of production and like a two-sector model in the composition of output. The correspondence between the terms in the original version of the model and in its two-sector version is:

Original Version		Two-Sector Version
A	$\longleftrightarrow$	A_c
P_o	$\longleftrightarrow$	A_c/A_I
β	$\longleftrightarrow$	λ_c/α
δ	$\longleftrightarrow$	$\lambda_I - \lambda_c$
$\frac{\alpha\beta + (1-\alpha)\delta}{\alpha}$	$\longleftrightarrow$	$\frac{\alpha\lambda_c + (1-\alpha)\lambda_I}{\alpha}$

The two-sector version of the model points up an interesting difference between aggregation procedures in the definitions of technical change. In definition II, rates of technical change in the industry making consumption goods and the industry making new capital goods are weighted by factor shares, α and $(1-\alpha)$ respectively. Definition I treats capital formation as if it were a second kind of consumption. When the time-series of real income used in estimating r in equation 1 is constructed by combining increments of consumption and new capital formation in a chain-link index-number formula, the rates of technical change in the industry making consumption goods and the industry making capital goods are weighted not by factor shares but by shares of expenditure, (1-s) and s respectively.

According to definition I the rate of aggregate technical change is

$$r = (1 - s)\,\lambda_c + s\,\lambda_I \tag{16}$$

According to definition II the rate of aggregate technical change is

$$G_{\frac{C}{L}} = G_{\frac{Y}{L}} = \frac{\alpha\lambda_c + (1 - \alpha)\,\lambda_I}{\alpha} \tag{17}$$

The α in the denominator of definition II is not a fundamental source of difference between these definitions because it merely converts a measure of disembodied technical change into a measure of technical change embodied in labour. The difference in the weighting systems is important. Domar[(12)] has shown that there are two fundamental ways of aggregating technical changes in sectors of an economy. Rates of technical change in the production of final products should be averaged over shares of expenditure. Rates of technical change in an industry producing a final product and an industry producing an intermediate produce are added; aggregate technical change is the rate in the industry making the final product <u>plus</u> $1 - \alpha$ times the rate in the industry making the intermediate product. The significance of the difference in the weighting systems in definitions I and II is that the weighting system in definition I is appropriate for aggregating over final products and the weighting

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(12) E.D. Domar "On the Measurement of Technological Change", <u>Economic Journal</u>, 1961.

system in definition II is appropriate for aggregating over a final product and an intermediate product. To see this point, consider Equations 13 and 5 as the principle equations in a two-sector model

$$C = A_c e^{\lambda ct} L_c^{\alpha} K_c^{1-\alpha} \qquad (13')$$

$$\Delta K = P_o e^{\delta t} \Delta C \qquad (5')$$

If we could ignore[13] the subscripts on L and K and the Δ on K, these equations would describe an economy in which consumption C is made with primary factors labour L and consumption foregone Δ C, and capital is intermediate. Eliminating capital yields one production function with a rate of disembodied technical change of $\lambda_c + (1-\alpha)\delta$, equivalent to a rate of

$$\frac{\alpha\lambda_c + (1-\alpha)\lambda_I}{\alpha} = G_{\frac{Y}{L}}$$

embodied in labour.

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(13) Using a vintage version of our model, it can be shown how the Δ in ΔK and the subscripts in L_c and K_c may be ignored without departing from rigour.

The Sensitivity of Technical Change to the Rate of Growth of The Capital Stock

To definition I

$$r = (1 - s)\lambda_c + s\lambda_I$$

and definition II

$$G_{\frac{C}{L}} = \frac{\alpha\beta + (1 - \alpha)\delta}{\alpha}$$

we add definition III which is simply

$$\lambda_c$$

and compare the sensitivity of the three measures to assumptions about changes over time in the relative price of capital goods. Definition III is not one that could be advocated seriously, and is introduced to show what happens when the quantity of capital is changed and the quantity of capital formation is not.

The measures of technical change derived from these definitions are compared in a simple calculation using American data for the years 1900 to 1965. It has been argued that in the long run what is important to consumers is the growth of income and the sacrifice of potential consumption to create capital; given these facts, the size of β and of the capital stock is immaterial, for a large capital stock corresponds to a low value of β and small capital stock corresponds to a high value of β. The calculation will show how sensitive each measure of technical change is to assumptions about the growth of the capital stock corresponding to a given history of the growth of income in consumption units and of the rate of saving.

The data(14) in the calculation are time series of:

(14) The data are from Long Term Economic Growth 1960-1965 U.S. Department of Commerce, Bureau of the Census, 1966. The series used were series A7, A8, B91, B65, B66, B1, B2, A108, A109, A127. The capital stock, originally valued at 1958 prices, is converted by the price deflator to 1900 prices.

$Y_\$$ gross national product in current dollars.

$S_\$$ saving in current dollars.

D the implicit price deflator for consumption goods normalized so that D (1900) = 1.

U the rate of unemployment of labour.

L the labour force.

and K(0) the cpaital stock in 1900.

Alternative assumptions were made about δ, the rate of cheapening of capital goods; δ was given values between 4% and -2% in steps of .5%. A number of rates of depreciation, d, and values of labour's share, α, were tried; the numbers used in constructing Table 1 were 10% for d and 2/3 for α.

For every value of δ, the capital stock in each year t + 1 was estimated recursively by the formula

$$K(t+1) = K(t)(1-d) + \frac{S_\$(t)}{D(t)\, e^{-\delta t}} \tag{18}$$

Income in consumption units, Y (t), was estimated as

$$Y(t) = Y_\$(t) \div D(t) \tag{19}$$

Real income, Y_R was estimated as

$$Y_R(t) = \{Y_\$(t) - S_\$(t) + S_\$(t)\, e^{\delta t}\} \div D(t) \tag{20}$$

the sum of consumption and new capital formation at 1900 prices.

By simple regressions, λ_c, equal to $\alpha\beta$, was estimated as the rate of growth of

$$Y(t) \div \{L(t)^{\alpha} K(t)^{1-\alpha}(1-U(t))\}$$

and r (equal to $\lambda_c (1-s) + \lambda_I s$) was estimated as the rate of growth of

$$Y_R(t) \div \{L(t)^{\alpha} K(t)^{1-\alpha}(1-U(t))\}$$

Results of the calculation are presented in Table 1.

Depending on the postulated value of δ , the growth rate of the capital stock varies from 4.8% down to -0.6%, and total factor productivity, λ_c , varies from 0.4% to 2.4%. Though Jorgenson and Griliches did not measure income in consumption units, they succeeded in reducing the estimate of technical change from 1.6% to 0.1% for much the same reason that λ_c varies in accordance with δ , namely that some changes in the size of the stock of human and non-human capital are not accounted for in new capital formation. When income is corrected to make capital formation correspond to the growth of capital, the estimate of the rate of technical change becomes much more stable, varying only between 1.6% to 2.2%. The estimate of technical change as sustainable growth of consumption per man employed is very stable, and would be completely invariant if the economy were in a state of steady growth. All three estimates of technical change are the same when $\delta = 0$, if allowance is made for the fact that definition II is of technical change embodied in labour while the other definitions are of disembodied technical change. The sensitivity of $G_{\frac{C}{L}}$ to changes in δ is a decreasing function of both α and d. When d = 2% and α = .5, $G_{\frac{C}{L}}$ varies between 3.35% and 2.04%. When d = 15% and = .75% $G_{\frac{C}{L}}$ varies between 2.323% and 2.343%. For every value of and d that I examined $G_{\frac{C}{L}}$ was less sensitive than r to changes in δ . The estimate of $G_{\frac{C}{L}}$, when converted to a rate of disembodied growth is only slightly larger than Solow's and Kendrick's estimates of total factor productivity.

TABLE I

A Comparison of Estimates of Technical Change when Labour's Share is 2/3 and Depreciation is 10%

Assumed Rate of Cheapening of Capital Goods δ (%)	Estimated Annual Growth Rate of the Capital Stock (%)	Definition III Rate of Growth of Total Factor Productivity when Income is in Consumption units λ_c (%)	Definition I Rate of Growth of Total Factor Productivity when Income is corrected for the Price of Capital Goods $\lambda_C(1-s)+\lambda_I s$ or r (%)	Definition III Rate of Growth of Consumption per Man Employed that can be Sustained at a Constant Rate of Investment (1) $\frac{\alpha\beta+(1-\alpha)\delta}{\alpha}$ or $G_{\frac{C}{L}}$ (%)
4	4.8	0.4	2.0	2.6
3.5	4.3	0.6	1.9	2.6
3	3.8	0.8	1.7	2.6
2.5	3.4	0.9	1.7	2.6
2	2.9	1.1	1.6	2.6
1.5	2.5	1.3	1.6	2.6
1	2.0	1.4	1.6	2.6
0.5	1.5	1.6	1.7	2.6
0	1.0	1.8	1.8	2.6
-0.5	0.6	1.9	1.9	2.6
-1	0.2	2.1	2.0	2.6
-1.5	-0.1	2.2	2.1	2.6
-2	-0.6	2.4	2.2	2.6

(1) The numbers to three decimal places decrease steadily from 2.647 to 2.615.

In this calculation the time-series of income, saving, and the labour force have been taken at their face value. The results would be somewhat different if education had been counted as investment instead of consumption, if the labour series had been adjusted to account for the reduction in hours worked per day, if some public service had been counted as cost of production instead of income, or if it had been supposed that the durability of capital had changed between 1900 and 1965.

Some Objections and Qualifications

(a) Specification Errors

It is both a strength and a weakness of definition II that it is closely connected to a simple theory of how the economy works. It is a strength because the full impact of technical change is captured in one, obviously desirable, property of economic growth. It is a weakness because the estimate of the rate of technical change is conditioned by the model and because another equally plausible model could give rise to a different estimate. In measuring technical change, the real economy with numerous products created and falling into disuse, with capital goods differing in gestation periods, durability and complementarity with labour, with production patterns in which goods are built up gradually and in which the distinction between capital and goods-in-process is an accounting convention, is looked upon *as if* consumption were a homogeneous stuff generated instantaneously from the services of homogeneous labour and homogeneous capital combined in a production function. A specification of technical change based on a period of production model, or a Von Neuman model, or a sophisticated real capital model with many capital goods or a model in which accumulated knowledge is treated as a factor or production that can be purchased just as new capital is purchased,(15) might well explain the observed time-series of consumption and saving in a manner implying a rate of steady-state growth of consumption per man employed different from 2.6%. Worse still, the rate of growth of consumption per man-employed may not be a property of technology at all. The use of definition II is based on the assumption that $G_{\frac{C}{L}} = 0$ in the long run if technical change does not occur. One can imagine an economy in which this is not true. For instance $G_{\frac{C}{L}}$ could take on any value is a fully-automated economy where income is produced by capital alone and not by labour. One can also imagine technical change that increases consumption for a time without generating a permanent increase in the rate of growth of consumption per head. Unless we impose a simple model onto the economy, it may not be possible to represent aggregate technical change by any single property of economic growth, and it may be necessary to describe technical change as a vector of changes of parameters, the effect of which an economic welfare depends upon non-technical factors like the rate of saving and the commodity-composition of consumption.

(b) Technical Change by Industry and in the Economy as a Whole

The concept of technical change as sustainable growth of consumption per man at a constant rate of investment might appear to be a return to the simple notion of income per man employed that has been rightly rejected as a measure of technical change by industry. In a comparison between industries, the correct measure of technical change is the growth of the ratio of output to all resources used, labour and capital together. It may be asked: If 'growth of output per unit of input' is correct and

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(15) For instance W.D. Nordhaus "An Economic Theory of Technological Change", The American Economic Review. Papers and Proceedings, May, 1969.

'growth of output per man' is incorrect as a measure of technical change among industries, why is the reverse true of technical change in an economy over time?

To answer this question, we must consider the reason for rejecting income per man as a measure of technical change by industry. The reason is that, on this definition, it would be possible to cause technical change to increase in one industry and to decrease in another by altering the distribution of the capital stock among industries. As increase in output per man in industry A caused by a transfer of capital from industry B is not technical change because the increase in output in industry A involves a cost borne by industry B.

It might be said that technical change is growth of consumption per man occurring at no alternative cost. This broad definition of technical change subsumes 'growth of output per unit of total factor input' for an industry and 'growth of consumption per man' in an economy as a whole. Capital in an industry has alternative cost while capital in an economy has not. A factory not used for making cars may be used for making radios, but a factory that the economy declines to use goes to waste. When a whole economy is looked at over time, the important trade-off is between consumption at different time periods, and the alternative cost that must be taken into account is not that of capital but of consumption foregone. Between industries, technical change is growth of output per man corrected for usage of capital, i.e. output per total factor input. Between time periods in a whole economy, technical change is growth of output per man corrected for the rate of saving.

Conclusion

The main points in this article are;

1) The rate of cheapening of capital goods with respect to consumption goods is implicit in time series of investment and of the capital stock. Jorgenson and Griliches succeeded in reducing the rate of technical change in the United States from 1.6% to 0.1% by increasing the growth rate of human and non-human capital without counting the cheapening of capital as part of technical change.

2) As long as the time-series of investment is kept consistent with the time-series of capital, the estimate of the rate of technical change is largely independent of how capital is measured. If a change in the definition of capital leads to an increase in the apparent growth of the capital stock, there must be a corresponding increase in the rate of growth of real investment, and the implicit price of capital goods must have fallen faster than was supposed. When technical change is defined the growth rate of total factor productivity, increases in inputs and in outputs tend to cancel out. When technical change is defined as sustainable growth of consumption per man employed, the decrease in β caused by an increase in the growth of capital is just compensated for by a rise in δ, and $\frac{\alpha\beta + (1 - \alpha)\delta}{\alpha}$ remains constant.

3) The two definitions of aggregate technical change lead to measures that differ only in their rules for combining rates of technical change in the consumption and investment sectors of the economy. Technical change as growth of total factor productivity weights technical change in consumption and investment by their shares of total expenditure. Technical

change as sustainable growth of income per man employed weights rates of technical change in the consumption and investment industries by the shares of labour and capital of the total national product. One measure of aggregate technical change treats capital as a final product and the other treats it as intermediate.

4) The choice between the two candidates for the definition of technical change depends ultimately what information about the economy one wants statistics on technical change to convey. I feel that sustainable growth of consumption per man employed represents what most users of the statistics mean by technical change and that growth of total factor productivity is too much like an index number formula, too open dispute over the definition of capital, and without sufficient social significance to serve as the primary definition of aggregate technical change.