Queen's Economics Department Working Paper No. 1129

Pooling Forecasts in Linear Rational Expectations Models

Gregor W. Smith
Department of Economics, Queen's University

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

6-2007

Pooling Forecasts in Linear Rational Expectations Models

Gregor W. Smith*

June 2007

first draft November 2006

Abstract

Estimating linear rational expectations models requires replacing the expectations of future, endogenous variables either with forecasts from a fully solved model, or with the instrumented actual values, or with forecast survey data. Extending the methods of McCallum (1976) and Gottfries and Persson (1988), I show how to pool these methods and also use actual, future values of these variables to improve statistical efficiency. The method is illustrated with an application using SPF survey data in the US Phillips curve, where the output gap plays a significant role but lagged inflation plays none.

JEL classification: E37, C53

Keywords: rational expectations, recursive projection, Phillips curve

* Department of Economics, Queen's University, Kingston Ontario Canada K7L 3N6; smithgw@econ.queensu.ca. I thank the Social Sciences and Humanities Research Council of Canada and the Bank of Canada fellowship programme for support of this research. The views expressed herein are not necessarily those of the Bank of Canada and are the author's alone. For helpful comments I thank Jason Allen, Paul Beaudry, and seminar participants at the Bank of Canada and the Canadian Economics Association meetings.

1. Introduction

Linear, expectational, difference equations serve as key building blocks in a range of macroeconomic models. Examples include versions of the Phillips curve, dynamic IS curves, or factor demand equations when there are adjustment costs. In these equations, the current value of an endogenous variable is partly explained by its expected future value. A large literature estimates these equations by (a) solving the difference equation using statistical forecasts for exogenous variables, or (b) the substitution method with instrumental variables, or (c) replacing the expectation with survey data. There does not seem to be a consensus on which method is best, though several studies have investigated this issue numerically for specific applications.

This paper outlines a method for gaining statistical efficiency by combining these two approaches. The method also uses actual data on future, endogenous variables but without the need for instrumental variables. As a by-product it yields estimates of the relative roles of various forecasts. The method is a simple application of the recursive projection formula and is a direct extension of an important but neglected contribution by Gottfries and Persson (1988).

Section 2 describes the problem and background on estimation methods. Section 3 outlines the combination of methods of modelling expectations. Section 4 applies the technique to the US new Keynesian Phillips curve. The economic findings are that survey data contain much of the information available on expected inflation. Moreover, inflation in the Phillips curve estimated with survey data is more forward-looking and more linked to the output gap than with traditional, instrumental variables methods. The economic findings are illustrated with a decomposition of US inflation from 1981 to 2006. Section 5 concludes.

2. Problem and Background

Suppose that a model links an endogenous variable, denoted y_t, to an exogenous variable, x_t. Thinking of y and x as scalars is only for simplicity. Suppose that $\{x_t, y_t\}$ are adapted to a filtration $F = \{\mathcal{F}_t : t \in [0, \infty)\}$ where $\mathcal{F}_t$ is a non-decreasing sequence of sub-tribes on a probability space $(\Omega, \mathcal{F}, P)$. Notice that this information set includes more that $\{x_t, y_t\}$ and is not simply generated from their histories. The economic model is:

$$y_t = \beta E[y_{t+1}|\mathcal{F}_t] + \lambda x_t. \tag{1}$$

Examples of equation (1) include the linearized, new Keynesian Phillips curve or a system that includes it as one equation. The econometrician observes y and x but does not observe $\mathcal{F}_t$, which is information available only to the market participants. Importantly, we assume that $x_t \in \mathcal{F}_t$. Notice that there is a standard, stochastic singularity (no error term) in the economic model (1). But error terms will arise in estimating equations due to projection on an econometrician's information set, just as in Hansen and Sargent's (1980) modelling approach.

For ease of reading, I refer to $\mathcal{F}_t$ as the market's information set. An econometrician also may forecast inflation. I use the traditional notation z_t for the instruments that the econometrician uses in forecasting. These lie in a set Z_t. Finally, some forecasters may be surveyed for their forecasts of y_{t+1}. The median forecast uses an information set $\mathcal{G}_t$, while another, individual forecaster uses an information set $\mathcal{G}_t^i$. Table 1 summarizes the information sets referred to.

Table 1: Information Sets

Set	Holder
$\mathcal{F}_t$	Market
$\mathcal{G}_t$	Median Forecaster
$\mathcal{G}_t^i$	Forecaster i
Z_t	Econometrician

A key set of assumptions is:

$$x_t \in Z_t \subseteq \mathcal{G}_t \subseteq \mathcal{F}_t. \tag{2}$$

The median forecaster has at least as much information as the econometrician, and the market has at least as much information as the median forecaster.

The three traditional approaches to estimating and testing the structure (1) are to (a) solve the model, forecast future values of x with a time-series model, replace the expectations with that model's forecasts, and estimate the system; (b) substitute actual, future y_{t+1}, an error-laden measure of the expectation, and then use instrumental variables estimation; or (c) substitute survey data on the forecasts of y_{t+1}.

In method (a) the difference equation can be solved, if accompanied by a time series model of x. The relationship (1) can be solved forwards in present-value form as:

$$y_t = \beta E[\sum_{k=1}^{\infty} \beta^k x_{t+k} | \mathcal{F}_t] + \lambda x_t, \tag{3}$$

provided a no-bubbles condition applies. Suppose that the econometrician forecasts x_{t+k} with an information set $Z_t \in \mathcal{F}_t$. For example, a specific instrument set z_t would involve past values of x alone. Then projecting the solved model (3) on z_t gives the estimating equation:

$$y_t = \beta E[\sum_{k=1}^{\infty} \beta^k x_{t+k} | z_t] + \lambda x_t + \epsilon_{t+1}, \tag{4}$$

where the error term reflects the information advantage of market participants over the econometrician. Equation (4) is estimated jointly with the forecasting equation for x. Hansen and Sargent (1980) is the classic reference. The drawback to this method is that it requires the specification of a stable forecasting equation.

In approach (b) we use the law of iterated expectations (the tower property of conditional expectations) and some instruments z_t as follows:

$$\begin{aligned} y_t &= \beta E[E(y_{t+1}|\mathcal{F}_t)|z_t] + \lambda x_t + \beta\big(E[y_{t+1}|\mathcal{F}_t] - E[E(y_{t+1}|\mathcal{F}_t)|z_t]\big) \\ &= \beta E[y_{t+1}|z_t] + \lambda x_t + \epsilon_{t+1} \end{aligned} \tag{5}$$

so that ϵ_t is an error term that is uncorrelated with the regressors. An alternate way to find the estimating equation (5) is to begin with the substitution of the actual value y_{t+1}:

$$\begin{aligned} y_t &= \beta y_{t+1} - \beta(y_{t+1} - E[y_{t+1}|\mathcal{F}_t]) + \lambda x_t \\ &= \beta y_{t+1} - \beta \eta_{t+1} + \lambda x_t \end{aligned}$$

The parameter β cannot be estimated consistently by least squares here because of the correlation between the regressor y_{t+1} and the forecast error η_{t+1}. But applying instrumental variables methods gives (5). This approach is of course the substitution method proposed by McCallum (1976) or a case of GMM estimation introduced by Hansen (1982).

A difficulty with approach (b) is that it sometimes is difficult to find strong instruments so that estimation and inference are reliable. Finding an instrument means forecasting y_{t+1} with a variable other than y_t or x_t. Roughly speaking, instruments may be weak when there is little time-series persistence in the problem. Andrews and Stock (2006) provide a comprehensive survey of this topic.

In method (c), suppose that we have survey data on the one-step-ahead, median forecasts: $E[y_{t+1}|\mathcal{G}_t]$. The law of iterated expectations gives us:

$$y_t = \beta E[y_{t+1}|\mathcal{G}_t] + \lambda x_t + \epsilon_{t+1}. \tag{6}$$

The error term reflects the fact that $\mathcal{G}_t \subseteq \mathcal{F}_t$; no individual forecaster has the complete information that drives the market. With this substitution, equation (6) can be estimated by ordinary least squares, for we directly collect data on $E[y_{t+1}|\mathcal{G}_t]$ and the error term is uncorrelated with the regressors.

Some researchers instrument forecast survey data, though the fact that $E[y_{t+1}|\mathcal{G}_t] \neq E[y_{t+1}|\mathcal{F}_t]$ does not create a traditional errors-in-variables problem. Instrumenting the survey data usually would reflect an errors-in-variables problem in which there is an additional source of noise – say denoted ξ_t – in the reported survey data. Suppose that the reported data are denoted $\hat{y}_{t+1}$, with:

$$\hat{y}_{t+1} = E[y_{t+1}|\mathcal{G}_t] + \xi_{t+1}, \tag{7a}$$

and with

$$E(\xi_{t+1}, E[y_{t+1}|\mathcal{G}_t]) = 0. \tag{7b}$$

This model implies that $\text{var}\hat{y}_{t+1} > \text{var}E[y_{t+1}|\mathcal{G}_t]$. Under this statistical model of the survey data, using the survey data as a regressor would lead to inconsistent parameter estimates, for the error term ξ_{t+1} would be correlated with the regressor $\hat{y}_{t+1}$.

We also can decompose the realized value of y_{t+1} into a forecast and a forecast error:

$$y_{t+1} = E[y_{t+1}|\mathcal{G}_t] + \eta_{t+1}, \tag{8a}$$

with

$$E(\eta_{t+1}, E[y_{t+1}|\mathcal{G}_t]) = 0. \tag{8b}$$

Combining (7a) and (8a) gives:

$$\hat{y}_{t+1} = y_{t+1} - \eta_{t+1} + \xi_{t+1}. \tag{9}$$

One might hope to test the EIV perspective on reported forecasts by regressing $\hat{y}_{t+1}$ on y_{t+1}, a method applied in different contexts by Mankiw, Runkle, and Shapiro (1984) and Milbourne and Smith (1989). But because η_{t+1} is correlated with y_{t+1}, equation (9) itself has an EIV problem. If the EIV model (7) holds, then the coefficient in the regression (9) will be biased towards zero, and the variance of $\hat{y}_{t+1}$ can be greater or less than the variance of y_{t+1}.

An alternative statistical perspective on the survey data is that they are reported without error and are rational forecasts based on limited information:
$\hat{y}_{t+1} = E[y_{t+1}|\mathcal{G}_t]$. In that case:

$$y_{t+1} = \hat{y}_{t+1} + \eta_{t+1}, \tag{10}$$

This model implies that $\mathrm{var}\hat{y}_{t+1} < \mathrm{var} y_{t+1}$. Under this view, there is no need to instrument the survey data. Of course, other statistical models of the survey data are possible. But meanwhile, regression (10) or the comparison of variances can be used as tests of the hypothesis that the survey data are rational forecasts.

3. Pooling Information on Expectations

With this background, I next show how to combine forecast methods optimally using the recursive projection formula and a simple extension of Gottfries and Persson's (1988) insight now to the economic model (1). I then discuss the findings and relate them to the research literature on forecast combination and on generated regressors. An extension shows that the method can be used to combine more than two sources of information on expectations.

3.1 Recursive Projection

There are two main methodological points in this paper. First, one can combine the estimation methods in one step and so improve estimates of the forecast value $E[y_{t+1}|\mathcal{F}_t]$ (gain statistical efficiency). Second, one also can include the actual, future value y_{t+1} in this combination without the traditional need to instrument it.

Methods (a) and (c) ignore a source of information on $E[y_{t+1}|\mathcal{F}_t]$ in the form of the actual value y_{t+1}. If we were trying to estimate the series of expectations $E[y_{t+1}|\mathcal{F}_t]$ as accurately as possible, then we certainly would use the series of realized values. For example, we could link the two in the observation equation of a Kalman filtering problem like the one adopted by Hamilton (1985). Instead, we can include this information in one step. This proposal is inspired by the neglected insight of Gottfries and Persson (1988) and also is a direct way of conducting the filtering exercise recommended by Hamilton (1985), although those authors did not discuss survey data or combining forecasts. Gottfries and Persson showed how to combine methods (a) and (b) with forecasts of an exogenous variable. I extend their method to the case with forecasts of endogenous variables and by allowing for survey data.

Suppose that one begins with a candidate to replace the unobservable $E[y_{t+1}|\mathcal{F}_t]$, either using econometric forecasts or professional forecasts of y_{t+1} as in equation (6). For simplicity, I use the latter case.

Proposition: A linear combination of $E[y_{t+1}|\mathcal{G}_t]$ and y_{t+1} is weakly exogenous for β. Thus the parameters $\{\beta, \lambda\}$ may be consistently estimated by least squares in:

$$y_t = \beta\big[(1-m)E[y_{t+1}|\mathcal{G}_t] + my_{t+1}\big] + \lambda x_t, \tag{11}$$

which yields statistical efficiency greater than or equal to that of the standard estimator that imposes $m = 0$. Alternately, actual y_{t+1} can be combined with the instrumented value $E[y_{t+1}|z_t]$ instead of the survey value $E[y_{t+1}|\mathcal{G}_t]$.

Proof: The proof uses the law of iterated expectations and the recursive projection formula given by Sargent (1987, chapter X) or Whittle (1983). From the economic model (1), y_t is generated by:

$$y_t = \beta E[y_{t+1}|\mathcal{F}_t] + \lambda x_t.$$

Consider the projection of y_t on $E[y_{t+1}|\mathcal{G}_t]$, y_{t+1}, and x_t:

$$\begin{aligned}
&E[y_t|E[y_{t+1}|\mathcal{G}_t], y_{t+1}, x_t] \\
&= E[\beta E[y_{t+1}|\mathcal{F}_t] + \lambda x_t|E[y_{t+1}|\mathcal{G}_t], y_{t+1}, x_t] \\
&= \beta E[E[y_{t+1}|\mathcal{F}_t]|E[y_{t+1}|\mathcal{G}_t], y_{t+1}] + \lambda x_t \\
&= \beta E[y_{t+1}|\mathcal{G}_t] + \beta E[E[y_{t+1}|\mathcal{F}_t] - E[y_{t+1}|\mathcal{G}_t]|y_{t+1} - E[y_{t+1}|\mathcal{G}_t]] + \lambda x_t \\
&= \beta E[y_{t+1}|\mathcal{G}_t] + \beta m(y_{t+1} - E[y_{t+1}|\mathcal{G}_t]) + \lambda x_t \\
&= \beta(1-m)E[y_{t+1}|\mathcal{G}_t] + \beta m y_{t+1} + \lambda x_t
\end{aligned} \tag{12}$$

where m is a least-squares projection coefficient given by:

$$m \equiv \frac{\mathrm{cov}(E[y_{t+1}|\mathcal{F}_t] - E[y_{t+1}|\mathcal{G}_t], y_{t+1} - E[y_{t+1}|\mathcal{G}_t])}{\mathrm{var}(y_{t+1} - E[y_{t+1}|\mathcal{G}_t])}. \tag{13}$$

There is an efficiency gain provided that the covariance in (13) is non-zero. The projection (12) applies with $Z_t \subseteq \mathcal{G}_t$ instead of $\mathcal{G}_t$, so that the estimator (11) also extends the standard instrumental-variables estimator.$\|$

3.2 Discussion

If one's initial forecast comes from survey data, then the parameters β, m, and λ can be estimated consistently by OLS with no need for instrumental variables estimation. Gottfries and Persson's insight was that a projection error is uncorrelated with the regressors, so that the assumptions of ordinary least squares apply. The error term is uncorrelated with $E[y_{t+1}|\mathcal{G}_t]$ and y_{t+1} by the recursive projection formula and with x_t given that $x_t \in \mathcal{G}_t$. (Recall from the assumption about information sets (2) that Z_t and $\mathcal{G}_t$ include x_t; thus projecting on either of these information sets leaves an error that is orthogonal to x_t.) The intuition is that least-squares selects the linear combination of the two series, $E[y_{t+1}|\mathcal{G}_t]$ and y_{t+1}, that best mimics the unobservable $E[y_{t+1}|\mathcal{F}_t]$ by selecting the combination that best explains y_t in the economic model. By combining information sources this approach yields more efficient estimates of β. It also yields interesting measurements of the coefficient m as a by-product; m measures the information on $E[y_{t+1}|\mathcal{F}_t]$ that is contained in y_{t+1} but missing from $E[y_{t+1}|\mathcal{G}_t]$.

If survey data are not available and one begins by constructing $E[y_{t+1}|z_t]$ econometrically, then valid instruments are needed as in any instrumental-variables application. As already noted, the information set must include x_t. And if there were a constant term in the model (1) then that would not be a valid instrument because it would not allow identification of β.

When one considers combining $E[y_{t+1}|z_t]$ with y_{t+1} in the estimating equation (11) the reader might wonder if one is projecting y_{t+1} on itself. That is not what happens. If one thought of this as two-stage least squares, then valid instruments would be used in the first stage to construct $E[y_{t+1}|z_t]$. In the second stage, the projection of y_t on this constructed forecast and y_{t+1} reproduces the projection of $E[y_{t+1}|\mathcal{F}_t]$ on those two variables.

It is possible that $m = 1$ so that there is a weight of zero on $E[y_{t+1}|\mathcal{G}_t]$ and a weight of one on y_{t+1}. But that outcome would not lead to an errors-in-variables problem familiar from McCallum's (1976) original contribution. Inspection of the formula for m (13) shows that $m = 1$ only if $E[y_{t+1}|\mathcal{F}_t] = y_{t+1}$, so that the unobserved forecast driving the market coincides with the realized value. In that case, y_{t+1} would indeed be the correct variable to include and estimation by least-squares would be appropriate.

One of the most striking results in research on economic forecasting is that there are gains from forecast combination. Perhaps these gains stem from diversification; the combination of forecasts based on different methods such as time series models or survey data. Whatever the explanation, there is considerable evidence that pooled forecasts outperform all individual forecasts. The classic paper by Bates and Granger (1969) described choosing weights in a linear combination of forecasts in order to minimize the forecast error variance of the combination. For example, the weights could be estimated by regressing the outcome on two, competing, past forecasts. Newbold and Harvey (2002) lucidly survey the large research literature on forecast combination.

The linear combination of information sources (11) is reminiscent of traditional forecast combination. But in the context of the economic relationship (1) studied here, we are not trying to predict y_{t+1} as accurately as possible but rather trying to mimic the unobserved forecast $E[y_{t+1}|\mathcal{F}_t]$ as accurately as possible in order to explain the current value y_t. The weights are chosen based on the observation equation (1). One cannot directly project the actual variable $E[y_{t+1}|\mathcal{F}_t]$ on the two information sources in order to find weights, but one can do this indirectly by projecting y_t and controlling for x_t. A zero weight in the combination means that the forecast from the corresponding information is encompassed.

The proposition holds that $\{\beta, m, \lambda\}$ may be estimated consistently by OLS. That method also yields correct standard errors when the projection uses survey data $E[y_{t+1}|\mathcal{G}_t]$. When, instead, the forecast of y_{t+1} is constructed by the econometrician, then the projection becomes:

$$\begin{aligned} y_t &= \beta\big[(1-m)E[y_{t+1}|z_t] + m y_{t+1}\big] + \lambda x_t \\ &= \beta E[y_{t+1}|z_t] + \beta m\big[y_{t+1} - E[y_{t+1}|z_t]\big] + \lambda x_t. \end{aligned} \tag{14}$$

Thus the projection used for estimation includes both the forecast and the 'surprise' relative to the information set Z_t, even though the economic model includes only the forecast relative to information set $\mathcal{F}_t$. (For that matter, so does the original version (11).) This is a case of observational equivalence: the additional term may be present because of extra information or because the surprise also determines y_t. Gottfries and Persson (1988, p 254) noted that m cannot be identified if the underlying economic model actually does include a surprise term.

Equation (14) obviously cannot be estimated by McCallum's substitution method, for the actual value y_{t+1} also enters as an explanatory variable. It can be estimated with a two-step method, in which one first estimates $E[y_{t+1}|z_t]$ by least squares, then substitutes the fitted values in the second step regression (14). Classic studies of rational expectations econometrics by Abel and Mishkin (1980) and Pagan (1984, theorem 7) show that the two-step estimator has the same limiting distribution as a system estimator but that it understates the standard errors because of the generated regressor. Thus standard errors must be constructed using two-stage least squares.

3.3 Extension

The same method may be used when one has more than one forecast of y_{t+1}. For example, suppose that one forms an econometric forecast $E[y_{t+1}|z_t]$ but also has a surveyed value $E[y_{t+1}|\mathcal{G}_t]$. Thus there are three sources of information on expectations: econometric forecasts, the median from a survey of professional forecasts, and actual outcomes. All three methods may be pooled. The recursive projection formula gives:

$$y_t = \beta\big[(1-m-n)E[y_{t+1}|z_t] + nE[y_{t+1}|\mathcal{G}_t] + my_{t+1}\big] + \lambda x_t. \tag{15}$$

Similarly, one could use several, individual, professional forecasts, rather than the median or mean from a survey, since the median or mean may not be the best representation of the expectation that drives the market. With two forecasters, say, one again could include the actual value along with each of the survey forecasts, $E[y_{t+1}|\mathcal{G}_t^1]$ and $E[y_{t+1}|\mathcal{G}_t^2]$:

$$\begin{aligned} y_t &= \beta\big[(1-m-n)E[y_{t+1}|\mathcal{G}_t^1] + nE[y_{t+1}|\mathcal{G}_t^2])] + my_{t+1}\big] + \lambda x_t \\ &= \beta\big[(1-m)(\omega E[y_{t+1}|\mathcal{G}_t^1] + (1-\omega)E[y_{t+1}|\mathcal{G}_t^2])] + my_{t+1}\big] + \lambda x_t, \end{aligned} \tag{16}$$

with

$$\omega \equiv \frac{1-m-n}{1-m}.$$

This approach yields measures of β and λ, the parameters of the economic model, ω, the relative role of the first forecaster among those surveyed, and m, the additional information on expectations that is correlated with y_{t+1}.

The next section applies the one-step pooling method to see what difference it makes in practice and how it affects economic conclusions from estimating a version of the US Phillips curve.

4. Application: The New Keynesian Phillips Curve

I illustrate the pooled estimators and the use of survey data with an application to the US Phillips curve. A great deal of recent research links inflation, π_t, to a measure of marginal cost or an output gap, x_t, like this:

$$\pi_t = \gamma_0 + \gamma_b\pi_{t-1} + \gamma_f E_t\pi_{t+1} + \lambda x_t. \tag{17}$$

This is the hybrid, new Keynesian Phillips curve (NKPC). Econometric findings on the relative importance of the backward and forward weights vary – see Fuhrer (1997) or Galí and Gertler (1999) – and these affect the economy's predicted response to shocks – see Woodford (2003, chapter 3.2)

Estimating the NKPC by method (b), substitution and instrumental variables, can be challenging, for instrumenting π_{t+1} involves forecasting it without using $\{\pi_t, \pi_{t-1}, x_t\}$. Mavroeidis (2005) and Nason and Smith (2005) provide assessments of this issue. Given this difficulty, and the importance of the Phillips curve in macroeconomic models, it is of interest to pool information sources on expected inflation.

In this application the inflation rate is the quarter-to-quarter rate of change in the CPI, while I use an off-the-shelf measure of the output gap using the potential output series constructed by the Congressional Budget Office. The survey series is the median, one-quarter-ahead forecast of quarter-to-quarter CPI inflation from the Survey of Professional Forecasters constructed by the Federal Reserve Bank of Philadelphia (previously the ASA/NBER survey).

Table 2 shows estimates of the hybrid NKPC parameters, based on several different statistical representations of expected inflation. The first row shows traditional instrumental-variables estimates. The weights on past inflation and expected future inflation are roughly equal. The output gap is insignificant in explaining current inflation.

The second row in table 2 shows the results from using the median survey forecast of inflation, denoted $\hat{\pi}_{t+1}$, instrumented as by Roberts (1995). He pioneered the use of surveys of forecasts in estimating the US NKPC. Roberts considered the possibility that forecasters might not provide thoughtful answers to surveys and that this might add measurement error. He therefore instrumented the forecast data.

His results are not directly comparable to those in table 2 because he used annual data from 1949-1950, with the Michigan or Livingston surveys of inflation that apply to that horizon of price changes. He also used McCallum's method of instrumenting the true value, and found similar results. In contrast, table 2 shows that – at least with this set of instruments – the results change when the instrumented survey data are used. The weights on inflation tilt away from the past and toward the future, and there is a larger and more significant role for the output gap.

Should we view the median survey data as containing measurement error or as rational forecasts? Figure 1 shows actual inflation and the median survey forecast one step ahead. Regressing $\hat{\pi}_{t+1}$ on π_{t+1} (as in (9)) gives a coefficient of 0.37, while the reverse regression (10) of π_{t+1} on $\hat{\pi}_{t+1}$ gives a coefficient of 0.58. Also $\hat{\pi}_{t+1}$ is about half as variable as the actual series π_{t+1}. Thus one can reject the hypothesis of unbiasedness. However, the variance ratio seems more representative of the rational forecast model than the errors-in-variables model.

Thomas (1999) and Croushore (2006) provide evidence on unbiasedness in survey data, and also report on other properties of SPF inflation forecasts. Mankiw, Reis, and Wolfers (2005) describe the disagreements about inflation in the SPF. Bonham and Cohen (2001) point out that one should test unbiasedness only with individual data, whereas here the identity of the median foreaster varies over the time series. Their argument suggests one should use individual forecast data in the Phillips curve model too.

The third row in table 2 shows what happens when the survey data are treated as rational forecasts. There now is no role for lagged inflation and the coefficient on expected future inflation is one. There is a positive and significant role for the output gap in explaining inflation. And the equation fits better than the traditional, instrumental-variables version in the first row.

It is striking that the best-fitting model involves the median professional forecast, for Ang, Bekaert, and Wei (2007) conclude this series also is the best predictor of annual US inflation. They ran a tournament among forecast models that included survey measures, time-series models, models with real-side variables, and arbitrage-free models of the term structure. Their main conclusion is that the median professional forecast (from the Livingston survey or SPF) is the best predictor of annual inflation. I reach a similar conclusion but selecting the forecast not for accuracy of prediction but to try to explain current inflation in the New Keynesian Phillips curve. Since the goal is to explain current inflation, my focus on measuring expected or forecast inflation arises in order to estimate parameters $\{\gamma_b, \gamma_f, \lambda\}$ of the Phillips curve.

The fourth row shows the new, pooled estimates based on combining the survey data $\hat{\pi}_{t+1}$ with the actual data. Here there is a small, positive role for the term $m\pi_{t+1}$ with a p-value of 0.12. There is little change in the estimates of the economic parameters $\{\gamma_b, \gamma_f, \lambda\}$ from the previous row that used $\hat{\pi}_{t+1}$ alone. The coefficient m takes a value of 0.139. It could be small because $\hat{\pi}_{t+1}$ is already close to $E[\pi_{t+1}|\mathcal{F}_t]$, or because π_{t+1} is not close to $E[\pi_{t+1}|\mathcal{F}_t]$ (*i.e.* because there is a large forecast error in trying to predict inflation). Ang, Bekaert, and Wei (2007) also allowed for forecast combination or pooling, using least-squares and other methods. They found that little weight was attached to any other candidate besides the median professional forecast in their pooling exercises.

The fifth row of table 2 shows what happens when one pools the traditional, IV estimator with the actual value. Now $m = 0.95$ (though with a p-value of 0.18). This point estimate suggests that there is substantial information on $E[\pi_{t+1}|\mathcal{F}_t]$ in π_{t+1} that is not contained in $E[\pi_{t+1}|z_t]$. However, the economic findings are quite similar to those in the original, IV estimation in the first row of table 2.

As a way of reporting an implication of the estimates, figure 2 shows the decomposition of quarterly US inflation since 1981 into its components, based on the IV estimates in row 1: $\hat{\gamma}_b\pi_{t-1}$, $\hat{\gamma}_f E(\pi_{t+1}|z_t)$, $\hat{\lambda}x_t$, and a residual. Standard error bands are omitted for legibility. The figure shows no contribution from the output gap, and roughly equal contributions from lagged and expected inflation.

Figure 3 shows the historical decomposition for the pooled estimator (row 4) that uses $(1-\hat{m})\hat{\pi}_{t+1} + \hat{m}\pi_{t+1}$. Here the history is dramatically different, with a significant role for the output gap, no role for lagged inflation, and an expected inflation series that parallels the low-frequency movement in inflation itself.

Of course expected inflation is an endogenous variable in any macroeconomic model, so I stress that the single equation estimates cannot be used for policy analysis. For example, they cannot tell us what output-gap path would produce a given inflation path.

See Sargent (1999), Nelson (2005), and Ireland (2007) for explanations of US disinflation. But in the historical data the expected inflation series is whatever it is. The estimates using survey data – and treating them as projections rather than EIV-laden estimates – suggest that US disinflation was driven mainly by a decline in expected inflation.

One minor puzzle remains from the combination of findings in this application. On the one hand, when I combine $\hat{\pi}_{t+1}$ and π_{t+1} to estimate $E[\pi_{t+1}|\mathcal{F}_t]$ the weight is almost entirely on the survey data, with a relatively small estimate for m. On the other hand, the hypothesis that the survey data are unbiased as forecasts of future inflation easily rejects. The resolution may simply be that $\hat{\pi}_{t+1}$ is not fully rational but nevertheless closely mimics the unobserved $E[\pi_{t+1}|\mathcal{F}_t]$ because that is not fully rational either.

5. Conclusion

The important but neglected contribution of Gottfries and Persson (1988) can be extended to linear models with expectations of future endogenous variables. At the same time, the pooling that results can include survey data. When there are competing ways to model unobserved expectations, the researcher does not need to choose based on forecast accuracy or some other external criterion, but rather can determine the pooling weights by least squares automatically using an economic model. And the pooling can include actual, future values of the endogenous variable.

An application of this pooling to the US new Keynesian Phillips curve shows that the median prediction from the Survey of Professional Forecasters best mimics the expected inflation series that influences current CPI inflation. In the same application, there is no role for lagged inflation but a significant role for the output gap.

Appendix: Data Sources

Real potential output is GDPPOT from FRED at the Federal Reserve Bank of St. Louis. The original source is the Congressional Budget Office. The series is quarterly in billions of chained 2000 dollars, beginning in 1949:1. Real GDP is series GDPC96 from the same source, also quarterly in billions of chained 2000 dollars, ending in 2006:4. The output gap is defined as 100(GDPC96/GDPPOT-1) *i.e.* the percent difference between output and potential output.

The price index is the CPI all items for urban consumers; CPIAUCSL from FRED. The original source is the BLS. Monthly values are averaged to quarterly frequency. The inflation rate is quarter-to-quarter at annual rates. The survey series on expected inflation is the one-quarter-ahead median forecast for quarter-to-quarter CPI inflation from the Survey of Professional Forecasters, conducted by the Federal Reserve Bank of Philadelphia. The series begins in 1981:3.

Table 2: US New Keynesian Phillips Curve

1981-2006

$$\pi_t = \gamma_0 + \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \lambda x_t$$

Expected Inflation Estimator	γ_b (p)	γ_f (p)	m (p)	λ (p)	$J(1)$ (p)	$\overline{R}^2$
$E(\pi_{t+1}\|z_t)$	0.387	0.475	–	0.015	0.361	0.32
	(0.00)	(0.00)		(0.87)	(0.55)	
$E(\hat{\pi}_{t+1}\|z_t)$	0.162	0.728	–	0.135	0.727	0.44
	(0.40)	(0.12)		(0.20)	(0.39)	
$\hat{\pi}_{t+1}$	0.046	0.994	–	0.177	–	0.46
	(0.60)	(0.00)		(0.03)		
$(1-m)\hat{\pi}_{t+1} + m\pi_{t+1}$	0.072	1.01	0.139	0.156	–	0.47
	(0.39)	(0.00)	(0.12)	(0.05)		
$(1-m)E(\pi_{t+1}\|z_t) + m\pi_{t+1}$	0.40	0.39	0.95	0.017	–	0.32
	(0.00)	(0.19)	(0.18)	(0.84)		

Notes: π_{t+1} is the value of future inflation while $\hat{\pi}_{t+1}$ is the median survey value. p-values are calculated with standard errors robust to heteroskedasticity. The instrument set is $z_t = \{\iota, \pi_{t-1}, \pi_{t-2}, x_t, x_{t-1}\}$.

References

Abel, A. and F.S. Mishkin (1980) On the econometric testing of rationality and market efficiency. unpublished paper, Department of Economics, University of Chicago.

Andrews, D.W.K and J.H. Stock (2006) Inference with Weak Instruments, in *Advances in Economics and Econometrics, Theory and Applications: Ninth World Congress of the Econometric Society*, Vol. III, ed. by R. Blundell, W. K. Newey, and T. Persson. Cambridge, UK: Cambridge University Press, forthcoming.

Ang, Andrew, Geert Bekaert and Min Wei (2007) Do macro variables, asset markets or surveys forecast inflation better? *Journal of Monetary Economics* 54, 1163-1212

Bates, J.M. and Clive W. Granger (1969) The combination of forecasts. *Operational Research Quarterly* 20, 451-468.

Bonham, Carl and Richard H. Cohen (2001) To aggregate, pool, or neither: Testing the rational expectations hypothesis using survey data. *Journal of Business and Economic Statistics* 3, 278-291.

Croushore, Dean (2006) An Evaluation of Inflation Forecasts from Surveys using Real-Time Data. Working Paper No. 06-19, Research Department, Federal Reserve Bank of Philadelphia.

Fuhrer, Jeffrey C. (1997) The (Un)importance of forward-looking behavior in price specifications. *Journal of Money, Credit, and Banking* 29, 338-350.

Galí, Jordi and Gertler, Mark, (1999) Inflation dynamics: A structural econometric analysis, *Journal of Monetary Economics* 44, 195-222.

Gottfries, Nils and Persson, Torsten (1988) Empirical examinations of the information sets of economic agents. *Quarterly Journal of Economics* 103, 251-259.

Hamilton, James (1985) Uncovering financial market expectations of inflation. *Journal of Political Economy* 93, 1224-1241.

Hansen, Lars Peter (1982) Large sample properties of generalized method of moments estimators. *Econometrica* 50, 1029-54.

Hansen, Lars Peter, and Sargent, Thomas J. (1980) Formulating and estimating dynamic linear rational expectations models. *Journal of Economic Dynamics and Control* 2, 7-46.

Ireland, Peter N. (2007) Changes in the Federal Reserve's inflation target: causes and consequences. Department of Economics, Boston College:
(`http://www2.bc.edu/~irelandp/targets.pdf`)

Mankiw, N. Gregory, Ricardo Reis, and Justin Wolfers (2005) Disagreement about inflation expectations. mimeo, Department of Economics, Harvard University

Mankiw, N.Gregory, David E. Runkle, and Matthew D. Shapiro (1984) Are preliminary announcements of the money stock rational forecasts? *Journal of Monetary Economics* 14, 15-27.

Mavroeidis, Sophocles (2005) Identification issues in forward-looking models estimated by GMM, with an application to the Phillips curve. *Journal of Money, Credit and Banking* 37, 421-448.

McCallum, Bennett T. (1976) Rational expectations and the estimation of econometric models: An alternative procedure. *International Economic Review* 17, 484-90.

Milbourne, Ross D. and Smith, Gregor W. (1989) How informative are preliminary announcement of the money stock in Canada? *Canadian Journal of Economics* 22, 595-606.

Nason, James M. and Gregor W. Smith (2005) Identifying the new Keynesian Phillips curve. Federal Reserve Bank of Atlanta working paper 2005-1.

Nelson, Edward (2005) The Great Inflation of the seventies: What really happened? *Advances in Macroeconomics*, 5(1), Article 3.

Newbold, Paul and David I. Harvey (2002) Forecast combination and encompassing. chapter 12 in *A Companion to Economic Forecasting*, eds. Michael P. Clements and David F. Hendry. Blackwell.

Pagan, Adrian (1984) Econometric issues in the analysis of regressions with generated regressors. *International Economic Review* 25, 221-247.

Roberts, John M. (1995). New Keynesian economics and the Phillips curve. *Journal of Money, Credit, and Banking* 27, 975-984.

Sargent, Thomas J. (1987) *Macroeconomic Theory* (second edition). New York: Academic Press.

Sargent, Thomas J. (1999) *The Conquest of American Inflation.* Princeton University Press.

Thomas, Lloyd B., Jr. (1999) Survey measures of expected US inflation. *Journal of Economic Perspectives* 13 (Autumn) 125-144.

Whittle, Peter (1983) *Prediction and Regulation by Linear Least-Square Methods* (second edition, revised). Basil Blackwell.

Woodford, Michael (2003) *Interest and Prices.* Princeton University Press.

Figure 1: US Inflation and Median Forecast

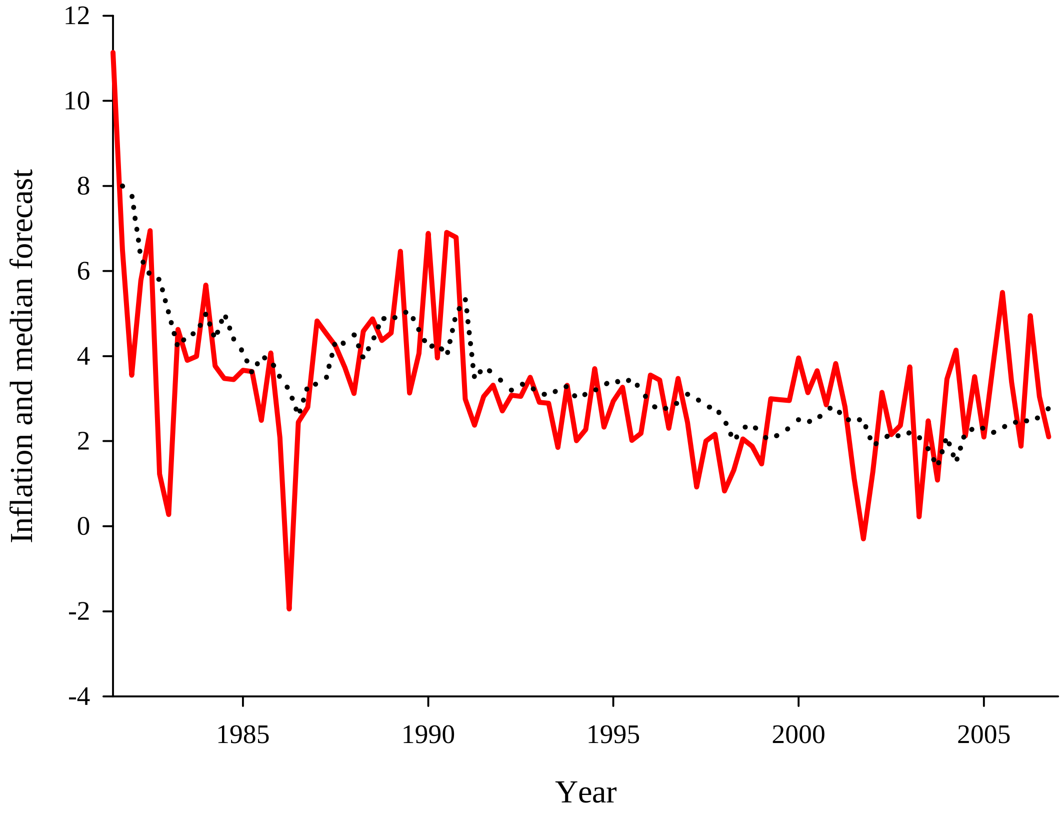

Figure 2: Contributions to US Disinflation (IVE)

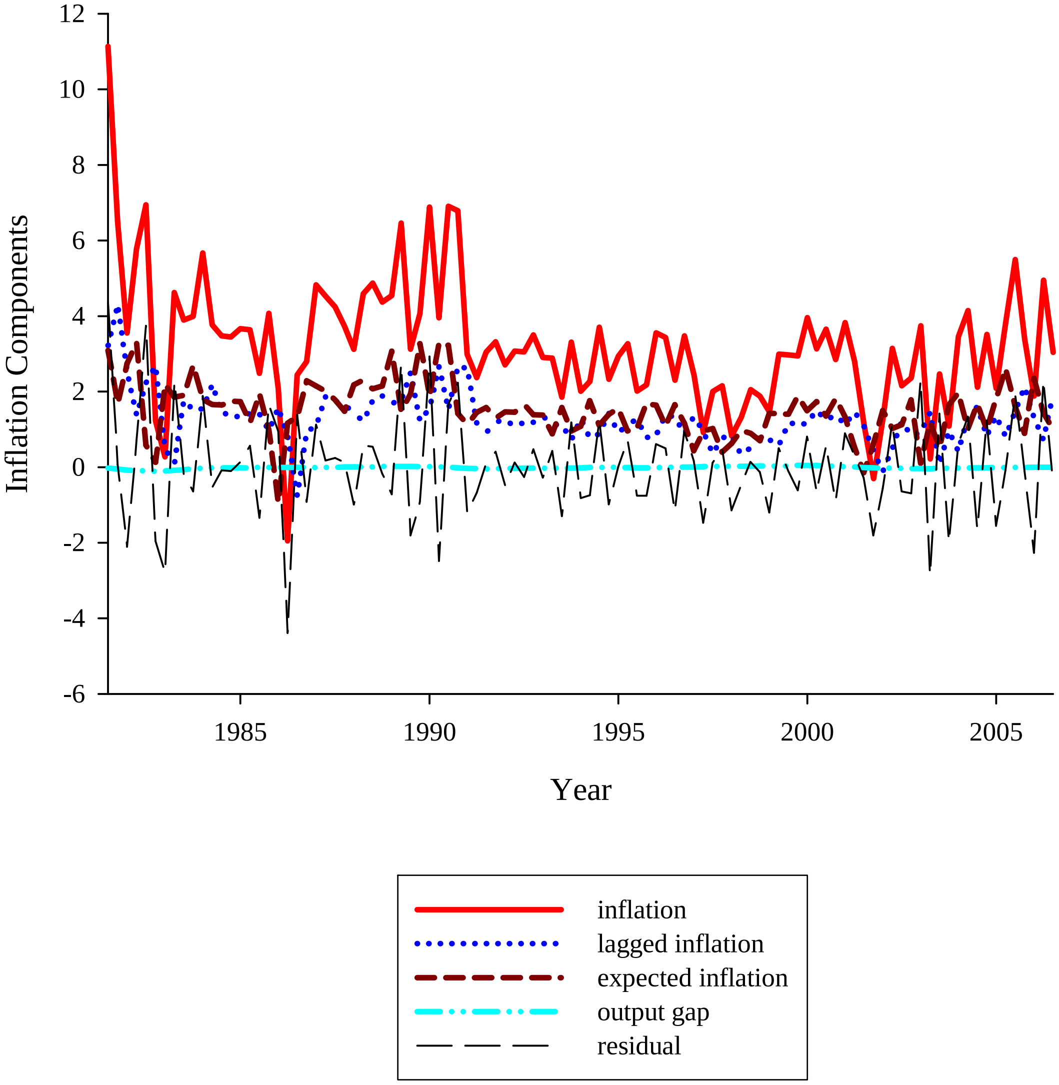

Figure 3: Contributions to US Disinflation (Survey)