Queen's Economics Department Working Paper No. 1177

A Lindahl Solution to International Emissions Trading

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8-2008

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August 14, 2008

*The author acknowledges Robin Boadway, Zhen Song, Kimiko Terai, Christopher Bidner, Christoph Luelfesmann, Michael Hoy, and Walid Marrouch for their comments and encouragement. Discussions with the participants of the 2008 CPEG (Simon Fraser University), CEA 42nd Annual Meeting (University of British Columbia), and 9th International Meeting of the Society for Social Choice and Welfare (Concordia University), and seminar participants at Queen's University are also very helpful. A usual disclaimer applies. Financial support from JSPS (Japan Society for the Promotion of Science) Postdoctoral Fellowships for Research Abroad is gratefully acknowledged.

Abstract

We consider international negotiations on the level of global pollution, and examine the Lindahl solution which determines the distribution of the pollution permits with unanimous agreement. We show various properties to clarify difficulties to achieve a Pareto efficient allocation as an agreement. The Lindahl solution may result in an unfair allocation, and it does not belong to the γ-core as in other solutions based on emissions trading. On the other hand, we provide mechanisms that implement the Lindahl solution as the subgame-perfect equilibrium. We also consider the market with region-specific prices as a device to induce second-best Pareto efficient allocations.

Keywords: International emissions trading, Global externality, Lindahl equilibrium, Efficiency, Equity, Core, Implementability, Second-best analysis

JEL classification: Q54, D61, D62, D63, D78, H87, Q58

1 Introduction

The signatories of the Kyoto Protocol set rules and obligations towards the reduction of greenhouse gas emissions, mostly among the developed countries, for the period of 2008-2012. However, the limitation of greenhouse gas emissions and its assignment to the signatories after this period is still under debate as an important issue for the future. The Kyoto framework also adopted schemes including an international system of *emissions trading* which is attractive to many for its potential for success. The idea is simple and appealing: The total number of pollution permits is first determined as the emission target; subsequently, trading on competitive markets assures this target with the minimum total cost (i.e., it achieves production efficiency). A vital issue is how to set up the initial pollution permits and their distribution at the stage prior to the market trading of the permits. Since the global environment is a public good, the total emission level should satisfy the Samuelson condition that balances aggregate marginal damages and marginal products of each producer.

A number of works modeled a regime with permit trading to examine the choice of the initial pollution permits and their distribution. Chichilnisky et al. (2000) and Heal (2000) considered the case where the total number of pollution permits is fixed, and showed that almost all distributions of the initial permits are incompatible with Pareto efficiency.[1] Tadenuma (2005), following the framework of Prat (2000), studied the complementary case where, given the distribution rule of the initial permits, the countries decide on the total emissions.[2] He showed that the locus of welfare vectors of the countries attained through a choice of the total emission level (called the bargaining frontier) is (i) mostly below the Pareto frontier, and (ii) not guaranteed to contain portions that Pareto dominate the noncooperative disagreement point. These results thus indicated difficulties for achieving a Pareto-efficient emission level as an agreement. In a different context, Caplan et al. (2003) and Caplan and Silva (2007) assumed that, accompanied with the emissions trading, an agency in charge of the international transfers maximizes a weighted sum of utilities. However, at the international level there is no benevolent planner.[3] As well, costless lump-sum transfers may not

[1]In this paper, instead of the case where the total number of pollution permits is fixed, we will examine a problem in which countries choose the Pareto efficient level of total emissions at the equilibrium of a noncooperative game.

[2]Prat (2000) showed that, under some regularity conditions, there exists the level of total emissions which is compatible with Pareto efficiency. However, in the absence of a benevolent planner, the total emission level has to be decided by self-interested countries. Prat (2000) also proved that the preferences of each country are single-peaked as to the levels of total emissions, so that a unique winning level of total emissions exists by the majority voting. However, as in the conventional case of the public goods, in general, the voting equilibrium will not be Pareto efficient.

[3] An example that Caplan et al. (2003) and Caplan and Silva (2007) suggested is the Global Environment

be available.

A Lindahl solution in this setting is a politico-economic system that induces unanimous support for the Pareto efficient level of emissions. Given the distribution of the initial permit, each country has its own preferred level of the total emissions. Since countries are different with respect to productivities and disutilities of global environmental damage, desired levels of total emissions are typically different across countries. Then the countries adjust the distribution of the permits, as individuals' personalized prices in the standard Lindahl equilibrium, in order to bring unanimity on the desired level of emissions. Therefore, in contrast with Chichilnisky et al. (2000), Heal (2000), Prat (2000) and Tadenuma's (2005) approach, it simultaneously determines the total pollution permits and their distribution. We will show that the solution achieves a Pareto efficient allocation (Propositions 1.(i) and 3). Notice also that it does not involve lump-sum transfers by a benevolent supreme body. The classic problem of strategic misrepresentation of preferences can be readily resolved in Section 6, by constructing simple mechanisms that give incentives to the countries to behave in accordance with their true preferences to implement the Lindahl solution as a noncooperative equilibrium outcome.

Although the Lindahl equilibrium is one of the central solutions in public-good problems, few papers discusses formally its properties in the present framework of multilateral externalities where production technologies are decentralized and yield private benefits. Exceptions

Facility (GEF) which is responsible to the operation of financial transfers in the United Nations Framework Convention of Climate Change. They also said that the weights of such aggregate welfare may be implied by the equilibrium of a political game, but they did not formalize such a game.

are Mäler and Uzawa (1995) and Uzawa (1995) who discussed existence and stability in a different setup from this paper. The present paper provides a comprehensive analysis on issues discussed in the literature of multilateral externalities, including the possibility of international cooperation (Helm (2003) and Shiell (2003)), core (Chander and Tulkens (1997) and Eyckmans (1997)), implementation (Walker (1981), Danziger and Schnytzer (1991), Varian (1994), and Duggan and Roberts (2002)), the use of the equilibrium with differential prices (Chichilnisky et al. (2000) and Sandmo (2003)), and thus illuminates various properties of the Lindahl equilibrium as a solution to global environmental problems.

We show that the Lindahl equilibrium is, if it exists, Pareto efficient. However, the countries' interactions taking account of the changes of the permit-prices in the subsequent stage may result in no unanimous agreement, which strengthens the existing results on the impossibility of achieving efficient allocations (Proposition 1.(ii), 1.(iii) and Corollary 1). Also, the pattern of international distribution of the initial permits may be contrary to our intuition of fair compensation, in that a country with higher damage and a country with lower emissions are endowed with fewer permits (Proposition 2). If we consider a market with respect to abatement rather than emissions, the conclusions are reversed (Proposition 3). As to the possibility of international cooperation, the Lindahl equilibrium does not belong to the γ-core as in other solutions based on emissions trading (Propositions 4 and 5). On the other hand, the Lindahl equilibrium belongs to the weaker notion of the stand-alone core. As to implementability, we provide mechanisms that implement the Lindahl solution as the subgame-perfect equilibrium (Proposition 6). We finally consider the market with region-specific prices as a device to induce second-best Pareto efficient allocations, and discuss the scope of the practical use of the Lindahl solution (Propositions 7 and 8).

2 A model with multilateral externalities

2.1 Technology, preferences, and Pareto efficiency

Consider a standard model of multilateral externalities. There are $n \geq 2$ countries, $N \equiv \{1, ..., n\}$. Let y_i denote the gross domestic product (GDP) of country $i \in N$, and c_i be the consumption of country i. Production generates emissions of greenhouse gases, $x_i \in I\!R_+$ for country i, which accrues a utility cost as global environmental damage. The relation between x_i and y_i is represented by the function $y_i = Y_i(x_i)$, where $Y_i(0) = 0$ and $Y_i'(x_i) > 0 > Y_i''(x_i)$ for all $x_i \in I\!R_+$.[4] We also assume $\lim_{x_i \to 0} Y_i'(x_i) = \infty$ for all i. Associated with a vector of emissions $(x_i)_{i \in N}$, feasibility is defined by $\sum_{i \in N} Y_i(x_i) \geq \sum_{i \in N} c_i$. Let $X \equiv \sum_{j \in N} x_j \in I\!R_+$ be the level of global emissions of greenhouse gases. Each country i has preferences over pairs (c_i, X) of its own consumption and a total emission level, represented by a utility function $u_i(\cdot, \cdot)$. For illustration, we assume quasi-linearity of the utility functions, a functional form widely used in the related literature (e.g., Clarke (1971), Chander and Tulkens (1997), and Helm (2003)):

$$u_i(c_i, X) = c_i - D_i(X), \tag{1}$$

[4] Alternatively, as considered in the literature including Hourcade and Gilotte (2000), Helm (2003) and Sandmo (2005), one can consider y_i as country i's benefits from emissions-generating activities such as transport and heating, with $Y_i(x_i)$ being the households' benefit function that results in pollution x_i.

where $D_i(X)$ represents a disutility from pollution X with $D_i'(X) > 0$ and $D_i''(X) \geq 0$ for all $X \in I\!R_+$.

Associated with any *Pareto efficient* allocation, the level of individual emissions and the total emissions are uniquely determined by $\arg\max_{x_1,\ldots,x_n} \sum_{i\in N}(Y_i(x_i) - D(\sum_{j\in N} x_j))$. The solution, denoted by x_i^* for country i and X^* for total emissions, is characterized by the familiar first-order condition that aggregate marginal damages and marginal products of each country are equalized:

$$Y_i'(x_i^*) = \sum_{j\in N} D_j'(X^*) \text{ for all } i. \tag{2}$$

2.2 Market-clearing conditions with allowance trading

With international emissions trading, countries' interaction can be illustrated as a multi-stage game. In the first stage, countries choose the total emission level X and a proportion of country i's tradable emission allowances $\theta_i \in (0, 1)$ for all i that satisfies $\sum_{j\in N} \theta_j = 1$. We will formalize the first stage of the game in the next section. In the second stage, production of each country takes place with the trading of allowances on an international permit market.

Given X and θ_i in the first stage, each country i chooses the emission level in a perfectly-competitive market of tradable permits. Given a price of the permit q, each country maximizes its own profit:

$$\max_{x_i} Y_i(x_i) + q(\theta_i X - x_i). \tag{3}$$

The first-order condition is:

$$Y_i'(x_i) = q. \tag{4}$$

Let the solution be $x_i(q)$. Given X, the market-clearing price $q(X)$ is uniquely determined by the following equation:

$$\sum_{j \in N} x_j(q(X)) = X. \tag{5}$$

2.3 Alternative formulation: abatement game

The literature including Eyckmans (1997), Chichilnisky et al. (2000) and Heal (2000) considered the case of the abatement game which we discuss in Sections 4 and 5. It turns out that some properties of the Lindahl equilibria in this framework are starkly different from the case of the emission game.

Suppose that, in the initial state without international cooperation, there is a given reference level of emissions of the pollution $\bar{x}_i$ for each country i. The $\bar{x}_i$ corresponds to a situation in which the countries do not care about global environment. Following the literature, we set $\bar{x}_i$ to be exogenous, but we will discuss it later. Abatement is defined by $a_i \equiv \bar{x}_i - x_i \geq 0$. Each country is endowed with w_i as an initial wealth. The abatement is costly and incurs the private cost $\phi_i(a_i)$ on country i. The cost functions satisfy $\phi_i(0) = 0$, $\phi_i'(a_i) > 0$ and $\phi_i''(a_i) > 0$ for all $a_i \in \mathbb{R}_+$. One can formulate the correspondence from the emission model as $w_i = Y_i(\bar{x}_i)$ and $\phi_i(a_i) = Y_i(\bar{x}_i) - Y_i(\bar{x}_i - a_i)$.

A multi-stage game of international abatement trading is defined in the same way as in the case of emissions-trading. Here, country i *receives* money with the abatement which incurs a private cost. Given the total abatement A and the proportion of initial allotment θ_i determined in the first stage, and a price of the abatement p, each country maximizes its own profit: $\max_{a_i} w_i - \phi_i(a_i) + p(a_i - \theta_i A)$. Let the solution be $a_i(p)$. Given A, the market-clearing price $p(A)$ is uniquely determined by the following equation:

$$\sum_{j \in N} a_j(p(A)) = A. \tag{6}$$

3 Lindahl equilibrium

We now formalize the first stage of the game. Suppose first that the distribution rule of the initial permits $(\theta_i)_{i \in N}$ is given. From the work of the tradable-emission market and the budget constraint $c_i = Y_i(x_i(q(X))) + q(X)(\theta_i X - x_i(q(X)) \equiv c_i(X, \theta_i)$, each country has its own preferred level of X:

$$\max_X \; Y_i(x_i(q(X))) + q(X)(\theta_i X - x_i(q(X))) - D_i(X). \tag{7}$$

Denote the solution of (7) as X_i. The properties of X_i's are important to analyze the nature of international negotiations by self-interested countries. Basically, a country with a higher θ_i has a higher revenue share from the emissions trading in the second stage. The countries may also be different with respect to the desired pollution-supply $(x_i(q(X)))$ and the disutilities from environmental damage $(D_i(X))$. As a result, for a given $(\theta_i)_{i \in N}$, (7) typically generates a different desired level of global emissions, i.e., $X_i \neq X_j$ for $i \neq j$. Also, the first-order conditions do not have any *a priori* relationship with (2), i.e., X_i is generally Pareto inefficient (Tadenuma (2005, Proposition 7)). These two properties indicate difficulties in achieving a Pareto-efficient emission level as an agreement.

A Lindahl solution in this setting is a politico-economic system that brings unanimous support for the Pareto efficient level of emissions. Given $(\theta_i)_{i \in N}$, each country has its own preferred level of X as in (7). The θ_i (country i's revenue share from tradable emission permits) is adjusted, as individuals' personalized prices in the standard Lindahl equilibrium, in order to induce $X_i = X$ for all i. Formally,

Definition 1 *The system of $(X, (\theta_i)_{i \in N}, (x_i, c_i)_{i \in N})$ satisfies a sequential Lindahl equilibrium iff: (i) $x_i = x_i(q(X))$ for all i with $\sum_{j \in N} x_j(q(X)) = X$, (ii) $\sum_{j \in N} \theta_j = 1$, (iii) $c_i = c_i(X, \theta_i)$ for all i, and (iv) $X = X_i$ for all i. Namely,*[5]

$$q(X)\theta_i + q'(X)(\theta_i X - x_i(q(X))) = D_i'(X) \text{ for all } i. \tag{8}$$

At the conventional Lindahl equilibrium, each agent maximizes her utility at the point where her marginal willingness to pay for the public good is equal to her personalized price. The present analysis deals with the case of the public bad, and each country receives the net revenue from the market trading, $q(X)(\theta_i X - x_i(q(X)))$, as a victim and a polluter.

[5] By supposition, $\lim_{X \to 0} q(X) > \sum_{j \in N} D_j'(0)$. Since $\sum_{i \in N} q'(X)(\theta_i X - x_i(q(X))) = 0$, $\lim_{X \to 0}(q(X)\theta_i + q'(X)(\theta_i X - x_i(q(X)))) \leq D_i'(0)$ for all i never happens, so that $X \neq 0$ in Definition 1. The other extreme of $X_i \to \infty$ would be easily eliminated by appropriate assumptions on the production functions and the damage functions. We therefore consider the case of the interior optimum.

Accordingly, in (8), each country balances (i) the marginal damage of the emission, $D_i'(X)$ on the right hand side, and (ii) the associated change in the net revenue, $q(X)\theta_i + q'(X)(\theta_i X - x_i(q(X)))$ on the left hand side.[6] The latter works as a personalized compensation against the utility loss incurred, which consists of: (ii-a) the increase in the revenue from pollution permits, $q(X)\theta_i > 0$, and (ii-b) the change in the net revenue due to the fall in q, $q'(X)(\theta_i X - x_i(q(X)))$, which is positive (resp. negative) if country i is an emission-permit buyer (resp. seller). The term "sequential" refers to the case where X is determined in the first stage, and the producer's optimization in the second stage is bound to the value of X, i.e., $q(X)$ and $(x_i(q(X)))_{i \in N}$. This framework follows recent contributions (e.g., Prat (2000), Helm (2003), and Tadenuma (2005)), and is more suitable to analyze the case of environmental treaties including the Kyoto Protocol. Mathematically, this setup yields countries' far-sighted behavior represented by (ii-b).

We adopt the Lindahl approach for the following reasons. Firstly, in contrast with Chichilnisky et al. (2000), Heal (2000), Prat (2000) and Tadenuma's (2005) approach outlined in the Introduction,[7] it simultaneously determines X and $(\theta_i)_{i \in N}$, inducing unanimity on the desired level of emissions. Secondly, making use of a classic politico-economic framework and the tradable-permits market, the solution achieves a Pareto efficient allocation (see Propositions 1.(i) and 3 below). Thirdly, it does not involve lump-sum transfers by a benev-

[6] The differentiation of the budget constraint yields $\partial c_i(X, \theta_i)/\partial X = Y_i'(x_i)x_i'(q)q'(X) + q(X)(\theta_i - x_i'(q)q'(X)) + q'(X)(\theta_i X - x_i(q(X))$. $Y_i'(x_i)x_i'(q)q'(X)$ offsets $q(X)x_i'(q)q'(X)$ by (4).

[7] In Chichilnisky et al. (2000) and Heal (2000), the countries first determine X, and then they decide on $(\theta_i)_{i \in N}$. In Prat (2000) and Tadenuma (2005), the countries decide X given $(\theta_i)_{i \in N}$. They pointed out difficulties for achieving a Pareto-efficient allocation.

olent supreme body. And fourthly, the problem of strategic misrepresentation of preferences can be readily resolved in Section 6: it is possible to construct simple mechanisms that give incentives to the self-interested countries to implement the Lindahl solution as a noncooperative equilibrium outcome (Proposition 6). The $(\theta_i)_{i\in N}$ in the Lindahl allocation can also be derived from the public-bad version of the matching contribution system by Guttman (1978) and Danziger and Schnytzer (1991), which is implementable as the subgame-perfect equilibrium as well (Appendix B).

4 Pareto efficiency, existence and welfare properties

4.1 Basic properties

Let $-\frac{q(X)}{X}\frac{1}{q'(X)} \equiv \varepsilon(X)$ be the elasticity of the demand for the pollution permits in the trading market. It is convenient to rewrite (8) as follows:

$$q(X)\theta_i\left(1-\frac{1}{\varepsilon(X)}\right) = D_i'(X) + q'(X)x_i(q(X)) \text{ for all } i. \tag{9}$$

We first show the following proposition on Pareto efficiency and existence:

Proposition 1 *(i) A sequential Lindahl equilibrium is, if it exists, Pareto efficient. (ii) If $\varepsilon(X^*) = 1$ and $D_i'(X^*) \neq -q'(X^*)x_i(q(X^*))$ for some i, there does not exist a sequential Lindahl equilibrium. (iii) Even if the equilibrium exists, the equilibrium proportion of the permits may not satisfy $\theta_i \in (0,1)$ for all i.*

Proof: We first prove part (i). Summing up (8) with respect to i, and using (4), (5) and

$\sum_{i\in N}\theta_i = 1$, we obtain:

$$\begin{aligned}\sum_{i\in N} D_i'(X) &= q(X)\sum_{i\in N}\theta_i + q'(X)(\sum_{i\in N}\theta_i X - \sum_{i\in N} x_i(q(X))) \\ &= q(X) \\ &= Y_j'(x_j) \text{ for all } j.\end{aligned}$$

Hence, we obtain (2), i.e, $X = X^*$ and $x_i = x_i^*$ for all i, at the sequential Lindahl equilibrium.

To prove (ii), suppose that the Lindahl equilibrium exists. By (i), it should be $X = X^*$. Then, when $\varepsilon(X^*) = 1$, (9) should be $D_i'(X^*) = -q'(X^*)x_i(q(X^*))$ for all i. This is a contradiction.

Part (iii) of the proposition is shown from the following numerical example:

$$n = 2,\ u_i(c_i, X) = c_i - \frac{a_i}{2}X^2,\ Y_i(x_i) = 2b_i(x_i)^{\frac{1}{2}},\ a_1 + a_2 = 1. \tag{10}$$

For example, when $a_1 = 0.9,\ b_1 = b_2 = 0.1$, the values of $(\theta_i)_{i\in N}$ consistent with $X_1 = X_2$ is $\theta_1 = 1.3$ and $\theta_2 = -0.3$. *Q.E.D.*

In (ii), the marginal revenue from the permit, $q(X_i)$, is cancelled out by the price decrease $q'(X_i)X_i$: graphically, the country's "demand function" on (X_i, θ_i)-space represented by (8) becomes vertical to the X_i axis, so that the standard Lindahl adjustment to reach $X_i = X^*$ for all i does not work. The case stated in (iii) is that, for *any* $(\theta_i)_{i\in N}$ such that $\theta_i \in (0, 1)$ for all i (the normal range of the permit distributions without lump-sum taxes and transfers), no level of the total emissions can receive unanimous support.[8, 9] These results strengthen

[8]However, regarding implementability, mechanisms shown in Section 6.1 and Appendix B can implement the Lindahl solution including the cases in Proposition 1.(iii) where $\theta_i < 0$ for some i.

[9]The second order conditions for the solution of (8) are $q'(X)(2\theta_i - (\sum_{j\in N}(Y_i''/Y_j''))^{-1}) + (\theta_i X -$

the previous works by Chichilnisky et al. (2000), Heal (2000) and Tadenuma (2005) on the impossibility of achieving efficient allocations which we outlined in the Introduction and Section 3.

Corollary 1 *The countries may not be able to find the allocation of initial permits,* $(\theta_i)_{i\in N}$*, with* $\theta_i \in (0,1)$ *for all* i *and* $\sum_{j\in N}\theta_j = 1$*, to have an agreed level of emissions* X *in the sense of* $X = X_i$ *for all* i*.*

We next examine the welfare properties of the Lindahl equilibrium. The inspection of (9) yields the following:

Proposition 2 *Suppose that* $\varepsilon(X^*) < 1$*. Under a sequential Lindahl equilibrium, (i) for* i *and* j *such that* $D_i'(X^*) = D_j'(X^*)$*,* $\theta_i > \theta_j$ *iff* $x_i(q(X^*)) > x_j(q(X^*))$*, and (ii) for* i *and* j *such that* $x_i(q(X^*)) = x_j(q(X^*))$*,* $\theta_i > \theta_j$ *iff* $D_i'(X^*) < D_j'(X^*)$*. These properties are reversed if* $\varepsilon(X^*) > 1$*.*

Proof: The results follow from (9), $q(X^*) > 0$ and $q'(X^*) < 0$. *Q.E.D.*

This proposition illustrates the pattern of redistribution through θ_i's among those which have same preferences on the one hand, and those which have the same production (functions) on the other hand. These properties critically depend on the elasticity of polluting activities with respect to the price change. The empirical literature typically shows low price

$x_i(q(X)))\sum_{j\in N} Y_j'''(x_j)(x_j'(q)q'(X))^3 - D_i''(X) < 0$. Sufficient conditions are: (i) the third derivative of the production functions, $Y_i'''(x_i)$ ($i \in N$), are sufficiently small (see Helm (2003, footnote 3)), and (ii) those which have high $-1/Y_i''(x_i^*)$ also have high θ_i (which can be compared with the properties in Proposition 2 below). They may not hold at $X = X^*$ for all i in general, but we are not sure how large is the class of functions in which the conditions fail to hold. It is, for example, satisfied in the economy in (10).

elasticities.[10] In our case, if $\varepsilon(X^*) < 1$, a country which suffers higher damage is endowed with *fewer* initial permits under the sequential Lindahl equilibrium. Also, a country with higher emissions is endowed with more permits: if developed countries emit higher pollution levels, Proposition 2 also predicts a regressive distribution of the initial permits (or a progressive distribution of the permits would not be consistent with a unanimous agreement). These are contrary to our intuition of the fair compensation[11] in a North-South framework.

4.2 Alternative formulations

Properties of the sequential Lindahl equilibrium discussed so far rest on the presence of $q'(X)$, i.e., agents determine X_i in (8) anticipating the effects on emissions-price in the subsequent stage. If, instead, we assume the existence of the self-fulfilling belief of the market-clearing price $q = q(X^*)$ in the first stage of the game, then the framework is equivalent to the one-shot game in which X from the consumers' side and $(x_i)_{i\in N}$ from the producers' side are simultaneously determined. Here we define a *simultaneous Lindahl equilibrium* in the following way:

[10] For demand for gasoline, Rogat and Sterner (1998) reported price elasticities of the Latin American countries over the period of 1960-94 that are, on average, 0.17 in the short run and 0.58 in the long run. Franzen and Sterner (1995) estimated the OECD countries over the period of 1963-85. The price elasticity is less than unity in the short-run but it exceeds unity in the long-run. In a survey by Goodwin (1992), the average long-run elasticity for petrol is 0.84 for cross-section studies and 0.71 for time-series studies.

[11] For example, according to a principle of *responsibility and compensation* (Fleurbaey and Maniquet (2002)), the redistribution should be sensitive to the factors beyond individuals' control (such as talent or ability), but individuals are held responsible for their genuine choices (ambitions or preferences). Suppose that the production technology is succeeded from the previous generations beyond countries' choice, but the countries are deemed responsible for their preferences for the damage, $D_i(X)$. Then the international transfers towards those with less efficient technology would be consistent with the compensation principle, whereas the higher transfers towards those with higher marginal damage make sense as a principle of responsibility.

Definition 2 *The system of* $(X, q, (\theta_i)_{i\in N}, (x_i, c_i)_{i\in N})$ *satisfies a simultaneous Lindahl equilibrium iff: (i) for* $x_i = x_i(q)$ $(i \in N)$ *determined in (4),* $\sum_{j\in N} x_j(q) = X$*, (ii)* $\sum_{j\in N} \theta_j = 1$*, (iii)* $c_i = Y_i(x_i(q)) + q(\theta_i X - x_i(q))$ *for all* i*, and (iv)* $X = \arg\max_{X'}\ Y_i(x_i(q)) + q(\theta_i X' - x_i(q)) - D_i(X')$ *for all* i*. Namely,*

$$q\theta_i = D_i'(X). \tag{11}$$

The difference between the simultaneous and the sequential Lindahl equilibrium is reflected on the first-order conditions. In the simultaneous Lindahl equilibrium, (11) is a benefit principle that the country's price θ_i is proportional to its marginal disutility $D_i'(X)$. For instance, in (10), $\theta_i = a_i$ for all i.

A simultaneous Lindahl equilibrium is clearly Pareto efficient, and it uniquely exists with $\theta_i \in (0,1)$ for all i.[12] In Section 6.2, we show that the simultaneous Lindahl allocation is equivalent to (and hence justified as) the allocation based on a classic solution of *pairwise-trading*, which is also implementable as the subgame-perfect equilibrium. In the following, we show various properties of these two Lindahl solutions. Many properties are in fact common in these solutions.

An alternative way to restore reasonable welfare properties is the case of the abatement game introduced in Section 2.3. With the utility function $v_i(c_i, A) \equiv c_i + B_i(A)$, $B_i'(A) > 0 > B_i''(A)$, associated with an equilibrium level of abatement A^*, under the sequential Lindahl

[12]The existence of the simultaneous Lindahl equilibrium was shown by Mäler and Uzawa (1995) and Uzawa (1995) under strict concavity of the utility functions and the production possibility set.

equilibrium,

$$(p(A^*) + p'(A^*)A^*)\theta_i = B_i'(A^*) + p'(A^*)a_i(p(A^*)) \text{ for all } i, \tag{12}$$

with $\sum_{i\in N} \theta_i = 1$. Suppose that the second order conditions are satisfied (sufficient conditions similar to those discussed in footnote 9). In contrast to Propositions 1 and 2, we obtain the following:

Proposition 3 *In the abatement game, the sequential Lindahl equilibrium exists, satisfies Pareto efficiency, and satisfies $\theta_i \in (0,1)$ for all i. Also, regardless of the value of elasticities of pollution supply, (i) for i and j such that $B_i'(A^*) = B_j'(A^*)$, $\theta_i > \theta_j$ iff $a_i(p(A^*)) > a_j(p(A^*))$, and (ii) for i and j such that $a_i(p(A^*)) = a_j(p(A^*))$, $\theta_i > \theta_j$ iff $B_i'(A^*) > B_j'(A^*)$.*

Proof: The proof of Pareto efficiency is the same as in Proposition 1, which implies that the sequential Lindahl equilibrium, if it exists, would induce the unique Pareto-efficient aggregate and individual abatement levels. To prove existence, in (12), all values except θ_i are uniquely determined as a function of A^*, which is unique. This uniquely determines θ_i. An inspection of (6) shows that $p'(A) = (\sum_{j\in N}(1/\phi_j''(a_j))^{-1} > 0$. Since $B_i'(A^*)$, $p'(A^*)$, and $a_i(p(A^*))$ are all positive, (12) indicates that $\theta_i > 0$ for all i. Since $\sum_{i\in N} \theta_i = 1$, then $\theta_i < 1$ has to be the case for all i. The properties (i) and (ii) follow from (12) as in Proposition 2. *Q.E.D.*

Therefore, the sequential Lindahl equilibrium of the abatement game overcomes the problem of non-existence of the equilibrium in $\theta_i \in (0,1)$ for all i, and also circumvents the pathological allocations of θ_i's in relation to $D_i'(X^*)$'s and $x_i(q(X^*))$'s in the emission game.

Given the symmetric structure, the difference of the properties is striking.

5 Possibility of international cooperation

5.1 γ-core

Prior to the first stage of the international emissions trading, countries decide on the establishment of a trading system. This decision may take place as a result of voluntary participation in the system. Helm (2003) and Tadenuma (2005) assumed that, if at least one country chooses not to participate for the establishment of a trading system, the outcome of the game is the following *Nash equilibrium allocation*, where each country maximizes its utility with respect to its own emissions while taking other countries' choices as given. The allocation, denoted by $(\hat{x}_i)_{i\in N}$ and $\hat{X} \equiv \sum_{j\in N} \hat{x}_j$, is determined by $\arg\max_{x_i}(Y_i(x_i) - D_i(x_i + \sum_{j\in N, j\neq i} \hat{x}_j))$. The first-order condition is:

$$Y_i'(\hat{x}_i) = D_i'(\hat{X}) \text{ for all } i. \tag{13}$$

One can easily derive $\hat{X} > X^*$.[13] This, however, does *not* mean that $\hat{x}_i > x_i^*$ for *all* i.

In this section we assume $n = 2$ for illustration. Associated with $\hat{X}$ and $\hat{u}_i \equiv u_i(Y_i(\hat{x}_i), \hat{X})$, we now introduce the following concept of the core:

Definition 3 *An allocation belongs to the γ-core if it is a Pareto efficient allocation that*

[13]Suppose that $\hat{X} \leq X^*$. Then, by convexity of D_i's, $\sum_{j\in N} D_j'(X^*) \geq \sum_{j\in N} D_j'(\hat{X})$. By (2) and (13), $Y_i'(x_i^*) \geq \sum_{j\in N} Y_j'(\hat{x}_j) > Y_i'(\hat{x}_i)$ for all $i \in N$. Then $X^* = \sum_{j\in N} x_j^* < \sum_{j\in N} \hat{x}_j = \hat{X}$. This is a contradiction.

Pareto dominates $(\hat{u}_i)_{i \in N}$.[14]

Table 1 illustrates the core property of the Lindahl equilibria, through the economy in (10) with varying the parameters $a_1,\ b_1$, and b_2:

Table 1 around here.

The table lists consumption, utility, $\theta_1(= 1 - \theta_2)$, and country 1's revenue from the emissions trading= $q(X^*)(\theta_1 X^* - x_1^*)$, in a sequential Lindahl equilibrium and a simultaneous Lindahl equilibrium, respectively. (8) implies that $\theta_i = 2a_i - (b_i^2 / \sum_{j \in N} b_j^2)$ in the sequential Lindahl equilibrium, and, as discussed before, (11) implies that $\theta_i = a_i$ in the simultaneous Lindahl equilibrium. Depending on the parameters under consideration, the two allocations give substantially different consumptions and utilities. However, in all cases, the Lindahl equilibria do not Pareto dominate the Nash equilibrium: in Cases 1 and 2, country 2 is worse-off at the Lindahl equilibria than the Nash equilibrium; in Case 3, country 1 is worse-off.

Proposition 4 *Generally, neither a simultaneous Lindahl equilibrium nor a sequential Lindahl equilibrium belongs to the* γ*-core.*

In Case 1, country 1 has a higher productivity but also has a higher marginal disutility of pollution than country 2. In a Nash equilibrium (13), consumers' high marginal disutility

[14] Chander and Tulkens (1997) considered a stronger concept of the core with respect to the γ-Characteristic Functions. Their notion of the γ-core is equivalent to our definition under $n = 2$.

voluntarily reduces emissions of pollution, given country 2's high pollution under the Nash equilibrium. In the market, on the other hand, a revenue-maximizing firm exploits the comparative advantage of the high productivity, following (4). Comparing these equilibria, q on the right hand side of (4) becomes lower than $D_1'(\hat{X})$ in (13), i.e., $x_1^* > \hat{x}_1$. The phenomenon of $x_1^* > \hat{x}_1$ is also observed in Case 2, where country 1 has a lower productivity. In both cases, country 2 is worse-off at the Lindahl equilibria. It is interesting to see that, in Case 1, country 2 is the emission-permit *seller* (so that it gains at the emissions trading) but it eventually becomes worse-off as a result of the market trading. In Case 3, country 1 is the emission-permit buyer, and its consumption is much lower than under the Nash equilibrium, although the consumption of country 2 is significantly higher. As a result, country 1 loses more by purchasing the emission-permits than the benefit from reducing pollutions.

5.2 Comparison with other solutions based on emissions trading

With the same numerical example as in Table 1, we now examine other solutions based on emissions trading in the literature: (i) Helm's (2003) equilibrium with noncooperative determination of initial permits,[15] (ii) a Pareto efficient allocation without a lump-sum transfer, i.e., $(c_i, X) = (Y_i(x_i^*), X^*)$ for all i, examined in Shiell (2003), and (iii) the case of $\theta_i = \hat{x}_i/\hat{X}$ for all i, namely, countries start negotiations at the point where the initial emission permits

[15] Helm (2003) considered the scenario that the values of θ_i and X are determined by noncooperative countries. Namely, each country announce $\theta_i X = \tilde{\omega}_i$ that would maximize its own utility, given the announcement of the others: $\max_{\omega_i} Y_i(x_i(q(\omega_i + \sum_{j\in N, j\neq i} \tilde{\omega}_j))) + q(\omega_i + \sum_{j\in N, j\neq i} \tilde{\omega}_j)(\omega_i - x_i(q(\omega_i + \sum_{j\in N, j\neq i} \tilde{\omega}_j))) - D_i(\omega_i + \sum_{j\in N, j\neq i} \tilde{\omega}_j)$.

are proportional to the Nash equilibrium,[16] examined in Tadenuma (2005).

Table 2 around here.

Several observations emerge. Firstly, Helm's (2003) equilibrium does not belong to the γ-core, either. The calculations in Table 2 complement Helm's (2003) Proposition 4. In Cases 1 and 2, the total emissions are reduced as a result of voluntary reduction of the permits and the work of the trading market. However, it does not result in the Pareto improvement. Case 3 is the so-called "hot air problem", in that a country with a low disutility from the environmental damage is willing to sell permits which in fact exceeds the business-as-usual emissions. Secondly, a Pareto efficient allocation without a lump-sum transfer does not belong to the γ-core. This property contrasts with Shiell's (2003) statement that such allocation brings cooperative gains.[17] An implication is significant: *for an allocation to belong to the γ-core, one generally needs international lump-sum taxes and transfers.* Thirdly, Tadenuma (2005, Proposition 8) showed that, when $\theta_i = \hat{x}_i/\hat{X}$ for all i, the bargaining frontier (the locus of utility vectors of the countries attained by changing X) contains allocations that Pareto dominate $(\hat{u}_i)_{i \in N}$.[18] In Table 2, the utility vector of (u_1^a, u_2^a)

[16]The choice of the weights $\theta_i = \hat{x}_i/\hat{X}$ to find efficiency gains resembles Roemer's (1989) solution concept in the context of the tragedy of commons.

[17]"In the absence of regulation, the countries will be located below the [Pareto] frontier, [called] point a for example, owing to the public-good nature of abatement. ... [T]here is one Pareto-efficient allocation associated with zero transfers ... This point is located to the north-east of a, since both countries benefit from some level of pollution control, even given the existing imbalance in the distribution of income." (Shiell (2003, pp. 42-43))

[18]In general, the Nash equilibrium $((\hat{u}_i)_{i \in N})$ may be outside the bargaining frontier, in which the outcome is $(\hat{u}_i)_{i \in N}$ — no cooperative agreement is made and the outcome is Pareto inefficient.

corresponds to an allocation with $X = \hat{X}$ and the emissions trading. The emissions trading improves production efficiency, so that this allocation Pareto dominates $(\hat{u}_1, \hat{u}_2)$, although it is not Pareto efficient. On the other hand, the utility vector of (u_1^b, u_2^b) corresponds to an allocation with $X = X^*$ and the emissions trading. It belongs to the bargaining frontier, but Table 2 suggests that (u_1^b, u_2^b) will not belong to the γ-core (except Case 1).

The above discussion is summarized into the following proposition:

Proposition 5 *(i) Helm's (2003) equilibrium does not belong to the γ-core. (ii) A Pareto efficient allocation without a lump-sum transfer does not belong to the γ-core. (iii) Suppose that $\theta_i = \hat{x}_i/\hat{X}$ for all i. An allocation in a bargaining frontier with $X = X^*$ does not belong to the γ-core.*

Chander-Tulkens' (1997) solution,

$$c_i = Y_i(\hat{x}_i) - r_i \sum_{j\in N}(Y_j(\hat{x}_j) - Y_j(x_j^*)),\ r_i \equiv \frac{D_i'(X^*)}{\sum_{j\in N} D_j'(X^*)} \text{ for all } i, \tag{14}$$

belongs to the γ-core under certain conditions. Eyckmans (1997, p. 324) stated that counterexamples can be found that this solution does not always belong to the γ-core. One issue about this solution in the present context is that it is hard to relate to the system of tradable permits. Neither producer-consumer's decentralized decision nor a market of tradable permits seems explicit.[19] The system may also have to be supported by appropriate international lump-sum transfers, which are usually excluded in the literature in the absence of

[19]Mechanisms proposed in the next section have decentralized decision making by the producer and the consumer in each country.

a central authority and costless transfers. Proposition 5 and this fact indicate a dilemma between the use of the decentralized market scheme without lump-sum transfers and the possibility of cooperation.

5.3 Stand-alone core

A weaker notion of the core is Foley's (1970) *stand-alone core.* The notion corresponds to the idea that the outsiders of the coalition do not contribute to the public good at all. In Appendix A, we provide a definition of the stand-alone core in the abatement-game, and conclude that a sequential Lindahl equilibrium belongs to the stand-alone core.[20] The property is the same as Eyckmans' (1997) Proposition 3 which states that the solution of the abatement game corresponding to Chander-Tulkens' (1997) allocation (14) belongs to the stand-alone core.

At this point we will discuss the issue of $\bar{x}_i$. There are three possibilities. The first possibility is that the emissions are carried out by self-interested producers so that the marginal product is *zero*. Both Mäler (1989) and Chander and Tulkens (1997) stated that such case is not interesting, since virtually all Pareto efficient allocations belong to the stand-alone core in such a case. The second possibility is to set $\bar{x}_i = \hat{x}_i$, the Nash equilibrium level defined in (13). In this case, however, since the stand-alone core is equivalent to the γ-core when $n = 2$, then the result of the above proposition cannot apply. Notice also that $x_i^* > \hat{x}_i$ may happen, as we showed above. The above proposition may not be extended to the cases

[20]In the emission game, a sequential Lindahl equilibrium belongs to the stand-alone core if $\bar{x}_i$'s are "sufficiently large". It is difficult to formalize this condition.

that allow $a_i < 0$ for some i, as in Eyckmans' (1997) Proposition 3. The third and a practical case is the one where *domestic* externality problems were not resolved so that $\bar{x}_i > \hat{x}_i$ for all i. The above proposition has some use in such a case.

6 Implementation of Lindahl equilibria

Suppose that the countries involved in the global environmental problem agreed to use the international system of emissions trading. Given that the Lindahl solution shares the nature with the classic politico-economic framework and is Pareto efficient, it is a good candidate solution.[21] Suppose that a neutral regulator is established by the negotiating parties for the enforcement of the Lindahl allocation. As in the conventional public-good problem, the direct use of the Lindahl system is subject to the countries' strategic behavior through misrepresentation of preferences. In a typical situation where information on the production functions and the damage functions of each country is private information that is not available for the regulator, how can she design a mechanism that will implement the Lindahl solution as a noncooperative equilibrium outcome through decentralized decisions by utility-maximizing consumers and profit-maximizing producers in each country? In this section, we provide simple and interpretable mechanisms that implement the sequential and the simultaneous Lindahl solutions as the subgame-perfect equilibrium, respectively.

[21] A failure to achieve the γ-core is shared with other candidate solutions through the emissions trading, as we saw in the previous section.

6.1 A mechanism that implements a sequential Lindahl equilibrium

We first consider a mechanism that implements a sequential Lindahl equilibrium.[22] We begin with the case of $n \geq 3$. The decentralized decisions are formulated as a $2n$-player game by n utility-maximizing consumers and n profit-maximizing producers. Corresponding to our setup in which the countries decide on X and $(\theta_i)_{i \in N}$ in the first stage, and the emissions trading takes place in the second stage, we consider the following two-stage game:

Stage 1 : The consumer in each country i announces $s_i \in I\!R$.

Stage 2 : The producer in each country i announces its own pollution-supply function $\tilde{x}_i^a : I\!R_{++} \to I\!R_+$ and a function $\tilde{x}_i^b : I\!R_{++} \to I\!R_+$ for its "neighbor," producer $i-1$, where we treat n as producer 1's neighbor. Each function must be continuous and decreasing, with $\tilde{x}_i^a(\infty) = \tilde{x}_i^b(\infty) = 0$.

The outcome of the game is defined as follows. Associated with the above strategies, the price q_i is assigned to producer i. It is given by:

$$\sum_{j \in N, j \neq i} \tilde{x}_j^a(q_i) + \tilde{x}_{i+1}^b(q_i) = \sum_{j \in N} s_j, \tag{15}$$

with the convention of $n+1 \equiv 1$. Here, we show the property of the mechanism for the case where such q_i exists for all i. See Appendix B.(a) for the construction of the outcome

[22] Hereafter we only examine the emission game, as the analogies to the abatement game is straightforward. Here, in order to exclude the situation of Proposition 1.(ii), we assume $\varepsilon(X) \neq 1$ for all X, which is known by the regulator as a minimal information on the domain: the information on elasticities could be available in practice at least in approximation (see footnote 10), and this assumption is not needed if we consider the abatement game.

when there is no q_i that satisfies (15) for some i, and the proof that such cases would not constitute an equilibrium. Let $q_c \equiv \min_{i \in N} q_i$ be the price assigned to the consumer. The regulator assigns an emission level of country i as $x_i = \tilde{x}_i^a(q_i)$, and:

$$\pi_i = Y_i(\tilde{x}_i^a(q_i)) - q_i \tilde{x}_i^a(q_i) - |\tilde{x}_{i-1}^a(q_{i-1}) - \tilde{x}_i^b(q_{i-1})|, \tag{16}$$

$$c_i = \pi_i + q_c((1/n) + s_{i+1} - s_{i+2}) \sum_{j \in N} \tilde{x}_j^a(q_j), \tag{17}$$

$$X = \sum_{j \in N} \tilde{x}_j^a(q_j), \tag{18}$$

for the profit, consumption, and the total emission, respectively, with the conventions of $i - 1 \equiv n$ for $i = 1$ and $n + 2 \equiv 2$. In (16), associated with $x_i = \tilde{x}_i^a(q_i)$, producer i has to pay $q_i \tilde{x}_i^a(q_i)$ to the regulator, and there is a penalty for misrepresenting the pollution-supply of the neighbor. In (17), the profit of producer i goes to consumer i as in the present model of emissions-trading. The consumer also receives the amount of $((1/n) + s_{i+1} - s_{i+2})q_c X$ from the regulator out of the revenue collected from the producers,[23] with the price q_c and the fraction $(1/n) + s_{i+1} - s_{i+2}$ of the permits for country i. The latter is determined by its neighbors' announcements and independent of its own, in the same manner as in Walker (1981). Producer i chooses strategies to maximize π_i in (16), and consumer i announces s_i to maximize her utility $u_i(c_i, X)$, with c_i and X determined by (17) and (18), respectively.

[23] The game form assures $\sum_{i \in N} q_i \tilde{x}_i^a(q_i) \geq \sum_{j \in N} q_c((1/n) + s_{j+1} - s_{j+2}) \sum_i \tilde{x}_i^a(q_i)$ at any strategy, which is complementary to the feasibility. The issue of ensuring the balance at nonequilibrium strategies would be readily handled, following the standard of the literature (e.g., Varian (1994), Tian (1994), and Duggan and Roberts (2002)). This is left as future research. A similar comment applies for the mechanism in the next section.

In order to see that the subgame-perfect equilibrium outcome coincides with the Lindahl solution, one must work backwards through the game. Begin with the second stage. Since q_i that producer i faces does not depend on producer i's own strategy, producer i's choice of $\tilde{x}_i^a(q_i)$ is simply profit-maximizing: $\tilde{x}_i^a(q_i) = x_i(q_i)$ as in (4). Neither does the choice of $\tilde{x}_i^b(q_{i-1}) \neq \tilde{x}_{i-1}^a(q_{i-1})$ affect q_i, so $\tilde{x}_i^b(q_{i-1}) = \tilde{x}_{i-1}^a(q_{i-1})$ for all i. In Appendix B.(b), we show that $q_i = q_c = q(\sum_{j\in N} s_j)$ for all i, as in (5). Substituting these values into (17) and (18), we obtain: (i) $\sum_{i\in N} x_i(q(\sum_{j\in N} s_j)) = \sum_{j\in N} s_j = X$, (ii) $\sum_{i\in N}((1/n) + s_{i+1} - s_{i+2}) = 1$, and (iii) $c_i = c_i(\sum_{j\in N} s_j, (1/n) + s_{i+1} - s_{i+2})$ for all i, corresponding to conditions (i)-(iii) of Definition 1.

We next examine Stage 1 of the game. Taking account of (i)-(iii) above, each consumer maximizes her utility $u_i(c_i, X) = u_i(c_i(\sum_{j\in N} s_j, (1/n) + s_{i+1} - s_{i+2}), \sum_{j\in N} s_j) \equiv U_i(s_i, s_{N\setminus\{i\}})$. From the Nash behavior, each consumer i treats $\sum_{j\in N, j\neq i} s_j$ and $(1/n) + s_{i+1} - s_{i+2}$ as given, so that the choice of s_i for the utility maximization is equivalent to the choice of X given $\theta_i = (1/n) + s_{i+1} - s_{i+2}$. Formally, the first-order condition of $U_i(s_i, s_{N\setminus\{i\}})$ with respect to s_i is equivalent to (8) with $(X, \theta_i) = (\sum_{j\in N} s_j, (1/n) + s_{i+1} - s_{i+2})$. Therefore, the equilibrium outcome is the Lindahl solution. Conversely, given $(\theta_i)_{i\in N}$ that satisfies (8) with $X = X^*$, the system of n linear equations, $\sum_{j\in N} s_j = X^*$ and $(1/n) + s_{i+1} - s_{i+2} = \theta_i$ $(i = 1, ..., n-1)$, has the unique solution $(s_i)_{i\in N}$ which is consistent with the equilibrium outcome. We therefore conclude that the subgame-perfect equilibrium outcome coincides with the Lindahl solution. Notice that we did not impose $(1/n) + s_{i+1} - s_{i+2} \geq 0$ for all i, so that this mechanism can implement the Lindahl solution when $\theta_i < 0$ for some i, as in the cases of Proposition 1.(iii).

Appendix B.(c) examines the case of $n = 2$, where we derive $(\theta_i)_{i\in N}$ as a version of Guttman (1978) and Danziger and Schnytzer's (1991) matching contribution game. The θ_i there has a particular interpretation as the compensation from the other consumer out of the revenues from the emissions-trading market.

6.2 A pairwise-trading mechanism

We next provide a mechanism to implement a simultaneous Lindahl equilibrium. It is based on a classic *pairwise-trading* system, an idea that each polluter (a producer) has to purchase the right of emissions from each victim (a consumer) per unit of pollution. The formal analysis for the work of this system also facilitates our discussion in the next section.

The proposed game form is a two-step compensation mechanism as in Varian (1994):

Stage 1 : The consumer in country i announces $(\hat{q}_{ik})_{k\in N} \in I\!R^n_+$, as the price that producer k pays for consumer i's damage. Simultaneously, the producer in country i announces $(\tilde{q}_{ki})_{k\in N, k\neq i} \in I\!R^{n-1}_+$, as the price that consumer k receives for producer i's emissions.

Stage 2 : The producer in country i chooses x_i to maximize the profit given by:

$$\pi_i = Y_i(x_i) - \sum_{k\in N} \hat{q}_{ki} x_i - \sum_{k\in N, k\neq i} |\hat{q}_{ki} - \tilde{q}_{ki}|. \tag{19}$$

Consumer i receives a payoff $u_i(c_i, \sum_{j\in N} x_j)$, with c_i determined by:

$$c_i = \pi_i + \sum_{k\in N, k\neq i} \tilde{q}_{ik} x_k + \hat{q}_{ii} x_i. \tag{20}$$

As in the previous section, the firms' profit is determined by its own announcement of the pollution-supply, the price determined by others, and the penalty for misrepresenting the prices of the bilateral-trade. In (20), π_i goes to consumer i again, and consumer i receives the compensation from the polluters. The π_i in (19) includes the term of $-\hat{q}_{ii}x_i$, which depends on consumer i's announcement. This term offsets the last term of (20), which eliminates the price-setting power of consumer i in the bilateral-trade against producer i.

Appendix C shows the following properties of the subgame-perfect equilibrium. (a) $\hat{q}_{ik} = \tilde{q}_{ik} = D_i'(X) = \hat{q}_{ii} \equiv d_i$ for all i and $k \neq i$: $\hat{q}_{ik}$ is independent of k, so the dimension of the bilateral-trade is reduced from n^2 to n.[24] (b) $Y_i'(x_i) = \sum_{k \in N} d_k \equiv q$ for all i: each polluter pays q per unit of emission, d_k to each victim k. (a) and (b) imply that the equilibrium is unique, with $X = X^*$ and $x_i = x_i^*$ for all i. Notice that d_i/q is equal to θ_i in (11), with $\sum_{i \in N} \theta_i = 1$. Substituting the equilibrium strategy into (19) and (20), the budget constraint of the consumer becomes:

$$c_i = Y_i(x_i) - \sum_{k \in N} d_k x_i + d_i \sum_{k \in N} x_k = Y_i(x_i) + q(\theta_i X - x_i). \tag{21}$$

(21) and the above discussion verified conditions (i)-(iv) of Definition 2. We conclude that the unique subgame-perfect equilibrium coincides with the unique simultaneous Lindahl equilibrium.

The analysis in this section concludes as follows:

[24] Consider the following simpler game. In Stage 1, the consumer in country i announces $(\tilde{q}_i, \hat{q}_{i-1})$. In Stage 2, the producer in country i chooses x_i. The outcome is that $\pi_i = Y_i(x_i) - \sum_{k \neq i} \tilde{q}_k x_i - \hat{q}_i x_i$, $c_i = \pi_i + \hat{q}_i \sum_{j \in N} x_j - |\tilde{q}_{i-1} - \hat{q}_{i-1}|$, $X = \sum_{j \in N} x_j$. This game also implements the simultaneous Lindahl allocation at the subgame-perfect equilibrium.

Proposition 6 *There exist mechanisms that implement the Lindahl solutions as the subgame-perfect equilibrium.*

7 Differentiated prices and second-best optimum

Costless international lump-sum transfers are not available in the second-best world. The previous literature suggested the use of a market solution with region-specific prices in such an environment. This may increase the scope of the Lindahl approach beyond a particular point in the first-best utility possibility frontier. In this section, we formally introduce the Lindahl equilibrium with region-specific prices, and examine the possible practical use of the system to induce the second-best Pareto optima. For an illustrative purpose, we assume $n = 2$.

7.1 Second-best Pareto efficiency

First, consider a welfare-maximization problem of $\max_{x_1,x_2} u_1(Y_1(x_1), x_1+x_2)+\mu u_2(Y_2(x_2), x_1+x_2)$ with $\mu > 0$. This corresponds to the case without lump-sum transfers. The solution, denoted by (x_1^μ, x_2^μ) and $X^\mu = x_1^\mu + x_2^\mu$, is characterized by:[25]

$$Y_1'(x_1^\mu) = D_1'(X^\mu) + \mu D_2'(X^\mu) = \mu Y_2'(x_2^\mu). \tag{22}$$

Once costless lump-sum transfers are ruled out, then we do not generally require production efficiency ($Y_1'(x_1) = Y_2'(x_2)$) in the social optimum. The allocations with emissions of

[25]Eliminating μ, (22) becomes $D_1'/Y_1' + D_2'/Y_2' = 1$, a general formula of the second-best optimum by Sandmo (2003, eq. (12)) and Chichilnisky et al. (2000, eq. (3.6)).

$(x_1^\mu, x_2^\mu, X^\mu)$ are called the second-best Pareto efficienct allocations.

7.2 Lindahl equilibrium with region-specific prices

Chichilnisky et al. (2000, p. 63) started with the introduction of the pairwise-trading model as in Section 6.2.[26] They proceeded to introduce *region-specific* prices $(q_i)_{i \in N}$ as a result of the pairwise-trading, with possibly a different price in each bilateral trade. Corresponding to their eq. (3.13), each country's budget constraint is characterized by:

$$c_i = Y_i(x_i) + q_i(\theta_i X - x_i), \tag{23}$$

where q_i is the price of pollution permits to country i. Having differential prices ($q_1 \neq q_2$) would be beneficial in achieving the second-best optima in (22), since the producers' revenue maximization, $Y_i'(x_i) = q_i$ ($i = 1, 2$), can lead to the condition of $Y_1'(x_1) = \mu Y_2'(x_2)$ with $\mu \neq 1$, by setting the differential prices appropriately. A similar discussion is found in Sandmo (2003, p. 122).[27]

To formalize their discussion, we introduce the following concept:

Definition 4 *The system of $(X, \mu, (\theta_i)_{i \in N})$ satisfies a Lindahl equilibrium with region-specific prices iff: (i) $\sum_{i \in N} \theta_i = 1$, (ii) $x_1(q(X)) + x_2(q(X)/\mu) = X$. (iii) Let $q_1(X) \equiv q(X)$ and*

[26] "In this context each pairwise externality is a separate commodity, separately priced. There are therefore ... n^2 prices, one between each pair of the n regions, as each is both a buyer and a seller of emission rights [$\hat{q}_{ki}$ $(k, i \in N)$ in Section 6.2], whereas with each charging a different price for a permit, there are only n prices [d_i]. By comparison, in the framework modeled previously, there is only one price [q]." (Chichilnisky et al. (2000, p. 63))

[27] "If [perfect international redistribution of income is not possible], one would like the marginal cost of contributing to the global public good to be less in the poor country than in the rich. ... [W]ith tradable quotas, *the price of a quota must be lower in the poor country.*" (Sandmo (2003, p. 122))

$q_2(X) \equiv q(X)/\mu$. *Then,*[28]

$$q_i(X)\theta_i + q_i'(X)(\theta_i X - x_i(q_i(X))) = D_i'(X) \text{ for all } i. \quad (24)$$

(24) is an extension of (8), which represents consumers' preferred level of X over the budget constraint (23). We now show the following:

Proposition 7 *The Lindahl-equilibrium with region-specific prices results in* $(x_1^\mu, x_2^\mu, X^\mu)$ *in (22).*

Proof: Summing up (24) with respect to 1 and 2, with country 2's equation being multiplied by μ, and using conditions (i)-(iii) of Definition 4, we obtain:

$$\begin{aligned} D_1'(X) + \mu D_2'(X) &= q(X)\sum_{i\in N}\theta_i + q'(X)(\sum_{i\in N}\theta_i X - x_1(q(X)) - x_2(q(X)/\mu)) \\ &= q(X) \\ &= Y_1'(x_1) = \mu Y_2'(x_2). \end{aligned}$$

Hence, we obtain (22) at the Lindahl equilibrium with region-specific prices. *Q.E.D.*

Proposition 7 formalizes the discussion in Chichilnisky et al. (2000, p. 63-64) which relates the pairwise-trading, the emissions trading with region-specific prices, and the second-best Pareto optima.[29] However, there are two problems to point out. Firstly, as a result

[28] The simultaneous Lindahl equilibrium can be defined analogously. In this section, we do not discuss the difference between the simultaneous and the sequential Lindahl equilibria, since the relevant qualitative properties to discuss are the same.

[29] The relationship is as follows. The firms' optimization condition, their eq. (3.14), corresponds to $x_i = x_i(q_i(X))$ in condition (ii) of Definition 4. The differential prices in their eq. (3.15) will result in the condition for second-best Pareto efficiency, eq. (3.6), corresponding to our (22). The present analysis makes the derivation of q_1, q_2 and X^μ in the Lindahl-process more explicit, connecting X^μ in (22) and the consumers' desired level of X in (24), which in turn derives $q_i = q_i(X^\mu)$ $(i = 1,\ 2)$ in the equilibrium.

of the pairwise-trading with n equilibrium prices, each polluter in fact pays the *same* price for each unit of pollution, as the result of Section 6.2 suggests.[30] Since Definition 4 involves differential prices among producers, *the pairwise-trading does not yield a Lindahl equilibrium with region-specific prices*, contrary to Chichilnisky et al.'s (2000) explanation quoted above.

The second and more serious problem with respect to our motivation in this section is the following:

Proposition 8 *Let c_i^μ be country i's consumption determined in (23) at the Lindahl equilibrium with region-specific prices. If $\mu \neq 1$ and $\theta_1 X^\mu \neq x_1^\mu$, then (23) implies $c_1^\mu + c_2^\mu \neq Y_1(x_1^\mu) + Y_2(x_2^\mu)$: one needs lump-sum taxes to cover a deficit or lump-sum transfers to refund a surplus to balance the world production and consumption.*

Proof: Adding up the net revenue from the emissions trading, we obtain $q_1(\theta_1 X^\mu - x_1^\mu) + q_2(\theta_2 X^\mu - x_2^\mu) = q_1(\theta_1 X^\mu - x_1^\mu) - q_2(\theta_1 X^\mu - x_1^\mu) = q_2(\mu - 1)(\theta_1 X^\mu - x_1^\mu) \equiv m^\mu$. Then $c_1^\mu + c_2^\mu = Y_1(x_1^\mu) + Y_2(x_2^\mu) + m^\mu$. *Q.E.D.*

Nothing in the equilibrium condition can ensure $\theta_1 X^\mu = x_1^\mu$, so that there generally arises a deficit or a surplus ($m^\mu \neq 0$) in the equilibrium. Such a deficit or a surplus has

[30] Consider a classic pairwise-trading, i.e., the model in Section 6.2 without individual strategic behaviors, where we assume from the beginning $\tilde{q}_{ik} = \hat{q}_{ik} \equiv q_{ik}$ and $\hat{q}_{ii} \equiv q_{ii}$ for all $i, k \in N$, $(k \neq i)$. The price-taking consumers' utility maximization from the budget constraint (20) results in $q_{ik} = D_i'(X) \equiv d_i$ for all i and k as long as $x_k > 0$: the consumers do not wish to differentiate the bilateral-prices in the process of the pairwise-trading. The Inada condition guarantees $x_k > 0$ for all k in the Warlasian equilibrium of the pairwise-trading. Therefore, in the equilibrium, each producer pays the same price $\sum_{j \in N} d_j \equiv q$ from a unit of pollution, as in Section 6.2. It is the *consumers* who receive different prices, $d_i = D_i'(X^*)$, from each polluter. The fraction d_i/q corresponds to θ_i of the simultaneous Lindahl equilibrium, not the Lindahl equilibrium with region-specific prices: see the difference between (21) and (23).

to be resolved by international transfers, *although we intended to find an efficiency gain of region-specific permit prices in the second-best world where such transfers are either costly or infrasible*! Neither Chichilnisky et al. (2000) nor Sandmo (2003) pointed out this issue.

Once we try to absorb the deficit or the surplus through international transfers, the mechanisms used in Section 6 do not work directly for an implementation of the Lindahl equilibria with differentiated prices: the m^μ above must enter the expression of π_i or c_i corresponding to (16), (17), (19) and (20), but the anticipation of the deficit or the surplus which depends on X and x_i distorts the behavior of producers and consumers. A more sophisticated mechanism is required (e.g., Moore and Repullo (1988)).

7.3 A framework with costly transfers

We instead consider the following framework. Suppose that country 1 is the rich country, and country 2 is the poor country. An international financial institute (such as the Global Environment Facility (GEF) in footnote 3) is established to absorb the possible deficit or the surplus. As a simple scenario, the deficit is financed by (taxing on) country 1 when $m^\mu > 0$, and the surplus is transferred to country 2 when $m^\mu < 0$.[31] As in the conventional welfare costs of the public finance, taxes incur the utility loss of $1 + \lambda_1 > 1$ per dollar to the taxpayers (due to the excess burden of the taxes or political oppositions against such expenditures), and only the fraction $1 - \lambda_2 \in (0, 1)$ of the transfer is recognized as the utility

[31] An example of the tax on the rich country is the Multilateral Fund by industrialized countries in the case of the Montreal Protocol. The transfer may be used for financing the range of global public goods, such as health, research, and development.

gain to the recipients (due to corruptions or technological inefficiencies of the foreign aids). Then the budget constraint of each country becomes:

$$c_1 = Y_1(x_1^\mu) + q^\mu(\theta_1^\mu X^\mu - x_1^\mu) - (1+\lambda_1)\max\{m^\mu, 0\}, \tag{25}$$

$$c_2 = Y_2(x_2^\mu) + \frac{q^\mu}{\mu}(\theta_2^\mu X^\mu - x_2^\mu) - (1-\lambda_2)\min\{m^\mu, 0\}, \tag{26}$$

where the superscript μ represents the Lindahl equilibrium with region-specific prices.

Suppose that a welfare weight μ^* is assigned on country 2, by a benevolent GEF or an equilibrium political bargaining. The present framework modifies the conclusion of Proposition 7, in that μ may not be equal to μ^* for a welfare maximization: the desire for global equity must be tempered by the direction of the permit trading (the second terms of (25) and (26)) and distortionary features of international transfers (the third terms of (25) and (26)).

In the context of *environmental taxation*, Hourcade and Gilotte (2000) and Sandmo (2005) showed that it is second-best optimal for the rich country to pay higher tax rates than the poor country in proportion to the weight of the social welfare. However, their analysis was from the perspective of a social-welfare maximizer. The present model can be applied to environmental taxation, as a politico-economic framework that allows an international tax-revenue sharing.

8 Conclusion

In this paper, we applied a Lindahl mechanism to the case of international interactions under global externalities. The proposed solution is appropriate to the Kyoto (or post-Kyoto) framework, since it has a built-in simultaneous choice of the total number of pollution permits (X) and their distribution ($(\theta_i)_{i \in N}$). It achieves the Samuelson condition, and furthermore, trading of the distribution through the competitive markets results in production efficiency. The present framework also admits (i) mechanisms for implementation (compliance with no misrepresentation of preferences), and (ii) extensions to the second-best environment with costly transfers.

Appendix A (Section 5.3)

Here, we introduce the definition of the stand-alone core in the abatement-game, and prove that a sequential Lindahl equilibrium belongs to the stand-alone core.

The set of feasible allocations for a nonempty coalition $S \subset N$ is given by: $F_S = \{(c_i, a_i)_{i \in S} \in I\!R_+^{|S|} \times \Pi_{j \in S}[0, \bar{x}_j] | \sum_{i \in S} w_i \geq \sum_{j \in S} c_j + \sum_{j \in S} \phi_j(a_j)\}$. An allocation $(c_i, a_i)_{i \in N} \in F_N$ belongs to the stand-alone core of the abatement game iff for all $S \subset N$, there does not exist $(c'_i, a'_i)_{i \in S} \in F_S$ such that for all $i \in S$, $v_i(c'_i, \sum_{j \in S} a'_j) \geq v_i(c_i, \sum_{j \in N} a_j)$, with strict inequality for at least one $i \in S$.

To prove the proposition, suppose that a coalition $S \subset N$ can find an allocation $(c'_i, a'_i)_{i \in S} \in F_S$ that is contrary to the condition stated in the previous footnote. Rewrite the utility function as $v_i(c_i, A) \equiv V_i(c_i, a_1, ..., a_n)$. Then the formulation of the Lindahl equilibrium is that every individual maximizes $V_i(c_i, a_1, ..., a_n)$ over the budget constraint of $c_i = w_i - \phi_i(a_i) + p(\sum_{j\in N} a_j)(a_i - \theta_i \sum_{j\in N} a_j)$. This implies that $c_i' \geq w_i - \phi_i(a_i') + p(\sum_{j\in S} a_j')(a_i' - \theta_i \sum_{j\in S} a_j')$ for all $i \in S$ with strict inequality for at least one $i \in S$. Then $\sum_{i\in S} c_i' > \sum_{i\in S}(w_i - \phi_i(a_i')) + p(\sum_{j\in S} a_j')(\sum_{i\in S} a_i' - \sum_{i\in S} \theta_i \sum_{j\in S} a_j') = \sum_{i\in S}(w_i - \phi_i(a_i')) + p(\sum_{j\in S} a_j') \sum_{i\in S} a_i'(1 - \sum_{j\in S} \theta_j) > \sum_{i\in S}(w_i - \phi_i(a_i'))$. This contradicts the definition of F_S. *Q.E.D.*

Appendix B (Section 6.1)

(a) First, we give full analysis for the cases when there is no q_i that satisfies (15). In such cases, either (i) $\sum_{j\in N, j\neq i} \tilde{x}_j^a(0) + \tilde{x}_{i+1}^b(0) < \sum_{j\in N} s_j$ or (ii) $\sum_{j\in N, j\neq i} \tilde{x}_j^a(q) + \tilde{x}_{i+1}^b(q) > \sum_{j\in N} s_j$ for all q. In (i), set the outcome as $q_i = 0$. In (ii), set the outcome as $q_i = \infty$ if $\sum_{j\in N} s_j > 0$, and, if $\sum_{j\in N} s_j \leq 0$, set directly the final outcome as $(\pi_i, x_i, c_i) = (0, 0, 0)$ for all i.

For case (i), $\tilde{x}_{i+1}^b(q_i) = \tilde{x}_i^a(q_i) = x_i(q_i)$ along the equilibrium, as shown in the text. And $x_i(0) = \infty$. This is not consistent with the original situation of $\sum_{j\in N, j\neq i} \tilde{x}_j^a(0) + \tilde{x}_{i+1}^b(0) < \sum_{j\in N} s_j$, so that it does not constitute an equilibrium. In (ii), if $\sum_{j\in N} s_j > 0$, then producers' optimization implies $\tilde{x}_j^a(q_i) = 0$ for all j and $\tilde{x}_{i+1}^b(q_i) = 0$, so that $\sum_{j\in N, j\neq i} \tilde{x}_j^a(q_i) + \tilde{x}_{i+1}^b(q_i) = 0$. This is a contradiction. Suppose that $\sum_{j\in N} s_j \leq 0$. Part (b) of the proof shows that, for any $\sum_{j\in N} s_j > 0$, $X = \sum_{j\in N} s_j$, $q_i = q(X)$ and $\tilde{x}_{i+1}^b(q_i) = \tilde{x}_i^a(q_i) = x_i(q(X))$ for all i, which implies $c_i = c_i(X, (1/n) + s_{i+1} - s_{i+2})$ for all i. If $\lim_{X\to 0} \varepsilon(X) \neq 1$ (see footnote 22), then $\lim_{X\to 0} c_i(X,(1/n)+s_{i+1}-s_{i+2})=0$ for all i, s_{i+1} and s_{i+2},[32] so that the consumers' payoff functions are continuous at any $(s_j)_{j\in N}$ where $\sum_{j\in N} s_j = 0$. Given $\sum_{j\in N} s_j \le 0$, for the expression of $u_i(c_i(\sum_{j\in N} s_j,(1/n)+s_{i+1}-s_{i+2}),\sum_{j\in N} s_j)$ in the text, there exists $k\in N$ such that $\partial u_k(c_i(0,(1/n)+s_{i+1}-s_{i+2}),0)/\partial s_k > 0$: the logic is analogous to footnote 5. Consumer k's best response against $(s_j)_{j\in N, j\neq k}$ is $\tilde{s}_k$ such that $\sum_{j\in N, j\neq k} s_j + \tilde{s}_k > 0$, so that it does not constitute an equilibrium.

(b) Next, we will show that $q_i = q_c$ for all i in the subgame-perfect equilibrium. Since the case of $\sum_{j\in N} s_j \le 0$ is dealt with in part (a), suppose that $\sum_{j\in N} s_j > 0$. Suppose to the contrary that $q_{i_1} > q_{i_2}$ for some i_1 and i_2. Notice that, $\tilde{x}_i^b(q_{i-1}) = \tilde{x}_{i-1}^a(q_{i-1}) = x_{i-1}(q_{i-1})$ for all i in equilibrium, and $q_{i_2} < \infty$ and the assumptions for the production functions together imply $\tilde{x}_{i_2}^b(q_{i_2-1}) > 0$. Since $\tilde{x}_i^a : I\!R_{++} \to I\!R_+$ and $\tilde{x}_i^b : I\!R_{++} \to I\!R_+$ must be decreasing, (15) implies $\sum_{j\in N} s_j = \sum_{j\in N, j\neq i_1} \tilde{x}_j^a(q_{i_1}) + \tilde{x}_{i_1+1}^b(q_{i_1}) < \sum_{j\in N, j\neq i_2} \tilde{x}_j^a(q_{i_2}) + \tilde{x}_{i_2+1}^b(q_{i_2}) = \sum_{j\in N} s_j$. This is a contradiction. Taking all properties together, (15) indicates that $q_i = q_c = q(\sum_{j\in N} s_j)$ for all i.

(c) For $n=2$, remaining the strategies in Stage 2 the same, we replace the strategies in Stage 1 with the following two-step process:

[32]First, $\lim_{X\to 0} Y_i(x_i(q(X))) = 0$ for all i. From de l'Hospital's rule, $\lim_{X\to 0} q(X)X = \lim_{X\to 0} \dfrac{q(X)}{1/X} = \lim\limits_{X\to 0} \dfrac{q'(X)}{-1/X^2} = \lim\limits_{X\to 0}(-q'(X)X^2) = \lim\limits_{X\to 0} q(X)X\dfrac{1}{\varepsilon(X)}$, so that $\lim_{X\to 0} q(X)X\left(1-\dfrac{1}{\varepsilon(X)}\right) = 0$. The assumption implies $\lim_{X\to 0} q(X)X = 0$. Also, $0 \le \lim_{X\to 0} q(X)x_i(q(X)) \le \lim_{X\to 0} q(X)X = 0$. Taking account of these, $\lim_{X\to 0} c_i(X,(1/n)+s_{i+1}-s_{i+2}) = \lim_{X\to 0}\{Y_i(x_i(q(X)))+q(X)(((1/n)+s_{i+1}-s_{i+2})X-x_i(q(X)))\} = 0$ for all i.

Stage 1.1 : The consumer in country i announces $(r_i^a, r_i^b) \in I\!R^2$.

Stage 1.2 : The consumer in country i announces $s_i \in I\!R_+$.

As to the outcome, q_i in (15), $q_c = \min_{i \in N} q_i$, π_i in (16), and X in (18) remain the same, and c_i in (17) is replaced with:

$$c_i = \pi_i + q_c((1 - r_k^a)s_i + r_k^b s_k) - |r_i^b - r_k^a|, \; i \neq k,$$

and consumers' utilities are given by $u_i(c_i, X)$.

Here, consumers announce a tolerable level of emissions (s_1, s_2), with $X = \sum_{j \in N} s_j$ in equilibrium. Out of the revenue $q_c s_i$ that consumer i receives from the emissions-trading market, the proportion r_k^a chosen by the opponent is used to compensate against consumer k, as a victim of multilateral externalities. Conversely, consumer i sets r_i^a, the rate of compensation from consumer k's announced revenue, and in equilibrium she receives $q_c r_k^b s_k = q_c r_i^a s_k$. This is a public-bad version of the matching contribution model by Guttman (1978) and Danziger and Schnytzer (1991), which has a particular interpretation in the present context. We will show that:

$$\theta_i s_i = (1 - r_k^a)s_i \text{ and } \theta_i s_k = r_i^a s_k \text{ for all } i \text{ and } k, \; i \neq k.$$

That is, the revenue share in the Lindahl solution represents: (i) the fraction of the revenue received from the emissions-trading market net of the compensation paid to the other consumer, and (ii) the compensation received from the other consumer.

Now, the equilibrium properties of Stage 2 remain the same as in the case of the text, since we made use of (15), (16) and (18) only in the derivation: we have $q_c = q(\sum_{j \in N} s_j)$, $X = \sum_{j \in N} s_j$, and $x_i = x_i(q(\sum_{j \in N} s_j))$ in equilibrium. The expression of c_i along the equilibrium path is:

$$Y_i(x_i(q(\sum_{j \in N} s_j))) + q(\sum_{j \in N} s_j)((1 - r_k^a)s_i + r_k^b s_k - x_i(q(\sum_{j \in N} s_j))) - |r_i^b - r_k^a| \equiv \tilde{c}_i(s_i, s_k; r), \; i \neq k, \tag{27}$$

where $r \equiv (r_i^a, r_i^b)_{i \in N}$. In Stage 1, consumers noncooperatively maximize $u_i(\tilde{c}_i(s_i, s_k; r), \sum_{j \in N} s_j)$.

In Stage 1.2, the choice of s_i is determined by $\max_{s_i \geq 0} u_i(\tilde{c}_i(s_i, s_k; r), \sum_{j \in N} s_j)$. Let $(1 - r_k^a)s_i + r_k^b s_k \equiv S_i$. The first-order conditions are:

$$q(X)(1 - r_k^a) + q'(X)(S_i - x_i(q(X))) = D_i'(X) \text{ or } s_i = 0, \quad i = 1, \; 2, \; i \neq k. \tag{28}$$

Consider Stage 1.1. As a minimal assumption, suppose that Local Invertibility of Varian (1994, p. 1286) holds.[33] We first show that $r_i^b = r_k^a$ $(i, k = 1, 2, \; i \neq k)$. Suppose to the contrary that i would choose $r_i^b \neq r_k^a$ which results in $(\hat{c}_i, \hat{s}_1 + \hat{s}_2)$. Consider an alternative strategy by consumer i where $r_i^b = r_k^a$ and use the compensation rate r_i^a to induce consumer k to announce $\hat{s}_j$. Then, this new allocation has the same pollution $\hat{s}_1 + \hat{s}_2$, the price of the permits $q(\hat{s}_1 + \hat{s}_2)$, productions, and the profits, but at a higher c_i since there is no penalty. This implies that the choice of $r_i^b \neq r_k^a$ does not constitute the equilibrium outcome. As to the total emissions, Varian's (1994) Theorem 1 applies to prove Pareto efficiency of

[33] Let $s = (s_1, s_2)$ be the outcome from announcement r. Let $\hat{s}_i$ be a choice close to s_i that consumer j prefers to s_i. Then there is some $\hat{r}_j^a$ that consumer j can announce that will make $\hat{s}_i$ an optimal choice for consumer i.

the equilibrium outcome ($s_1 + s_2 = X^*$). Then (s_1, s_2) has two possibilities. First, it is an interior solution so that s_1 and s_2 are both positive. In this case, since s_1 and s_2 are perfect substitutes in consumption, they must have the same price in equilibrium (otherwise there is a profitable deviation): we now have $1 - r_i^a = r_i^b = r_k^a$ $(i, k = 1, 2,\ i \neq k)$. Then the condition of the interior optimum in (28) is equivalent to (8), where the only solution is $r_i^a = \theta_i$ $(i = 1, 2)$. Second, when $s_k = 0$ for $k = 1$ or 2, then $s_i = X^*$ $(i \neq k)$, and the first-order condition $q(X^*)(1 - r_k^a) + q'(X^*)((1 - r_k^a)X^* - x_i(q(X^*))) = D_i'(X^*)$ is equivalent to $1 - r_k^a = \theta_i$ corresponding to (8). Since $\theta_k = 1 - \theta_i$, so $r_k^a = \theta_k$. This yields $S_i = (1 - r_k^a)s_i = \theta_i X^*$ and $S_k = r_i^b X^* = r_k^a X^* = \theta_k X^*$. Substituting the equilibrium conditions into (27), we obtain $\tilde{c}_i(s_i, s_k; r) = c_i(X^*, \theta_i)$ and $\tilde{c}_k(s_i, s_k; r) = c_k(X^*, \theta_k)$ in both cases. As in the text, we obtained one-to-one correspondence between conditions (i)-(iv) of Definition 1 and the subgame-perfect equilibrium outcome. *Q.E.D.*

Appendix C (Section 6.2)

We work through backward-induction to examine the equilibrium of the game in Section 6.2. Start with the second stage. Producer $i \in N$ maximizes its profits π_i in (19), given $(\hat{q}_{ki})_{k \in N}$ announced by each consumer $k \in N$ in the first stage. This results in $Y_i'(x_i) = \sum_{k \in N} \hat{q}_{ki}$ for all i, which determines the relationship between x_i and $\sum_{k \in N} \hat{q}_{ki}$. The function $x_i(q)$ is exactly the same as in (4), with $x_i'(q) < 0$.

In Stage 1, the firms' choice is to minimize the penalties, similar to Section 6.1: $\tilde{q}_{ki} = \hat{q}_{ki}$ for all i and $k \neq i$. On the other hand, consumer i maximizes $u_i(c_i, \sum_{k \in N} x_k)$ with c_i given by (20), noting that her choice of $\hat{q}_{ik}$ has an indirect effect through the influence on polluter k's choice of x_k in Stage 2. The first-order conditions of the utility maximization with respect to $\hat{q}_{ik}$ $(k \neq i)$ yield $\partial(\pi_i(\sum_{j\in N}\hat{q}_{ji}, \sum_{j\in N, j\neq i}|\hat{q}_{ji} - \tilde{q}_{ji}|) + \sum_{l\in N, l\neq i}\tilde{q}_{il}x_l(\sum_{j\in N}\hat{q}_{jl}) + \hat{q}_{ii}x_i(\sum_{j\in N}\hat{q}_{ji}) - D_i(\sum_{l\in N}x_l(\sum_{j\in N}\hat{q}_{jl}))/\partial\hat{q}_{ik} = (\tilde{q}_{ik} - D_i'(X))x_k'(\sum_{j\in N}\hat{q}_{jk}) = 0$. As to $\hat{q}_{ii}$, noting that $\partial(\pi_i + \hat{q}_{ii}x_i(\sum_{j\in N}\hat{q}_{ji}))/\partial\hat{q}_{ii} = -x_i(\sum_{j\in N}\hat{q}_{ji}) + x_i(\sum_{j\in N}\hat{q}_{ji}) + \hat{q}_{ii}x_i'(\sum_{j\in N}\hat{q}_{ji}) = \hat{q}_{ii}x_i'(\sum_{j\in N}\hat{q}_{ji})$, the utility maximization yields $(\hat{q}_{ii} - D_i'(X))x_i'(\sum_{j\in N}\hat{q}_{ji}) = 0$. Since $x_j'(q) < 0$ for all j, one must conclude $\tilde{q}_{ik} = D_i'(X) = \hat{q}_{ii}$ for all i and $k \neq i$ as the equilibrium prices. *Q.E.D.*

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Table1: Numerical Example (1)

		Case 1	Case 2	Case 3
a_1		0.9	0.6	0.52
a_2		0.1	0.4	0.48
b_1		0.5	0.23	0.9
b_2		0.1	0.4	0.2
x^*_1		0.613702	0.148367	0.902691
x^*_2		0.024548	0.448748	0.044577
X^*		0.63825	0.597116	0.947268
Sequential Lindahl equilibrium	c_1	0.733255	0.427857	0.93321
	c_2	0.081473	0.285238	0.861424
	u_1	0.549941	0.320893	0.699907
	u_2	0.061105	0.213928	0.646068
	θ_1	0.838462	0.951527	0.087059
	country 1's revenue	-0.05014	0.250672	-0.77697
Simultaneous Lindahl equilibrium	c_1	0.758323	0.302521	1.321695
	c_2	0.056404	0.410574	0.472939
	u_1	0.575009	0.195557	1.088393
	u_2	0.036036	0.339264	0.257583
	θ_1	0.9	0.6	0.52
	country 1's revenue	-0.02507	0.125336	-0.38849
Nash equilibrium (disagreement point)	$\hat{X}$	1.093806	1.046761	1.468865
	c_1	0.507911	0.168456	2.120946
	c_2	0.182848	0.764262	0.113466
	$\hat{x}_1$	0.257973	0.134109	1.388399
	$\hat{x}_2$	0.835833	0.912652	0.080466
	$\hat{u}_1$	-0.03047	-0.16026	1.559979
	$\hat{u}_2$	0.123027	0.545121	-0.40435

Table 2: Numerical Example (2)

		Case 1	Case 2	Case 3
a_1		0.9	0.6	0.52
a_2		0.1	0.4	0.48
b_1		0.5	0.23	0.9
b_2		0.1	0.4	0.2
x^*_1		0.613702	0.148367	0.902691
x^*_2		0.024548	0.448748	0.044577
X^*		0.63825	0.597116	0.947268
Nash equilibrium	$\hat{X}$	1.093806	1.046761	1.468865
	$\hat{u}_1$	-0.03047	-0.16026	1.559979
	$\hat{u}_2$	0.123027	0.545121	-0.40435
Helm (2003)	X	1.013159	0.947862	1.503695
	u_1	-0.2961	-0.22598	1.476364
	u_2	0.809349	0.675204	-0.34581
	country 1's revenue	-0.82119	-0.17969	-0.09044
Shiell (2003)	u_1	0.600078	0.070221	1.476878
	u_2	0.010967	0.4646	-0.1309
Tadenuma (2005)	θ_1	0.235849	0.128118	0.945219
	u_1^a	0.10016	-0.15093	1.559997
	u_2^a	0.368198	0.54723	-0.40402
	u_1^b	0.304458	0.027309	1.469949
	u_2^b	0.306587	0.507512	-0.12397